Modeling neutrino-nucleus interaction
R. Gonzalez-Jimenez (UGent)
Wrocław, 8 Dec 2017

Modeling neutrino-nucleus interaction for neutrino-oscillation experiments

Raúl González Jiménez

**Department of Physics and Astronomy,
Ghent University, Belgium**

Wrocław, 8 December, 2017

In collaboration with...

J. Enrique Amaro

Maria. B. Barbaro

Juan A. Caballero

T. William Donnelly

Martin V. Ivanov

Guillermo D. Megias

José M. Udías

Natalie Jachowicz

Kajetan Niewczas

Alexis Nikolakopoulos

Jannes Nys

Vishvas Pandey

Jan Ryckebusch

Tom Van Cuyck

Nils Van Dessel

3

Outline

I   History

II   Some Nobel Prize experiments

III New era

IV   Nuclear physics in modern neutrino oscillation experiments

V   Conclusions

4

Offener Brief an die Gruppe der Radioaktiven bei der
Gauvereins-Tagung zu Tübingen.

Absohrift

Physikalisches Institut
der Eidg. Technischen Hochschule   Zürich, 4. Dez. 1930
Zürich   Gloriastrasse

     Liebe Radioaktive Damen und Herren,

     Wie der Ueberbringer dieser Zeilen, den ich huldvollst
anzuhören bitte, Ihnen des näheren auseinandersetzen wird, bin ich
angesichts der "falschen" Statistik der N- und Li-6 Kerne, sowie
des kontinuierlichen beta-Spektrums auf einen verzweifelten Ausweg
verfallen um den "Wechselsatz" (1) der Statistik und den Energiesatz
zu retten. Nämlich die Möglichkeit, es könnten elektrisch neutrale
Teilchen, die ich Neutronen nennen will, in den Kernen existieren,
welche den Spin 1/2 haben und das Ausschliessungsprinzip befolgen und
sich von Lichtquanten ausserdem noch dadurch unterscheiden, dass sie
nicht mit Lichtgeschwindigkeit laufen. Die Masse der Neutronen
müsste von derselben Grössenordnung wie die Elektronenmasse sein und
jedenfalls nicht grösser als 0,01 Protonenmasse.- Das kontinuierliche
beta- Spektrum wäre dann verständlich unter der Annahme, dass beim
beta-Zerfall mit dem Elektron jeweils noch ein Neutron emittiert
wird, derart, dass die Summe der Energien von Neutron und Elektron
konstant ist.

5

In that time (1930), the **nucleus was made of protons and electrons:** A nucleus of mass A was made of A protons and Z electrons.

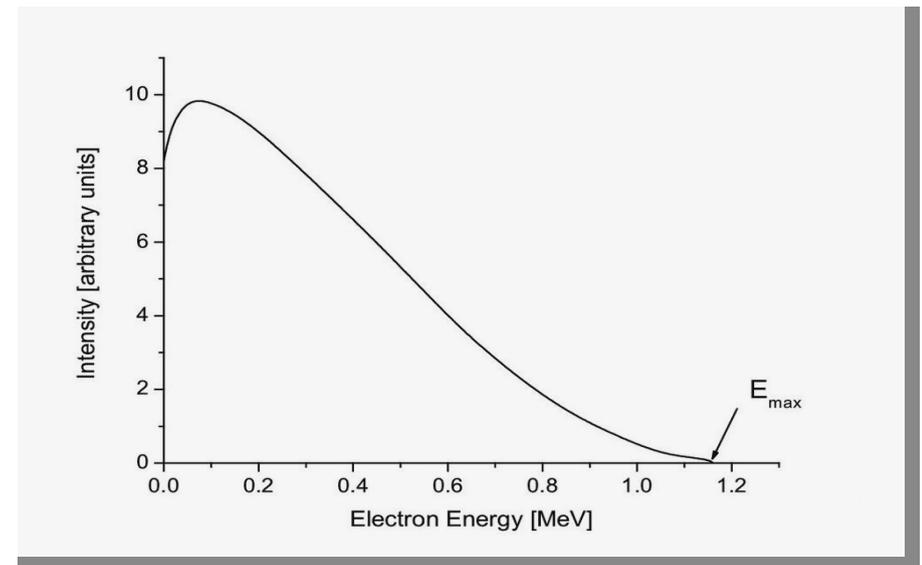Among others, there were two clear inconsistencies that needed to be solved:

+ Nuclear Spin
(1928) Nuclear Spin of Nitrogen 14 = 1 (hbar)

An integer spin cannot be explained by coupling an odd number of spin ½ particles (14 protons and 7 electrons)

+ Beta decay Spectrum

n → p + e- + ?

The spectrum cannot be continuous if only one particle is emitted.

Pauli's letter of the 4th of December 1930

Dear Radioactive Ladies and Gentlemen,

As the bearer of these lines, to whom I graciously ask you to listen, will explain to you in more detail, how **because of the "wrong" statistics of the N and Li6 nuclei and the continuous beta spectrum, I have hit upon a desperate remedy to save the "exchange theorem" of statistics and the law of conservation of energy.** *Namely,* **the possibility that there could exist in the nuclei electrically neutral particles, that I wish to call neutrons,** *which have spin 1/2 and obey the exclusion principle and which further differ from light quanta in that they do not travel with the velocity of light. The mass of* **the neutrons should be of the same order of magnitude as the electron mass** *and in any event not larger than 0.01 proton masses.* **The continuous beta spectrum would then become understandable by the assumption that in beta decay a neutron is emitted in addition to the electron** *such that the sum of the energies of the neutron and the electron is constant... I agree that* **my remedy could seem incredible because one should have seen those neutrons much earlier if they really exist. But only the one who dare can win** *and the difficult situation, due to the continuous structure of the beta spectrum, is lighted by a remark of my honoured predecessor, Mr Debye, who told me recently in Bruxelles: "Oh, it's well better not to think to this at all, like new taxes". From now on, every solution to the issue must be discussed. Thus, dear radioactive people, look and judge.*

Unfortunately, I cannot appear in Tubingen personally since I am indispensable here in Zurich because of a ball on the night of 6/7 December. With my best regards to you, and also to Mr Back.

Your humble servant . **W. Pauli, December 1930**

The Nobel Prize in Physics 1995


Martin L. Perl
Prize share: 1/2

© University of California Regents
Frederick Reines
Prize share: 1/2

The Nobel Prize in Physics 1995 was awarded *"for pioneering experimental contributions to lepton physics"* jointly with one half to Martin L. Perl *"for the discovery of the tau lepton"* and with one half to Frederick Reines *"for the detection of the neutrino"*.

The neutrino is detected for the first time

In the **mid 1950's, Frederick Reines and Clyde L. Cowan, Jr.** came up with an experiment to verify the existence of the neutrino. It made use of the fact that nuclear reactors were expected to produce neutrino fluxes on the order of 10^{12}-10^{13} neutrinos per second per cm2, far higher than any attainable flux from radioactive sources.

1. Nuclear reactor produces antineutrinos

$$n \rightarrow p + e^- + \bar{\nu}_e$$
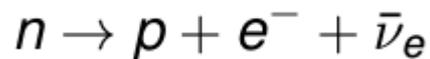
2. The antineutrino interacts with a proton in the detector

$$\bar{\nu}_e + p \rightarrow n + e^+$$
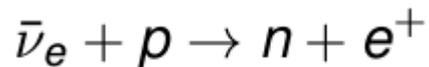
3. The positron annihilates immediately with an electron of the media

$$e^+ + e^- \rightarrow 2\gamma$$
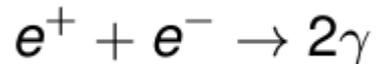
4. If the two gammas and the neutron are detected within a time windows → **antineutrino**
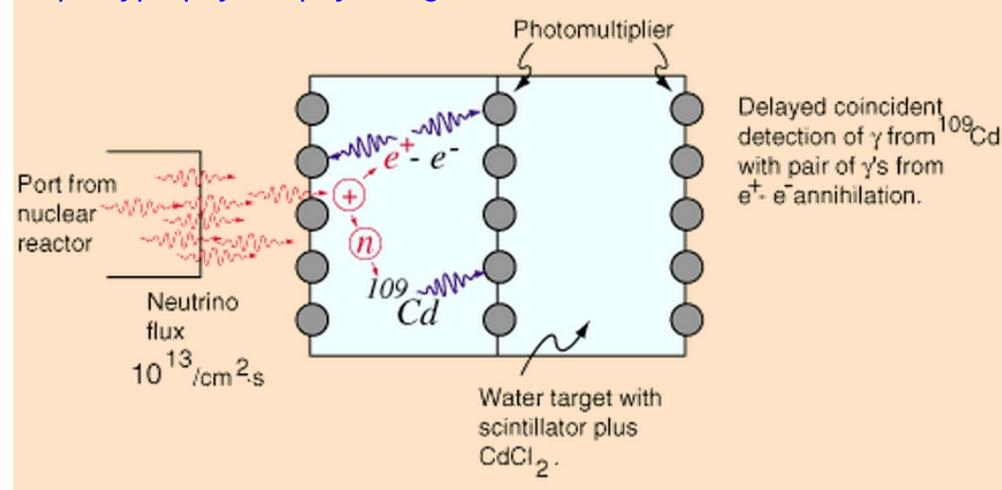
After months of data collection, they had accumulated data on about three neutrinos per hour in their detector.

Raymond Davis Jr.
Prize share: 1/4

Masatoshi Koshiba
Prize share: 1/4

Riccardo Giacconi
Prize share: 1/2

The Nobel Prize in Physics 2002 was divided, one half jointly to Raymond Davis Jr. and Masatoshi Koshiba *"for pioneering contributions to astrophysics, in particular for the detection of cosmic neutrinos"* and the other half to Riccardo Giacconi *"for pioneering contributions to astrophysics, which have led to the discovery of cosmic X-ray sources"*.
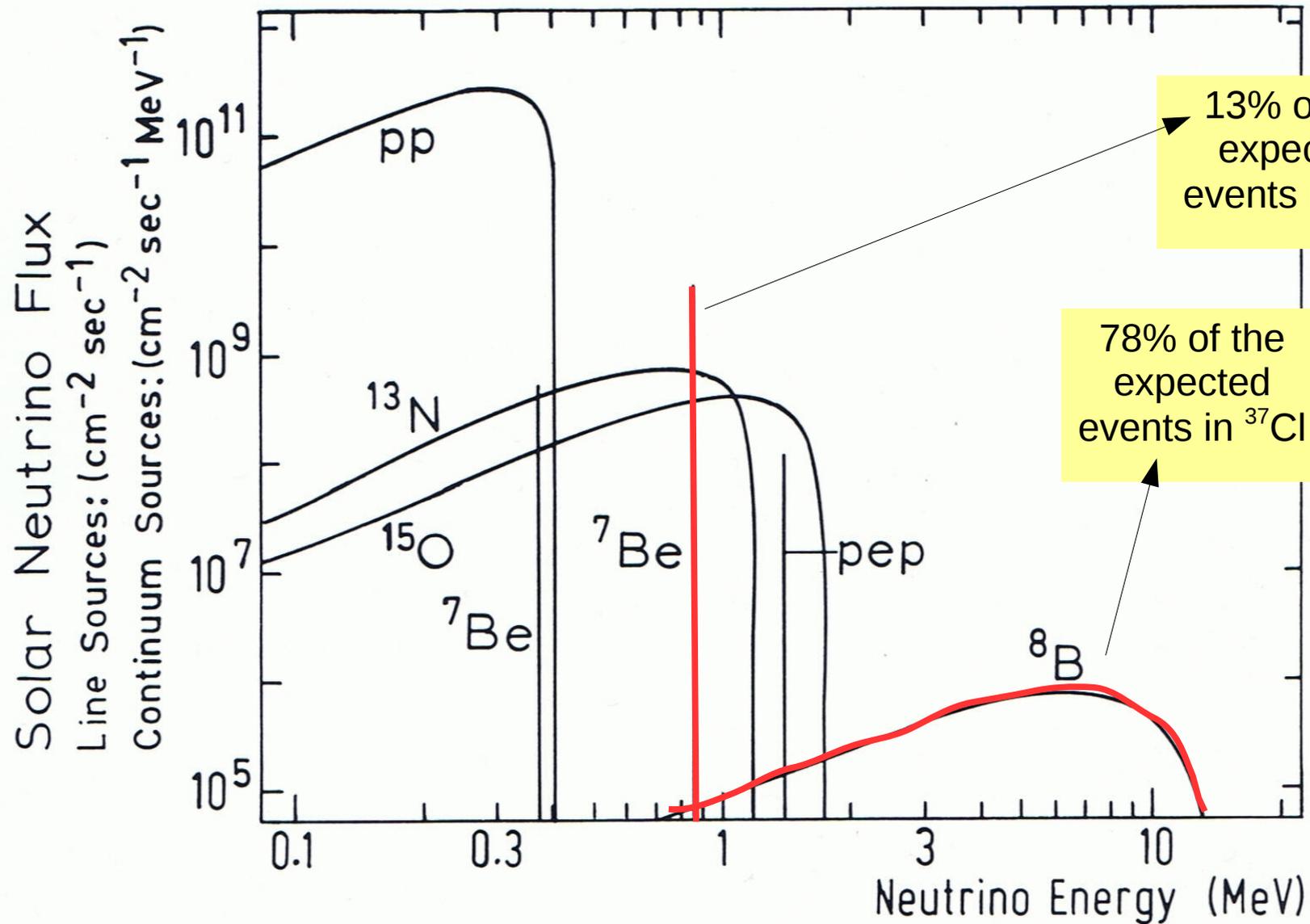
10

2 Homestake Solar Neutrino Detector

➤ It was built in the period 1965-1967.

➤ Originally, the goal was to measure the total flux of solar neutrinos with an energy above 0.814 MeV.

➤ The technique to count the neutrinos consists in an original idea by Pontecorvo (1946).
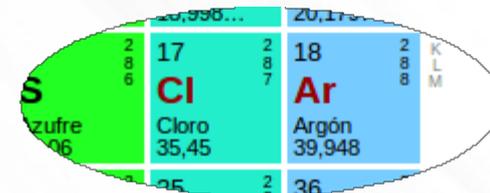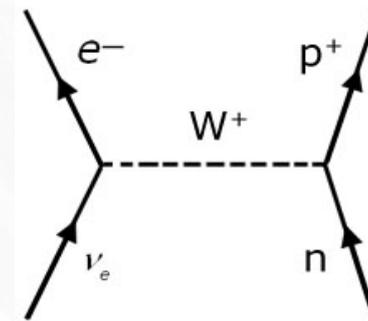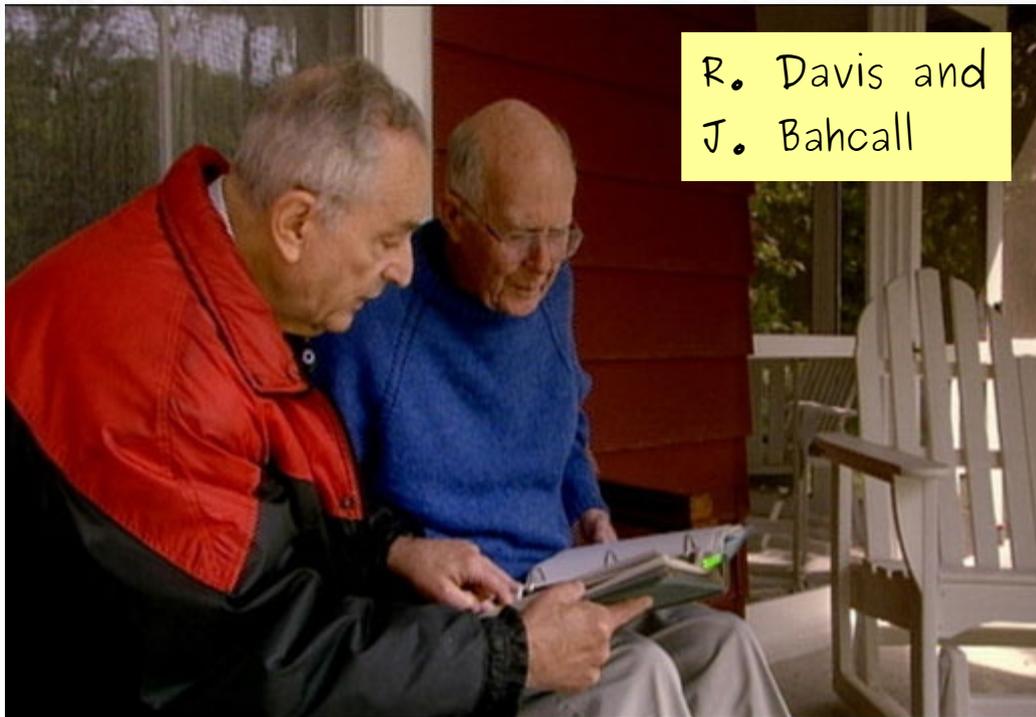
$$^{37}Cl + \nu_e \rightarrow \ ^{37}Ar + e$$

R. Davis and J. Bahcall

Relevant solar neutrino fluxes

$$^{37}Cl + \nu_e \rightarrow \,^{37}Ar + e$$

13% of the expected events in ^{37}Cl

78% of the expected events in ^{37}Cl

Neutrinos from the Sun [pp chain]

[also: CNO cycle]

[pp-neutrinos]
$$p + p \rightarrow {}^2H + e^+ + \nu_e$$

[pep-neutrinos]
$$p + e^- + p \rightarrow {}^2H + \nu_e$$

99.75%   0.25%

$${}^2H + p \rightarrow {}^3He + \gamma$$

85%   ~15%   ~10^{-5}%

[hep-neutrinos]
$${}^3He + p \rightarrow {}^4He + e^+ + \nu_e$$

$${}^3He + {}^3He \rightarrow {}^4He + 2p$$ $${}^3He + {}^4He \rightarrow {}^7Be + \gamma$$

15.07%   0.02%

$${}^7Be + e^- \rightarrow {}^7Li + \gamma + \nu_e$$
[^{7}Be-neutrinos]

$${}^7Be + p \rightarrow {}^8B + \gamma$$

$${}^7Li + p \rightarrow \alpha + \alpha$$

$${}^8B \rightarrow {}^8Be^* + e^+ + \nu_e$$
[^{8}B-neutrinos]

13

CONDENSER

NEUTRON SOURCE TUBE

VAPOR TIGHT DOOR

COOLING JACKETS

GAS PROCESSING & CONTROL ROOM

LIQUID NITROGEN STORAGE

VESSEL (100,000 GALLONS)

WATER TIGHT DOOR

WATER FOR NEUTRON SHIELDING

VAPOR TIGHT DOOR

PERCHLOROETHYLENE PUMPS

TO YATES SHAFT

14

THE EXPERIMENT

➢ 615 tons of tetrachloroethylene, C_2Cl_4, (natural abundance of ^{37}Cl is ~24%).

615 tons of C_2Cl_4

THE EXPERIMENT

ν_e

615 tons of C$_2$Cl$_4$

^{37}Ar

615 tons of C_2Cl_4

THE EXPERIMENT

ν_e

^{37}Ar

615 tons of C₂Cl₄

$$615 \text{ tons of } C_2Cl_4$$

^{37}Ar

^{37}Ar

615 tons of C$_2$Cl$_4$

19
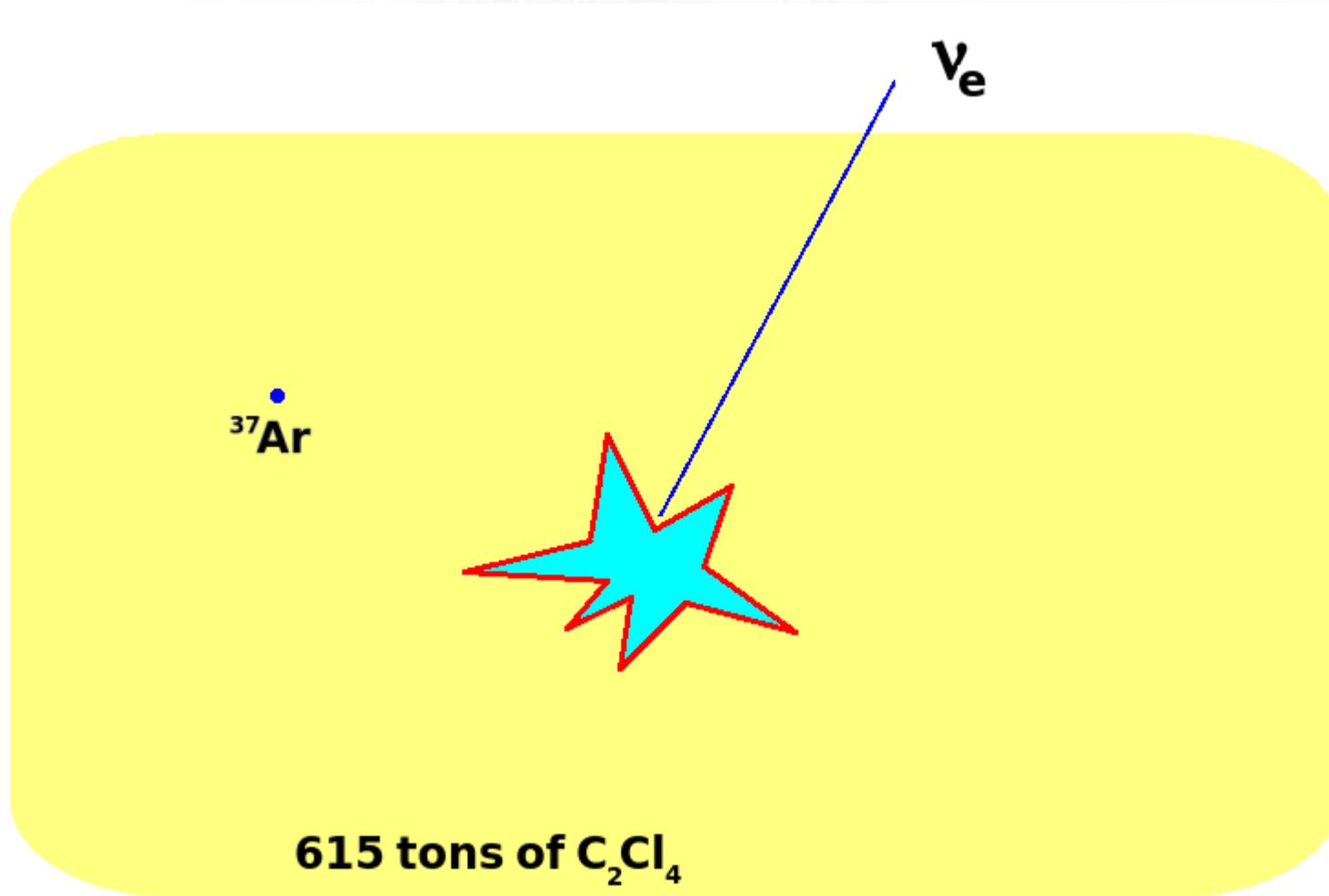
After 2 month...

615 tons of C_2Cl_4
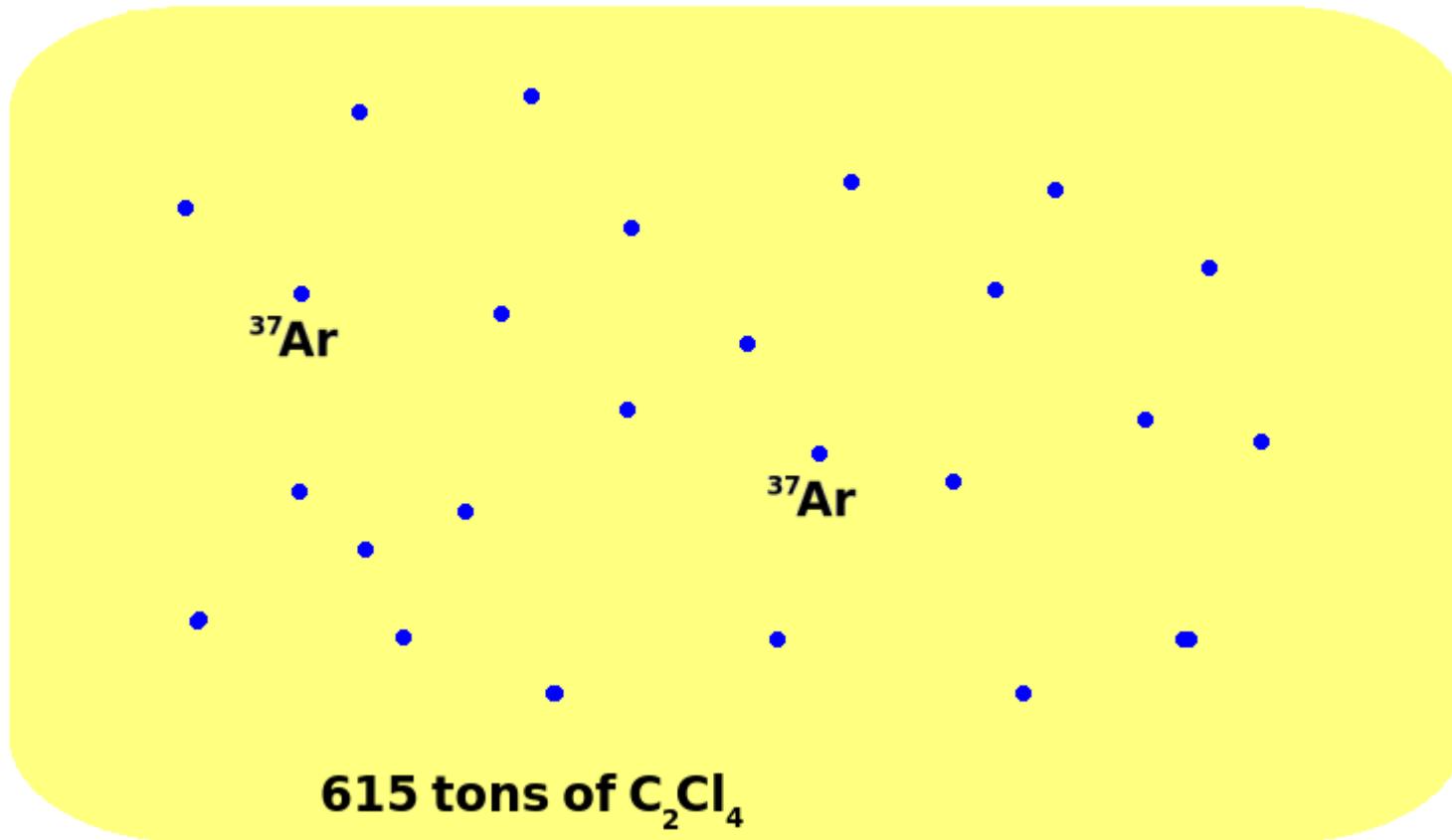
... we have ~ 18 atoms of ^{37}Ar !!!

^{37}Ar half-live ~ 35 days.

➢ The produced argon starts decaying immediately after the production so there exist a saturation level.

➢ ***Saturation level in the tank is ~25 ^{37}Ar atoms.***

➢ A 60 days exposure results in 70% of saturation.

➢ Typical run is 2 month duration.

615 tons of C$_2$Cl$_4$

THE EXPERIMENT
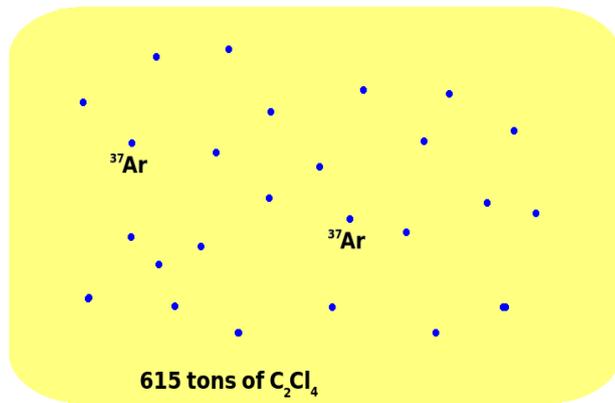
Extraction of Argon

^{37}Ar

^{37}Ar

615 tons of C_2Cl_4
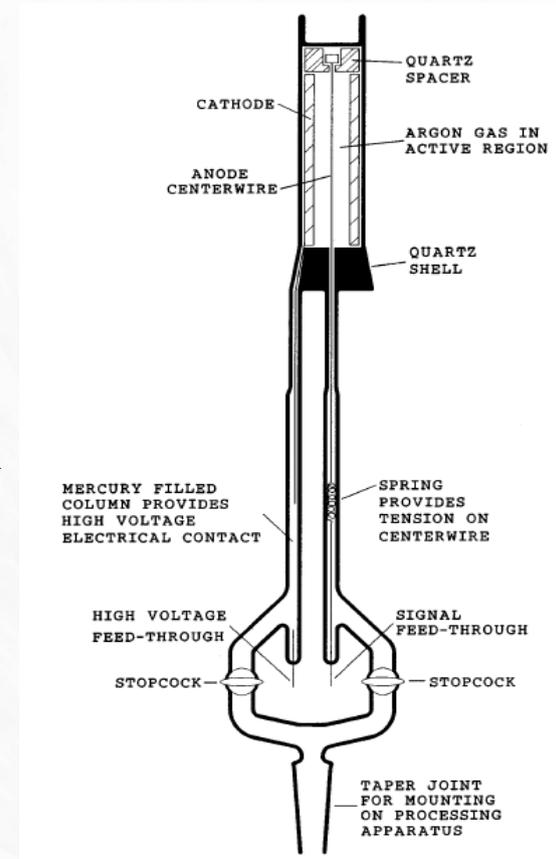
^{37}Ar decays in ^{37}Cl by **electron capture** producing 3-5 **Auger electrons** which deposit an energy of 2.82 KeV in the counter.

