

Challenges in cross section measurements and global fits

C. Wilkinson, L. Pickering, P. Stowell, C. Wret



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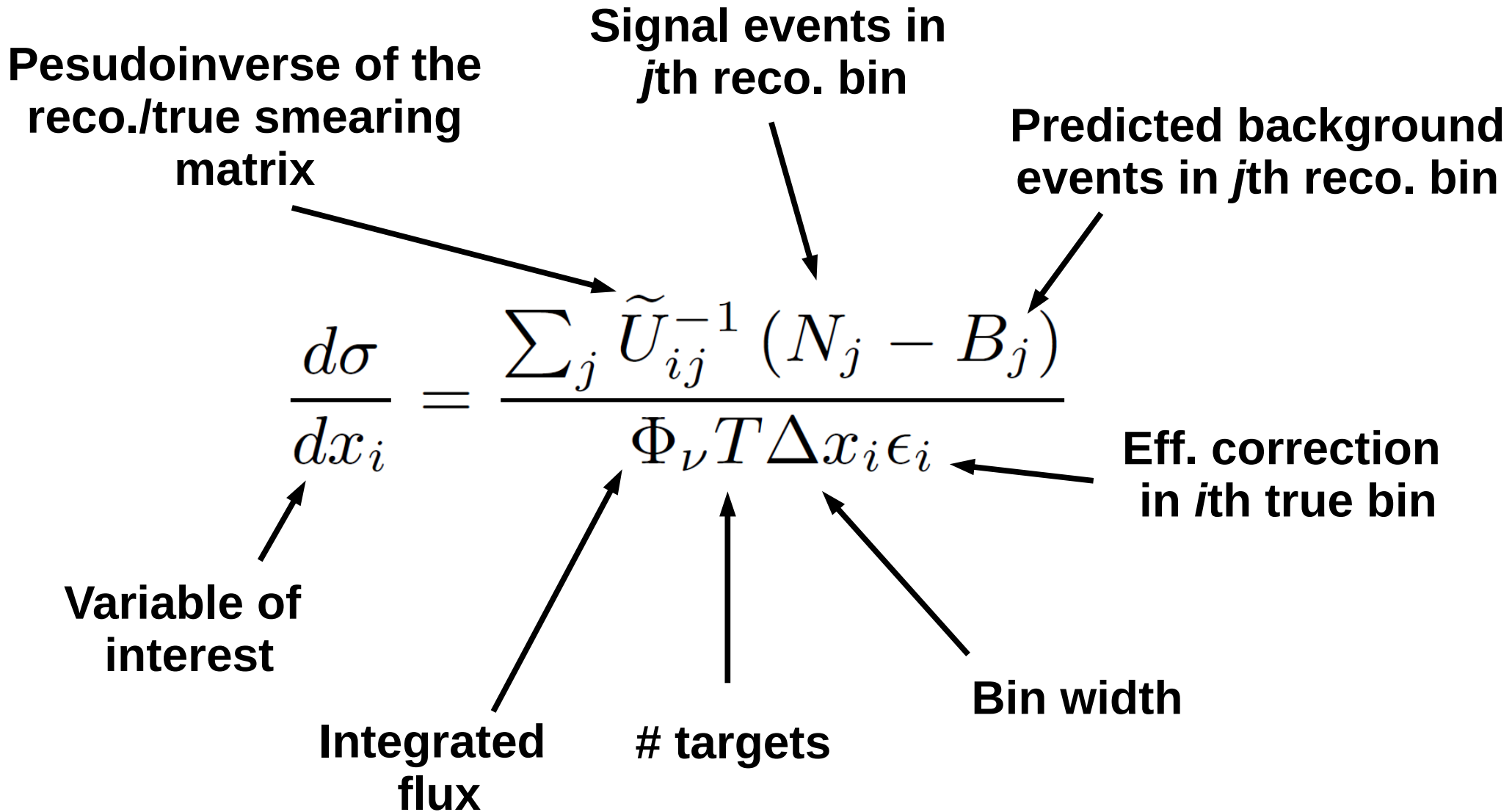
^b
UNIVERSITÄT
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FOR FUNDAMENTAL PHYSICS

Challenges for cross section measurements

Model dependence, and how we bias our data

Cross section extraction





Cross section model dependence



Some unfolding methods introduce bias

The signal definition and background subtraction can be model dependent

$$\frac{d\sigma}{dx_i} = \frac{\sum_j \tilde{U}_{ij}^{-1} (N_j - B_j)}{\Phi_\nu T \Delta x_i \epsilon_i}$$

Choice of variables

Efficiency corrections couple to model in complex ways

Choice of variables

- **Model-independent:** final state particle kinematics, or some combination of them (e.g., Q^2_{QE}). *Combinations are prone to subtle efficiency issues!*
- **Model-dependent:** interaction-level kinematics, Q^2 , E_ν , W ...
- Perception that theorists will prefer interaction-level variables because they are easier to use... very shortsighted view...
- Often it's unclear to experimentalists which variables are of interest to the theory community, *and can be measured...*



Cross section model dependence



Some unfolding methods introduce bias

The signal definition and background subtraction can be model dependent

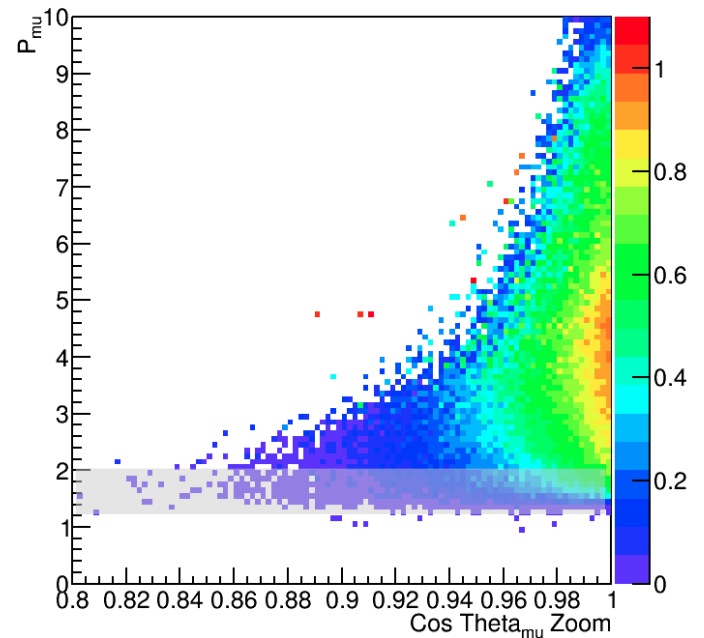
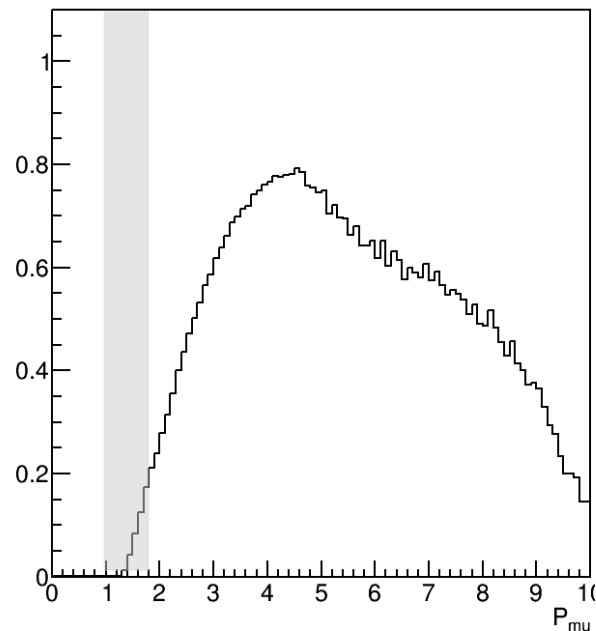
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Choice of variables

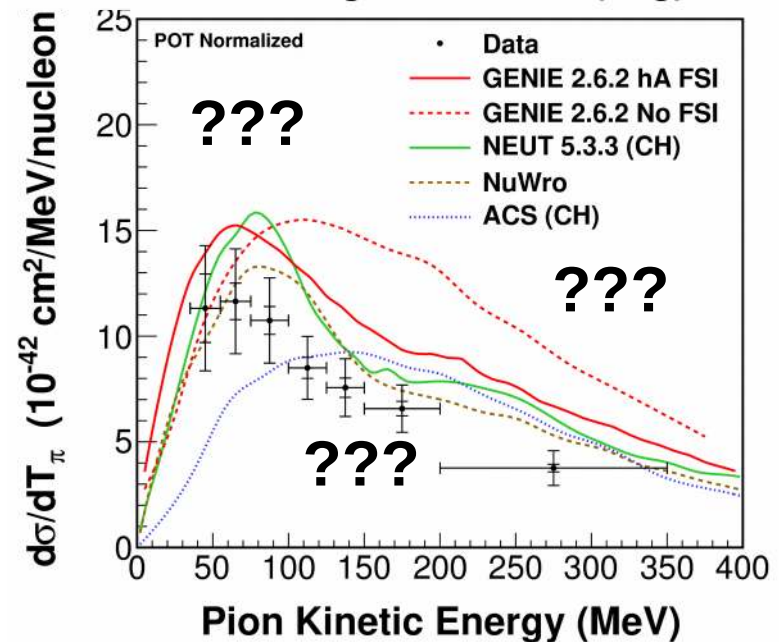
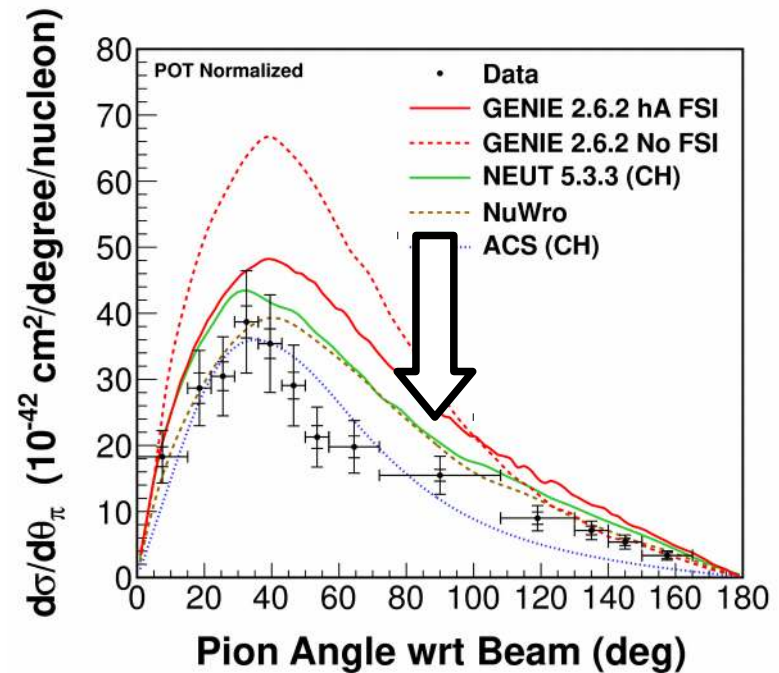
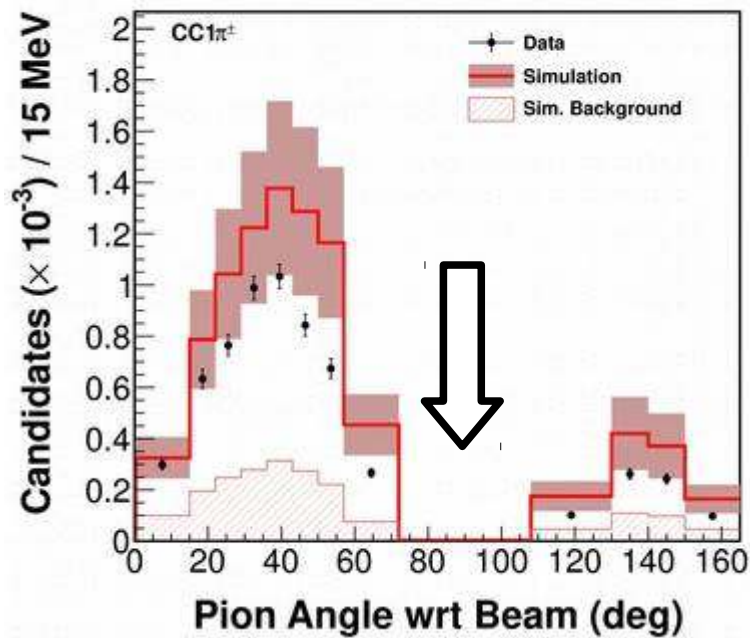
Efficiency corrections couple to model in complex ways

