Gravitational-wave (GW) Astronomy of Compact Objects

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Karpacz winter school , 27th (1/3) March 2019

(3D-GR simulation in Kuroda, Kotake et al. in prep)

Menu:

1st . General Introduction

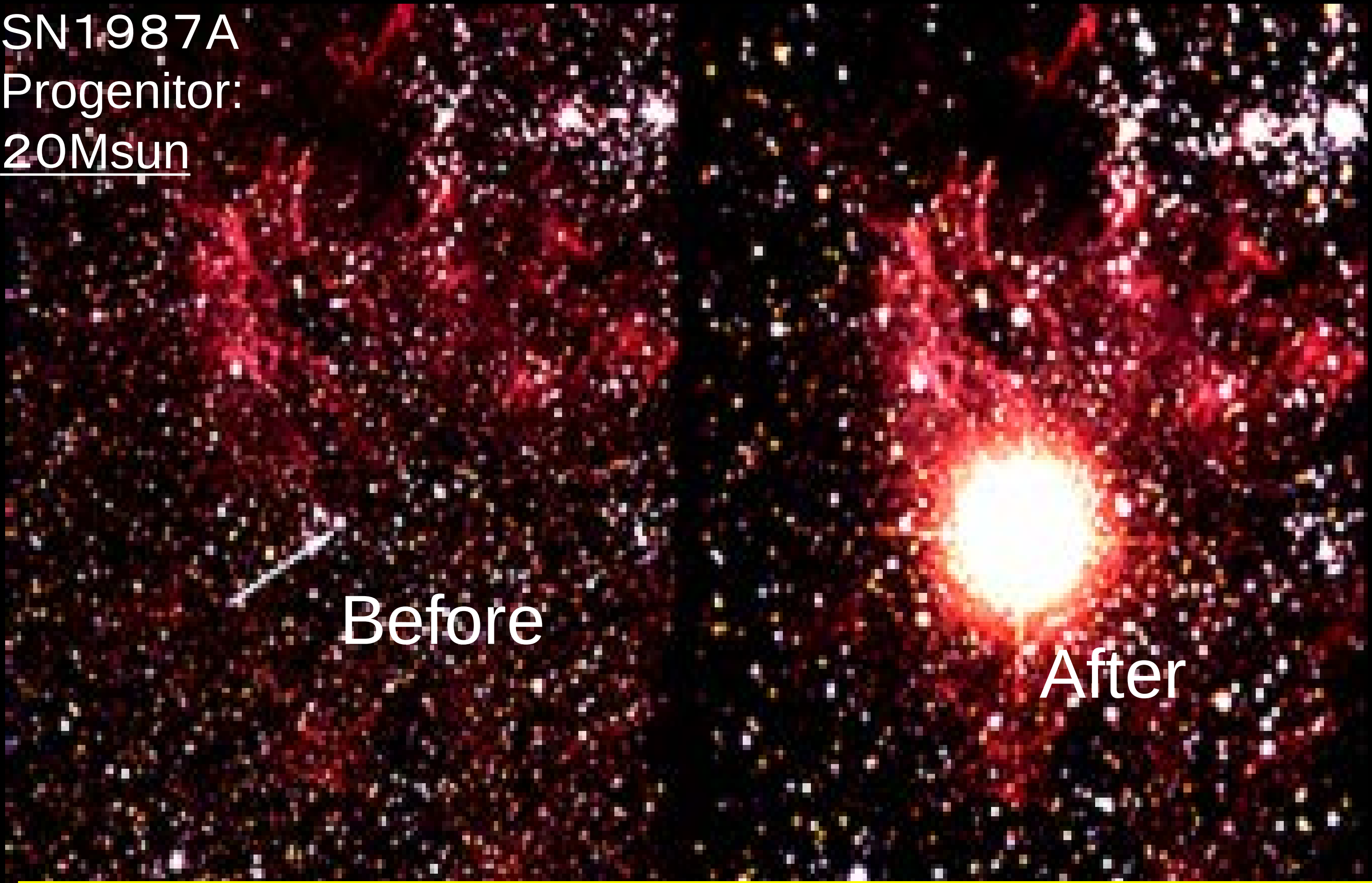
- ✓ **Why multi-messengers (inc. GW)?**
- ✓ **Basics of GW Physics and Detection**
- ✓ **First detection of GW150914**

2nd . Core-collapse supernova theory: how to solve "*numerically*" the space-time evolution of dying stars

3rd . GW signatures from core-collapse supernovae: what we can learn from future GW observation ?

Bottom-line **: Story about …. ?**

"Core-Collapse Supernova (CCSN)" is explosion of massive stars (> ~9 M_{sun})



- ✓ Origin of explosion asymmetry
- ✓ Origin of heavy elements
- ✓ Origin of explosion energy (∼ 10^{51} erg = 1 Bethe)

⟺ Explosion Mechanism

Bottom-line : Story about …. ?

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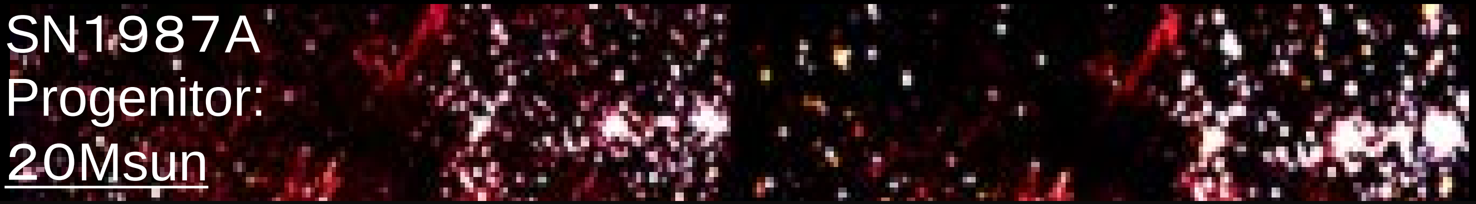

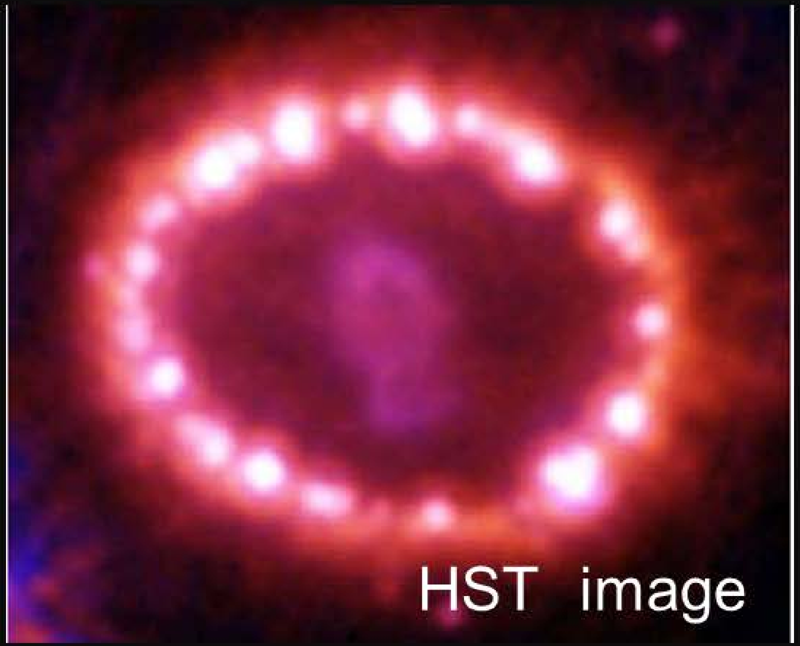

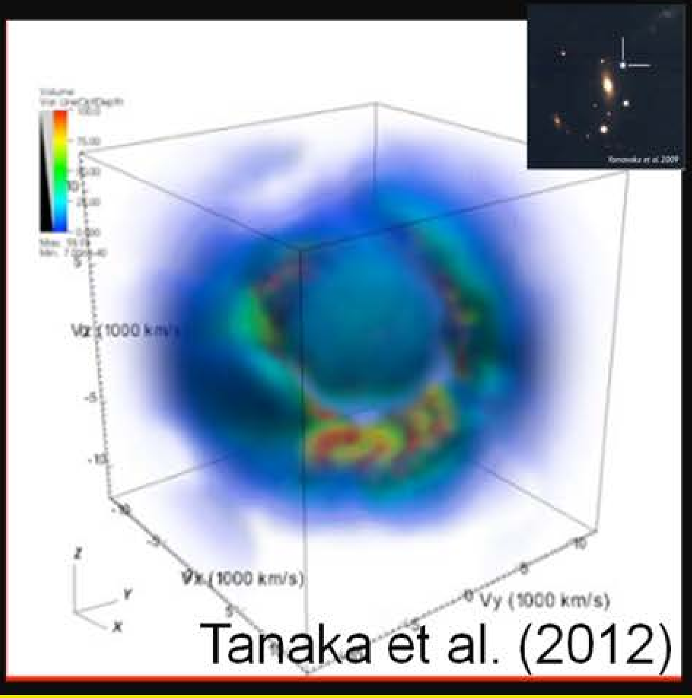

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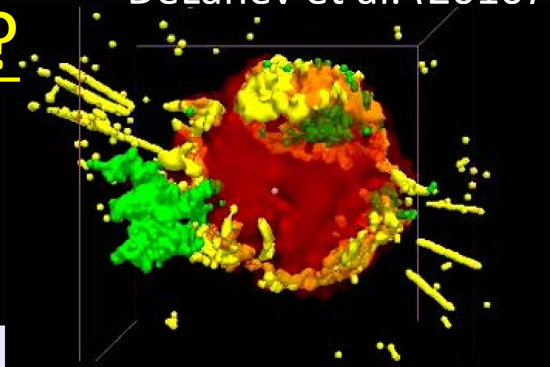
⟺ Explosion Mechanism

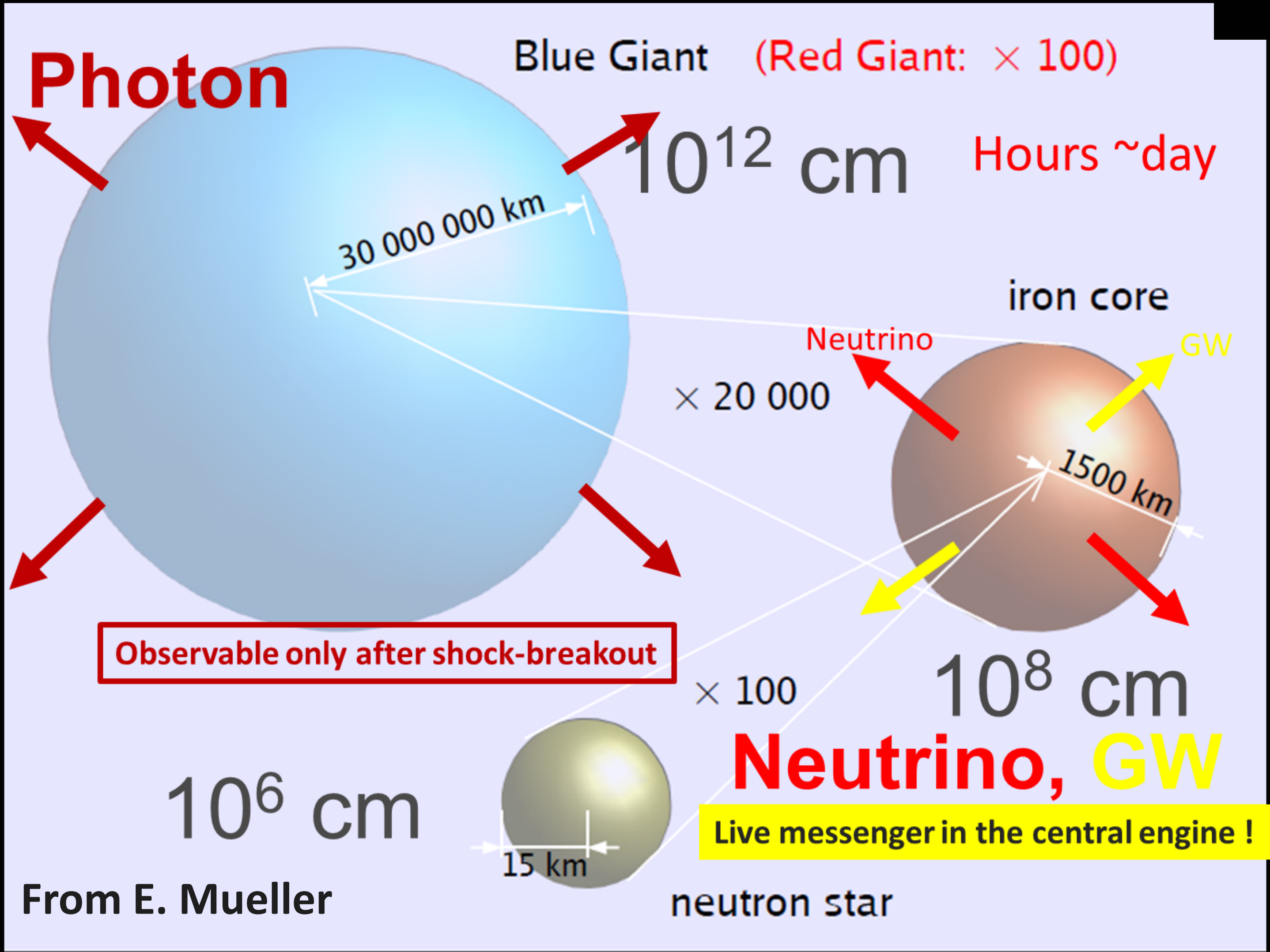
Not clear… over 50 years !
(lecture by Foglizzo)

Why "Multi-Messenger" observations ?

✓ Conventionally, "Astronomy" means electromagnetic-wave (EM) observation.

DeLanev et al. (2010)





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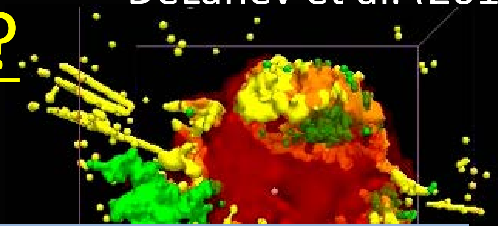

ears,
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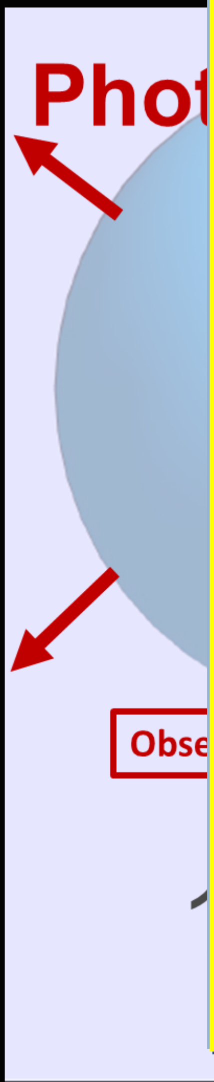


Multidimensionality
(origin of anisotropy)

Self Consistent models

Signal Prediction

Exp. Mechanism
Thermodynamics
Dynamics

Data analysis

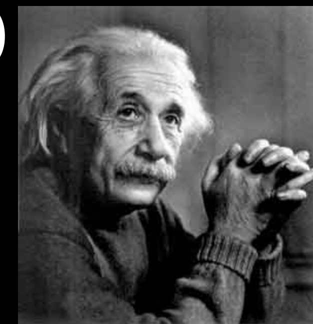
Neutrino signals
(Lecture by Mirizzi)
Gravitational wave(GW)
EM waves

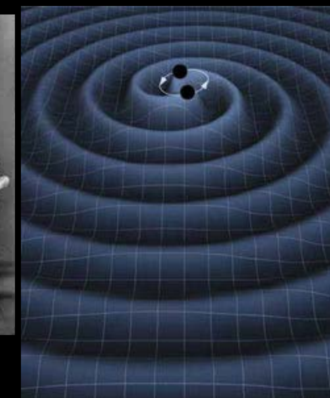
✓ A **Final goal of this field…**

⇒Clarify the formation mechanism of compact objects (Neutron stars and black holes) via muti-messenger observables (GW, neutrinos, multi-wavelength EM waves) (nucleosynthesis, lecture by Diehl !)

Gravitational wave (GW) homework from …?

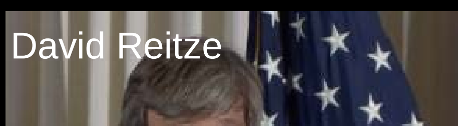
Einstein's theory of General relativity (1915)





GWs : a ripple of space-time
propagate at the speed of light

First announcement by LIGO (Laser Interferometer Gravitational Wave Observatory)



David Reitze

2016, Feb,12th 2 am (JST)

Second announcement
GW151226, third (GW1701204)

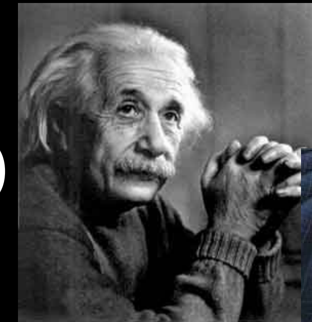
GWs from merging
BHs (**GW150914**) !!!

