Uncertainties of Compact Star Observables from Nuclear Matter Model Parameters

Gergely Gábor Barnaföldi, Péter Pósfay, Antal Jakovác

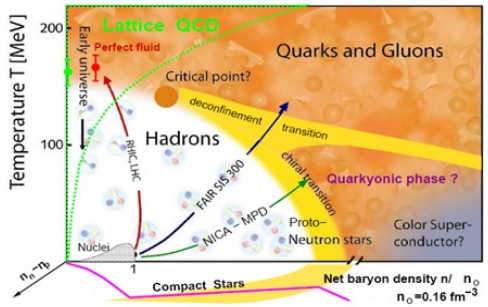
References:
Universe 5 (2019) **6**, 153; *PASA* **35** (2018) 19; PRC **97** (2018) 025803

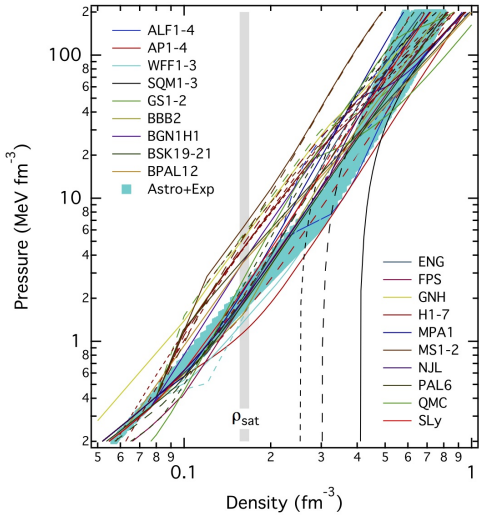
Support: *Hungarian OTKA grants, NK123815, K120660, Wigner GPU Laboratory, the PHAROS MP16214 and THOR CA15213 COST actions.*

40th Max Born Symposium, Wroclaw, Poland, 11th October 2019

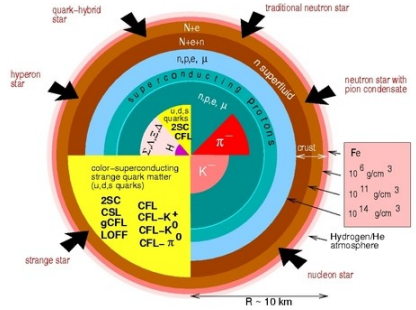
Motivation

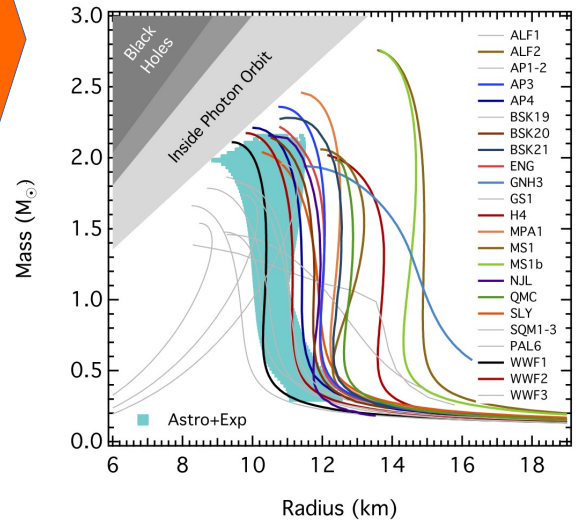
EoS from exp & theory



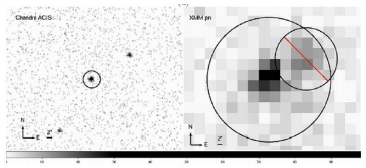


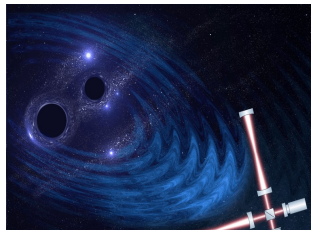
Application in compact stars

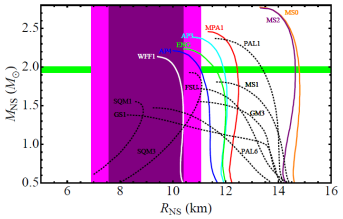


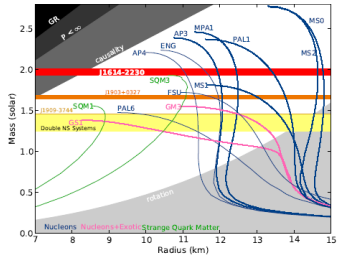


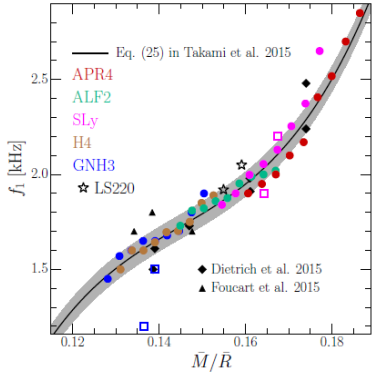
Constraints by astrophysical observations











Motivation

Weih & Most & Rezzolla: ApJ 881,73 (2019)

Optimal neutron-star mass ranges to constrain the equation of state of nuclear matter with electromagnetic and gravitational-wave observations

L. R. Weih,[1] E. R. Most,[1] and L. Rezzolla[1]

[1] *Institut für Theoretische Physik, Goethe Universität Frankfurt am Main, Germany*

ABSTRACT

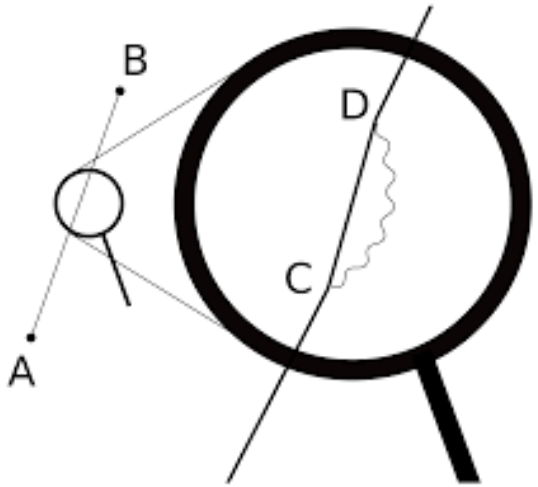
Exploiting a very large library of physically plausible equations of state (EOSs) containing more than 10^7 members and yielding more than 10^9 stellar models, we conduct a survey of the impact that a neutron-star radius measurement via electromagnetic observations can have on the EOS of nuclear matter. Such measurements are soon to be expected from the ongoing NICER mission and will complement the constraints on the EOS from gravitational-wave detections. Thanks to the large statistical range of our EOS library, we can obtain a first quantitative estimate of the commonly made assumption that the high-density part of the EOS is best constrained when measuring the radius of the most massive, albeit rare, neutron stars with masses $M \gtrsim 2.1\,M_\odot$. At the same time, we find that radius measurements of neutron stars with masses $M \simeq 1.7 - 1.85\,M_\odot$ can provide the strongest constraints on the low-density part of the EOS. Finally, we quantify how radius measurements by future missions can further improve our understanding of the EOS of matter at nuclear densities.

1) How much difference arise from the different levels of approximations?

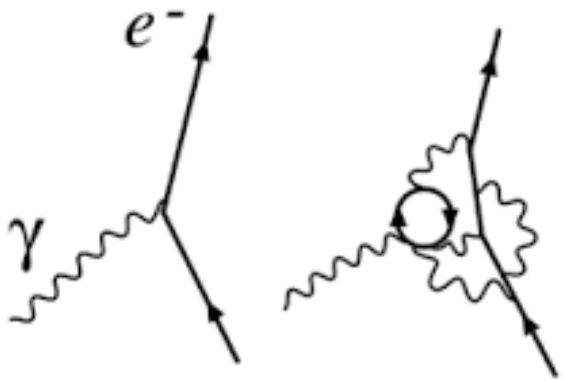
P. Pósfay, GGB, A. Jakovác: PASA **35** (2018) 19, PRC **97** (2018) 025803

Motivation for FRG

- It is hard to get effective action for an interacting field theory: e.g.: EoS for superdense cold matter ($T \to 0$ and finite μ)
- Taking into account quantum fluctuations using a scale, k
  - Classical action, $S = \Gamma_{k \to \Lambda}$ in the UV limit, $k \to \Lambda$
  - Quantum action, $\Gamma = \Gamma_{k \to 0}$ in the IR limit, $k \to 0$
- FRG Method
  - Smooth transition from macroscopic to microscopic
  - RG method for QFT
  - Non-perturbative description
  - Not depends on coupling
  - BUT: Technically it is NOT simple



$\Lambda \xrightarrow{\text{scale, } k} 0$



Functional Renormalization Group (FRG)

- FRG is a general non-perturbative method to determine the effective action of a system.
- **Scale dependent effective action (k scale parameter)**

$$\partial_k \Gamma_k = \frac{1}{2} \int dp^D \; STr \left[\frac{\partial_k R_k}{\Gamma_k^{(2)} + R_k} \right]$$

Wetterich equation

k=Λ
Classical action

Integration

k=0
Quantum fluctuations included

Ansatz: Interacting Fermi-gas model

Ansatz for the effective action:

$$\Gamma_k[\varphi,\psi] = \int d^4x \left[\bar{\psi}\left(i\partial\!\!\!/ - g\varphi\right)\psi + \frac{1}{2}\left(\partial_\mu\varphi\right)^2 - U_k(\varphi)\right]$$

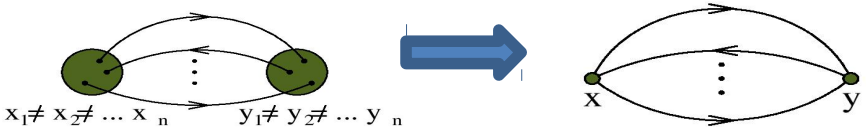
Fermions : *m=0,* **Yukawa-coupling** generates mass

Bosons: the **potential** contains self interaction terms

We study the scale dependence of the potential only!!