(electron capture: $p + e \rightarrow n + \nu_e$)

Active ^{37}Ar Energy Spectrum

Test sample: ^{37}Ar spectrum

Chlorine Data Energy Spectrum (Early)

First 105 days: Background + ^{37}Ar decays

(a)

(b)

Chlorine Data Energy Spectrum (Late)

More than 175 days: Background

(c)
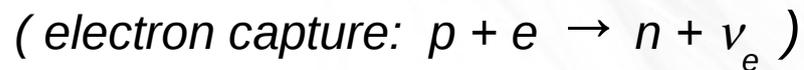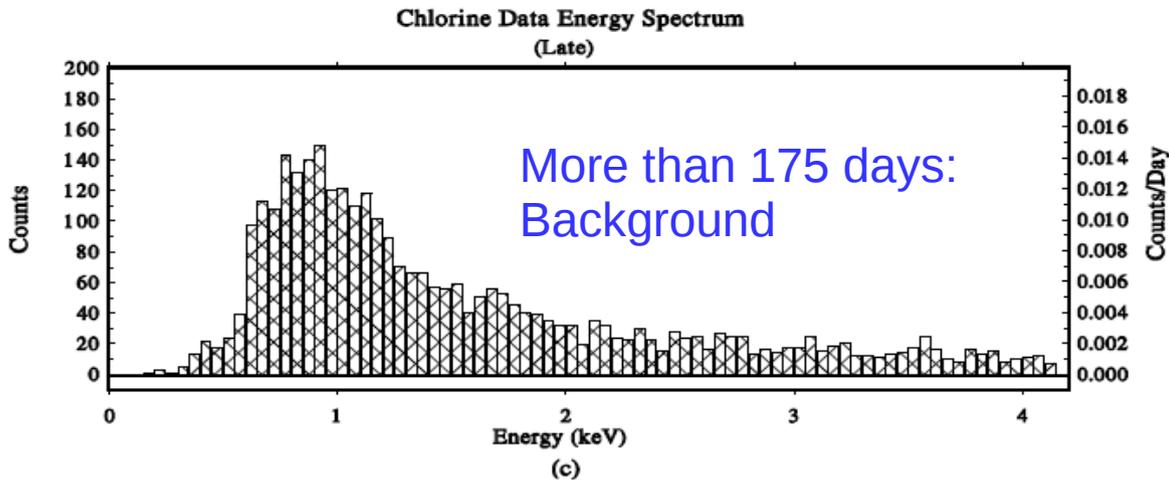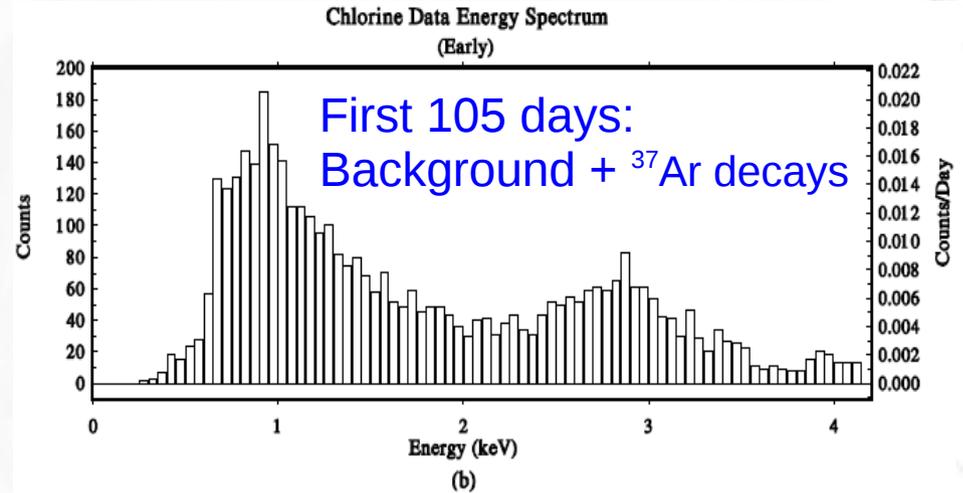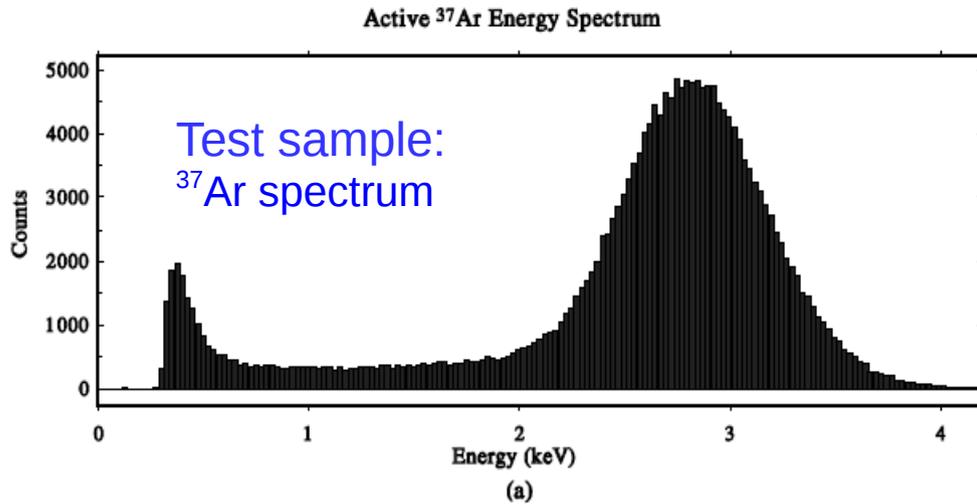
After ~ 2 months the argon is extracted and placed in a tiny proportional counter.
- ^{37}Ar half-live ~ 35 days.
- Within 105 (3 half-lives) days 87.5% of the atoms have decayed.
- After 175 days (5 half-lives) only ~3% ^{37}Ar atoms remain in the counter.

FIG. 9.—(a) Energy spectrum from a counter filled with active ^{37}Ar. (b) Cumulative energy spectrum from 93 solar neutrino observations (early counting data: 0–105 days following the end of an extraction). (c) Cumulative energy spectrum from 93 solar neutrino observations (late counting data: 175–350 days following the end of an extraction). Plots (b) and (c) are of events with normalized ADP between 0.85 and 1.02 (i.e., fast events).

23

Results after 25 years
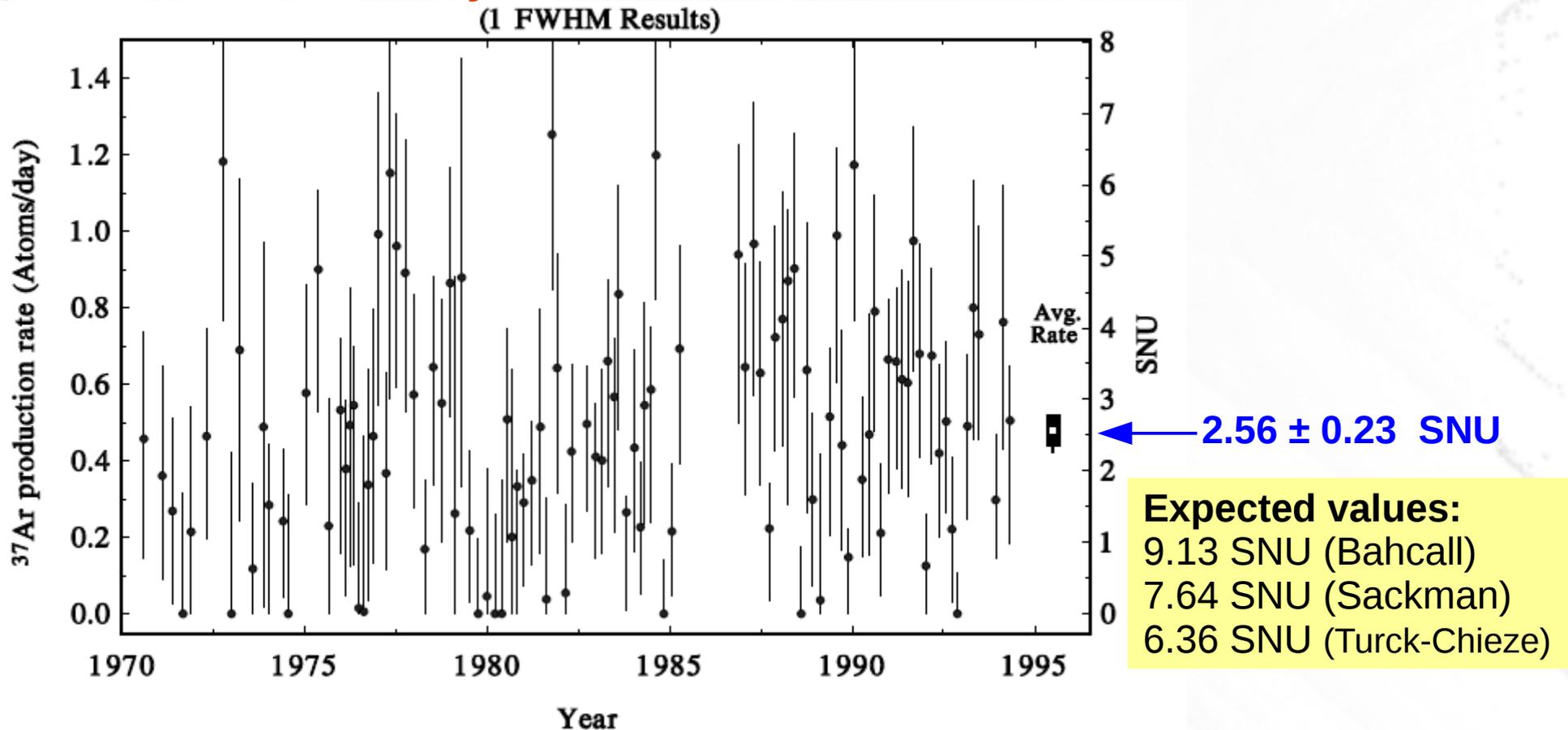
FIG. 13.—Homestake Experiment—one FWHM results. Results for 108 individual solar neutrino observations made with the Homestake chlorine detector. The production rate of ^{37}Ar shown has already had all known sources of nonsolar ^{37}Ar production subtracted from it. The errors shown for individual measurements are statistical errors only and are significantly non-Gaussian for results near zero. The error shown for the cumulative result is the combination of the statistical and systematic errors in quadrature.

2.56 ± 0.23 SNU

Expected values:
9.13 SNU (Bahcall)
7.64 SNU (Sackman)
6.36 SNU (Turck-Chieze)

24

SNU = Solar Neutrino Unit = one interaction per 10^{36} target atoms per second

Takaaki Kajita
Prize share: 1/2
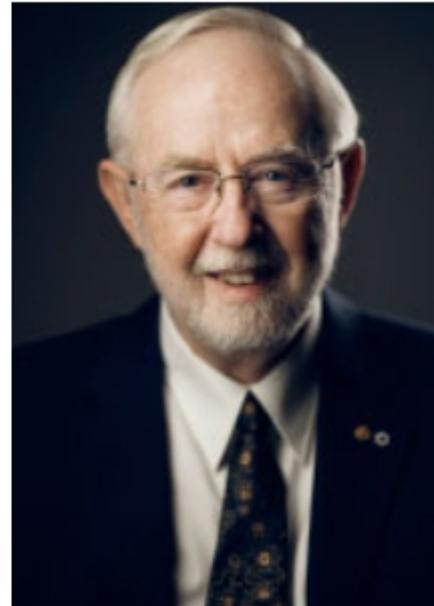
Arthur B. McDonald
Prize share: 1/2

The Nobel Prize in Physics 2015 was awarded jointly to Takaaki Kajita and Arthur B. McDonald *"for the discovery of neutrino oscillations, which shows that neutrinos have mass"*

25

http://www.nobelprize.org

3 KamiokaNDE

KamiokaNDE experiment was designed to search for proton decay.

➔ A proton decay detector must be extremely well isolated from backgrounds since the lower limit for proton half-live is ~10^{34} years.

$$p+ \rightarrow e+ + \pi 0$$
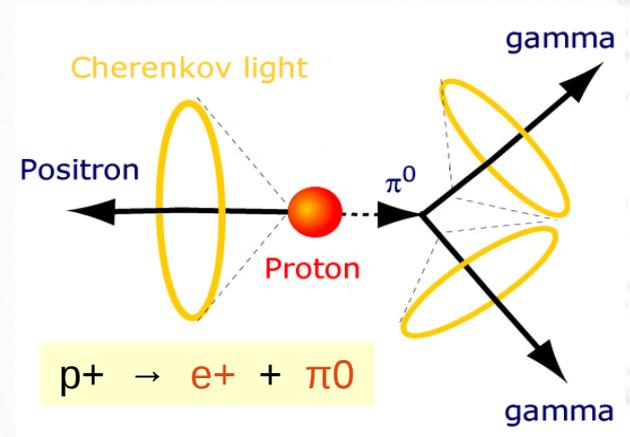
Kamiokande detector:

1000 m underground (2700 m.w.e.).

Start operating on July 1983.

Cylinder ~ 15.6 m diameter x 16 m height.

Thousands of photomultiplier tubes fill the inner surface of the cylinder.

Tons of water **Cerenkov detector**.

Event in Super-Kamiokande Cerenkov detector

Event in Super-Kamiokande Cerenkov detector

28
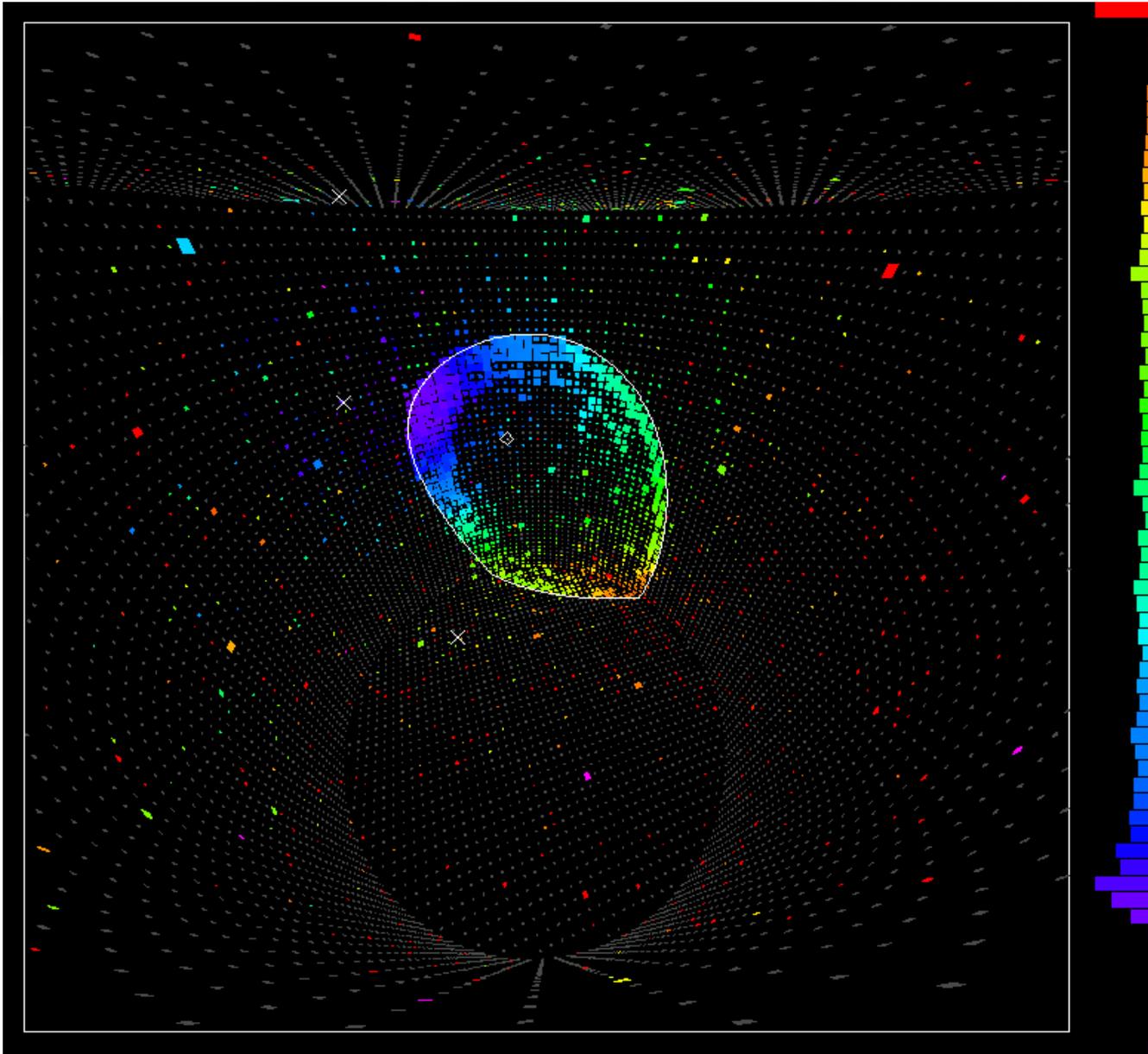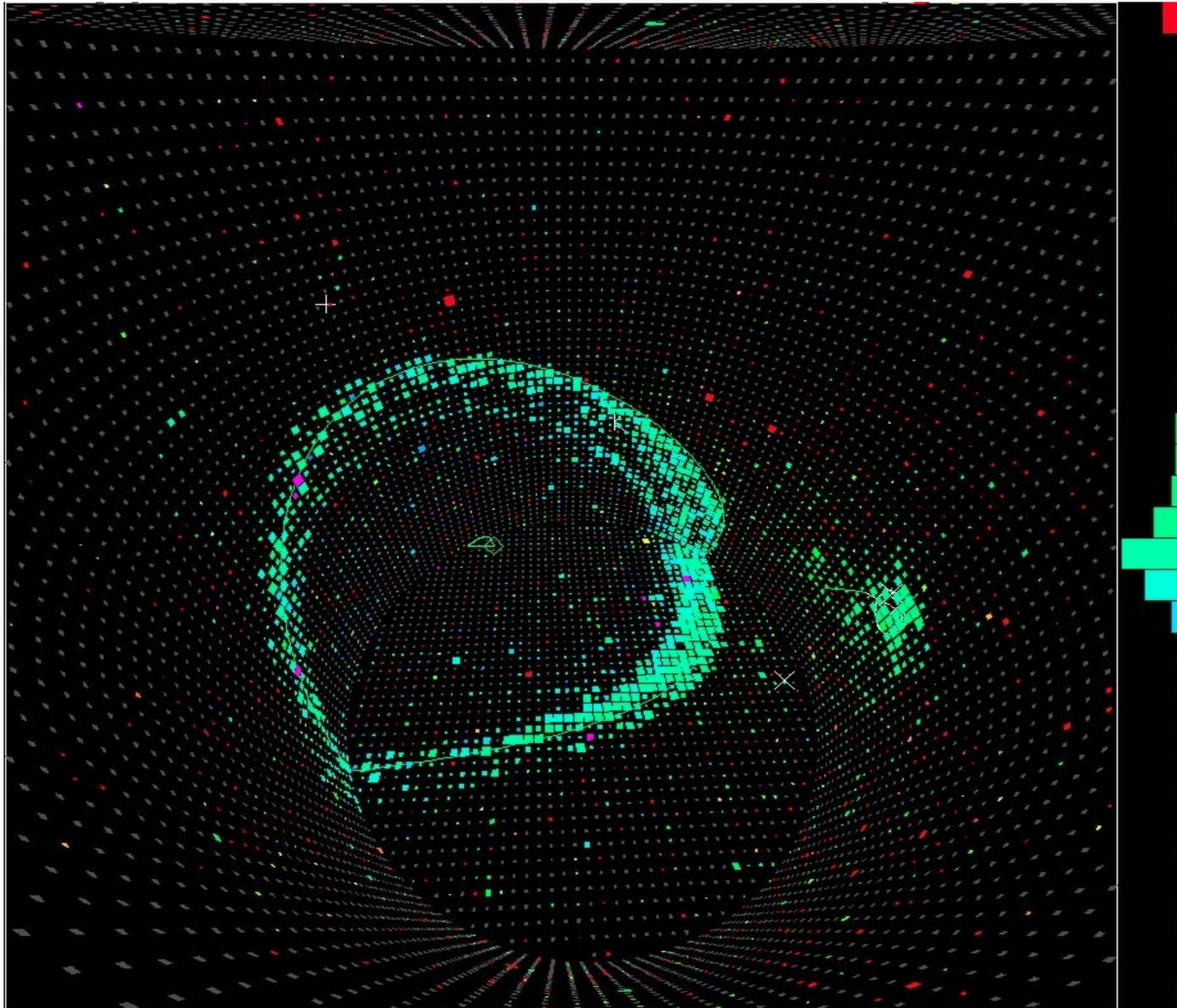
Super-Kamiokande

50 000 tons of water.
11 456 photomultipliers
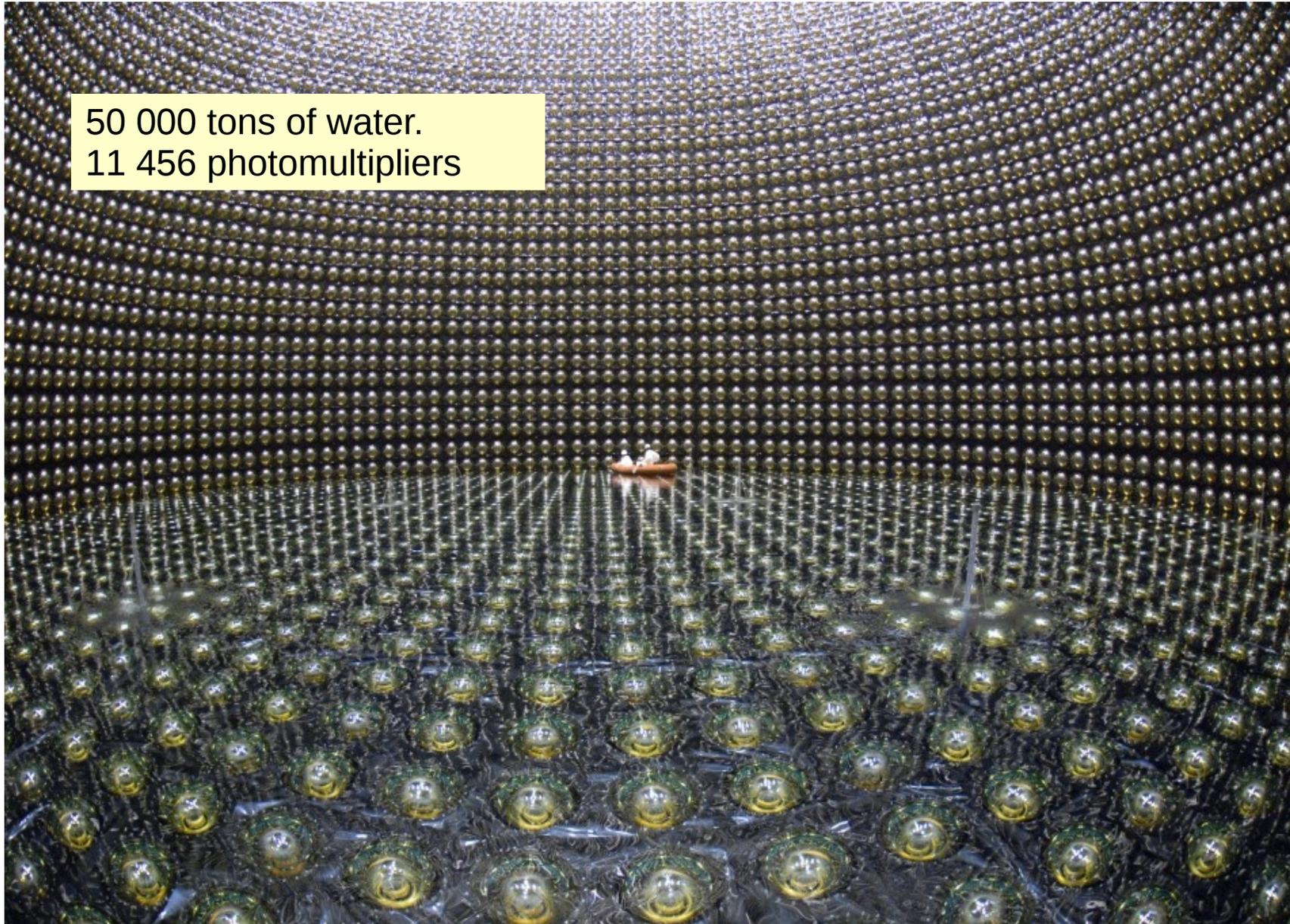
Neutrino production in the atmosphere

http://www.twinkletoesengineering.info/neutrinos.htm

http://hep.bu.edu/~superk/atmnu/

Atmospheric neutrinos enter the detector from every direction.
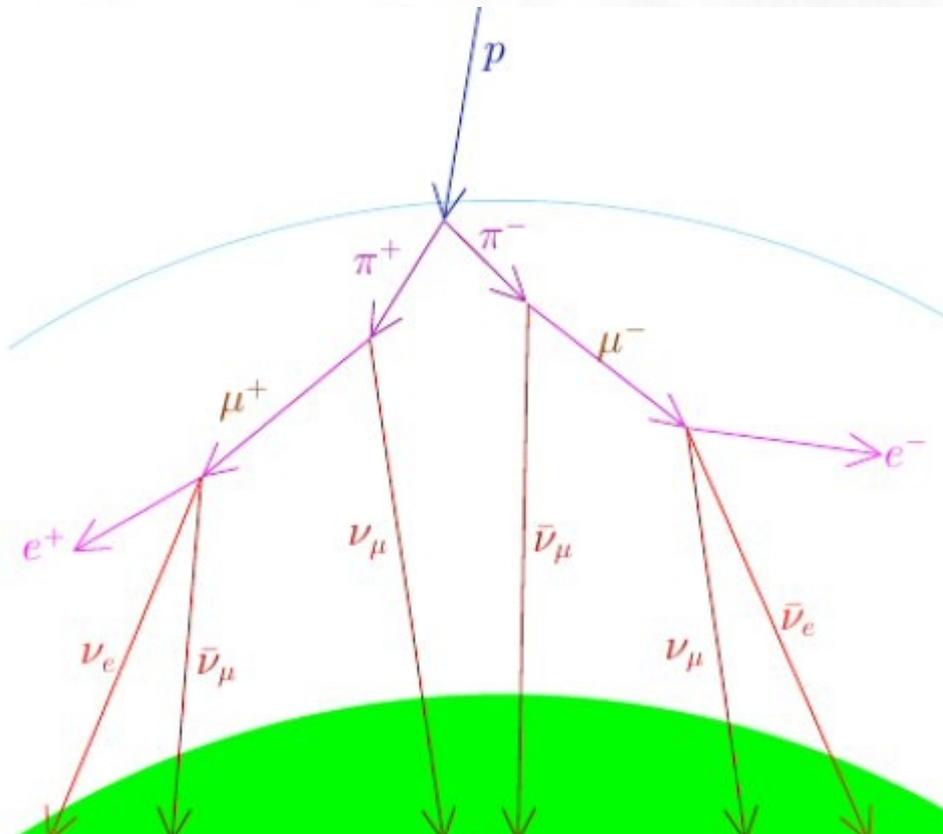
Prediction for the neutrino fluxes:

$$R_{Theor} = (\nu_\mu + \overline{\nu}_\mu) / (\nu_e + \overline{\nu}_e) \sim 2$$

Experiment (1992, Kam-I and Kam-II):

$$\mathbf{R_{data} / R_{Theor} = 0.60 \pm 0.09}$$

Far from 1 !!!

Atmospheric ν_μ/ν_e ratio in the multi-GeV energy range
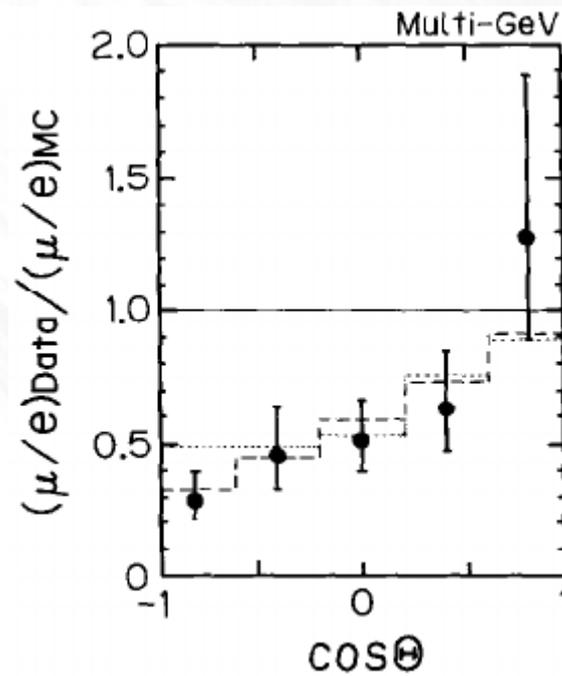
Strong dependence on the zenith angle

Results in the multi-GeV region (E_{lepton} > 1.3 GeV)

31

4   Solar Neutrinos at the Sudbury Neutrino Observatory (SNO)
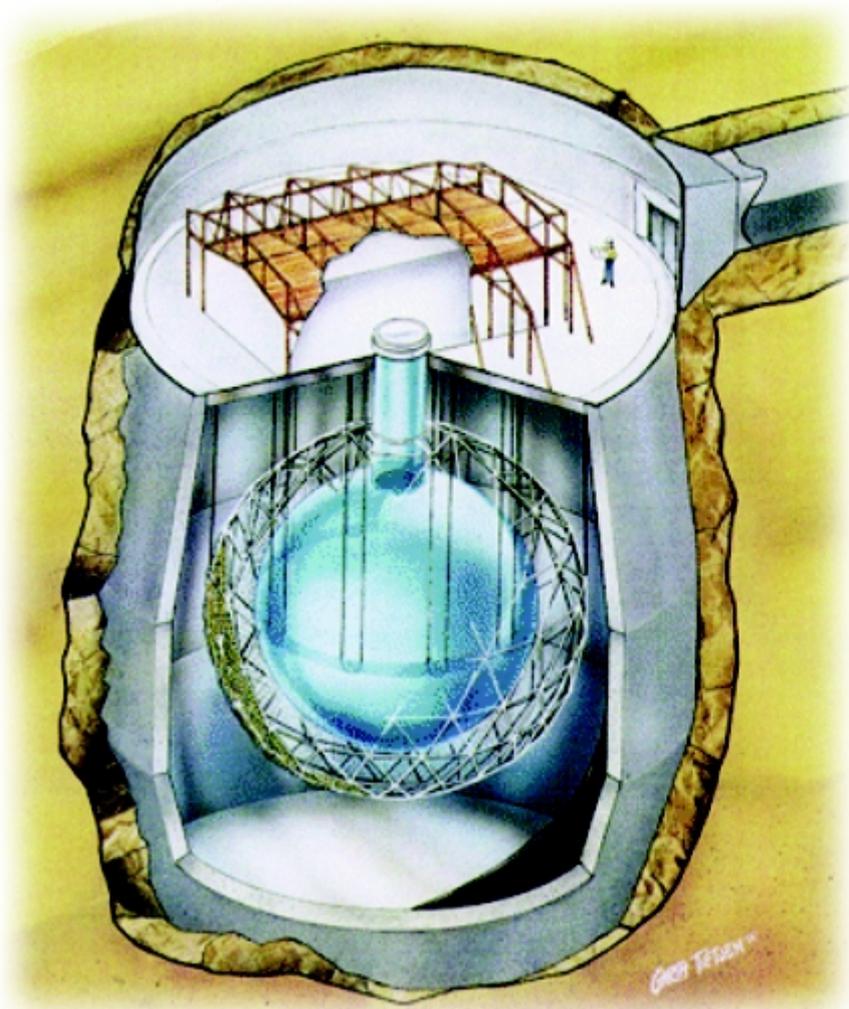
Image from http://www.kip.uni-heidelberg.de/~coulon/Lectures/SM/Free_PDFs/Lecture_HC_13.pdf