Credit: LIGO

Once is a chance, twice is a coincidence, thrice is a pattern !

Gravitational wave (GW) ?

Einstein's theory of General relativity (1915)



GWs : a ripple of space-time
propagate at the speed of light

✓ How to generate GW

Electromagn

charge

- Dynamical motion of electric-magnetic (EM) fiel

⇒ Electro-magnetic wa

Näherungsweise Integration der Feldgleichungen der Gravitation.

Von A. Einstein.

Bei der Behandlung der meisten speziellen (nicht prinzipiellen) Probleme auf dem Gebiete der Gravitationstheorie kann man sich damit begnügen, die $g_{\mu\nu}$ in erster Näherung zu berechnen. Dabei bedient man sich mit Vorteil der imaginären Zeitvariable $x_4 = it$ aus denselben Gründen wie in der speziellen Relativitätstheorie. Unter »erster Näherung« ist dabei verstanden, daß die durch die Gleichung

$$g_{\mu\nu} = -\delta_{\mu\nu} + \gamma_{\mu\nu} \qquad (1)$$

Basics of Gravitational-wave (GW) emission (1/3)

Naive analogy with Electromagnetics: **Quiz1: "leading term of EM emission" ?!**

Ans: Electric dipole radiation !

$$L_{\mathrm{dipole}} = \frac{2}{3}\frac{d^2}{dt^2}\boldsymbol{d}^2$$

Quiz 2: Dipole radiation from moving "uncharged" particles ?!

$$\boldsymbol{d} = \sum_{\mathrm{particles}\ i} m_i \boldsymbol{x}_i$$

Matter dipole moment

$$\frac{d}{dt}\boldsymbol{d} = \sum_{\mathrm{particles}\ i} m_i \frac{d}{dt}\boldsymbol{x}_i = P_i$$

$\Rightarrow$ $L_{\mathrm{dipole}} = 0$ (because $\frac{d}{dt}P_i = 0$)

Next-order emission in EM; Magnetic dipole radiation from electric current.

Matter-current dipole moment:

$$\boldsymbol{\mu} = \sum_{\mathrm{particles}\ i} \boldsymbol{x}_i \times m_i \boldsymbol{v}_i$$

magnetic dipole moment

$$\mathbf{m} = \int \boldsymbol{\mathcal{M}} d^3x = \frac{1}{2c}\int (\mathbf{x} \times \mathbf{J})\, d^3x$$

$\frac{d^2\mu}{dt^2} = 0$ (ang. momentum conservation) $\Rightarrow$ **No dipole emissions from matter motions**

$\Rightarrow \frac{d^2}{dt^2}$ (Quadrupole (>) matter moments) leads to Gravitational-wave (GW emission) !

Basics of Gravitational-wave (GW) emission (2/3)

(e.g., “Gravitation”, Misner, Thorne, Wheeler, 1973)

Standard Quadrupole formula (SQF)

Einstein equations

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \frac{8\pi G}{c^4}T_{\mu\nu}$$

Metric: expand up to 1st order deviation from Minkowski metric

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

→ Minkowski metric

Weak-field approx, $|h_{\mu\nu}| \ll 1.$

Note $R_{\mu\nu}\ (: curvature) \sim \frac{|g|}{r^2} \sim \left(\frac{\partial}{\partial x}\right)^2 h_{\mu\nu}$

$$\Box \bar{h}^{\mu\nu} = -\frac{16\pi G}{c^4}T^{\mu\nu}$$

Retarded green function

$$h_{jk}^{TT}(t, \boldsymbol{x}) = \frac{4G}{c^4}\int d^3x' \frac{T_{jk}^{TT}(t - \frac{|\boldsymbol{x}-\boldsymbol{x}'|}{c}, \boldsymbol{x}')}{|\boldsymbol{x}-\boldsymbol{x}'|}$$

Slow-motion approximation

SQF :

$$h_{i,j}^{\mathrm{TT}}(R) = \frac{2G}{c^4}\frac{1}{R}\frac{d^2}{dt^2}I_{i,j}^{\mathrm{TT}}\left(t - \frac{R}{c}\right)$$

Quadrupole formula

R: source source

here **Mass quadrupole moment**

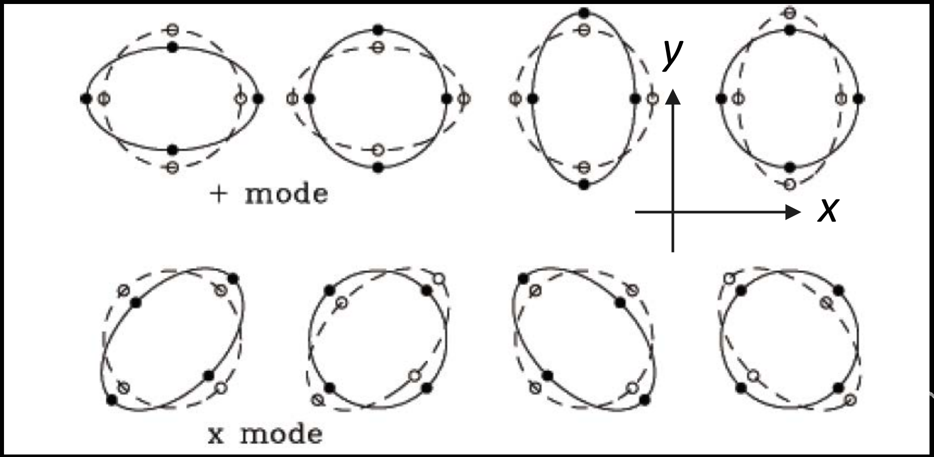
$$I_{i,j} = \int \rho(x)(x_i x_j - \frac{1}{3}x^2\delta_{i,j})d^3x$$

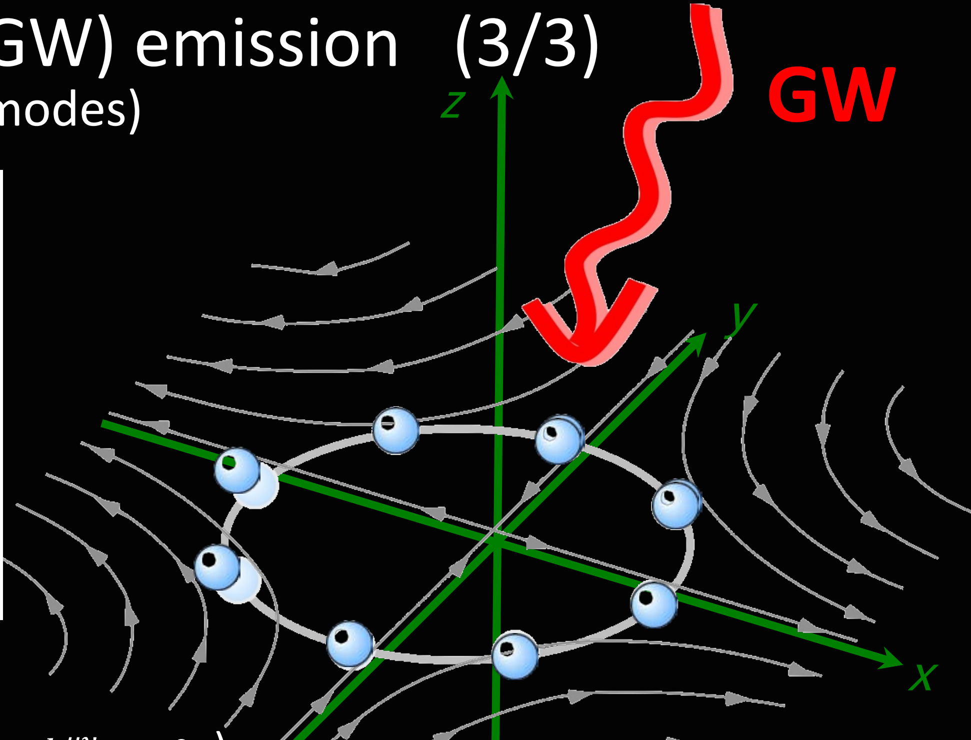
Order-of-magnitude

$$|h| \sim \frac{GM\,R_*^2}{c^4 R T_*^2} \sim \frac{GMv_*^2}{c^4 R}$$

Basics of Gravitational-wave (GW) emission (3/3)

- ✓ GWs have two polarization states (+, x modes)

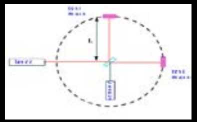




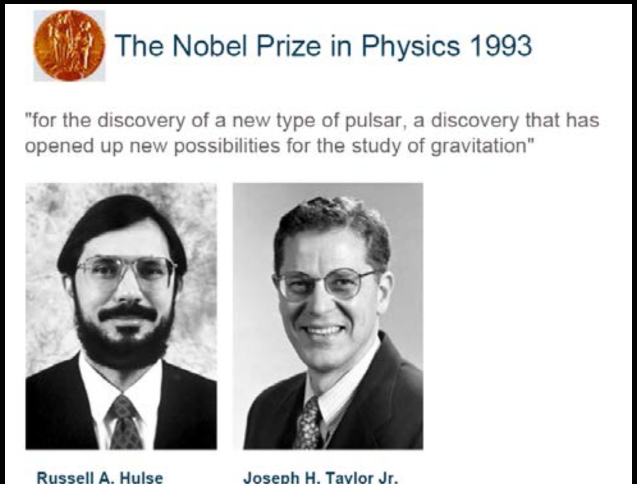
- ✓ why "two" ?: $g_{\mu\nu}(symmetric\ tensor)$

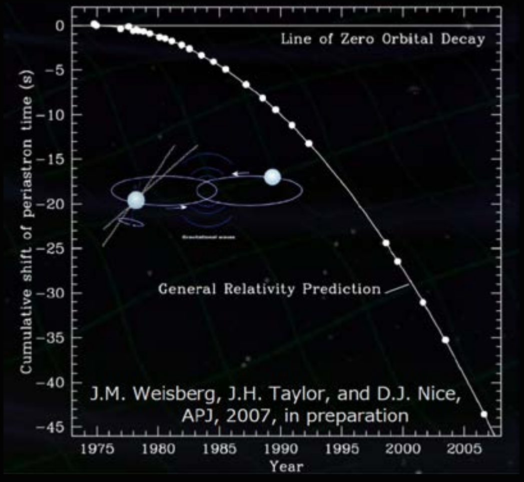
  $= \frac{4\times 5}{2} =$ 10 - 4 (choice of the gauge freedom: $h^{\mu\nu}{}_{,\nu} = 0$)

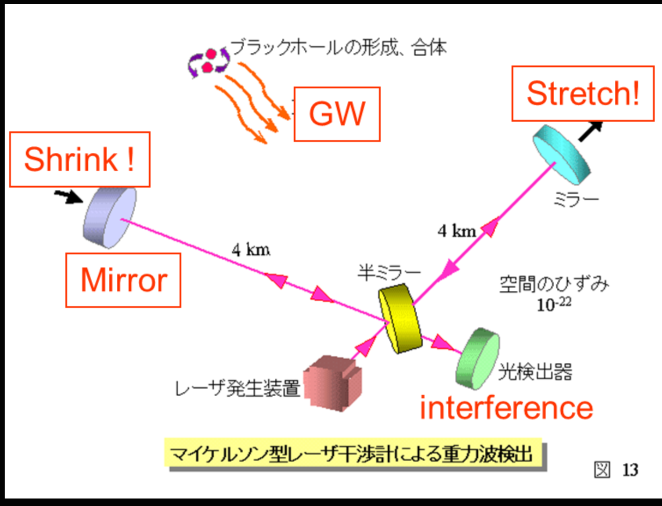
  - 4 (choice of the coordinates) = **「2」**



- ✓ *Indirect* proof of GWs: Hulse-Taylor pulsar

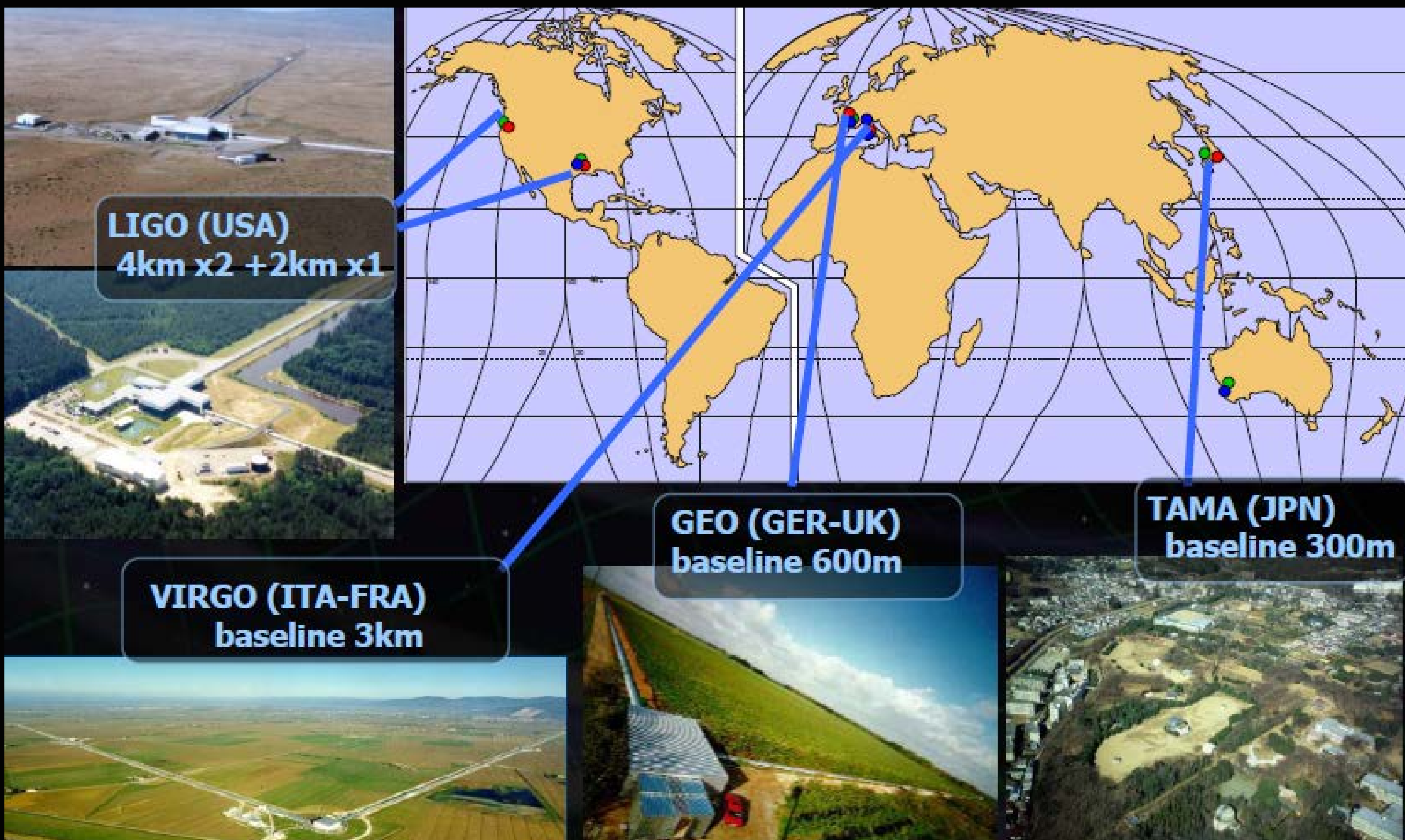






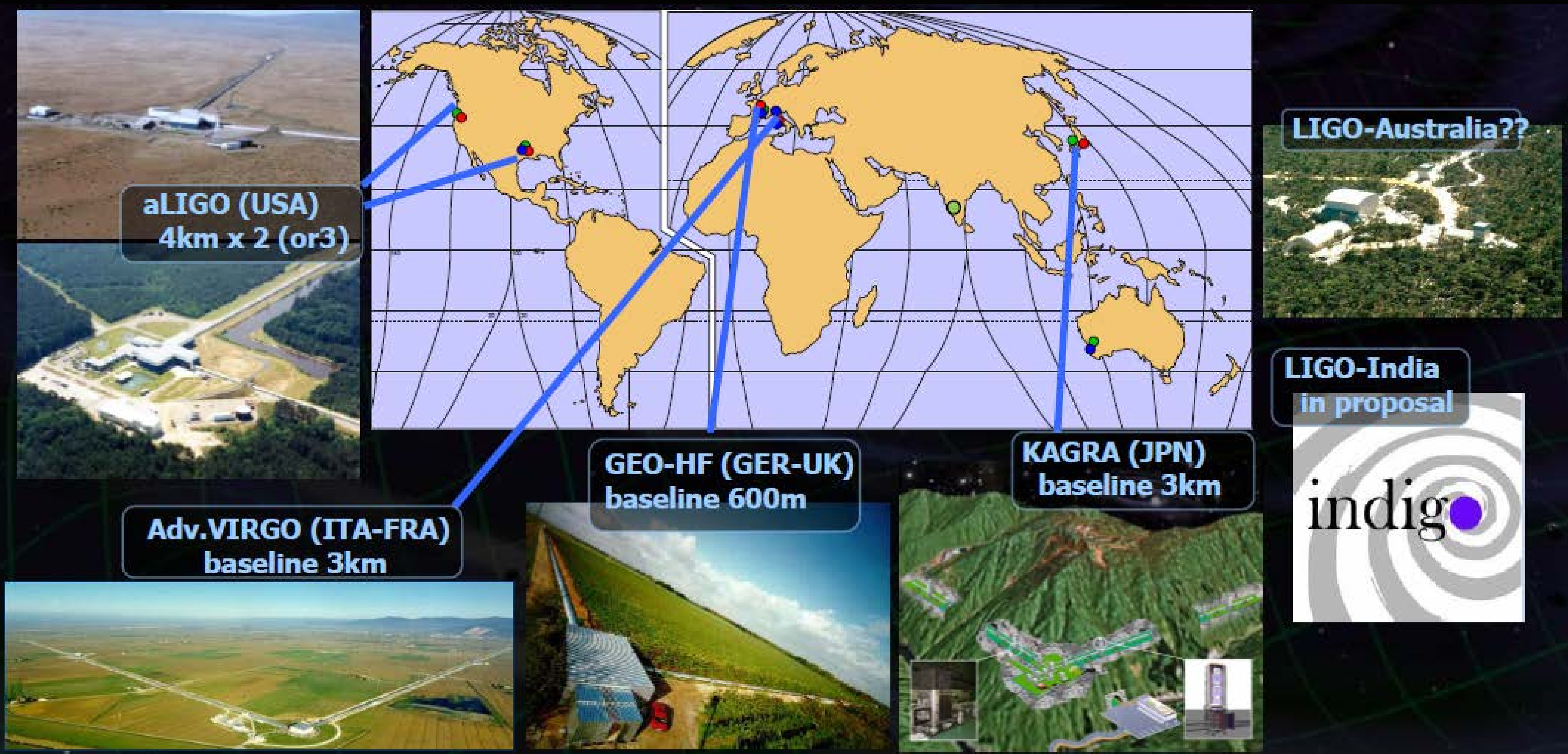
Short History: 1st generations of Laser-interferometers