Interacting Fermi-gas at finite temperature

Ansatz for the effective action in LPA:



$$\boxed{\Gamma_k[\varphi,\psi] = \int d^4x \left[\bar{\psi}\left(i\partial\!\!\!/ - g\varphi\right)\psi + \frac{1}{2}\left(\partial_\mu\varphi\right)^2 - U_k(\varphi)\right]}$$

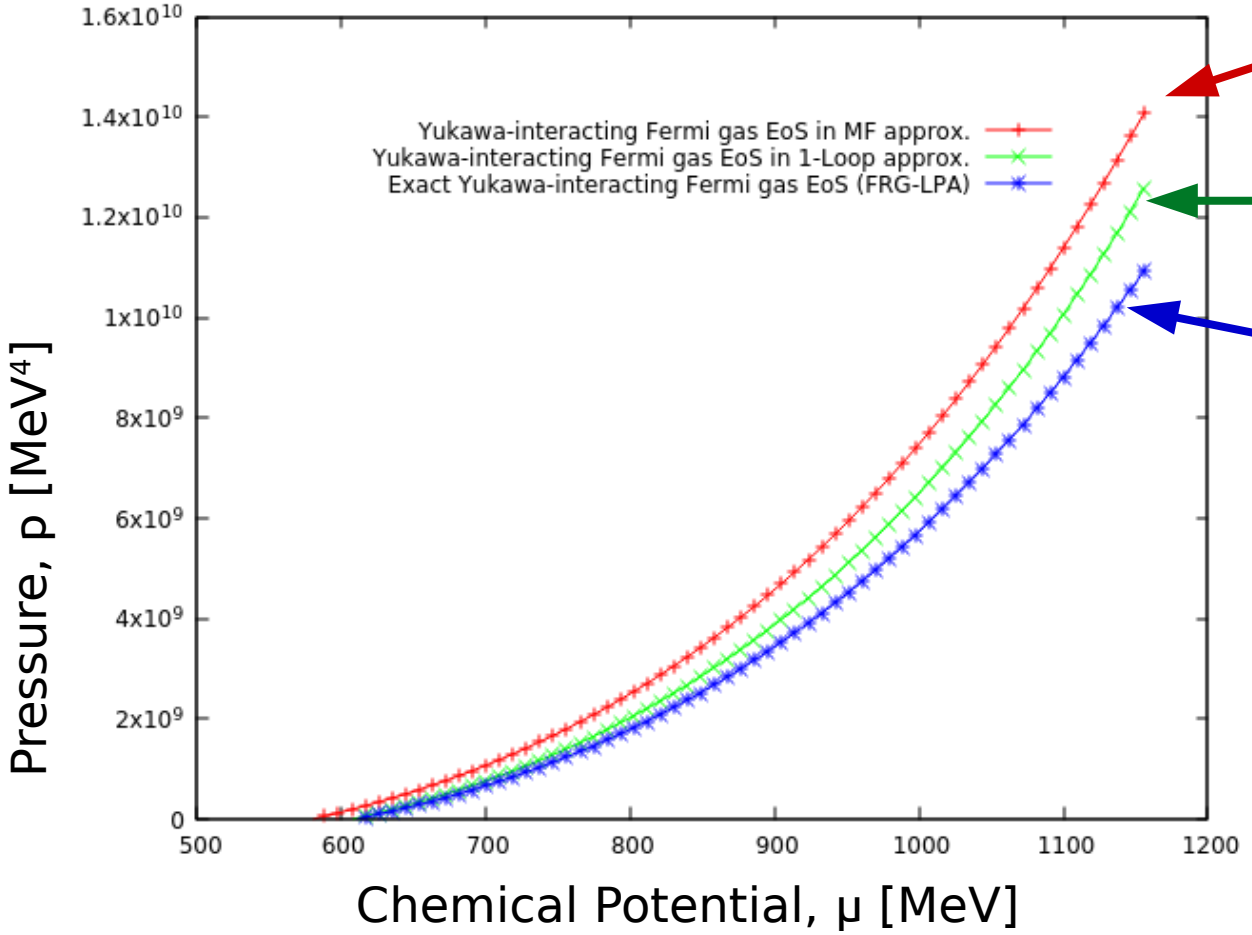
$$\Gamma_k[\psi] = \int d^4x \left[\frac{1}{2}\psi_i K_{k,ij}\psi_j + U_k(\psi)\right]$$

Wetterich-equation in LPA

$$\partial_k U_k = \frac{k^4}{12\pi^2}\left[\underbrace{\frac{1+2n_B(\omega_B)}{\omega_B}}_{\text{Bosonic part}} + 4\underbrace{\frac{-1+n_F(\omega_F-\mu)+n_F(\omega_F+\mu)}{\omega_F}}_{\text{Fermionic part}}\right]$$

$$U_\Lambda(\varphi) = \frac{m_0^2}{2}\varphi^2 + \frac{\lambda_0}{24}\varphi^4 \qquad \omega_F^2 = k^2 + g^2\varphi^2 \qquad \omega_B^2 = k^2 + \partial_\varphi^2 U \qquad n_{B/F}(\omega) = \frac{1}{1\mp e^{-\beta\omega}}$$

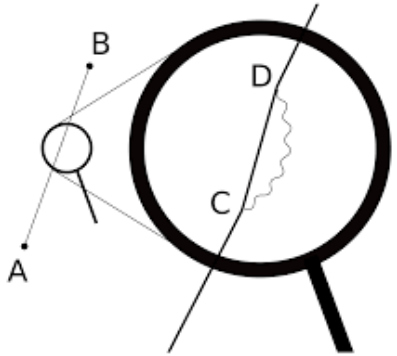
Result: Comparison of MF, 1L, & FRG-based EoS



