Bayesian analysis for extracting properties of the nuclear equation of state from observational data

#### Alexander Ayriyan

Division of Computational Physics Laboratory of IT, Joint Institute for Nuclear Research (JINR)

Computational Physics and IT Division, A. Alikhanyan National Science Laboratory (AANL)

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Hovik Grigorian (LIT & YSU & AANL), David Blaschke (UW & JINR & MEPhI), Dmitry Voskresensky (MEPhI & JINR), Konstantin Maslov (MEPhI & JINR) David-Edwin Alvarez-Castillo (JINR), Nobutoshi Yasutake (Chiba Institute of Technology), Vahagn Abgaryan (JINR & AANL)

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## Outline

#### I. Motivation

II. Mixed phase construction for cold and dense nuclear matter

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- III. Bayesian analysis for extracting properties of the nuclear equation of state from observational data
- IV. Conclusions

# Motivation

#### What if we have twins





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- Does hybrid neutron star exist?
- Does NS twin exist?
- Does CEP exist on QCD phase diagram?
- etc.

#### Neutron star mass-radius relation



Seidov criterion for instability:

$$\frac{\Delta\varepsilon}{\varepsilon_{crit}} \geq \frac{1}{2} + \frac{3}{2} \frac{P_{crit}}{\varepsilon_{crit}}$$

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Credit: Mark G. Alford, Sophia Han, and Madappa Prakash. Phys. Rev. D 88, 083013 (2013)

Finite-size effects



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The surface tension  $\sigma$  is unknown and used as free parameter.

Yasutake, Maruyama, Tatsumi, Phys. Rev. D80 (2009) 123009

## Mimicking the Pasta phase. The idea



Schematic representation of the interpolation function  $P_M(\mu)$ , it has to go though three points:  $P_H(\mu_H)$ ,  $P_c + \Delta P$  and  $P_Q(\mu_Q)$ .

#### The Interpolation Method

$$P_{M}(\mu) = \sum_{q=1}^{N} \alpha_{q} \left(\mu - \mu_{c}\right)^{q} + \left(1 + \Delta_{P}\right) P_{c}$$

where  $\Delta_P$  is a free parameter representing additional pressure of the mixed phase at  $\mu_c$ .

$$P_{H}(\mu_{H}) = P_{M}(\mu_{H}) \qquad P_{Q}(\mu_{Q}) = P_{M}(\mu_{Q})$$
$$\frac{\partial^{q}}{\partial \mu^{q}} P_{H}(\mu_{H}) = \frac{\partial^{q}}{\partial \mu^{q}} P_{M}(\mu_{H}) \qquad \frac{\partial^{q}}{\partial \mu^{q}} P_{Q}(\mu_{Q}) = \frac{\partial^{q}}{\partial \mu^{q}} P_{M}(\mu_{Q})$$
where  $q = 1, 2, ..., k$ . All  $N + 2$  parameters  $(\mu_{H}, \mu_{Q} \text{ and } \alpha_{q}, \text{ for } q = 1, ..., k$ .

q = 1, ..., N) can be found by solving the above system of equations, leaving one parameter ( $\Delta P$ ) as a free one.

Ayriyan and Grigorian, *EPJ Web Conf.* **173**, 03003 (2018) Abgaryan, Alvarez-Castillo, Ayriyan et al. *Universe* **4(9)**, 94 (2018)

# The Interpolation Method



The squared speed vs chemical potential given by the interpolation with k = 1 (upper left) k = 2 (upper right) and k = 3 (right).