- ✓ Start Observation ~ 1999 (TAMA of NAOJ)
- ✓ 6 interferometers by 4 projects -> International Newtorks
- ✓ Observational range (binary NS, lecture by Bauswein)-> ~20 Mpc:Event rate ~0.01/yr



✓ 2nd generations of Laser-interferometers

✓ International network of LIGO-Virgo (LVC) is working.
-> First joint GW detection of BH-BH merger (Abbott et al. (2017), PRL)
✓ Obs. Range > 200 Mpc -> Event rate ~ 10/ yr !

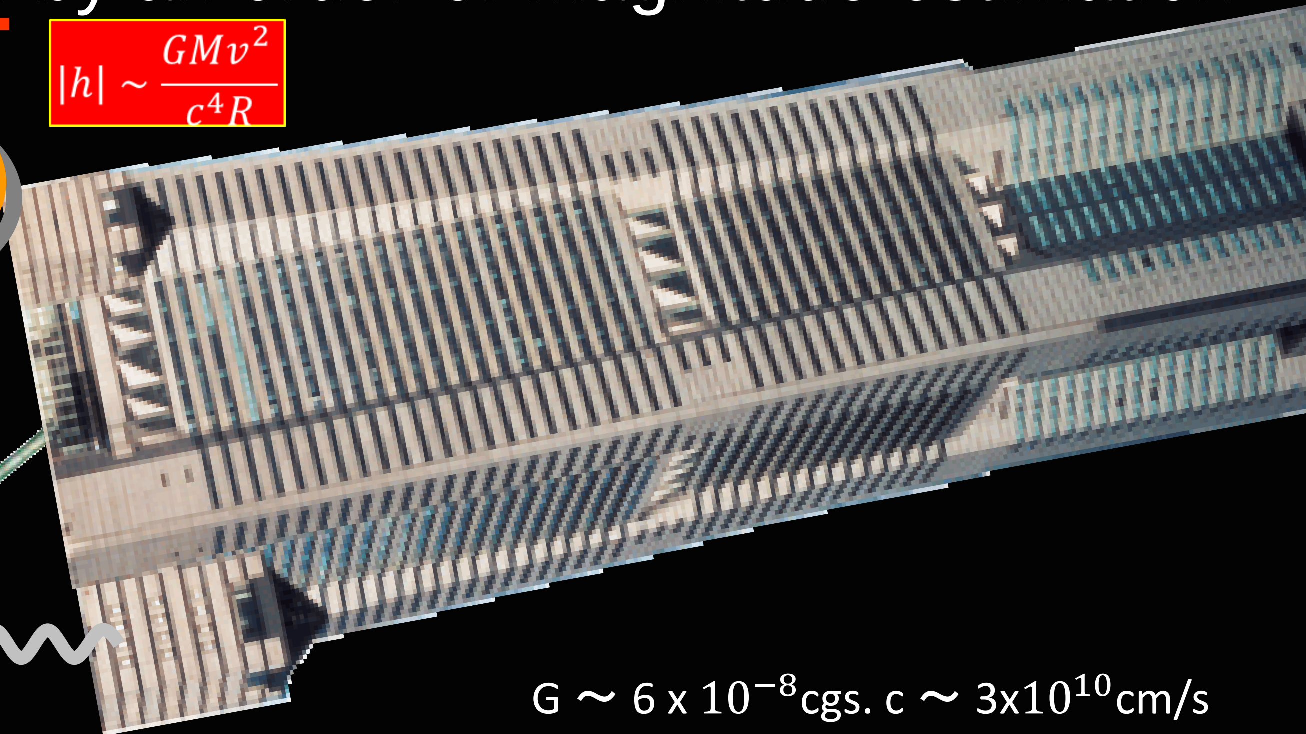


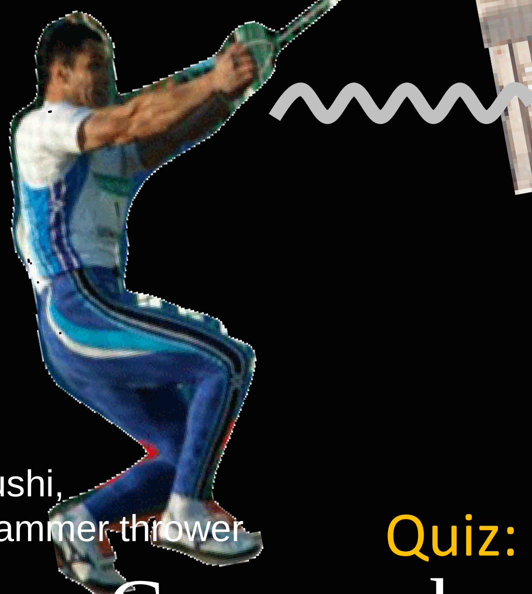
KAGRA =
Kamioka Gravitational Wave
Detector

✓GW amplitudes by an order-of-magnitude estimation

$$|h| \sim \frac{GMv^2}{c^4 R}$$

$$h \approx \frac{GMv^2}{rc^4}$$





Koji Murofushi,
Medalist Hammer thrower

G ~ 6 x 10^{-8}cgs. c ~ 3x10^{10}cm/s

Mass(M) : **600000** ton

Length : 300 m

Frequency: 2/s

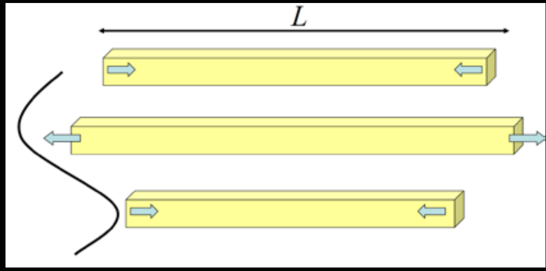
Observer distance(r): ? m

Quiz: How much is your "*h*" ?

Can we observe the GWs ??

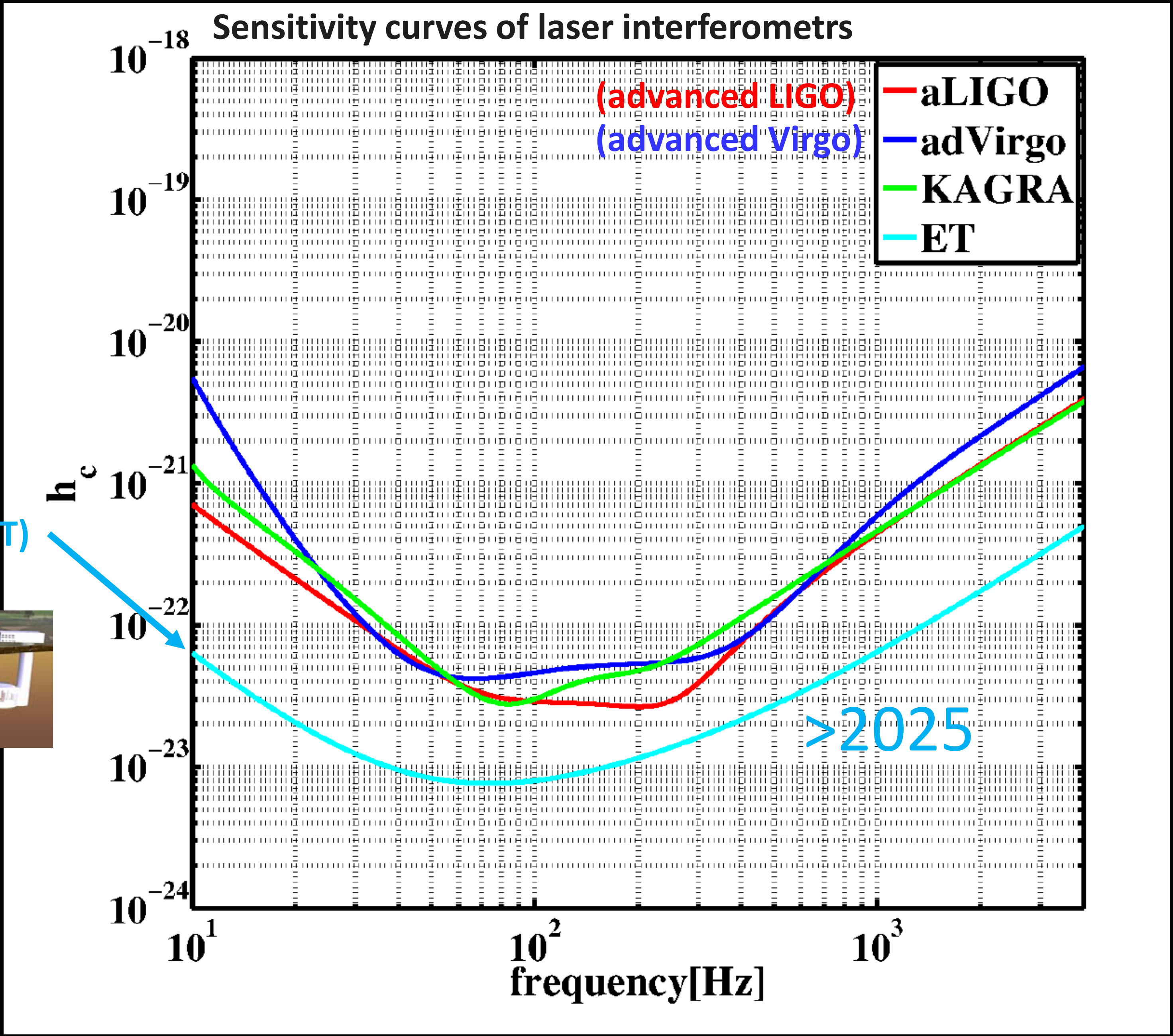
"*h*" is a strain.

$$\Delta L = h \times L$$

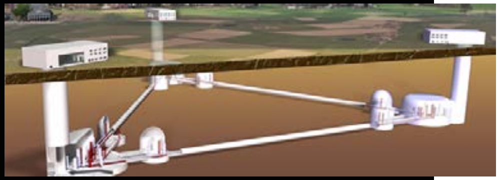


Can we observe the GW emission from the rotating building ?

$$f_{\mathrm{GW}} \sim \frac{1}{T_{\mathrm{dyn}}}$$



10 km long:
Einstein Telescope (ET)
could start ~2025.



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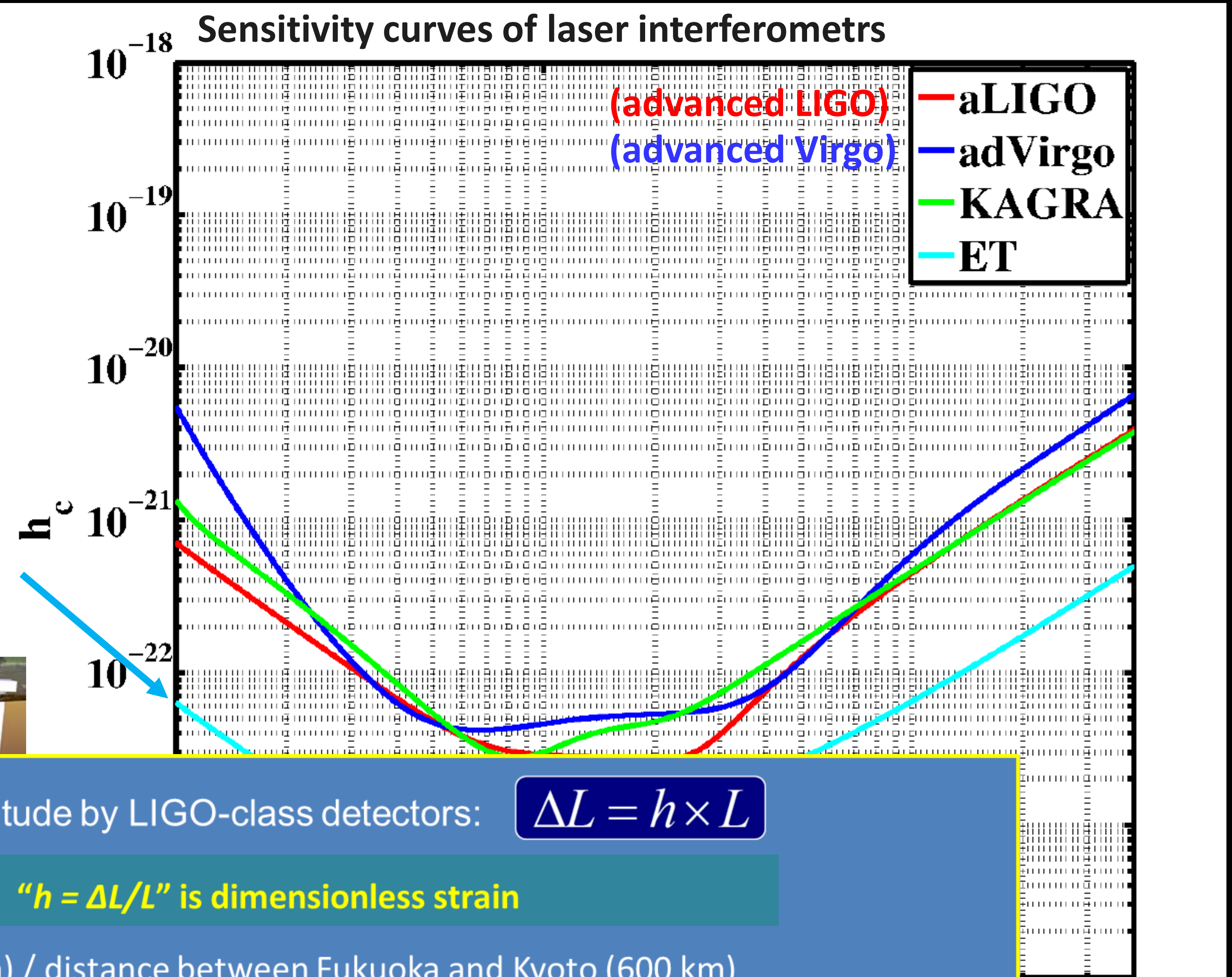


10 km long:
Einstein Telescope (ET)
could start ~2025.

