1\textsuperscript{st}. General Introduction

✓ Why multi-messengers (inc. GW)?
✓ Basics of GW Physics and Detection
✓ First detection of GW150914

2\textsuperscript{nd}. Core-collapse supernova theory: how to solve “\textit{numerically}” the space-time evolution of dying stars (40 min)

3\textsuperscript{rd}. GW signatures from core-collapse supernovae: what we can learn from future GW observation? (40 min)
Gravitational Waves (GWs) from Stellar Collapse

GW amplitude from the quadrupole formula

$$h_{ij} = \frac{2G}{c^4 R} \frac{\partial^2}{\partial t^2} Q_{ij} \sim \frac{R_s}{R} \frac{\left(\frac{\nu}{c}\right)^2}{c}$$

Quadrupole moment

$$h \sim 10^{-20}$$

Typical values at the formation of Neutron Star (NS)

$$R_s = 3 \text{ km} \left(\frac{M}{M_\odot}\right) \quad \nu/c = 0.1 \quad R = 10 \text{ kpc}$$

Good news! (Future)

10 km long: Einstein Telescope (ET) could start \(\sim 2025\) (?)

40 km long: Cosmic Explore (CE) could operate \(\sim 2035\) (?)

\(\checkmark\) CCSN event in our galaxy (several/century) is primary target!

More correctly:

$$h_{ij} = \epsilon \frac{R_s}{R} \left(\frac{\nu}{c}\right)^2$$

\(\epsilon\) : the degree of anisotropy.

If collapse proceeds spherically, \(\epsilon = 0\) no GWs!

What makes the SN-dynamics deviate from spherical symmetry is essential for the GW emission mechanism!
Two candidate mechanisms of core-collapse supernovae

(Lecture by T. Foglizzo, reviews in Janka ('17), Müller ('16), Foglizzo+('15), Burrows('13), Kotake+ ('12))

<table>
<thead>
<tr>
<th>Progenitor</th>
<th>Neutrino mechanism</th>
<th>MHD mechanism</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Non- or slowing- rotating star</td>
<td>Rapidly rotation with strong B</td>
</tr>
<tr>
<td></td>
<td>(\Omega_0 &lt; \sim 0.1 \text{ rad/s})</td>
<td>(\Omega_0 &gt; \sim \pi \text{ rad/s}, B_0 &gt; \sim 10^{11} \text{ G})</td>
</tr>
<tr>
<td>Key ingredients</td>
<td>Turbulent Convection and SASI</td>
<td>Field winding and the MRI</td>
</tr>
<tr>
<td></td>
<td>(e.g., Kazeroni, Guilet, Foglizzo, (2017))</td>
<td>(e.g., Obergaulinger &amp; Aloy (2017), Rembiasz et al.</td>
</tr>
<tr>
<td></td>
<td>Prescollapse Inhomogenities/structures</td>
<td>(2016), Moesta et al. (2016), Masada + (2015))</td>
</tr>
<tr>
<td></td>
<td>Novel microphysics: Bollig+(17), Fischer+(18)</td>
<td>Non-Axisymmetric instabilities</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(e.g., Takiwaki, et al. (2016), Summa et al. (2017))</td>
</tr>
<tr>
<td>Progenitor fraction</td>
<td>Main players</td>
<td>~&lt;1% (Woosley &amp; Heger (07), ApJ):</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(hypothetical link to magnetar, collapsar)</td>
</tr>
</tbody>
</table>

(see also, Burrows et al. ('17), Melson et al. ('15), Lentz et al. ('15), Roberts et al. ('16), B. Mueller ('15), Takiwaki et al. ('16))

20 \(M_\odot\) from Melson et al. ('16)

11.2 \(M_\odot\) from Nakamura et al. in prep.

