String Theory and Nonrelativistic Gravity

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work done in collaboration with

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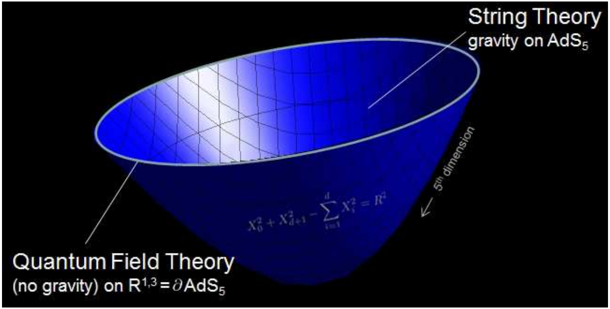
Motivation

Holography



Gravity is not only used to describe the gravitational force!

Non-relativistic Holography

two approaches

- Keep general relativity in the bulk but take background geometry with non-relativistic isometries

  Christensen, Hartong, Kiritsis, Obers and Rollier (2013-2015)

- Take non-relativistic gravity in the bulk

  Gomis, Ooguri (2001); Gopakumar, Bagchi (2009)

Outline

String Newton-Cartan Gravity

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4D Galilei Symmetries

- time translations : $\delta t = \xi^0$ <u>but not</u> $\delta t = \lambda^i x^i$!
- space translations : $\delta x^i = \xi^i \qquad i = 1, 2, 3$
- spatial rotations : $\delta x^i = \lambda^i{}_j \, x^j$
- Galilean boosts : $\delta x^i = \lambda^i t$

'Conventional' Constraints

symmetry	generators	gauge field	#	curvatures	#
time translations	H	τ_μ		$\tau_{\mu\nu} = \partial_{[\mu}\tau_{\nu]}$	6
space translations	P^A	${E_\mu}^A$		${R_{\mu\nu}}^A(P)$	18
Galilean boosts	G^A	${\Omega_\mu}^A$	12	${R_{\mu\nu}}^A(G)$	
spatial rotations	J^{AB}	${\Omega_\mu}^{AB}$	12	${R_{\mu\nu}}^{AB}(J)$	

$\mu = 0, 1, 2, 3; A = 1, 2, 3$

${R_{\mu\nu}}^A(P) = 0 \quad (18)$: does only solve for part of ${\Omega_\mu}^A, {\Omega_\mu}^{AB}$

From Galilei to Bargmann

the zero commutator

$$[G_A, P_B] = 0$$

implies that a massive particle with non-zero spatial momentum P_B cannot by any boost transformation G_A be brought to a rest frame $\Rightarrow$

$$[G_A, P_B] = \delta_{AB} Z \qquad \rightarrow \qquad \text{extra gauge field } m_\mu$$

and additional 6 conventional constraints: $R_{\mu\nu}(Z) = 0$

'Geometric' Constraints

$$\tau_{\mu\nu} \equiv \partial_{[\mu}\tau_{\nu]} = 0 \quad \rightarrow \quad \tau_\mu = \partial_\mu \rho \quad \text{with} \quad \tau_\mu \quad \text{clock function}$$

$$\Delta T = \int_{\mathcal{C}} \mathrm{d}x^\mu \tau_\mu = \int_{\mathcal{C}} \mathrm{d}\rho \ \text{ is path-independent} \quad \rightarrow \quad \text{absolute time}$$

NC Gravity couples to particles

what about strings?

Comparing Limits

The NR limit of a particle coupled to general relativity is finite provided that the particle is coupled to a (central charge) gauge field B_μ which is expressed in terms of the timelike NC Vierbein field τ_μ as

$$B_\mu = \omega \tau_\mu$$

with $\omega \to \infty$ in the NR limit

Similarly, the string should be coupled to a 2-form gauge field $B_{\mu\nu}$ with

$$B_{\mu\nu} = \omega^2 \epsilon_{AB} \tau_\mu{}^A \tau_\nu{}^B \,, \qquad A = 0, 1$$

defining a string NC geometry with generalized clock functions $\tau_\mu{}^A$

see also Gomis, Ooguri (2001)