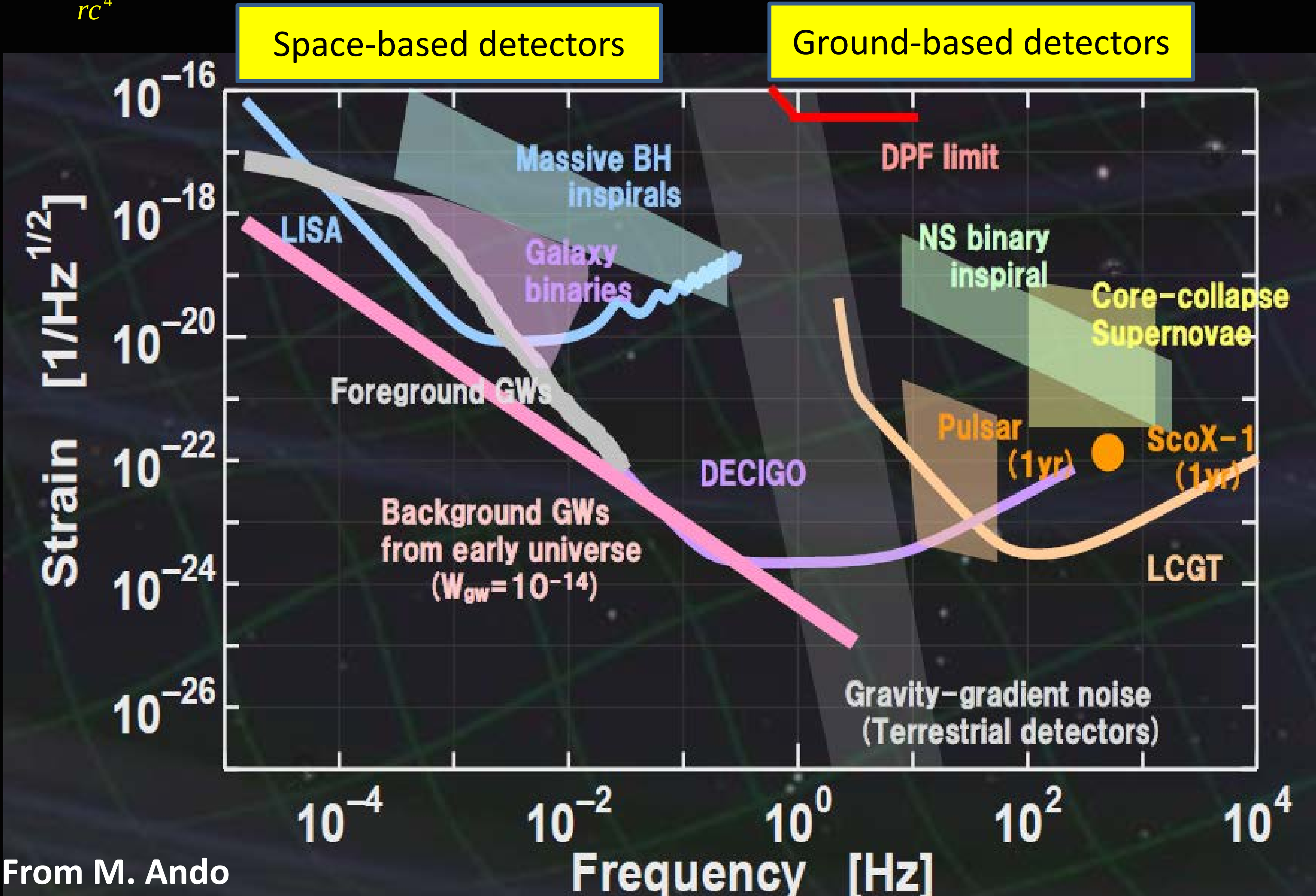
✓Detectable GW amplitude by LIGO-class detectors: $\Delta L = h \times L$

$h = 10^{-21}$ **"h = ΔL/L" is dimensionless strain**

- ~ Size of nuclei (5 fm) / distance between Fukuoka and Kyoto (600 km)
- ~ Size of atom (0.1 nm) / distance from Earth and Sun (1.5 hundred million km)
- ~ Can measure the change of the galaxy (0.1 million light year) with accuracy of 1 m !

More mass ! More velocity ! : Astrophysical GW sources

$$h \approx \frac{GMv^2}{rc^4}$$



From M. Ando

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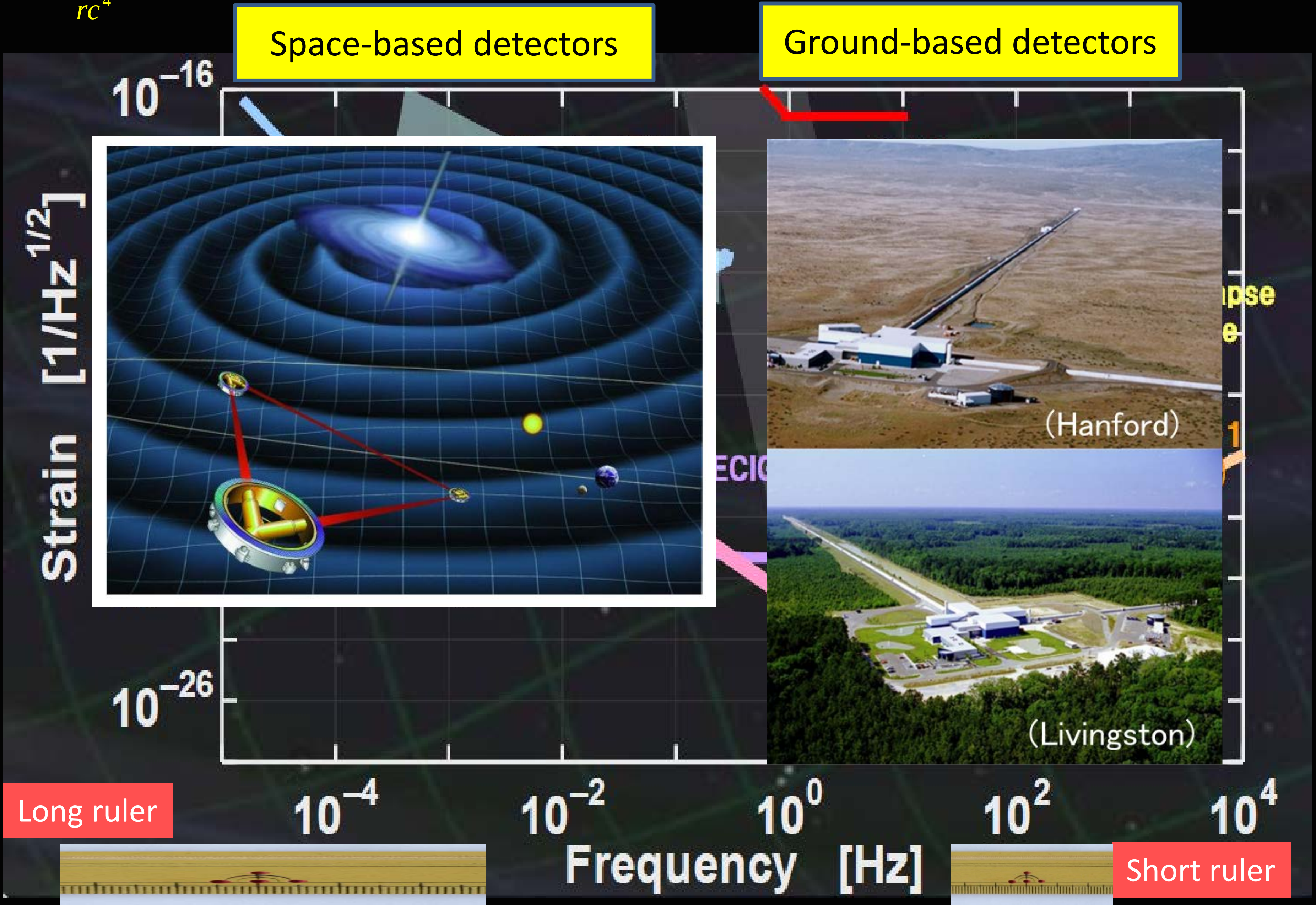
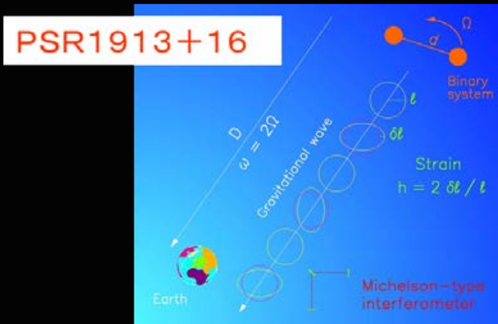
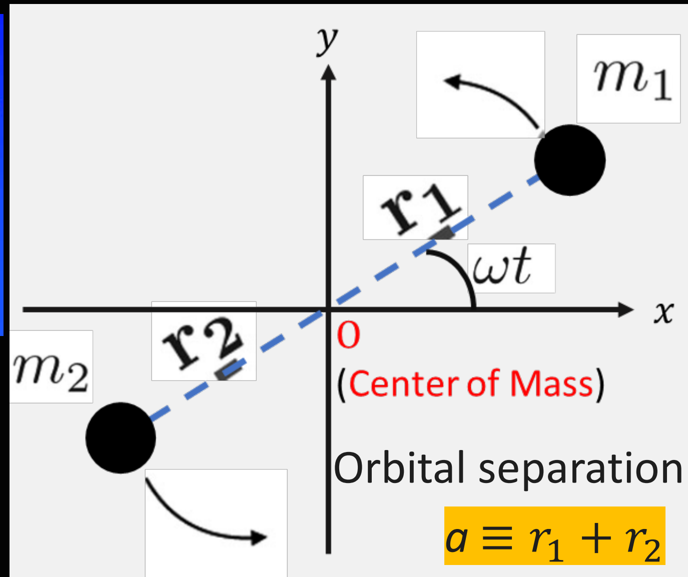


Why even now, difficult to detect GWs from Hulse-Taylor pulsar (1/4)?





$m_1 = (1.4398 \pm 0.0002) M_\odot$
$m_2 = (1.3886 \pm 0.0002) M_\odot$
Period: P = 7.75 hours
eccentricity: e = 0.671334(5)
Orbital separation: a

$$a = \left(\frac{G(m_1+m_2)P^2}{4\,\pi^2}\right)^{\frac{1}{3}} \sim 1.95 \times 10^{11}\ \mathrm{cm}$$

Distance to the source:
d = 6500 pc $\sim 6500 \times 3.1 \times 10^{13}$ km
$= 2 \times 10^{22}$ cm

Weisberg, Nice, Taylor (2010) **ApJ**

For simplicity, let's consider a circular orbit (e = 0) !

(1). Positions of the two stars (NSs)

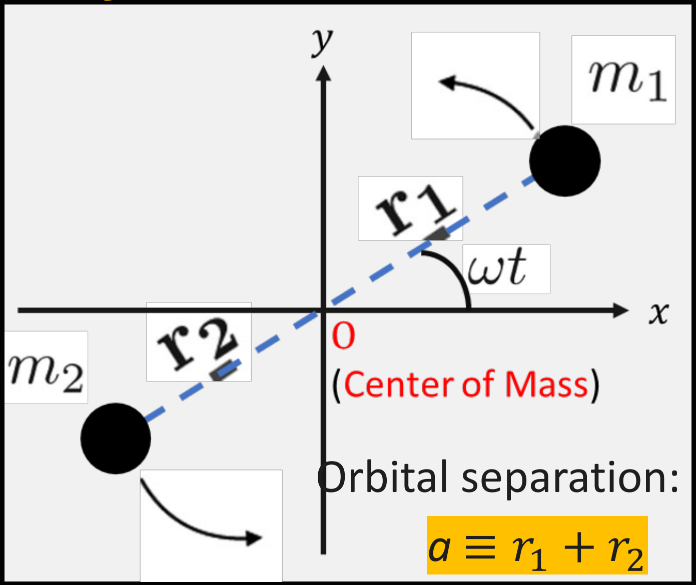
$$x_1 = \frac{m_2}{m_1+m_2} a\cos(\omega t), \qquad y_1 = \frac{m_2}{m_1+m_2} a\sin(\omega t),$$
$$x_2 = -\frac{m_1}{m_1+m_2} a\cos(\omega t), \qquad y_2 = -\frac{m_1}{m_1+m_2} a\sin(\omega t)$$

(here we used the relations, $m_1\, r_1 = m_2\, r_2, a = r_1 + r_2, \Rightarrow r_1 = \frac{m_2}{m_1+m_2} a, r_2 = \frac{m_1}{m_1+m_2} a$)

(2). Kepler's (third) law: $F = \frac{G m_1 m_2}{a^2} = \frac{m_1 v_1^2}{r_1^2} = \frac{m_2 v_2^2}{r_2^2}, v_1 = r_1 \omega, v_2 = r_2 \omega \Rightarrow$

$$G(m_1 + m_2) = \omega^2 a^3$$

Why even now, difficult to detect GWs from Hulse-Taylor pulsar (2/4)?



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(3). Quadrupole moments

$$I_{xx} = m_1 x_1^2 + m_2 x_2^2 = \frac{m_1 m_2}{m_1+m_2} a^2 \cos^2(\omega t) = \frac{m_1 m_2}{2(m_1+m_2)} a^2 [1+\cos(2\omega t)],$$
$$I_{yy} = m_1 y_1^2 + m_2 y_2^2 = \frac{m_1 m_2}{m_1+m_2} a^2 \sin^2(\omega t) = \frac{m_1 m_2}{2(m_1+m_2)} a^2 [1-\cos(2\omega t)],$$
$$I = I_{xx} + I_{yy} = \frac{m_1 m_2}{m_1+m_2} a^2$$

$$I_{xy} = I_{yx} = m_1 x_1 y_1 + m_2 x_2 y_2 = \frac{m_1 m_2}{2(m_1+m_2)} a^2 \sin(2\omega t)$$

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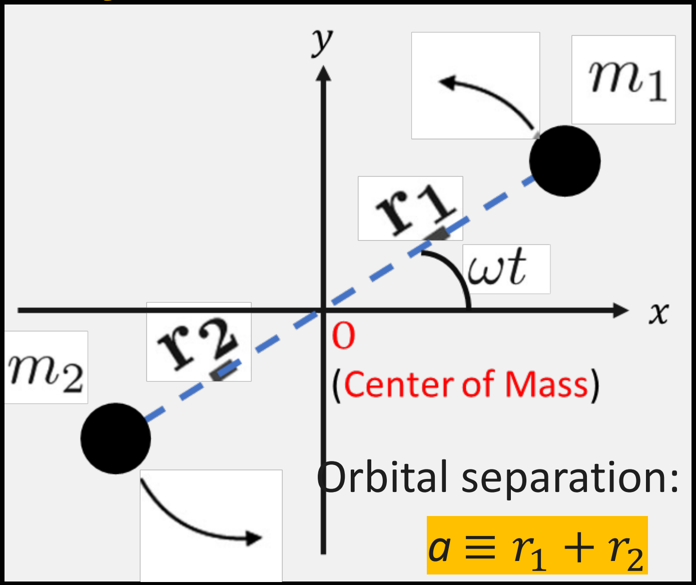

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$$I_{yy} = m_1 y_1^2 + m_2 y_2^2 = \frac{m_1 m_2}{m_1+m_2} a^2 \,\mathrm{si}\ \ldots\ a^2[1-\cos(2\omega t)],$$

$$\cos^2\theta = \frac{1}{2}(1+\cos 2\theta)$$

$$I = I_{xx} + I_{yy} = \frac{m_1 m_2}{m_1+m_2} a^2$$

$$I_{xy} = I_{yx} = m_1 x_1 y_1 + m_2 x_2 y_2 = \frac{m_1 m_2}{2(m_1+m_2)} a^2 \sin(2\omega t)$$

Why even now, difficult to detect GWs from Hulse-Taylor pulsar (2/4)?

For simplicity, let's consider a circular orbit (e = 0) !