15 \(M_\odot\) star from Lentz et al. ('15)
GW signatures from 2D neutrino-driven explosion (1/2)

Three generic phases in neutrino-driven models:

1. **Prompt-convection phase**: within ~50 ms post-bounce
2. **Non-linear phase (Convection/SASI)**: Downflows hit the PNS surface
3. **Explosion phase**: Long-lasting signal but terminates if BH forms


Waveforms have no template character: stochastic explosion processes.
GW signatures from 2D neutrino-driven explosion (2/2)

✓ GWs by anisotropic neutrino emission ~ bigger than the matter contribution!
GWs from anisotropic neutrino emission

\[ h_{\mu\nu}(t, \mathbf{x}) = 4 \int \frac{T_{\mu\nu}(t - |\mathbf{x} - \mathbf{x}'|, \mathbf{x}')}{|\mathbf{x} - \mathbf{x}'|} d^3 x' \]

\[ T_{\mu\nu} = T_{\mu\nu}^{\text{matter}} + T_{\mu\nu}^{\text{neutrino}} \]

Epstein(78), Mueller & Janka (97)

\[ h_{\nu}(t) = \frac{2G}{c^4 R} \int_0^t dt L_{\nu}(t') \alpha(t') \]

Neutrino anisotropy: degree of anisotropic neutrino emission (zero if spherical)

In 2D,

\[ \alpha(t) = \frac{1}{L_{\nu}(t)} \int_{4\pi} d\Omega' \Phi(\theta') \frac{dL_{\nu}(\Omega', t')}{d\Omega'} \]

In the opaque region, neutrinos emission is isotropic, little GWs from neutrinos.

The neutrino anisotropy, coming out from the neutrino sphere, is the source of the neutrino GWs.
GWs from anisotropic neutrino emission

\[
h_{\mu\nu}(t, \mathbf{x}) = 4 \int \frac{T_{\mu\nu}(t - |\mathbf{x} - \mathbf{x}'|, \mathbf{x}')}{|\mathbf{x} - \mathbf{x}'|} d^3 x'
\]

\[
T_{\mu\nu} = T_{\mu\nu}^{\text{matter}} + T_{\mu\nu}^{\text{neutrino}}
\]

Epstein(78), Mueller & Janka (97)

\[
h_\nu(t) = \frac{2G}{c^4 R} \int_0^t dt L_\nu(t') \alpha(t')
\]

Neutrino anisotropy: degree of anisotropic neutrino emission (zero if spherical)

In 2D,

\[
\alpha(t) = \frac{1}{L_\nu(t)} \int_{4\pi} d\Omega' \Phi(\theta') \frac{dL_\nu(\Omega', t')}{d\Omega'}
\]

Typical amplitude:

\[
|h_\nu| \sim 10^{-21} \left( \frac{\alpha}{0.01} \right) \left( \frac{L_\nu}{10^{52}\text{erg/s}} \right) \left( \frac{\delta t}{1\text{ sec}} \right) \left( \frac{R}{10\text{kpc}} \right)^{-1}
\]

\[
|h_\nu| \sim h_{\text{bounce}} \sim 10^{-21} (10\text{kpc})
\]

Typical frequency:

\[
t_\nu \sim \frac{1}{\sqrt{G\rho}} \geq 10\text{ msec} \left( \frac{\rho_{\text{trap}}}{10^{11}\text{ g/cm}^3} \right)^{-1/2}
\]

\[
\nu_\nu \sim \frac{1}{t_\nu} \leq 100\text{ Hz}
\]

Frequencies of GWs from neutrinos are typically lower than ~ 100 Hz.
How to detect GWs with no-template features…


⇒ Decompose data-stream into time-frequency domains
⇒ Search for “hot” regions with excess power in the spectrogram!

✓ GW spectrogram from Murphy et al. (‘09) ApJ.

☆ Increase of typical frequency
⇒ the g-mode frequency of PNS

\[ f_p = \frac{N}{2\pi} \frac{1}{2\pi} \frac{GM}{R^2} \sqrt{\frac{(\Gamma - 1)m_n}{\Gamma k_b T}} \left( 1 - \frac{GM}{Rc^2} \right)^{3/2} \]

M, R, T : mass, radius & temperature of PNS, \( \Gamma \) : stiffness of EOS
Due to mass accretion, M ↑, R ↓
⇒ \( f_p \) ↑

(see complete derivation in B. Mueller et al. (‘13), ApJ)

✓ (With no template character…) Three generic phases are in the spectrogram!
✓ Secular increase of typical GW frequency (\( f_p \)) reflects the PNS evolution.
✓ On top of \( f_p \), the high frequency component comes from strong downflows to PNS.
✓ These qualitative features: Common to more recent 2D and 3D models!
"PNS" asteroseismology

How to derive $f_p$?