Abgaryan, Alvarez-Castillo, Ayriyan, Blaschke and Grigorian. Universe 4(9) (2018), 94







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#### The results of pasta effects



#### Third family robust against $\Delta_P$ up to around 5%!

Abgaryan, Alvarez-Castillo, Ayriyan et al. Universe 4(9), 94 (2018)

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#### The realistic hadron and quark matter models



The quark EoS model SFM with available volume fraction parameter



Maslov, Kolomeitsev, Voskresensky, Nucl.Phys. A950 (2016) Kolomeitsev & Voskresensky, Nuc. Phys. A 759 (2005)

Kaltenborn, Bastian, Blaschke, Phys. Rev. D 96, 056024 (2017)

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#### Robustness of third family solutions



Ayriyan, Bastian, Blaschke, Grigorian, Maslov, Voskresensky. PRC 97, 045802 (2018)

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#### Robustness of third family solutions



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#### Relation with surface tension parameter



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#### Relation with surface tension parameter



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#### Relation with surface tension parameter



Maslov, Yasutake, Blaschke, Ayriyan, Grigorian, Maruyama, Tatsumi, Voskresensky. PRC100, 025802 (2019)

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How to mimic pasta phase for given  $\sigma$ 

- 1. Maxwell constructio to obtain  $P_c$  and  $\mu_c$
- 2. Glendaning construction to find  $\Delta_P(0)$
- 3. Use the fit formulas of **Maslov et al PRC100 (2019)** to obtain  $\Delta_P(0)$ .
- 4. Use the interpolation formulas of **Ayriyan et al PRC98** (2018) to obtain the pasta-equivalent hybrid EoS with a mixed phase for a given choice of the surface tension  $\sigma$

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D. Blaschk. **Cooking book for mimicking the pasta phase** presented at MPCS2019 (20.09.2019, Yerevan)

## **Bayesian Analysis**

Bayesian analysis is a statistical paradigm that shows the most expected hypotheses using probability statements and current knowledge.

One of the most frequent case is analysis of probable values of model parameters.



Prior: knowledge before experiment (logically) Likelihood: Probability for data if the hypothesis was true Posterior: Probability that the hypothesis is true given the data Evidence: normalization; important for model comparison

Generally, maximum likelihood (parameters which maximize the probability for data) **does not** give the most likely parameters!!!

## **Bayesian Analysis**



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## Equation of State



Equation of State at M - R diagram



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Equation of State at  $\Lambda_1 - \Lambda_2$  diagram



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The same phenomena were found in Montana, Tolos, Hanauske, Rezzolla. PRD99, 103009 (2019) for politropic models More interesting results have been achieved by Prof. Armen Sedrakian for triplet of compact stars produced by the forth family. The region  $\Lambda_2 < \Lambda_1$  was called unphysical at Abbott *et al.* PRL121 (2018). The set of parameters of models could be represented in the parameter space with introduction of the vector of parameters, each vector is one fixed model from considered types of EoS model and transition construction:

$$\overrightarrow{\pi}_{i} = \left\{ \mu_{<(j)}, \Delta_{P(k)} \right\},\,$$

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where i = 0..N - 1 and  $i = N_2 \times j + k$  and  $j = 0..N_1 - 1$ ,  $k = 0..N_2 - 1$  and  $N_1$  and  $N_2$  are number of values of model parameters  $\mu_{<}$  and  $\Delta_P$  correspondingly.

## Likelihood of a EoS model for the mass constraint



Cromartie et. al. Nature Astronomy (2019), doi: 10.1038/s41550-019-0880-2

Likelihood of a EoS model for the mass constraint

$$P(E_M | \pi_i) = \Phi(M_i, \mu_A, \sigma_A)$$

here  $M_i$  is maximum mass of the given by  $\pi_i$ , and  $\mu_A = 2.17 \text{ M}_{\odot}$ and  $\sigma_A = 0.105 \text{ M}_{\odot}$  is the mass measurement of PSR J0740+6620  $2.17^{+0.11}_{-0.10} \text{ M}_{\odot}$  [Cromartie *et al.*, arXiv:1904.06759 (2019)].



Note, that here we replace previously used mass measurement for two solar mass pulsar J0348+0432  $2.01^{+0.04}_{-0.04}$  M<sub> $\odot$ </sub> [Antoniadis *et al.*, Science **340**, 6131 (2013)].