(4). The GW amplitude

$$\bar{h}^{jk}(t,\boldsymbol{x}) \approx \frac{2G}{r}\frac{d^2 I^{jk}(t-r)}{dt^2}$$

$$= -\frac{4Ga^2\omega^2 m_1 m_2}{(m_1+m_2)r}\begin{pmatrix}\cos[2\omega(t-r)] & \sin[2\omega(t-r)] & 0 \\ \sin[2\omega(t-r)] & -\cos[2\omega(t-r)] & 0 \\ 0 & 0 & 0\end{pmatrix}$$

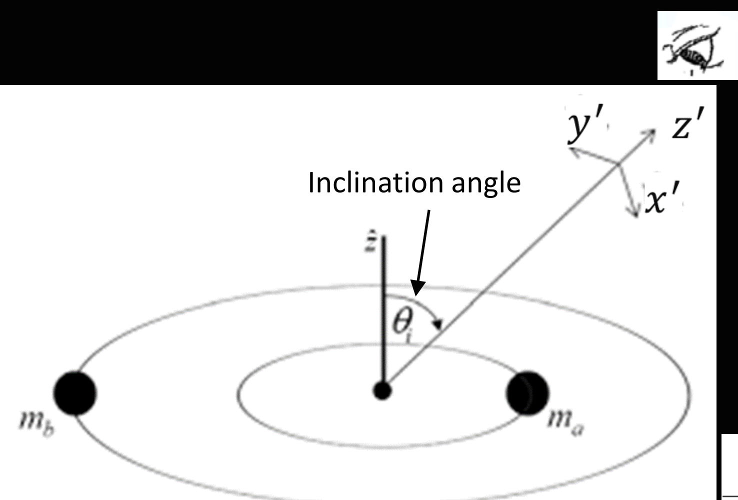
Traceless-transverse

✓ The frequency of GW emission (2ω) is twice of the orbital frequency of a binary star (ω)

(5). Angular dependence of GW emission

$$h_{ij}'^{TT} = \frac{\partial x_i'}{\partial x_k}\frac{\partial x_j'}{\partial x_l}h_{kl}^{TT} = \begin{pmatrix} c & 0 & -s \\ 0 & 1 & 0 \\ s & 0 & c\end{pmatrix}\begin{pmatrix} c^2H_+ & cH_\times & -scH_+ \\ cH_\times & -H_+ & -sH_\times \\ -scH_+ & -sH_\times & s^2H_+\end{pmatrix}\begin{pmatrix} c & 0 & s \\ 0 & 1 & 0 \\ -s & 0 & c\end{pmatrix}$$

$$= \begin{pmatrix} cH_+ & H_\times & -sH_+ \\ cH_\times & -H_+ & -sH_\times \\ 0 & 0 & 0\end{pmatrix}\begin{pmatrix} c & 0 & s \\ 0 & 1 & 0 \\ -s & 0 & c\end{pmatrix} = \begin{pmatrix} H_+ & H_\times & 0 \\ H_\times & -H_+ & 0 \\ 0 & 0 & 0\end{pmatrix}.$$



+ mode

$$H_+ = \frac{\cos^2\theta h_{xx} - h_{yy}}{2}$$

x mode

$$H_\times = \cos\theta h_{xy}$$

$$h_+^2 + h_\times^2 = \text{Const}$$

Circular polarization

Seen from the spin axis

$$\bar{h}_{ij}^{TT}(t,\boldsymbol{x}) = \begin{pmatrix}(h_{xx}-h_{yy})/2 & h_{xy} & 0 \\ h_{xy} & -(h_{xx}-h_{yy})/2 & 0 \\ 0 & 0 & 0\end{pmatrix}$$

Quiz:
Seen from equator?

Why even now, difficult to detect GWs from Hulse-Taylor pulsar (4/4)?

(4). The GW amplitude

$$\bar{h}^{jk}(t,\boldsymbol{x}) \approx \frac{2G}{r}\frac{d^2 I^{jk}(t-r)}{dt^2}$$

$$= -\frac{4Ga^2\omega^2 m_1 m_2}{(m_1+m_2)r}\begin{pmatrix}\cos[2\omega(t-r)] & \sin[2\omega(t-r)] & 0\\ \sin[2\omega(t-r)] & -\cos[2\omega(t-r)] & 0\\ 0 & 0 & 0\end{pmatrix}$$

Kepler's law

$$h = \frac{4Ga^2\omega^2 m_1 m_2}{(m_1+m_2)r} = \frac{4Ga^2 \times G(m_1+m_2)/a^3 \times m_1 m_2}{(m_1+m_2)r} = \frac{2Gm_1 \times 2Gm_2}{ar}$$

$$h = \frac{\frac{2Gm_1}{c^2} \times \frac{2Gm_2}{c^2}}{ad} \approx \frac{(1.4\times 3)^2}{2\times 10^6 \times 2\times 10^{17}} \approx 4\times 10^{-23}$$

✓The GW is not that small for ground detectors. Important: The GW amp. increases with time!

✓The GW frequency is, Period: P = 7.75 hours

$$2\omega = 0.45\text{mHz}$$

Too low to detect by interferometers on the earth!

Why did LIGO make it to detect GWs (from BHs) ?

Why even now, difficult to detect GWs from Hulse-Taylor pulsar (4/4)?

(4). The GW amplitude

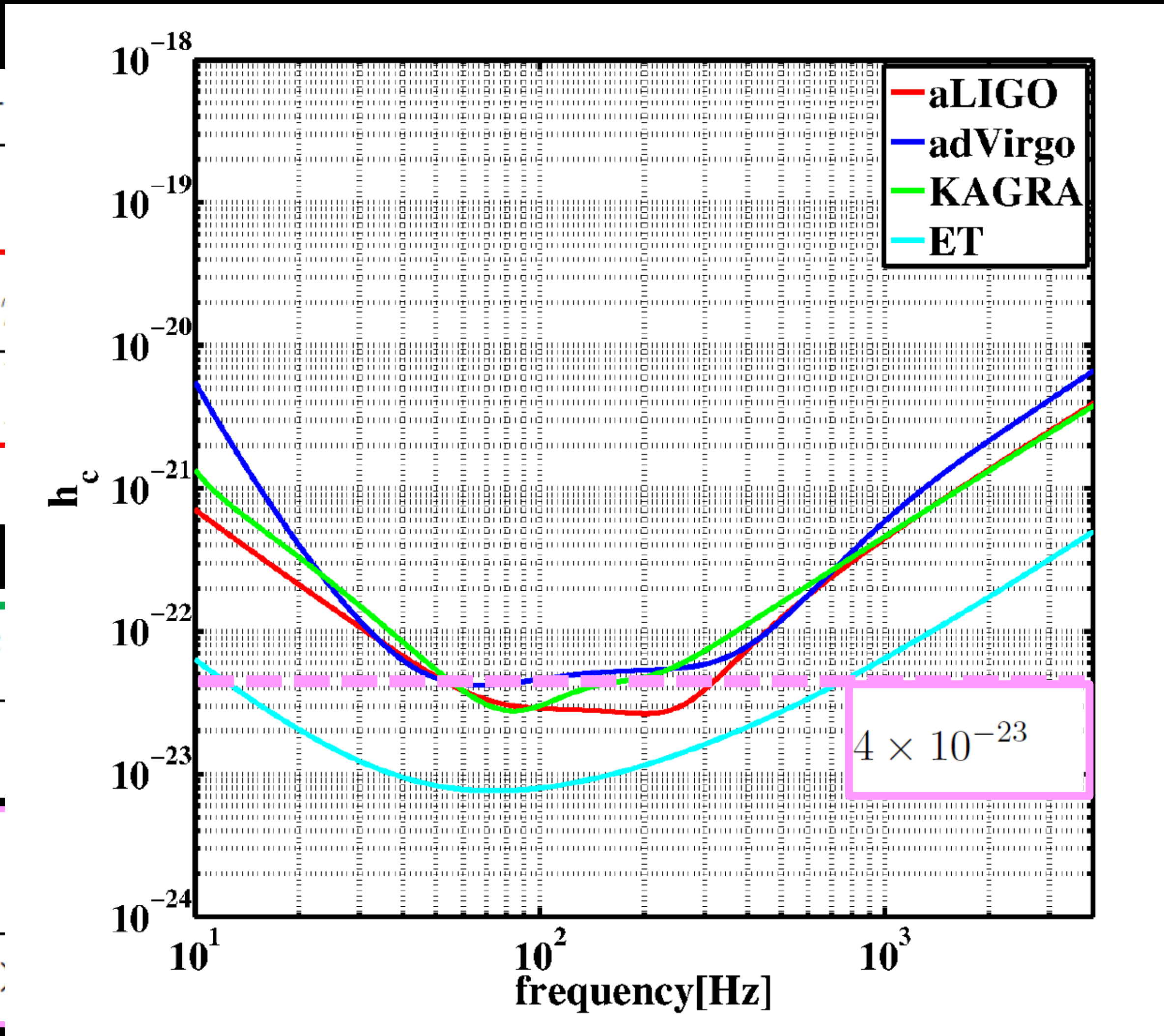
$$\bar{h}^{jk}(t,\boldsymbol{x}) \approx \frac{2G}{r}\frac{d^2 I^{jk}(t -}{dt^2}$$

$$= -\frac{4Ga^2\omega^2 m_1}{(m_1+m_2}$$

Kepler's law

$$h = \frac{4Ga^2\omega^2 m_1 m_2}{(m_1+m_2)r} = \frac{4Ga^2}{}$$

$$h = \frac{\frac{2Gm_1}{c^2}\times\frac{2Gm_2}{c^2}}{ad} \approx \frac{}{2}$$



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Success of GR in Hulse-Taylor pulsar (1/5)

(5). The GW luminosity

GW flux: $[\dot{\bar{h}}^{jk}]^2 \propto [\dddot{I}^{jk}]^2/r^2$ X 4 π r^2

$$L_{\rm GW} \propto \langle \dddot{I}_{jk}\dddot{I}^{jk}\rangle$$

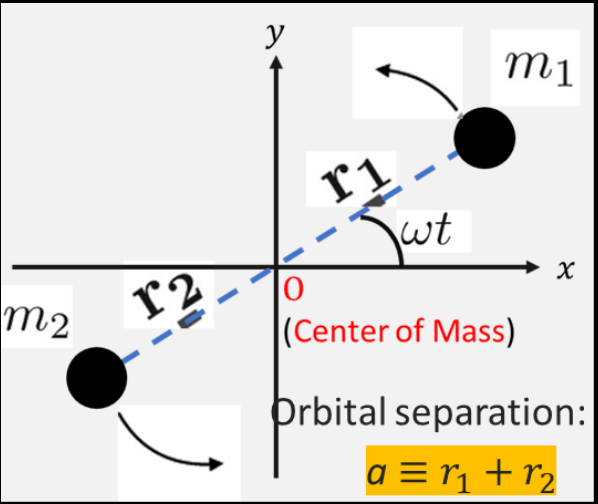
$$L_{\rm GW} = \frac{1}{5}\frac{G}{c^5}\langle \dddot{\mathcal{I}}_{jk}\dddot{\mathcal{I}}^{jk}\rangle$$

[erg/s]
Quadrupole
(Issacson's)formula

Third time-derivatives of quadrupole moment

$$\dddot{\mathcal{I}}^{jk}(t,\boldsymbol{x}) = \frac{4a^2\omega^3 m_1 m_2}{m_1+m_2}\begin{pmatrix} \sin[2\omega(t-r)] & -\cos[2\omega(t-r)] & 0 \\ -\cos[2\omega(t-r)] & -\sin[2\omega(t-r)] & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\begin{aligned} L_{\rm GW} &= \frac{1}{5}\frac{G}{c^5}\langle \dddot{\mathcal{I}}_{jk}\dddot{\mathcal{I}}^{jk}\rangle \\ &= \frac{G}{5c^5}\left(\frac{4a^2\omega^3 m_1 m_2}{m_1+m_2}\right)^2 \langle 2\sin^2[2\omega(t-r)] + 2\cos^2[2\omega(t-r)]\rangle \\ &= \frac{32G}{5c^5}\frac{a^4\omega^6 m_1^2 m_2^2}{(m_1+m_2)^2} = \frac{32G^4}{5c^5}\frac{m_1^2 m_2^2(m_1+m_2)}{a^5}. \end{aligned}$$



✓Energy loss rate due to GW emission

$$\frac{dE}{dt} = \frac{Gm_1m_2}{2a^2}\frac{da}{at} = -L_{\rm GW}$$

✓Decreasing rate of the orbital separation

$$\frac{da}{dt} = -\frac{64G^3}{5c^5}\frac{m_1m_2(m_1+m_2)}{a^3}$$

✓Using the Kepler law,

$$G(m_1+m_2)P^2 = 4\pi^2 a^3$$

$$2G(m_1+m_2)P\frac{dP}{dt} = 12\pi^2 a^2\frac{da}{dt} = -\frac{64\times 12G^3\pi^2}{5c^5}\frac{m_1m_2(m_1+m_2)}{a}$$

✓Total gravitational energy

$$E = -\frac{Gm_1m_2}{2a}$$

Success of General Relativity in Hulse-Taylor pulsar (2/5)

(5). Decrease rate of the orbital spin period (e.g., increase of the orbital frequency)

$$\frac{dP}{dt} = -\frac{64 \times 6 G^2 \pi^2}{5c^5} \frac{m_1 m_2}{aP} \approx 2.0 \times 10^{-13} \approx 6.3\ \mu\mathrm{s/yr}$$

On the other hand, 30-year of observation of the Hulse-Taylor pulsar:

$$\frac{dP}{dt} = (-2.423 \pm 0.001) \times 10^{-12} \approx -76.5\ \mu\mathrm{sec/year}$$

~12 times bigger !

The orbit has eccentricity !

$$L_{\mathrm{GW}} = \frac{32}{5} \frac{G^3 \mu^2 M^3}{a^5 (1-e^2)^{7/2}} \left(1 + \frac{73}{24} e^2 + \frac{37}{96} e^4\right)$$

$$= \frac{32}{5c^5} \frac{G^4 m_1^2 m_2^2 (m_1 + m_2)}{a^5 (1-e^2)^{7/2}} \left(1 + \frac{73}{24} e^2 + \frac{37}{96} e^4\right)$$

Taking " e = 0.671334(5)" -> 11.8

Accuracy of GR prediction confirmed !

$$0.997 \pm 0.002$$

(confirmed also by perihelion shift of Mercury, deflection of light by the Sun …)

(6) Important quantity: The Chirp mass

The GW luminosity; L_{GW}

$$\frac{dE_{\mathrm{GW}}}{dt} = \frac{32G}{5c^5} \mu^2 a^4 (2\pi f_b)^6$$

$f_b = 1/P_b$

$\mu = m_1 m_2 / (m_1 + m_2)$

Kepler's law

$$\frac{dE_{\mathrm{GW}}}{dt} = \frac{32 G^{7/3}}{5c^5} M_c^{10/3} (2\pi f_b)^{10/3}$$

The Chirp mass; M_c

$$M_c \equiv \mu^{3/5} M^{2/5} = q^{3/5} M, \quad q = \mu / M$$

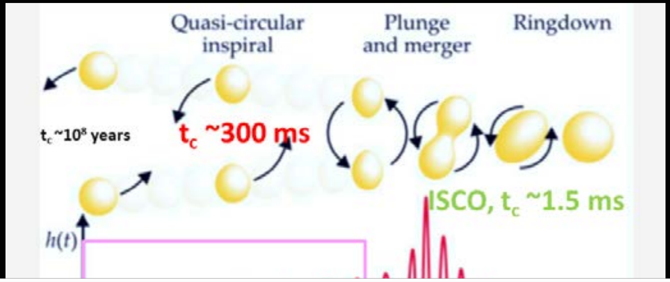
Success of General Relativity in Hulse-Taylor pulsar (3/5)

✓ The Chirp signal

by 3.5 Post-Newtonian (PN) techniques

(e.g., Blanchet, Living Reviews, 2013,)