String Galilei Symmetries

Gomis, Ooguri (2001); Andringa, Gomis, de Roo + E.B. (2012)

$$D+1 \text{ flat indices} \rightarrow \begin{cases} 2 \ \text{longitudinal indices } A \\ D-1 \ \text{transverse indices } A' \end{cases}$$

longitudinal translations	H_A
transverse translations	$P_{A'}$
string Galilei boosts	$G_{AB'}$
longitudinal Lorentz rotations	M_{AB}
transverse spatial rotations	$J_{A'B'}$

Conventional versus Geometric Constraints

$$R_{\mu\nu}{}^{A}(H) = D_{[\mu}(\Omega)\tau_{\nu]}{}^{A} = 0 \quad \text{with} \quad \tau_\mu{}^A \ \text{generalized clock function}$$

4D : 12 components

- 4 conventional constraints : solves for $\Omega_\mu{}^{AB} = \epsilon^{AB}\Sigma_\mu$
- 8 geometric constraints

Noncentral Extension

$$[G_{AA'}, P_{B'}] = 0 \qquad \rightarrow \qquad [G_{AA'}, P_{B'}] = \delta_{A'B'} Z_A$$

The independent string NC fields $\{\tau_\mu{}^A, e_\mu{}^{A'}, m_\mu{}^A\}$ transform as follows:

$$\begin{aligned}
\delta\tau_\mu{}^A &= \Lambda^A{}_B \tau_\mu{}^B\,, \\
\delta E_\mu{}^{A'} &= \Lambda^{A'}{}_{B'} E_\mu{}^{B'} - \Sigma_A{}^{A'} \tau_\mu{}^A\,, \\
\delta m_\mu{}^A &= D_\mu \sigma^A + \Sigma^A{}_{A'} E_\mu{}^{A'}
\end{aligned}$$

longitudinal metric: $\quad \tau_{\mu\nu} \equiv \tau_\mu{}^A \tau_\nu{}^B \eta_{AB}$

transverse 'metric': $\quad H_{\mu\nu} \equiv E_\mu{}^{A'} E_\nu{}^{B'} \delta_{A'B'} + \left(\tau_\mu{}^A m_\nu{}^B + \tau_\nu{}^A m_\mu{}^B\right) \eta_{AB}$

Outline

NR Limit Polyakov Particle

$$S_{\mathrm{Pol.}} = -\tfrac{1}{2}\int d\tau \Big\{ -\frac{1}{e} E_\mu{}^A \dot{x}^\mu E_\nu{}^B \dot{x}^\nu \eta_{AB} + M^2 e - 2M B_\mu \dot{x}^\mu \Big\}$$

$$\begin{aligned} S_{\mathrm{Pol.}}(c^2) &= -\tfrac{1}{2}\int d\tau \frac{1}{e} c^2 \big[\tau_\mu \dot{x}^\mu - me\big]^2 \\ &= -\tfrac{1}{2}\int d\tau \frac{1}{e} \Big\{\lambda(\tau_\mu \dot{x}^\mu - me) - \frac{1}{4c^2}\lambda^2\Big\} \quad \Rightarrow \end{aligned}$$

$$S_{\mathrm{Pol.}}(\mathrm{N.R.}) = -\tfrac{1}{2}\int d\tau \frac{1}{e} \Big\{\dot{x}^\mu \dot{x}^\nu H_{\mu\nu} + \lambda\big(\tau_\mu \dot{x}^\mu - me\big)\Big\}$$

Nambu-Goto (NG)

- relativistic NG is non-linear in longitudinal and transverse embedding coordinates

- nonrelativistic NG is nonlinear in longitudinal embedding coordinates only

The Nonrelativistic Polyakov String

$$h_{\alpha\beta} = e_\alpha{}^a e_\beta{}^b \eta_{ab}$$

$$e_\alpha \equiv e_\alpha{}^0 + e_\alpha{}^1\,, \qquad \bar{e}_\alpha \equiv e_\alpha{}^0 - e_\alpha{}^1$$

$$\tau_\mu \equiv \tau_\mu{}^0 + \tau_\mu{}^1\,, \qquad \bar{\tau}_\mu \equiv \tau_\mu{}^0 - \tau_\mu{}^1$$

$$S_{\text{Pol.}} = -\frac{T}{2}\int d^2\sigma \left[\sqrt{-h}\, h^{\alpha\beta}\,\partial_\alpha x^\mu \partial_\beta x^\nu H_{\mu\nu} + \epsilon^{\alpha\beta}\left(\lambda\, e_\alpha \tau_\mu + \bar{\lambda}\, \bar{e}_\alpha \bar{\tau}_\mu\right)\partial_\beta x^\mu\right]$$

$$-\frac{T}{2}\int d^2\sigma\, \epsilon^{\alpha\beta}\partial_\alpha x^\mu \partial_\beta x^\nu B_{\mu\nu}$$