Efficiency corrections

- **Basic problem:** efficiency corrections are typically done on a bin-by-bin basis. Other degrees of freedom have been integrated out.
- Binning efficiency in p_μ only integrates out $\cos\theta_\mu$ variation: all events in a p_μ bin are assigned the same efficiency correction.
- If data more/less forward than MC, result biased up/down
- Sometimes regions of zero efficiency are “corrected for”



MINERvA CC1 π^\pm

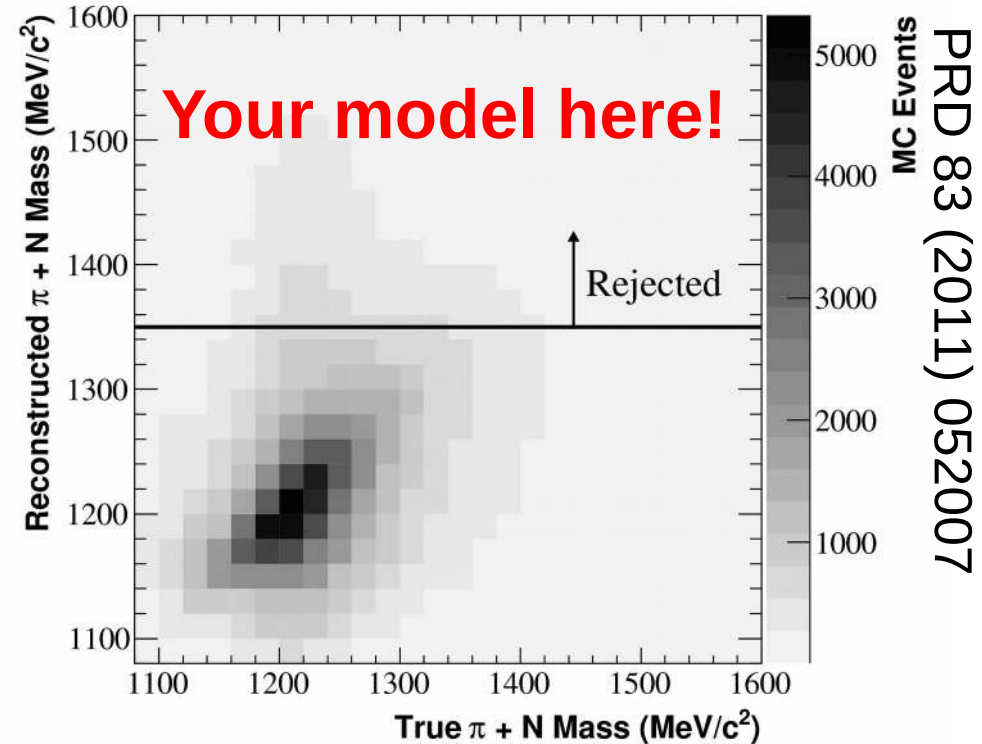
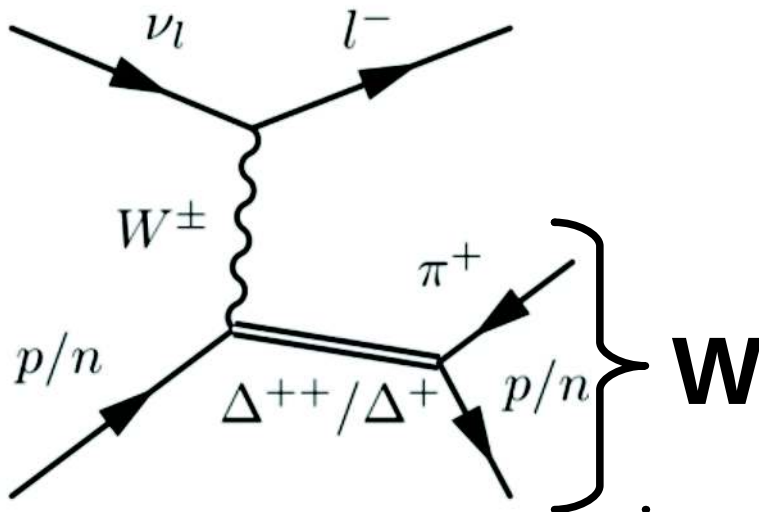


- One angular bin filled with MC... unclear where this affects the KE spectrum
- Also, signal defined as π^\pm . But, Michel required! So π^- effectively filled in with MC...
- Unclear how much MC (GENIE) has biased the result...

PRD 92 (2015) 092008

MiniBooNE CC1 π^+

- Cut made on reconstructed invariant mass, but not reflected in signal definition
- ~30% correction to published cross section comes from MB MC.



PRD 83 (2011) 052007

- Cannot assess where this bias lives... it might dominate some kinematic bins.
- **Hidden efficiency correction!**



Cross section model dependence



Some unfolding methods introduce bias

The signal definition and background subtraction can be model dependent

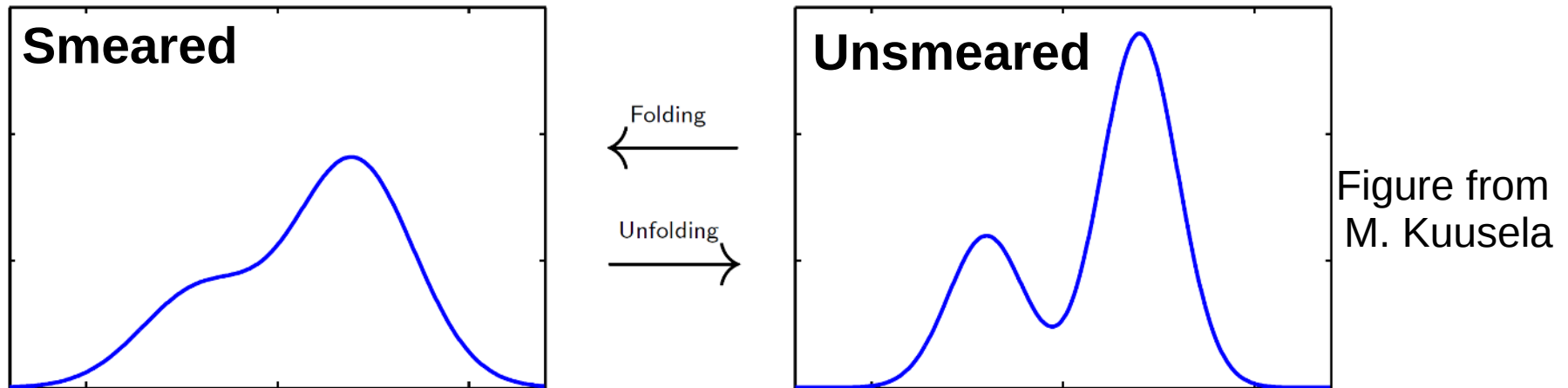
$$\frac{d\sigma}{dx_i} = \frac{\sum_j \tilde{U}_{ij}^{-1} (N_j - B_j)}{\Phi_\nu T \Delta x_i \epsilon_i}$$

Choice of variables

Efficiency corrections couple to model in complex ways

Unfolding methods

- Discussion at PhyStat-nu conferences highlighted unfolding as another critical source for model-dependence
- Unfolding is an inverse problem:



- Some commonly used methods lead to biased results:
 - D'Agostini unfolding with low (< 10) # iterations (**common!**)
 - Bin-by-bin efficiency corrections
 - ***Impossible to quantify bias after the fact***

Unfolding methods

- **Solution #1:** use data driven regularization methods.
 - Regularization makes *some* assumptions about expected result
 - No regularization be harder to interpret (large bin-bin variation)
- **Solution #2:** not unfolding (MB NCEL, MINERvA CC-inc ratios)
 - Present results in reconstructed variables, provide smearing matrix to smear theory to match data
 - Harder to use... some concern that this will lead to results *not being used*
 - Unambiguous advice from statisticians...



Cross section model dependence



Some unfolding methods introduce bias

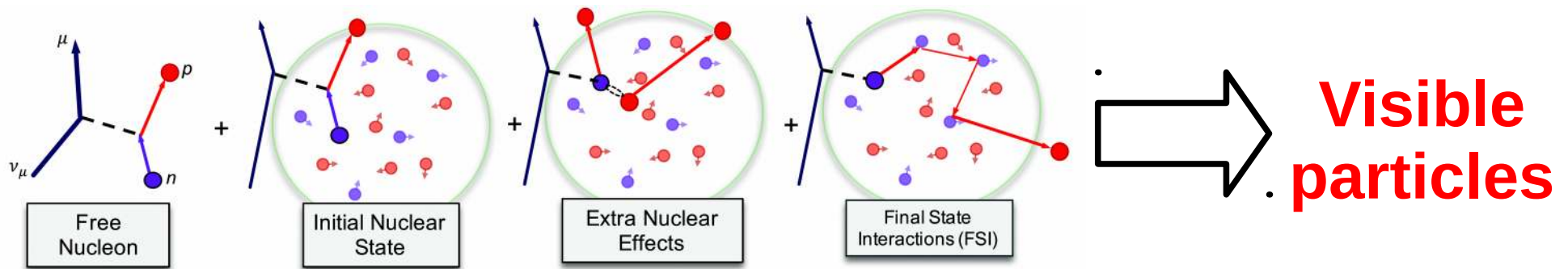
The signal definition and background subtraction can be model dependent

$$\frac{d\sigma}{dx_i} = \frac{\sum_j \tilde{U}_{ij}^{-1} (N_j - B_j)}{\Phi_\nu T \Delta x_i \epsilon_i}$$

Choice of variables

Efficiency corrections couple to model in complex ways

Model-independent signal definitions



- Experiments can only measure final state particles, e.g., $1\mu 0\pi$:

$$\mathbf{CC0\pi} = \mathbf{CCQE} + \mathbf{n\rho nh(0\pi)} + \mathbf{CC1\pi(+abs)} + \dots$$

- Many previous measurements try to correct for irreducible backgrounds to make the result easier to use...

