For the next Galactic event (several/century..),
CCSN GW signatures should be clarified in advance
: Hydrodynamics modeling of exploding stars !

from Takiwaki, KK, Suwa (2014), ApJ

1st . General Introduction

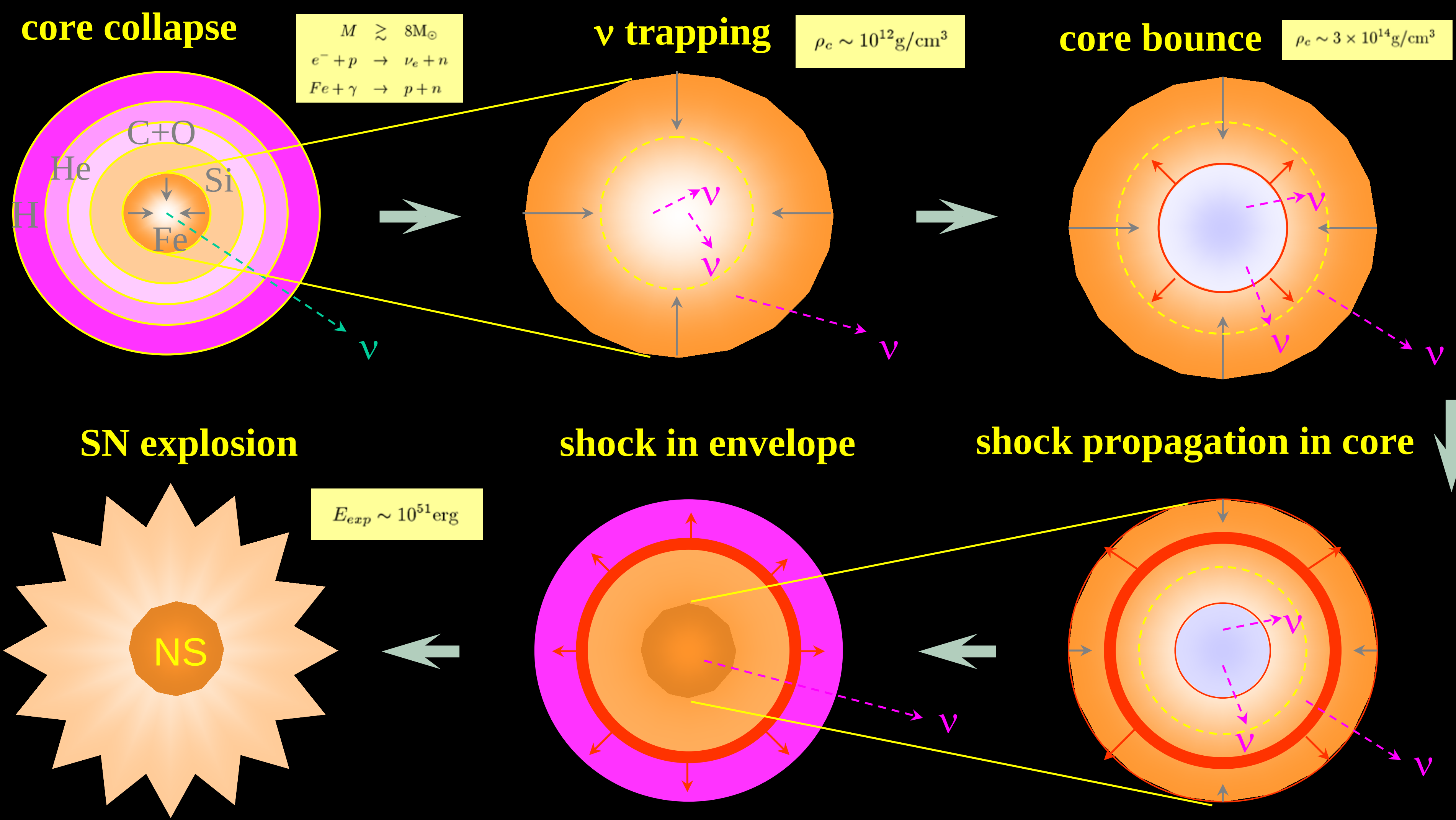
- ✓ Why multi-messengers (inc. GW)?
- ✓ Basics of GW Physics and Detection
- ✓ First detection of GW150914

2nd . Core-collapse supernova theory:
how to solve "*numerically*"
the space-time evolution of dying stars
(40 min)

3rd . GW signatures from core-collapse supernovae: what we can learn from future GW observation ?
(60 min)

Standard scenario of core-collapse SNe

(e.g., Kotake+06, Foglizzo+14, Mezzacappa+15, Janka17 for a review)



Standard scenario of core-collapse SNe

(e.g., Kotake+06, Foglizzo+14, Mezzacappa+15, Janka17 for a review)

core collapse

$M \gtrsim 8M_\odot$
$e^- + p \rightarrow \nu_e + n$

ν trapping $\rho_c \sim 10^{12}\mathrm{g/cm^3}$

core bounce $\rho_c \sim 3 \times 10^{14}\mathrm{g/cm^3}$

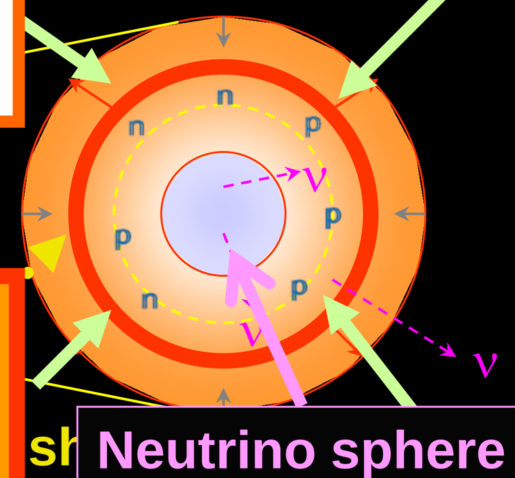
✓Have to find the way how to revive the stalled bounce shock !
✓ The best-studied & most promising:
the neutrino-heating mechanism (Bethe, Wilson 1985)
: **neutrinos heat material to produce explosions !**

SN explosion

NS

propagation in core

$$\nu_e + n \rightarrow e^- + p$$
$$\bar{\nu}_e + p \rightarrow e^+ + n$$



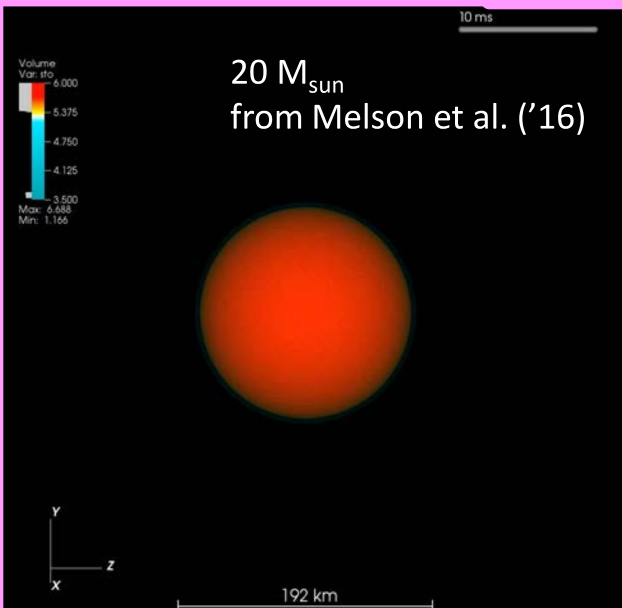
The Neutrino-heating mechanism
(Bethe, Wilson, Mayle 1985)

Two candidate mechanisms of core-collapse supernovae

(**Lecture by T. Foglizzo,** reviews in Janka ('17), Müller ('16), Foglizzo+('15), Burrows('13), Kotake+ ('12))

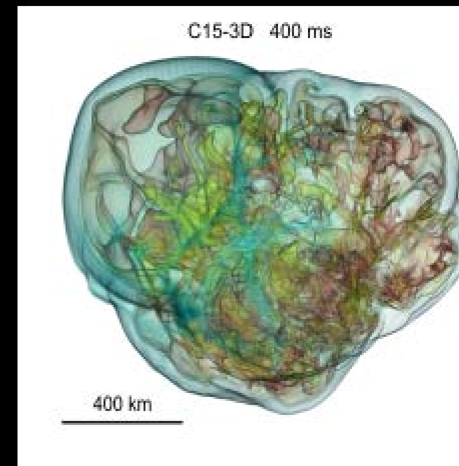
	Neutrino mechanism	**MHD mechanism**
Progenitor	Non- or slowing- rotating star ($\Omega_0 < \sim 0.1$ rad/s)	Rapidly rotation with strong B ($\Omega_0 > \sim\pi$ rad/s, $B_0 > \sim 10^{11}$ G)
Key ingredients	✓ **Turbulent Convection and SASI** (e.g., Kazeroni, Guilet, Foglizzo, (2017)) ✓ **Precollapse Inhomogenities/structures** (e.g., B.Mueller et al. (17), Suwa & Mueller (16)) ✓ **Novel microphysics:** Bollig+(17), Fischer+(18)	✓ **Field winding and the MRI** (e.g., Obergaulinger & Aloy (2017), Rembiasz et al. (2016), Moesta et al. (2016), Masada + (2015)) ✓ **Non-Axisymmetric instabilities** (e.g., Takiwaki, et al. (2016), Summa et al. (2017))
Progenitor fraction	Main players	~<1% (Woosley & Heger (07), ApJ): (hypothetical link to magnetar, collapsar)