Equation of motion of m_1



$$a_1 = -\frac{Gm_2}{r_{12}^2}n_{12}$$

$$+\frac{1}{c^2}\Bigg\{\Bigg[\frac{5G^2m_1m}{r_{12}^3}$$

$$+\frac{Gm_2}{r_{12}^2}\Big(4$$

$$+\frac{1}{c^5}\Bigg\{\Bigg[\frac{208G^3m_1m_2^2}{15r_{12}^4}$$

$$+\Bigg[\frac{8G^3m_1^2m}{5r_{12}^4}$$

$$
\begin{aligned}
&+\frac{1}{c^6}\Bigg\{\Bigg[\frac{Gm_2}{r_{12}^2}\Big(\frac{35}{16}(n_{12}v_2)^6-\frac{15}{8}(n_{12}v_2)^4v_1^2+\frac{15}{2}(n_{12}v_2)^4(v_1v_2)+3(n_{12}v_2)^2(v_1v_2)^2\\
&\quad-\frac{15}{2}(n_{12}v_2)^4v_2^2+\frac{3}{2}(n_{12}v_2)^2v_1^2v_2^2-12(n_{12}v_2)^2(v_1v_2)v_2^2-2(v_1v_2)^2v_2^2\\
&\quad+\frac{15}{2}(n_{12}v_2)^2v_2^4+4(v_1v_2)v_2^4-2v_2^6\Big)\\
&+\frac{G^2m_1m_2}{r_{12}^3}\Big(-\frac{171}{8}(n_{12}v_1)^4+\frac{171}{2}(n_{12}v_1)^3(n_{12}v_2)-\frac{723}{4}(n_{12}v_1)^2(n_{12}v_2)^2\\
&\quad+\frac{383}{2}(n_{12}v_1)(n_{12}v_2)^3-\frac{455}{8}(n_{12}v_2)^4+\frac{229}{4}(n_{12}v_1)^2v_1^2\\
&\quad-\frac{205}{2}(n_{12}v_1)(n_{12}v_2)v_1^2+\frac{191}{4}(n_{12}v_2)^2v_1^2-\frac{91}{8}v_1^4-\frac{229}{2}(n_{12}v_1)^2(v_1v_2)\\
&\quad+244(n_{12}v_1)(n_{12}v_2)(v_1v_2)-\frac{225}{2}(n_{12}v_2)^2(v_1v_2)+\frac{91}{2}v_1^2(v_1v_2)\\
&\quad-\frac{177}{4}(v_1v_2)^2+\frac{229}{4}(n_{12}v_1)^2v_2^2-\frac{283}{2}(n_{12}v_1)(n_{12}v_2)v_2^2\\
&\quad+\frac{259}{4}(n_{12}v_2)^2v_2^2-\frac{91}{4}v_1^2v_2^2+43(v_1v_2)v_2^2-\frac{81}{8}v_2^4\Big)\\
&+\frac{G^2m_2^2}{r_{12}^3}\Big(-6(n_{12}v_1)^2(n_{12}v_2)^2+12(n_{12}v_1)(n_{12}v_2)^3+6(n_{12}v_2)^4\\
&\quad+4(n_{12}v_1)(n_{12}v_2)(v_1v_2)+12(n_{12}v_2)^2(v_1v_2)+4(v_1v_2)^2\\
&\quad-4(n_{12}v_1)(n_{12}v_2)v_2^2-12(n_{12}v_2)^2v_2^2-8(v_1v_2)v_2^2+4v_2^4\Big)\\
&+\frac{G^3m_2^3}{r_{12}^4}\Big(-(n_{12}v_1)^2+2(n_{12}v_1)(n_{12}v_2)+\frac{43}{2}(n_{12}v_2)^2+18(v_1v_2)-9v_2^2\Big)\\
&+\frac{G^3m_1m_2^2}{r_{12}^4}\Big(\frac{415}{8}(n_{12}v_1)^2-\frac{375}{4}(n_{12}v_1)(n_{12}v_2)+\frac{1113}{8}(n_{12}v_2)^2-\frac{615}{64}(n_{12}v_{12})^2\pi^2\\
&\quad+18v_1^2+\frac{123}{64}\pi^2v_{12}^2+33(v_1v_2)-\frac{33}{2}v_2^2\Big)\\
&+\frac{G^3m_1^2m_2}{r_{12}^4}\Big(-\frac{45887}{168}(n_{12}v_1)^2+\frac{24025}{42}(n_{12}v_1)(n_{12}v_2)-\frac{10469}{42}(n_{12}v_2)^2+\frac{48197}{840}v_1^2\\
&\quad-\frac{36227}{420}(v_1v_2)+\frac{36227}{840}v_2^2+110(n_{12}v_{12})^2\ln\Big(\frac{r_{12}}{r'_1}\Big)-22v_{12}^2\ln\Big(\frac{r_{12}}{r'_1}\Big)\Big)\\
&+\frac{16G^4m_2^4}{r_{12}^5}+\frac{G^4m_1^2m_2^2}{r_{12}^5}\Big(175-\frac{41}{16}\pi^2\Big)+\frac{G^4m_1^3m_2}{r_{12}^5}\Big(-\frac{3187}{1260}+\frac{44}{3}\ln\Big(\frac{r_{12}}{r'_1}\Big)\Big)\\
&+\frac{G^4m_1m_2^3}{r_{12}^5}\Big(\frac{110741}{630}-\frac{41}{16}\pi^2-\frac{44}{3}\ln\Big(\frac{r_{12}}{r'_2}\Big)\Big)\Bigg]n_{12}\\
&+\Bigg[\frac{Gm_2}{r_{12}^2}\Big(\frac{15}{2}(n_{12}v_1)(n_{12}v_2)^4-\frac{45}{8}(n_{12}v_2)^5-\frac{3}{2}(n_{12}v_2)^3v_1^2+6(n_{12}v_1)(n_{12}v_2)^2(v_1v_2)
\end{aligned}
$$

Nightmare...

$$
\begin{aligned}
&\quad-6(n_{12}v_2)^3(v_1v_2)-2(n_{12}v_2)(v_1v_2)^2-12(n_{12}v_1)(n_{12}v_2)^2v_2^2+12(n_{12}v_2)^3v_2^2\\
&\quad+(n_{12}v_2)v_1^2v_2^2-4(n_{12}v_1)(v_1v_2)v_2^2+8(n_{12}v_2)(v_1v_2)v_2^2+4(n_{12}v_1)v_2^4\\
&\quad-7(n_{12}v_2)v_2^4\Big)\\
&+\frac{G^2m_2^2}{r_{12}^3}\Big(-2(n_{12}v_1)^2(n_{12}v_2)+8(n_{12}v_1)(n_{12}v_2)^2+2(n_{12}v_2)^3+2(n_{12}v_1)(v_1v_2)\\
&\quad+4(n_{12}v_2)(v_1v_2)-2(n_{12}v_1)v_2^2-4(n_{12}v_2)v_2^2\Big)\\
&+\frac{G^2m_1m_2}{r_{12}^3}\Big(-\frac{243}{4}(n_{12}v_1)^3+\frac{565}{4}(n_{12}v_1)^2(n_{12}v_2)-\frac{269}{4}(n_{12}v_1)(n_{12}v_2)^2\\
&\quad-\frac{95}{12}(n_{12}v_2)^3+\frac{207}{8}(n_{12}v_1)v_1^2-\frac{137}{8}(n_{12}v_2)v_1^2-36(n_{12}v_1)(v_1v_2)\\
&\quad+\frac{27}{4}(n_{12}v_2)(v_1v_2)+\frac{81}{8}(n_{12}v_1)v_2^2+\frac{83}{8}(n_{12}v_2)v_2^2\Big)\\
&+\frac{G^3m_2^3}{r_{12}^4}\big(4(n_{12}v_1)+5(n_{12}v_2)\big)\\
&+\frac{G^3m_1m_2^2}{r_{12}^4}\Big(-\frac{307}{8}(n_{12}v_1)+\frac{479}{8}(n_{12}v_2)+\frac{123}{32}(n_{12}v_{12})\pi^2\Big)\\
&+\frac{G^3m_1^2m_2}{r_{12}^4}\Big(\frac{31397}{420}(n_{12}v_1)-\frac{36227}{420}(n_{12}v_2)-44(n_{12}v_{12})\ln\Big(\frac{r_{12}}{r'_1}\Big)\Big)\Bigg]v_{12}\Bigg\}\\
&+\frac{1}{c^7}\Bigg\{\Bigg[\frac{G^4m_1^3m_2}{r_{12}^5}\Big(\frac{3992}{105}(n_{12}v_1)-\frac{4328}{105}(n_{12}v_2)\Big)\\
&\quad+\frac{G^4m_1^2m_2^2}{r_{12}^6}\Big(-\frac{13576}{105}(n_{12}v_1)+\frac{2872}{21}(n_{12}v_2)\Big)-\frac{3172}{21}\frac{G^4m_1m_2^3}{r_{12}^6}(n_{12}v_{12})\\
&\quad+\frac{G^3m_1^2m_2}{r_{12}^4}\Big(48(n_{12}v_1)^3-\frac{696}{5}(n_{12}v_1)^2(n_{12}v_2)+\frac{744}{5}(n_{12}v_1)(n_{12}v_2)^2-\frac{288}{5}(n_{12}v_2)^3\\
&\qquad-\frac{4888}{105}(n_{12}v_1)v_1^2+\frac{5056}{105}(n_{12}v_2)v_1^2+\frac{2056}{21}(n_{12}v_1)(v_1v_2)\\
&\qquad-\frac{2224}{21}(n_{12}v_2)(v_1v_2)-\frac{1028}{21}(n_{12}v_1)v_2^2+\frac{5812}{105}(n_{12}v_2)v_2^2\Big)\\
&\quad+\frac{G^3m_1m_2^2}{r_{12}^4}\Big(-\frac{582}{5}(n_{12}v_1)^3+\frac{1746}{5}(n_{12}v_1)^2(n_{12}v_2)-\frac{1954}{5}(n_{12}v_1)(n_{12}v_2)^2\\
&\qquad+158(n_{12}v_2)^3+\frac{3568}{105}(n_{12}v_{12})v_1^2-\frac{2864}{35}(n_{12}v_1)(v_1v_2)\\
&\qquad+\frac{10048}{105}(n_{12}v_2)(v_1v_2)+\frac{1432}{35}(n_{12}v_1)v_2^2-\frac{5752}{105}(n_{12}v_2)v_2^2\Big)\\
&\quad+\frac{G^2m_1m_2}{r_{12}^3}\Big(-56(n_{12}v_{12})^5+60(n_{12}v_1)^3v_{12}^2-180(n_{12}v_1)^2(n_{12}v_2)v_{12}^2\\
&\qquad+174(n_{12}v_1)(n_{12}v_2)^2v_{12}^2-54(n_{12}v_2)^3v_{12}^2-\frac{246}{35}(n_{12}v_{12})v_1^4\\
&\qquad+\frac{1068}{35}(n_{12}v_1)v_1^2(v_1v_2)-\frac{984}{35}(n_{12}v_2)v_1^2(v_1v_2)-\frac{1068}{35}(n_{12}v_1)(v_1v_2)^2
\end{aligned}
$$

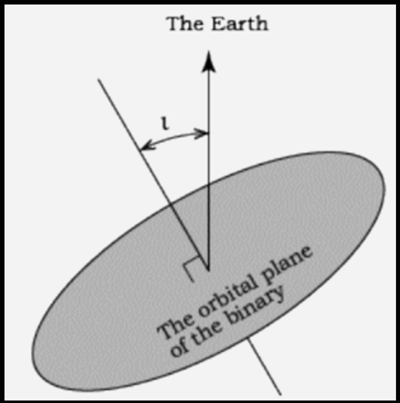
Success of General Relativity in Hulse-Taylor pulsar (4/5)

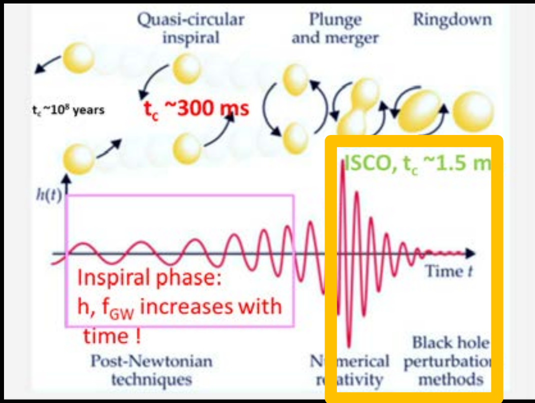
✓ The Chirp GW signal

(e.g., Blanchet, Living Reviews Relativity, 17, 2 (2014))

$$h_+(t) = \frac{A}{r}\left(\frac{\pi f_{gw}(\tau)}{c}\right)^{2/3} \frac{1+\cos^2\iota}{2}\cos\Phi(\tau)$$

$$h_\times(t) = \frac{A}{r}\left(\frac{\pi f_{gw}(\tau)}{c}\right)^{2/3} \cos\iota\sin\Phi(\tau)$$





$$A = 4\left(\frac{GM_c}{c^2}\right)^{5/3} \quad f_{gw}(\tau) = \frac{1}{\pi}\left(\frac{5}{256\tau}\right)^{3/8}\left(\frac{GM_c}{c^2}\right)^{-5/8} \quad \Phi(\tau) = -2\left(\frac{5GM_c}{c^2}\right)^{-5/8}\tau^{5/8} + \Phi_0 \quad \tau = t_{coal} - t$$

By observing f_{GW} and $\dot{f}_{GW}$ → The Chirp mass M_c via $\dot{f}_{gw} = \frac{96}{5}\pi^{8/3}\left(\frac{GM_c}{c^3}\right)^{5/3} f_{gw}^{11/3}$

By observing h_+ and $h_\times$ → The source distance r, inclination angle ι

By observing the ring-down phase (quasi-normal mode) → The BH mass M, BH spin, a

$$h(t) = e^{-(t-t_0)\pi f_R/Q}\cos[2\pi f_R(t-t_0)]$$

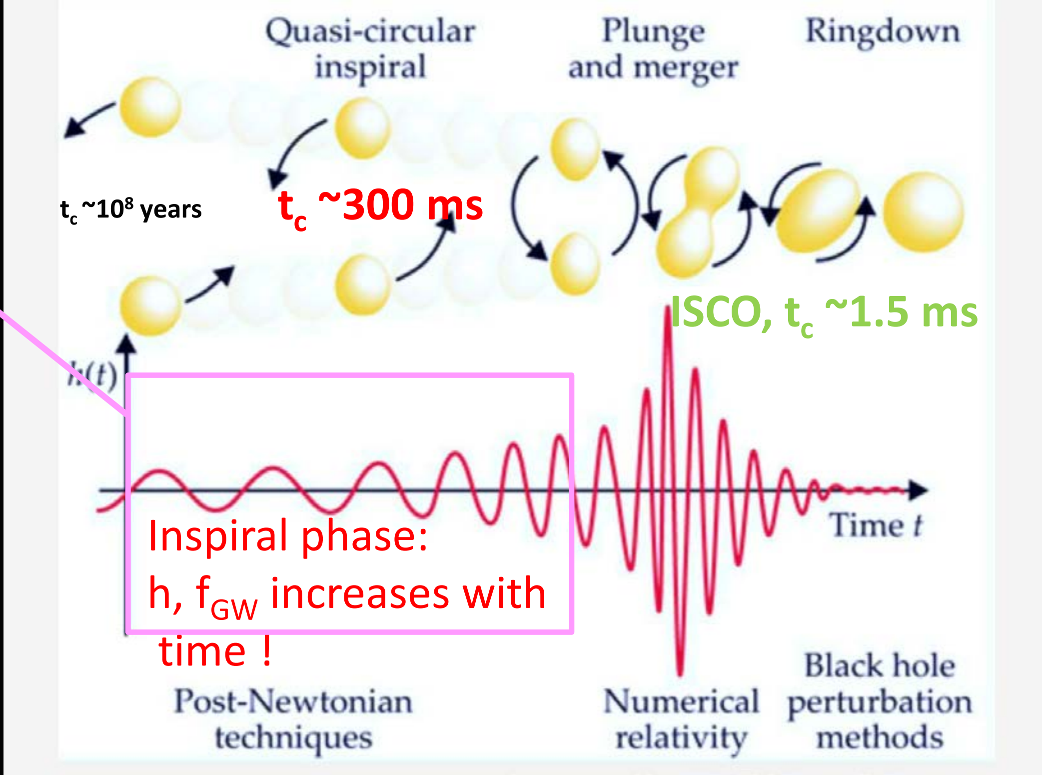
$$f_R = \frac{c^3}{2\pi GM}\{1.5251 - 1.1568(1-a)^{0.1292}\}$$
$$Q = 0.7000 + 1.4187(1-a)^{-0.4990} \quad (4)$$

Damping rate, “a” is Kerr parameter (J/M)

Success of General Relativity in Hulse-Taylor pulsar (5/5)

✓Three evolution phases of binary merger
(see lecture by Bauswein !)