... but trying to recover CCQE introduces model dependence

$$\mathbf{CCQE} = \mathbf{CC0\pi} - \mathbf{n\rho nh(0\pi)} - \mathbf{CC1\pi(+abs)} - \dots \mathbf{???$$

Data

Data

Generator

What can we actually measure?

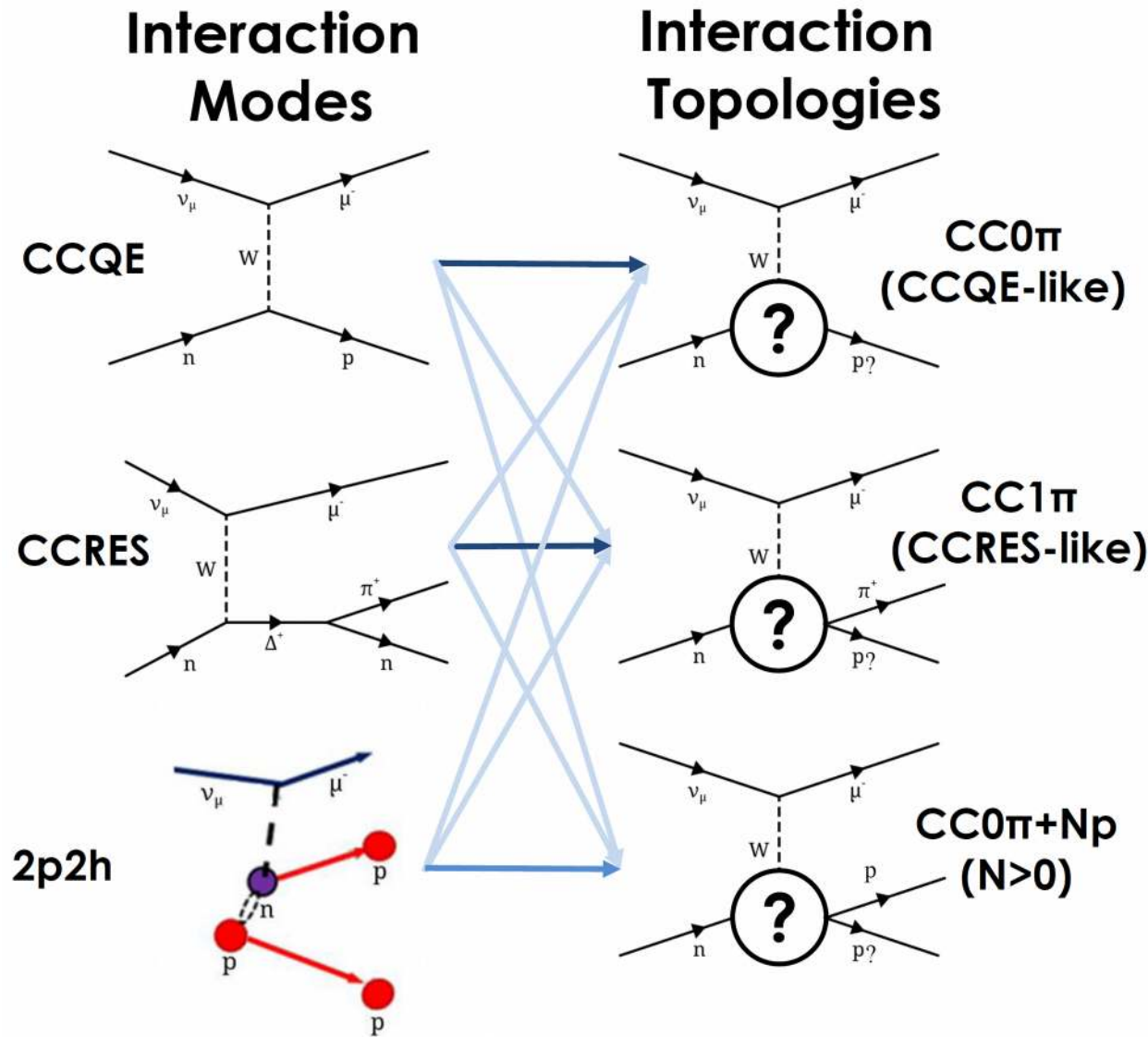
- Only **post-FSI** cross sections are model-independent:

$$\tilde{\sigma}_k(\vec{y}) = \sum_i \int_{E_{min}}^{E_{max}} \sigma_i(E_\nu, \vec{x}) \times \text{FSI}(\vec{x}, \vec{y}) dE_\nu$$

$$\mathbf{CC0}\pi = 1p1h + 2p2h + \mathbf{CC1}\pi(+abs) + \dots$$

- Need to integrate out all degrees of freedom other than \mathbf{y}
FSI makes this difficult analytically
- Some results will become very difficult to compare to outside a generator (e.g., MINERvA $1\pi^+$, with any number of π^0 s allowed)

What can we actually measure?



Many modes contribute to any measurement

Integrated over broad ω region

Difficult to tune models!

Challenges for global fits

Tuning models to data

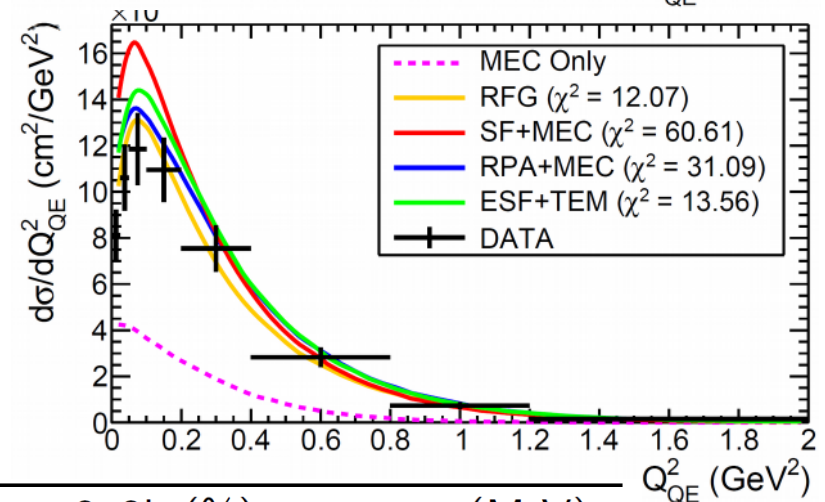
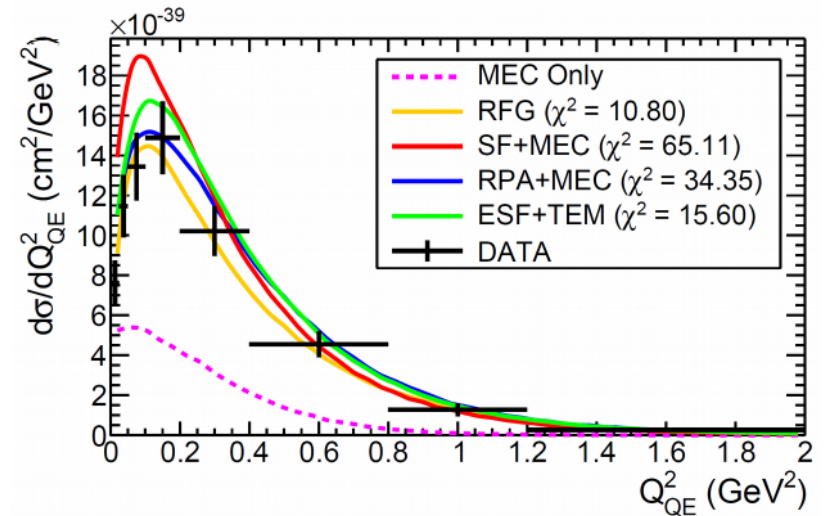
- Tuning σ_i parameters requires many post-FSI datasets to break degeneracies!
 - Multiple fluxes
 - Different acceptance
 - Detector technologies
 - Multiple targets
- Cannot fit parameters of a single interaction channel, without making assumptions about (or fitting) others
- **Tuning or validation of a model is really challenging!**





Previous T2K CC0 π attempt

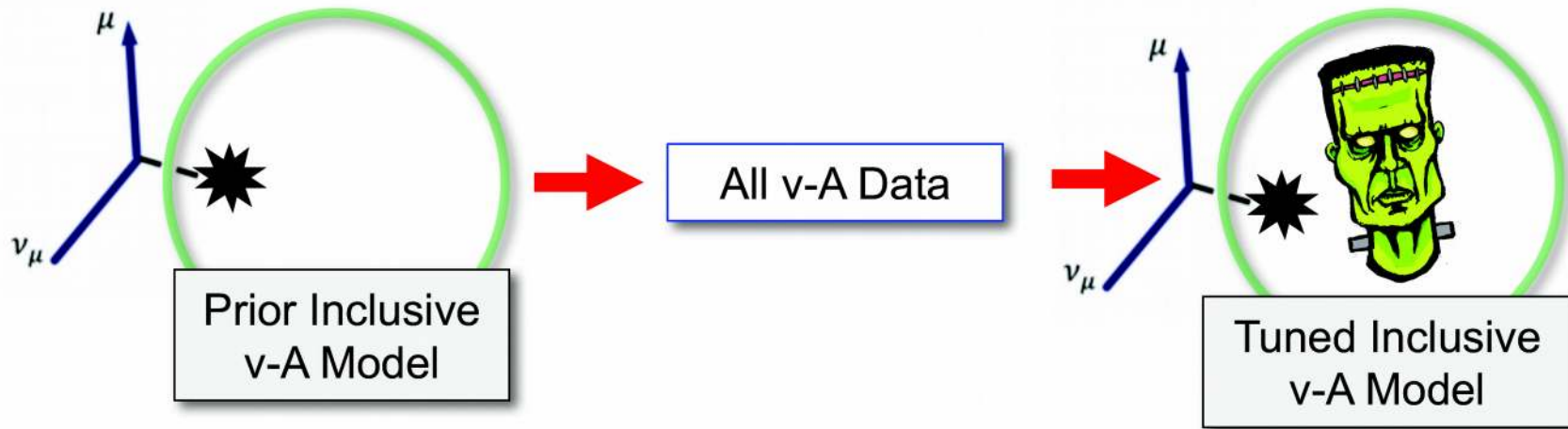
- Attempt to fit all CC0 π data:
 - MiniBooNE $T_{\mu} - \cos\theta_{\mu} \nu_{\mu}$
 - MiniBooNE $T_{\mu} - \cos\theta_{\mu} \bar{\nu}_{\mu}$
 - MINERvA $Q^2 \nu_{\mu}$ & $\bar{\nu}_{\mu}$ (with corr.)
- Many NEUT model improvements: SF, 2p2h, ...
- Unable to fit the data, **surprising** and **unsatisfactory** results.