Knock a "watermelon" the best time to eat

Accretion downflows

$PNS$ in hydrostatic equilibrium: "1PN" ToV equation

$\frac{dP}{dr} = -\frac{\rho GM}{R^2 \alpha}$

$\alpha = 1 - \frac{GM}{R}$, $\phi = \alpha^{-2}$

Buoyancy frequency (Brunt-Vaisälä (BV) frequency, or $g$-mode)

$N^2 = \frac{1}{\rho} \frac{\partial \Phi}{\partial r} \left[ \frac{1}{c_s^2} \frac{\partial P}{\partial r} - \frac{\partial \rho}{\partial r} \right]$ [check the unit!]

$\Phi \sim \frac{cm^2}{s^2}, N^2 \sim \frac{1}{r^2}, f_p = \frac{N}{2\pi} [\text{Hz}]$ (characteristic frequency)

Relativistic Extension of $f_p$

$P = \rho k_B T/m_n$ (non-relativistic baryon)

$\frac{d\rho}{dr} = m_n \frac{dP}{dr}$

$c_s^2 = \Gamma k_B T/m_u$

$\gamma_{ij} = \phi^4 \delta_{ij}, \Gamma = \rho_{1+2}$

Gravitational redshift
“PNS” asteroseismology (2/2)

✓ GR (1PN) correction important!

Morozova et al. (2018), MNRAS
“PNS” asteroseismology (2/2)

✓ GR (1PN) correction important!

Toward "template-based" GW search!
Just started!
(e.g., Sotani et al. (2017). PRD)

Detectability: yet to be understood.

Morozova et al. (2018), MNRAS

Torres-Forné et al. (2017, 2018), MNRAS

![Graphs showing seismic waves for 20 M_{\odot} and 30 M_{\odot} stars.](graph.png)
How to detect GWs with no-template features...

✓ **Excess power method**: Flanagan & Hugh (1998)

⇒ Decompose data-stream into time-frequency domains
⇒ Search for “hot” regions with excess power in the spectrogram!

✓ **GW spectrogram** from Murphy et al. (‘09) ApJ.

- Increase of typical frequency
  ⇒ the g-mode frequency of PNS

\[
f_p = \frac{N}{2\pi} = \frac{1}{2\pi} \frac{GM}{R^2} \sqrt{\left(\frac{T-1}{\Gamma k_b T}\right) \left(1 - \frac{GM}{R c^2}\right)^{3/2}}
\]

M, R, T: mass, radius & temperature of PNS, Γ: stiffness of EOS
Due to mass accretion, M ↑, R ↓
⇒ \( f_p \) ↑

(see complete derivation in B. Mueller et al. (‘13), ApJ)

✓ (With no template character…) **Three generic phases are in the spectrogram**!
✓ Secular increase of typical GW frequency \((f_p)\) reflects the PNS evolution.
✓ On top of \( f_p \), the high frequency component comes from strong downflows to PNS.
✓ **These qualitative features**: Common to more recent 2D and 3D models!
Recent GW predictions from 3D CCSN models with neutrino transport

- Yakunin, Mezzacappa et al. (2017)
  ✓ “Three generic phases” also seen in 3D
  ✓ 2D overestimates GW amp. relative to 3D

+ mode

+ mode

- mode

Based on 15 $M_{\odot}$ model from Lentz et al. (2015), ApJL

- Andresen, B & E Müller and Janka (2017) MNRAS
  ✓ Wave amplitudes; rather insensitive to direction (to the observer).

- mode

- mode

especially when convection dominates over SASI.

✓ The horizon of LIGO is limited to nearby events.
Third generation detectors (ET) could detect any Galactic events!