Reaction channels, detection and identification

CC $\nu_e + d \rightarrow p + p + e^-$

NC $\nu_x + d \rightarrow p + n + \nu_x$
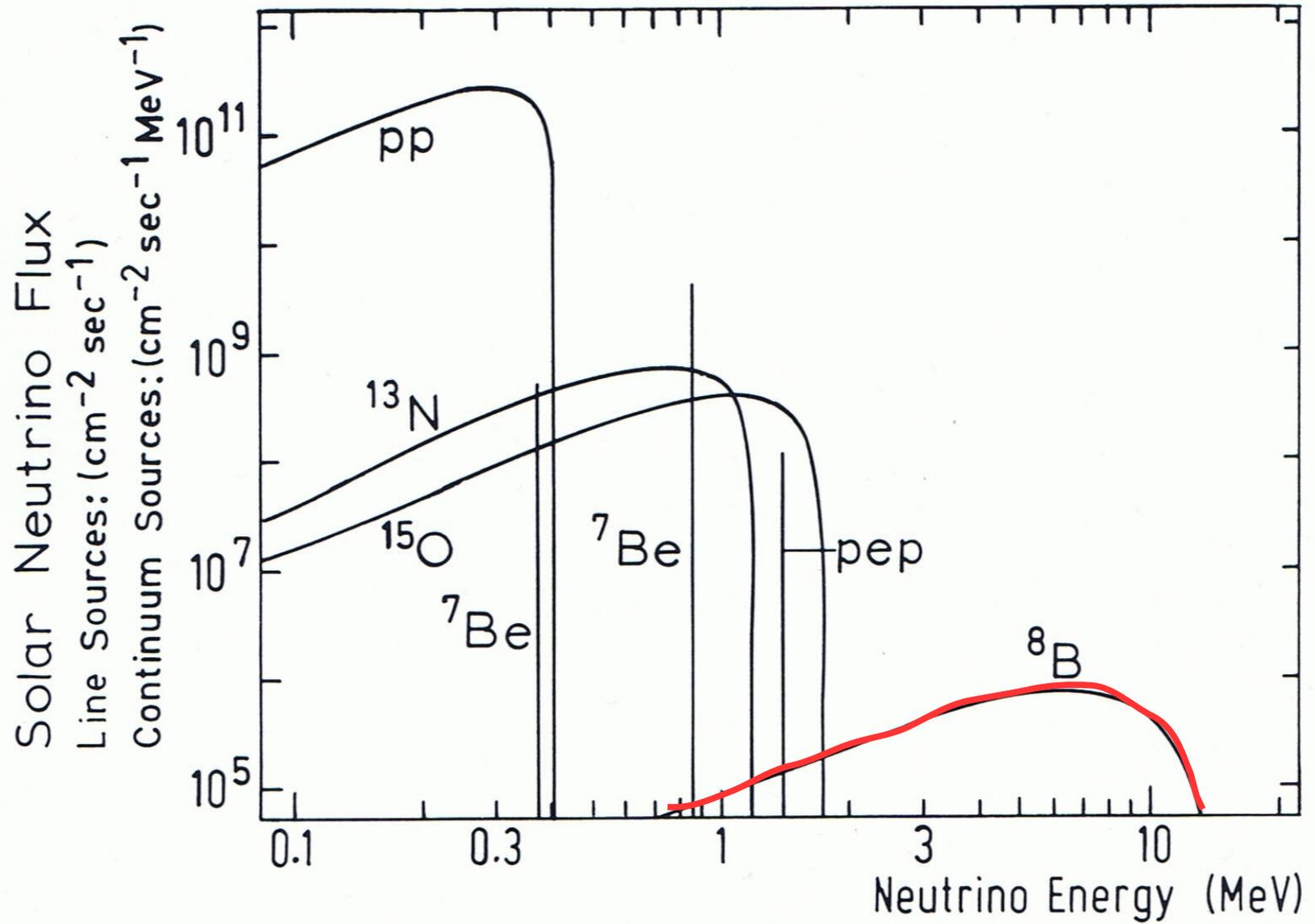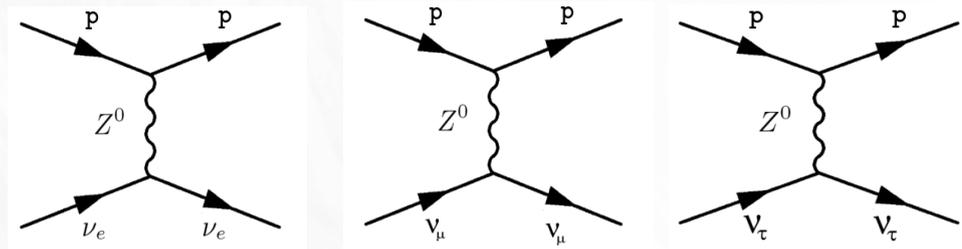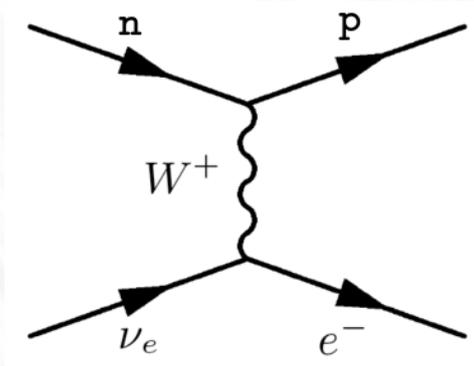
+ interactions with the neutron from deuterium

ES $\nu_x + e^- \rightarrow \nu_x + e^-$

34

Reaction channels, detection and identification

CC $\nu_e + d \rightarrow p + p + e^-$
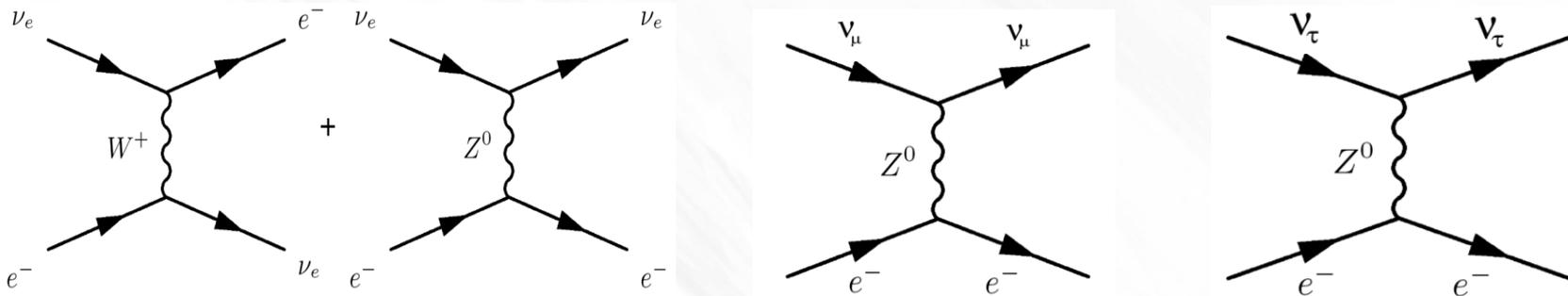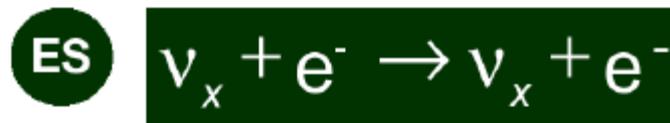
- Measurement of ν_e energy spectrum
- Weak directionality: $1 - 0.340 \cos\theta$

NC $\nu_x + d \rightarrow p + n + \nu_x$
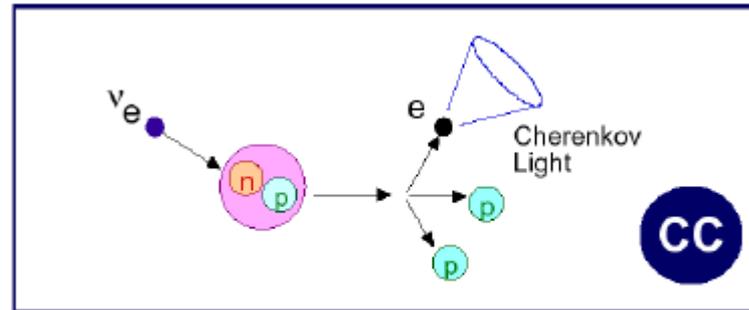
- Measure total ^8B ν flux from the sun
- $\sigma(\nu_e) = \sigma(\nu_\mu) = \sigma(\nu_\tau)$

ES $\nu_x + e^- \rightarrow \nu_x + e^-$
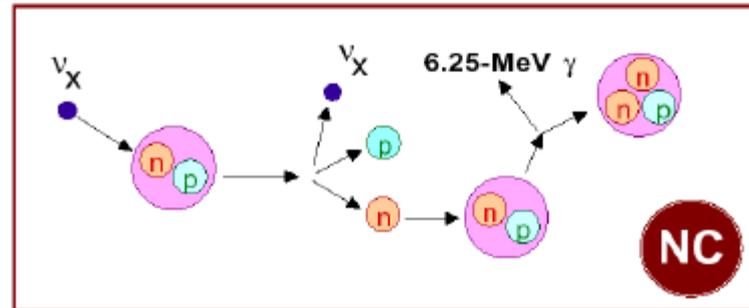
- Low Statistics
- $\Sigma\phi = \phi(\nu_e) + 0.154\,\phi(\nu_\mu + \nu_\tau)$
- Strong directionality:

$$\theta_e \leq 18^\circ \quad (T_e = 10\ \text{MeV})$$
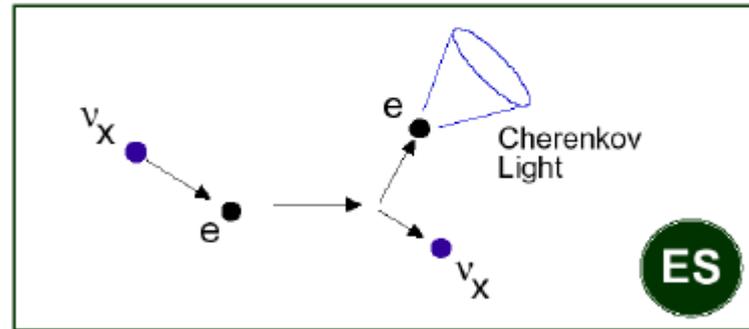
- CC and ES channel. Total amount of Cerenkov light is correlated with the energy of the incoming neutrino.
- NC channel. Detection of neutrons: 6.25 MeV gamma rays due to neutron capture on deuterium. The gamma rays produce Compton electrons or e+e- pairs. The Cerenkov light from these is detected.

Image from http://www.kip.uni-heidelberg.de/~coulon/Lectures/SM/Free_PDFs/Lecture_HC_13.pdf

35

Oscillations??

n events via CC interaction → Solar neutrino Flux from CC channel

n events via NC interaction → Solar neutrino Flux from NC channel

Theoretical Inputs: cross sections

Same ???

YES → no oscillations

NO → Oscillations!!!

36

RESULTS

Data recorded between 2 November 1999 and 28 May 2001.

After analyses:

All fluxes in units: 10^6 cm^{-2} s^{-1}

$1967.7^{+61.9}_{-60.9}$ CC events,

$263.6^{+26.4}_{-25.6}$ ES events,

$576.5^{+49.5}_{-48.9}$ NC events

These events correspond to the following fluxes

if <u>no-oscillations</u> are considered

$$\phi^{SNO}_{CC} = 1.76^{+0.06}_{-0.05}(\text{stat})^{+0.09}_{-0.09}(\text{syst}),$$

$$\phi^{SNO}_{ES} = 2.39^{+0.24}_{-0.23}(\text{stat})^{+0.12}_{-0.12}(\text{syst}),$$

$$\phi^{SNO}_{NC} = 5.09^{+0.44}_{-0.43}(\text{stat})^{+0.46}_{-0.43}(\text{syst}).$$

The excess of the NC flux over the CC and ES fluxes implies neutrino flavor transformations

Note that the NC is in excellent agreement with the prediction for the 8B flux by the Standard Solar Model (SSM).

$$\phi_{SSM} = 5.05^{+1.01}_{-0.81}$$

37

RESULTS

Data recorded between 2 November 1999 and 28 May 2001.

After analyses:

$1967.7^{+61.9}_{-60.9}$ CC events,

$263.6^{+26.4}_{-25.6}$ ES events,

$576.5^{+49.5}_{-48.9}$ NC events

These events corres-pond to the following fluxes

if _no-oscillations_ are considered

$\phi^{SNO}_{CC} = 1.76^{+0.06}_{-0.05}(\text{stat})^{+0.09}_{-0.09}(\text{syst})$,

$\phi^{SNO}_{ES} = 2.39^{+0.24}_{-0.23}(\text{stat})^{+0.12}_{-0.12}(\text{syst})$,

$\phi^{SNO}_{NC} = 5.09^{+0.44}_{-0.43}(\text{stat})^{+0.46}_{-0.43}(\text{syst})$.

The excess of the NC flux over the CC and ES fluxes implies neutrino flavor transformations

Note that the NC is in excellent agreement with the prediction for the 8B flux by the Standard Solar Model (SSM).

$\phi_{SSM} = 5.05^{+1.01}_{-0.81}$

38

Accelerator-based neutrino oscillation experiments
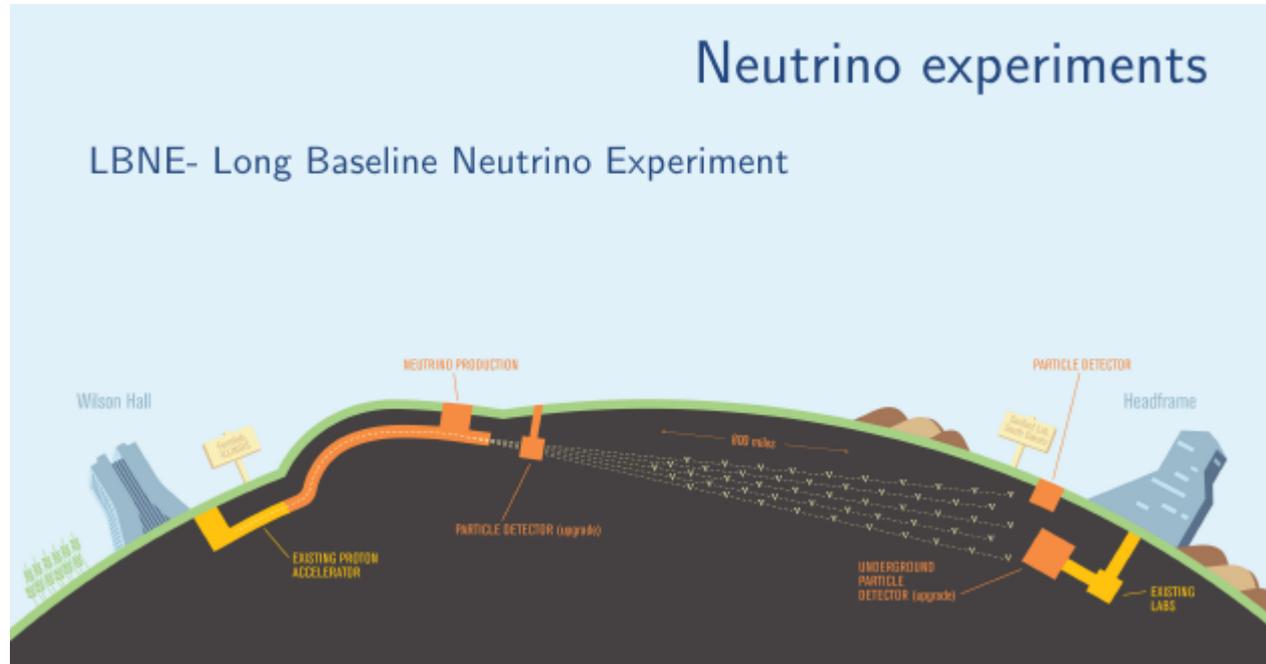
$$P(\nu_x \rightarrow \nu_y) = \sin^2 2\theta \sin^2 \left(1.27 \frac{\Delta m^2 (eV^2) L(km)}{E(GeV)} \right)$$

Some accelerator-based neutrino experiments

Experiment	Physics studied	Status
MiniBooNE	Neutrino oscillations	Finnished
T2K	θ_{13}, θ_{23}, Δm^2_{13}, Δm^2_{23}	Running
ArgoNeuT	Neutrino interactions, LArTPC	Analysis
Minerνa	Neutrino interactions	Running
MINOS	θ_{13}, θ_{23}, Δm^2_{13}, Δm^2_{23}	Running
Noνa	θ_{13}, θ_{23}, Δm^2_{13}, Δm^2_{23}, δ_{CP}	Under construction
LBNE	θ_{13}, θ_{23}, Δm^2_{13}, Δm^2_{23}, δ_{CP}	In development
Hyper-K	θ_{13}, θ_{23}, Δm^2_{13}, Δm^2_{23}, δ_{CP}	Proposed

(2015)

Goals of the Neutrino Oscillation Programme

NuSTEC[a] White Paper: Status and Challenges of Neutrino-Nucleus Scattering

L. Alvarez-Ruso,[1] M. Sajjad Athar,[2] M. B. Barbaro,[3] D. Cherdack,[4] M. E. Christy,[5] P. Coloma,[6] T. W. Donnelly,[7] S. Dytman,[8] A. de Gouvêa,[9] R. J. Hill,[10,6] P. Huber,[11] N. Jachowicz,[12] T. Katori,[13] A. S. Kronfeld,[6] K. Mahn,[14] M. Martini,[15] J. G. Morfín,[6] J. Nieves,[1] G. Perdue,[6] R. Petti,[16] D. G. Richards,[17] F. Sánchez,[18] T. Sato,[19,20] J. T. Sobczyk,[21] and G. P. Zeller[6]

1. establish whether nature violates CP in the lepton sector and, if so, measure δ_{CP};

2. improve the accuracy on θ_{23} and, if not maximal, a determination of the octant it belongs to: $\theta_{23} < \pi/4$ vs. $\theta_{23} > \pi/4$;

3. determine the neutrino mass ordering at high confidence level: $m_1 < m_2 < m_3$ vs. $m_3 < m_1 < m_2$.

	θ_{12}	θ_{13}	θ_{23}	$\Delta m_{21}^2/10^{-5}$	$\Delta m_{3j}^2/10^{-3}$	δ_{CP}
Normal Ordering	$33.56^{+0.77}_{-0.75}$	$8.46^{+0.15}_{-0.15}$	$41.6^{+1.5}_{-1.2}$	$7.50^{+0.19}_{-0.17}$	$2.524^{+0.039}_{-0.040}$	261^{+51}_{-59}
Inverted Ordering	$33.56^{+0.77}_{-0.75}$	$8.49^{+0.15}_{-0.15}$	$50.0^{+1.1}_{-1.4}$	$7.50^{+0.19}_{-0.17}$	$-2.514^{+0.038}_{-0.041}$	277^{+40}_{-46}

Two major problems slow down this programme:

Neutrino fluxes

1. Neutrino beams, contrary to electron beams, are **not mono-energetic.** One needs to model all reactions channels in the broad energy band covered by the neutrino beam.
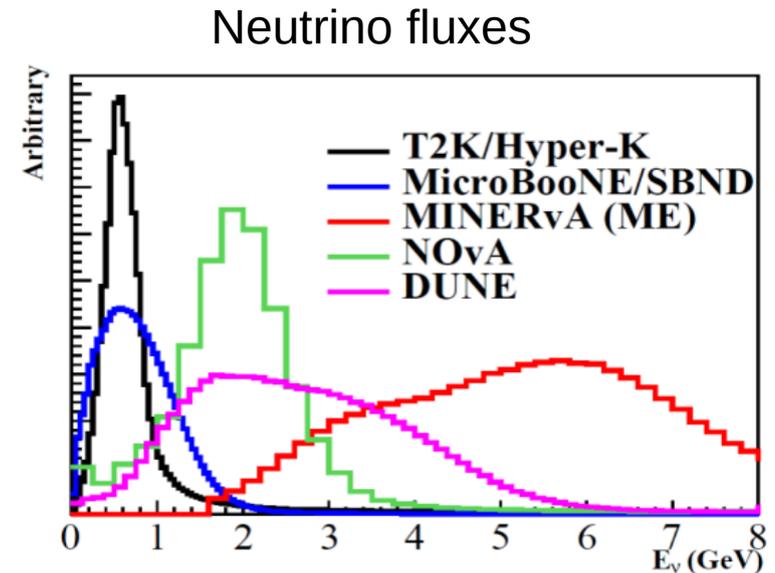
2. "Complex" nuclear targets (compared to the 'easy-to-model' H_2 target).

The necessity to accumulate huge amounts of detector material makes it impossible to use the (explosive!) hydrogen target. The use of other molecules, made of more complex nuclei, such as mineral oils (CH_x)**, water,** or liquid **argon**, allows for the accumulation of tons of detector material, thereby notably increasing the number of events in the detectors.
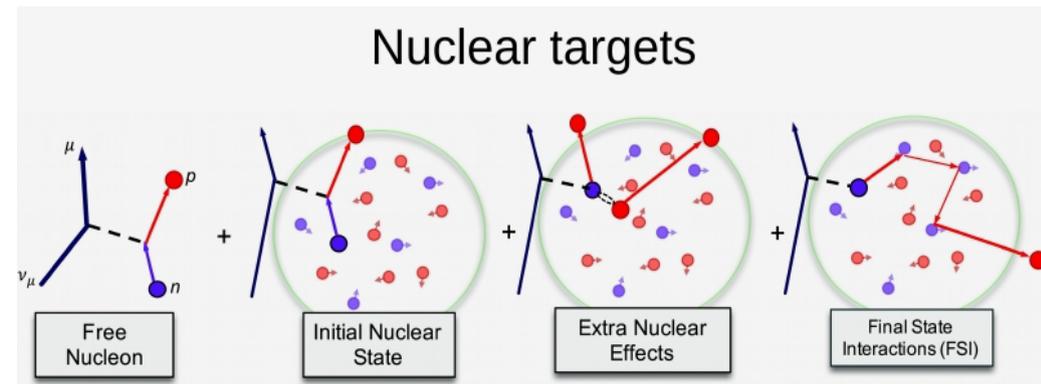
plot by C. Wilkinson (NUISANCE)

42

Determining the oscillation probability

Number of events

$$\nu_\alpha \to \nu_\beta$$

$$N_\beta (E_\nu ?) \sim \Phi_{\nu_\alpha}(E_\nu)\, \sigma_{\nu_\beta}(E_\nu)\, \varepsilon_{det.}\, P_{\nu_\alpha \to \nu_\beta}(\{\Theta\}, E_\nu)$$

| ν flux | ν cross section | Detector efficiency | ν Energy in the oscillation probability |

1) 'Number of event' corresponding to a given topology are measured in the detector (event distribution).

2) The energy of the incident neutrino that caused that event is unknown.

3) **This neutrino energy is reconstructed** combining theoretical information (cross sections) and experimental data.
The reconstruction procedure introduce important uncertainties in the oscillation analyses.

4) The probability $P_{\alpha \to \beta}$ is determined for each neutrino energy.

43

What we know from (e,e')

One needs to model all neutrino-nucleus reaction channels covered by the neutrino beam
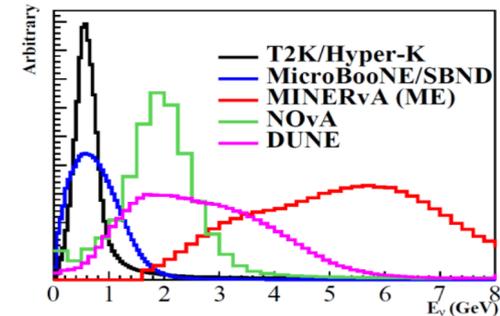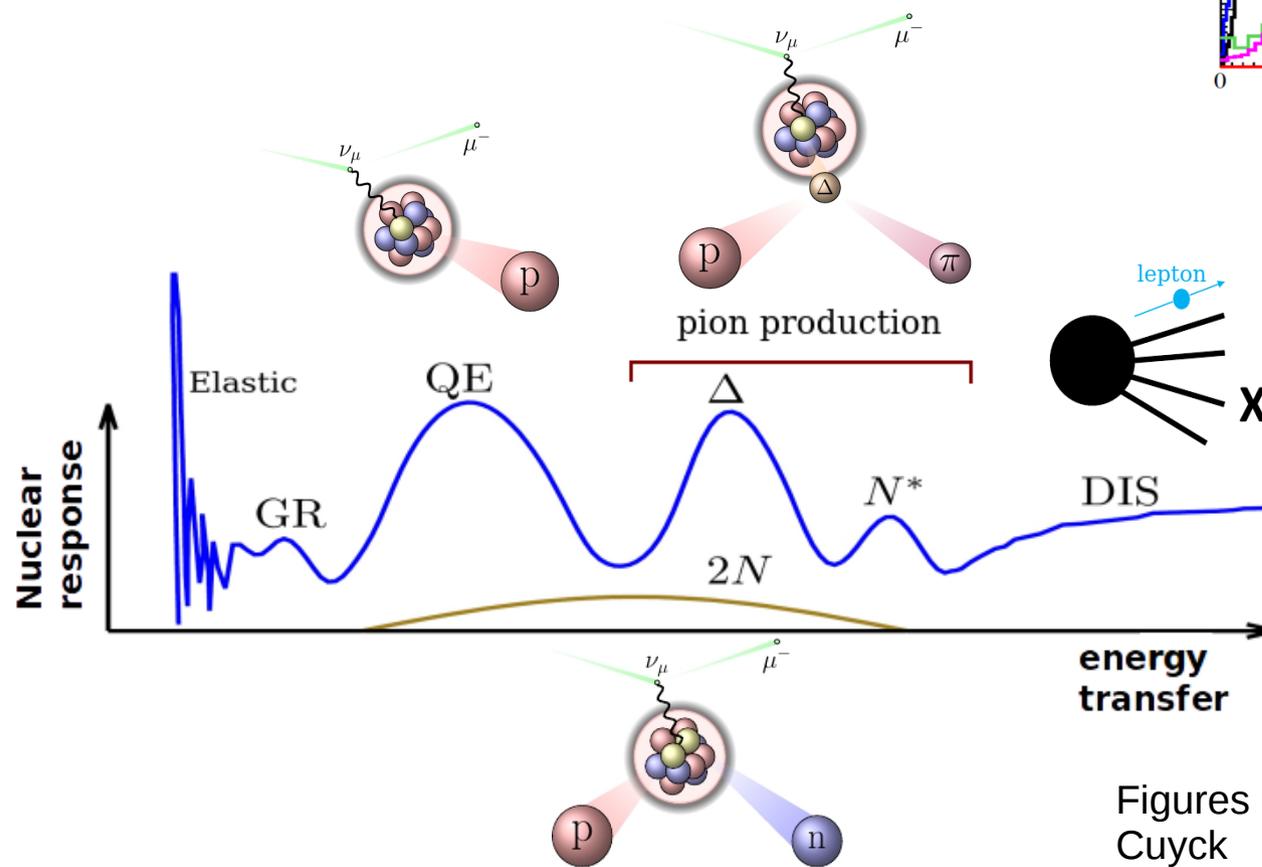
Figures by T. Van Cuyck

44

What we know from (e,e')

Superscaling approach (Megias et al., Phys. Rev. D 91, 073004 (2015))
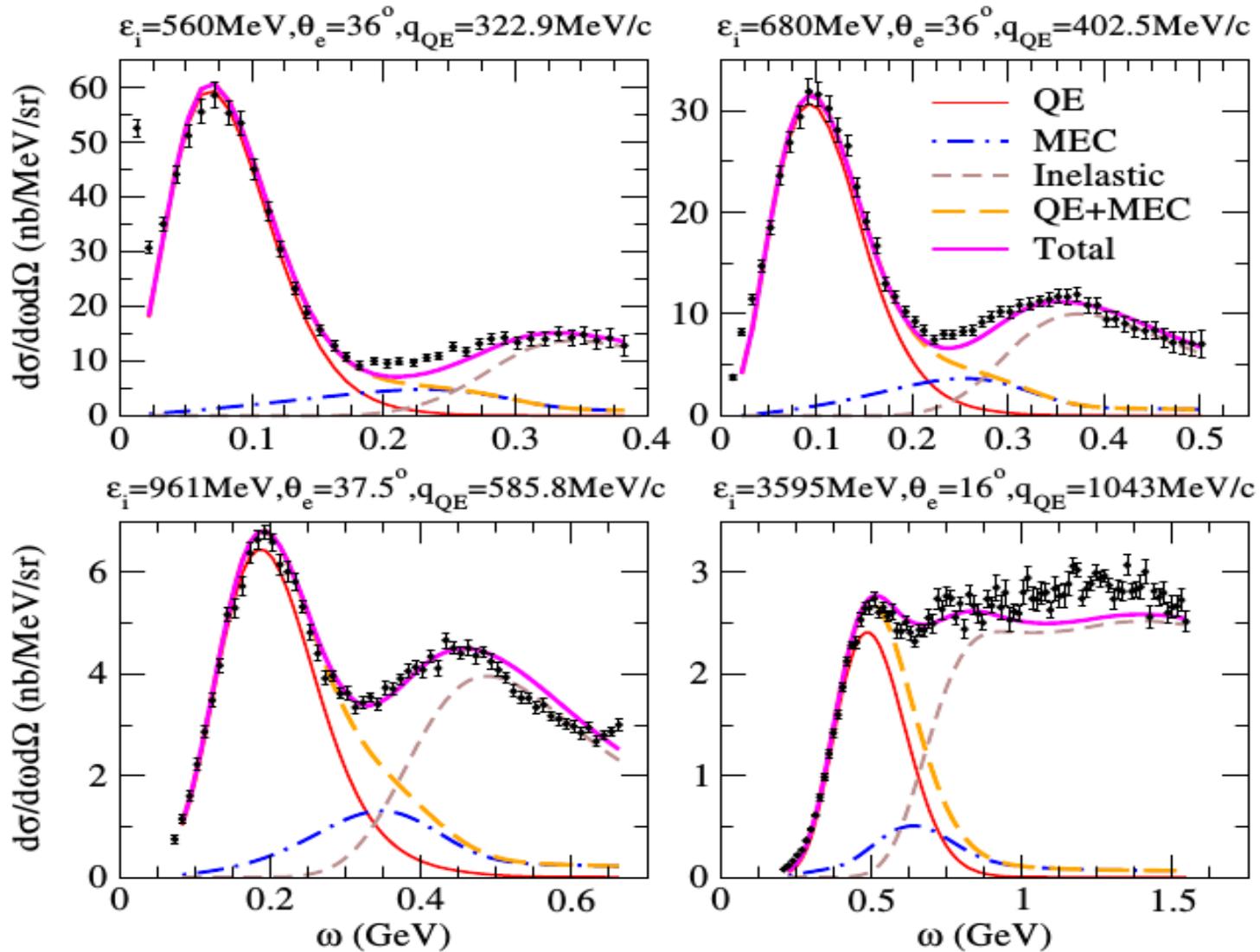
^{12}C(e,e')

What we know from (e,e')
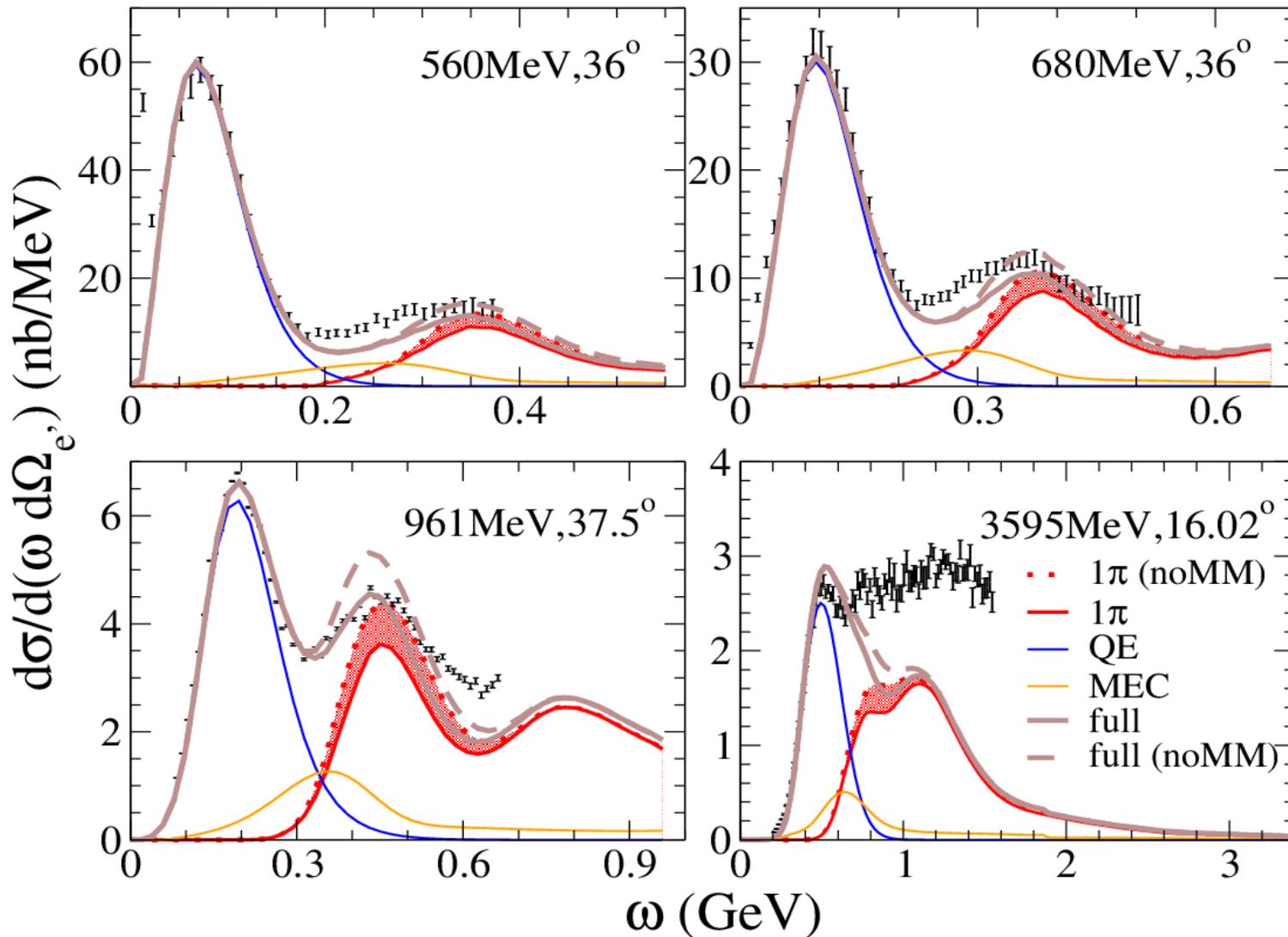
^{12}C(e,e')