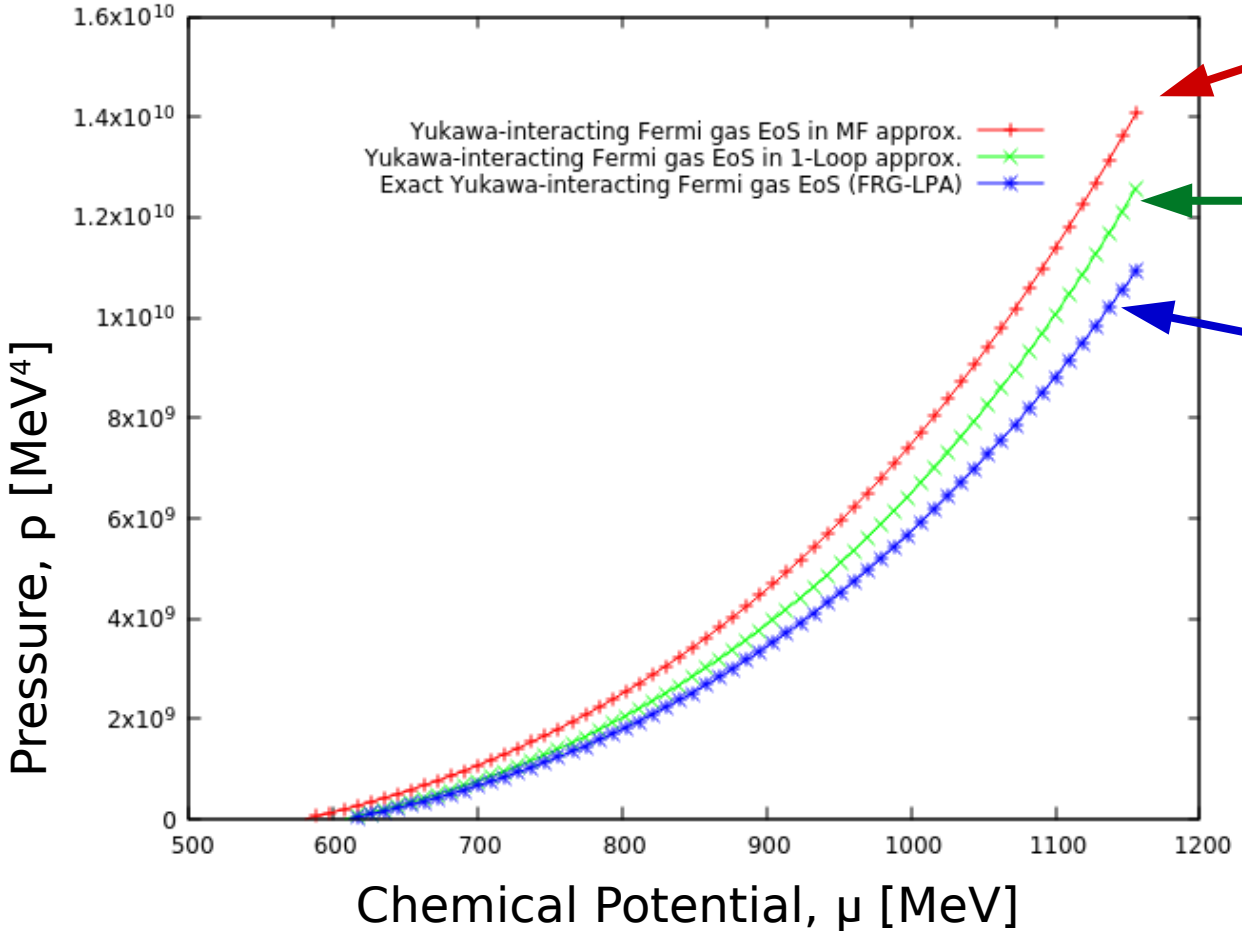
Mean Filed is the stiffest

1-Loop approximation

Exact FRG solution softest



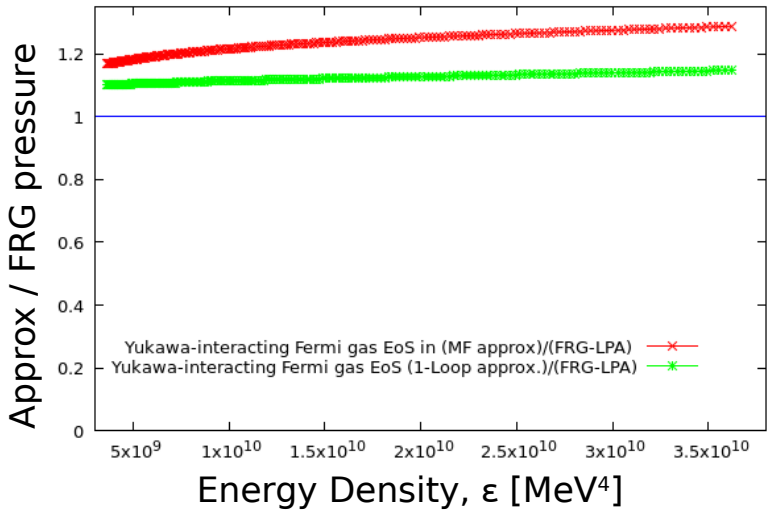
Result: Comparison of MF, 1L, & FRG-based EoS