<table>
<thead>
<tr>
<th></th>
<th>s27</th>
<th>s20</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Low</td>
<td>High</td>
</tr>
<tr>
<td>AdvLIGO</td>
<td>3.7</td>
<td>4.5</td>
</tr>
<tr>
<td>ET-C</td>
<td>50.0</td>
<td>64.0</td>
</tr>
<tr>
<td>ET-B</td>
<td>78.5</td>
<td>73.7</td>
</tr>
</tbody>
</table>
GW signatures from 3D-GR models with strong SASI vs. weak SASI activity
(from Kuroda, KK, & Takiwaki ApJL (2016), see also Andresen, B, E Müller and Janka (2017))

✓ Two EOSs $\rightarrow$ SFHx (Steiner et al. (2013), fits well with experiment/NS radius, Steiner+(2011)),
  HS(TM1) (Shen et al. (1998)).
✓ 15 M$_{\text{sun}}$ star (Woosley & Weaver (1995))

SFHx : softer

TM1 : stiffer

✓ SASI activity higher for softer EOS (due to high growth rate, e.g., Foglizzo et al. (‘06)).
GW Spectrograms from 3D-GR models with strong SASI vs. weak SASI activity

- Two EOSs → SFHx (Steiner et al. (2013), fits well with experiment/NS radius, Steiner + (2011)), HS(TM1) (Shen et al. (1998)).
- 15 Msun star (Woosley & Weaver (1995))

- TM1 : stiffer
- SFHx : softer

- The quasi-periodic modulation is associated with SASI, clearly visible with realistic EOS.
- By coherent network analysis of LIGO, VIRGO, and KAGRA, the detection horizon is only 2~3 kpc, but could extend out to 100 kpc when ET and CE are on-line (>2035).
- Detection of neutrinos (Super-K, IceCube) important to get timestamp of GW detection.
- The SASI activity, if very high, results in characteristic signatures in both GWs and neutrino signals (e.g., Tamborra et al. (2012) for SASI-induced neutrino signals).
GW Spectrograms from 3D-GR models with strong SASI vs. weak SASI activity

- **SFHx:** softer
- **TM1:** stiffer


- The quasi-periodic modulation is associated with SASI, clearly visible with realistic EOS.
- By coherent network analysis of LIGO, VIRGO, and KAGRA, the detection horizon is only 2~3 kpc, but could extend out to 100 kpc when ET and CE are on-line (>2035).
- Detection of neutrinos (Super-K, IceCube) important to get timestamp of GW detection.
- The SASI activity, if very high, results in characteristic signatures in both GWs and neutrino signals (e.g., Tamborra et al. (2012) for SASI-induced neutrino signals).
GW Spectrograms from 3D-GR models with strong SASI vs. weak SASI activity

SFHx : softer

TM1 : stiffer

Sensitivity curves and model predictions

The reconstructed GW spectrogram

✓ The quasi-periodic modulation is associated with SASI, clearly visible with realistic EOS.

✓ By coherent network analysis of LIGO, VIRGO, and KAGRA, the detection horizon is only 2~3 kpc, but could extend out to 100 kpc when ET and CE are on-line (>2035).

✓ Detection of neutrinos (Super-K, IceCube) important to get timestamp of GW detection.

✓ The SASI activity, if very high, results in characteristic signatures in both GWs and neutrino signals (e.g., Tamborra et al. (2012) for SASI-induced neutrino signals).
GW Spectrograms from 3D-GR models with strong SASI vs. weak SASI activity

- **SFHx** : softer
- **TM1** : stiffer

- The quasi-periodic modulation is associated with SASI, clearly visible with realistic EOS.
- By **coherent network analysis** of LIGO, VIRGO, and KAGRA, the detection horizon is only 2~3 kpc, but could extend out to 100 kpc when ET and CE are on-line (>2035).
- **Detection of neutrinos** (Super-K, IceCube) important to get timestamp of GW detection.
- The SASI activity, if very high, results in characteristic signatures in both GWs and neutrino signals (e.g., Tamborra et al. (2012) for SASI-induced neutrino signals).
"New" GW messenger is Circular Polarization of GW: Non-axisymmetric instabilities (incl. low T/|W|, spiral SASI) 

Hayama et al. (2016), PRL (see also Klimenko et al. (2015) PRD)

