Exercises Karpacz School

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1 I. A simple r-process calculation

1.1 Rationale

The astrophysical r-process is thought to occur in an environment with such high neutron densities and temperatures that neutron captures and photodissociation reactions operate on a time scale much shorter than beta decays. Under these conditions, an equilibrium between neutron captures and photodissociation develops, $(n,\gamma) \rightleftarrows (\gamma,n)$. In this exercise, we will show that this equilibrium condition approximately sets the r-process path and the nuclei abundances along it.

1.2 Requests

Deduce the expression on slide 45 from the lecture that relates the abundance of two neighbouring nuclei with even neutron number:

$$\frac{Y(Z,A+2)}{Y(Z,A)} = n_n^2 \left(\frac{2\pi\hbar^2}{m_u k_B T}\right)^3 \left(\frac{A+2}{A}\right)^{3/2} \frac{G(Z,A+2)}{4G(Z,A)} \exp\left[\frac{S_{2n}(Z,A+2)}{k_B T}\right] \tag{1}$$

Assuming a temperature of 0.69 GK and a neutron density of 2.5×10^{24} cm^{-3}, determine the r-process path.

As a first approximation one can assume that for each element the path consists of a single nucleus corresponding to the maximum of equation (1), i.e. the most neutron-rich nucleus with even neutron-number that fullfills the condition

$$S_{2n}(Z,A) \gtrsim S_{2n}^0 = \frac{2T_9}{5.04}\left(34.075 - \log n_n + \frac{3}{2}\log T_9\right)$$

Use the two neutron separation energies as provided in the file `frdm.dat`

Alternatively one can iterate equation (1) starting with an arbitrary value of $Y(Z,A)$ for the most neutron deficient nucleus. This defines the abundances of the different isotopes up to a normalization factor.

In order to fix the normalization we can use the beta-flow approximation discussed on slide 49 that gives the condition for each Z value:

$$\sum_A \lambda_\beta(Z,A)Y(Z,A) = \text{constant} = 1 \tag{2}$$

where $\lambda_\beta(Z,A) = \ln 2/t_{1/2}(Z,A)$ and the half-lives are given in the file `moeller-beta-rates.dat`. Compare your results with those obtained by full dynamical r-process calculations given in the file `abundances.dat`. Notice that he abundances in the file are normalized such

$$\sum_{A,Z} A\,Y(Z,A) = 1 \tag{3}$$

2 II. a kilonova light curve model

2.1 Rationale

The detection of an electromagnetic counterpart, AT 2017gfo, associated to the gravitational wave detection GW170817 (compatible with the merger of two neutron stars) opened the era of multimessenger astrophysics. Eleven hours after the arrival of the GW signal (and for many days after it), telescopes all over the world and in space detected ultraviolet (UV), visible (V) and infrared (IR) radiation from the merger remnant, compatible with the predictions of the kilonova/macronova electromagnetic transient expected from a so called kilonova. In particular, the signal showed a peak in UV-V less than one day after the merger (for example, in V and R bands), and a peak at IR frequences around 5 days after the merger (in K_s bands). These rich light curves point to a complex and structured ejection of matter, in which weak reactions play a key role in setting the properties of the ejecta. The goal of the exercise is to deduce from the observed properties of AT 2017gfo information about the binary merger ejecta.

The time (t_p), the photon luminosity (L_p) and the black body temperature (T_p), determined from the Stefan-Boltzmann law $L_p = 4\pi\sigma R_p^2 T_p^4$, at kilonova peak can be expressed as (see slide 67 from the lecture):

$$t_p \approx 1.53 \text{ day} \left(\frac{\kappa}{1 \text{ cm}^2 \text{ g}^{-1}}\right)^{1/2} \left(\frac{M}{0.01\, M_\odot}\right)^{1/2} \left(\frac{v}{0.1\, c}\right)^{-1/2} \tag{4}$$

$$L_p \sim 1.14 \times 10^{41} \text{ erg s}^{-1} \left(\frac{\kappa}{1 \text{ cm}^2 \text{ g}^{-1}}\right)^{-\alpha/2} \left(\frac{M}{0.01\, M_\odot}\right)^{1-\alpha/2} \left(\frac{v}{0.1\, c}\right)^{\alpha/2} \tag{5}$$

$$T_p \sim 5600 \text{ K} \left(\frac{\kappa}{1 \text{ cm}^2 \text{ g}^{-1}}\right)^{-(\alpha+2)/8} \left(\frac{M}{0.01\, M_\odot}\right)^{-\alpha/8} \left(\frac{v}{0.1 c}\right)^{(\alpha-2)/8} \tag{6}$$

The parameter α describes the evolution of the nuclear heating rate coming from the decay of r-process elements:

$$\dot{Q} = 10^{10} \text{ erg s}^{-1} \text{ g}^{-1} \left(\frac{t}{1 \text{ day}}\right)^{-\alpha} . \tag{7}$$

Detailed nuclear network calculations show that $\alpha \approx 1.3$ on timescales of a few days. Model the ejecta as two components. One component has $L_{p,1} \approx 4.4 \times 10^{41}$ erg s^{-1} and $T_{p,1} \approx 6000$ K at $t_{p,1} \approx 1$ day; the second one $L_{p,2} \approx 1.0 \times 10^{41}$ erg s^{-1} and $T_{p,2} \approx 2400$ K at $t_{p,2} \approx 5.6$ day. Determine the combination of ejected mass velocity and opacity that give reproduce those values of luminosity, temperature and peak time.