Model	χ^2/DOF	$p_1: M_A$ (GeV)	$p_2: 2p2h$ (%)	$p_3: p_F$ (MeV)
Theory expectation		~ 1.00	~ 100	~ 217
A: SF+2p2h	97.5/228	1.33 ± 0.02	0 (at limit)	234 ± 4
B: RFG+RPA+2p2h	97.8/228	1.15 ± 0.03	27 ± 12	223 ± 5

See PRD 072010 (2016) for the gory details!

Frankenmodels



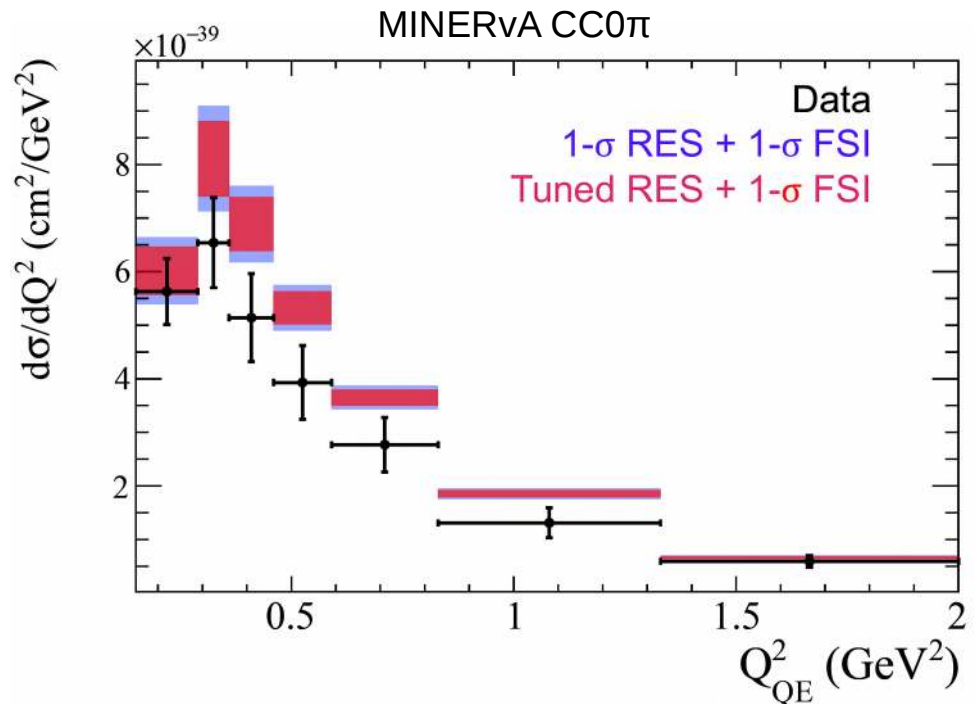
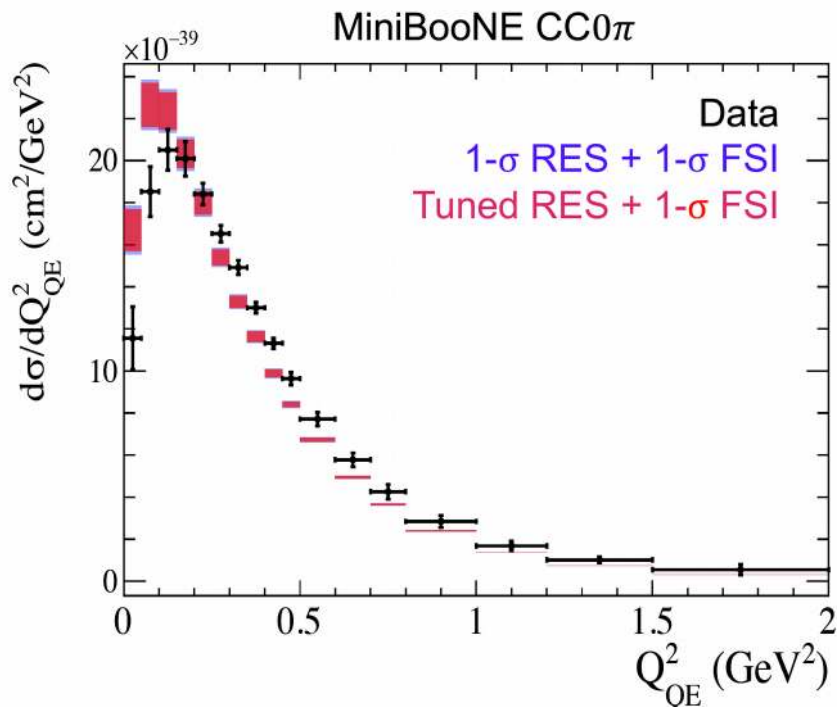
- Incomplete models lead to unphysical **effective** parameters (large axial mass!)
- Not clear where the deficiency lies
- Common issue for all neutrino experiments and model tunings!





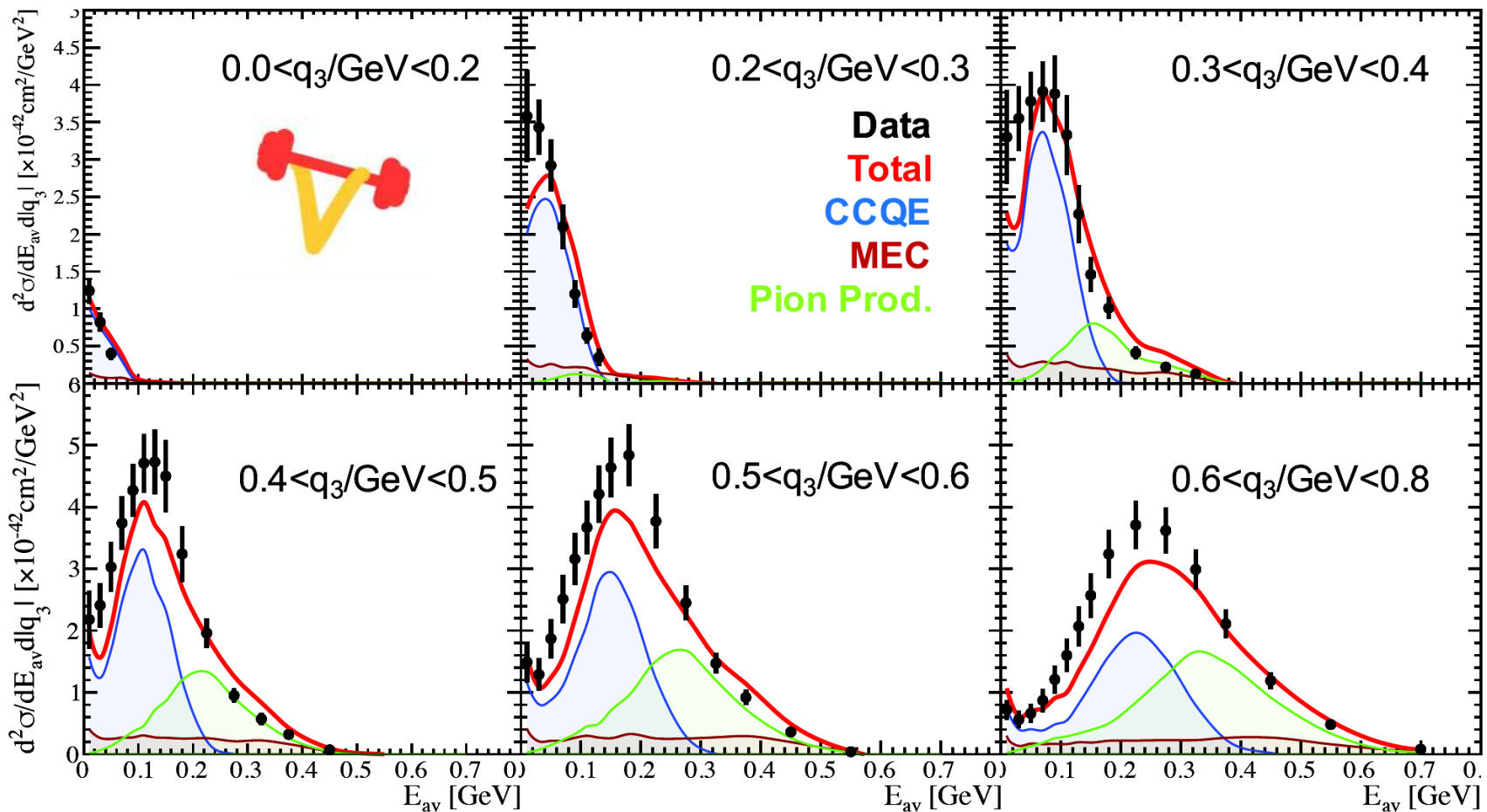
Unclear where deficiency lies

- We know that we can't isolate CCQE for ν -A data
→ CC0 π
- But there's still a temptation to fit CCQE parameters to CC0 π data



- RES parameters in NEUT clearly have a large effect on CC0 π data
- But, this makes theory challenges outside generators very challenging!

Another example (sort of)



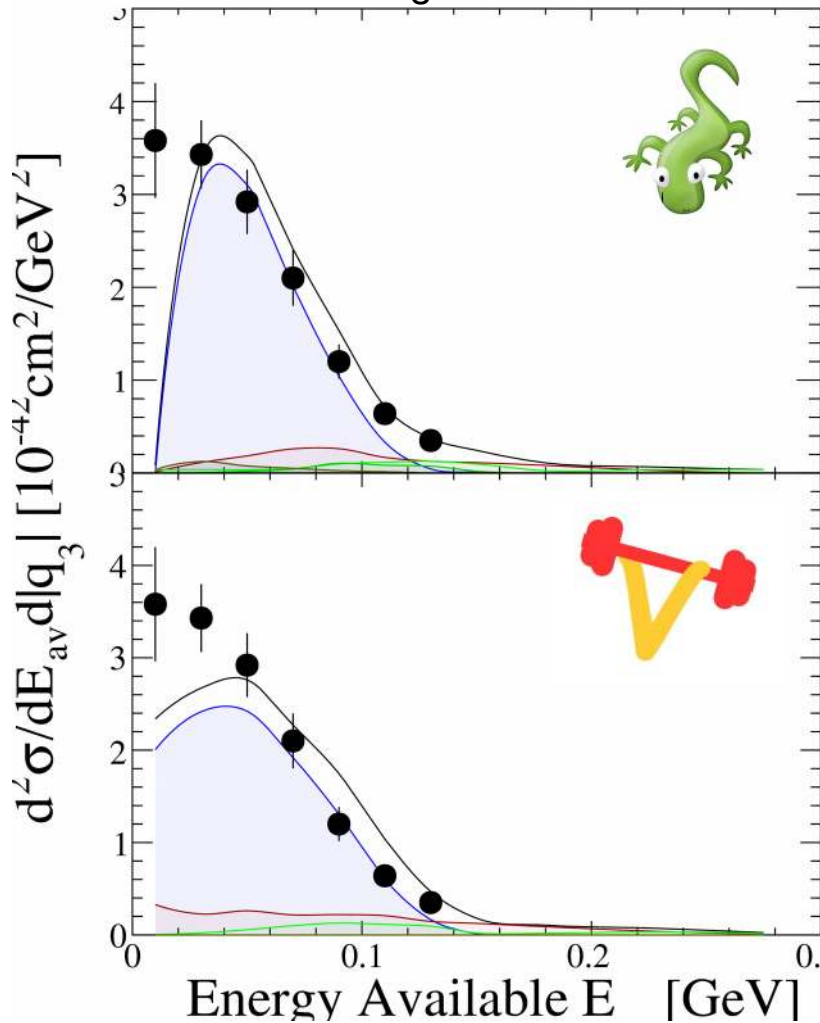
PRL 116 (2016) 071802

$$E_{av} = \sum_{i=p,\pi^\pm} T_i^K + \sum_{i=\pi^0,e,\gamma} E_i$$

Is disagreement due to the CCQE model? MEC? Pion production?