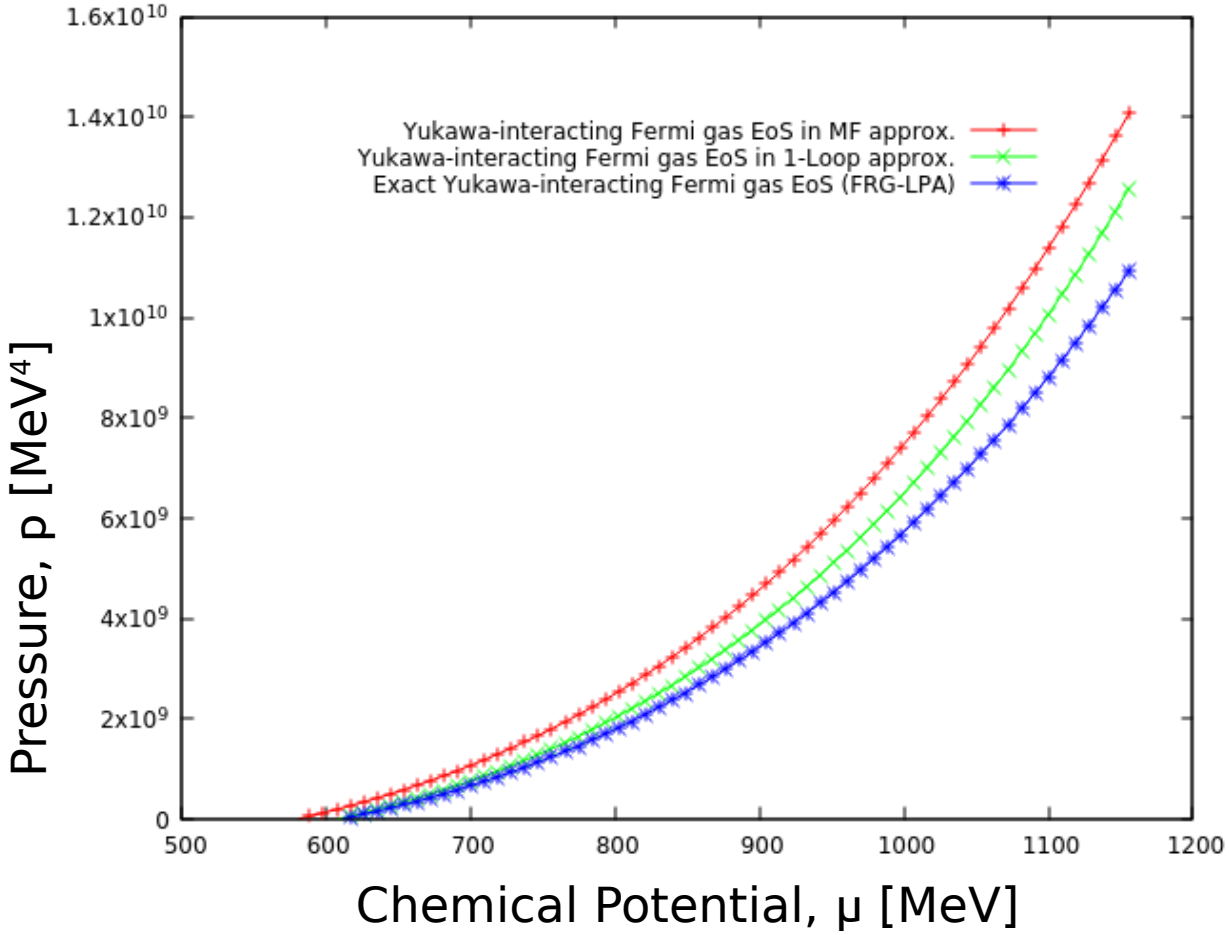
MF is 25% stiffer than the FRG

1L is 10% stiffer than the FRG

Exact FRG solution softest

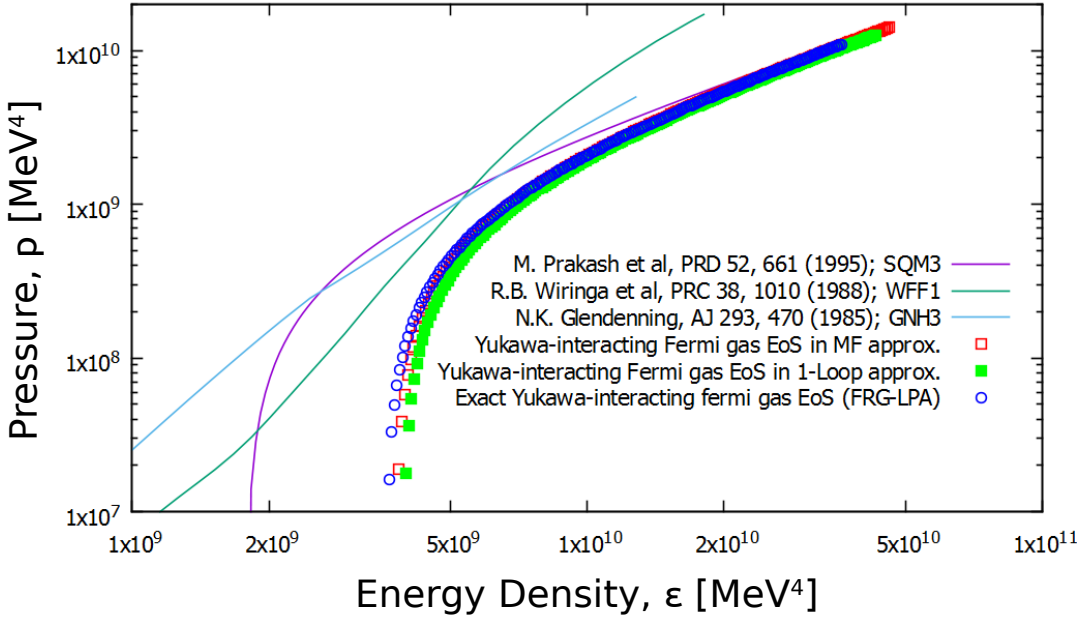


Result: Comparison to other EoS models



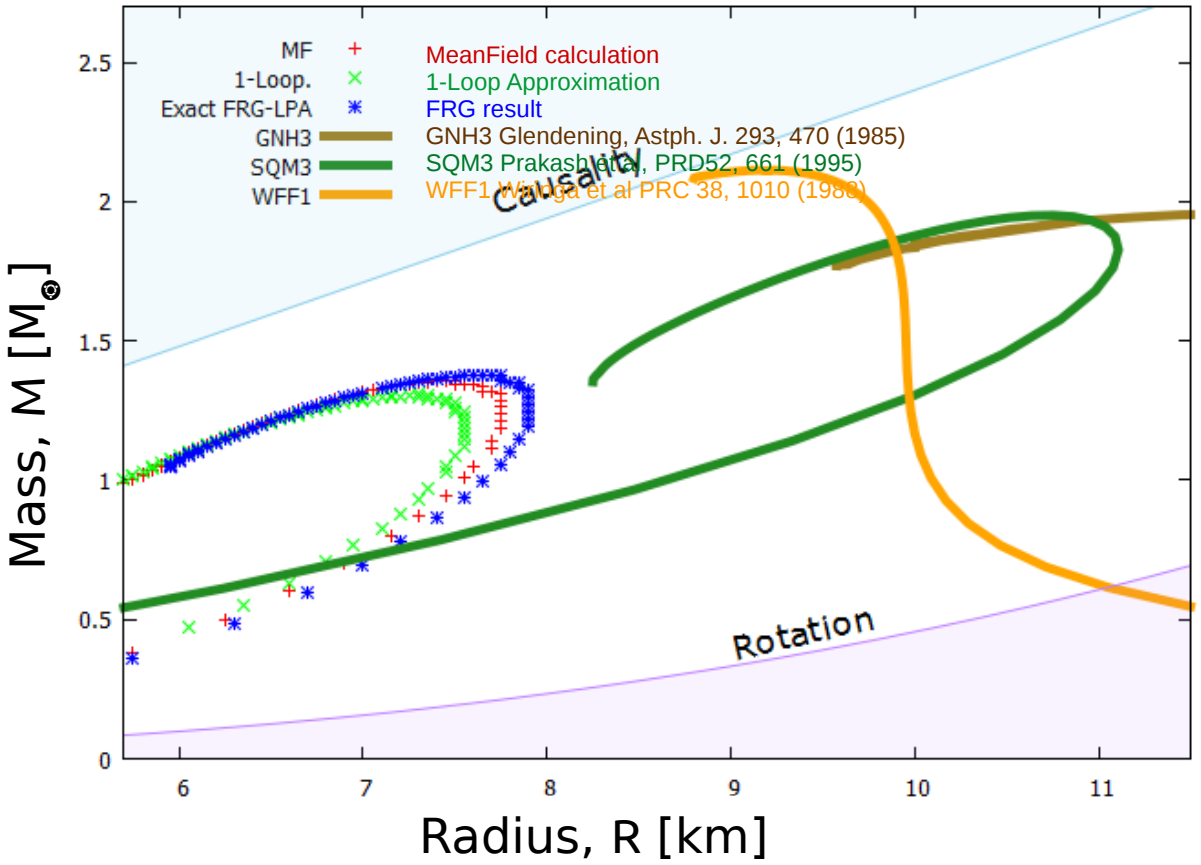
Compare FRG to SQM3, GNH3, WFF1

- Overlap with SQM3 at high ε
- Cutoff, ε_{Cut} is also higher
- Approximations differ slightly



Result: Test in a Compact Star

- Compare FRG EoS to SQM3, GNH3, WFF1 → TOV result on M(R) diagram



Compare FRG to 1L and MF

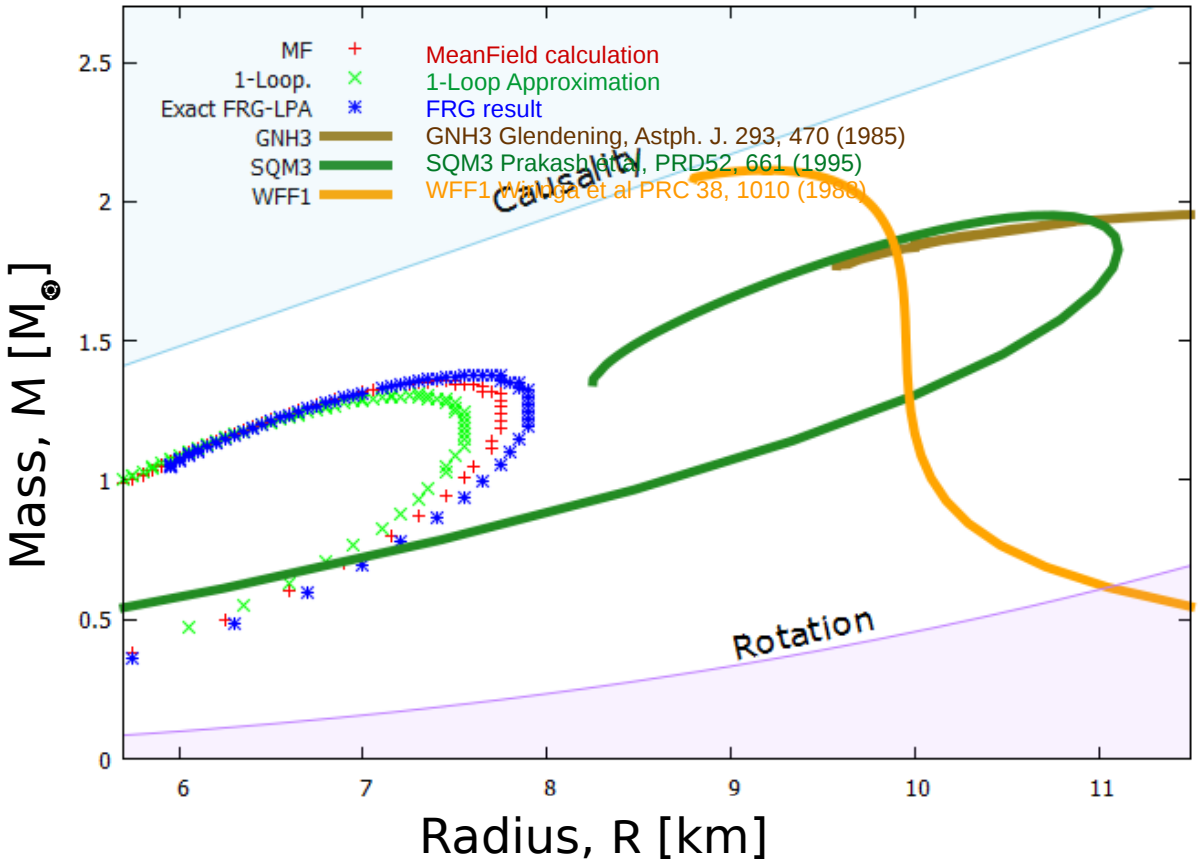
- Soft FRG make biggest star
- High-ε part is similar for all
- Difference: ~5% (.1 $M_\odot$ and .5 km)