20 M_{sun} from Melson et al. ('16)



11.2 M_{sun} from Nakamura et al. in prep.

15 M_{sun} star from Lentz et al. ('15)

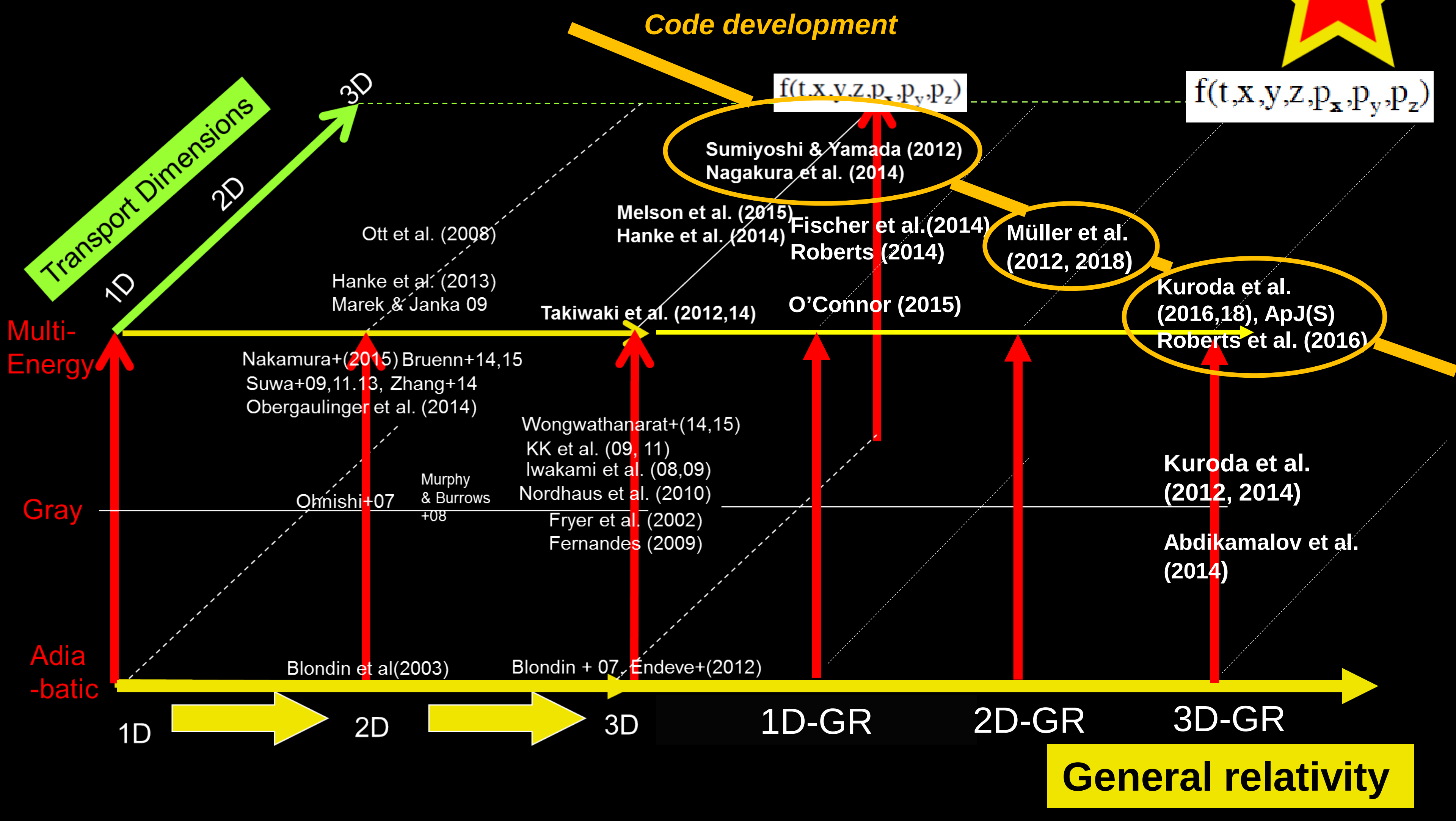


(see also, Burrows et al. ('17), Melson et al. ('15), Lentz et al. ('15), Roberts et al. ('16), B. Mueller ('15), Takiwaki et al. ('16))

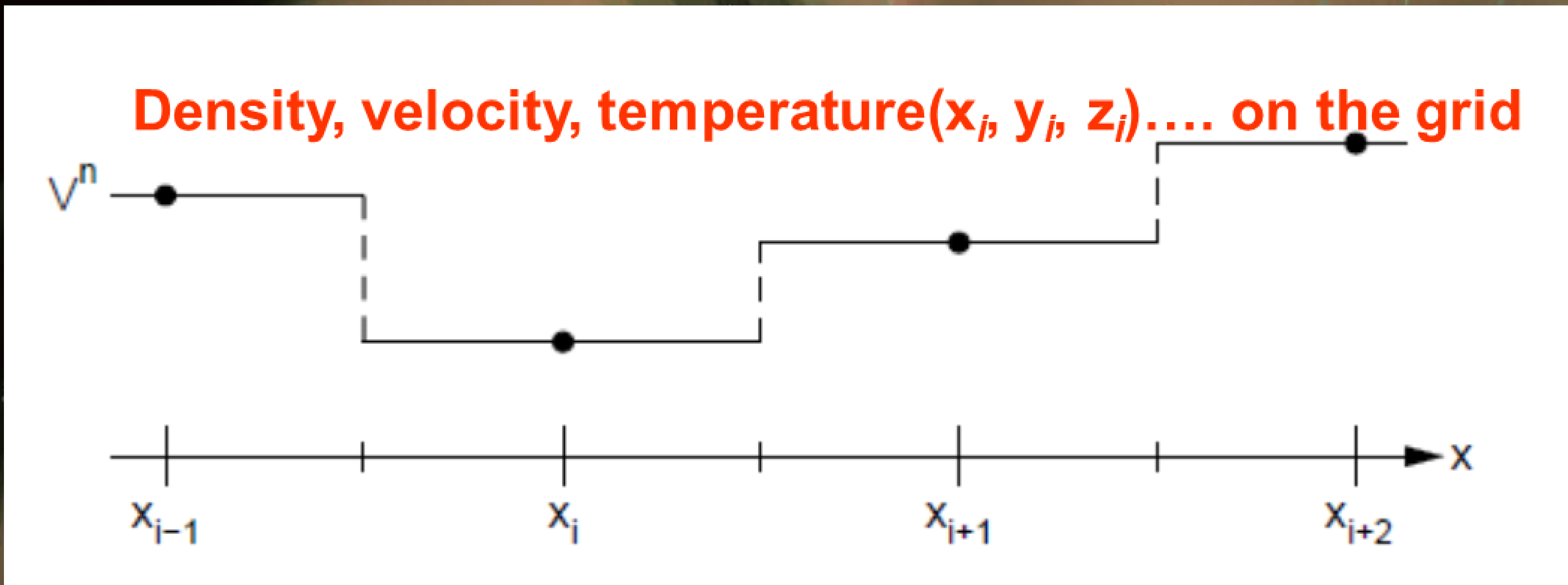
Requirements of CCSN simulations

Disclaimer: only CCSNs

Ultimate goal:
7D Boltzmann transport in full GR Magneto-hydrodynamics (MHD) with accurate microphysical inputs



Quick review; how to evolve hydrodynamics equations (1/3)



Hydrodynamics equations: Non-linear ⇒ Computational Fluid Dynamics (CFD)

Quick review; how to evolve hydrodynamics equations (1/3)

Closed set of hydro equations:

Mass conservation

1 $$\frac{\partial \rho}{\partial t} + \mathrm{div}\,\rho\mathbf{v} = 0$$

Momentum conservation

3 $$\frac{\partial \rho v_i}{\partial t} + \frac{\partial \pi_{ij}}{\partial x_j} = \rho g_i$$

$$\pi_{ij} = \rho v_i v_j + \delta_{ij} p$$

Energy conservation

1 $$\frac{\partial}{\partial t}(\rho e) + \mathrm{div}\,[(\rho e + p)\,\vec{v}] = -\rho \vec{v}\,\mathrm{grad}\,\Phi$$