Stokes Parameters:

\[
\begin{align*}
\left( \langle h_R(f, \hat{n})h_R(f', \hat{n}') \rangle \langle h_L(f, \hat{n})h_R(f', \hat{n}') \rangle \right)
&= \frac{1}{4\pi} \delta_D(\hat{n} - \hat{n}') \delta_D(f - f') \\
&\times \left( I(f, \hat{n}) + V(f, \hat{n}) \right) \left( Q(f, \hat{n}) - iU(f, \hat{n}) \right)
\end{align*}
\]

\( h_R := (h_+ - ih_\times) / \sqrt{2} \)

\( h_L := (h_+ + ih_\times) / \sqrt{2} \)

(See definitions in Seto and Taruya (2007), PRL)

Rapidly rotating 15 M\(_{\odot}\) (early postbounce phase) from Kuroda, Takiwaki, KK (2014) PRD

@10kpc

CP (if seen from the spin axis): evidence of "rapid rotation".
What about Circular GW polarization in “Non-rotating” progenitors?

Non-rotating 11.2 M\textsubscript{sun} star; Convection dominant

- If the core is convection-dominant (likely for low $\xi$ stars), no clear signature of CP!

Non-rotating 15 M\textsubscript{sun} star (SFHx EOS); SASI dominant

- If the SASI dominant (likely for high $\xi$ stars), clear signature of CP!

$\Rightarrow$ indication of SASI motions non-spherical mass accretion (Hayama, KK et al. 2018)
The detection of GW amplitude is within several kpc using LIGO (e.g., Andresen et al. (2017))

The detection of CP could extend (far) beyond the detection horizon of GW waveform!

The CP would provide new window to detect GW signals!

The Origin of the Nobel-Prize-awarded BHs (7 ~40 M\(_{\odot}\))?

The Nobel Prize in Physics 2017
Rainer Weiss, Barry C. Barish, Kip S. Thorne

The Nobel Prize in Physics 2017 was divided, one half awarded to Rainer Weiss, the other half jointly to Barry C. Barish and Kip S. Thorne. "for decisive contributions to the LIGO detector and the observation of gravitational waves".

Low metallicity environment needed for large stellar mass BH formation.
(e.g., Kinugawa et al. (2014,2016))
3D-GR results of $70 \ M_{\text{sun}} \ (M_{\text{CO}} \sim 28.5 \ M_{\text{sun}})$ (progenitor from Takahashi et al. (2014))
3D-GR results of 70 $M_{\text{sun}}$ ($M_{\text{CO}} \sim 28.5 M_{\text{sun}}$)

The first BH forming simulation in 3D!

Before the BH formation, **monotonic increase** of neutrino luminosity and rms energy.

(consistent with 1D, e.g., Sumiyoshi+ (2006), Fischer+ (2009), Huedepohl+(2016))

Strong GW emission is visible to 1 Mpc, **but not** O(100) Mpc...

Kuroda, KK et al. (2018), MNRAS Letters
Switching gears to **MHD mechanism (rapid rotation required !!)** My research life....

Magnetohydodynamics Simulations

\[ B_0 = 10^{12} \, G \]
\[ \Omega_0 = 2 \, \text{rad/s} \]

See recent developments in Moesta et al.(2015)
Masada et al. (2015), Ramirez et al. (2016), Sawai et al. (2014)
Switching gears to **MHD mechanism (rapid rotation required !!)**

**GW from Rapidly Rotating Core-Collapse and Bounce**

(Dimmelmeier et al. (07, PRL), Scheidegger et al. (10, A&A) Ott et al. (12, ApJ), Abdikamalov+(14, PRD), Kuroda+(14,PRD))

- **Infall phase:**
  - Rotational flattening of the core

- **Core bounce:**
  - stiffening of nuclear EOS, making big change in the mass quadrupole

- **Ring-down phase:**
  - settles down to stationary state, with amplitudes decreasing with time.