QE (SuSAv2):
RGJ et al., PRC 90,
035501 (2014)

MEC: Megias et al.,
PRD 91, 073004
(2015)

1pion: RGJ et al.,
PRD 95, 113007
(2017);
arXiv:1710.08374

Quasielastic scattering
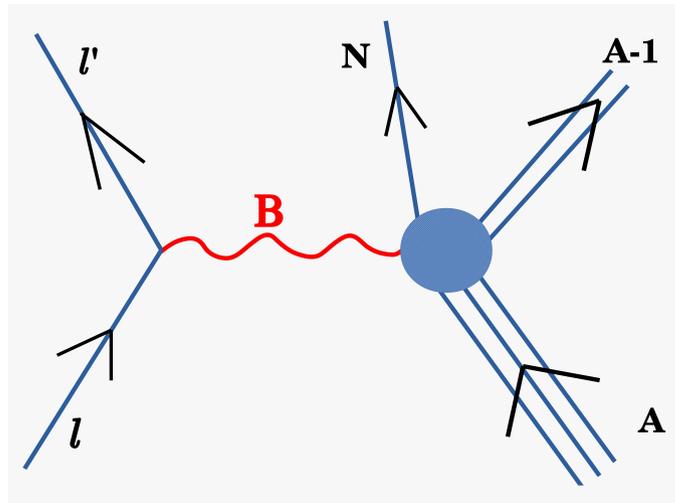
Impulse approximation

$$J^\mu_{had} = \langle N, A-1 | \hat{\mathcal{O}}^\mu_{many\text{-}body} | A \rangle$$

Impulse Approximation

$$J^\mu_{had} = \sum_i^A \int d\mathbf{r}\, \overline{\Psi}_F(\mathbf{r})\, \hat{\mathcal{O}}^\mu_{one\text{-}body}\, \Psi_B(\mathbf{r})\, e^{i\mathbf{q}\cdot\mathbf{r}}$$

where $\qquad \hat{\mathcal{O}}^\mu_{one\text{-}body} = F_1 \gamma^\mu + i \dfrac{F_2}{2M_N} \sigma^{\mu\alpha} Q_\alpha$

48
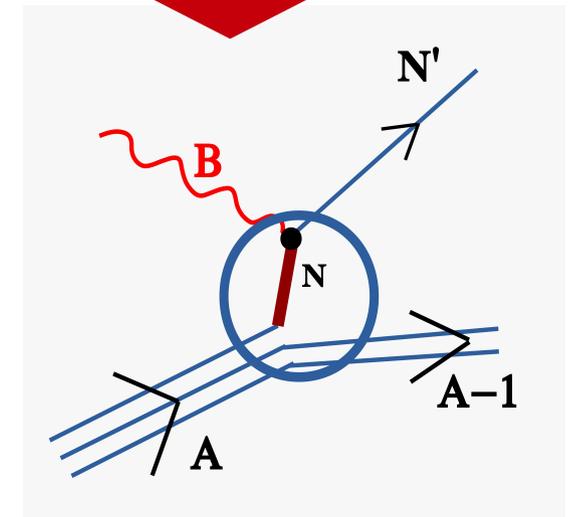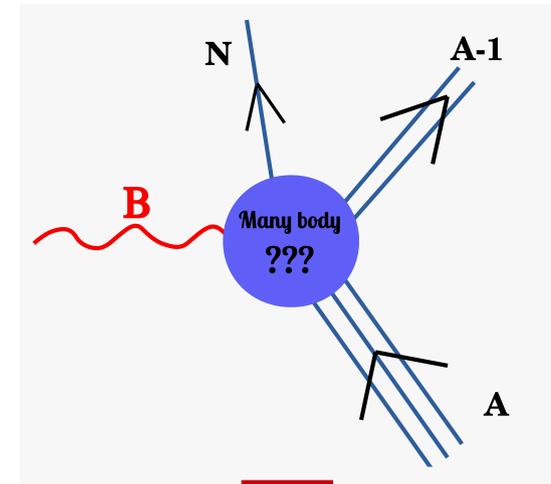
Impulse approximation

$$J^\mu_{had} = \langle N, A - 1 | \hat{\mathcal{O}}^\mu_{many\text{-}body} | A \rangle$$

Impulse Approximation

Relativistic mean-field wave functions

$$J^\mu_{had} = \sum_i^A \int d\mathbf{r}\, \overline{\Psi}_F(\mathbf{r})\, \hat{\mathcal{O}}^\mu_{one\text{-}body}\, \Psi_B(\mathbf{r})\, e^{i\mathbf{q}\cdot\mathbf{r}}$$

where $\quad \hat{\mathcal{O}}^\mu_{one\text{-}body} = F_1 \gamma^\mu + i\dfrac{F_2}{2M_N}\sigma^{\mu\alpha}Q_\alpha$

49
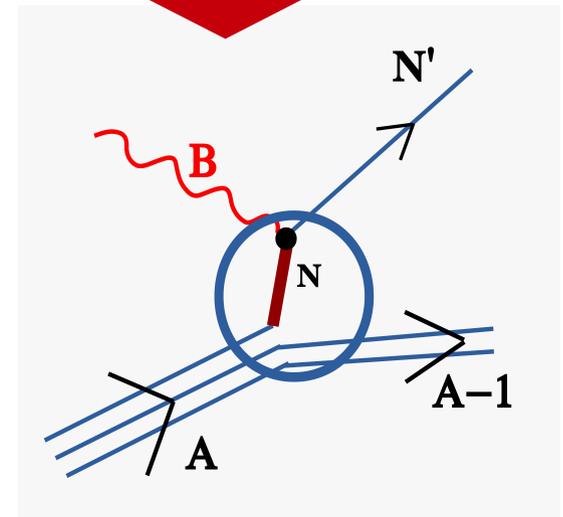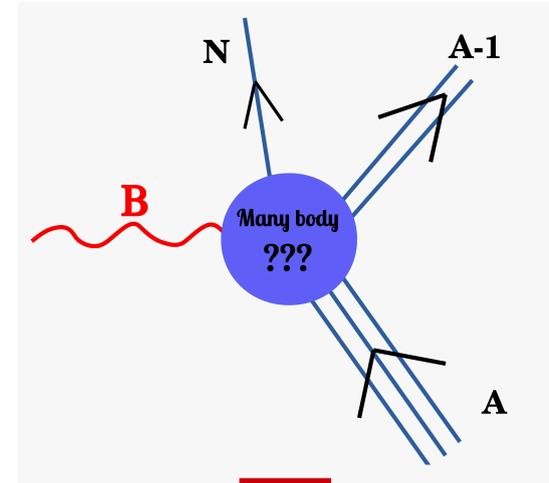
Mean-field vs plane waves

Intermediate energies (typical QE regime)
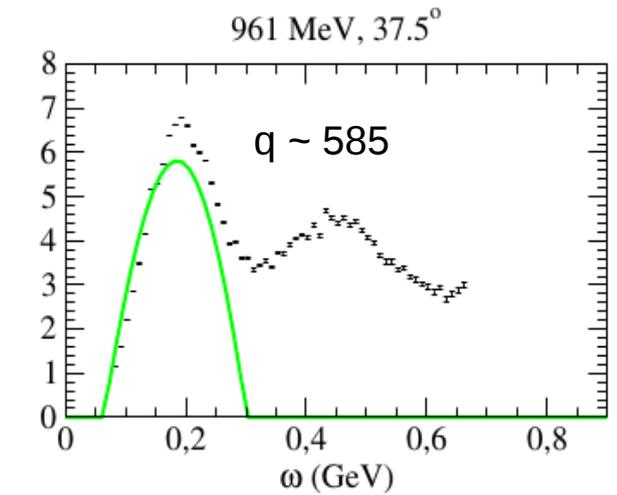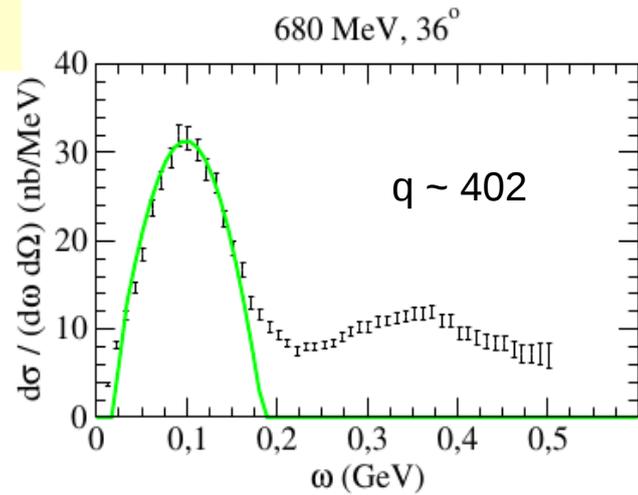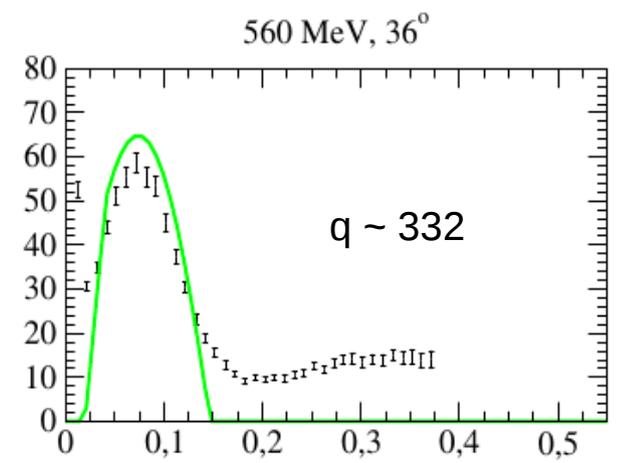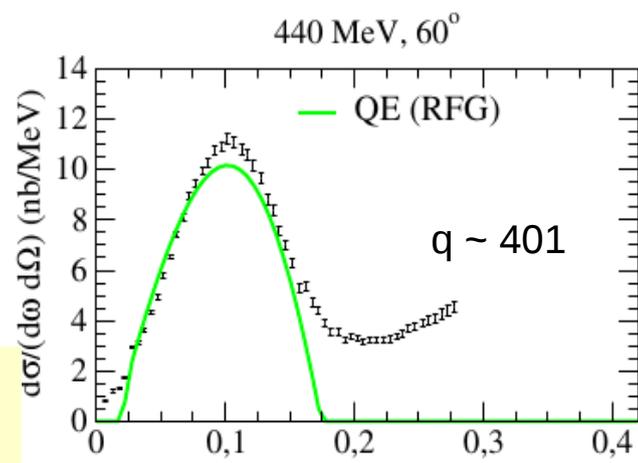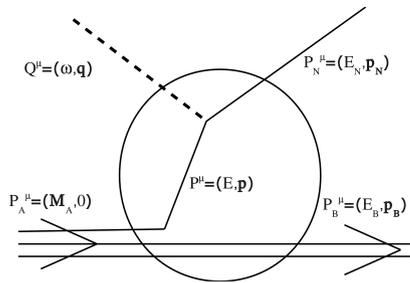
440 MeV, 60° — QE (RFG) — q ~ 401

560 MeV, 36° — q ~ 332

680 MeV, 36° — q ~ 402

961 MeV, 37.5° — q ~ 585

$d\sigma/(d\omega\, d\Omega)$ (nb/MeV)

ω (GeV)

50

Mean-field vs plane waves

$$[-i\boldsymbol{\alpha} \cdot \boldsymbol{\nabla} + V(r) + \beta(M + S(r))]\Psi_i(\boldsymbol{r}) = E_i\Psi_i(\boldsymbol{r})$$

$$J_{had}^\mu = \sum_i^A \int d\boldsymbol{r}\,\overline{\Psi}_F(\boldsymbol{r})\,\hat{\mathcal{O}}_{one\text{-}body}^\mu\,\Psi_B(\boldsymbol{r})\,e^{i\boldsymbol{q}\cdot\boldsymbol{r}}$$

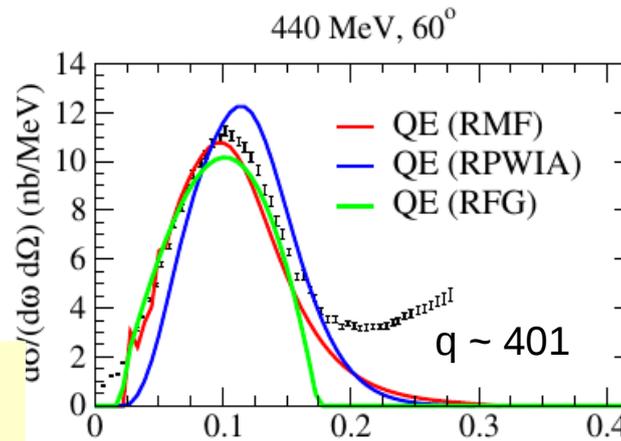**RPWIA**: Scattered nucleon wf is described as a Dirac plane wave.

RMF-FSI: Scattered nucleon wf is solution of Dirac eq. in presence of the same potentials used to describe the bound nucleon wf.

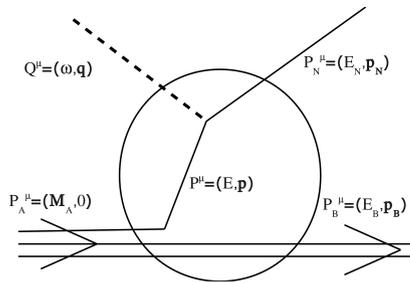Intermediate energies (typical QE regime)

Mean-field vs plane waves

**RPWIA**: Scattered nucleon wf is described as a Dirac plane wave.
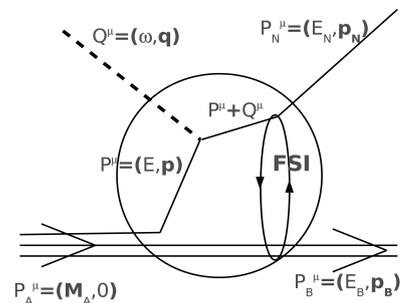
RMF-FSI: Scattered nucleon wf is solution of Dirac eq. in presence of the same potentials used to describe the bound nucleon wf.

$$[-i\boldsymbol{\alpha}\cdot\boldsymbol{\nabla}+V(r)+\beta(M+S(r))]\Psi_i(\boldsymbol{r})=E_i\Psi_i(\boldsymbol{r})$$

$$J_{had}^{\mu}=\sum_i^A\int d\mathbf{r}\,\overline{\Psi}_F(\mathbf{r})\,\hat{\mathcal{O}}_{one\text{-}body}^{\mu}\,\Psi_B(\mathbf{r})\,e^{i\mathbf{q}\cdot\mathbf{r}}$$

Intermediate energies (typical QE regime)
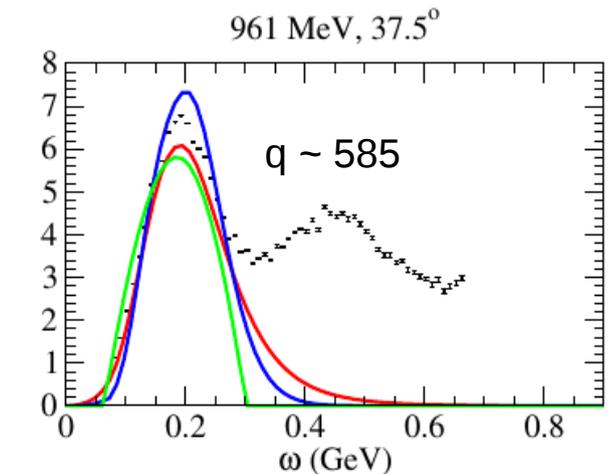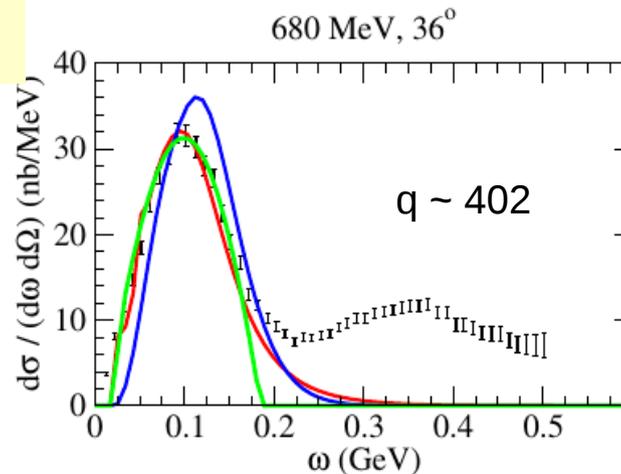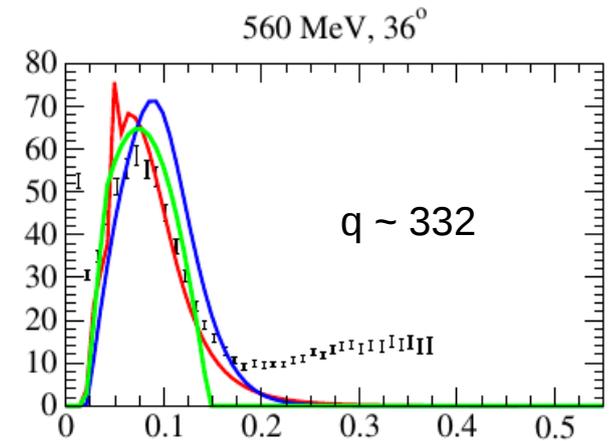
Mean-field vs plane waves

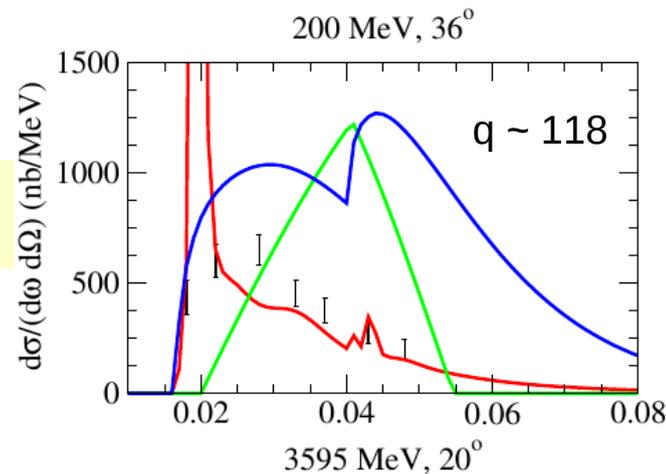**RPWIA**: Scattered nucleon wf is described as a Dirac plane wave.
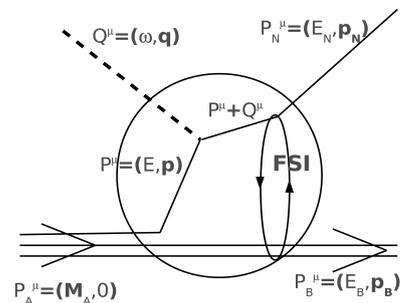
RMF-FSI: Scattered nucleon wf is solution of Dirac eq. in presence of the same potentials used to describe the bound nucleon wf.
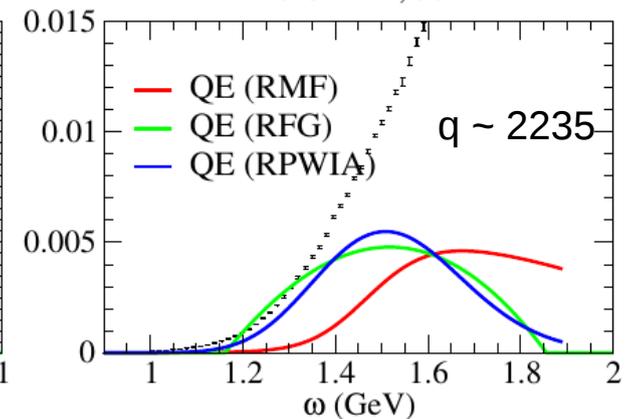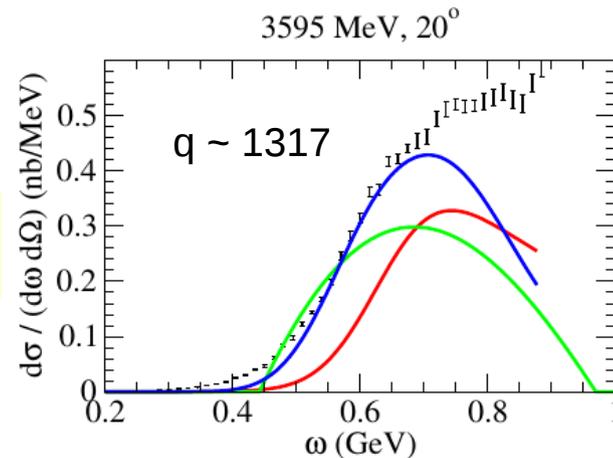
$$[-i\boldsymbol{\alpha}\cdot\boldsymbol{\nabla}+V(r)+\beta(M+S(r))]\Psi_i(\boldsymbol{r})=E_i\Psi_i(\boldsymbol{r})$$

$$J^\mu_{had}=\sum_i^A\int d\boldsymbol{r}\,\overline{\Psi}_F(\boldsymbol{r})\,\hat{\mathcal{O}}^\mu_{one\text{-}body}\,\Psi_B(\boldsymbol{r})\,e^{i\boldsymbol{q}\cdot\boldsymbol{r}}$$
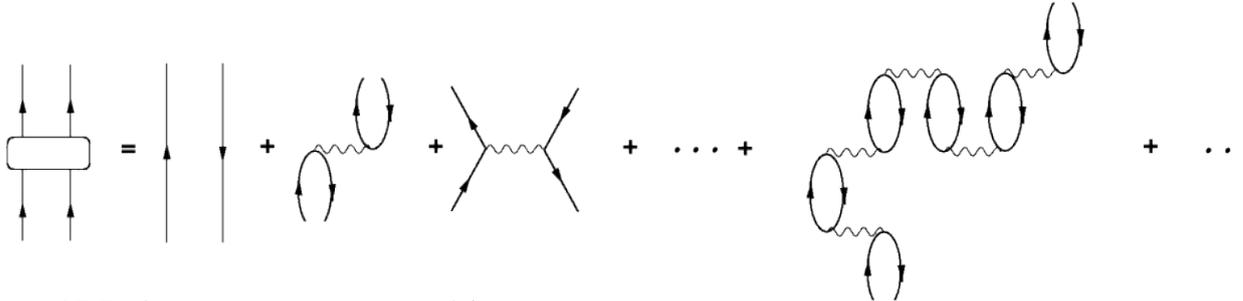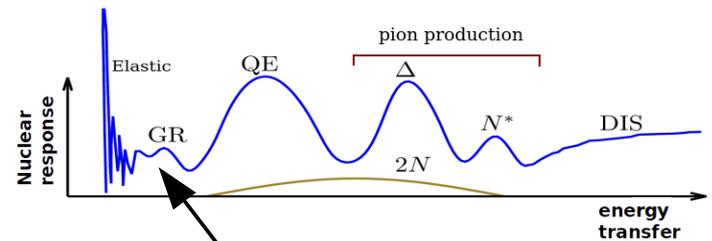
Low energies
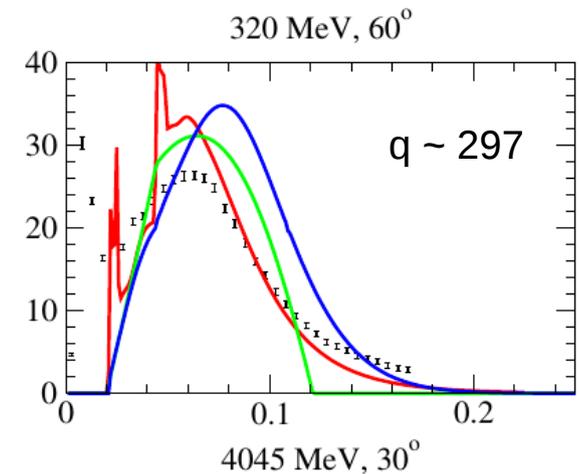
High energies

Long-range correlations: CRPA model

+ Long-range correlations between the nucleons are introduced through a **continuum Random Phase Approximation** (CRPA).

+ RPA equations are solved using a Green's function approach.

Giant Resonance (GR) region: collective nuclear effects

$$\Pi^{(RPA)}(x_1, x_2; \omega) = \Pi^{(0)}(x_1, x_2; \omega)$$
$$+ \frac{1}{\hbar} \int dx \int dx' \; \Pi^{(0)}(x_1, x; \omega) \; \tilde{V}(x, x') \; \Pi^{(RPA)}(x', x_2; \omega)$$
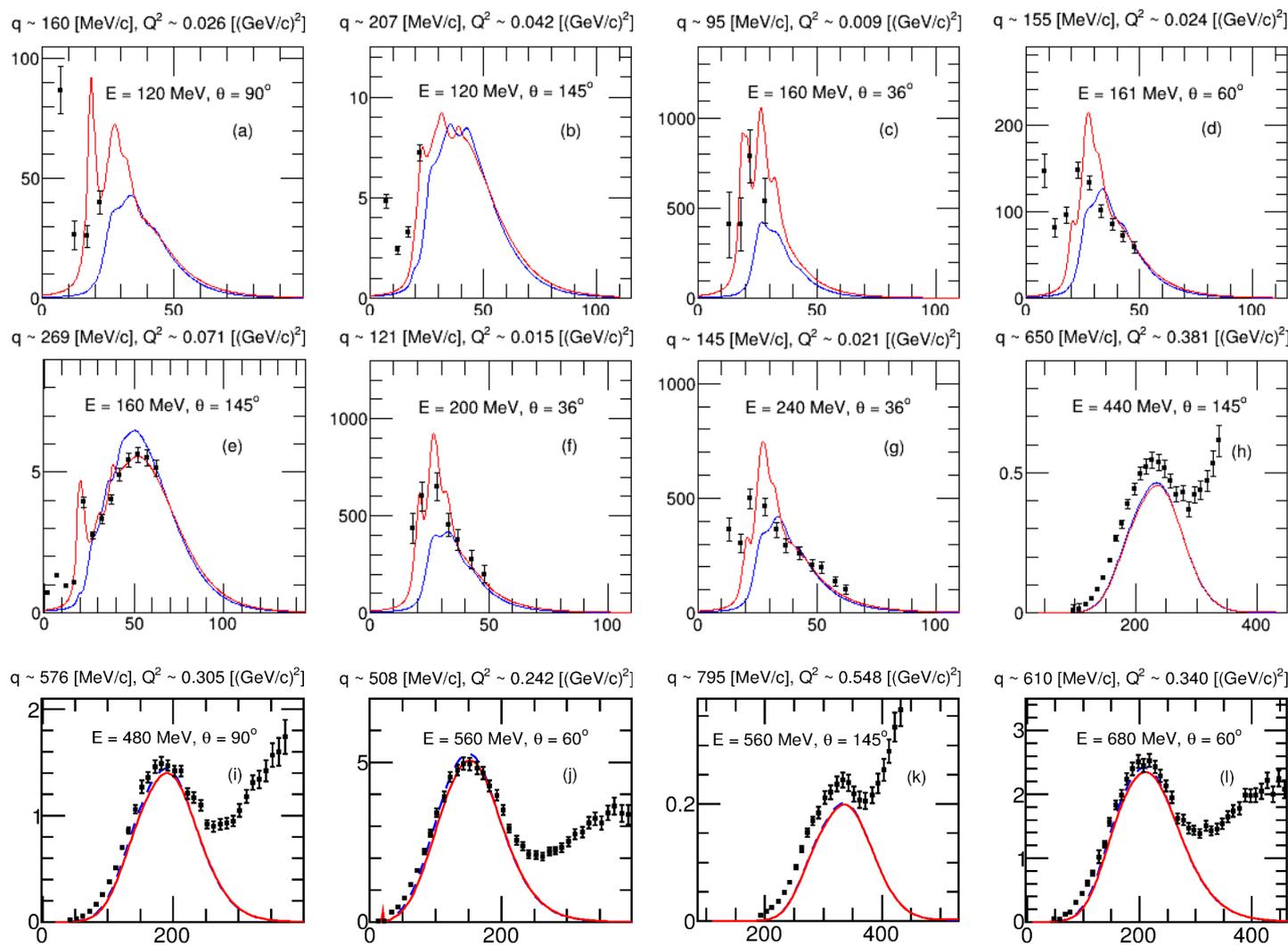
Excitations are obtained as linear combinations of different particle-hole configurations.

$$|\Psi_{RPA}\rangle = \sum_c \left\{ X_{(\Psi, C)} \left| ph^{-1} \right\rangle - Y_{(\Psi, C)} \left| hp^{-1} \right\rangle \right\}$$
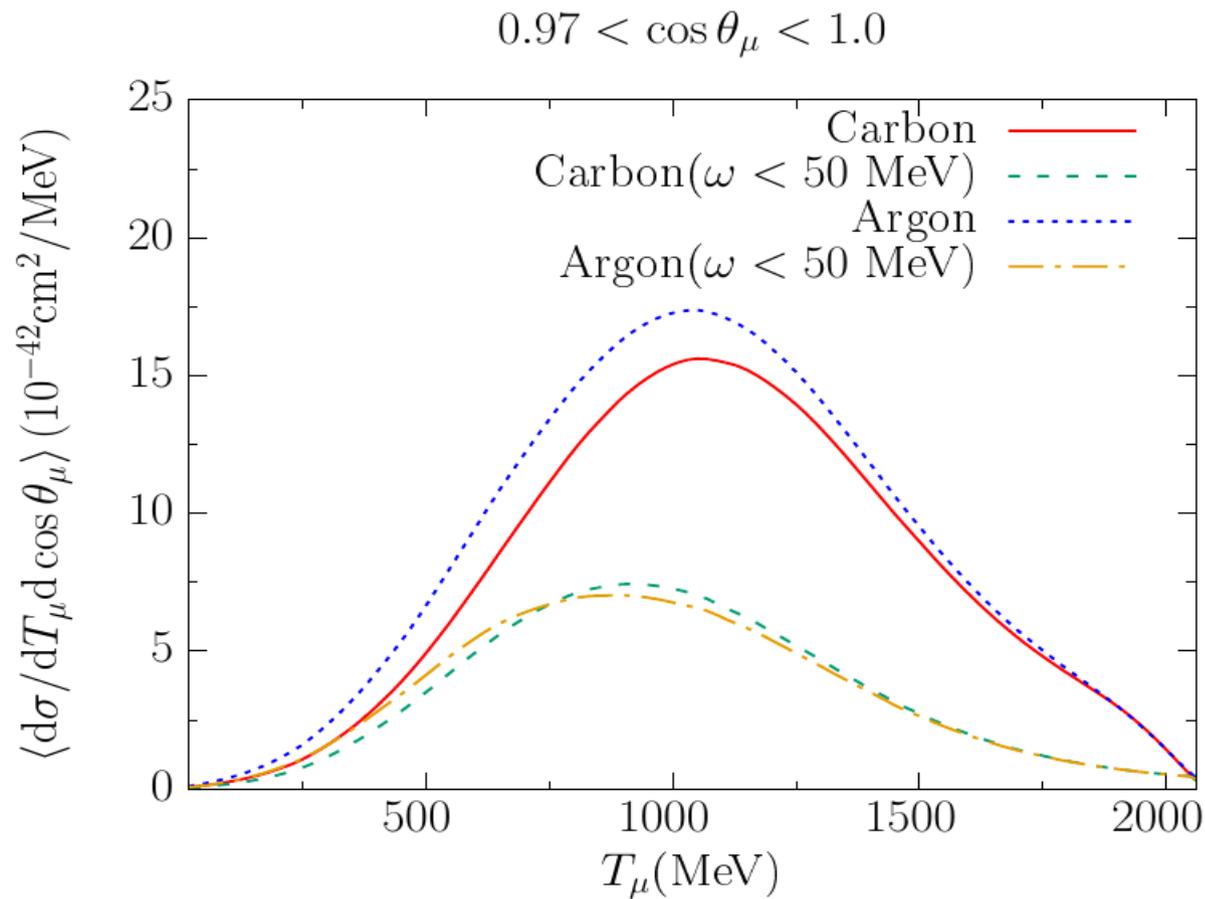
54