1 Equation of State: EOS $: P(\rho, T, Y_e)$

1 Poission eq. $\Delta\Phi = 4\pi G\rho$

variables

ρ 1, $\mathbf{v}$ 3

p 1, g_i 1

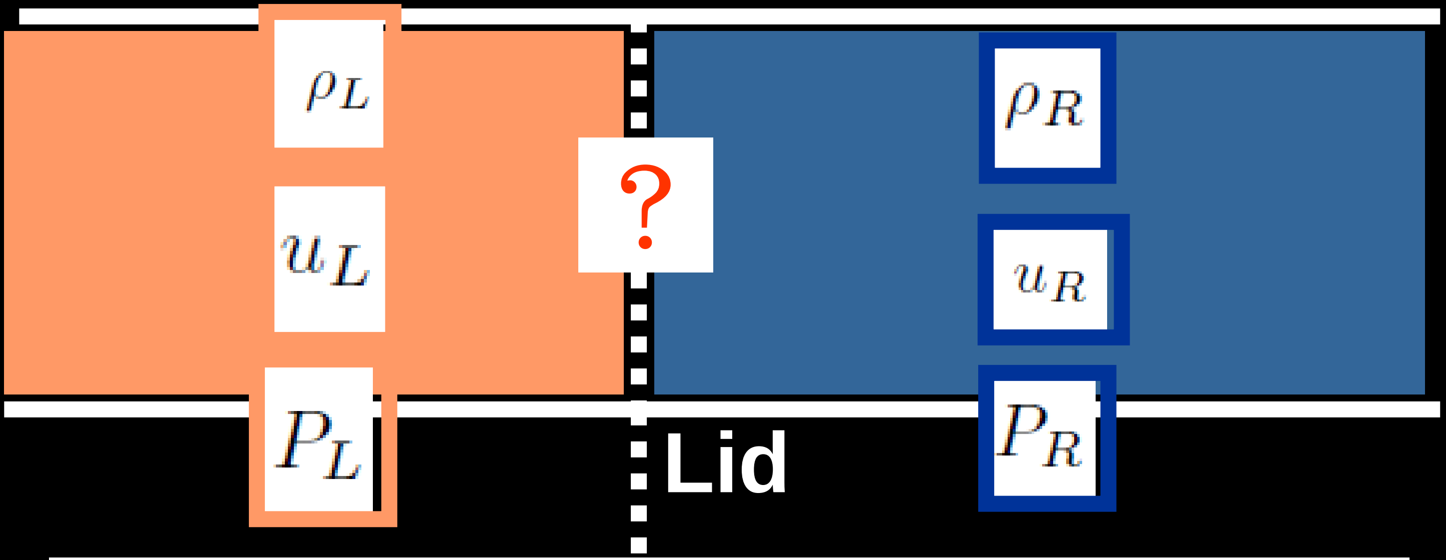
e 1, Φ 1

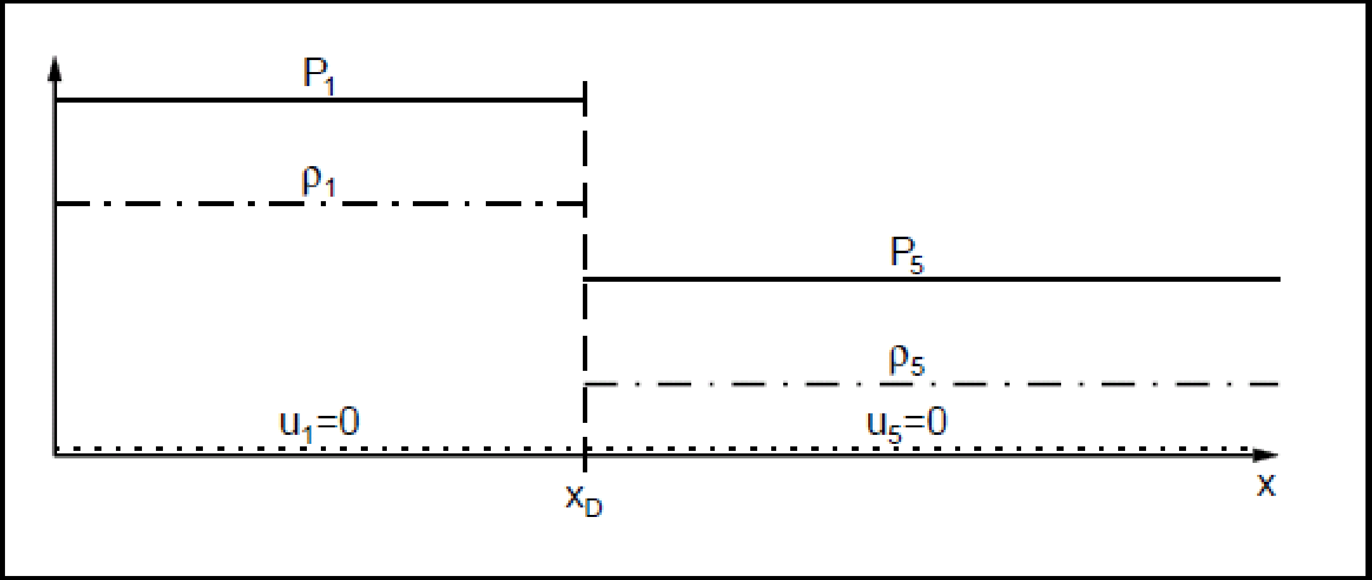
1

$g_i = -\nabla_i \Phi$

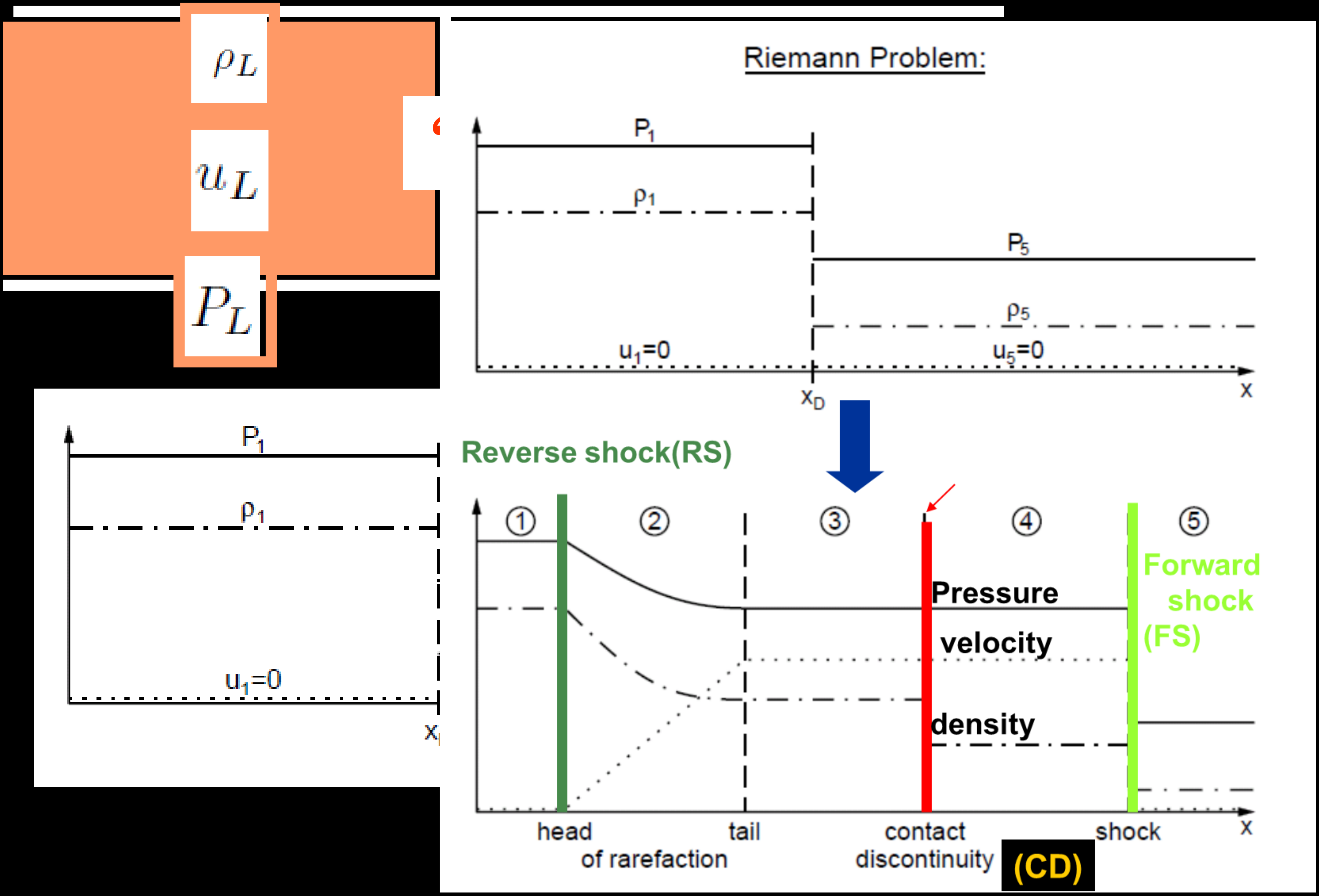
Hydrodynamics equations: Non-linear ⇒ Computational Fluid Dynamics (CFD)

CFD: in essence.. The Riemann problem





CFD: in essence.. The Riemann problem



Quick review; how to evolve hydrodynamics equations (3/3)

$$\mathbf{U}_t + \mathbf{F}(\mathbf{U})_x = 0$$

$$\mathbf{U} = \begin{pmatrix} \rho \\ \rho u \\ \rho E \end{pmatrix}$$

✓Discretization

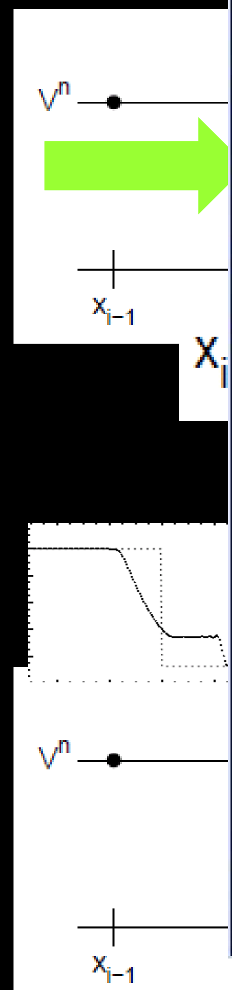
$$\frac{\mathbf{U}_i^{n+1} - \mathbf{U}_i^n}{\Delta t} = -\frac{\mathbf{F}_{i-1/2}^n - \mathbf{F}_{i+1/2}^n}{\Delta x}$$

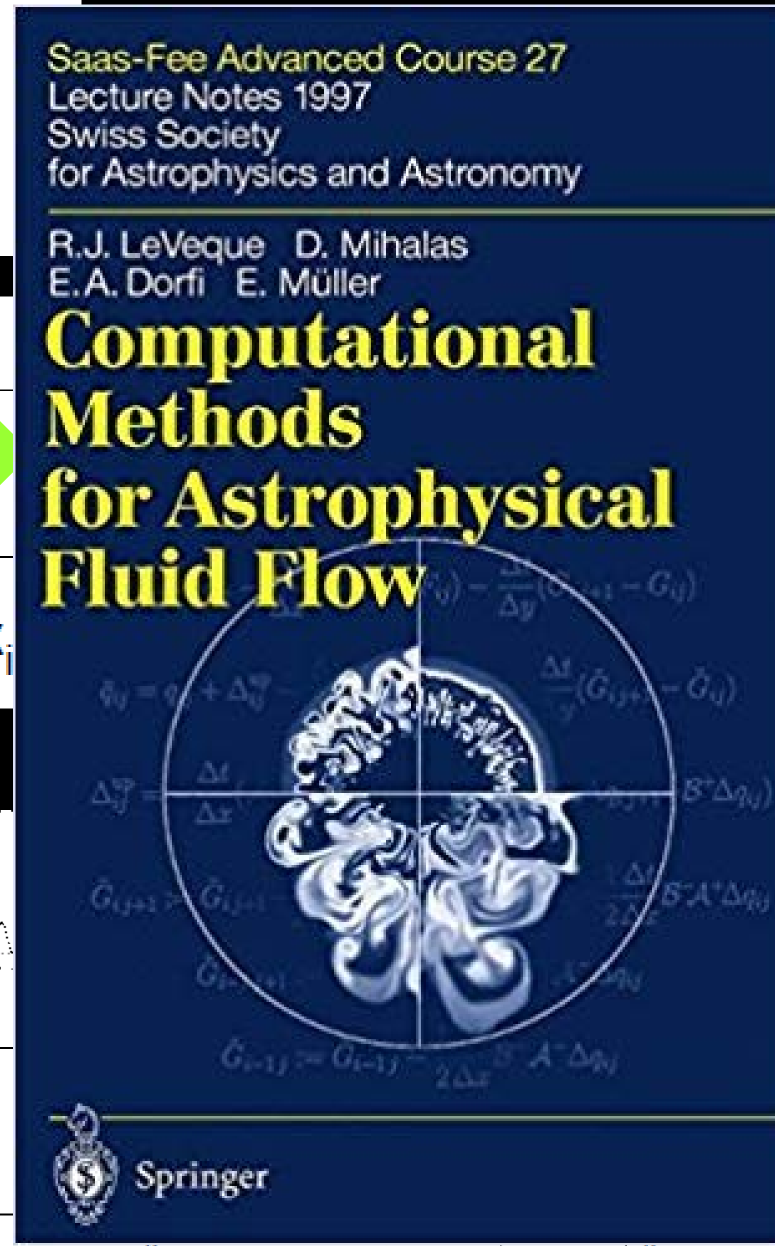
$$\mathbf{U}_i^{n+1} = \mathbf{U}_i^n + \frac{\Delta t}{\Delta x}\left(\mathbf{F}_{x_{i-1/2}} - \mathbf{F}_{i+1/2}\right)$$