Likelihood of a EoS model for the  $\Lambda_1 - \Lambda_2$  constraint

$$\begin{split} P\left(E_{GW} | \pi_i\right) &= \int_{I_{22}} \beta(\Lambda_1(\tau), \Lambda_2(\tau)) d\tau + \int_{I_{23}} \beta(\Lambda_1(\tau), \Lambda_2(\tau)) d\tau \\ &+ \int_{I_{32}} \beta(\Lambda_1(\tau), \Lambda_2(\tau)) d\tau + \int_{I_{33}} \beta(\Lambda_1(\tau), \Lambda_2(\tau)) d\tau, \end{split}$$

where  $l_{ps}$  are the length of the line at  $\Lambda_1 - \Lambda_2$ , the indecies p and s determine to which family of compact stars the GW170817 components belong. The parameter  $\tau$  is, for instance, central density of a star.



# $\beta(\Lambda_1,\Lambda_2)$



The PDF  $\beta(\Lambda_1, \Lambda_2)$  has been reconstructed by the method Gaussian kernel density estimation with  $\Lambda_1 - \Lambda_2$  data given at LIGO web-page https://dcc.ligo.org/LIGO-P1800115/public.



[Guillot. Talk at the Workshop "NSs and their environments", (April 8, 2019)]

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[Guillot. Talk at the Workshop "NSs and their environments", (April 8, 2019)]



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The fictitious M-R measurement has been implemented, inspired by the preliminary results of NICER observation of M-R of the PSR J0030+0451.

$$P(E_{MR} | \pi_i) = \int_{l_2} \mathcal{N}(\mu_R, \sigma_R, \mu_M, \sigma_M, \rho) d\tau + \int_{l_3} \mathcal{N}(\mu_R, \sigma_R, \mu_M, \sigma_M, \rho) d\tau,$$

where  $\mu_R = 13.84$ ,  $\sigma_R = 1.2276$ ,  $\mu_M = 1.44$ ,  $\sigma_M = 0.18$ , and the correlation parameter  $\rho = 0.9566$ , winch corresponds to 8° of th ellipse rotation.  $I_2$  and  $I_3$  are length of the lines at *M*-*R* diagram of the second and third families correspondingly.

#### Posterior distribution

The full likelihood for the given  $\pi_i$  can be calculated as a product of all likelihoods, since the considered constraints are independent of each other

$$P(E | \overrightarrow{\pi}_i) = \prod_m P(E_m | \overrightarrow{\pi}_i).$$

where m is index of the constraints.

The posterior distribution of models on parameter diagram is given by Bayes' theorem

$$P(\overrightarrow{\pi}_{i}|E) = \frac{P(E|\overrightarrow{\pi}_{i})P(\overrightarrow{\pi}_{i})}{\sum_{j=0}^{N-1}P(E|\overrightarrow{\pi}_{j})P(\overrightarrow{\pi}_{j})},$$

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where  $P(\vec{\pi}_j)$  is a prior distribution of a models taken to be uniform:  $P(\vec{\pi}_j) = 1/N$ .

## BA results



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Ayriyan, Alvarez-Castillo, Blaschke, Grigorian. In preparation.

## Conclusions

The mixed phase interpolation method is very simple and well describes quark-hadron pasta phase for any give possible surface tension value.

The third family survives against pasta phase for the considered EoS models.

 $\Lambda_1 - \Lambda_2$  relation from GW170817 favours softer EoS and hybrid stars with strong phase order transitions (even with no third family due to the mixed phase).

The region  $\Lambda_2 < \Lambda_1$  has physical meaning in case of low-mass twins, when heavier companion belongs to the second family and the lighter one to the third family.

If NICER will approve the "fictitious radius measurement" it will support the low-mass twin stars around 1.4  $M_{\odot}$  for considered models.

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#### Olga Tokarczuk Nobel laureate in Literature

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## **Great Days of Wroclaw!**



Olga Tokarczuk Nobel laureate in Literature



David Blaschke 60th Anniversary

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