Long-range correlations: CRPA model
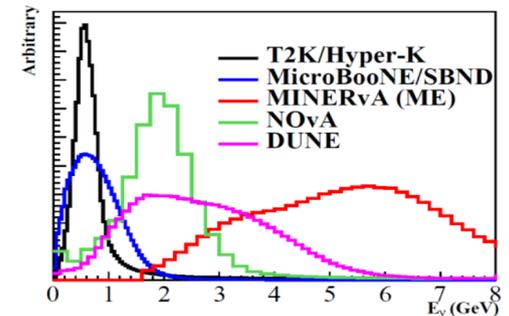
$^{12}C(e, e')$

HF (blue) vs CRPA (red)

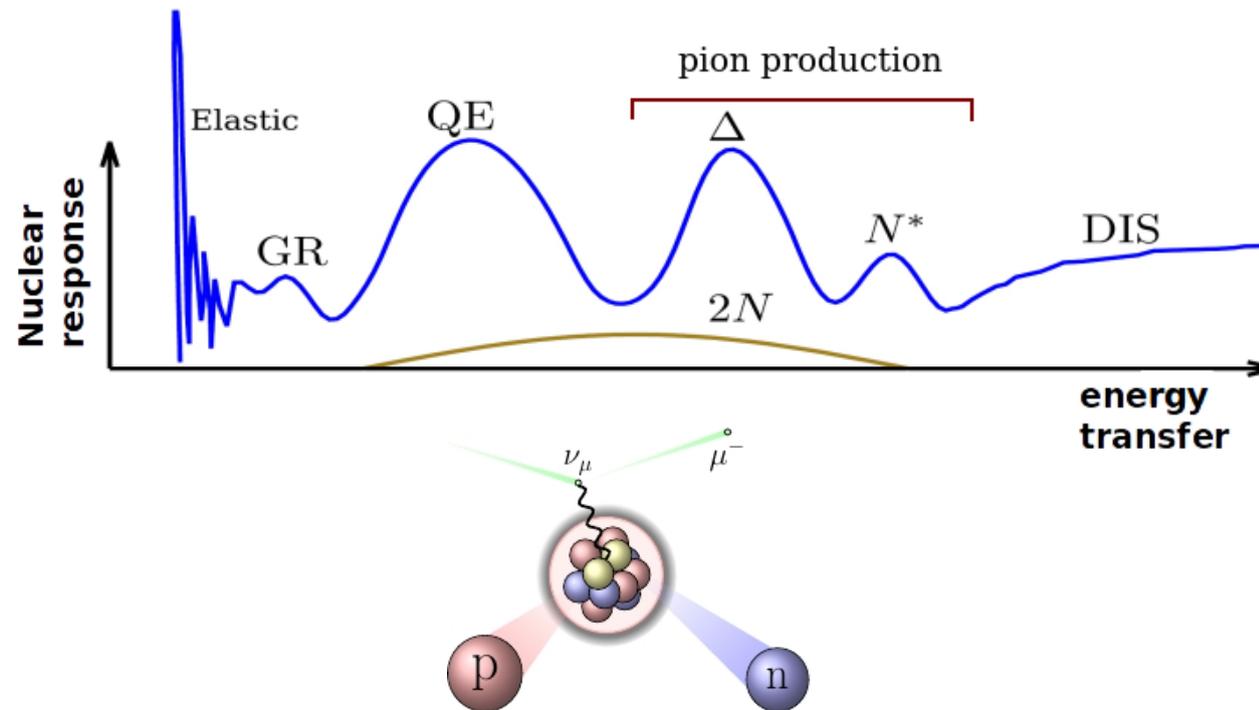[V. Pandey PhD Thesis; PRC 92, 024606 (2015)]

55

Low-energy contributions in flux-folded XS
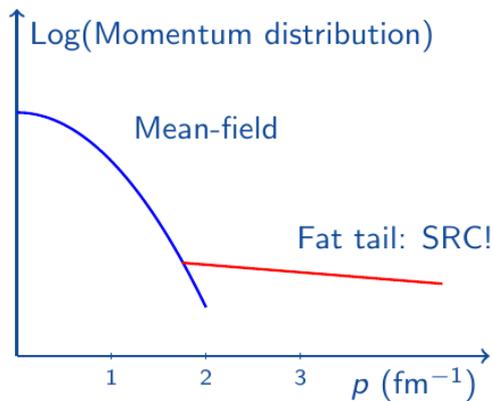

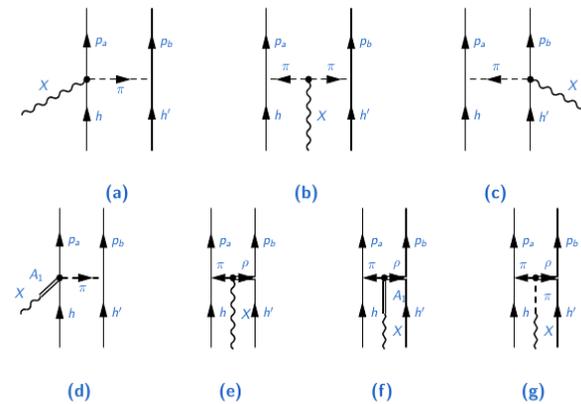MicroBooNE flux-folded cross section.

Two-nucleon knockout processes

Two-nucleon knockout processes

In our approach, two mechanisms give rise to the emission of two nucleons:
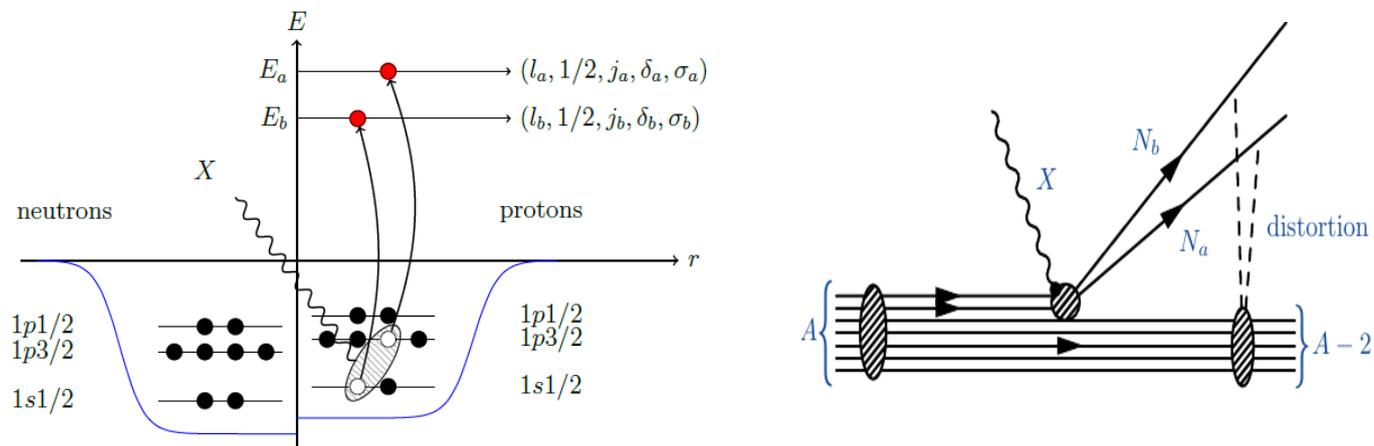
Short-range correlations
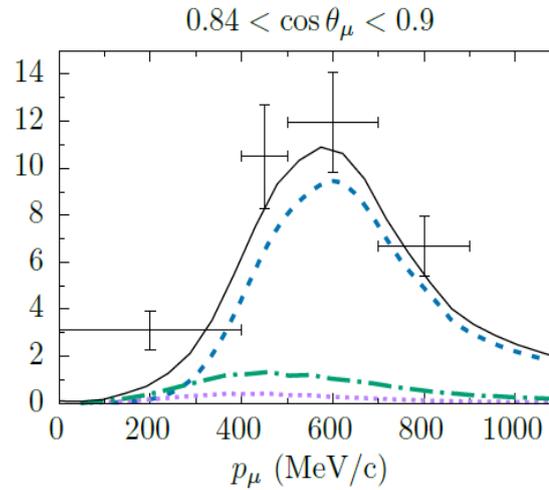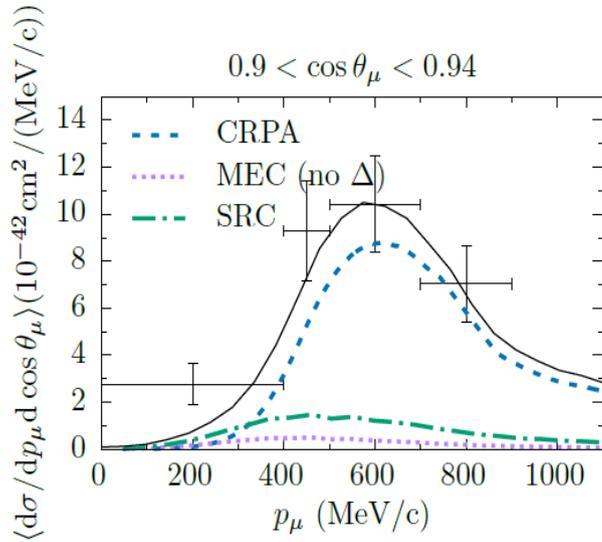
Meson-exchange currents

T. Van Cuyck's PhD Thesis

The same **mean-field** model is used to describe the **bound and scattered nucleons**:

Flux folded xs: MiniBooNE & T2K
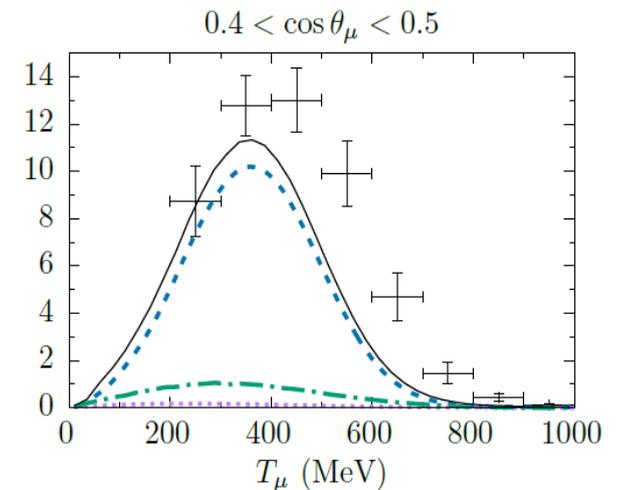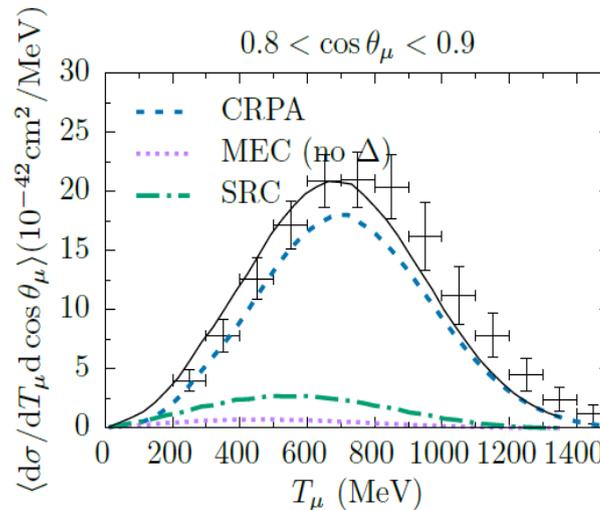

T2K
(inclusive data:
QE, 2p2h, pions, DIS)

Van Cuyck et al., arXiv:1708.03723

MiniBooNE

(QE-like: QE+2p2h)

Meson-exchanged currents

Other approaches (Superscaling coll.) consider MEC as the only contribution to the 2N-nucleon knockout responses. Fully relativistic calculation that includes both vector and axial current contributions.
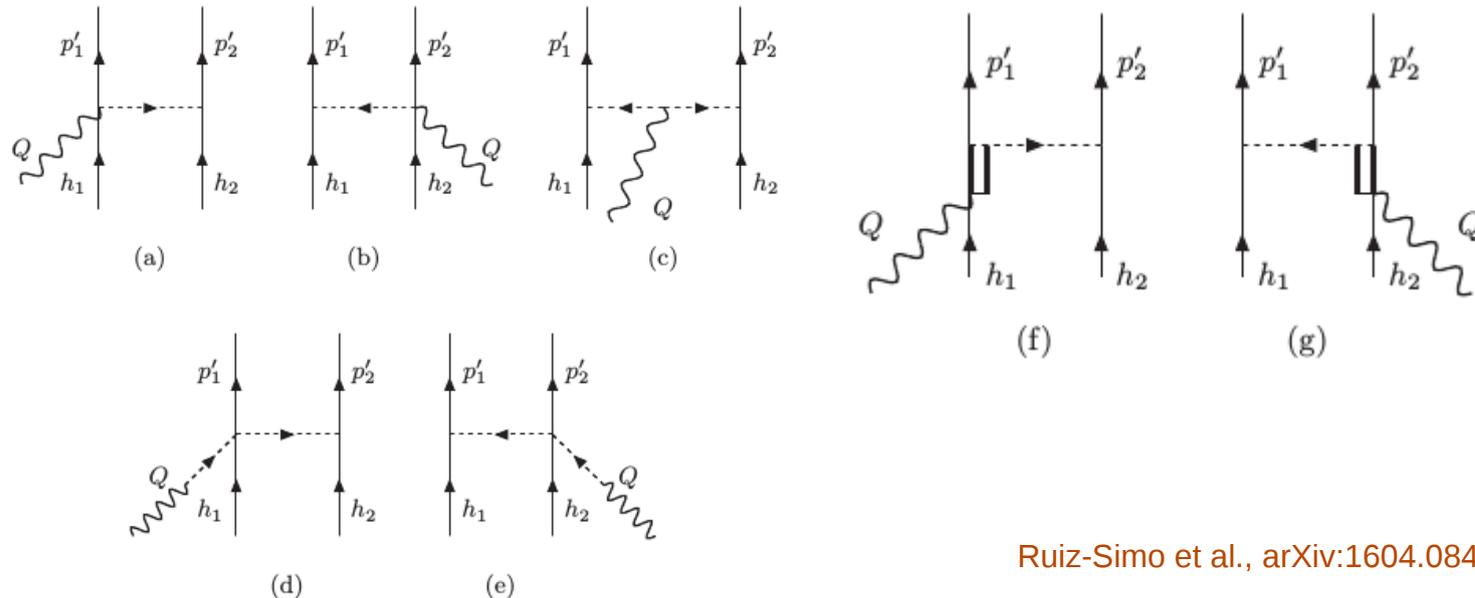
Ruiz-Simo et al., arXiv:1604.08423

Meson-exchanged currents

Other calculations (Superscaling coll.) consider MEC as the only contribution to the 2N-nucleon knockout responses. Fully relativistic calculation that includes both vector and axial current contributions.

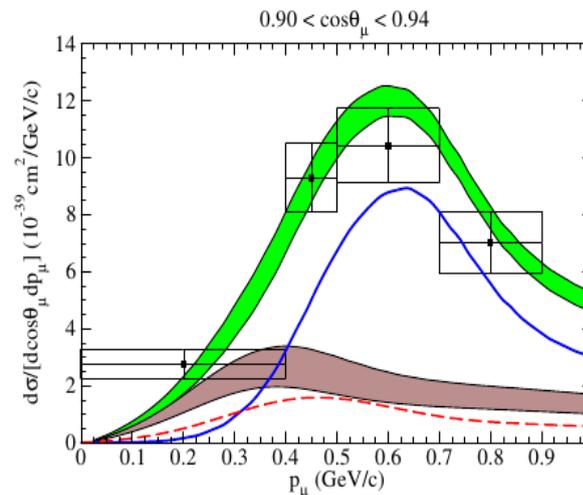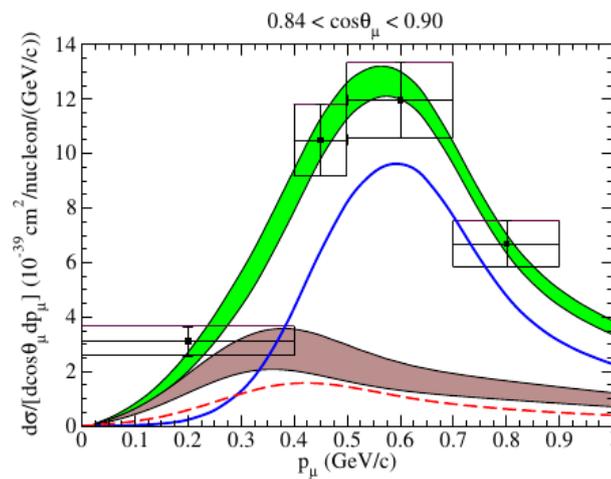Megias et al., arXiv:1607.08565v2

T2K
(inclusive data: QE, 2p2h, pions, DIS)

MiniBooNE

(QE-like: QE+2p2h)

61
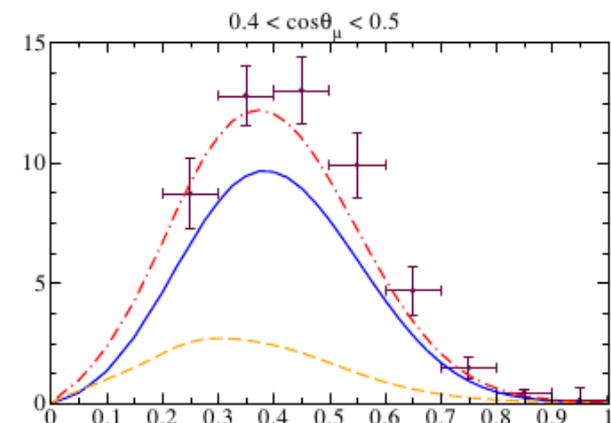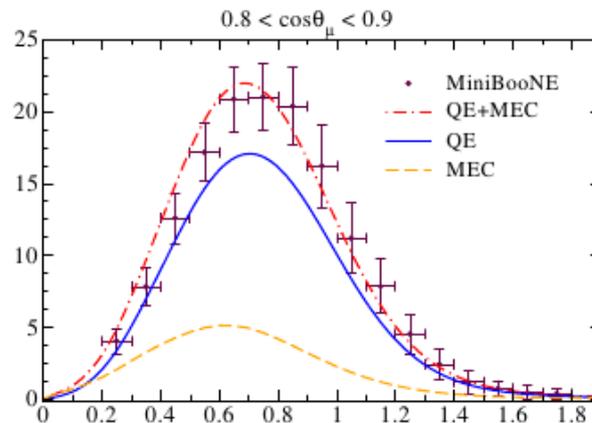
Single-Pion Production on the nucleon
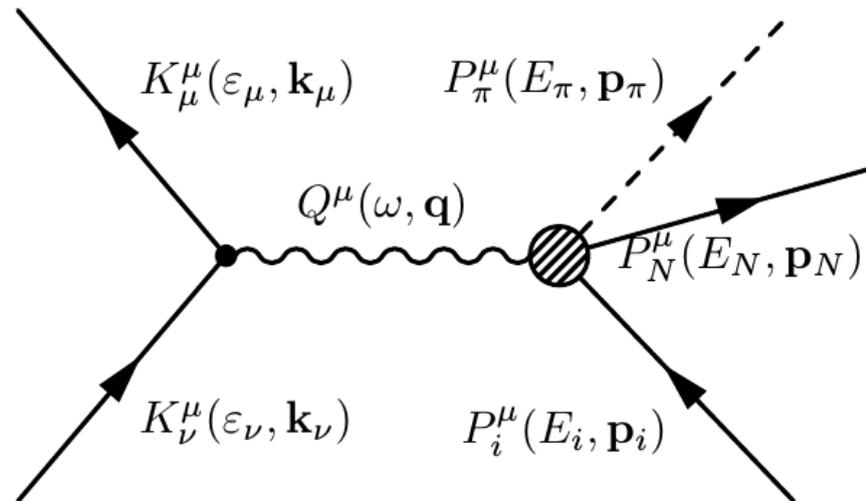
González-Jiménez et al., PRD 95, 113007 (2017)

Single-Pion Production on the nucleon

Low-energy model

$K_\mu^\mu(\varepsilon_\mu, \mathbf{k}_\mu)$ $P_\pi^\mu(E_\pi, \mathbf{p}_\pi)$

$Q^\mu(\omega, \mathbf{q})$

$P_N^\mu(E_N, \mathbf{p}_N)$

$K_\nu^\mu(\varepsilon_\nu, \mathbf{k}_\nu)$ $P_i^\mu(E_i, \mathbf{p}_i)$
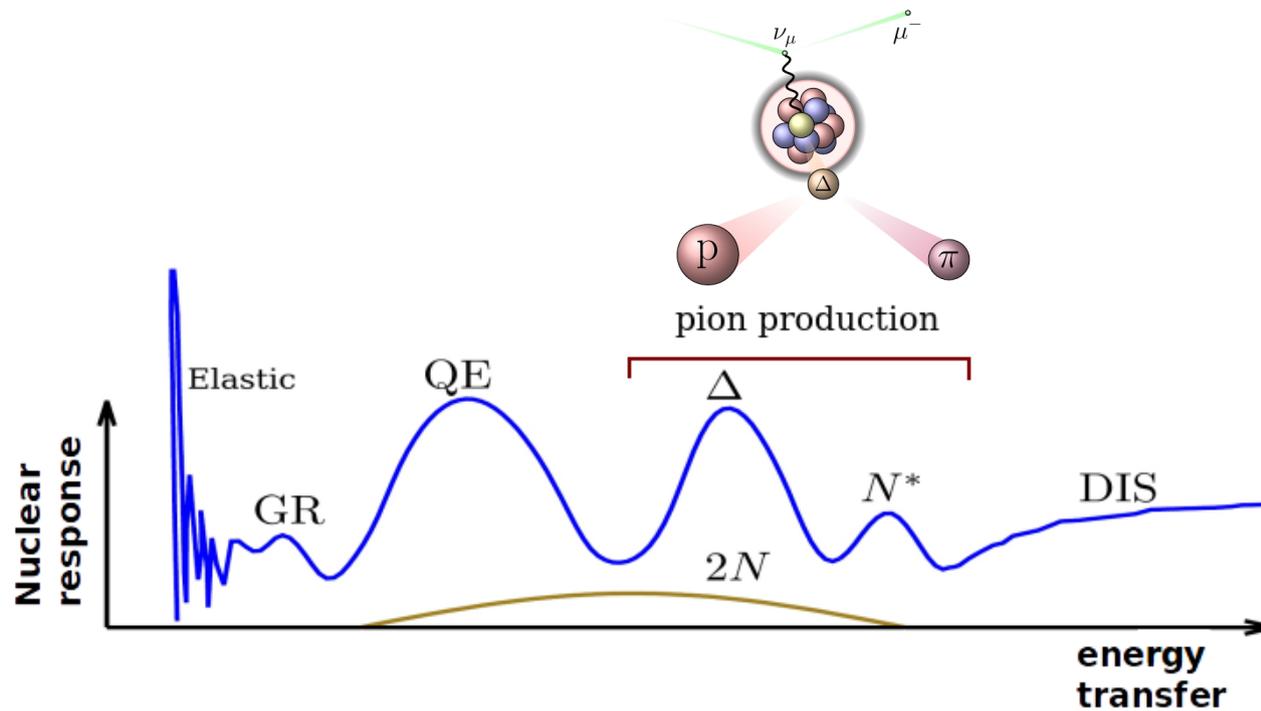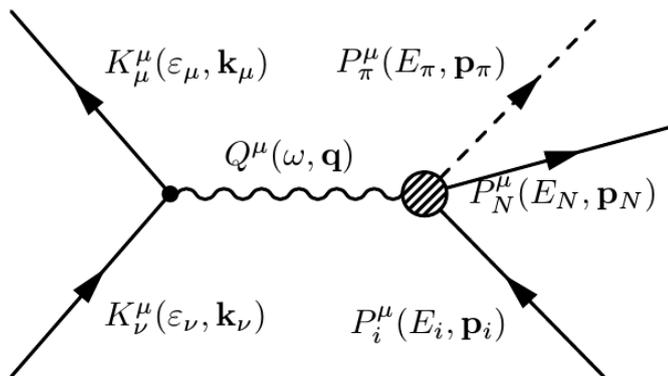
Low-energy model for pion-production on the nucleon:
ChPT background + resonances
Valencia model
(PRD 76 (2007) 033005; PRD 87 (2013) 113009)

Resonances:
P33(1232), D13(1520),
S11(1535), P11(1440)

ChPT background:

64
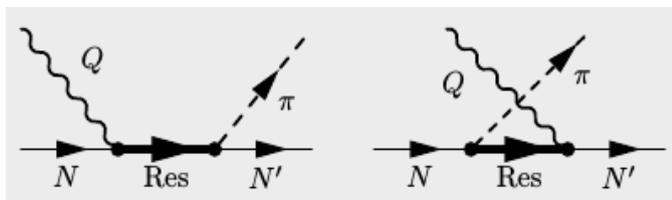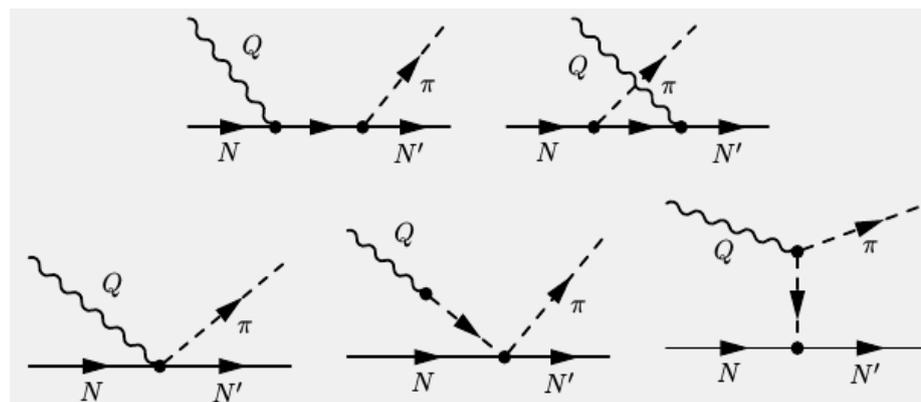
The Problem

Low-energy model
(resonances + ChPT bg)
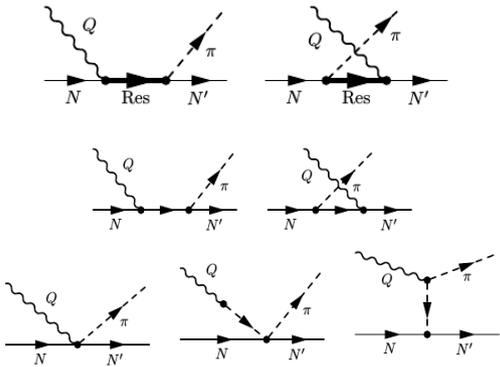
Unphysical predictions at large invariant masses.
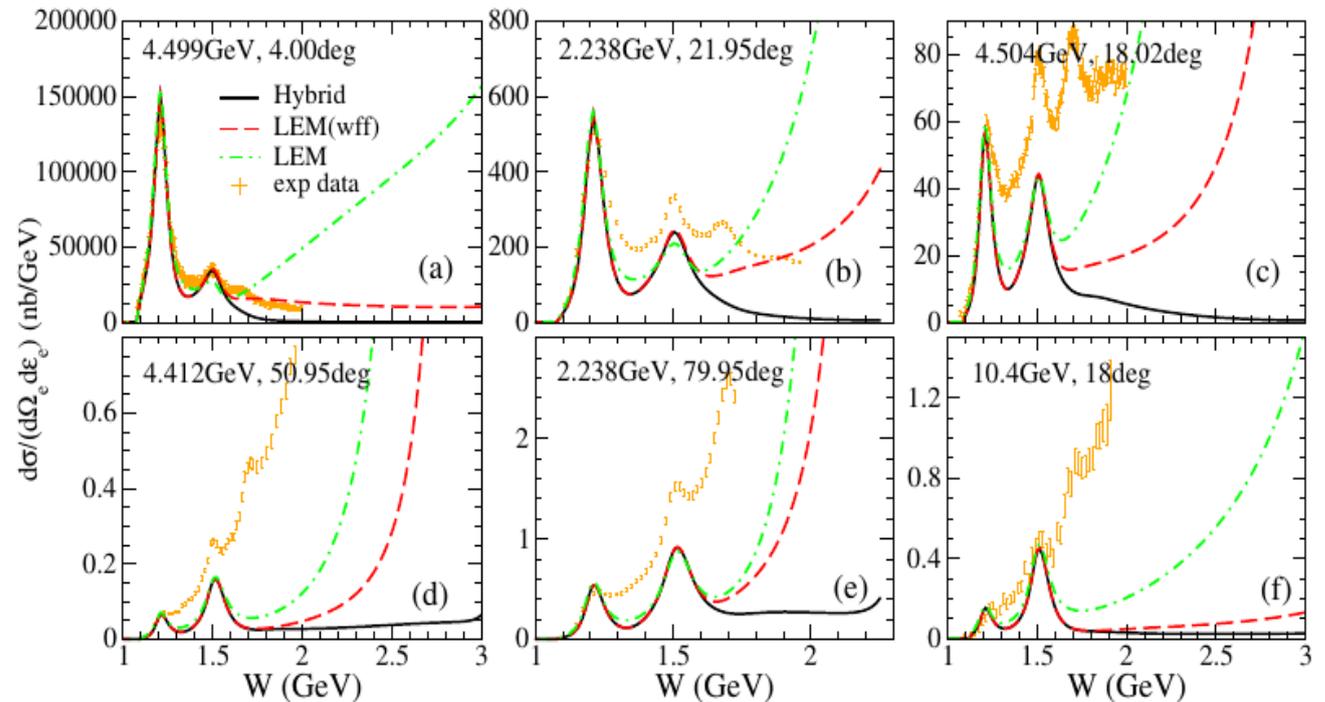
Figure: The model overshoots **inclusive electron-proton** scattering data.