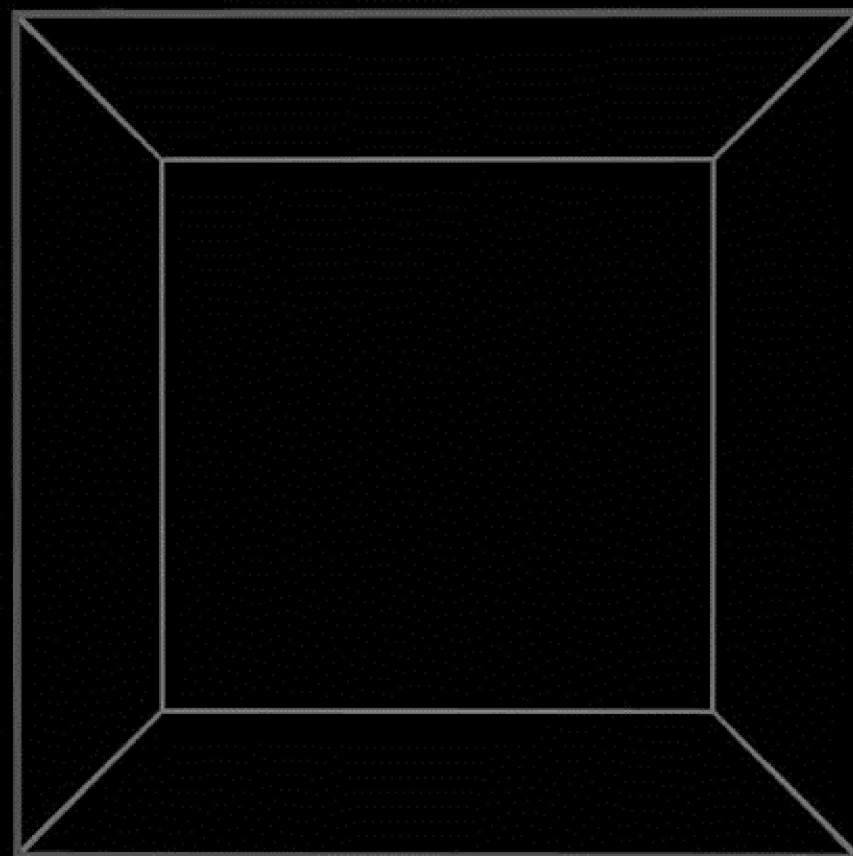
Solve the Riemann problem

accurate

Godunov method (Godunov, 1959)
Roe (Roe, 1981)
HLLD・C (Toro et al. 1994)
HLL (Einfeldt, 1988)





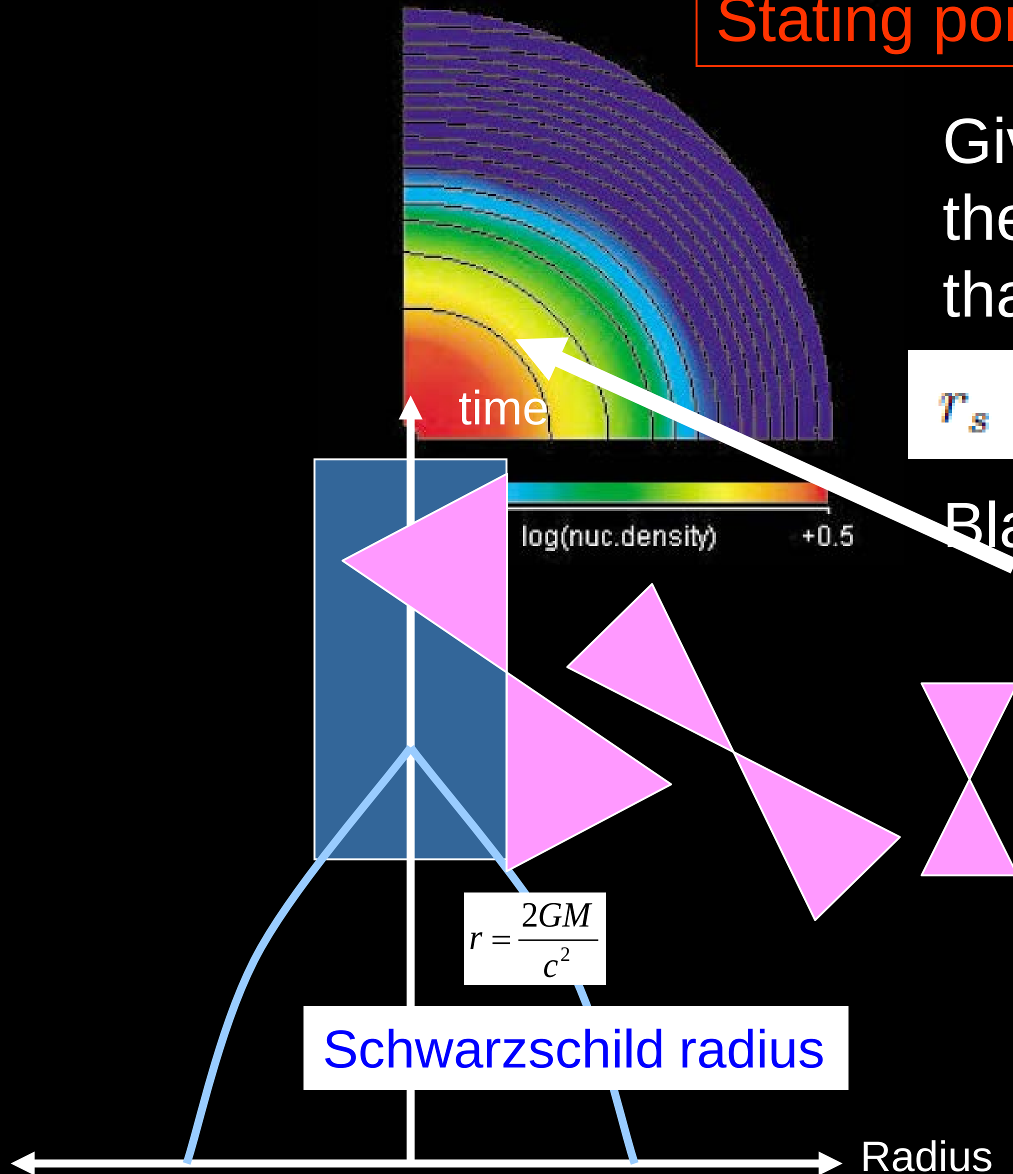


3D Newtonian simulations of rapidly rotating core-collapse of a 27 M_{sun} star (Ω_0 = 2 rad/s) (from Takiwaki & Kotake, 2018, MNRAS Letters)

Why GR ? Introduction to Numerical Relativity

See textbooks by Baumgarte and Shapiro, Shibata, Rezzolla

Stating point;



Given the stellar core mass M, if the core radius(R) is smaller than Schwarzshild radius

$$r_s = 2GM/c^2 = 2.95(M/M_\odot)\,\text{km}$$

Black hole (BH) forms.

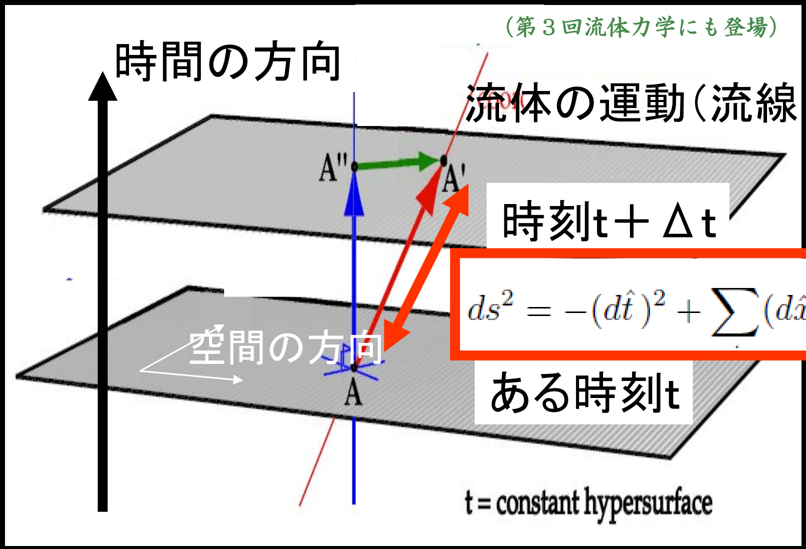
(P)NS radius ＝R~10-50km
mass~ $M_\odot$

R_s/R ～10%: GR effect is significant in the stellar center !

Solving dynamics of space-time(1/4)

For a flat spacetime: Minkowski

$$\eta_{\mu\nu} \equiv \mathrm{diag}(-1, 1, 1, 1)$$

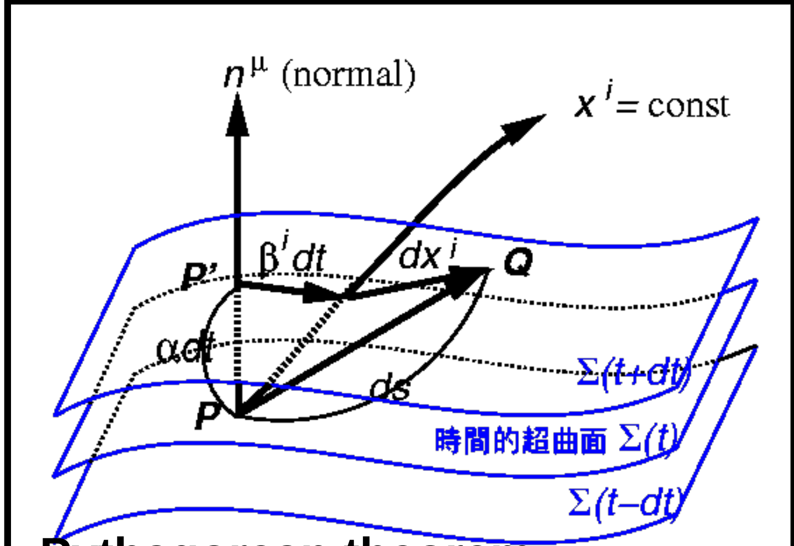


$$ds^2 = -(d\hat{t})^2 + \sum (d\hat{x}^i)^2$$

Flat space-time : $d^2s = \eta_{\mu\nu} dx^\mu dx^\nu$

Minkowski $dx^0 = dt, dx^i = dx$

Curved spacetime



Pythagorean theorem

In the curved spacetime,

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu$$

$g_{\mu\nu}(x^\alpha)$: **depends on space-time**

$d\hat{t} = \alpha dt$ α : lapse $d\hat{x}^i = dx^i + \beta^i dt$: **shift vector** β^i

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = -\alpha^2 dt^2 + \gamma_{ij}(dx^i + \beta^i dt)(dx^j + \beta^j dt)$$

$$g_{\mu\nu} = \left(\begin{array}{c|ccc} -\alpha^2 + \beta_i \beta^i & \beta_1 & \beta_2 & \beta_3 \\ \hline \beta_1 & & & \\ \beta_2 & & \gamma_{ij} & \\ \beta_3 & & & \end{array} \right)$$

Need to determine 10 (variables or d.o.f.)

Need “Numerical relativity” .

（Analytic solution only in special cases, Schwarzschild, Kerr）

Solving dynamics of space-time (2/4)

Need to determine $g_{\mu\nu}$

Again, the Minkowski metric.