The chirp GW signal:
The waveforms generally well-modeled by Post-Newtonian techniques (theory).
(see, Sathyaprakash & Schutz (2011) Living review)



For the "real-signal" detection:

$$s(t) = h(t) + n(t)$$

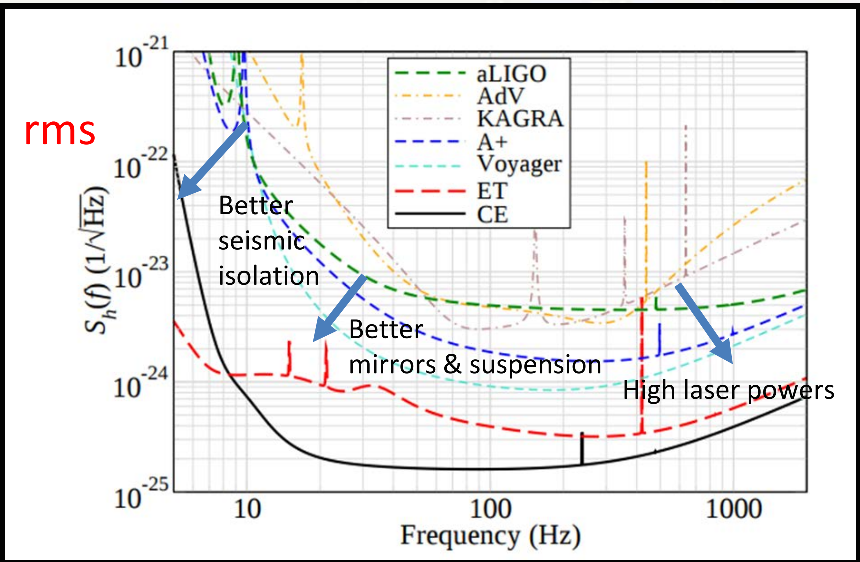
@ the detector noise

✓The signal-to-noise ratio: SNR or S/N

$$S/N \sim \frac{s(t)}{n(t)},$$ not accurate.. more correctly...

$$\frac{(s|h)}{\sqrt{<(n|h)^2>}} \sim 10$$

$$(s\,|\,h) \approx \int \frac{df}{S_n(f)} s(f)\, h^*(f)$$



h (f) :template (= $h\sqrt{N}$ (Thorne (1987))
(for quasi-periodic signals of N cycles
, e.g., chirp, **matched filtering**)

S_n (f)= spectral noise density

Success of Ge

✓Three evolution phases o

(see lecture by Bauswei

The chirp GW signal:

The waveforms generally

by Post-Newtonian techn

(see, Sathyaprakash & Schutz (

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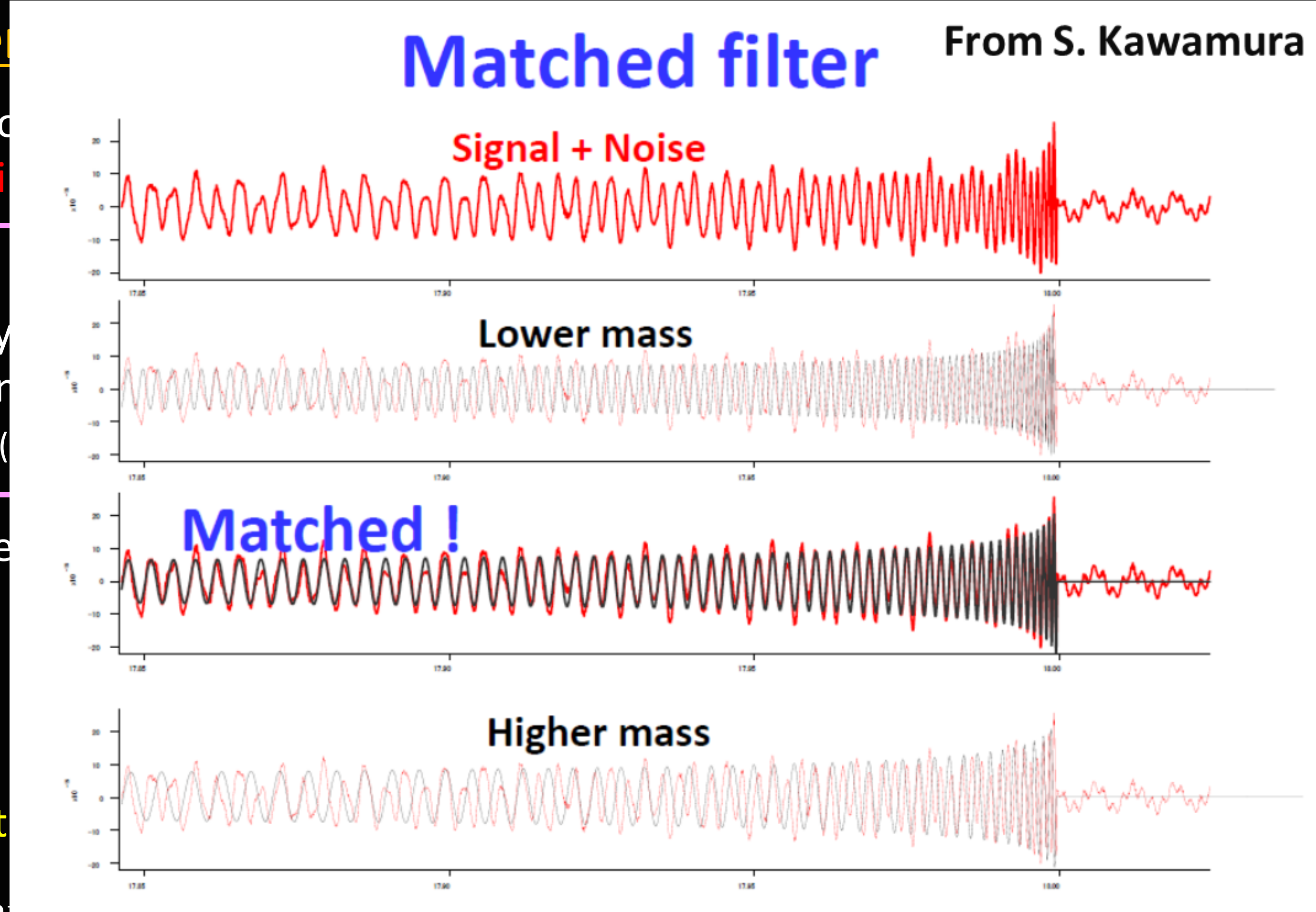
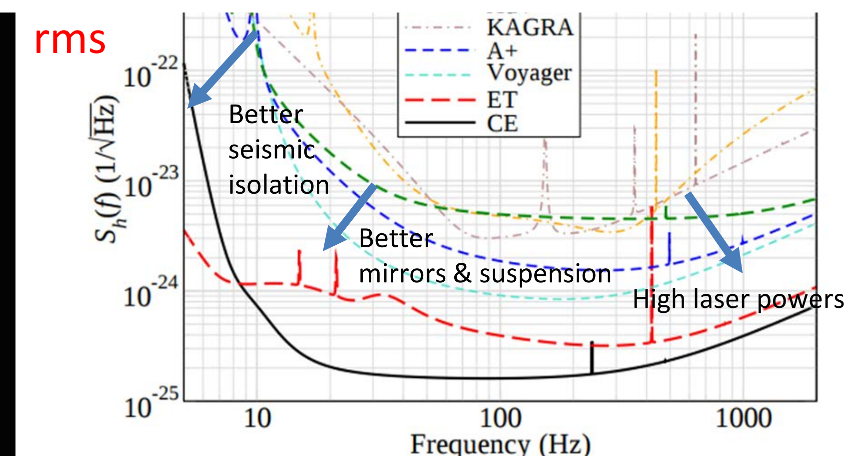
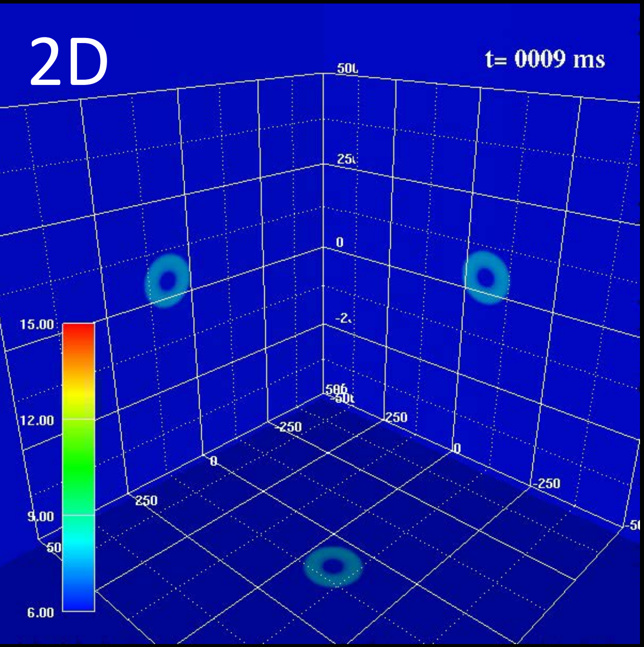




Quiz: Can one expect GW emission from axisymmetric (2D) stars ?



(e.g., Takiwaki,KK, Suwa (2012,2014), ApJ)

$$I_{xx} = \int \rho(R,Z)\, R^2 \cos^2\phi \; R\, \mathrm{d}R\, \mathrm{d}z\, \mathrm{d}\phi = \pi \int \rho(R,Z)\, R^3\, \mathrm{d}R\, \mathrm{d}z,$$

$$I_{yy} = \int \rho(R,Z)\, R^2 \sin^2\phi \; R\, \mathrm{d}R\, \mathrm{d}z\, \mathrm{d}\phi = I_{xx},$$

$$I_{xy} = \int \rho(R,Z)\, R^2 \sin\phi\; \cos\phi \; R\, \mathrm{d}R\, \mathrm{d}z\, \mathrm{d}\phi = 0.$$

$$I_{zz} = \int \rho(R,Z)\, z^2 \; R\, \mathrm{d}R\, \mathrm{d}z\, \mathrm{d}\phi = 2\pi \int \rho(R,Z)\, z^2 \; R\, \mathrm{d}R\, \mathrm{d}z$$

$$I_{xz} = I_{yz} = 0.$$

$$h_+ = \frac{h_{\theta\theta}}{r^2}, \qquad h_\times = \frac{h_{\theta\phi}}{r^2 \sin\theta}$$

$$h_{\theta\theta} = r^2 \Big[(h^Q_{xx} - h^Q_{yy}) \frac{(\cos^2\theta + 1)}{4} \cos 2\phi - \frac{h^Q_{xx} + h^Q_{yy} - 2h^Q_{zz}}{4} \sin^2\theta + h^Q_{xy} \frac{\cos^2\theta + 1}{2} \sin 2\phi - h^Q_{xz} \sin\theta\cos\theta\cos\phi - h^Q_{yz} \sin\theta\cos\theta\sin\phi \Big]$$

$$\frac{h_{\theta\phi}}{r^2\sin\theta} = -\frac{h^Q_{xx} - h^Q_{yy}}{2} \cos\theta \sin 2\phi + h^Q_{xy} \cos\theta\cos 2\phi + h^Q_{xz} \sin\theta\sin\phi - h^Q_{yz} \sin\theta\cos\phi,$$

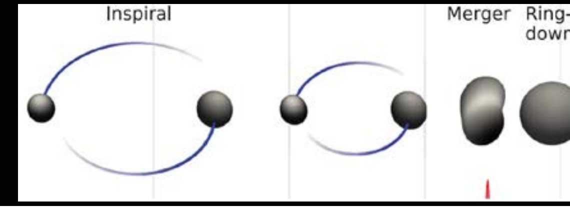
$$h^Q_{ij} = \frac{2}{r} \ddot{I}^{TT}_{ij}$$

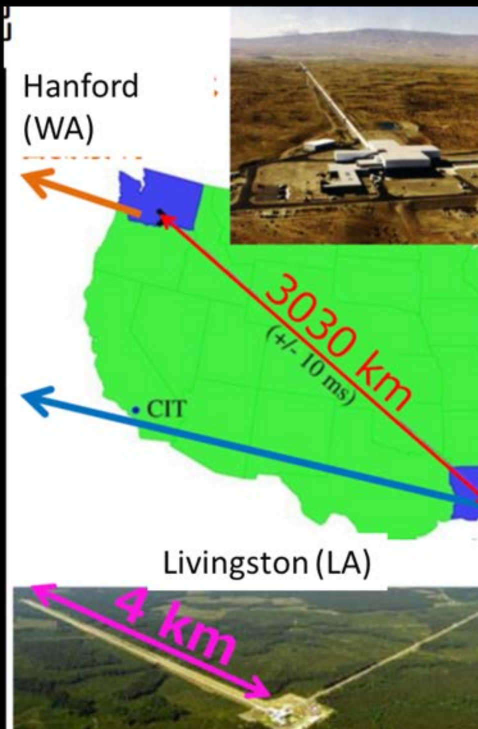
$$h_+ = -\frac{1}{r}(\ddot{I}_{xx} - \ddot{I}_{zz}) \sin^2\theta,$$

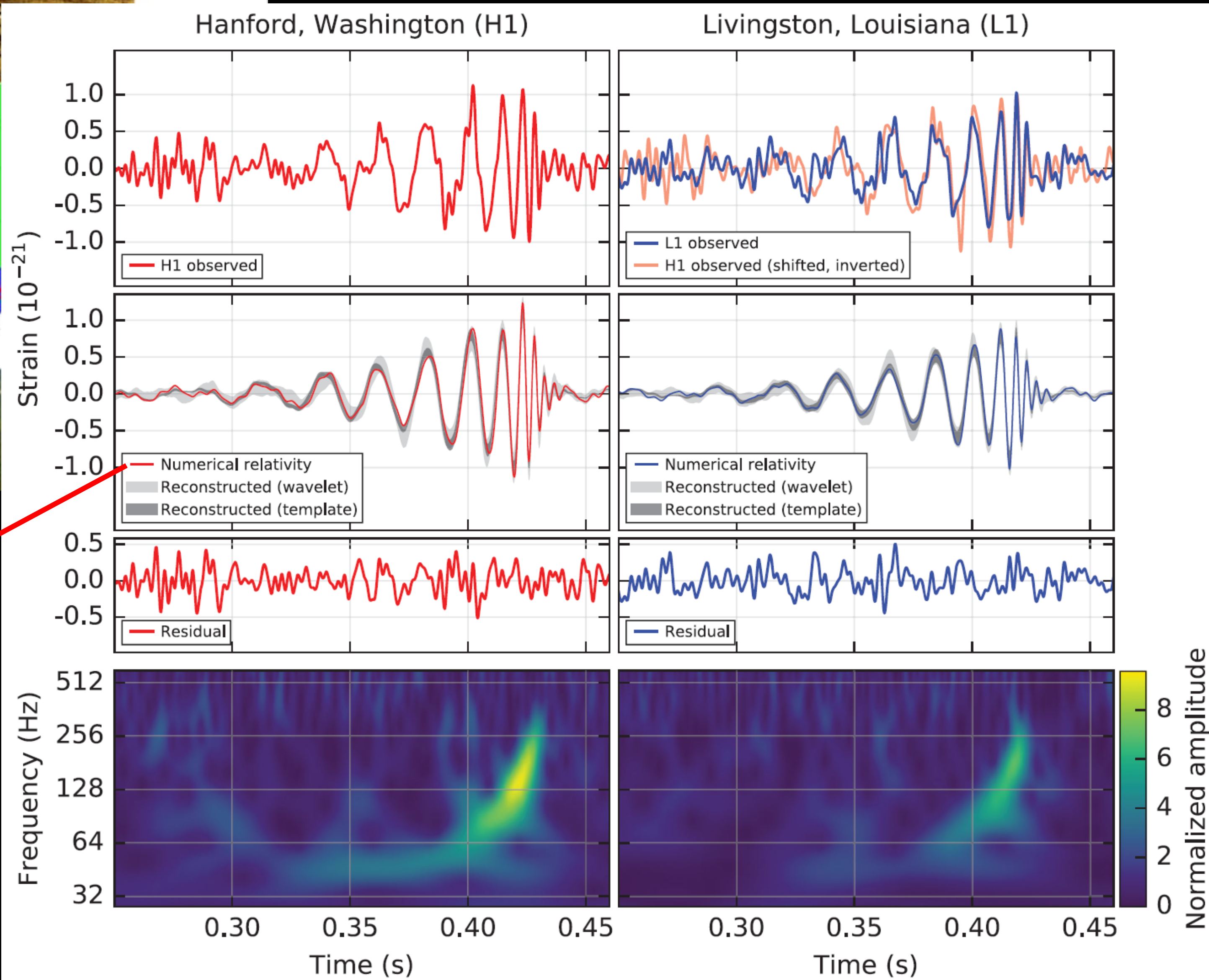
$$h_\times = 0.$$

1st discovery: GW150914

Abbott et al. (PRL), 2016, 116, 061102







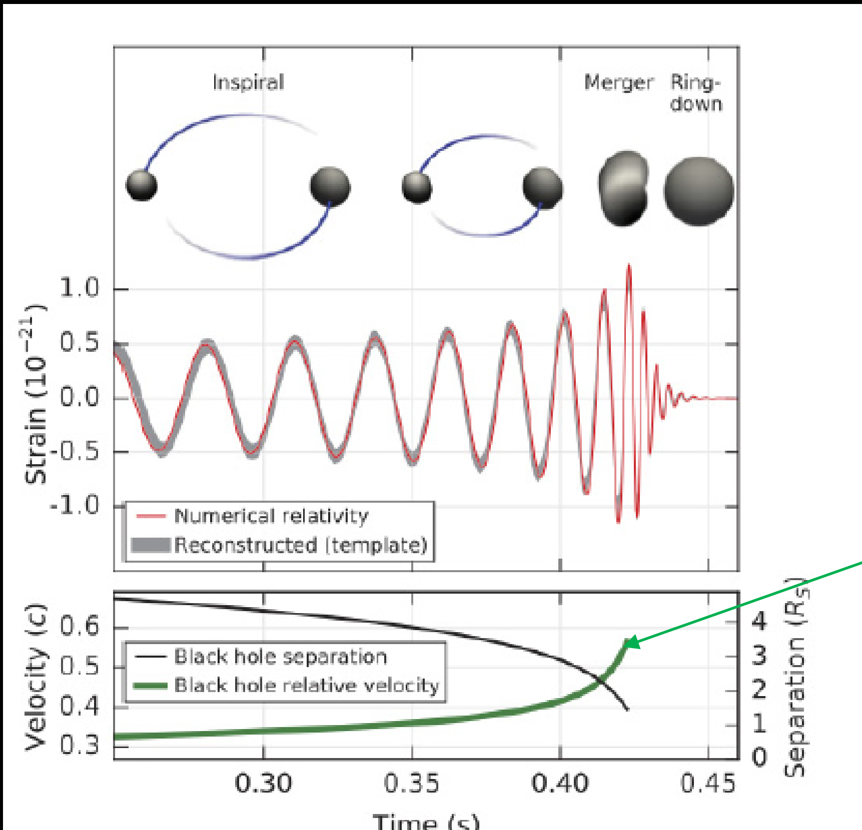
Numerical Relativity Simulation by Caltech-Cornell-CITA group