Partial exception

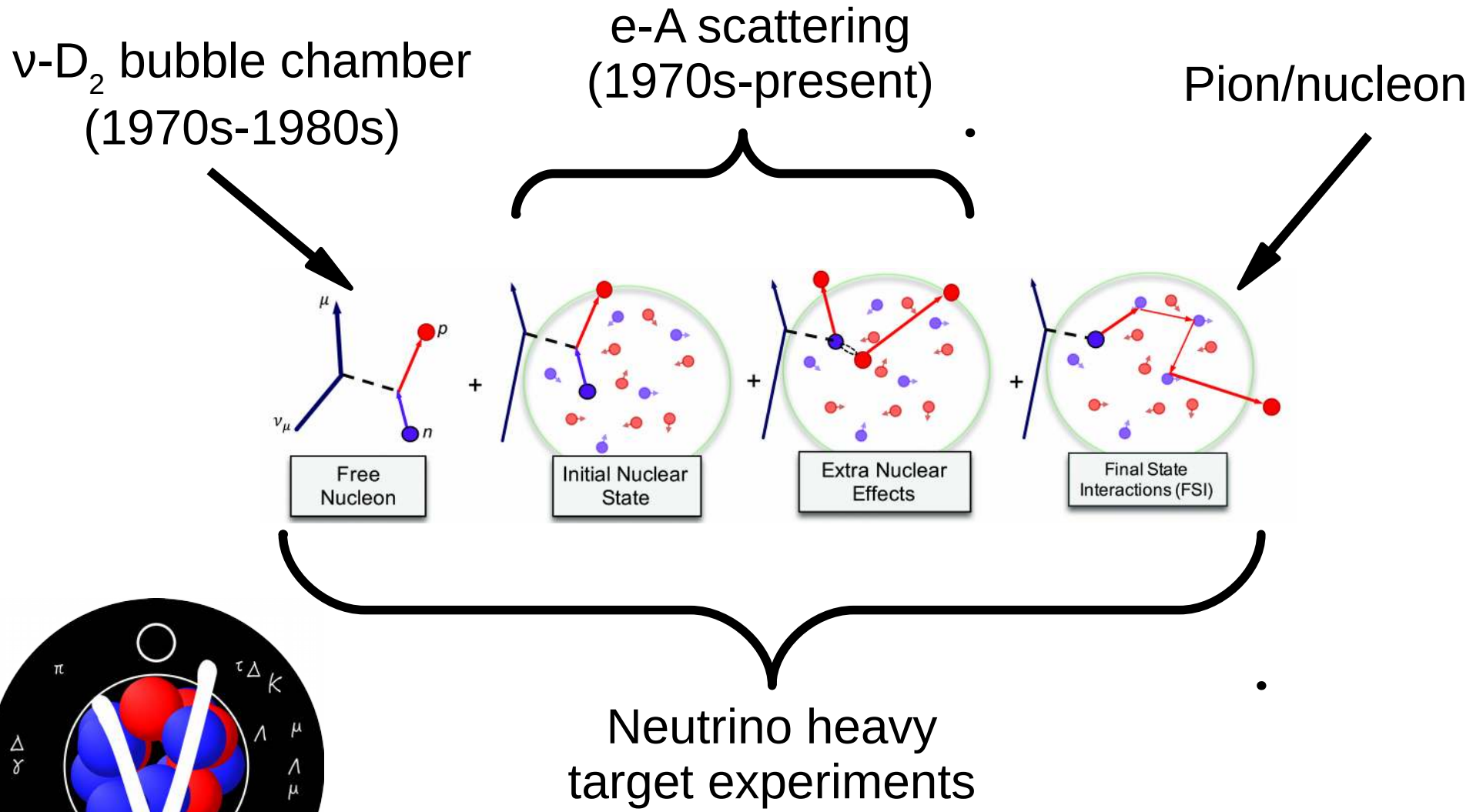
$0.2 < q_3/\text{GeV} < 0.3$



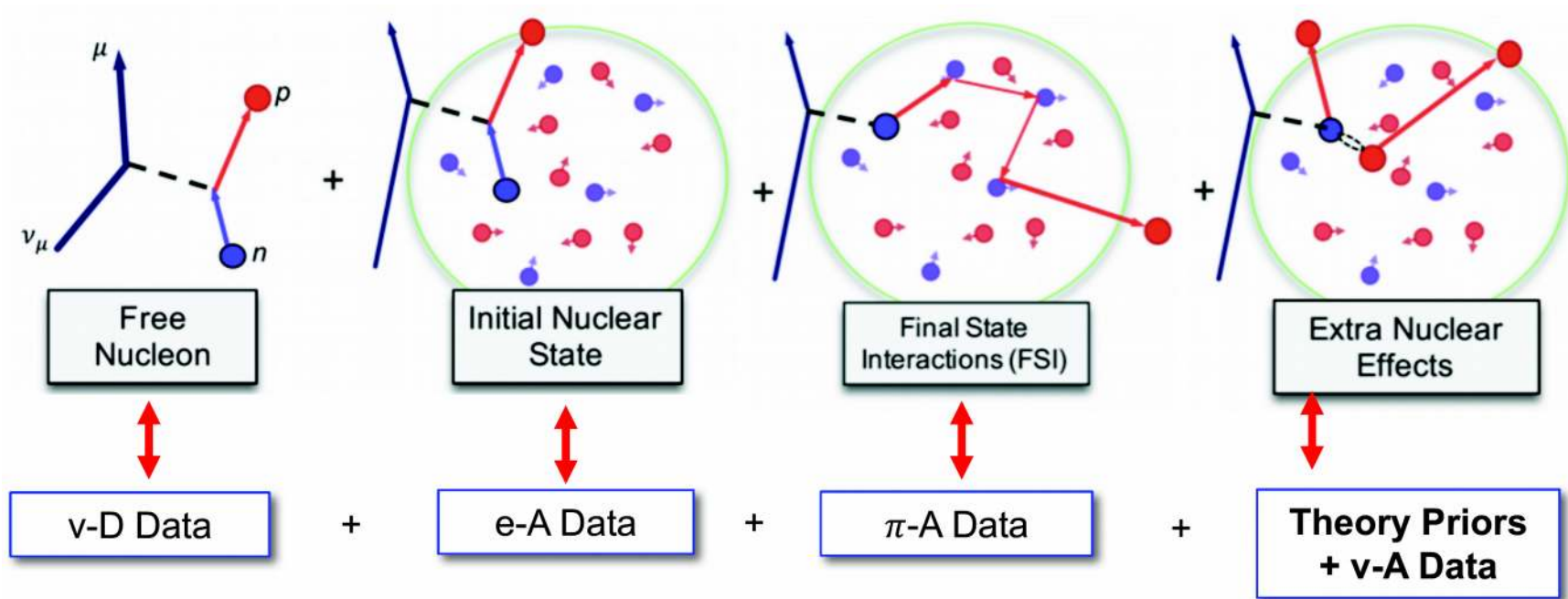
- Inclusive data can still highlight model deficiencies
- NEUT clearly deficient at very low energy transfers (QE-dominated)
- Difference in the nuclear model → motivated further NEUT development
- Limited use, but important cross check!

$$E_{av} = \sum_{i=p,\pi^\pm} T_i^K + \sum_{i=\pi^0,e,\gamma} E_i$$

External cross section model constraints



Modular approach on T2K



- Tune different aspects of the model to *appropriate data*
- Develop *ad hoc* models in the generators (NEUT) to include new theory where possible / fill in the gaps
- **Theory and generator co-ordination essential to improve this horrible situation!**

Outlook

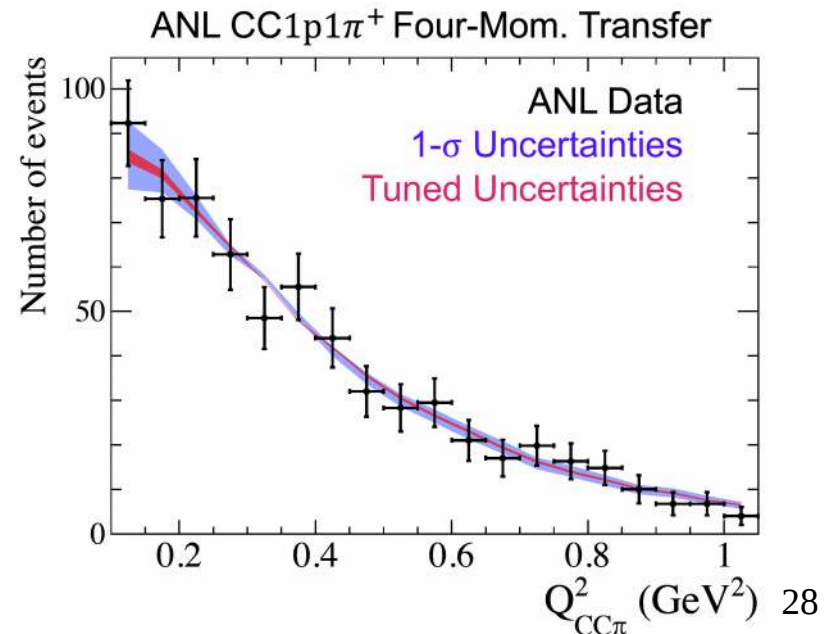
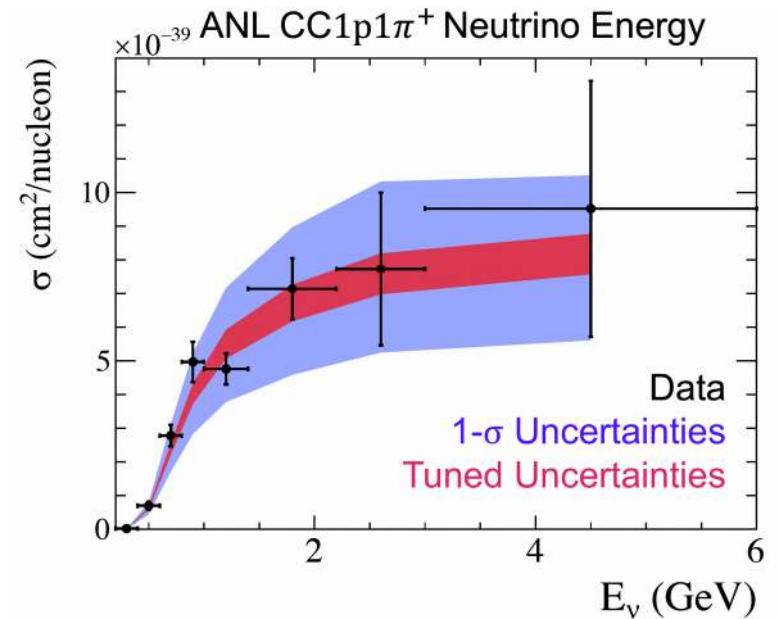
- Difficult road ahead to reduce cross systematics for future experiments.
- Situation is improving, a lot of new, higher quality data.
- Alternative generators and models essential for that
- Challenge to using that data to constrain models. Bad data may also spoil the picture...
- Need new theory models in generators rather than simply fiddling with effective generator parametrizations.