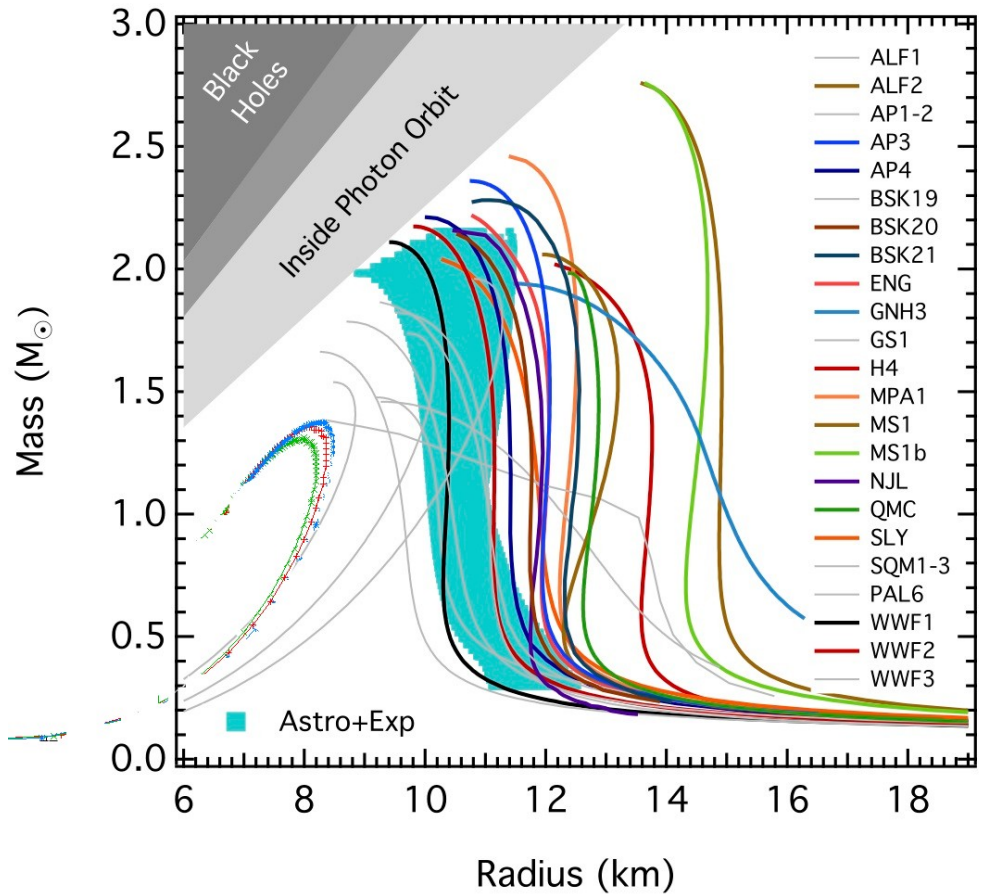
FRG to SQM3, GNH3, WFF1

- Small stars 1.4 $M_\odot$ and 8 km
- Overlap with SQM3 at high ε
- Interaction (ω) will increase

Result: Test in a Compact Star

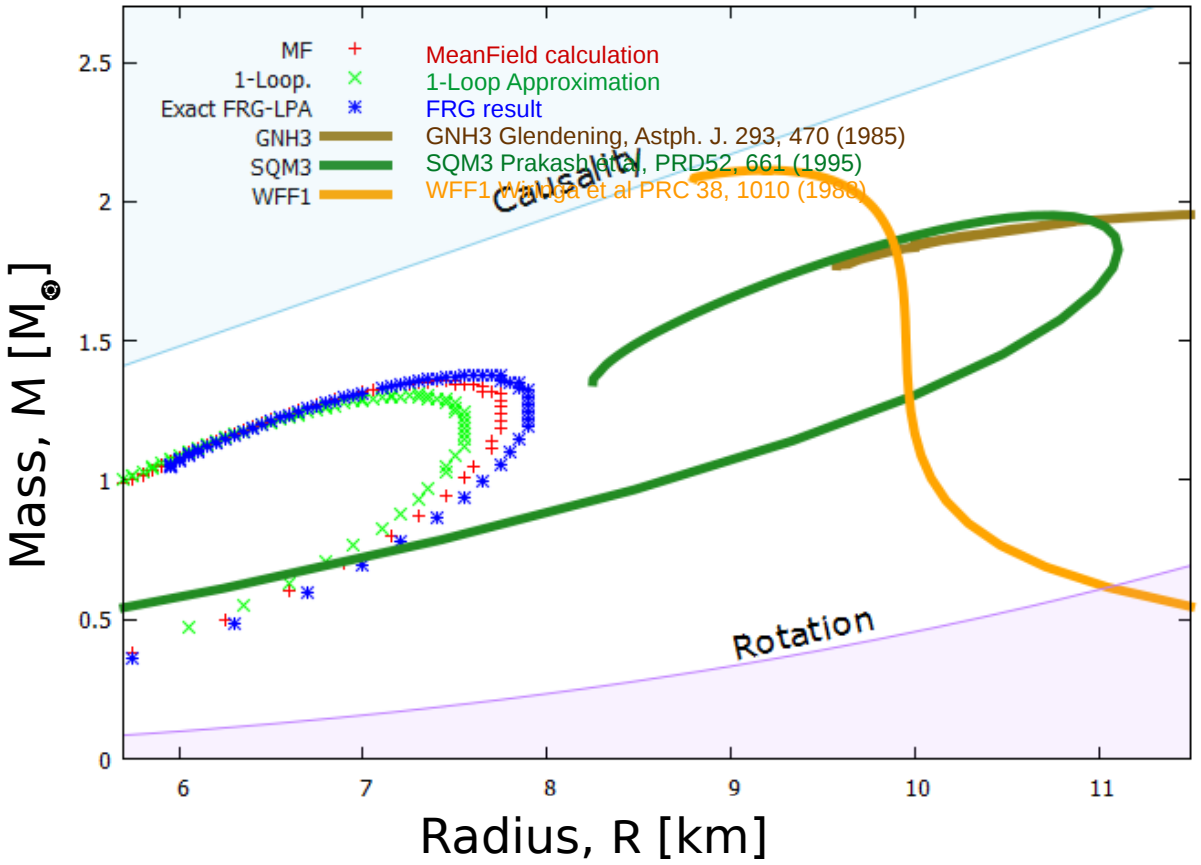
- Compare FRG EoS to SQM3, GNH3, WFF1 → TOV result on M(R) diagram