Nambu-Goto : $e_\alpha{}^a \neq \tau_\alpha{}^A$ but $h_{\alpha\beta} = \tau_{\alpha\beta}$ up to a scale factor

World-sheet Analysis

Gomis, Ooguri (2001)

Conformal Gauge

$$\sqrt{-h}\, h^{\alpha\beta} = \eta^{\alpha\beta}$$

Flat Spacetime

$$\tau_\mu{}^A = \delta_\mu^A\,, \qquad E_\mu{}^{A'} = \delta_\mu^{A'}\,, \qquad m_\mu{}^A = 0$$

$$\Downarrow$$

$$S = -\frac{T}{2}\int d^2\sigma \left(\partial x^{A'}\,\bar\partial x^{B'}\,\delta_{A'B'} + \lambda\,\bar\partial X + \bar\lambda\,\partial\bar X\right)$$

Outline

Relativistic T-duality

Buscher (1987,1988); Roček, Verlinde (1992)

adapted coordinates: $x^\mu = (y, x^i)$ $\qquad k^\mu \partial_\mu = \partial_y$

$$S_{\text{parent}} = \underbrace{S_{\text{Pol.}}(\partial_\alpha y \to v_\alpha)}_{\text{quadratic in } v_\alpha!} - T \int d^2\sigma \epsilon^{\alpha\beta} \tilde{y} \partial_\alpha v_\beta$$

$$\frac{\delta S_{\text{parent}}}{\delta v_\alpha} = 0 \qquad \to \qquad v_\alpha \text{ is solved for in terms of dual coordinate } \tilde{y}$$

$$\tilde{G}_{yy} = \frac{1}{G_{yy}} \qquad \Rightarrow \qquad R \Leftrightarrow \frac{1}{R}$$

Non-relativistic Longitudinal T-duality

$$\tau_\mu{}^0 k^\mu = 0, \qquad \tau_\mu{}^1 k^\mu \neq 0\,, \qquad E_\mu{}^{A'} k^\mu = 0$$

adapted coordinates: $x^\mu = (y, x^i) \qquad k^\mu \partial_\mu = \partial_y$

$$S_{\text{parent}} = S_{\text{Pol.}}(\partial_\alpha y \to v_\alpha) - T\int d^2\sigma \epsilon^{\alpha\beta} \tilde{y} \partial_\alpha v_\beta$$

$\dfrac{\delta S_{\text{parent}}}{\delta v_\alpha} = 0 \quad \to \quad v_\alpha$ is solved for in terms of dual coordinate $\tilde{y}$ and the Lagrange multipliers $\lambda, \bar{\lambda}$ if we dualize in the longitudinal direction

$\to \quad \lambda, \bar{\lambda}$ are no Lagrange multipliers anymore!

Dual Action

$$\tilde{S}_{\text{long.}} = -\frac{T}{2}\int d^2\sigma \left(\sqrt{-h}\, h^{\alpha\beta}\,\partial_\alpha \tilde{x}^\mu \partial_\beta \tilde{x}^\nu\, \tilde{G}_{\mu\nu} + \epsilon^{\alpha\beta}\,\partial_\alpha \tilde{x}^\mu \partial_\beta \tilde{x}^\nu\, \tilde{B}_{\mu\nu}\right)$$

$$\text{with} \quad \tilde{x}^\mu = (\tilde{y}, x^i) \quad \text{and} \quad \tilde{G}_{yy} = 0: \ \text{lightlike direction}$$

- The longitudinal spatial T-dual of the NR string is the Polyakov string moving in a GR background with a lightlike direction
    - DLCQ relativistic string
    - Rigid susy in Lorentzian curved spacetimes

See, e.g., Cassani, Klare, Martelli, Tomasiello, Zaffaroni (2014)

- The transverse spatial T-dual of the NR string is again a NR string with a transverse spatial isometry direction à la Buscher