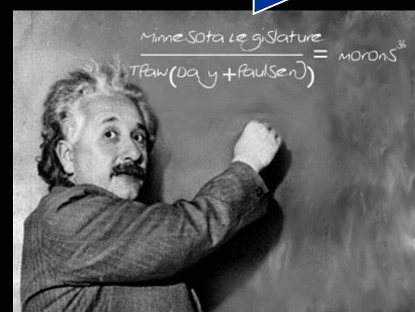
$$g^{\mu\nu} \longrightarrow \eta^{\mu\nu} \equiv \mathrm{diag}(-1,1,1,1)$$

Need to solve Einstein equation

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} T_{\mu\nu}$$

In the limit of $R_s/R \rightarrow 0$

$$\nabla^2 \phi = 4\pi G \rho$$



$R_{\mu\nu}$ Ricci tensor

Energy-momentum tensor

$$T^{\mu\nu} = (e+p) u^\mu u^\nu + p g^{\mu\nu}$$

$R_{\mu\nu}$, R are the functional of $g_{\mu\nu}$

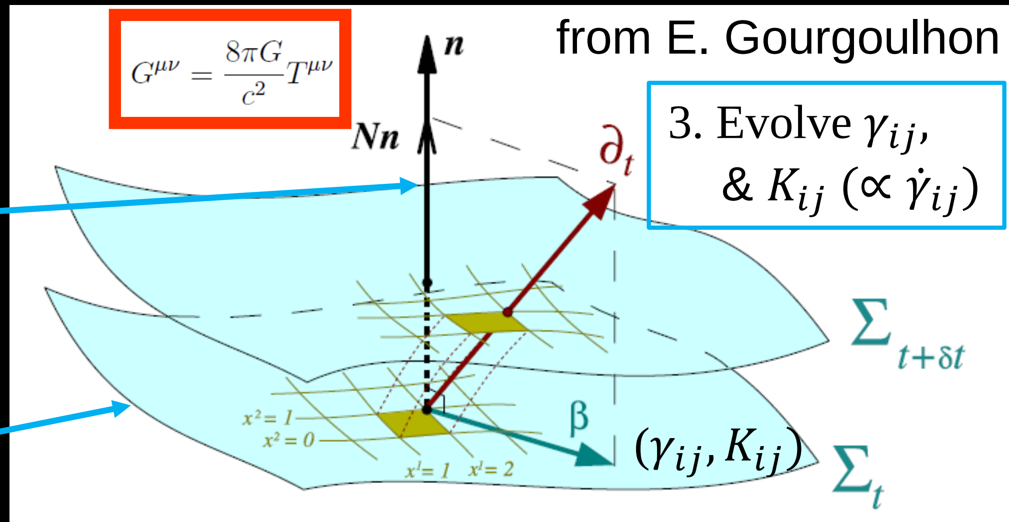
$$R_{\mu\nu} = \Gamma^\alpha{}_{\mu\nu,\alpha} - \Gamma^\alpha{}_{\mu\alpha,\nu} + \Gamma^\alpha{}_{\gamma\alpha}\Gamma^\gamma{}_{\mu\nu} - \Gamma^\alpha{}_{\gamma\nu}\Gamma^\gamma{}_{\mu\alpha}$$

$$\Gamma^\alpha{}_{\beta\gamma} \equiv \frac{1}{2} g^{\alpha\mu}(g_{\mu\beta,\gamma} + g_{\mu\gamma,\beta} - g_{\beta\gamma,\mu})$$

$R = R^\beta{}_\beta$ Ricci scalar

Solving dynamics of space-time (3/4)

✓3+1 decomposition (see textbooks by E.Gougoulhon, M. Shibata, L. Rezzolla..)



2. Time-like unit vector :n defined

1. Set space-like hyper surface at *t*

$$(\partial_t - \mathcal{L}_\beta)\gamma_{ij} = -2\alpha K_{ij}$$

$$G_{\mu\nu} n^\mu n^\nu = 8\pi T_{\mu\nu} n^\mu n^\nu \text{ : Hamiltonian constraint}$$

$$G_{\mu\nu} n^\mu \gamma^\nu_k = 8\pi T_{\mu\nu} n^\mu \gamma^\nu_k \text{ : Momentum constraint}$$

$$G_{\mu\nu} \gamma^\mu_i \gamma^\nu_j = 8\pi T_{\mu\nu} \gamma^\mu_i \gamma^\nu_j \text{ : Evolution equation}$$

γ_{ij} : 3-metric, n^μ : timelike normal

Analogy with EM

$$\begin{cases} \nabla_i E^i = 4\pi\rho_e \\ \nabla_i B^i = 0 \end{cases}$$

Constraint equation

$$\begin{cases} \dot{E}_i = (\nabla \times B)_i - 4\pi j_i \\ \dot{B}_i = -(\nabla \times E)_i \end{cases}$$

Evolution equation

✓Initial ... bolic)

Einstein eqns in 3+1 formalism

$$\begin{aligned} \partial_t \gamma_{ij} &= -2\alpha K_{ij} + \nabla_i \beta_j + \nabla_j \beta_i \\ \partial_t K_{ij} &= -\nabla_i \nabla_j \alpha + \alpha(R_{ij} + K K_{ij} - 2K_{ik}K^k_j) \\ &\quad + \beta^k \nabla_k K_{ij} + K_{ik} \nabla_j \beta^k + K_{jk} \nabla_i \beta^k \\ &\quad - 8\pi\alpha\left(S_{ij} - \frac{\gamma_{ij}}{2}(S - \rho_{\rm H})\right) \\ 0 &= R + K^2 - K_{ij}K^{ij} - 16\pi\rho_{\rm H} \\ 0 &= \nabla_i (K^{ij} - \gamma^{ij} K) - 8\pi S^j \end{aligned}$$

Degree of freedom =
10
– 4 (Hamiltonian 1, Momentum 3)
– 4 (α, β^i)
= 2 (GW: +, x mode)

Solving dynamics of space-time (4/4)

✓ With $g_{\mu\nu}$, one can solve the (radiation-)hydrodynamics equations !

Mass cons.

$$\frac{\partial\sqrt{\gamma}\rho W}{\partial t} + \frac{\partial\sqrt{-g}\rho W\hat{v}^i}{\partial x^i} = 0$$

Momentum cons.

$$\frac{\partial\sqrt{\gamma}\rho h W^2 v_j}{\partial t} + \frac{\partial\sqrt{-g}\left(\rho h W^2 v_j \hat{v}^i + \delta^i_j P\right)}{\partial x^i} = \frac{1}{2}\sqrt{-g}T^{\mu\nu}\frac{\partial g_{\mu\nu}}{\partial x^j} - \left(\nabla_\beta T^{i\beta}_{(\nu)}\right)_C$$

Energy cons.

$$\frac{\partial\sqrt{\gamma}\tau}{\partial t} + \frac{\partial\sqrt{-g}\left(\tau\hat{v}^i + P v^i\right)}{\partial x^i} = \alpha\sqrt{-g}\left(T^{\mu 0}\frac{\partial \ln\alpha}{\partial x^\mu} - T^{\mu\nu}\Gamma^0_{\mu\nu}\right) - \left(\nabla_\beta T^{0\beta}_{(\nu)}\right)_C$$

Newtonian limit: $D = \rho W \overset{\text{Newton}}{\longrightarrow} \rho$

$$W = \frac{1}{\sqrt{1-v^2}} = 1 + \frac{1}{2}v^2 + \mathcal{O}(v^4)$$

$$S^i = \rho h W^2 v^i = \rho(1 + \epsilon + \frac{p}{\rho})v^i \overset{\text{Newton}}{\longrightarrow} \rho v^i$$

$$\rho_{,t} + (\rho v^i)_{,i} = 0$$

Neutrino energy momentum tensor

$$T^{\alpha\beta}_{(\nu)} \equiv \rho_b E_{(\nu)} u^\alpha u^\beta + F^\alpha_{(\nu)} u^\beta + F^\beta_{(\nu)} u^\alpha + p^{\alpha\beta}_{(\nu)}$$

✓Baumgarte-Shibata-Shapiro-Nakamura (BSSN) formalism : ADM numerically **unstable**

BSSN variables:

$$\phi \equiv \frac{1}{12}\ln[\det(\gamma_{ij})] ,$$

$$\tilde{\gamma}_{ij} \equiv e^{-4\phi}\gamma_{ij} ,$$

$$K \equiv \gamma^{ij}K_{ij} ,$$

$$\tilde{A}_{ij} \equiv e^{-4\phi}\left(K_{ij} - \frac{1}{3}\gamma_{ij}K\right) ,$$

$$\tilde{\Gamma}^i \equiv -\tilde{\gamma}^{ij}{}_{,j} .$$

$$(\partial_t - \mathcal{L}_\beta)\tilde{\gamma}_{ij} = -2\alpha\tilde{A}_{ij} \quad (11)$$

$$(\partial_t - \mathcal{L}_\beta)\phi = -\frac{1}{6}\alpha K \quad (12)$$

$$(\partial_t - \mathcal{L}_\beta)\tilde{A}_{ij} = e^{-4\phi}\left[\alpha(R_{ij} - 8\pi\gamma_{i\mu}\gamma_{j\nu}T^{\mu\nu}_{(\text{total})} - D_i D_j\alpha\right]^{\text{trf}} + \alpha(K\tilde{A}_{ij} - 2\tilde{A}_{ik}\tilde{\gamma}^{kl}\tilde{A}_{jl}) \quad (13)$$

$$(\partial_t - \mathcal{L}_\beta)K = -\Delta\alpha + \alpha(\tilde{A}_{ij}\tilde{A}^{ij} + K^2/3) + 4\pi\alpha\left(n_\mu n_\nu T^{\mu\nu}_{(\text{total})} + \gamma^{ij}\gamma_{i\mu}\gamma_{j\nu}T^{\mu\nu}_{(\text{total})}\right) \quad (14)$$

$$(\partial_t - \beta^k\partial_k)\tilde{\Gamma}^i = 16\pi\tilde{\gamma}^{ij}\gamma_{i\mu}n_\nu T^{\mu\nu}_{(\text{total})} - 2\alpha\left(\frac{2}{3}\tilde{\gamma}^{ij}K_{,j} - 6\tilde{A}^{ij}\phi_{,j} - \tilde{\Gamma}^i_{jk}\tilde{A}^{jk}\right) + \tilde{\gamma}^{jk}\beta^i_{,jk} + \frac{1}{3}\tilde{\gamma}^{ij}\beta^k_{,kj} - \tilde{\Gamma}^j\beta^i_{,j} + \frac{2}{3}\tilde{\Gamma}^i\beta^j_{,j} + \beta^j\tilde{\Gamma}^i_{,j} - 2\tilde{A}^{ij}\alpha_{,j} , \quad (15)$$

General Relativistic Simulations (**limited to CCSN context**) :

✓AEI-Southampton-Amsterdam Caltech collaboration:
(e.g., Cactus code http://cactuscode.org)

✓ Our team:
Kuroda, KK, Takiwaki

✓ Monash-Garching group
(Conformally flatness approximation)
Mueller, Janka et al.