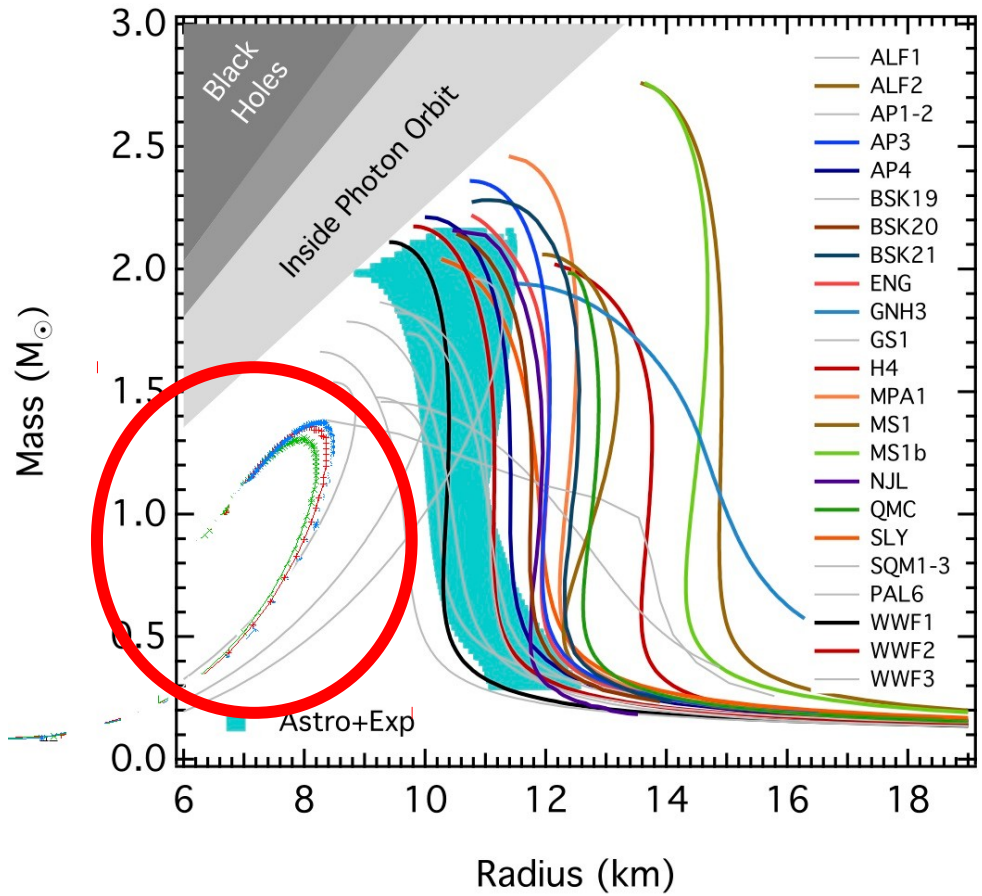




Result: Test in a Compact Star

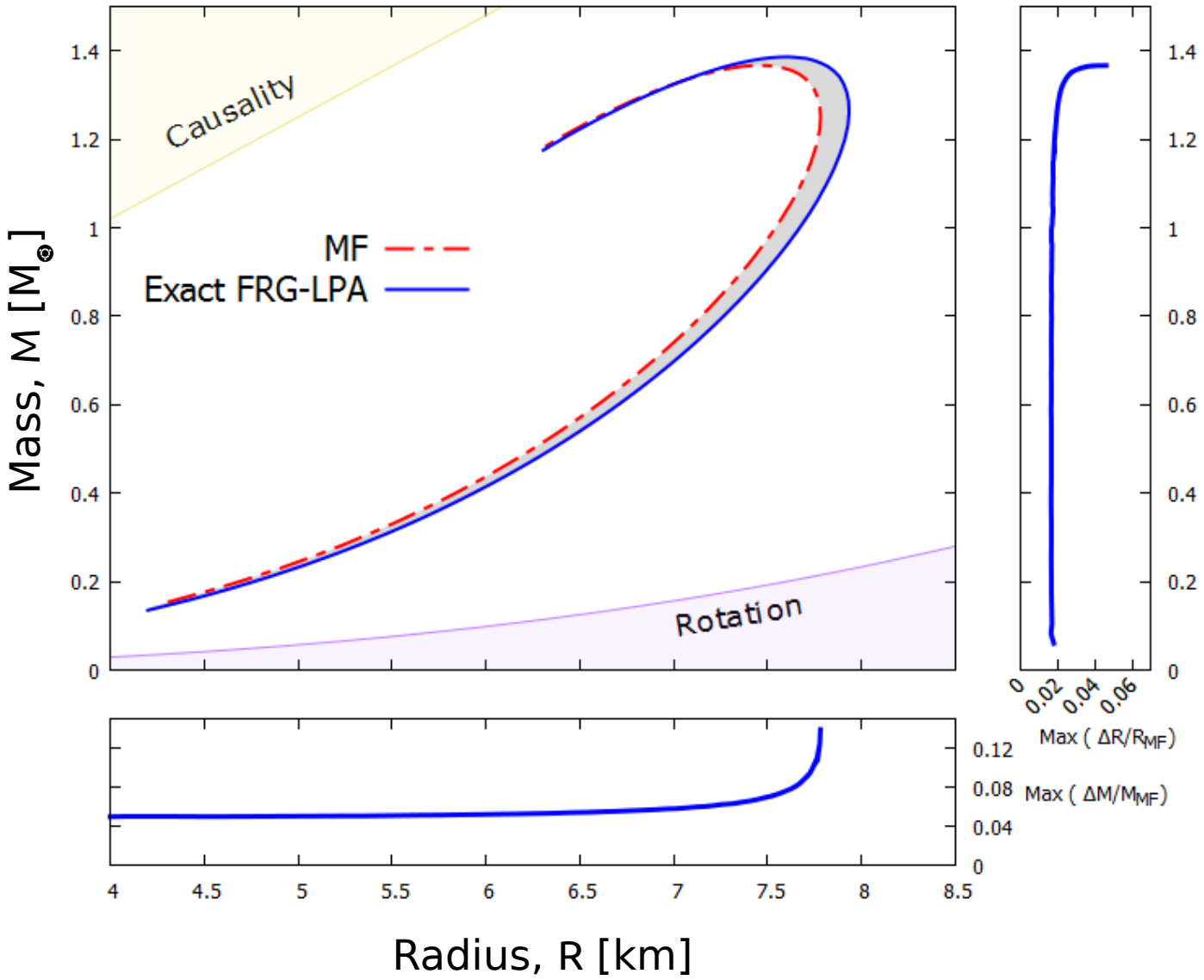
- Compare FRG EoS to SQM3, GNH3, WFF1 → TOV result on M(R) diagram





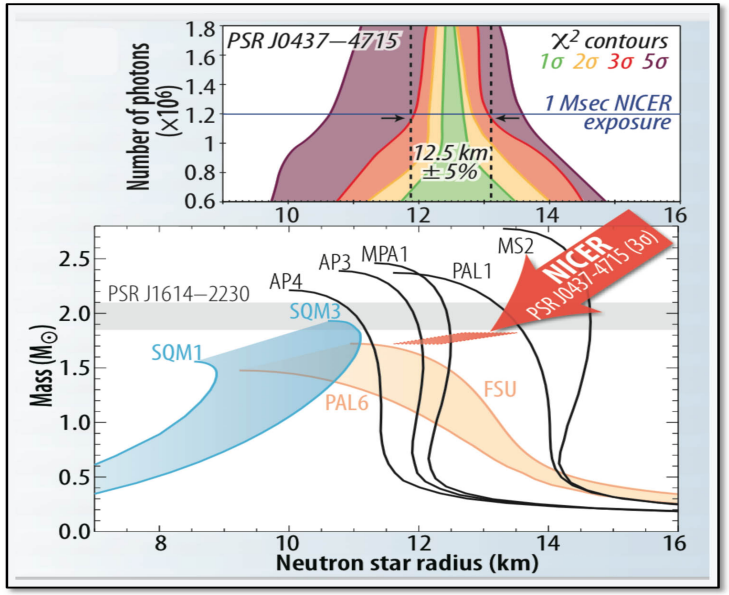
Test: Can we test this by observations?

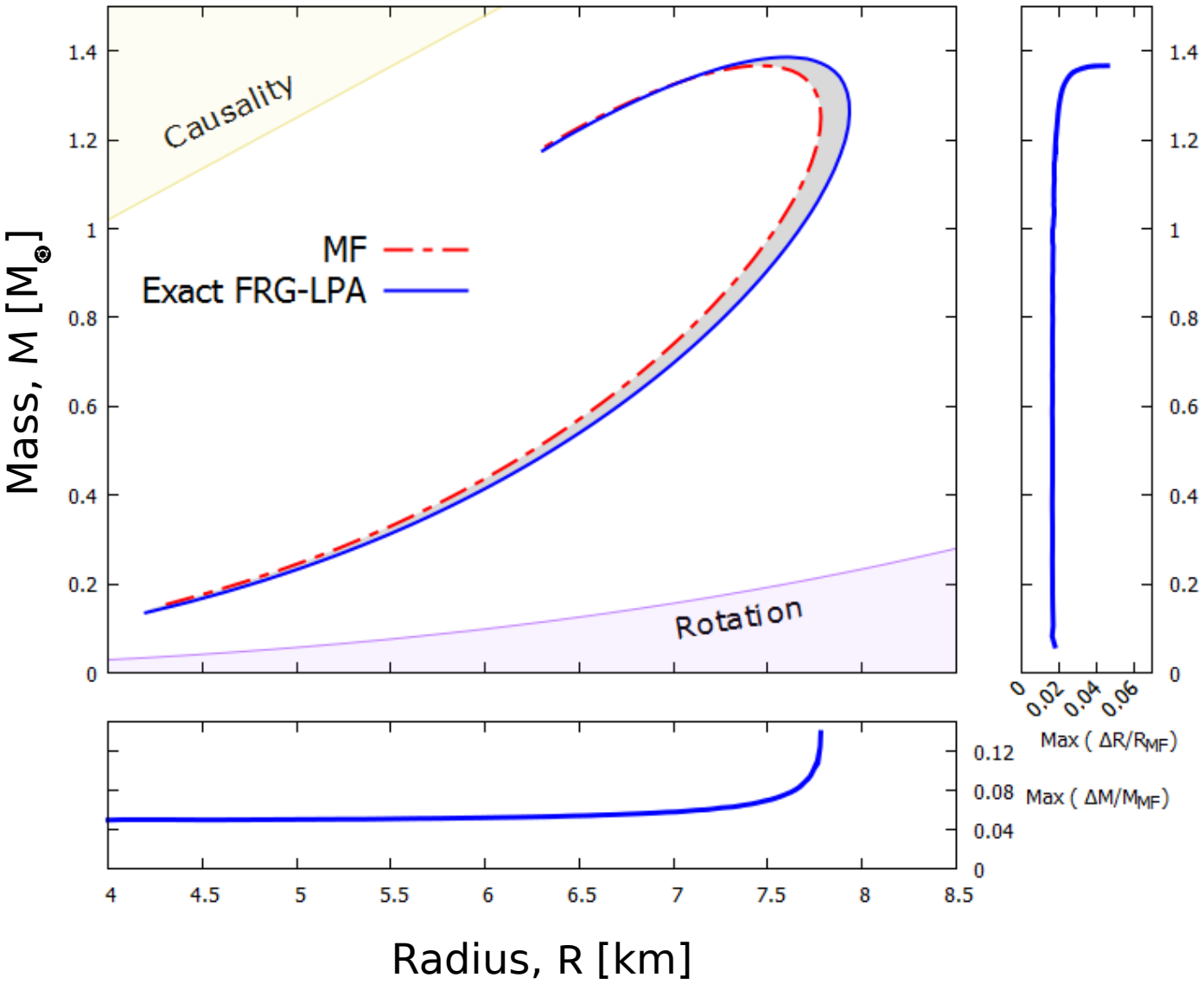
- Compare different EoS results on M(R) diagram: MF & FRG
- Maximal relative differences are also plotted



Test: Can we test this by observations?