Outline

3D Extended Bargmann Gravity

$$S = \frac{1}{2\kappa^2}\int d^3x\,\left(E\,R(\Omega) - \epsilon^{\mu\nu\rho}B_\mu\partial_\nu A_\rho\right) \quad \Rightarrow$$

Chern-Simons action for 3D Extended Bargmann Algebra

Rosseel + E.B. (2016); Hartong, Obers (2016)

- The 2 central charges originate from the 2 vector fields

- 3D Extended Bargmann Gravity $\neq$ 3D NC gravity

4D Extended String Bargmann Gravity

$$S = \frac{1}{2\kappa^2}\int d^4x\,(E\,R(\Omega) - \epsilon^{\mu\nu\rho\sigma}B_{\mu\nu}\partial_\rho A_\sigma)$$

Grosvenor, Şimşek, Yan + E.B., work in progress

- The 2-form gives rise to the 2 noncentral extensions Z_A $(A = 0, 1)$ of the String Bargmann Algebra

- The 1-form gives rise to the extra central extension S of the Extended String Bargmann Algebra

A 4D First-Order Action

Grosvenor, Şimşek, Yan + E.B. (to appear)

$$S = \frac{1}{2\kappa^2}\int d^4x\,\epsilon^{\mu\nu\rho\sigma}\Big[-\tfrac{1}{2}\epsilon_{A'B'}E_\mu{}^{A'}E_\nu{}^{B'}\mathcal{R}_{\rho\sigma}(M) - \epsilon_{AB}\epsilon_{A'B'}\tau_\mu{}^{A}E_\nu{}^{A'}\mathcal{R}_{\rho\sigma}{}^{BB'}(G)$$

$$+\,\epsilon_{AB}\tau_\mu{}^{A}m_\nu{}^{B}\mathcal{R}_{\rho\sigma}(J) + \tfrac{1}{2}\epsilon_{AB}\tau_\mu{}^{A}\tau_\nu{}^{B}\mathcal{R}_{\rho\sigma}(S)\Big]$$

Relation to 3D Extended Bargmann Gravity

The 3D Chern-Simons action of Extended Bargmann gravity is obtained after a dimensional reduction over the longitudinal y-direction with $\mu = (y, i), i = 0, 1, 2$ followed by a truncation. For instance:

$$\tau_\mu{}^A = \begin{pmatrix} \tau_y{}^0 & \tau_y{}^1 \\ \tau_i{}^0 & \tau_i{}^1 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ \tau_i & 0 \end{pmatrix} \qquad \Rightarrow$$

$$S_{\rm CS} = \frac{k}{4\pi} \int d^3x\, \epsilon^{ijk} \Big[\epsilon_{A'B'} E_i{}^{A'} \mathcal{R}_{jk}{}^{B'}(G) - m_i \mathcal{R}_{jk}(J) - \tau_i \mathcal{R}_{jk}(S) \Big]$$

Outline

Nonrelativitic String Theory

- Nonrelativistic String Theory can be defined independent of any limit of relativistic string theory
    - T-duality
    - Target Space Actions

- String NC Geometry is to NR string theory what Riemannian geometry is to relativistic string theory

Generalization to $p+3$ dimensions

In $p+3$ dimensions the divergencies originating from the Einstein-Hilbert term are cancelled by introducing a $p+1$-form gauge field B and a 1-form gauge field A:

$$\epsilon^{\mu_1\cdots\mu_{p+3}}\mathcal{B}_{\mu_1\cdots\mu_{p+1}}\partial_{\mu_{p+2}}\mathcal{A}_{\mu_{p+3}}$$

The non-relativistic limit leads to an action for extended p-brane Bargmann gravity or extended co-dimension 2 Bargmann gravity

They are all related by dimensional reduction of a spatial direction of the p-brane followed by a truncation.

Open Issues

- Does β-function calculation leads to consistent backgrounds?

- Nonrelativistic holography?

Gopakumar, Bagchi (2009)

- Anyonic strings?

cp. to Duval, Horvathy (2000); Jackiw, Nair (2000); Mezincescu, Townsend (2010)

See also, Gußmann, Sarkar amd Wintergerst (2018)

- Lie algebra expansions

Izquierdo, private communication

- Double Field Theory?

S. M. Ko, C. Melby-Thompson, R. Meyer and J.-H. Park (2015)

Take Home Message

Nonrelativistic String Theory can be studied!

Gomis, Ooguri (2001)