RGJ et al., PRD 95, 113007 (2017)

The Problem
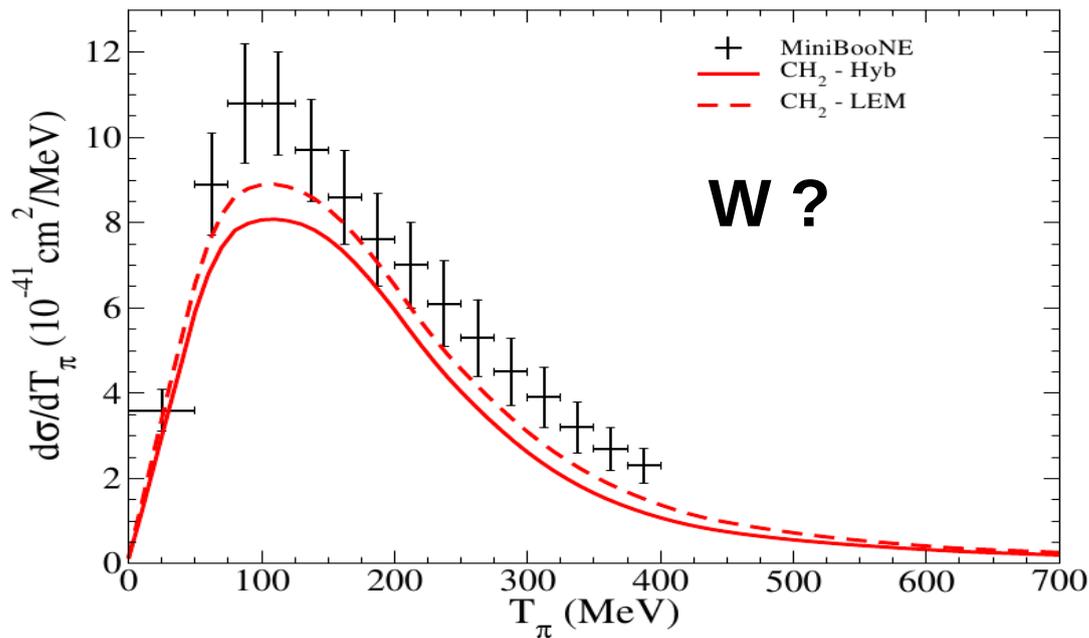
W values? We don't know...
+ Fermi motion
+ Flux-folding

Therefore, we need reliable predictions in:

+ the **resonance region** W < 2 GeV,

+ the **high-energy** energy region W > 2 GeV

66

Hybrid model: results

W < 1.4 GeV
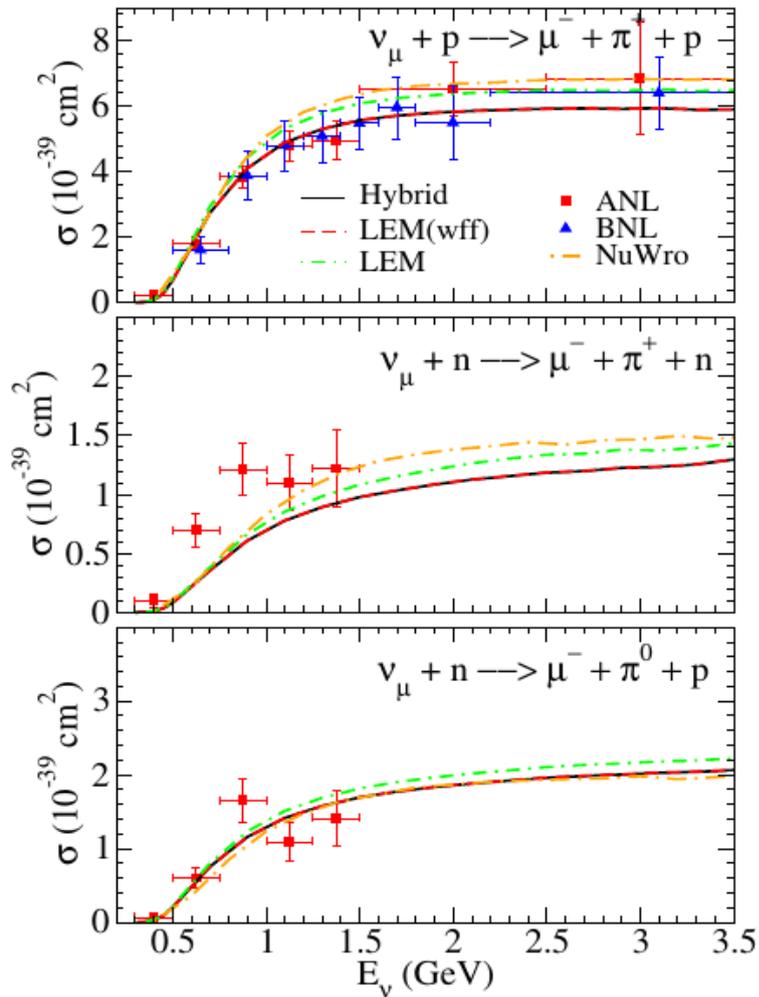
No cut in W
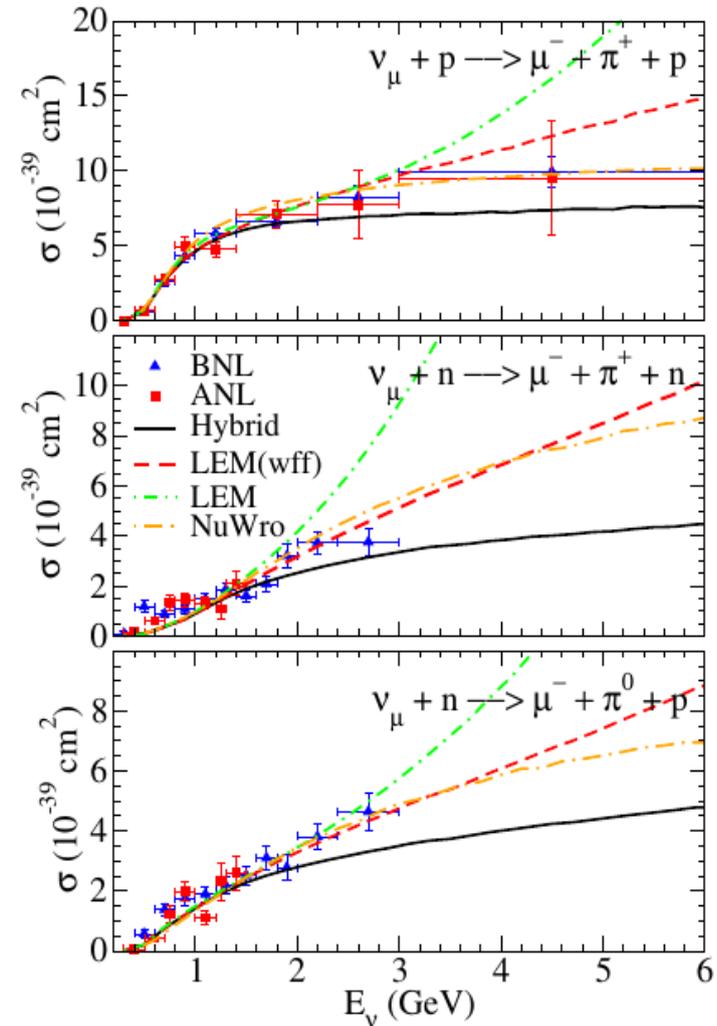
Electroweak one-pion production on nuclei
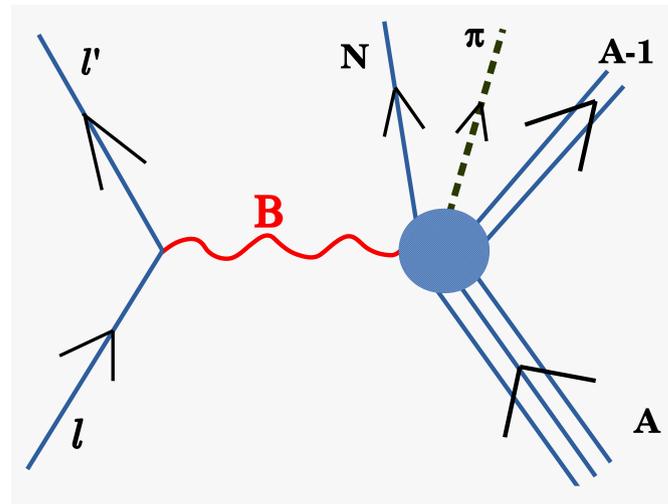
RGJ et al., arXiv:1710.08374 [nucl-th]

Relativistic mean field model
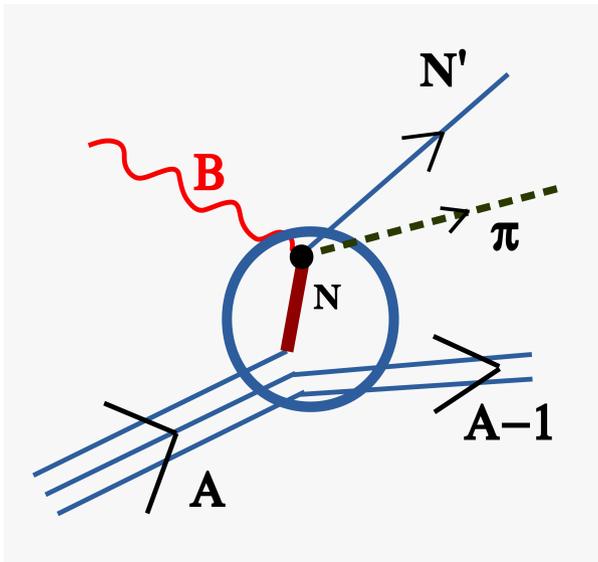
$$J^{\mu}_{had} = \sum_{i}^{A} \int d\mathbf{r}\, \overline{\Psi}_F(\mathbf{r})\, \phi^*(\mathbf{r})\, \hat{\mathcal{O}}^{\mu}_{one\text{-}body}(\mathbf{r})\, \Psi_B(\mathbf{r})\, e^{i\mathbf{q}\cdot\mathbf{r}}$$

$$\frac{d^8\sigma}{d\varepsilon_f d\Omega_f dE_\pi d\Omega_\pi d\Omega_N} = \frac{m_i m_f}{(2\pi)^8} \frac{M_N\, p_N\, k_\pi}{E_N\, f_{rec}} \frac{k_f}{\varepsilon_i} \sum_{fi} \overline{|\mathcal{M}_{fi}|^2}$$

8-fold differential cross section: Computationally very demanding
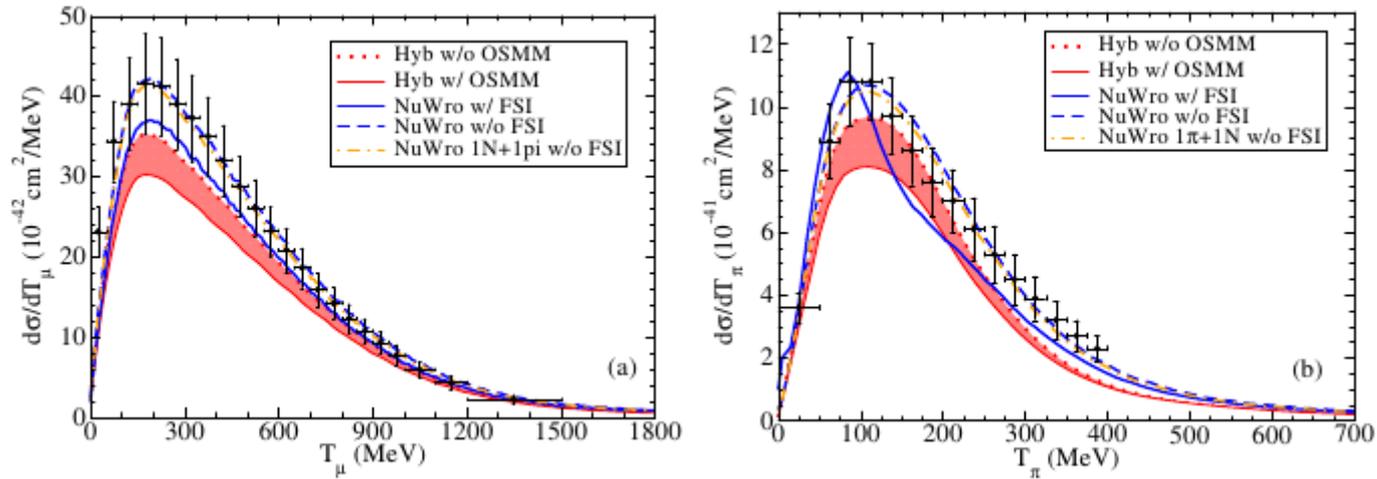
69

MiniBooNE neutrino CC 1pion+.

MINERvA antineutrino CC 1pion0.
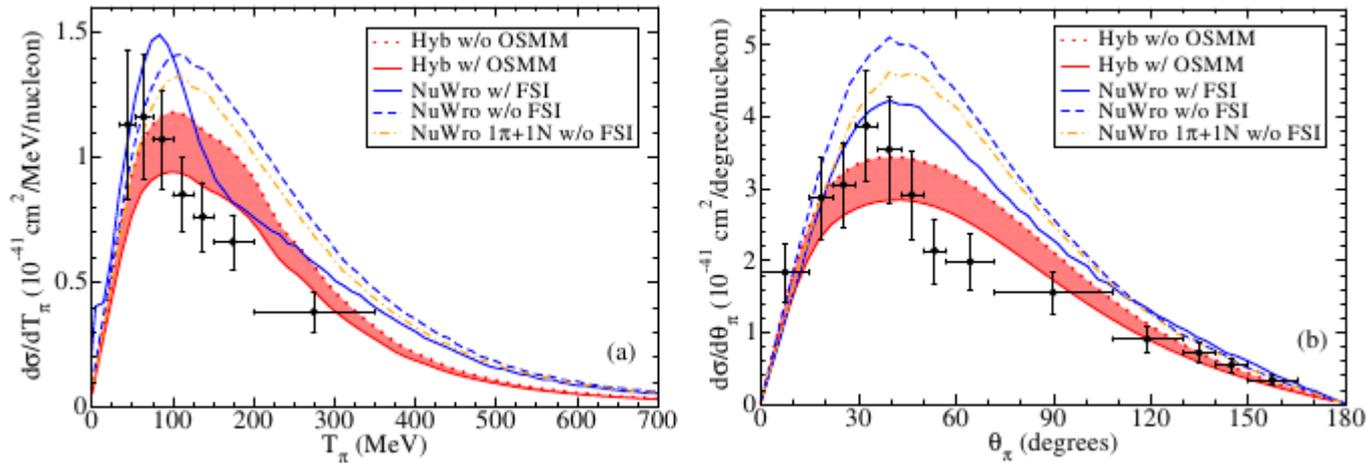
RGJ et al., arXiv:1710.08374 [nucl-th]

70

MiniBooNE neutrino CC 1pion0.
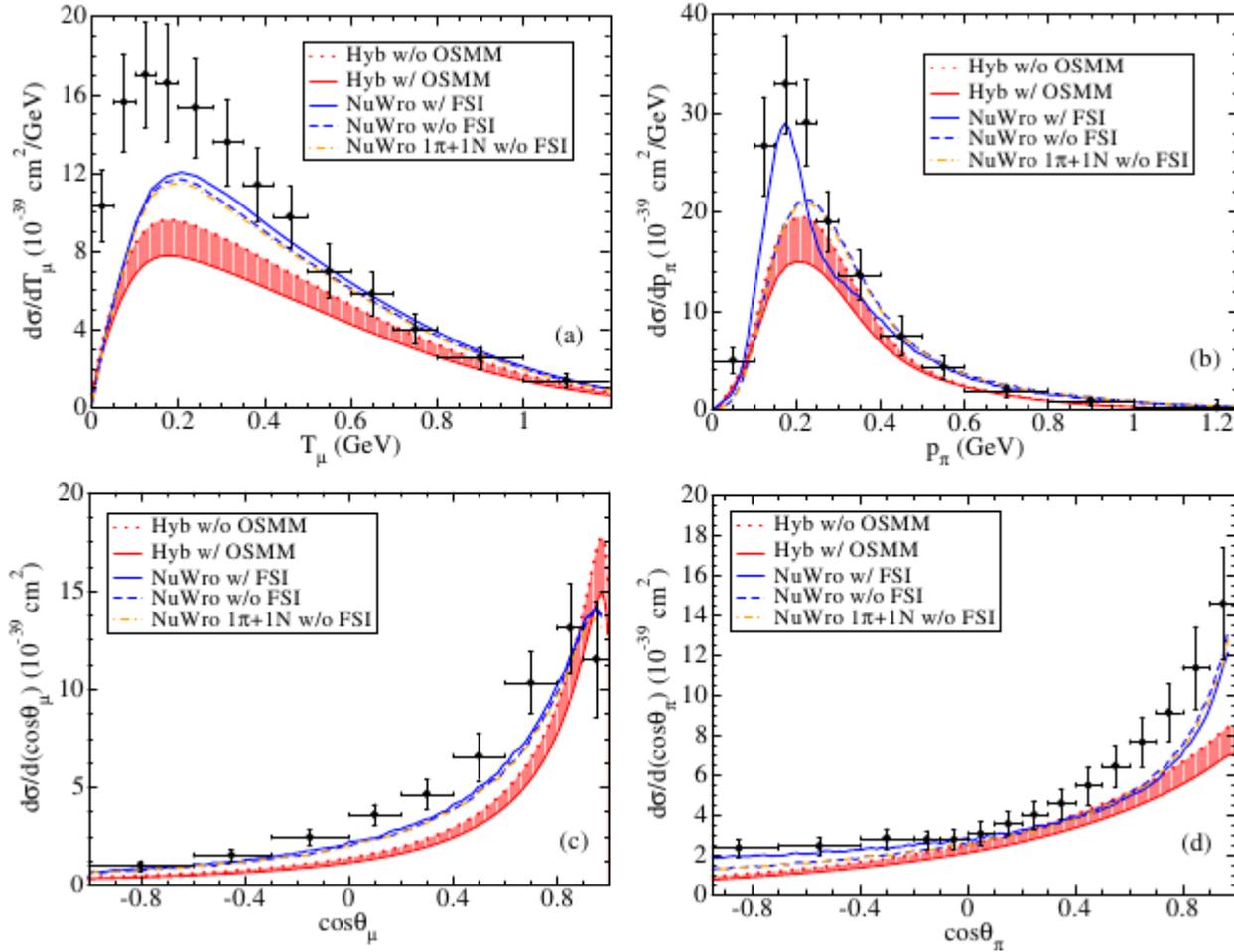
RGJ et al., arXiv:1710.08374 [nucl-th]

MINERvA neutrino CC 1pion0.
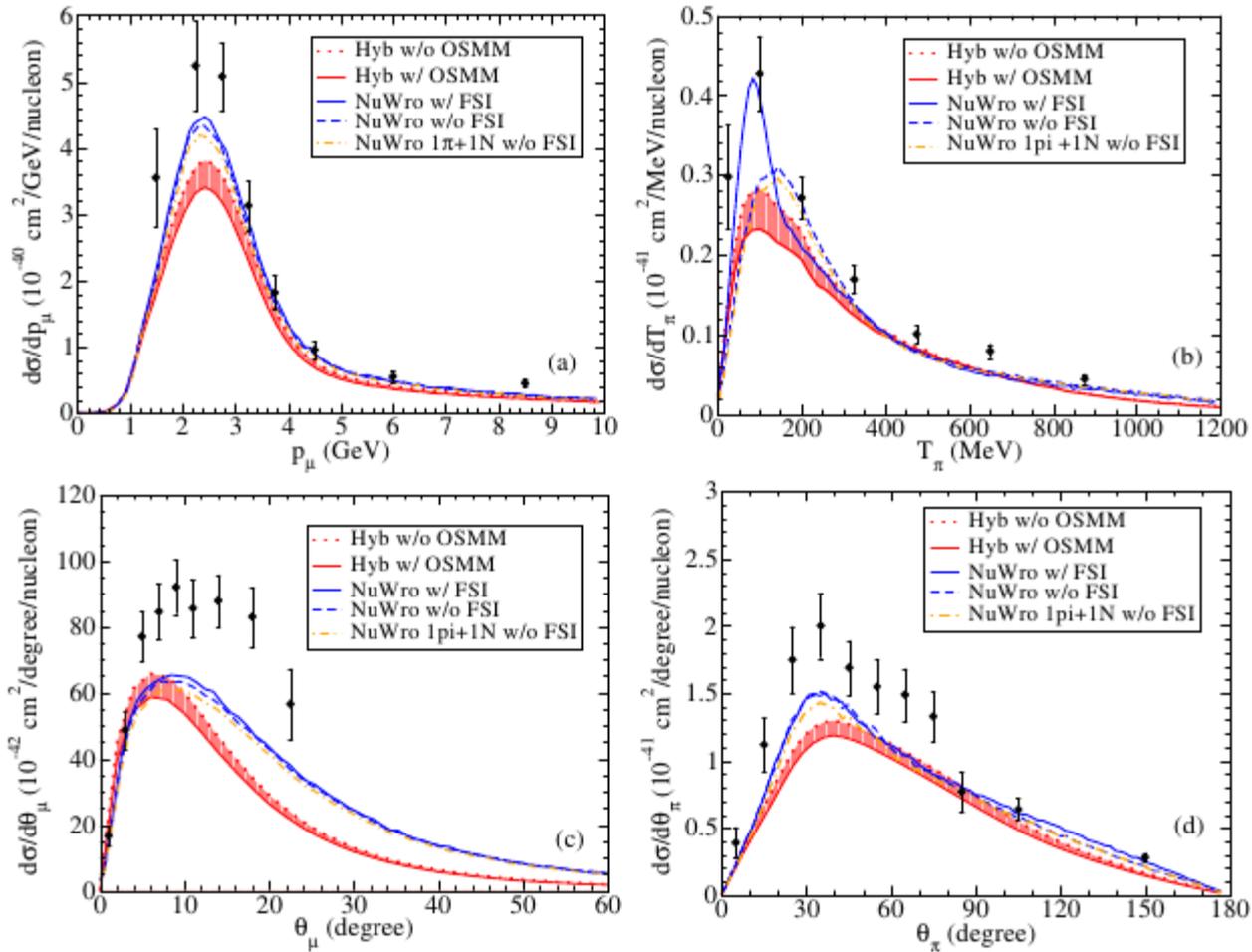
RGJ et al., arXiv:1710.08374 [nucl-th]

MINERvA antineutrino CC 1pion0.
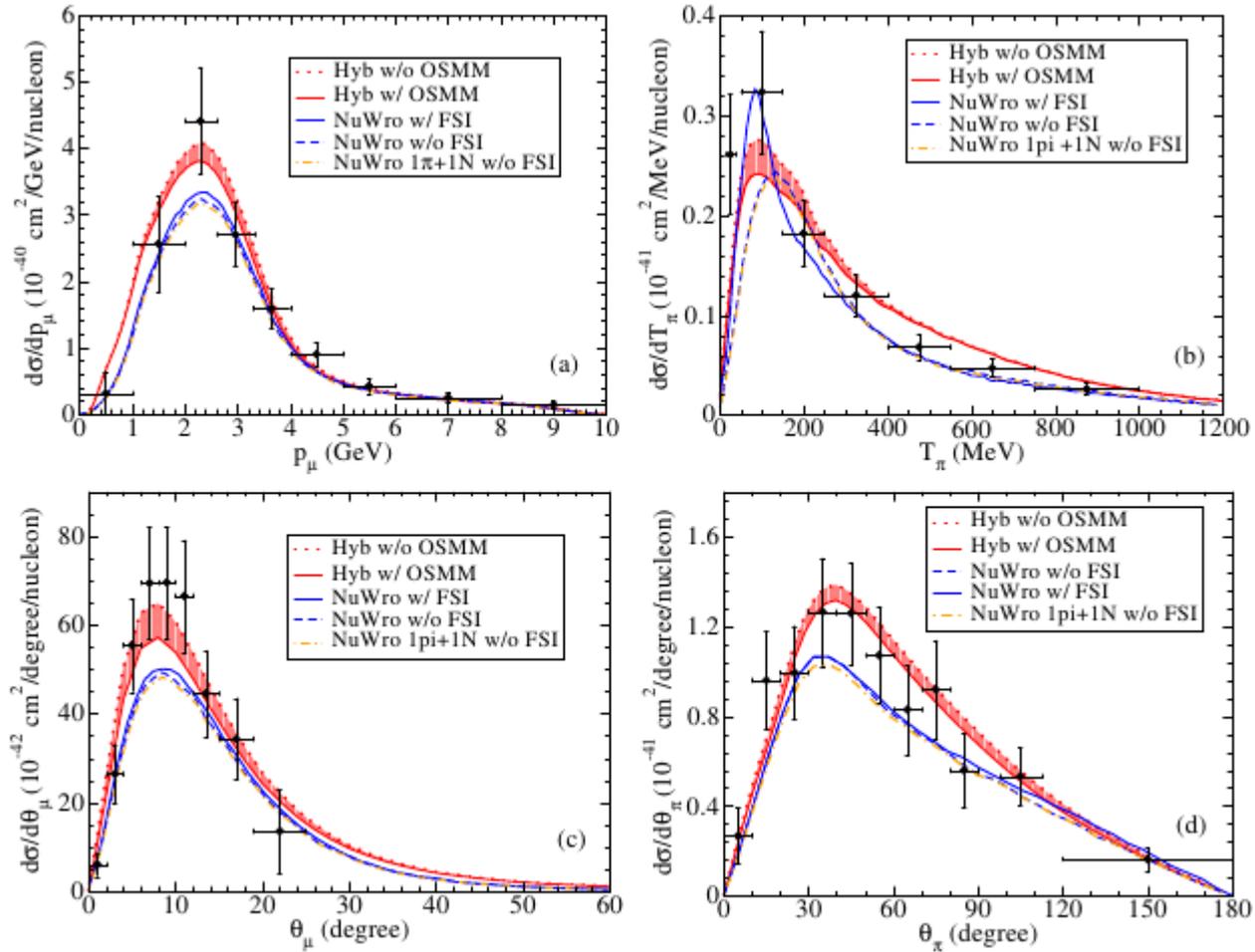
Conclusions

✔ **Neutrino-nucleus cross sections** are important to reduce systematics in neutrino-oscillation analyses. We need to improve our current understanding of the neutrino-nucleus and neutrino-nucleon interaction.

✔ New theoretical **developments should be implemented in the MC event generators** that are used in the oscillation analyses.

✔ We are working to provide a **microscopic description** of the main reaction mechanisms at intermediate energies: QE scattering, 2-nucleon emission, pion production.

Lot of work to do...

74

Dzi**ę**kuję za uwag**ę**!

Backup slides

Results in the sub-GeV region ($E_{lepton} < 1.3$ GeV)
Kam I and Kam II

4.92 kton yr sample

Observation of a small atmospheric ν_μ/ν_e ratio in K

The estimation of the expected **event rate distribution in the detector** requires knowledge of the neutrino flux, cross section and experimental conditions.
Theoretical predictions of the fluxes differ by as much as **30%**. HOWEVER, the ratio

$$R = (\nu_\mu + \overline{\nu}_\mu) / (\nu_e + \overline{\nu}_e),$$

where $\nu_{\mu,e}$ represent the event rates distribution, agrees upon by all authors to within **5%**.
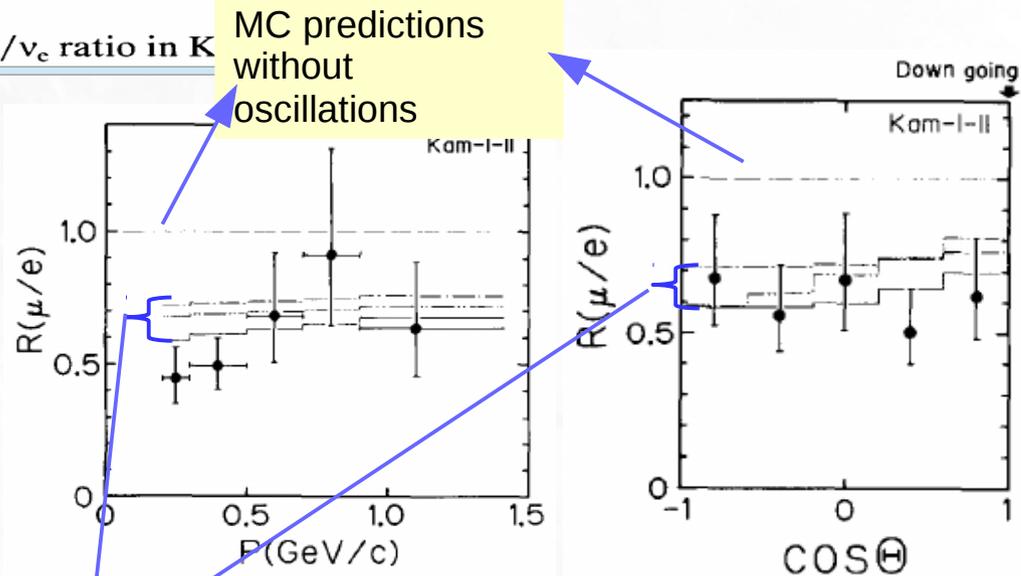
MC predictions without oscillations

MC predictions within different oscillation scenarios

They found:

$$R(\mu/e) = R_{data} / R_{Theory} = 0.60 \pm 0.07$$

Far from 1 !!!
(actually, more than 4σ below 1.)

Conclusion

decays relative to all single ring events. Neutrino os-
cillations, with the totality of atmospheric neutrino
and solar neutrino data favoring the channel $\nu_\mu \leftrightarrow \nu_\tau$,
might account for the observations.

Results in the multi-GeV region ($E_{lepton} > 1.3$ GeV)

Atmospheric ν_μ / ν_e ratio in the multi-GeV energy range

8.2 kton yr sample

They found:
$R(\mu/e) = R_{data} / R_{Theory} = 0.58 \pm 0.08$
(in agreement with the sub-GeV result)
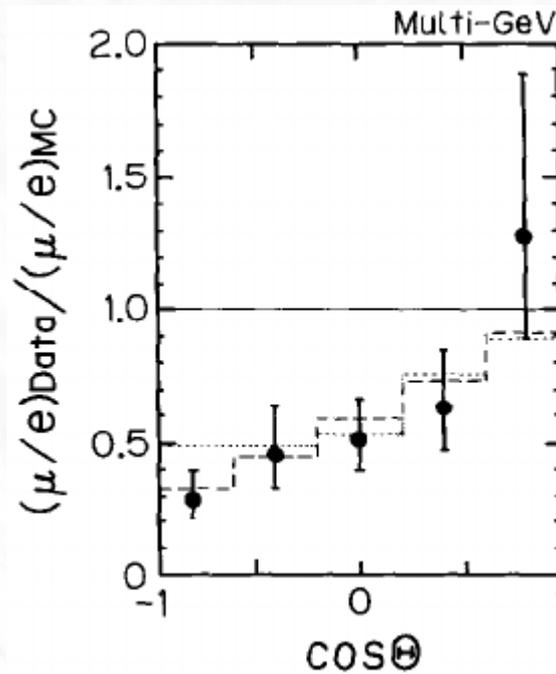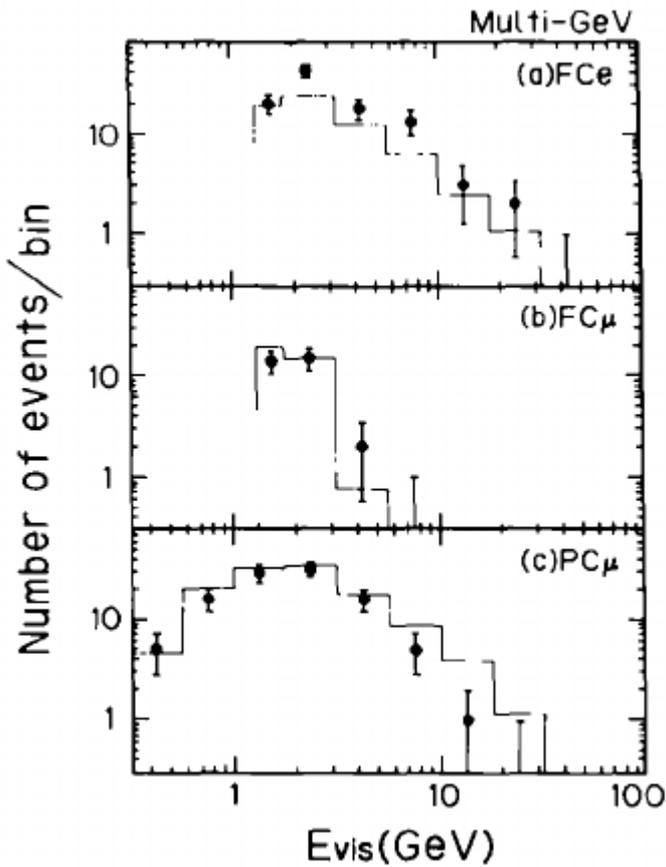
Strong dependence on the zenith angle

Strong dependence on the zenith angle that increases with the energy

⟹ **OSCILLATIONS**

$$P_{a \to b} = \sin^2 2\theta \sin^2 \left(\frac{1.27 \Delta m^2 (\text{eV}^2) L (\text{km})}{E_\nu (\text{GeV})} \right),$$

$\sin^2 2\theta = 0.95$
$\Delta m^2 = 4 \times 10^{-4}\, \text{eV}^2$

NEUTRINOS FROM COSMIC RADIATION

SUPER-KAMIOKANDE
KAMIOKA, JAPAN

COSMIC RADIATION

ATMOSPHERE

ν_μ

SUPER-KAMIOKANDE

ν_μ

MUON-NEUTRINO

1 000 m

PROTECTING ROCK

Muon-neutrinos give signals in the water tank.