- Compare different EoS results on M(R) diagram: MF & FRG
- Maximal relative differences are also plotted





Take-away from theoretical uncertainties

- The magnitude of the uncertainties of (astro)physical observables
  - Microscopical observables are maximum: 10-25%
  - Macroscopical astrophysical ones are maximum: 5-10%
  - Measurement resolution limit is about: 10%

Observable	Max theory uncertainty (%)
Potential, U(φ)	< 25%
Phase diagram (g_c)	< 25%
EoS p(μ),p(ε)	< 25%
Compressibility	< 10%
ε(R)	~ 5%
M(R) diagram	< 10% (M) < 5% (R)
Compactness	< 10% (M) < 5% (R)

2) Uncertainties from the parameters of realistic nuclear matter

P. Pósfay, GGB, A. Jakovác: Universe 5 (2019) no.6, 153 (symmetric case)

Modified σ-ω model in mean field

$$\mathcal{L}_{MF} = \boxed{\sum_{i=1,2} \bar{\psi}_i \left(i\partial\!\!\!/ \underbrace{- m_N + g_\sigma \overline{\sigma}}_{\text{Nucleon effective mass}} - g_\omega \gamma^0 \overline{\omega}_0 \right) \psi_i}$$

Proton and neutron

$$\boxed{-\frac{1}{2} m_\sigma^2 \overline{\sigma}^2 - \lambda_3 \overline{\sigma}^3 - \lambda_4 \overline{\sigma}^4}$$

Scalar meson self interaction terms

Extra terms

$$\boxed{+\frac{1}{2} m_\omega^2 \overline{\omega}_0^2}$$

Vector meson

$$\boxed{+ \frac{1}{2} m_\rho^2 \rho_\mu^a \rho^{\mu\, a}}$$

Tensor meson

$$\boxed{+ \overline{\Psi}_e \left(i\partial\!\!\!/ - m_e \right) \Psi_e}$$

Electron in β-equilibrium

$$\boxed{\mu_n = \mu_p + \mu_e}$$

Modified σ-ω model in mean field

$$\mathcal{L}_{MF} = \sum_{i=1,2} \bar{\psi}_i \left(i\partial\!\!\!/ - \underbrace{m_N + g_\sigma \overline{\sigma}}_{\text{Nucleon effective mass}} - g_\omega \gamma^0 \overline{\omega}_0 \right) \psi_i$$

Proton and neutron

$$-\frac{1}{2} m_\sigma^2 \overline{\sigma}^2 - \lambda_3 \overline{\sigma}^3 - \lambda_4 \overline{\sigma}^4$$

Scalar meson self interaction terms

$$+\frac{1}{2} m_\omega^2 \overline{\omega}_0^2$$

Vector meson

Extra terms

p – n Nuclear force

$$+ \frac{1}{2} m_\rho^2 \rho_\mu^a \rho^{\mu\, a}$$

Tensor meson

$$+ \overline{\Psi}_e \left(i\partial\!\!\!/ - m_e \right) \Psi_e$$

Electron in β-equilibrium

$$\mu_n = \mu_p + \mu_e$$

Modified σ-ω model in mean field

$$\mathcal{L}_{MF} = \sum_{i=1,2} \bar{\psi}_i \left(i\partial\!\!\!/ - \underbrace{m_N + g_\sigma \overline{\sigma}}_{\text{Nucleon effective mass}} - g_\omega \gamma^0 \overline{\omega}_0 \right) \psi_i$$

Proton and neutron

$$- \frac{1}{2} m_\sigma^2 \overline{\sigma}^2 - \lambda_3 \overline{\sigma}^3 - \lambda_4 \overline{\sigma}^4$$

Scalar meson self interaction terms

$$+ \frac{1}{2} m_\omega^2 \overline{\omega}_0^2$$

Vector meson

$$+ \frac{1}{2} m_\rho^2 \rho_\mu^a \rho^{\mu\, a}$$

Isospin asymmetry

Tensor meson

$$+ \overline{\Psi}_e \left(i\partial\!\!\!/ - m_e \right) \Psi_e$$

Electron in β-equilibrium

$$\mu_n = \mu_p + \mu_e$$

Modified σ-ω model in mean field

$$\mathcal{L}_{MF} = \sum_{i=1,2} \bar{\psi}_i \left(i\partial\!\!\!/ - \underbrace{m_N + g_\sigma \overline{\sigma}}_{\text{Nucleon effective mass}} - g_\omega \gamma^0 \overline{\omega}_0 \right) \psi_i$$

Proton and neutron

$$- \frac{1}{2} m_\sigma^2 \overline{\sigma}^2 - \lambda_3 \overline{\sigma}^3 - \lambda_4 \overline{\sigma}^4$$

Scalar meson self interaction terms

$$+ \frac{1}{2} m_\omega^2 \overline{\omega}_0^2$$

Vector meson

$$+ \frac{1}{2} m_\rho^2 \rho_\mu^a \rho^{\mu\, a}$$

Tensor meson

$$+ \overline{\Psi}_e \left(i\partial\!\!\!/ - m_e \right) \Psi_e$$

Electron in β-equilibrium

$$\mu_n = \mu_p + \mu_e$$

Modified σ-ω model in mean field

$$\mathcal{L}_{MF} = \sum_{i=1,2} \bar{\psi}_i \left(i\partial\!\!\!/ - \underbrace{m_N + g_\sigma \overline{\sigma}}_{\text{Nucleon effective mass}} - g_\omega \gamma^0 \, \overline{\omega}_0 \right) \psi_i$$

Proton and neutron

$$- \frac{1}{2} m_\sigma^2 \overline{\sigma}^2 - \lambda_3 \, \overline{\sigma}^3 - \lambda_4 \, \overline{\sigma}^4$$

Scalar meson self interaction terms

Extra terms

$$+ \frac{1}{2} m_\omega^2 \, \overline{\omega}_0^2$$

Vector meson

$$+ \frac{1}{2} m_\rho^2 \, \rho_\mu^a \, \rho^{\mu \, a}$$

Tensor meson

$$+ \overline{\Psi}_e \left(i\partial\!\!\!/ - m_e \right) \Psi_e$$

Electron in β-equilibrium

$$\mu_n = \mu_p + \mu_e$$

Modified σ-ω model in mean field

- **Theoretical mean field model:**
  - Symmetric case: 3 combinations with the higher-order scalar meson self-interaction terms to original Walecka:
  - Asymmetric case: tensor force is added to the interaction in addition to the electrons, for β-equilibrium: $\mu_n = \mu_p + \mu_e$

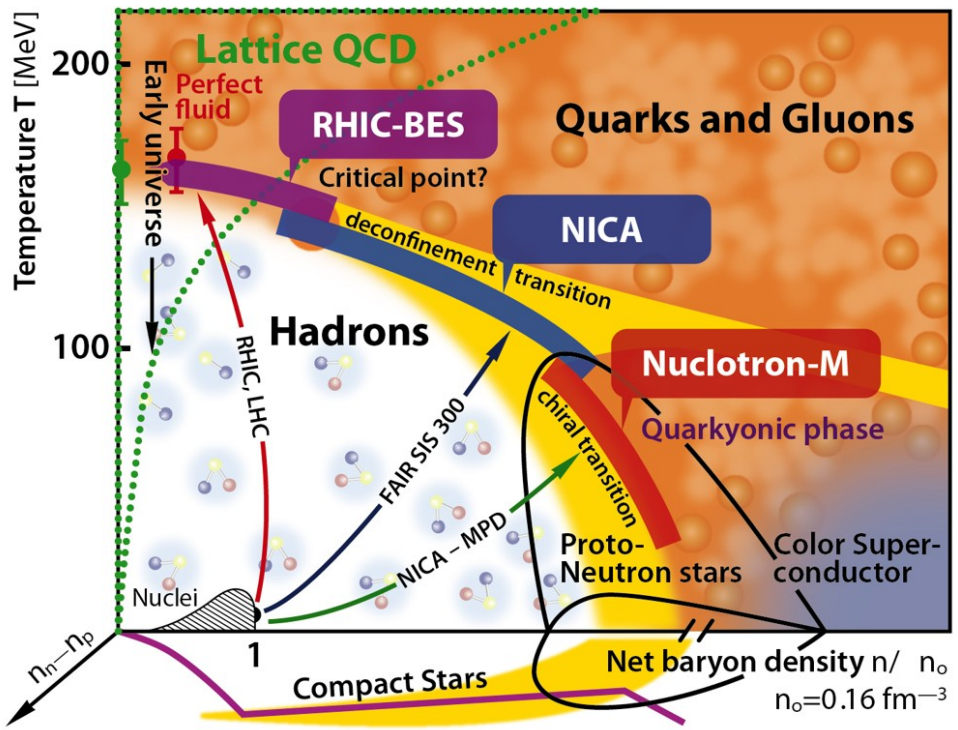
$$- \lambda_3 \overline{\sigma}^3 - \lambda_4 \overline{\sigma}^4$$