Solving dynamics of neutrino(ν) radiation field (1/3)

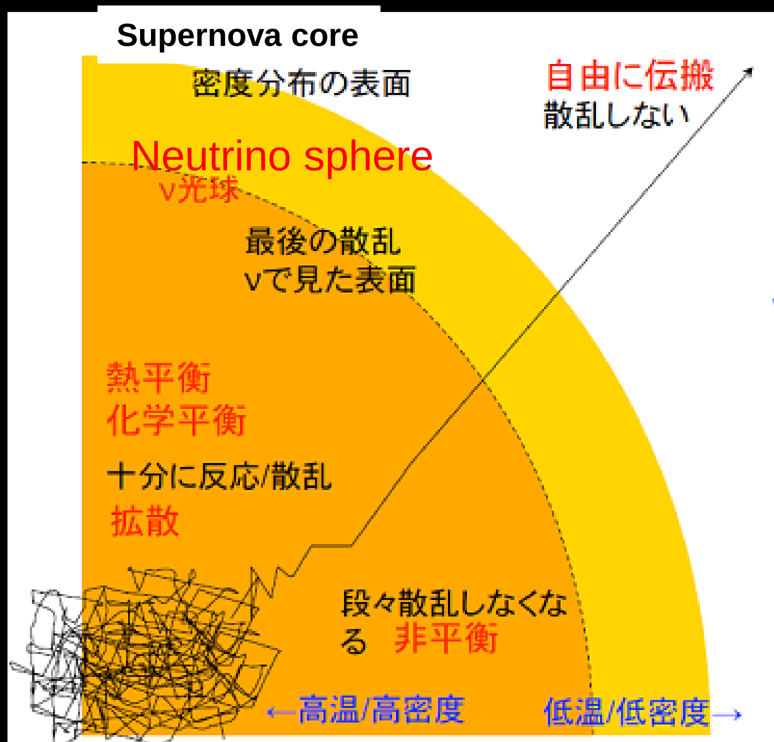
$f_{(\nu)}(\sout{t, r, \theta, \phi}, E_\nu, \sout{\theta_\nu, \phi_\nu})$

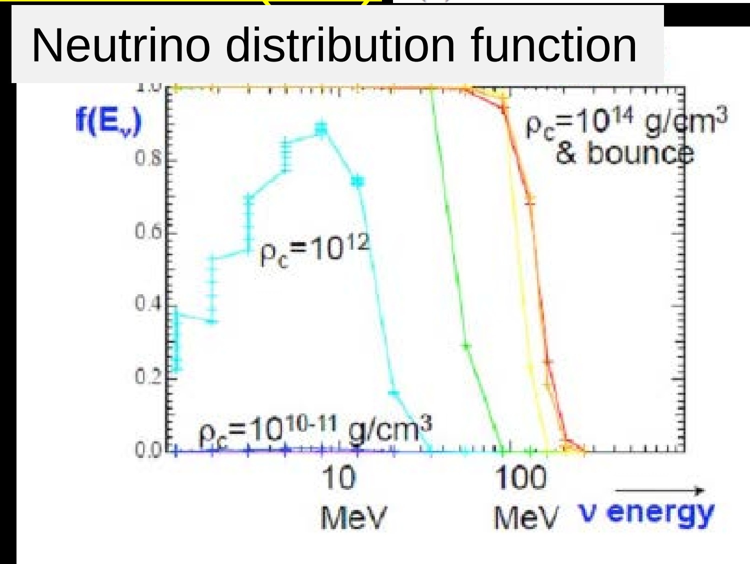
✓Neutrino propagation in supernova core

Free-streaming limit

Semi-transparent

Diffusion limit





✓**β-equilibrium** is achieved

$$f_\nu(E_\nu) = \frac{1}{\exp\left(E_\nu - \mu_\nu\right)/k_{\rm B}T + 1}$$

only in the high-density region !

⇒ **Neutrino occupation probability : f_ν in the energy space needs to be accurately treated (bottom line).**

Levels of approximations:

$$f(t, r, \theta, \phi, E, \theta_p, \phi_p)$$

“MGMA”(6 dimensional problem)

$$E_R(t, r, \theta, \phi, E) = \int d\theta_p \, d\phi_p \; f$$

“MG”(Multi energy-Group)
(e.g., MGFLD, M1, IDSA)

$$E_R(t, r, \theta, \phi) = \int dE \, d\theta_p \, d\phi_p \; f$$

“Gray (no energy-dependence)”

Solving dynamics of neutrino(ν) radiation field (1/3)

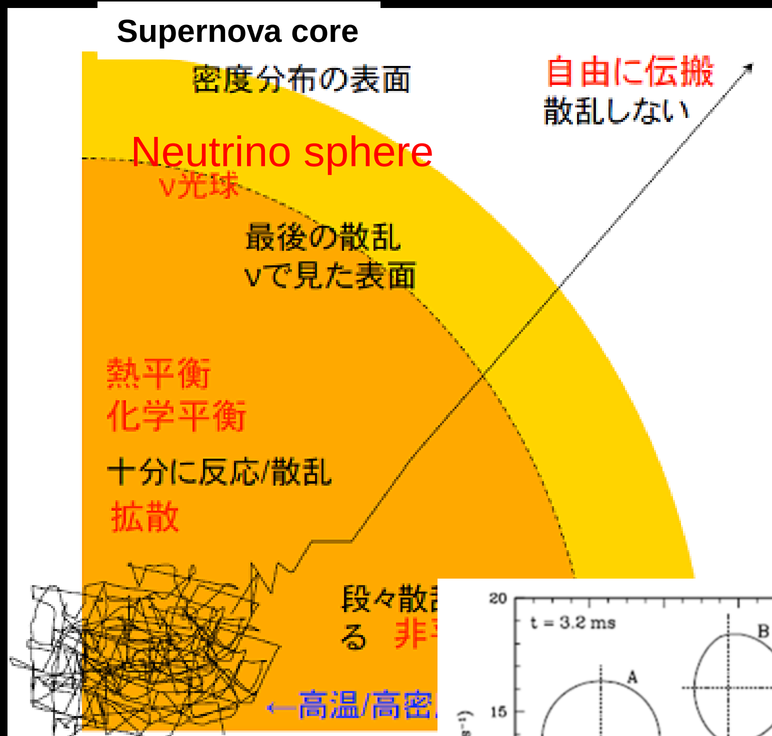
$f_{(\nu)}(t, r, \theta, \phi, E_\nu, \theta_\nu, \phi_\nu)$

✓Neutrino propagation in supernova core

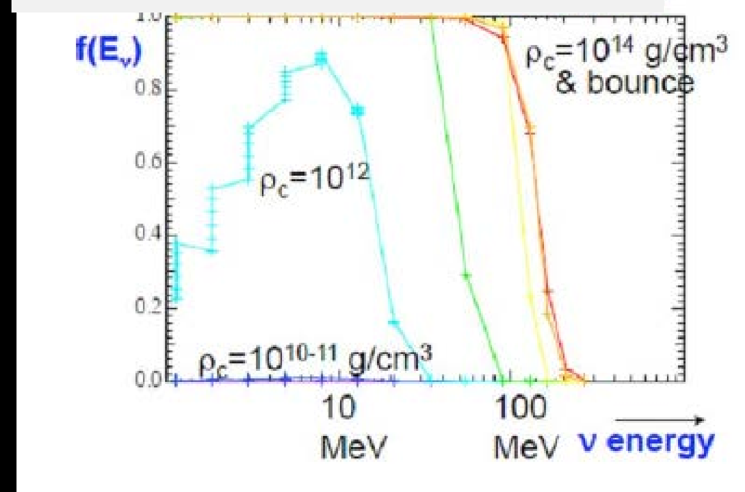
Free-streaming limit

Semi-transparent

Diffusion limit



Neutrino distribution function



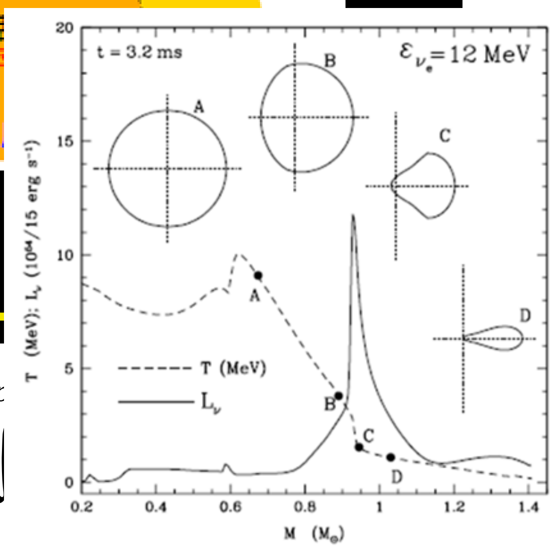
✓**β-equilibrium** is achieved

Levels of approximations:

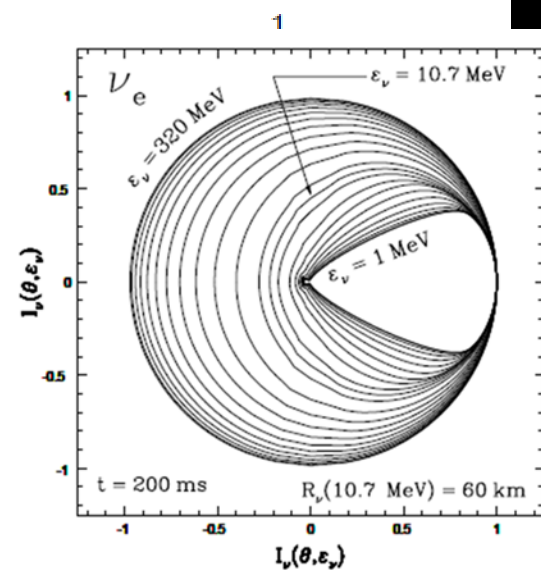
$f(t, r, \theta, \phi, E, \theta_p, \phi_p$

$E_R(t, r, \theta, \phi, E) = \int$

$E_R(t, r, \theta, \phi) = \int d$



Multi-angle, Multi-energy transport : Ultimate goal !



Thompson et al. (2003), ApJ

Solving dynamics of neutrino(ν) radiation field (2/3)

Neutrino-matter interaction

Neutrino Boltzmann equation $f_{(\nu)}(t, r, \theta, \phi, E_\nu, \theta_\nu, \phi_\nu)$

$$T^{\alpha\beta}_{(\nu)} \equiv \rho_b E_{(\nu)} u^\alpha u^\beta + F^\alpha_{(\nu)} u^\beta + F^\beta_{(\nu)} u^\alpha + p^{\alpha\beta}_{(\nu)}$$

$$\frac{\partial f_{(\nu)}}{\partial t} + \frac{d\vec{r}}{dt} \cdot \frac{\partial f_{(\nu)}}{\partial \vec{r}} + \frac{d\vec{p}}{dt} \cdot \frac{\partial f_{(\nu)}}{\partial \vec{p}} = \left.\frac{d f_{(\nu)}}{dt}\right|_{\mathrm{coll}}$$

0th, energy density, $E_{(\nu)} = E_\nu^3 \int d\Omega\, f_{(\nu)}\, \hat{p}^0_{(\nu)} = E_\nu^3 \int d\Omega$

1st moment, Flux: $F^\alpha_{(\nu)} = E_\nu^3 \int d\Omega\, f_{(\nu)}\, \hat{p}^\alpha_{(\nu)}$

(note: comoving vs. laboratory frame)

✓A roadmap how to implement neutrino heating/cooling in your code!