Muon-neutrinos arriving directly from the atmosphere

ν_μ

μ

40 m

Light detectors measuring Cherenkov radiation

μ

Muon-neutrinos that have travelled through the Earth

ν_μ

CHERENKOV RADIATION

Illustration: © Johan Jarnestad/The Royal Swedish Academy of Sciences

Example: the reconstruction procedure at the MiniBooNE experiment:

1) In MiniBooNE a **QE-like event** happens when a muon and no pions are detected. The event is characterized by the scattering angle and the energy of the muon.

Example: the reconstruction procedure at the MiniBooNE experiment:

1) In MiniBooNE a **QE-like event** happens when a muon and no pions are detected. The event is characterized by the scattering angle and the energy of the muon.

2) The neutrino energy is reconstructed using the formula:

$$E_\nu^{QE} = \frac{m_p^2 - m'^2_n - m_\mu^2 + 2m'_n E_\mu}{2(m'_n - E_\mu + p_\mu \cos\theta_\mu)}$$

So, we go from

N(E ?) to N(E^{QE})

81

Example: the reconstruction procedure at the MiniBooNE experiment:

1) In MiniBooNE a **QE-like event** happens when a muon and no pions are detected. The event is characterized by the scattering angle and the energy of the muon.

2) The neutrino energy is reconstructed using the formula:

$$E_\nu^{QE} = \frac{m_p^2 - m'^2_n - m_\mu^2 + 2m'_n E_\mu}{2(m'_n - E_\mu + p_\mu \cos\theta_\mu)}$$

So, we go from

N(E ?) to N(E^{QE})

3) Assuming pure QE interaction one can compute the **probability density of the reconstructed energy E^{QE} matching the true energy E: P(E^{QE}|E)**
(MiniBooNE used a Fermi gas model to simulate nuclear system)

Example: the reconstruction procedure at the **MiniBooNE experiment**:

1) In MiniBooNE a **QE-like event** happens when a muon and no pions are detected. The event is characterized by the scattering angle and the energy of the muon.
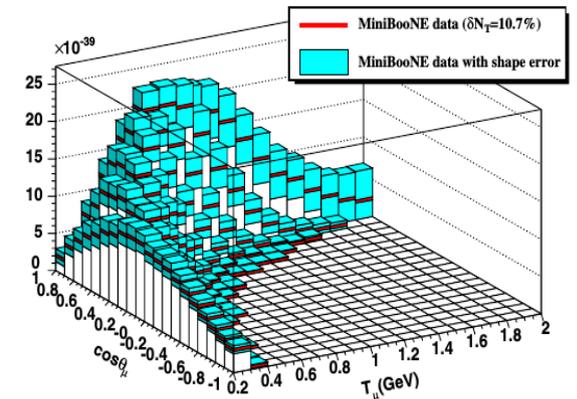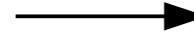

2) The neutrino energy is reconstructed using the formula:

$$E_\nu^{QE} = \frac{m_p^2 - m'^2_n - m_\mu^2 + 2m'_n E_\mu}{2(m'_n - E_\mu + p_\mu \cos\theta_\mu)}$$

So, we go from

N(E ?) to N(E^{QE})

3) Assuming pure QE interaction one can compute the **probability density of the reconstructed energy E^{QE} matching the true energy E: P(E^{QE}|E)**
(MiniBooNE used a Fermi gas model to simulate nuclear system)

4) The **event distribution as a function of the true neutrino** energy results from:

N(E) ~ ∫ N(E^{QE}) P(E^{QE}|E) dE^{QE}

83

Sources of **ERRORS**:

1) Other processes, like 2-nucleon emission and pionless delta decay, may contribute to the QE-like signal.

2) Fermi gas model is not good enough

Probability distributions (reconstructed energy → true energy)

<div align="center">

pure QE with Fermi gas ≠ QE+2p2h with better nuclear models

</div>

L. Alvarez-Ruso , arXiv:1012.3871 (Neutrino 2010)

Summary: A **major source of systematic uncertainties** in neutrino oscillation analyses are the neutrino-nucleus cross sections.

Better description of neutrino-nucleus interaction is needed and has to be implemented in the experimental analyses.

84

Sources of **ERRORS**:

1) Other processes, like 2-nucleon emission and pionless delta decay, may contribute to the QE-like signal.

2) Fermi gas model is not good enough

Probability distributions (reconstructed energy → true energy)

pure QE with Fermi gas ≠ QE+2p2h with better nuclear models

L. Alvarez-Ruso , arXiv:1012.3871 (Neutrino 2010)

Summary: A **major source of systematic uncertainties** in neutrino oscillation analyses are the neutrino-nucleus cross sections.

Better description of neutrino-nucleus interaction is needed and has to be implemented in the experimental analyses.

85

Sources of **ERRORS**:

1) Other processes, like 2-nucleon emission and pionless delta decay, may contribute to the QE-like signal.

2) Fermi gas model is not good enough

Probability distributions (reconstructed energy → true energy)

pure QE with Fermi gas ≠ QE+2p2h with better nuclear models

Summary: A **major source of systematic uncertainties** in neutrino oscillation analyses are the neutrino-nucleus cross sections.

Better description of neutrino-nucleus interaction is needed and has to be implemented in the experimental analyses.

86

Relativistic mean-field model

RMF model provides a microscopic description of the ground state of finite nuclei which is consistent with Quantum Mechanic, Special Relativity and symmetries of strong interaction.
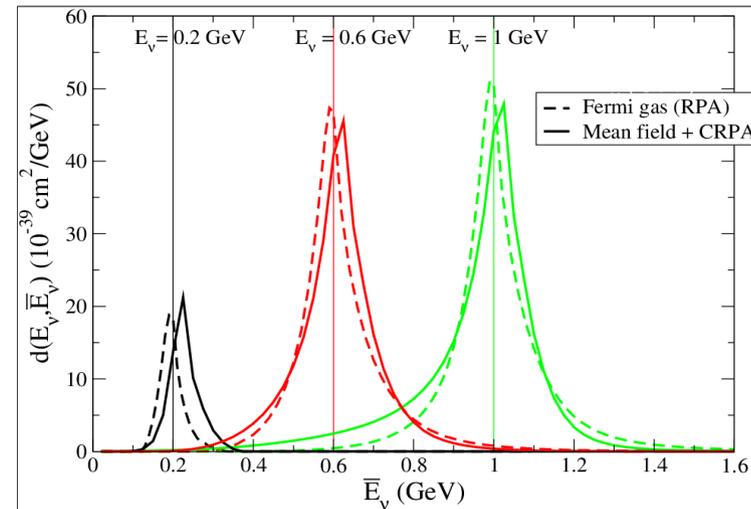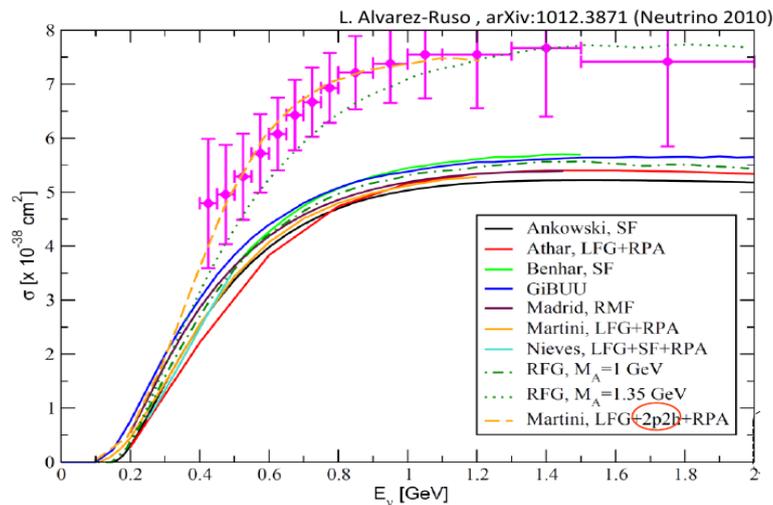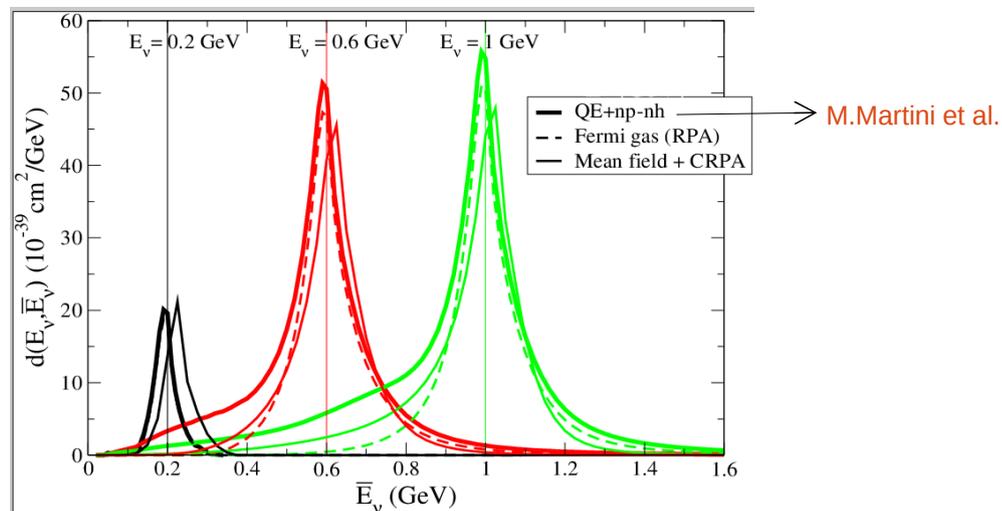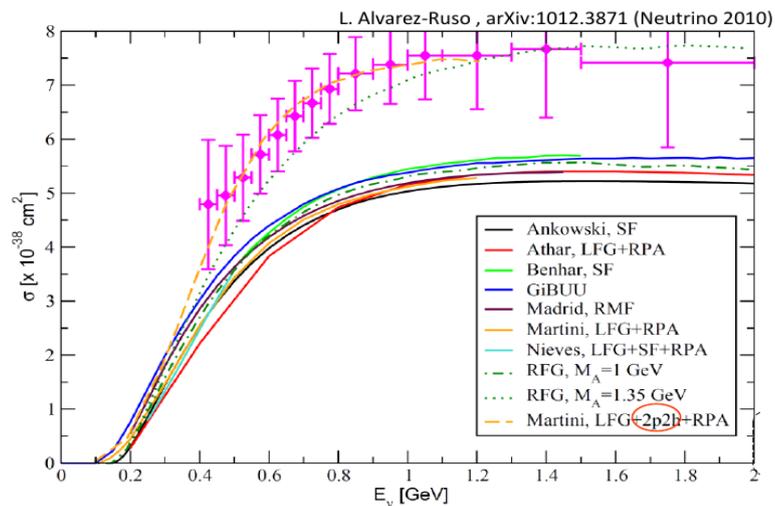
The starting point is a Lorentz covariant Lagrangian density

$$\mathcal{L} = \overline{\Psi}\left(i\gamma_\mu\partial^\mu - M\right)\Psi + \frac{1}{2}\left(\partial_\mu\sigma\partial^\mu\sigma - m_\sigma^2\sigma^2\right) - U(\sigma)$$

$$- \frac{1}{4}\Omega_{\mu\nu}\Omega^{\mu\nu} + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu - \frac{1}{4}\mathbf{R}_{\mu\nu}\mathbf{R}^{\mu\nu} + \frac{1}{2}m_\rho^2\boldsymbol{\rho}_\mu\boldsymbol{\rho}^\mu - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

$$- g_\sigma\overline{\Psi}\sigma\Psi - g_\omega\overline{\Psi}\gamma_\mu\omega^\mu\Psi - g_\rho\overline{\Psi}\gamma_\mu\boldsymbol{\tau}\boldsymbol{\rho}^\mu\Psi - g_e\frac{1+\tau_3}{2}\overline{\Psi}\gamma_\mu A^\mu\Psi.$$

where

$$\Omega^{\mu\nu} = \partial^\mu\omega^\nu - \partial^\nu\omega^\mu,$$

$$\mathbf{R}^{\mu\nu} = \partial^\mu\boldsymbol{\rho}^\nu - \partial^\nu\boldsymbol{\rho}^\mu,$$

$$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu.$$

$$U(\sigma) = \tfrac{1}{3}g_2\sigma^3 + \tfrac{1}{4}g_3\sigma^4$$

Main approximations:

1) Mean-field approximation:

$$\omega_\mu \rightarrow \langle\omega_\mu\rangle \quad \sigma \rightarrow \langle\sigma\rangle \quad \boldsymbol{\rho}_\mu \rightarrow \langle\boldsymbol{\rho}_\mu\rangle$$

2) Static limit:

$$\partial^0\omega_0 = \partial^0\boldsymbol{\rho}_0 = \partial^0\sigma = 0 \quad \omega_\mu = \delta_{\mu 0}\omega_0, \quad \boldsymbol{\rho}_\mu = \delta_{\mu 0}\boldsymbol{\rho}_0$$

3) Spherical symmetry for finite nuclei:

$$\omega_0 = \omega_0(r) \quad \boldsymbol{\rho}_0 = \boldsymbol{\rho}_0(r) \quad \sigma = \sigma(r)$$

Relativistic mean-field model

Dirac equation for nucleons (eq. of motion for the barionic fields):

$$[-i\boldsymbol{\alpha} \cdot \boldsymbol{\nabla} + V(r) + \beta(M + S(r)]\Psi_i(\boldsymbol{r}) = E_i\Psi_i(\boldsymbol{r})$$

where the scalar (S) and vector (V) potential are given by:

$$
\begin{aligned}
S(r) &= g_\sigma\sigma(r)\,, \\
V(r) &= g_\omega\omega^0(r) + g_\rho\tau_3\rho_3^0(r) + e\frac{1+\tau_3}{2}A^0(r)
\end{aligned}
$$

Eqs. of motion for the mesons and the photon:

$$
\begin{aligned}
\left[-\nabla^2 + m_\sigma^2\right]\sigma(r) &= -g_\sigma\rho_s(r) - g_2\sigma^2(r) - g_3\sigma^3(r)\,, \\
\left[-\nabla^2 + m_\omega^2\right]\omega^0(r) &= -g_\omega\rho_B(r)\,, \\
\left[-\nabla^2 + m_\rho^2\right]\rho_3^0(r) &= -g_\rho\rho_\rho(r)\,, \\
-\nabla^2 A^0 &= e\rho_c\,,
\end{aligned}
$$

Current densities

$$
\begin{aligned}
\rho_s(r) &= \sum_i^A \overline{\Psi}_i(\boldsymbol{r})\Psi_i(\boldsymbol{r})\,, \\
\rho_B(r) &= \sum_i^A \Psi_i^\dagger(\boldsymbol{r})\Psi_i(\boldsymbol{r})\,, \\
\rho_\rho(r) &= \sum_i^A \Psi_i^\dagger(\boldsymbol{r})\tau_3\Psi_i(\boldsymbol{r}) \\
\rho_c(r) &= \sum_i^A \Psi_i^\dagger(\boldsymbol{r})\frac{1+\tau_3}{2}\Psi_i(\boldsymbol{r})
\end{aligned}
$$

Solution of the couple equations for the fields in a self-consistent way.

Relativistic mean-field model

In general, the parameters are fit to reproduce some general properties of some closed shell spherical nuclei and nuclear matter.

Parameters for the NLSH model (fitted to the mean charge radius, binding energy and neutron radius of the ^{16}O, ^{40}Ca, ^{90}Zr, ^{116}Sr, ^{124}Sn and ^{208}Pb.

6 free parameters

M_N	m_σ	m_ω	m_ρ	g_σ	g_ω	g_ρ	g_2	g_3
939.0	526.059	783.0	763.0	10.444	12.945	4.3830	-6.9099	-15.8337

$$[-i\boldsymbol{\alpha} \cdot \boldsymbol{\nabla} + V(r) + \beta(M + S(r)]\Psi_i(\boldsymbol{r}) = E_i\Psi_i(\boldsymbol{r})$$

$$\Psi_k^{m_j}(\boldsymbol{r}) = \begin{pmatrix} g_k(r)\varphi_k^{m_j}(\Omega_r) \\ if_k(r)\varphi_{-k}^{m_j}(\Omega_r) \end{pmatrix},$$

$$\varphi_k^{m_j}(\Omega_r) = \sum_{m_\ell s}\langle \ell m_\ell \frac{1}{2}s|jm_j\rangle Y_\ell^{m_\ell}(\Omega_r)\chi^s$$
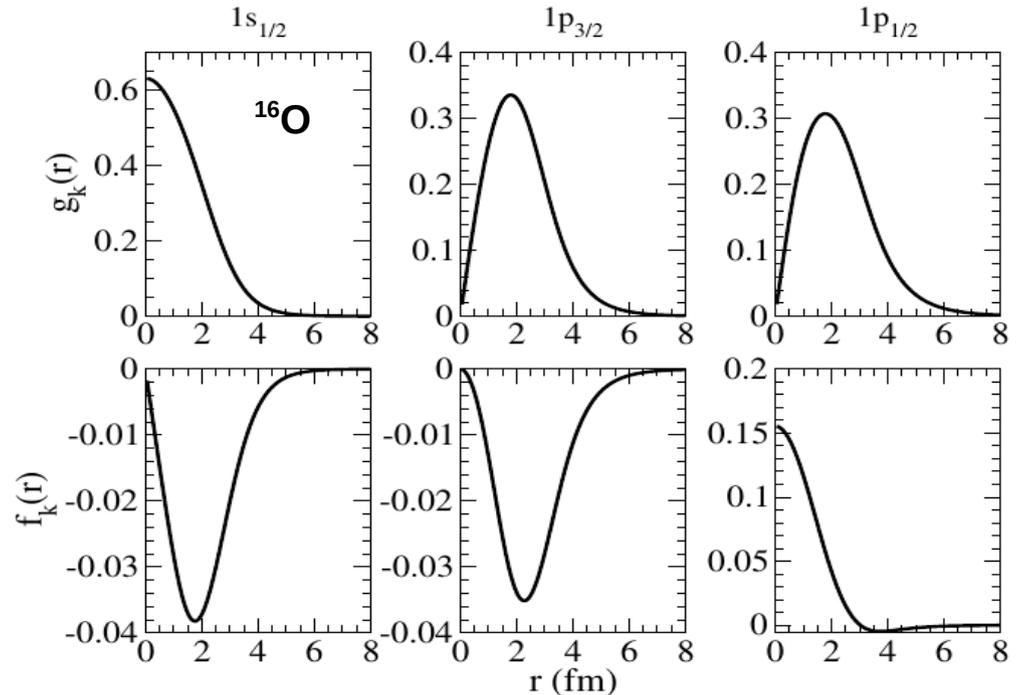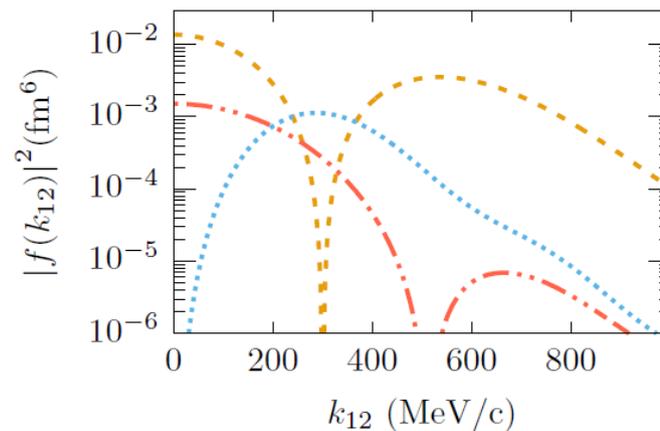
Short range correlations

$$|\Psi\rangle = \frac{1}{\sqrt{\mathcal{N}}}\widehat{\mathcal{G}}|\Phi\rangle \qquad \text{with} \qquad \widehat{\mathcal{G}} \approx \widehat{\mathcal{S}}\left(\prod_{i<j}^{A}\left[1 + \widehat{l}(i,j)\right]\right)$$

$$\widehat{l}(i,j) = -\,g_c(r_{ij}) + f_{\sigma\tau}(r_{ij})\,(\vec{\sigma}_i \cdot \vec{\sigma}_j)\,(\vec{\tau}_i \cdot \vec{\tau}_j)$$

$$+\,f_{t\tau}(r_{ij})\widehat{S}_{ij}\,(\vec{\tau}_i \cdot \vec{\tau}_j)\,,$$

The complexity induced by correlations is shifted from the wave functions to the operators

90
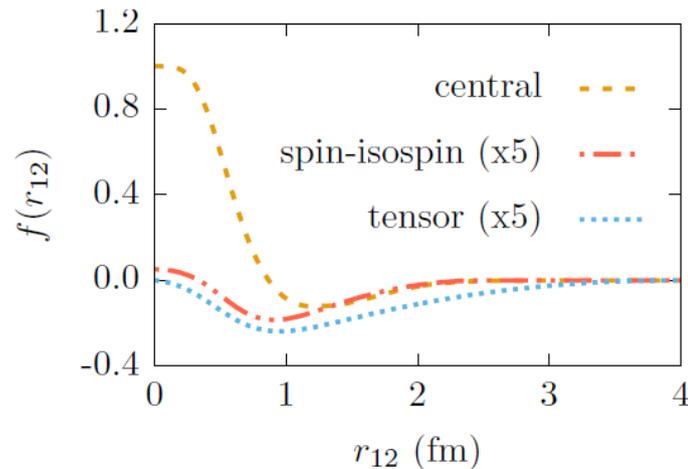
Short range correlations
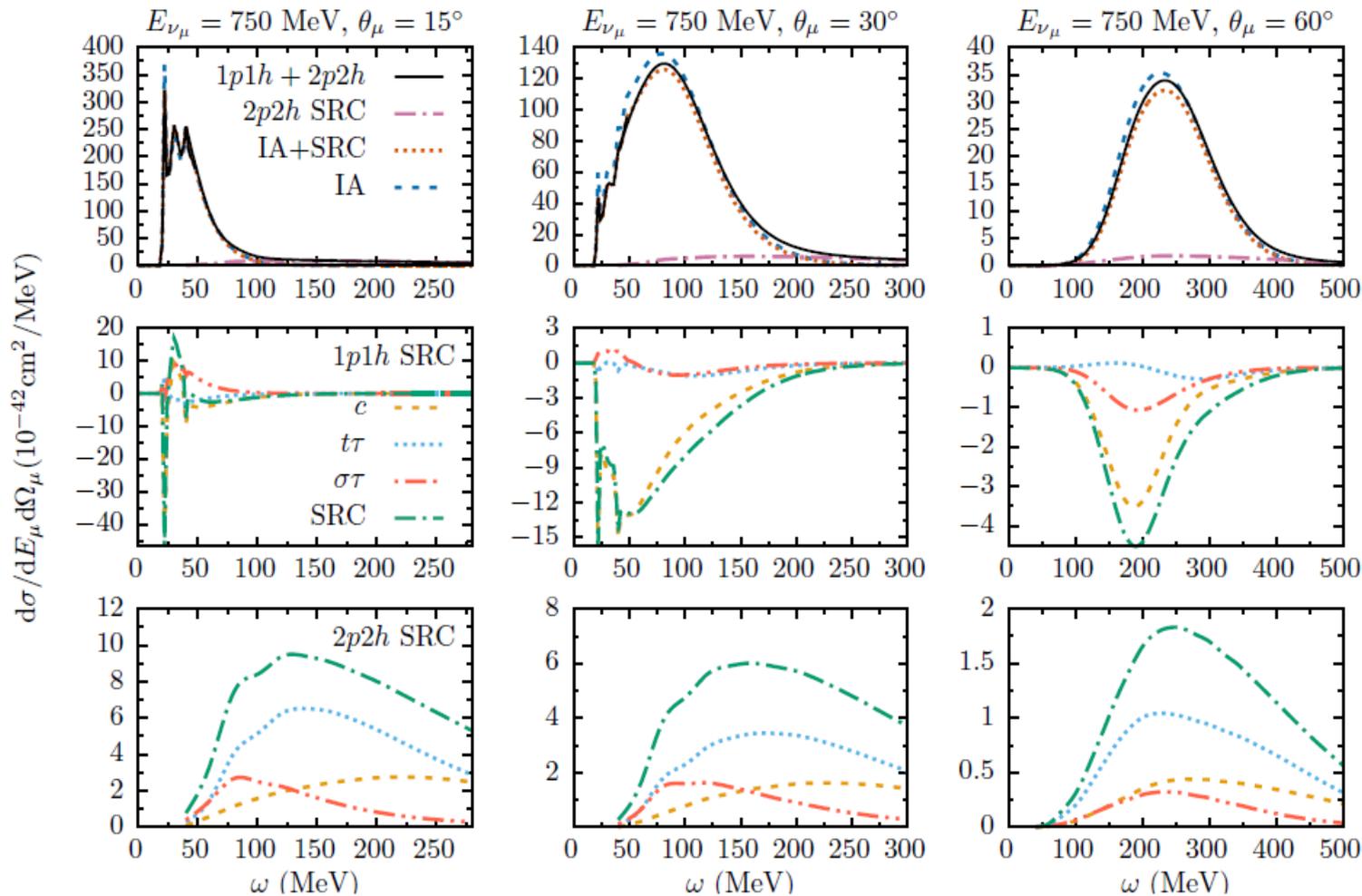
SRC affect the 1p1h and the 2p2h responses

Van Cuyck et al., Phys. Rev. C 94, 024611 (2016)

Meson-exchange currents
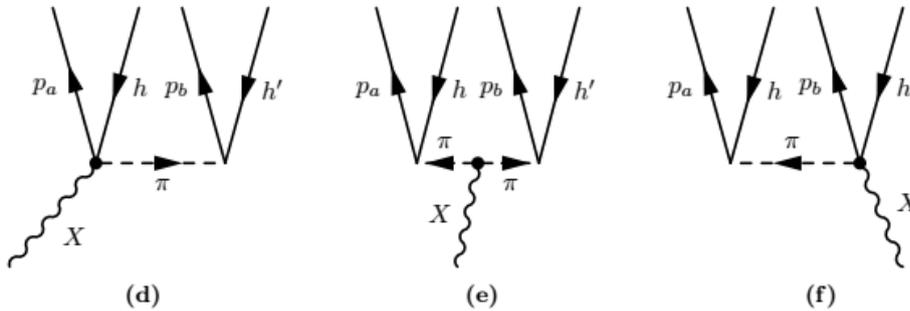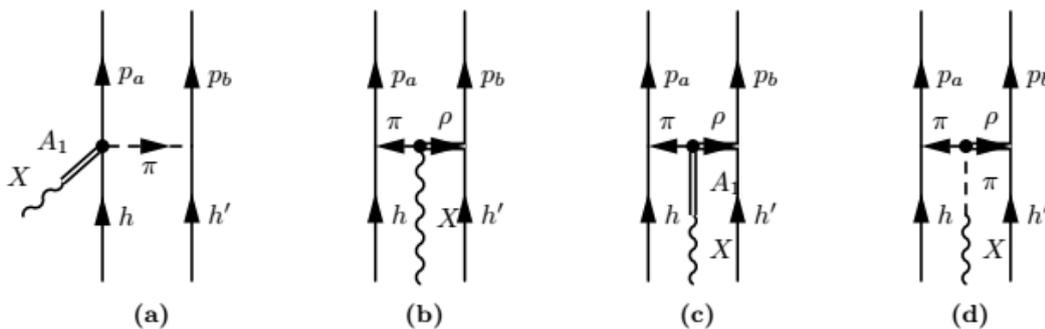
Van Cuyck et al., PRC 95, 054611 (2017)
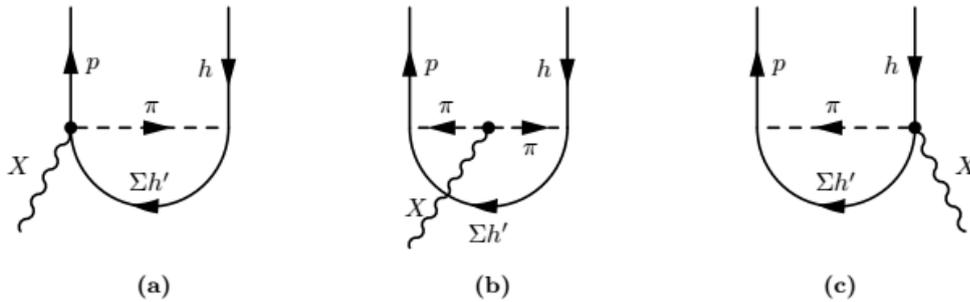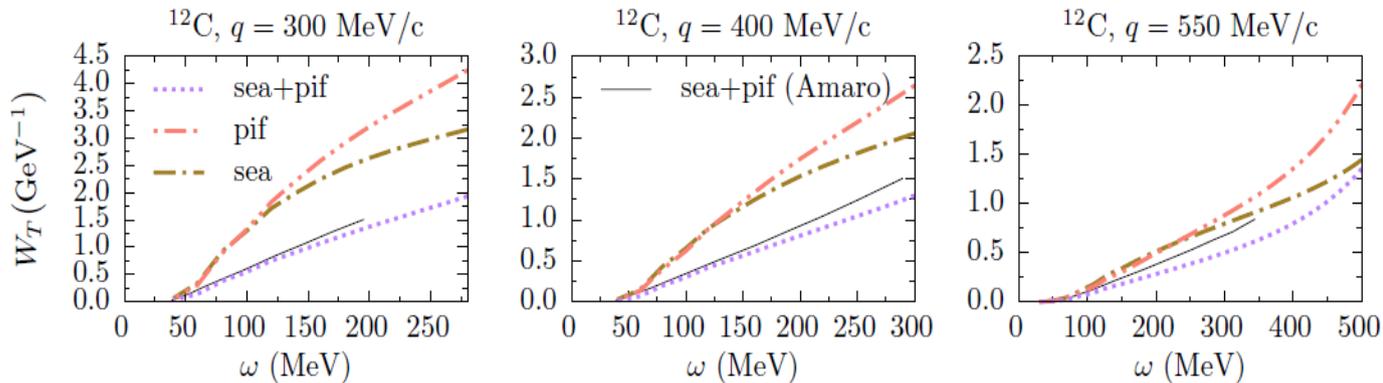

1p1h contributions (vector current)

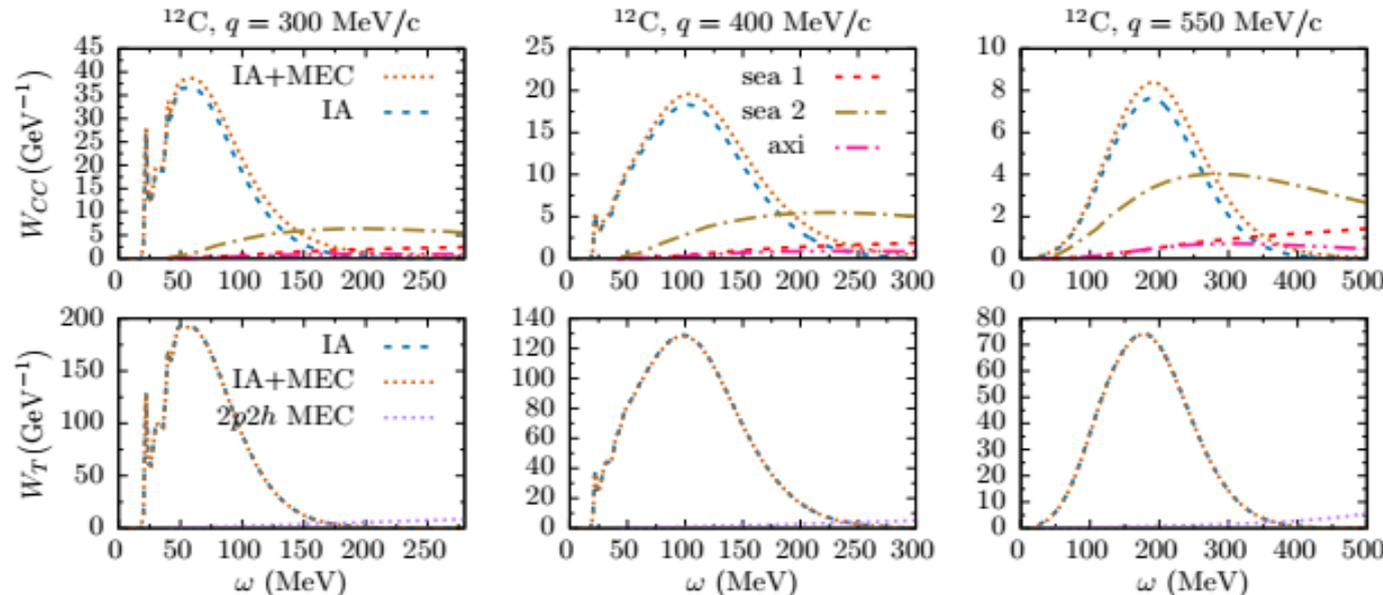2p2h contributions (vector current)

Axial current (1p1h and 2p2h)

Meson-exchange currents

Good agreement with other predictions [Amaro et al., NPA578, 365 (1994)]:

1p1h and 2p2h contributions to the longitudinal and transverse responses:

93

Medium modifications of the Delta

Delta propagator:

$$S_{\Delta,\alpha\beta} = \frac{-(K_\Delta + M_\Delta)}{K_\Delta^2 - M_N^2 + iM_\Delta\Gamma_{\text{width}}}\left(g_{\alpha\beta} - \frac{1}{3}\gamma_\alpha\gamma_\beta - \frac{2}{3M_\Delta^2}K_{\Delta,\alpha}K_{\Delta,\beta} - \frac{2}{3M_\Delta}(\gamma_\alpha K_{\Delta,\beta} - K_{\Delta,\alpha}\gamma_\beta)\right)$$

with the energy dependent Delta width:

$$\Gamma_{\text{width}}(W) = \frac{1}{12\pi}\frac{(f_{\pi N\Delta})^2}{m_\pi^2 W}(p_{\pi,cm})^3(M + E_{N,cm})$$

$$\Gamma_{\text{width}}^{\text{free}} \longrightarrow \Gamma_{\text{width}}^{\text{in-medium}} = \Gamma_{\text{Pauli}} - 2\Im(\Sigma_\Delta), \quad M_\Delta^{\text{free}} \longrightarrow M_\Delta^{\text{in-medium}} = M_\Delta^{\text{free}} + \Re(\Sigma_\Delta).$$

+ Γ_{Pauli}: some nucleons from Δ-decay are Pauli blocked (the Δ-decay width decreases).