- **”Optimal” detection horizon using matched filtering**

- Waveform:
  - (seen from equator)

- Coherent network analysis using L-H-V-K

---

**Bounce GW signal (in the context of rapidly rotating collapse and bounce):**

- Characterized by “one” big spike at bounce followed by smaller peaks: “type I” signal
- Matched filtering (or PCA) likely applicable.
- Horizon distance can reach beyond LMC (50kpc)
GWs from (Rotation-induced) Non-Axisymmetric Instabilities

✓ Low $T/|W|$ instability is most likely to develop (Ott + (05, ApJL), Scheidegger + (10, A&A))

GW emissivity:

- Low $T/|W|$ instability is most likely to develop (Ott + (05, ApJL), Scheidegger + (10, A&A))

- Circular polarization can be evidence of “rapid rotation”.

- “Quasi-periodicity” enhances the chance of detection.

- Strong emission from one-armed spiral wave

GW from non-axisym. instabilities (incl. low $T/|W|$, spiral SASI) : Quasi-Periodicity

(Ott + (07, PRL), Scheidegger + (10, A&A), Kuroda + (14, PRD))

⇒ The effective amplitude scales as the # of GW cycles as

$h_{\text{eff}} \propto h \sqrt{N}$
### Summary

<table>
<thead>
<tr>
<th>Neutrino mechanism</th>
<th>MHD mechanism</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Progenitor</strong></td>
<td>Rapidly rotating star with strong B fields $(\Omega_0 &gt; \sim \pi \text{ rad/s}, B_0 &gt; \sim 10^{11} \text{ G})$</td>
</tr>
<tr>
<td><strong>Main GW signatures</strong></td>
<td>Rotating bounce (&lt; 20 ms p.b.) and non-axisymmetric instabilities (&lt; ? ms)</td>
</tr>
</tbody>
</table>

**Detection Prospect**

- **Requires 3rd generation detector** to see every Galactic event (with high SNR).
- **Closeby events** (2~3kpc) detectable, LIGO/Virgo/KAGRA
- **If detected**, critical information about SN engine (convection-dominant vs. SASI dominant) can be obtained.
- **Detection of circular polarization**: important probe of SASI.

- **Bounce GW signal**: detection horizon of LIGO, depending on $\Omega_0$, can cover our Milky way and beyond.
- **GWs from non-axisymmetric instabilities**: “quasi-periodicity” enhances chance of detection.
- **Detection of circular polarization**: important probe of core rotation.
Next 10 years: Where are we and are we going?

“A” self-consistent 3D model

Gray-transport simulation
Nucleosynthesis

To-do-1: Long-term evolution in self-consistent 3D (GR) models
⇒ confront CCSN theory with observation ⇒ Pragmatism

For progenitors (11.2, 15, 20, 27 M$_{\odot}$), the stalled shock revived!
(5D/4D with approximate transport)

Melson+15, Takiwaki+16, Ott+18

Hydrodynamic model:
Mixing, RT, RM instabilities

DeLaney et al. (2010)

To-do-2: Full GR and Boltzmann project:
⇒ ultimately test whether the stalled shock would revive. ⇒ Perfectionism
SN 20xx! in the Galactic center: End-to-End Bridging Simulations

Log (day)
- 4 - 3 - 2 - 1 - 0 - 2 - 3

SK detects ~ 10,000 neutrinos
< 15 min SURGE meeting (Supernova Urgent Response Group of Experts)
< 1 hour SK provide alert: Astronomers telegram (onset of neutrino burst, duration, event #)

Gravitational Waves
KAGRA
GAZOOKS (SK + Gd); Indispensable for choosing telescope
⇒ MNi, Eexp, M\#*, R\#*

Multi-messenger research in steady progress!
LIGO, Virgo, SK, KamLAND, K, IceCube, INTEGRAL
KAGRA, Hyper-K, and Post-K ET, CE, DUNE.....
Useful references

1. Review on GW signatures from CCSNe

~1000 $ !

Chapter 7
“Gravitational waves From Core-Collapse Supernovae”
By Kotake and Kuroda

2. Recent publications on CCSN GWs

Summary of publication lists (by Ewald Mueller):
https://wwwmpa.mpa-garching.mpg.de/rel_hydro/GWlit_catalog.shtml

Asteroseismology:
Torres-Forne et al. (2019), MNRAS
Sotani et al. (2017), PRD