Backup



Tuning the R-S model

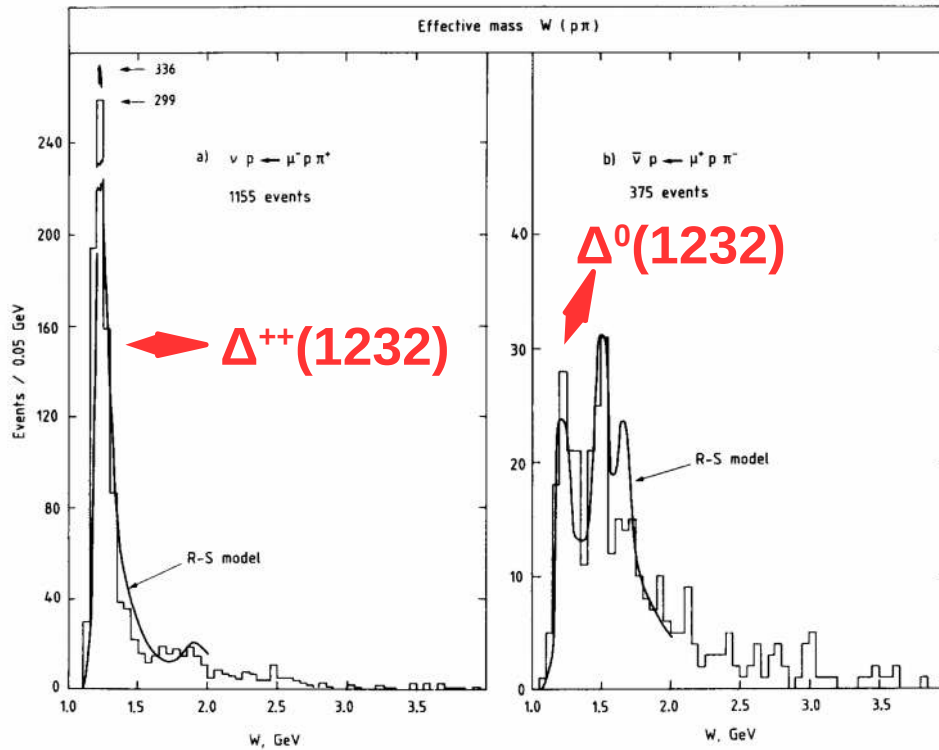
- Parameters tuned to 10% level by limited set of BC data:
 - ANL and BNL, E_ν and Q^2
 - $\nu_\mu + p \rightarrow \mu^- + \pi^+ + p$
 - $\nu_\mu + n \rightarrow \mu^- + \pi^0 + p$
 - $\nu_\mu + n \rightarrow \mu^- + \pi^+ + n$
- Caveats:**
 - D_2 data, FSI may not be negligible
 - CC used to predict NC model
 - Many known limitations to R-S model!





ν -N resonance production

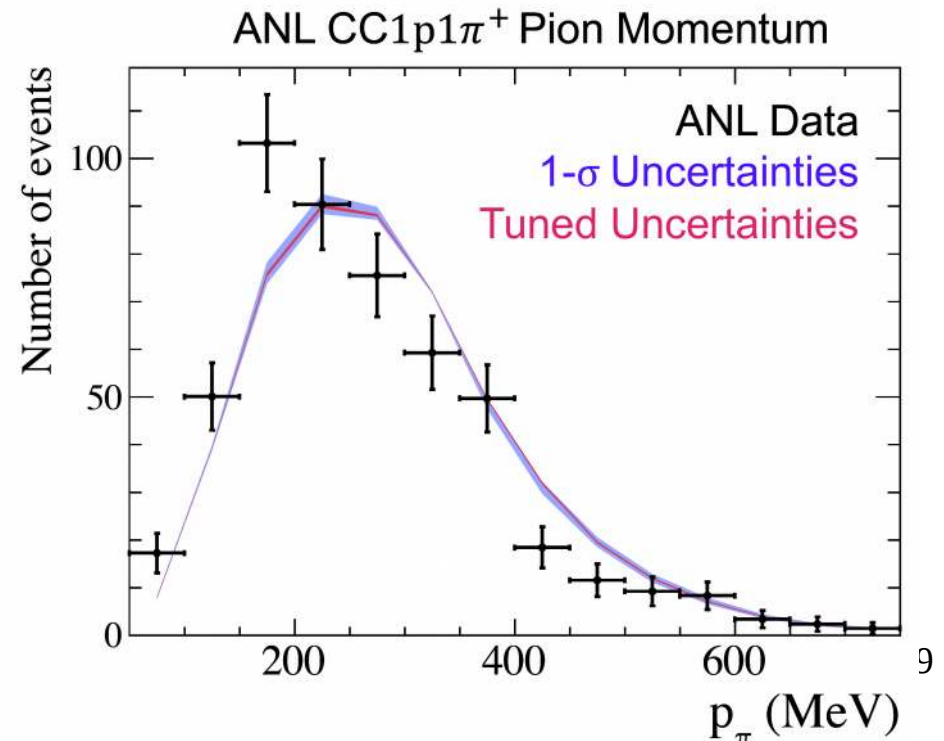
Nucl. Phys. B264 221 (1986)



$$W^2 = (\Sigma E)^2 - |\Sigma \mathbf{p}|^2$$

- Not clear that the R-S model fits all available kinematics.
- Little flexibility in model to cover possible discrepancies.

- Higher order resonances also contribute, insufficient data to constrain model for them
- Current experiments can't do better, can't reconstruct W well enough.

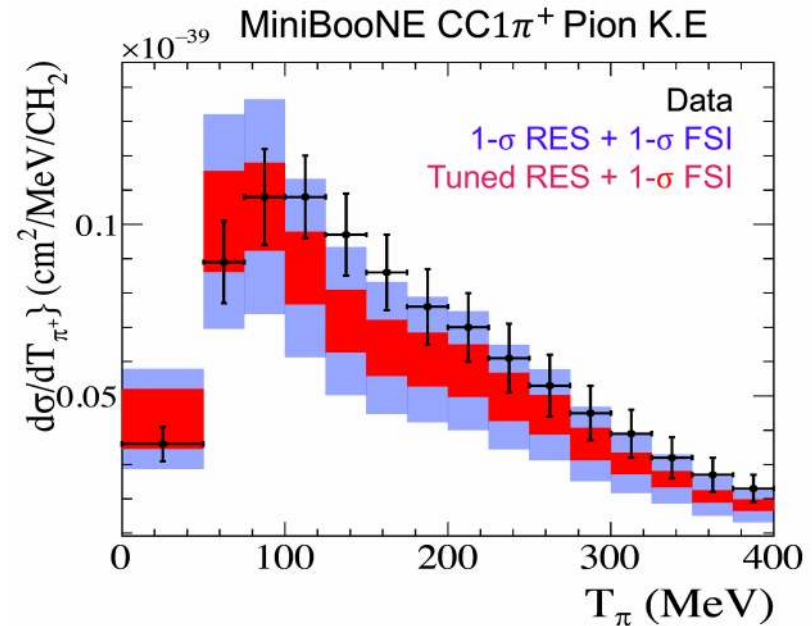
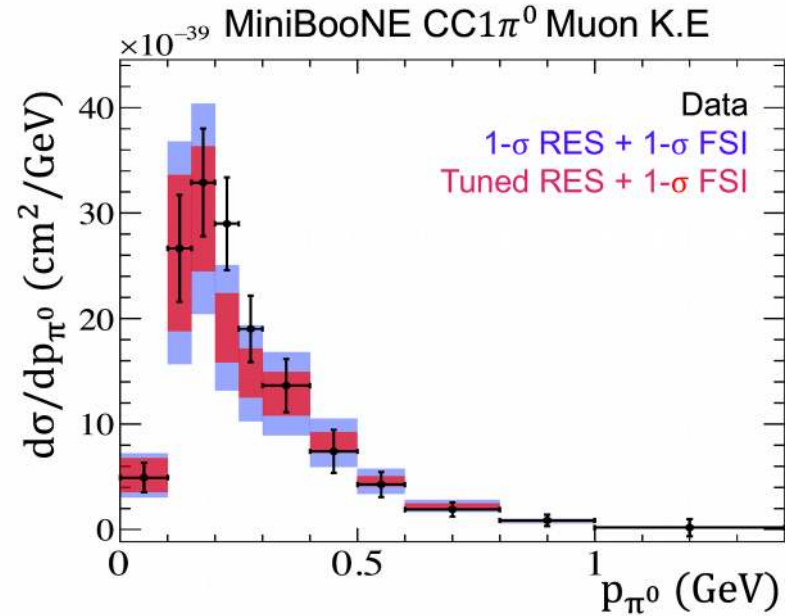
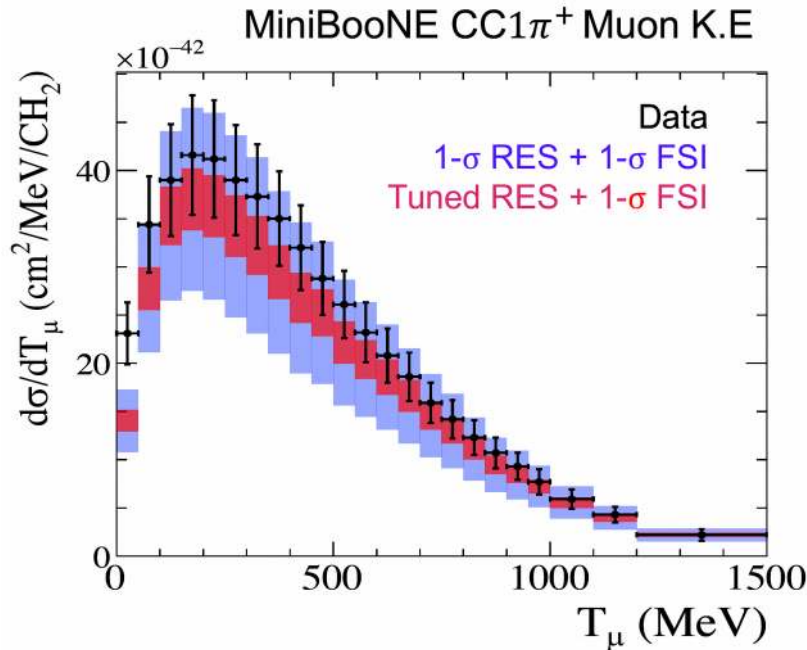