Easy

1. Simple deleptonization
   (e.g., Liebendoerfer (2005), ApJ)
2. Neutrino leakage scheme
   (e.g., Kotake et al. (2003), ApJ, Sekiguchi (2009), PTP)
3. Simple deleptonization + Light-bulb scheme
   (e.g., Janka and Mueller (1996) A &A , Murphy et al. (2009, ApJ))

Neutrino cooling (on)

6 months "only if" your hydro is robust

"Manual" Neutrino heating (on)

4. Single energy flux-limited diffusion $f_{(\nu)}(t, r, \theta, \phi)$
   (e.g., Keil et al. (1996) A & A, Fryer (2001))
5. Multi-energy flux-limited diffusion, M1 $f_{(\nu)}(t, r, \theta, \phi, E_\nu)$
   Isotropic Diffusion Source Approximation
   (e.g., Bruenn (1985), Burrows et al. (2006), Liebendoerfer et al. (2009), Obergaulinger et al. (2016)
6. Ray-by-ray Boltzmann transport (VEF) $f_{(\nu)}(t, r, \theta, \phi, E_\nu, \bar{\theta}_\nu)$
   (e.g., Rampp and Janka (1998), Buras et al. (2006), Mueller et al. (2009))
7. Full multi-angle Boltzmann transport $f_{(\nu)}(t, r, \theta, \phi, E_\nu, \theta_\nu, \phi_\nu)$
   (e.g., Sumiyoshi and Yamada (2012), Nagakura et al. (2016))

✓Self-consistent modeling (no parameters for heating/cooling)

✓ Implicit scheme needed !

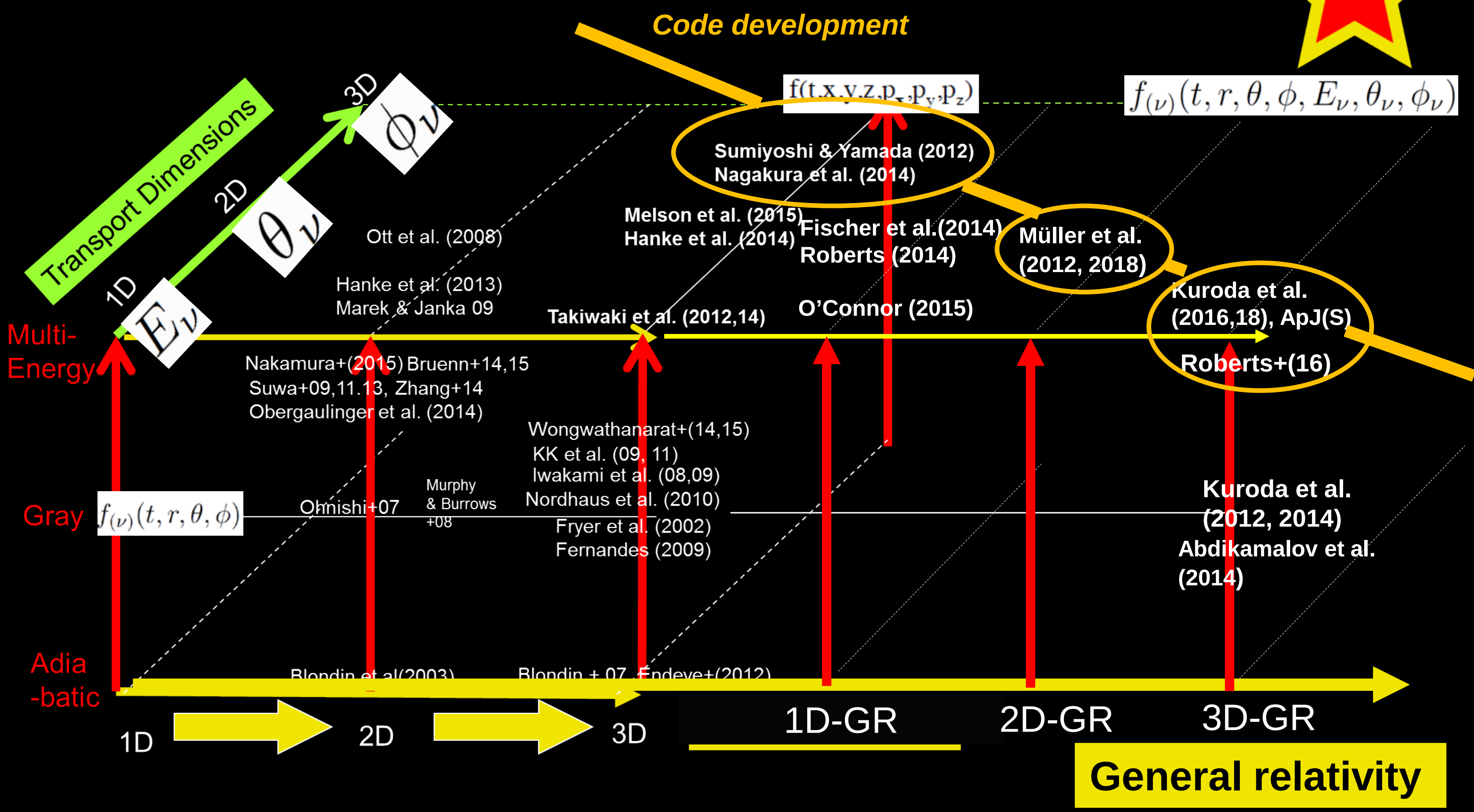
> a few years (or a life work!)

Hard

Current Status of CCSN simulations

Disclaimer: only CCSNs

Ultimate goal:
7D Boltzmann transport in full GR Magneto-hydrodynamics (MHD) with accurate microphysical inputs



General relativistic neutrino transport with detailed ν transport: Vertex-CoCoNuT code

B. Mueller et al (2012), ApJ

✓ Conformal flatness approximation (+)

$$ds^2 = -\alpha(t,r)^2 dt^2 + \phi(t,r)^4 \left[(dr + \beta^r(t,r)\,dt)^2 + r^2 d\theta^2 + r^2 \sin^2\theta\, d\varphi^2\right]$$

✓ L.H.S. of Boltzmann eq. is super messy…

$$W\left[\frac{\xi}{\alpha}\left(\frac{\partial f}{\partial t} - \beta^r \frac{\partial f}{\partial r}\right) + \frac{\nu}{\phi^2}\frac{\partial f}{\partial r}\right] - \frac{\varepsilon W^3}{r\alpha\phi^3}\frac{\partial f}{\partial \varepsilon}\left\{\beta^r\phi^3\left(-\psi - r\mu\frac{\partial v_r}{\partial r}\right) + v_r^2\phi\left[\beta^r\phi\left(2r\frac{\partial \phi}{\partial r} - \psi\phi\right) + r\left(-\mu\frac{\partial \alpha}{\partial r} + \mu^2\phi^2\frac{\partial \beta^r}{\partial r} - \frac{\partial \phi^2}{\partial t}\right)\right] + v_r^3\left[r\mu\phi\left(-\mu\frac{\partial \alpha}{\partial r} + \frac{\partial \beta^r\phi^2}{\partial r} - \frac{\partial \phi^2}{\partial t}\right) - \psi\frac{\alpha}{\phi}\frac{\partial r\phi^2}{\partial r}\right] + \phi\left[r\mu\left(\mu\alpha\frac{\partial v_r}{\partial r} + \frac{\partial \alpha}{\partial r} + \phi^2\left(-\mu\frac{\partial \beta^r}{\partial r} + \frac{\partial v_r}{\partial t}\right)\right) + r\frac{\partial \phi^2}{\partial t} - r\beta^r\frac{\partial \phi^2}{\partial r}\right] + v_r\alpha\left[\phi\left(\psi + r\mu\frac{\partial v_r}{\partial r}\right) + 2r\psi\frac{\partial \phi}{\partial r} + \phi^2\left(\mu\frac{\partial v_r}{\partial t} - \frac{\partial \beta^r}{\partial r}\right) + \frac{\partial \phi^2}{\partial t}\right]\right\} + \frac{W^3(1-\mu^2)}{r\alpha\phi^3}\frac{\partial f}{\partial \mu}\left\{\alpha\left[\phi\left(\frac{\xi}{W^2} - r\nu\frac{\partial v_r}{\partial r}\right) + 2r\frac{\xi}{W^2}\frac{\partial \phi}{\partial r}\right] + \phi\left[\beta\phi^2\left(r\xi\frac{\partial v_r}{\partial r} - \frac{\nu}{W^2}\right) - \frac{r}{W^2}\left(\xi\frac{\partial \alpha}{\partial r} - \nu\phi^2\frac{\partial \beta^r}{\partial r}\right) - r\xi\phi^2\frac{\partial v_r}{\partial t}\right]\right\} = \mathfrak{C}[f],$$

Full-3D-GR code with multi-energy neutrino transport (M1)

✓ "FGR" : Fully General Relativistic code with multi-energy neutrino transport

Kuroda, Takiwaki, and KK, ApJS. (2016) (see, **Zelmani code** by Robert et al. (2016))

The marriage of **BSSNOK formalism** (3D GR code, Kuroda & Umeda (2010, ApJS))

+ **M1 scheme**; Shibata+2011, Thorne 1981, (see also, Just et al. (2015), O'Connor (2015) for recent work)

✓ Evolution equation of neutrino radiation energy

$$\partial_t\sqrt{\gamma}E_{(\varepsilon)} + \partial_i\sqrt{\gamma}(\alpha F^i_{(\varepsilon)} - \beta^i E_{(\varepsilon)}) + \sqrt{\gamma}\alpha\partial_\varepsilon(\varepsilon\tilde{M}^\mu_{(\varepsilon)}n_\mu) = \sqrt{\gamma}(\alpha P^{ij}_{(\varepsilon)}K_{ij} - F^i_{(\varepsilon)}\partial_i\alpha - \alpha S^\mu_{(\varepsilon)}n_\mu),$$

✓ Evolution equation of radiation flux

$$\partial_t\sqrt{\gamma}F_{(\varepsilon)i} + \partial_j\sqrt{\gamma}(\alpha P_{(\varepsilon)i}{}^j - \beta^j F_{(\varepsilon)i}) - \sqrt{\gamma}\alpha\partial_\varepsilon(\varepsilon\tilde{M}^\mu_{(\varepsilon)}\gamma_{i\mu}) = \sqrt{\gamma}[-E_{(\varepsilon)}\partial_i\alpha + F_{(\varepsilon)j}\partial_i\beta^j + (\alpha/2)P^{jk}_{(\varepsilon)}\partial_i\gamma_{jk} + \alpha S^\mu_{(\varepsilon)}\gamma_{i\mu}]$$

✓ Analytic Closure with the use of Minerbo-type Eddington factor **(Murchikova, Abdikamalov + (2017))**

$$P^{ij}_{(\varepsilon)} = \frac{3\chi_{(\varepsilon)} - 1}{2}P^{ij}_{\mathrm{thin}(\varepsilon)} + \frac{3(1-\chi_{(\varepsilon)})}{2}P^{ij}_{\mathrm{thick}(\varepsilon)}$$

$$\chi_{(\varepsilon)} = \frac{5 + 6\bar{F}^2_{(\varepsilon)} - 2\bar{F}^3_{(\varepsilon)} + 6\bar{F}^4_{(\varepsilon)}}{15}$$