+ The parametrization of $\Im(\Sigma_\Delta)$ and $\Re(\Sigma_\Delta)$ is given in terms of the nuclear density ρ:

$$
\begin{aligned}
-\Im(\Sigma_\Delta) &= C_{QE}(\rho/\rho_0)^\alpha + C_{A2}(\rho/\rho_0)^\beta + C_{A3}(\rho/\rho_0)^\gamma, \\
\Re(\Sigma_\Delta) &= 40\ \text{MeV}\ (\rho/\rho_0).
\end{aligned}
$$

We modify the free $\Delta\pi N$-decay constant ($f_{\Delta\pi N}$) to take into account the E-dependent medium modification of the Δ width:

$$f_{\Delta\pi N}^{\text{in-medium}}(W) = f_{\Delta\pi N}\sqrt{\frac{\Gamma_{\text{Pauli}} + 2C_{QE}(\rho/\rho_0)^\alpha}{\Gamma_{\text{width}}^{\text{free}}}}$$

94

Medium modifications of the Delta
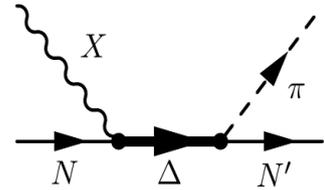
$$-\Im(\Sigma_\Delta) = C_{QE}\,(\rho/\rho_0)^\alpha + C_{A2}\,(\rho/\rho_0)^\beta + C_{A3}\,(\rho/\rho_0)^\gamma$$

Each contribution corresponds to a different process:

- QE $\Longrightarrow \Delta N \to \pi NN$ (still one pion in the final state)

- A2 $\Longrightarrow \Delta N \to NN$ (no pions in the final state)

- A3 $\Longrightarrow \Delta NN \to NNN$ (no pions in the final state)

We modify the free Delta decay constant to take into account the E-dependent medium modification of the Delta-width

$$\Gamma^\alpha_{\Delta\pi N} = \frac{f_{\pi N\Delta}}{m_\pi}\,P^\alpha_\pi$$

$$f^{\text{in-medium}}_{\Delta\pi N}(W) = f_{\Delta\pi N}\sqrt{\frac{\Gamma_{\text{Pauli}} + 2C_{QE}\,(\rho/\rho_0)^\alpha}{\Gamma^{\text{free}}_{\text{width}}}}$$
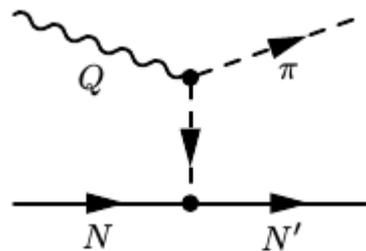
References: [*] E. Oset and L. L. Salcedo, Nucl. Phys. A 468, 631 (1987).

R. González-Jiménez   Ghent University

High-energy model

Regge approach for the vector amplitudes.

We use the approach of **Guidal, Laget, and Vanderhaeghen** [NPA627, 645 (1997)], originally developed for pion photoproduction ($Q^2 = 0$):

1) Feynman **meson-exchange diagrams** are reggeized.

Kaskulov and Mosel, PRC (2010)

$$\frac{1}{t - m_\pi^2}$$

The pion propagator is replace by the Regge trajectory of the pion family

$$\mathcal{P}_\pi(t, s) = -\alpha'_\pi \varphi_\pi(t) \Gamma[-\alpha_\pi(t)](\alpha'_\pi s)^{\alpha_\pi(t)}$$

High-energy model

Regge approach for the vector amplitudes.

We use the approach of **Guidal, Laget, and Vanderhaeghen** [NPA627, 645 (1997)], originally developed for pion photoproduction ($Q^2 = 0$):
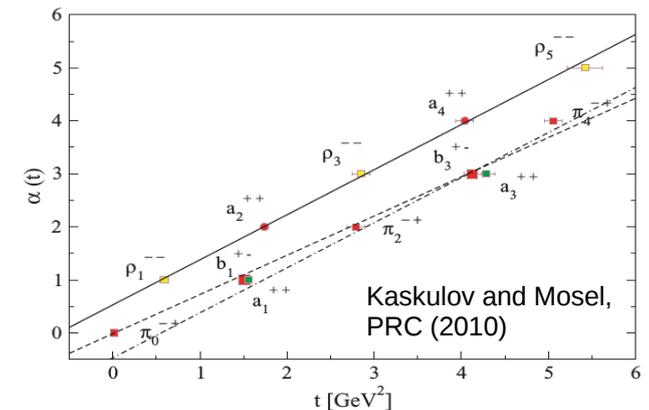
1) Feynman **meson-exchange diagrams** are reggeized.

2) s-channel and u-channel diagrams are included to keep **Conservation of Vector Current**.

$$\frac{1}{t - m_\pi^2}$$

The pion propagator is replace by the Regge trajectory of the pion family

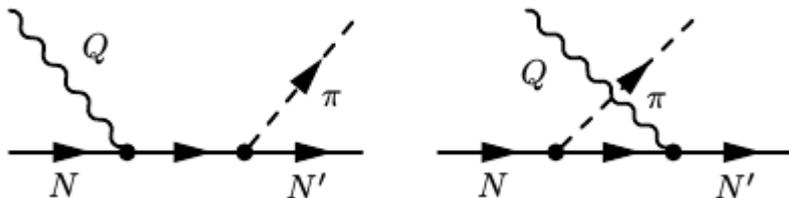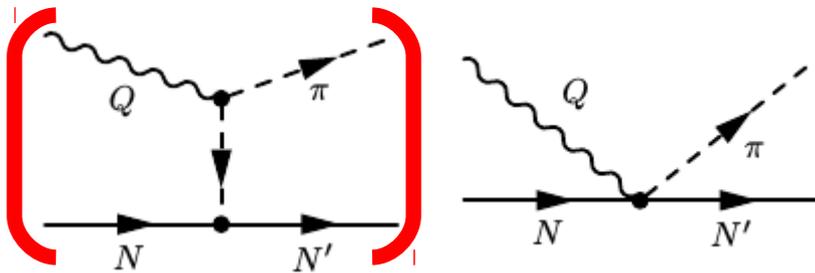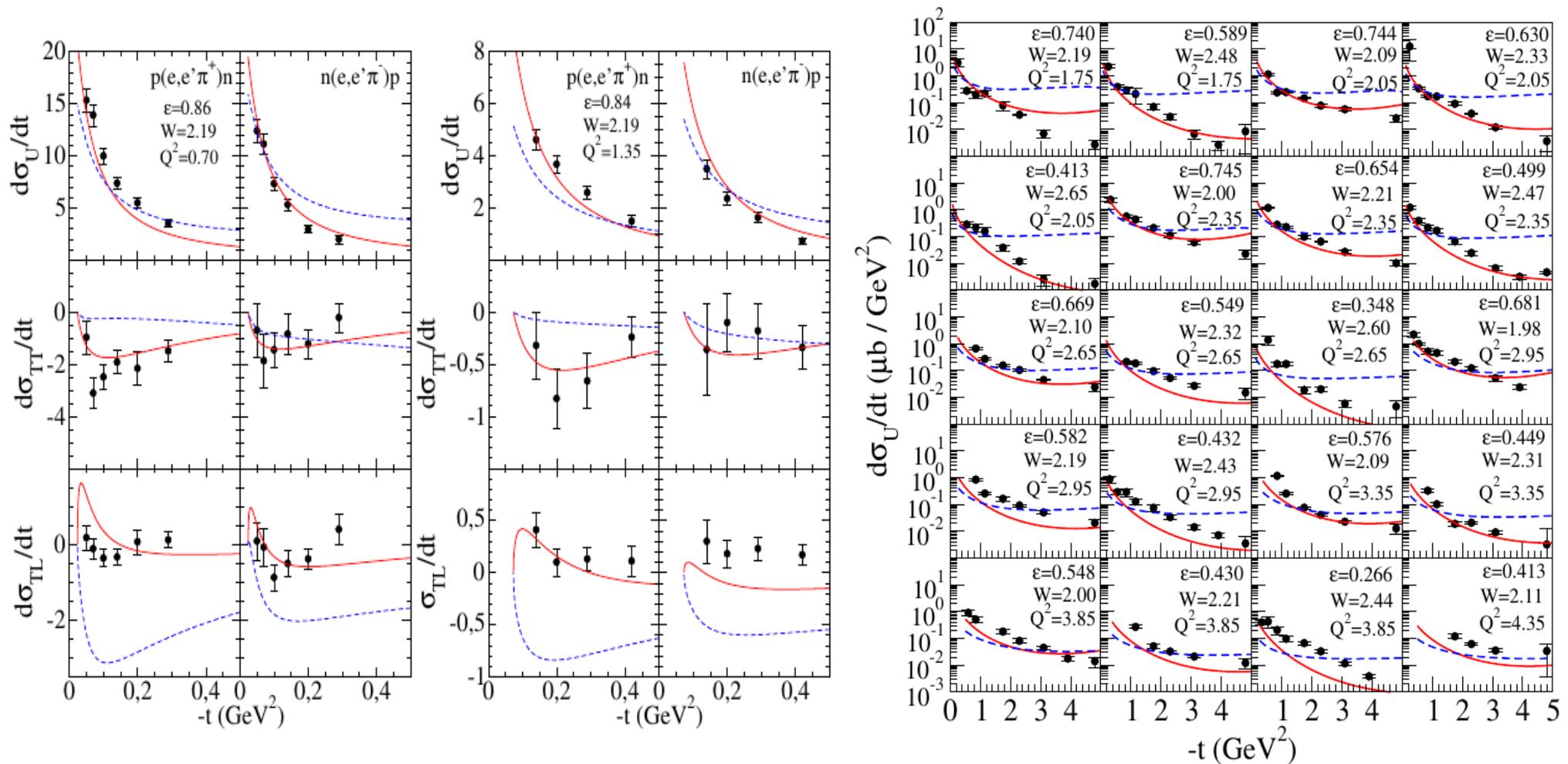$$\mathcal{P}_\pi(t,s) = -\alpha'_\pi \varphi_\pi(t)\Gamma[-\alpha_\pi(t)](\alpha'_\pi s)^{\alpha_\pi(t)}$$
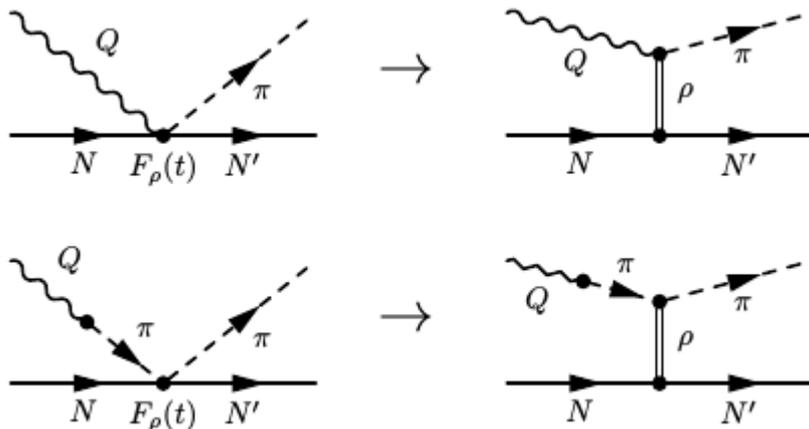
97

High-energy model: results (EM current)

Figure: High-energy model (red lines), low-energy model (blue lines) and **electron-induced single-pion production** data.

High-energy model

Regge approach for the axial amplitudes.

We need meson exchange diagrams to apply the reggeization procedure of the current.

Effective rho-exchange diagrams. This allows us to consider the rho-exchange as the main Regge trajectory in the axial current.

$$\mathcal{O}^{\mu}_{CT\rho} = i\mathcal{I}\,\frac{m_{\rho}^2}{m_{\rho}^2 - t}F_{A\rho\pi}(Q^2)\frac{1}{\sqrt{2}f_{\pi}}$$
$$\times\left(\gamma^{\mu} + i\frac{\kappa_{\rho}}{2M}\sigma^{\mu\nu}K_{t,\nu}\right)\,.$$

We consider $\kappa_{\rho} = 0$ so that the low-energy model amplitude is recovered.

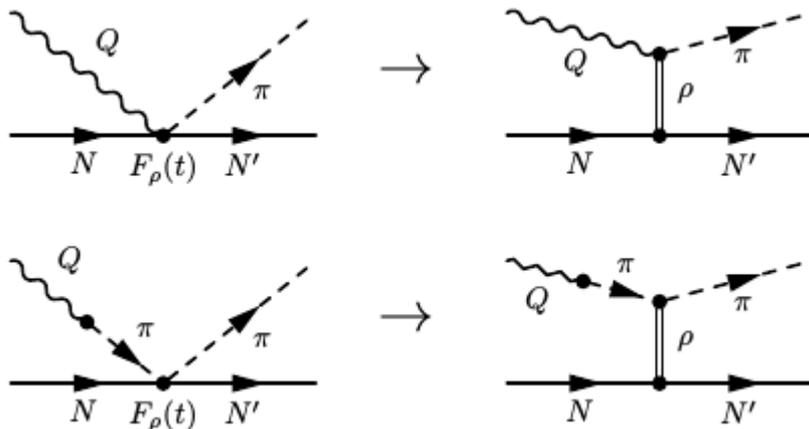The propagator of the rho is replaced by the Regge trajectory of the **rho family**:

$$\mathcal{P}_{\rho}(t,s) = -\alpha'_{\rho}\varphi_{\rho}(t)\Gamma[1 - \alpha_{\rho}(t)](\alpha'_{\rho}s)^{\alpha_{\rho}(t)-1}$$

99

High-energy model

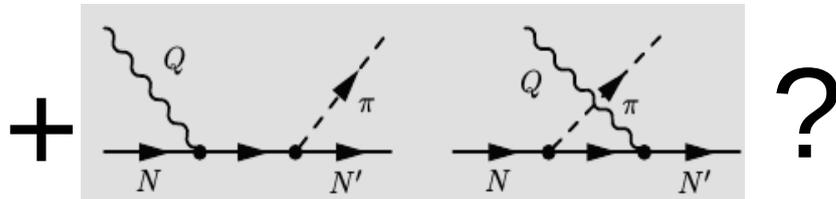**Regge approach for the axial amplitudes.**

We need meson exchange diagrams to apply the reggeization procedure of the current.

Effective rho-exchange diagrams. This allows us to consider the rho-exchange as the main Regge trajectory in the axial current.

$$\mathcal{O}^{\mu}_{CT\rho} = i\mathcal{I}\,\frac{m_{\rho}^2}{m_{\rho}^2 - t}F_{A\rho\pi}(Q^2)\frac{1}{\sqrt{2}f_{\pi}}$$

$$\times \left(\gamma^{\mu} + i\frac{\kappa_{\rho}}{2M}\sigma^{\mu\nu}K_{t,\nu}\right).$$

We consider $\kappa_{\rho} = 0$ so that the low-energy model amplitude is recovered.

High-energy model

"Reggeizing" the ChPT background:

$$\mathcal{O}^{\mu}_{ReChi,V} = \mathcal{O}^{\mu}_{ChPT,V} \, \mathcal{P}_{\pi}(t,s)(t - m_{\pi}^2)$$
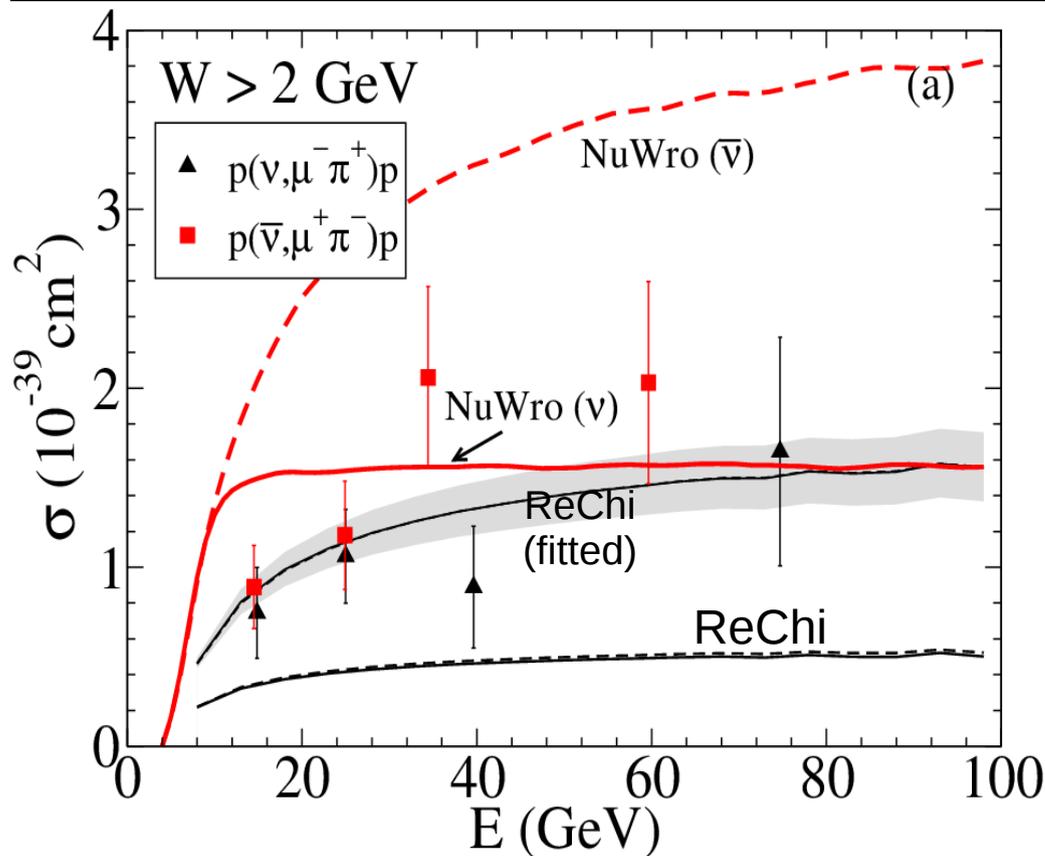
high-energy model:
ReChi (from Reggeized
ChPT background)

low-energy model (only
the ChPT background)

The pion propagator is replaced
by the **Regge propagator** of the
pion trajectory

$$\mathcal{O}^{\mu}_{ReChi,A} = \mathcal{O}^{\mu}_{ChPT,A} \, \mathcal{P}_{\rho}(t,s)(t - m_{\rho}^2)$$

High-energy model: results

Figure: ReChi model and NuWro predictions are compared with high energy cross section data for neutrino and antineutrino reactions (Note the high energy cut W>2 GeV !!). Data from Allen et al. NPB264, 221 (1986).

ReChi model: One free parameter in the boson-nucleon-nucleon vertex
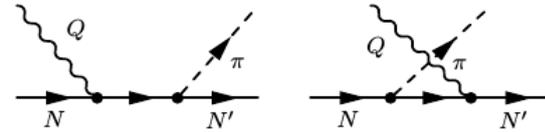
$$G_A[Q^2, s(u)] = g_A \left(1 + \frac{Q^2}{\Lambda_{Apn^*}[s(u)]^2}\right)^{-2}$$

$$\Lambda_{Anp^*}(s) = \Lambda_{Apn} + (\Lambda_\infty^A - \Lambda_{Apn}) \left(1 - \frac{M^2}{s}\right)$$

$$\Lambda_\infty^A = (7.20 \pm {}_{1.32}^{2.09}) \text{ GeV} \text{ !!!}$$

NuWro: Based on DIS formalism and PYTHIA for hadronization.

Antineutrino cross section is ~2 the neutrino one:

$$\bar{\nu} + \overbrace{uud}^{p} \to \mu^+ + \overbrace{\bar{u}d}^{\pi^-} + uud,$$

$$\nu + uud \to \mu^- + \underbrace{u\bar{d}}_{\pi^+} + uud.$$

R. González-Jiménez   Ghent University

Hybrid model

1) Regularizing the behavior of resonances (u- and s-channel contributions): we multiply the resonance amplitude by a dipole-Gaussian form factor
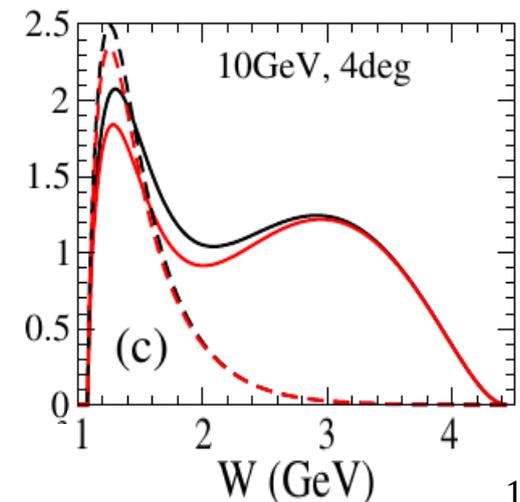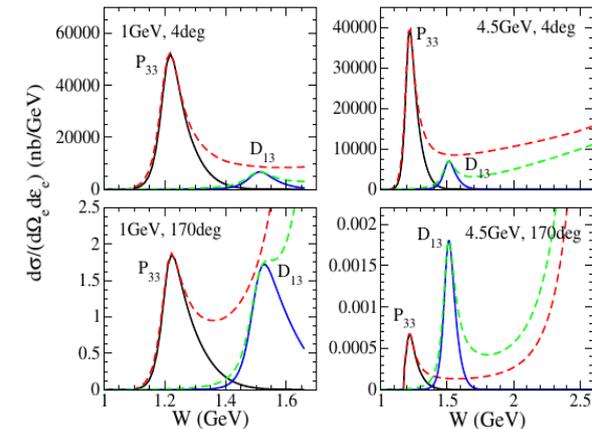
$$F(s,u) = F(s) + F(u) - F(s)F(u)$$

$$F(s) = \exp\left(\frac{-(s - M_R^2)^2}{\lambda_R^4}\right) \frac{\lambda_R^4}{(s - M_R^2)^2 + \lambda_R^4}$$

2) Gradually replacing the ChPT background by the High-energy (ReChi) model: we use a phenomenological transition function

$$\widetilde{\mathcal{O}} = \cos^2\phi(W)\mathcal{O}_{ChPT} + \sin^2\phi(W)\mathcal{O}_{ReChi}$$

$$\phi(W) = \frac{\pi}{2}\left(1 - \frac{1}{1 + \exp\left[\frac{W - W_0}{L}\right]}\right) \, ,$$

$$W_0 = 1.7 \text{ GeV}$$
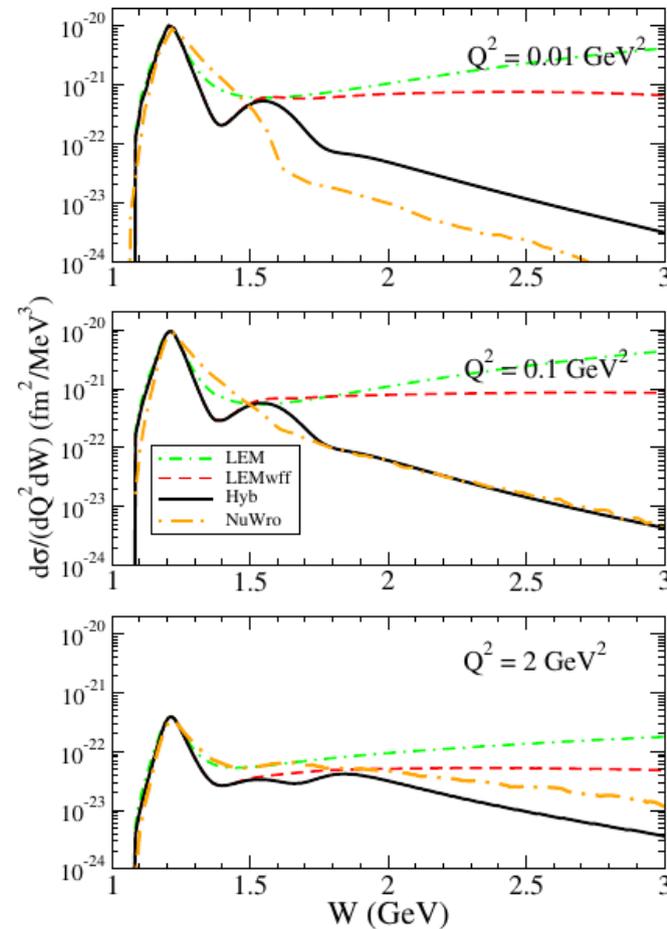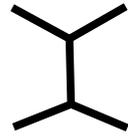$$L = 100 \text{ MeV}$$

103

Hybrid model: results

FIG. 21. (Color online) Different model predictions for the differential cross section $d\sigma/(dQ^2 dW)$, for the channel $p(\nu_\mu, \mu^- \pi^+)p$. The incoming neutrino energy is fixed to $E_\nu = 10$ GeV.

104

Why does this happen?

Cross channels:

$$\mathcal{A}(t,s) = \sum_{\ell} (2\ell + 1)\, A_\ell(t)\, P_\ell(z_t)$$
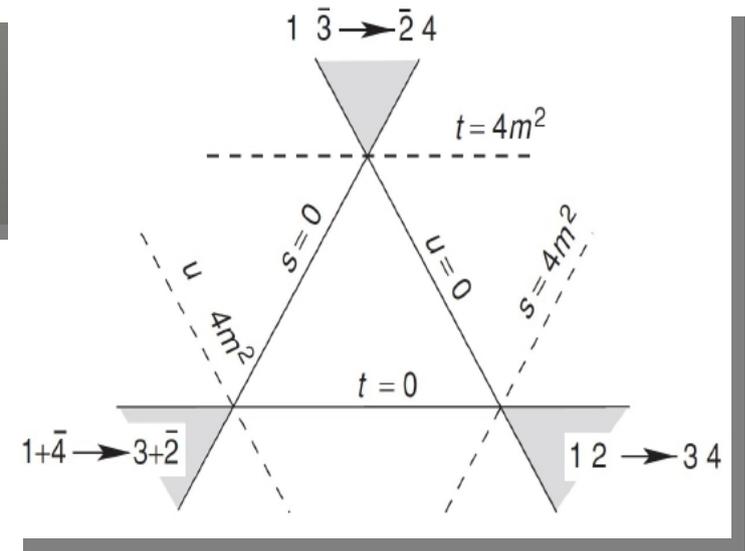
$$P_\ell(z_t) \xrightarrow{\;s\to\infty\;} (2s)^\ell$$

$$z_t \equiv \cos\theta_t = 1 + \frac{2s}{t - 4m^2}$$

Direct channels:

$$\mathcal{A}(s,t) = \sum_{\ell} (2\ell + 1)\, A_\ell(s)\, P_\ell(z_s)$$

$$z_s \equiv \cos\theta_s = 1 + \frac{2t}{s - 4m^2}$$

$$A_\ell(s) \sim \left(\frac{s - 4m^2}{2}\right)^\ell$$

Behavior at threshold (barrier factor).
Feynman diagrams provide the right
behavior at threshold but not at high s

105

Regge Theory

Based on unitarity, causality and crossing symmetry, Regge Theory predicts the following **high energy (s → ∞) behavior** for the invariant amplitude:

$$A(s,t) \sim \beta(t)\, s^{\alpha(t)}$$

$\alpha(t)$: Families or Regge trajectories

Regge theory does not predict the **t-dependence** of the amplitude.

For that, one needs a model.

Kaskulov and Mosel, PRC (2010)