$M_1 = 36 M_{sun}$

$M_2 = 29\ M_{sun}$

(10^{-24} for binary NSs)

Applications to GW formulae to GW150914



Abbott et al. (PRL), 2016, 116, 061102

$$L_{\rm GW} = \frac{1}{5}\frac{G}{c^5}\langle \dddot{\mathcal{I}}_{jk}\dddot{\mathcal{I}}^{jk}\rangle$$

[erg/s]

Quadrupole (Issacson's)formula

$$\sim \frac{1}{5}\frac{G}{c^5}\left(\frac{M\,r^2}{T^3}\right)^2 \sim \frac{1}{5}\frac{G}{c^5}M^2 r^4\,\omega^6$$

@ $h_{\rm max,}$ $r = 3\,r_s\,, v = r\omega_{max} \sim 0.5\,c$

$$L_{GW}|_{max} \sim 10^{-4}\,L_{\rm Plank}$$

$$L_{\rm Plank} \equiv \frac{c^5}{G} \sim 3.6\times 10^{52}\ {\rm W}$$

$$L_{GW}\Big|_{max} = \frac{c^3}{16\pi G}\iint \Big|\dot{h}\Big|_{max}\Big|^2 d_L^2\, d\Omega$$

$$\sim \frac{c^3 d_L^2}{4G}\Big|\dot{h}\Big|_{max}\Big|^2 \sim \frac{c^5}{4G}\left(\frac{\omega_{GW}|_{max}\, d_L\, h|_{max}}{c}\right)^2$$

$$\Rightarrow d_L \sim 300{\rm Mpc}$$

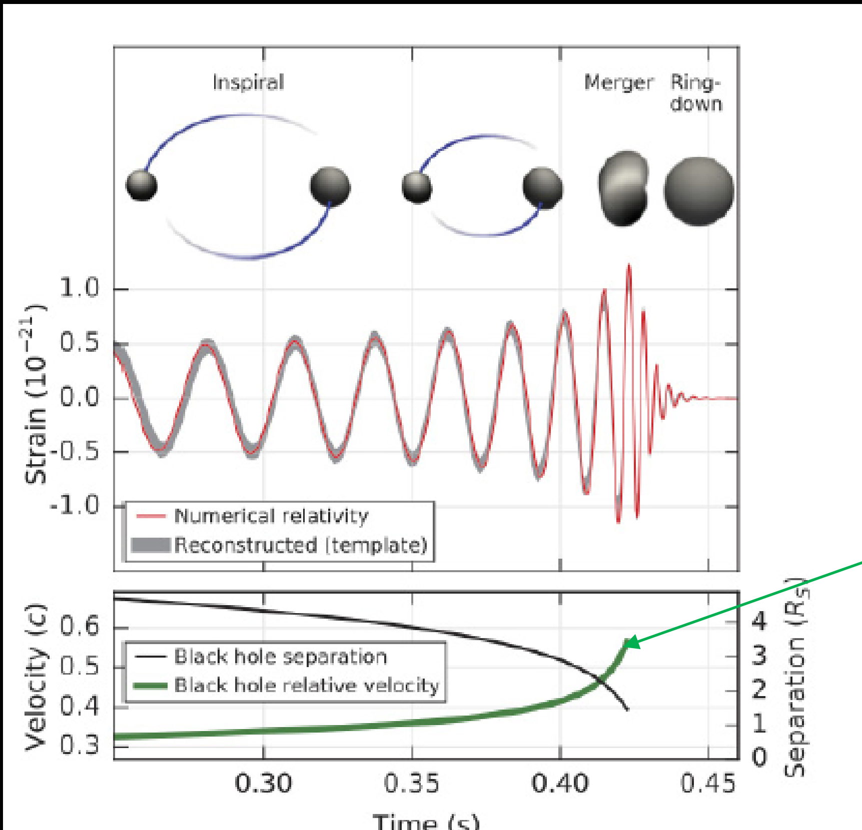
$$M_{Chirp} = \frac{c^3}{G}\left[\left(\frac{5}{96}\right)^3 \pi^{-8}(f_{GW})^{-11}(\dot{f}_{GW})^3\right]^{1/5}$$

$$M_{Chirp} \equiv \frac{(m_1 m_2)^{3/5}}{(m_1+m_2)^{1/5}}$$

Assuming equal mass

M ~ 30 $M_\odot$

Applications to GW formulae to GW150914



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[erg/s]

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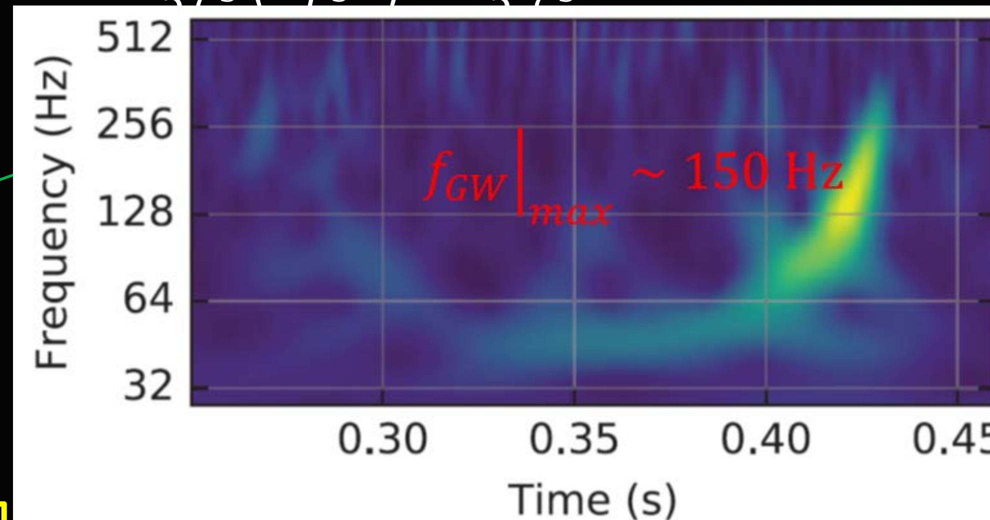


$$M_{Chirp} = \frac{c^3}{G}\left[\left(\frac{5}{96}\right)^3 \pi^{-8}(f_{GW})^{-11}(\dot{f}_{GW})^3\right]^{1/5}$$

$$M_{Chirp} \equiv \frac{(m_1 m_2)^{3/5}}{(m_1+m_2)^{1/5}}$$

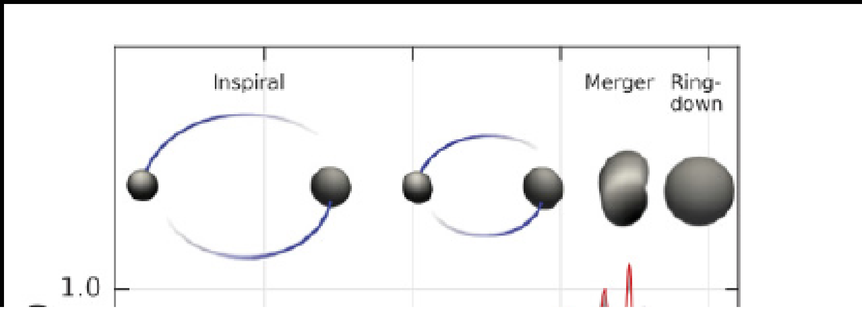
Assuming equal mass

M ~ 30 $M_\odot$

$$\sim \frac{c^3 d_L^2}{4G}\left|\dot{h}\right|_{max}^2 \sim \frac{c^5}{4G}\left(\frac{\omega_{GW}|_{max}\,d_L\,h|_{max}}{c}\right)^2$$

$$\Rightarrow d_L \sim 300\mathrm{Mpc}$$

Applications to GW formulae to GW150914



$$L_{\rm GW} = \frac{1}{5}\frac{G}{c^5}\langle \dddot{I}_{jk}\dddot{I}^{jk}\rangle$$

[erg/s]

Quadrupole

(Issacson's)formula

Selected for a Viewpoint in *Physics*

PRL **116,** 061102 (2016) PHYSICAL REVIEW LETTERS week ending 12 FEBRUARY 2016

The real answers:

Observation of Gravitational Waves from a Binary Black Hole Merger

B. P. Abbott *et al.**

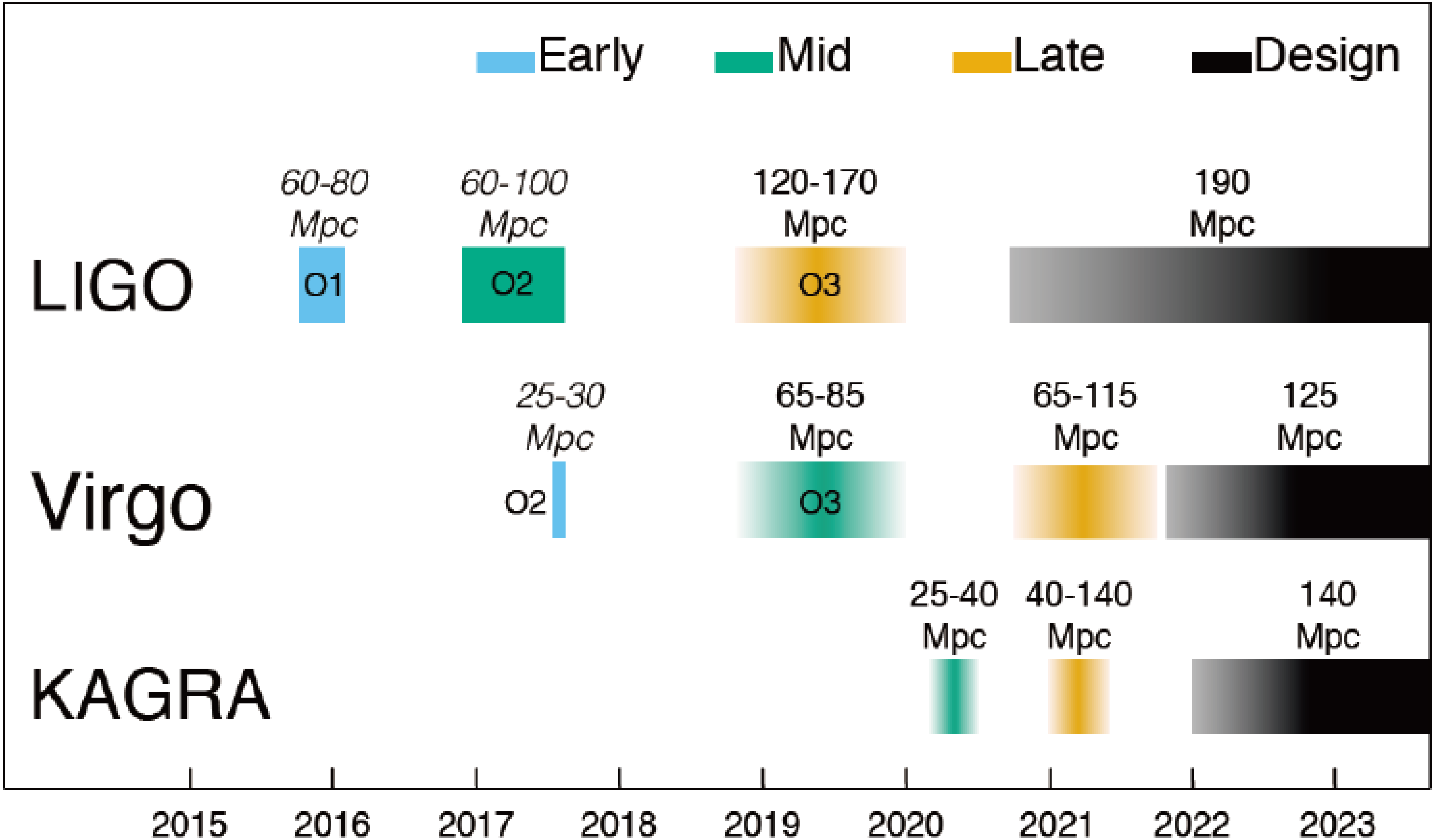
(LIGO Scientific Collaboration and Virgo Collaboration)

(Received 21 January 2016; published 11 February 2016)

On September 14, 2015 at 09:50:45 UTC the two detectors of the Laser Interferometer Gravitational-Wave Observatory simultaneously observed a transient gravitational-wave signal. The signal sweeps upwards in frequency from 35 to 250 Hz with a peak gravitational-wave strain of 1.0×10^{-21}. It matches the waveform predicted by general relativity for the inspiral and merger of a pair of black holes and the ringdown of the resulting single black hole. The signal was observed with a matched-filter signal-to-noise ratio of 24 and a false alarm rate estimated to be less than 1 event per 203 000 years, equivalent to a significance greater than 5.1σ. The source lies at a luminosity distance of 410^{+160}_{-180} Mpc corresponding to a redshift $z = 0.09^{+0.03}_{-0.04}$. In the source frame, the initial black hole masses are $36^{+5}_{-4} M_\odot$ and $29^{+4}_{-4} M_\odot$, and the final black hole mass is $62^{+4}_{-4} M_\odot$, with $3.0^{+0.5}_{-0.5} M_\odot c^2$ radiated in gravitational waves. All uncertainties define 90% credible intervals. These observations demonstrate the existence of binary stellar-mass black hole systems. This is the first direct detection of gravitational waves and the first observation of a binary black hole merger.

Future Roadmaps

☆ **Note; distance to binary-NSs**



✓ 4-detector era -> Sky-localization ~100-> 2~3 deg², Polarization

One-sentence summary

1st . General Introduction

✓**Why GWs** ?

To see the inner-workings of BH/NS forming cites !

✓**Basics of GW Physics and Detection**

From the Einstein equations (quadrupole formulae), to applications how to extract basic information of binary parameters (Hulse-Taylor pulsar)

✓**First detection of GW150914**

The GW astronomy started !

(multimessenger analysis important,

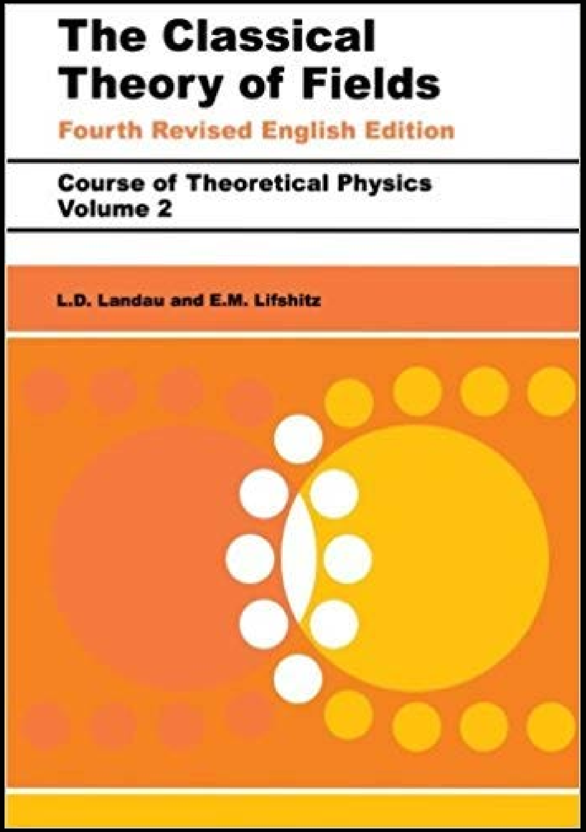
Lectures:**G.M. Pinedo, A Bauswein, R.Diehl** !)

2nd . Core-collapse supernova theory:

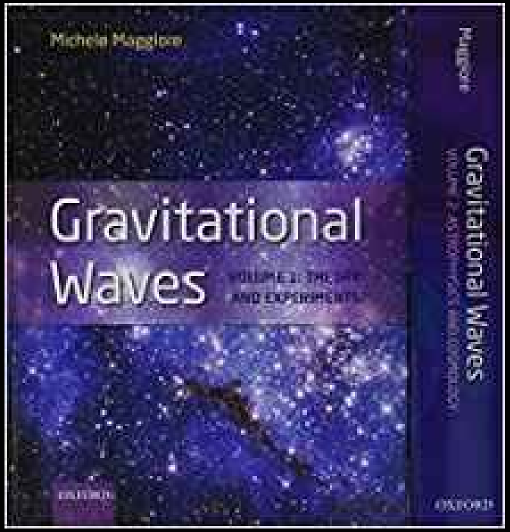
The space-time evolution of dying stars

Useful references:

1. General relativity: textbook by Landau, Lifshitz



2. Concise review of GW physics and detection: by Michele Maggiore



3. Brief overview of GW150914


Ann. Phys. (Berlin) **529**, No. 1–2, 1600209 (2017) / DOI 10.1002/andp.201600209

annalen der physik

Original Paper

The basic physics of the binary black hole merger GW150914

*LIGO Scientific and VIRGO Collaborations**,**

Received 5 August 2016, revised 21 September 2016, accepted 22 September 2016
Published online 4 October 2016