Comparison with nuclear data

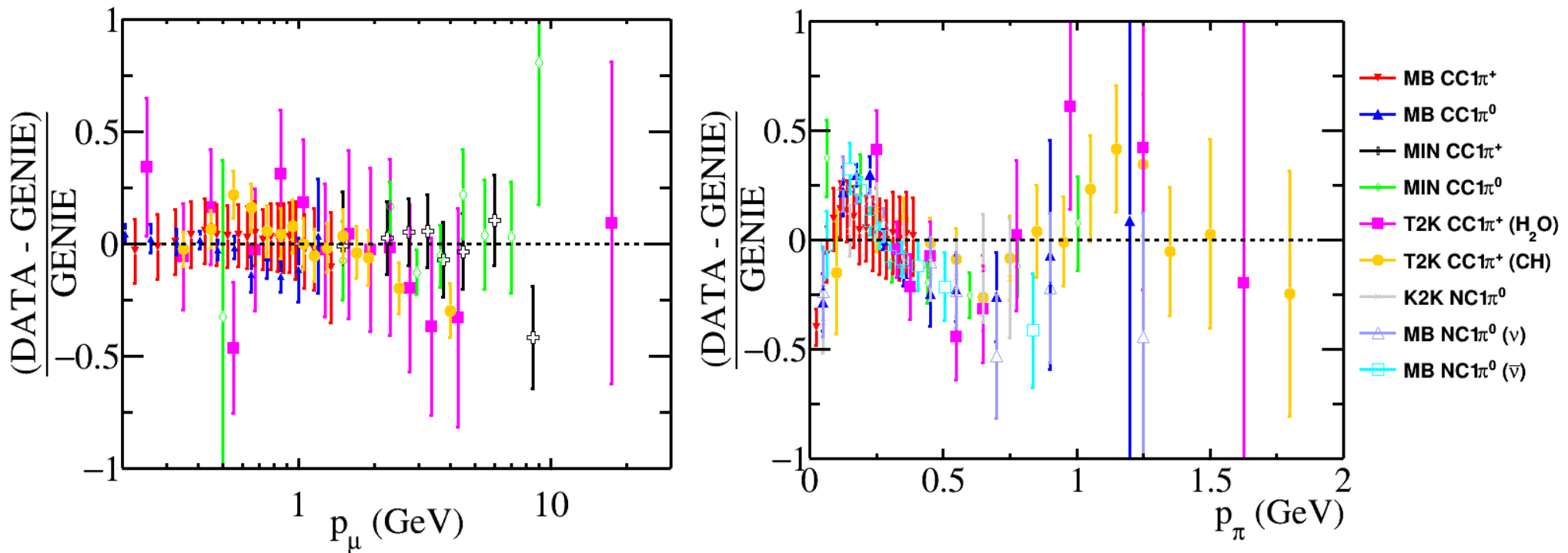
- Reasonable agreement with outgoing **muon kinematics**
- Not for pion kinematics. Inadequate FSI model?
... but poor for ν -N data too!

MB ν_{μ} -CH₂ CC1 π^+
PRD 83 (2011) 052007



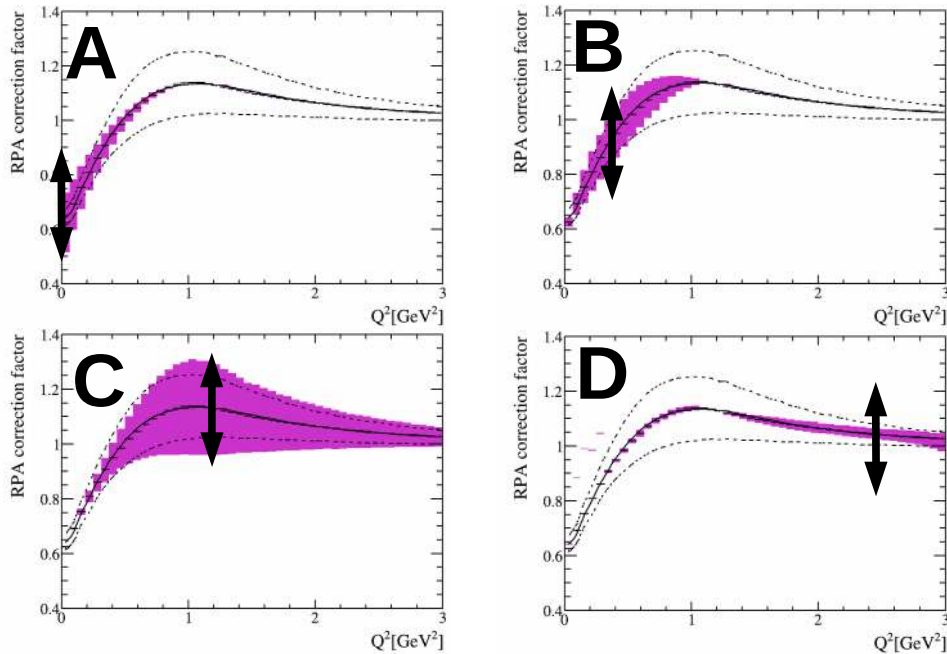
MB ν_{μ} -CH₂ CC1 π^0
PRD 83 (2011) 052009

ν -A data-MC disagreement



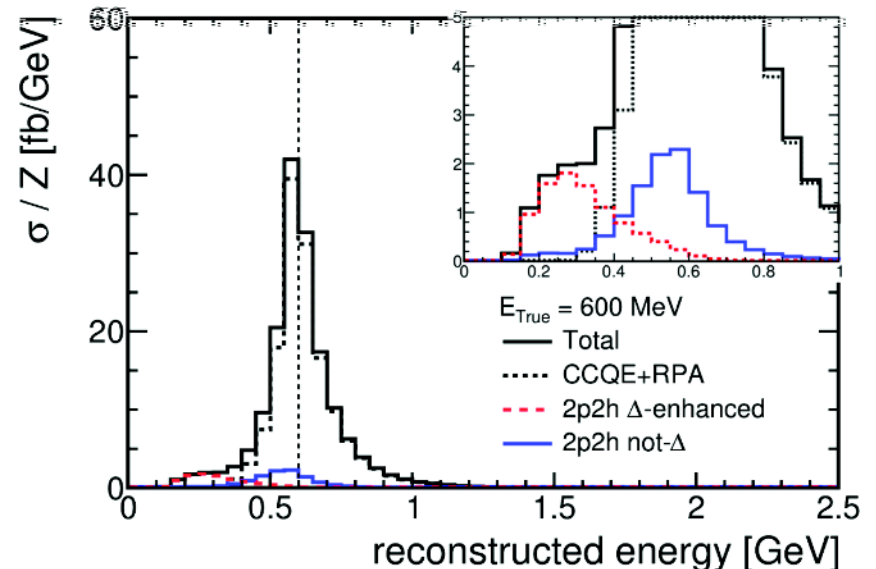
- Fractional deviation of data from reference model (shape-only)
- Good agreement for muon kinematics, poor for pion kinematics
- Difficult to resolve... simply tuning cascade model parameters is clearly inadequate...

Developing new parameters



- **2p2h shape: separate terms**
 - **Delta-component (PDD)**
 - **Non-Delta**
 - **Interference**
- Vary relative strength of the Delta-component, but preserve total 2p2h norm.

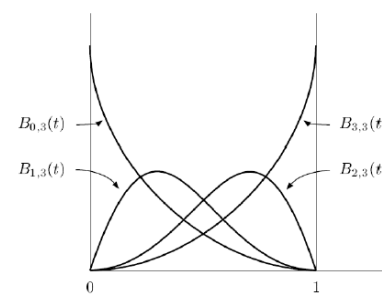
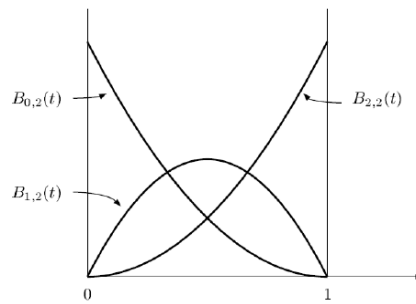
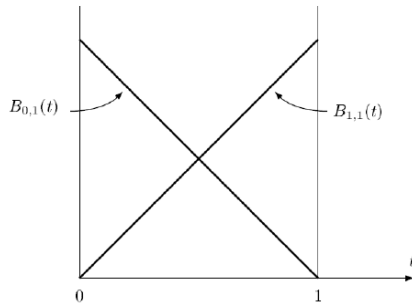
- **BeRPA**: effective model to mock up Nieves RPA, but introduce flexibility.
- Approximates theoretical error band.



Bernstein polynomials

- Bernstein polynomials of degree n form a basis for the power polynomials of order n :

$$B_{i,n}(x) = \binom{n}{i} x^i (1-x)^{n-i}$$



- The n Bernstein polynomials peak at different values of x and can reproduce any n th order power polynomial by varying their normalizations.
- The same cubic to exponential form expressed with Bernstein polynomials gives:

$$f(x) = \begin{cases} A(1-x')^3 + 3B(1-x')^2x' + 3C(1-x')x'^2 + Dx'^3, & x < U \\ 1 + E \exp(-F(x-U)), & x > U \end{cases}$$