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✓ Conformal flatness approximation (+)

$ds^2 = -$

✓ L.H.S. of Boltzmann eq. is super messy…

$$W\left[\frac{\xi}{\alpha}\left(\frac{\partial f}{\partial t}-\beta^r\frac{\partial f}{\partial r}\right)+\frac{\nu}{\phi^2}\frac{\partial f}{\partial r}\right]-\frac{\varepsilon W^3}{r\alpha\phi^3}\frac{\partial f}{\partial \varepsilon}\left\{\beta^r\phi^3\left(-\psi-r\mu\frac{\partial v_r}{\partial r}\right)+v_r^2\phi\left[\beta^r\phi\left(2r\frac{\partial\phi}{\partial r}-\psi\phi\right)\right.\right.$$

$$\left.r\left(-\mu\frac{\partial\alpha}{\partial r}+\mu^2\phi^2\frac{\partial\beta^r}{\partial r}-\frac{\partial\phi^2}{\partial t}\right)\right]+v_r^3\left[r\mu\phi\left(-\mu\frac{\partial\alpha}{\partial r}+\frac{\partial\beta^r\phi^2}{\partial r}-\frac{\partial\phi^2}{\partial t}\right)-\psi\frac{\alpha}{\phi}\frac{\partial r\phi^2}{\partial r}\right]+$$

$$\phi\left[r\mu\left(\mu\alpha\frac{\partial v_r}{\partial r}+\frac{\partial\alpha}{\partial r}+\phi^2\left(-\mu\frac{\partial\beta^r}{\partial r}+\frac{\partial v_r}{\partial t}\right)\right)+r\frac{\partial\phi^2}{\partial t}-r\beta^r\frac{\partial\phi^2}{\partial r}\right]+v_r\alpha\left[\phi\left(\psi+r\mu\frac{\partial v_r}{\partial r}\right)+\right.$$

$$\left.\left.2r\psi\frac{\partial\phi}{\partial r}+\phi^2\left(\mu\frac{\partial v_r}{\partial t}-\frac{\partial\beta^r}{\partial r}\right)+\frac{\partial\phi^2}{\partial t}\right]\right\}+\frac{W^3\left(1-\mu^2\right)}{r\alpha\phi^3}\frac{\partial f}{\partial\mu}\left\{\alpha\left[\phi\left(\frac{\xi}{W^2}-r\nu\frac{\partial v_r}{\partial r}\right)+2r\frac{\xi}{W^2}\frac{\partial}{\partial}\right.\right.$$

$$\left.\left.\phi\left[\beta\phi^2\left(r\xi\frac{\partial v_r}{\partial r}-\frac{\nu}{W^2}\right)-\frac{r}{W^2}\left(\xi\frac{\partial\alpha}{\partial r}-\nu\phi^2\frac{\partial\beta^r}{\partial r}\right)-r\xi\phi^2\frac{\partial v_r}{\partial t}\right]\right]\right\}=\mathfrak{C}[f],$$

Table 2
Neutrino Physics Input

Process	Full Rates (G11, G15, M15, N15)
$\nu A \rightleftharpoons \nu A$	Horowitz (1997; ion–ion correlations)
	Langanke et al. (2008; inelastic contribution)
$\nu\, e^{\pm} \rightleftharpoons \nu\, e^{\pm}$	Mezzacappa & Bruenn (1993)
$\nu\, N \rightleftharpoons \nu\, N$	Burrows & Sawyer (1998)[a]
$\nu_e\, n \rightleftharpoons e^-\, p$	Burrows & Sawyer (1998)[a]
$\bar{\nu}_e\, p \rightleftharpoons e^+\, n$	Burrows & Sawyer (1998)[a]
$\nu_e\, A' \rightleftharpoons e^-\, A$	Langanke et al. (2003)
$\nu\bar{\nu} \rightleftharpoons e^-\, e^+$	Bruenn (1985); Pons et al. (1998)
$\nu\bar{\nu}\, NN \rightleftharpoons NN$	Hannestad & Raffelt (1998)
$\nu_{\mu,\tau}\bar{\nu}_{\mu,\tau} \rightleftharpoons \nu_e\bar{\nu}_e$	Buras et al. (2003)
$\overset{(-)}{\nu}_{\mu,\tau}\overset{(-)}{\nu}_e \rightleftharpoons \overset{(-)}{\nu}_{\mu,\tau}\overset{(-)}{\nu}_e$	Buras et al. (2003)

Full-3D-GR code with multi-energy neutrino transport (M1)

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$$\partial_t\sqrt{\gamma}E_{(\varepsilon)}+\partial_i\sqrt{\gamma}(\alpha F^i_{(\varepsilon)}-\beta^i E_{(\varepsilon)})+\sqrt{\gamma}\alpha\partial_\varepsilon(\varepsilon\tilde{M}^\mu_{(\varepsilon)}n_\mu)$$
$$=\sqrt{\gamma}(\alpha P^{ij}_{(\varepsilon)}K_{ij}-F^i_{(\varepsilon)}\partial_i\alpha-\alpha S^\mu_{(\varepsilon)}n_\mu),$$

✓ Analytic Closure with the use of Minerbo-typ

$$P^{ij}_{(\varepsilon)}=\frac{3\chi_{(\varepsilon)}-1}{2}P^{ij}_{\mathrm{thin}(\varepsilon)}+\frac{3(1-\chi_{(\varepsilon)})}{2}P^{ij}_{\mathrm{thick}(\varepsilon)}$$

Table 1
The Opacity Set Included in this Study and their References

Process	Reference
$n\nu_e \leftrightarrow e^-p$	Bruenn (1985), Rampp & Janka (2002
$p\bar{\nu}_e \leftrightarrow e^+n$	Bruenn (1985), Rampp & Janka (2002
$\nu_e A \leftrightarrow e^-A'$	Bruenn (1985), Rampp & Janka (2002
$\nu p \leftrightarrow \nu p$	Bruenn (1985), Rampp & Janka (2002
$\nu n \leftrightarrow \nu n$	Bruenn (1985), Rampp & Janka (2002
$\nu A \leftrightarrow \nu A$	Bruenn (1985), Rampp & Janka (2002
$\nu e^{\pm} \leftrightarrow \nu e^{\pm}$	Bruenn (1985)
$e^-e^+ \leftrightarrow \nu\bar{\nu}$	Bruenn (1985)
$NN \leftrightarrow \nu\bar{\nu}NN$	Hannestad & Raffelt (1998)

✓ Base-line opacity (t.b.updated)

One-sentence Summary

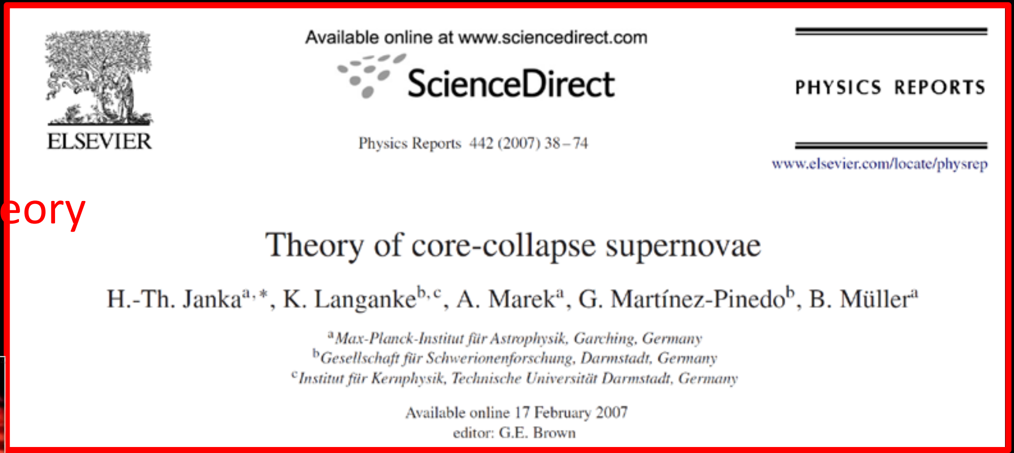
2nd . Core-collapse supernova theory: how to solve "*numerically*" the space-time evolution of dying stars

⇒ Numerical relativity (space-time) + CFD (hydrodynamics) + Neutrino Boltzmann equation (with approximations) self consistently !

3rd . GW signatures from core-collapse supernovae: what we can learn from future GW observation ?

Useful references

1. Review on Core-Collapse Supernova Theory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Physics Reports 442 (2007) 38–74

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Theory of core-collapse supernovae

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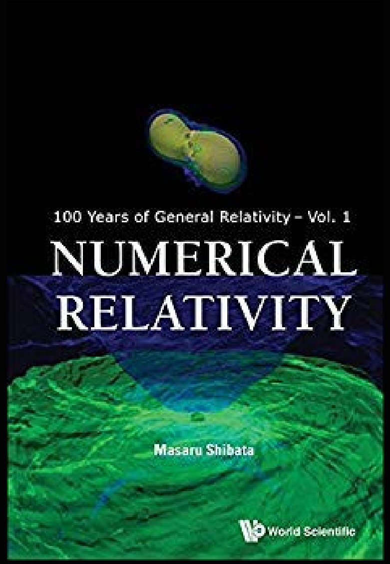
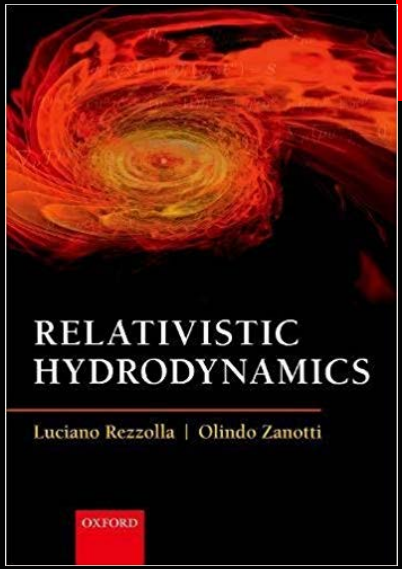
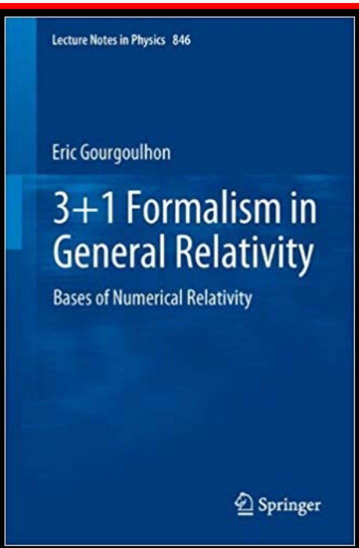
2. Books on numerical relativity







3. Books on radiation hydrodynamics