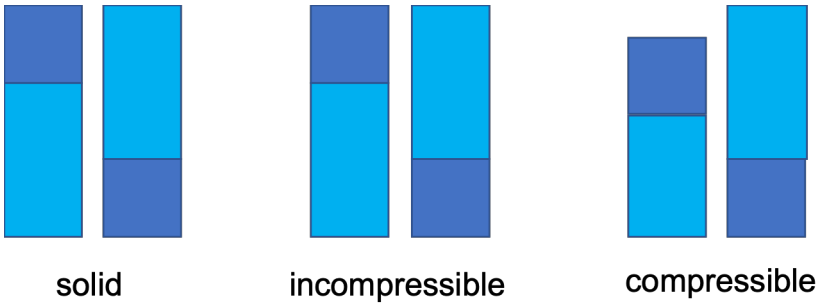
Session of exercises on hydrodynamics, turbulence and instabilities

Thierry Foglizzo *Friday 1/3/2019 12h30-13h30 and 15h30-16h30*

1. Comparing solid and fluid mechanics

Consider the hydrostatic equilibrium of two materials in a uniform vertical gravity g



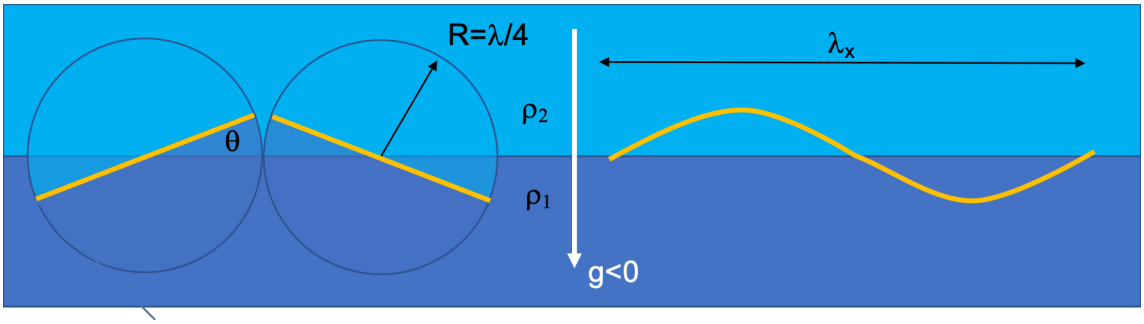
1.a Calculate the difference of potential energy between top-heavy and top-light configurations when the two materials are made of:

-2 solids with same mass and densities ρ_1 ρ_2

-2 incompressible fluids with same mass and densities ρ_1 ρ_2

-2 perfect isothermal gases with same mass and different temperatures T_1, T_2

-2 perfect adiabatic gases with same mass and different entropies S_1, S_2

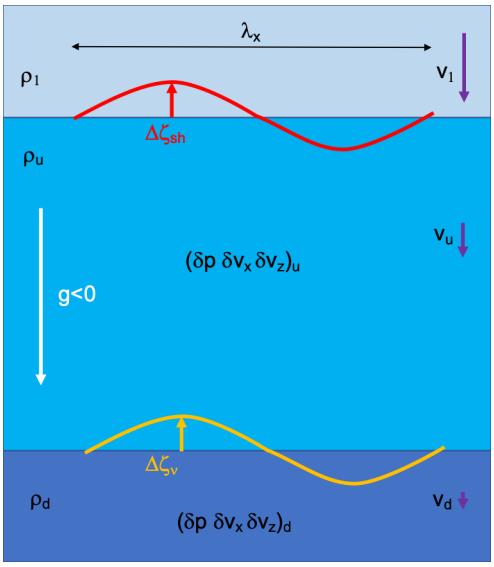


1.b Calculate the oscillation frequency of the interface and compare the dependence with respect to the horizontal wavelength. What is the vertical length-scale of the perturbations? In the case of a solid, consider the motion of a disc around its geometric center.

1.c Calculate the same with a linear profile of density

2. Calculate the frequency and growth rate of SASI oscillations in the simplest setup and interpret is through the coupling of pressure and advected perturbations

Consider a plane parallel flow in a uniform gravity g with a stationary shock at the altitude z_{sh} followed by a condensation from the density ρ_u to ρ_d at the altitude z_ν



2.a Write the conservation equations across the perturbed shock at $z(x,t)=z_{sh}+\Delta\zeta_{sh}$ for a perfect gas with adiabatic index γ.

2.b Write the conservation equations across the perturbed surface of phase transition

2.c Decompose the post-shock perturbations into pressure, entropy, vorticity perturbations

2.d Write the lower boundary condition describing the downward advection of perturbations

2.e Write the global equation defining the complex eigenfrequencies ω

2.f Write the coupling coefficients Q_{sh}, R_{sh} between advected and pressure perturbations with a real frequency ω_r at the shock

2.g Write the coupling coefficients Q_ν, R_ν between advected and pressure perturbations at the condensation surface z_ν

2.h Relate the amplification factor Q of the advective-pressure cycle to the imaginary part ω_i of the complex eigenfrequency solution of 2.e

2.i Discuss the saturation mechanism