A kilonova light curve `model` is supplied developed by Albino Perero [1]. A description of the model is provided in slides 32–34 of the following `lecture`. The model provides a 2-component, anisotropic description of the ejecta, and gives light curves as a function of time. Play with its free parameters to constraint the properties of the ejecta to reproduced the observed features of AT 2017gfo. Use the masses, opacity and velocities of the previous point as a first guess. The code is written in python. Before running it, open the file mkn.py and replace all the six strings "@@ ... @@ " with the user-defined values. Then, to run it, type in a shell "python mkn.py" .

3 III. the chirp mass from a GW signal

3.1 Rationale

The emission of GWs from compact binaries carries away energy and angular momentum, leading to a reduction of the eccentricity and relative distance, and eventually to the merger of the system. The corresponding GW signal is often described as a chirp signal: its frequence and amplitude increase with time up to merger. At the first-order in post-Newtonian theory, the chirp is directly related with a combination of the masses of the merging bodies, known as chirp mass, $\mathcal{M}_{\text{chirp}}$. Goal of this exercise is the deduction of the chirp mass from the published data of GW detections.

Assume that two bodies, of masses m_1 and m_2, are orbiting each other and are still widely separated (i.e., still not merging), on a circular ($e = 0$, where e is the eccentricity), Keplerian orbit of radius $a \gg R_{\text{NS}}$, and located at a distance $r \gg a$ from the detector. The system emits GW radiation.

- Advanced Ligo and Virgo are sensible to GW frequences in the interval between 50 and 1000 Hz. Using the Kepler law, compute the distance between the two bodies at the edges of the band assuming a total mass of $2.8M_\odot$ (BNS case), and compare with with typical NS radii.
- Deduce the expression for the evolution of the GW frequency $f_{\rm GW}$ as a function of time

$$\frac{\mathrm{d}f_{\rm GW}}{\mathrm{d}t} = \frac{96}{5}\pi^{8/3}\left(\frac{G\mathcal{M}_{\rm chirp}}{c^3}\right)^{5/3} f_{\rm GW}^{11/3} \tag{8}$$

from the expression of the (orbit averaged) reduction of the semi-major axis a and eccentricity e of a compact binary [2]:

$$\frac{\mathrm{d}a}{\mathrm{d}t} = -\frac{64}{5}\frac{G^3 m_1 m_2 (m_1+m_2)}{c^5 a^3\,(1-e^2)^{7/2}}\left(1+\frac{73}{24}e^2+\frac{37}{96}e^4\right) \tag{9}$$

$$\frac{\mathrm{d}e}{\mathrm{d}t} = -\frac{304}{15}\,e\,\frac{G^3 m_1 m_2 (m_1+m_2)}{c^5 a^4\,(1-e^2)^{5/2}}\left(1+\frac{121}{304}e^2\right) \tag{10}$$

where G and c the Newton constant and the speed of light. Notice the following relation between the Gravitational wave frequency and the orbital frequency $f_{\rm GW} = 2f_{\rm orb}$. Inside the resulting expression, identify the chirp mass $\mathcal{M}_{\rm chirp}$ defined as

$$\mathcal{M}_{\rm chirp} = \frac{(m_1 m_2)^{3/5}}{(m_1+m_2)^{1/5}} \tag{11}$$

- Peters' relation for $\mathrm{d}a/\mathrm{d}t$ provide also a direct estimate of the merger time scale due to the emission of GW, $\tau_{\rm GW}$, for observed binary NS systems at astrophysical distances, much larger than the distances "visible" by LIGO and Virgo:

$$\frac{\mathrm{d}a}{\mathrm{d}t} = \frac{a}{\tau_{\rm GW}} \tag{12}$$

Compute $\tau_{\rm GW}$ for the Hulse and Taylor BNS ($a = 0.013\,\mathrm{AU}$, $e = 0.6$, where $1\,\mathrm{AU} = 1.50 \times 10^{13}\,\mathrm{cm}$). Compare the estimated value with the result of integrating equation (12) assuming constant eccentricity or the full system of differential equations (9) and (10) that accounts to the change of eccentricity due to the emission of Gravitational Waves.

- Given the data set `ligo-virgo.dat` with the frequences measures by Advanced Ligo and Virgo as a function of time just before the merger, deduce the chirp mass. At which GW event does it correspond? How good can you guess the masses of the colliding body from the chirp mass?
Hint: Equation (8) can be rewritten as:

$$\frac{\mathrm{d}f_{\rm GW}}{\mathrm{d}t} = \Lambda f_{\rm GW}^{11/3} \tag{13}$$

with

$$\Lambda = \frac{96}{5}\pi^{8/3}\left(\frac{G\mathcal{M}_{\rm chirp}}{c^3}\right)^{5/3}. \tag{14}$$

Assuming the merger occurs at $t = 0$ it can be integrated for $t < 0$ to give

$$f_{\rm GW}^{-8/3}(t) + \frac{8}{3}\Lambda t - f_{\rm GW}^{-8/3}(0) = 0 \tag{15}$$

We can make a linear regression between $x = t$ and $y = f_{\rm GW}^{-8/3}$ assuming $y(t=0) = 0$ to find $\frac{8}{3}\Lambda$ and, in turn, to deduce $\mathcal{M}_{\rm chirp}$

References

[1] A. Perego, D. Radice, and S. Bernuzzi, Astrophys. J. **850**, L37 (2017).

[2] P. C. Peters, Phys. Rev. 136, B1224 (1964).