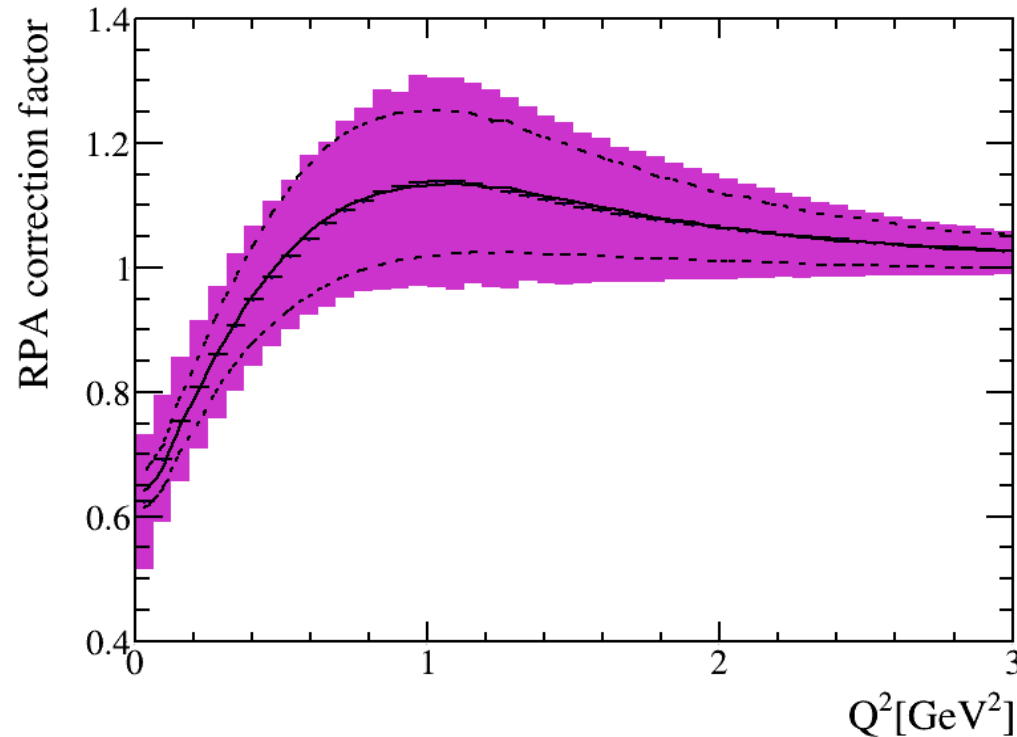
where $x = Q^2$, $x' = x/U$

- Looks awful! But, the continuity conditions are much less problematic:

$$E = D - 1$$

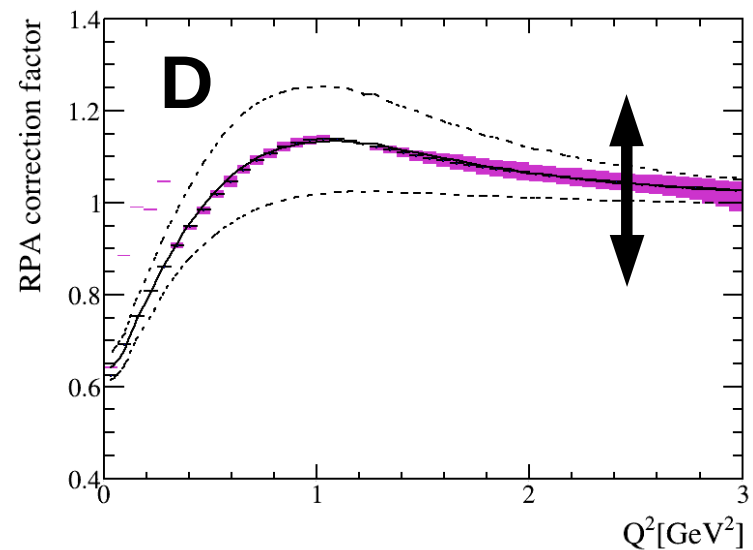
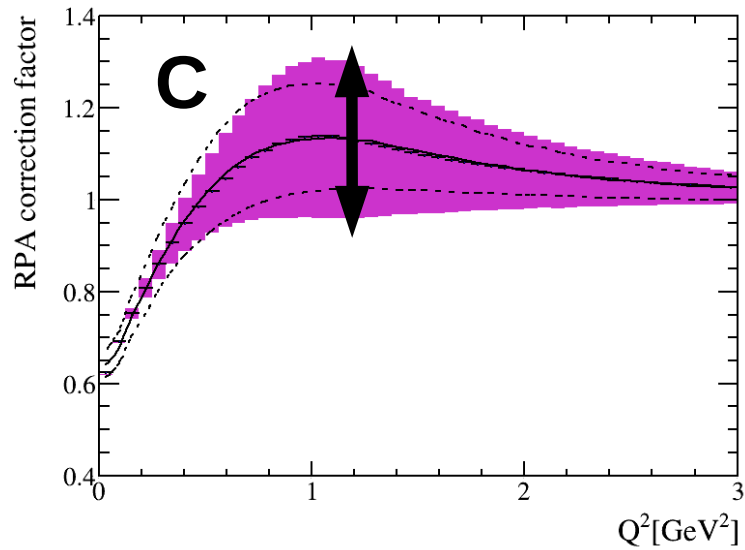
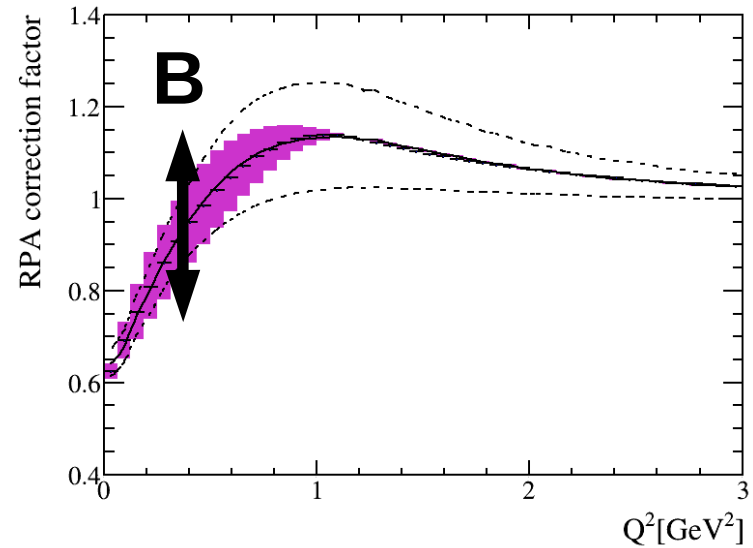
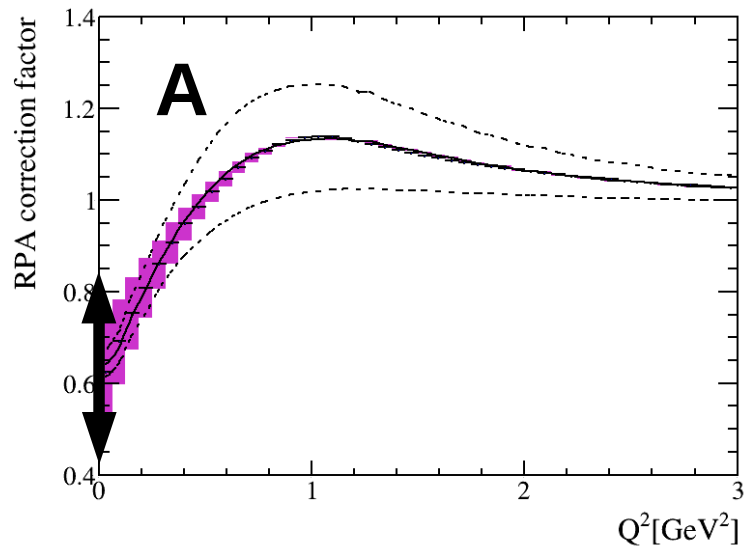
$$C = D + \frac{UF(D-1)}{3}$$

BeRPA (1)



- **B**ernstein polynomials used to approximate the Nieves **RPA** model, with additional freedom.
- Fit (blue) to nominal Nieves RPA (solid black).
- Guesstimate errors (purple band) which cover the Nieves error (dashed).

BeRPA (2)

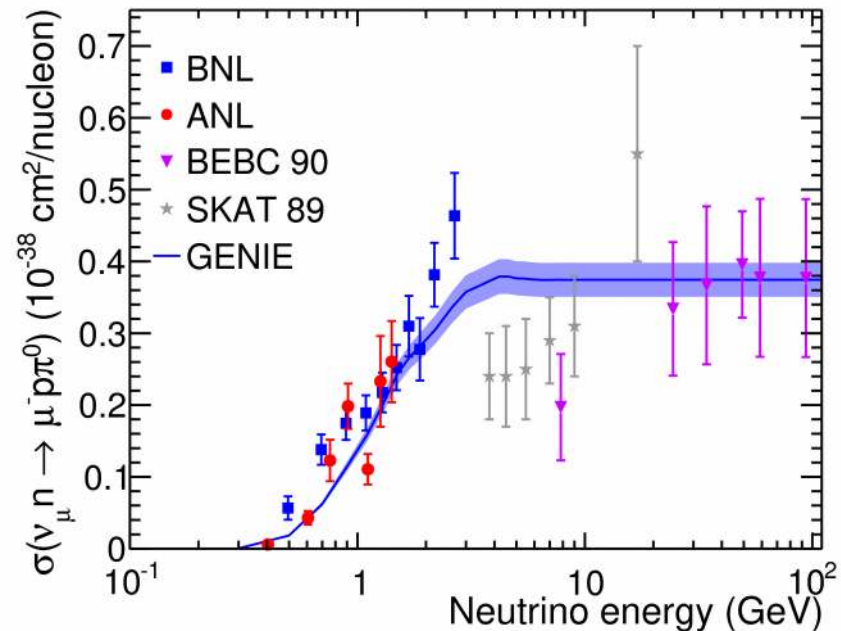
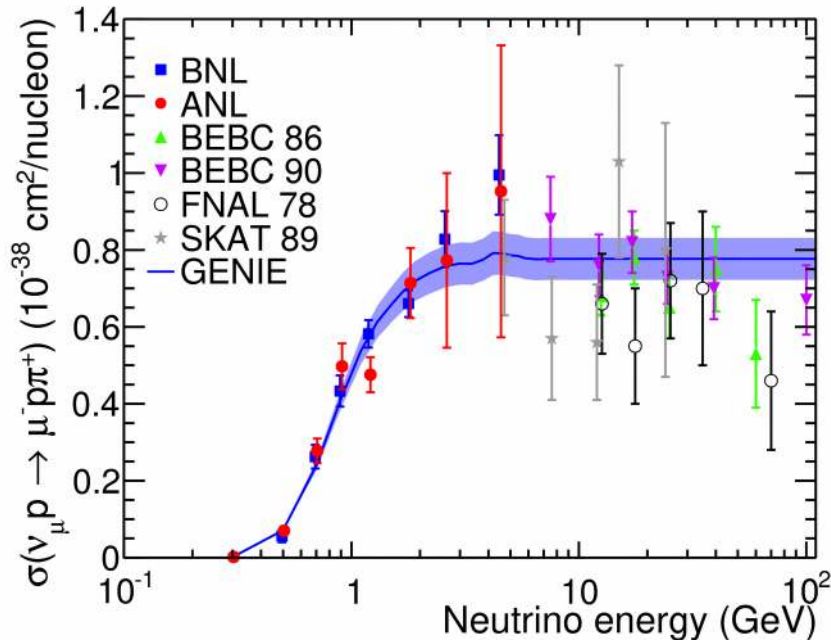


BeRPA parameters affect different regions of Q^2 → more flexibility



Bubble chamber tuning

- Parameters tuned to a limited set of bubble chamber data:
 - ANL and BNL, E_ν and Q^2 distributions
 - $\nu_\mu + p \rightarrow \mu^- + \pi^+ + p$
 - $\nu_\mu + n \rightarrow \mu^- + \pi^0 + p$
 - $\nu_\mu + n \rightarrow \mu^- + \pi^+ + n$
- Similar to tuning of the GENIE model for MINERvA (pictured)



EPJC 76 (2016) 76:474