$$+ \frac{1}{2} m_\rho^2 \rho_\mu^a \rho^{\mu\, a}$$

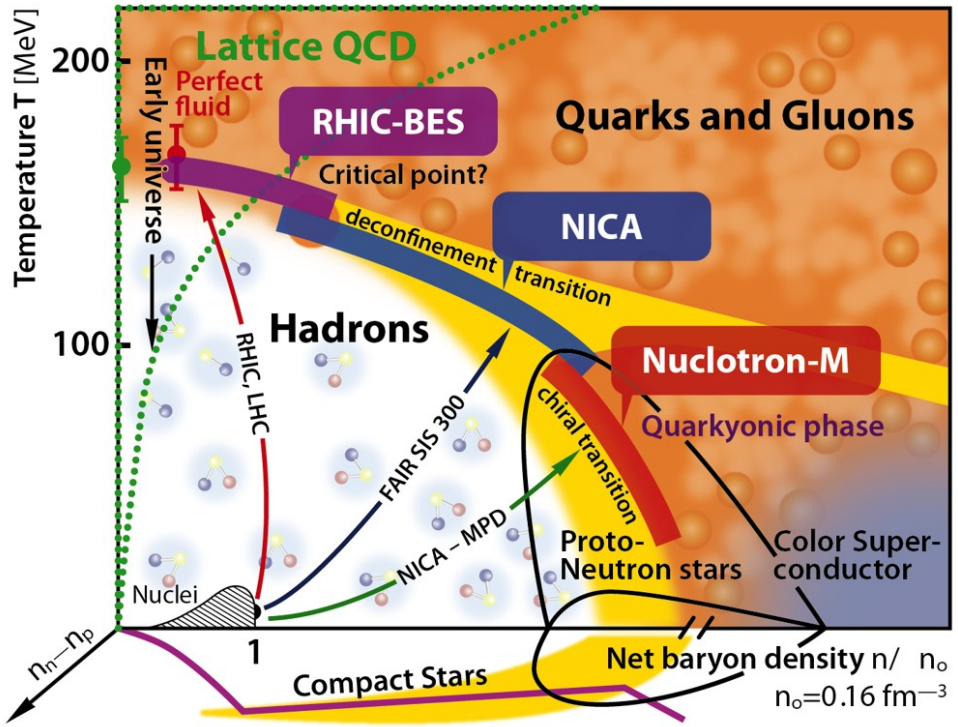
$$+ \overline{\Psi}_e \left(i \partial\!\!\!/ - m_e \right) \Psi_e$$

- **Parameters of the theoretical model**
  - Fit couplings/masses/etc. according to the Rhoades–Ruffini theorem in agreement with experimental data.
  - Parameters are usually non-independent: optimalization of the parameters need to perform → similar EoS
- **Cross check the consistency with the the existing EM, GR, HIC, etc data + errors → Theoretical uncertainties**

Parameters to fit normal nuclear matter

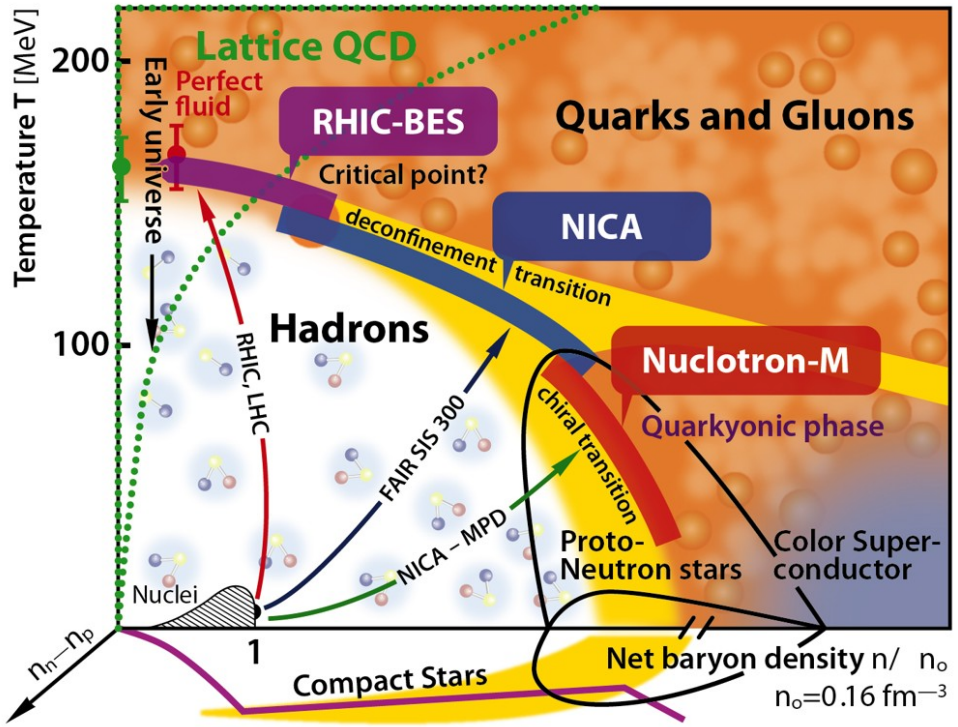


Parameters to fit normal nuclear matter



Parameter	Value
Saturation density	0.156 1/fm^3
Binding energy	-16.3 MeV
Nucleon effective mass	0.6 m_N
Nucleon Landau mass	0.83 m_N
incompressibility	240 MeV
Asymmetry energy	32.5 MeV

Parameters to fit normal nuclear matter

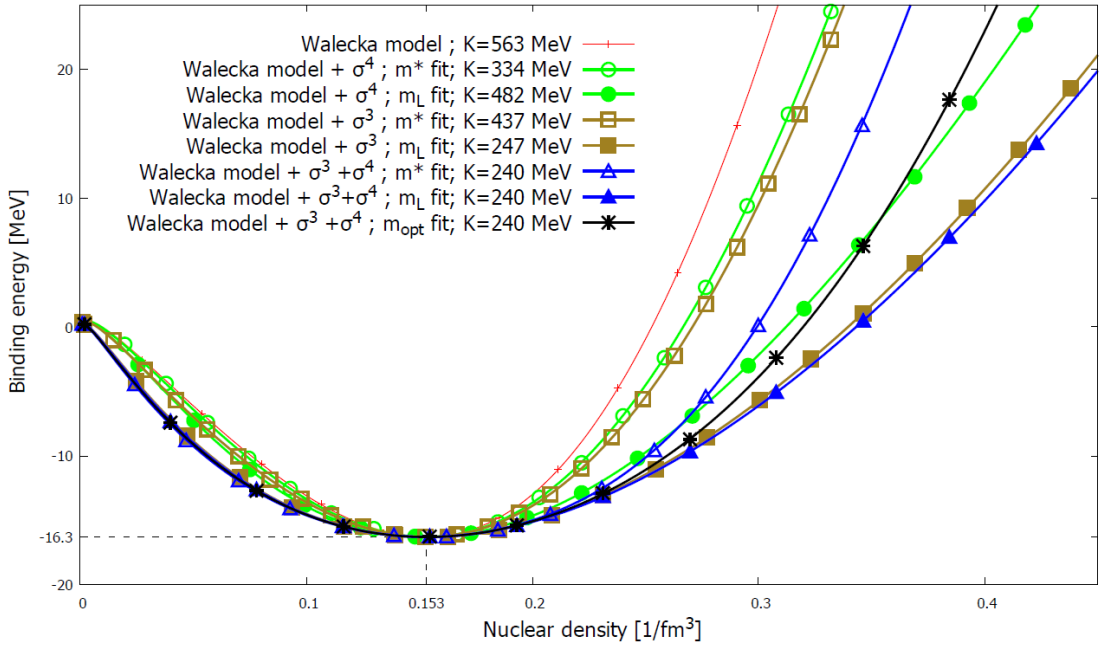


Parameter	Value
Saturation density	0.156 1/fm^3
Binding energy	-16.3 MeV
Nucleon effective mass	0.6 m_N
Nucleon Landau mass	0.83 m_N
incompressibility	240 MeV
Asymmetry energy	32.5 MeV

Incompressibility $$K = k_F^2 \frac{\partial^2(\epsilon/n)}{\partial k_F^2} = 9\,\frac{\partial p}{\partial n}$$

Landau mass $$m_L = \frac{k_F}{v_F} = \sqrt{k_F^2 + m_{N,eff}^2}$$

Parameters to fit normal nuclear matter



Parameter	Value
Saturation density	0.156 1/fm^3
Binding energy	-16.3 MeV
Nucleon effective mass	0.6 m_N
Nucleon Landau mass	0.83 m_N
incompressibility	240 MeV
Asymmetry energy	32.5 MeV

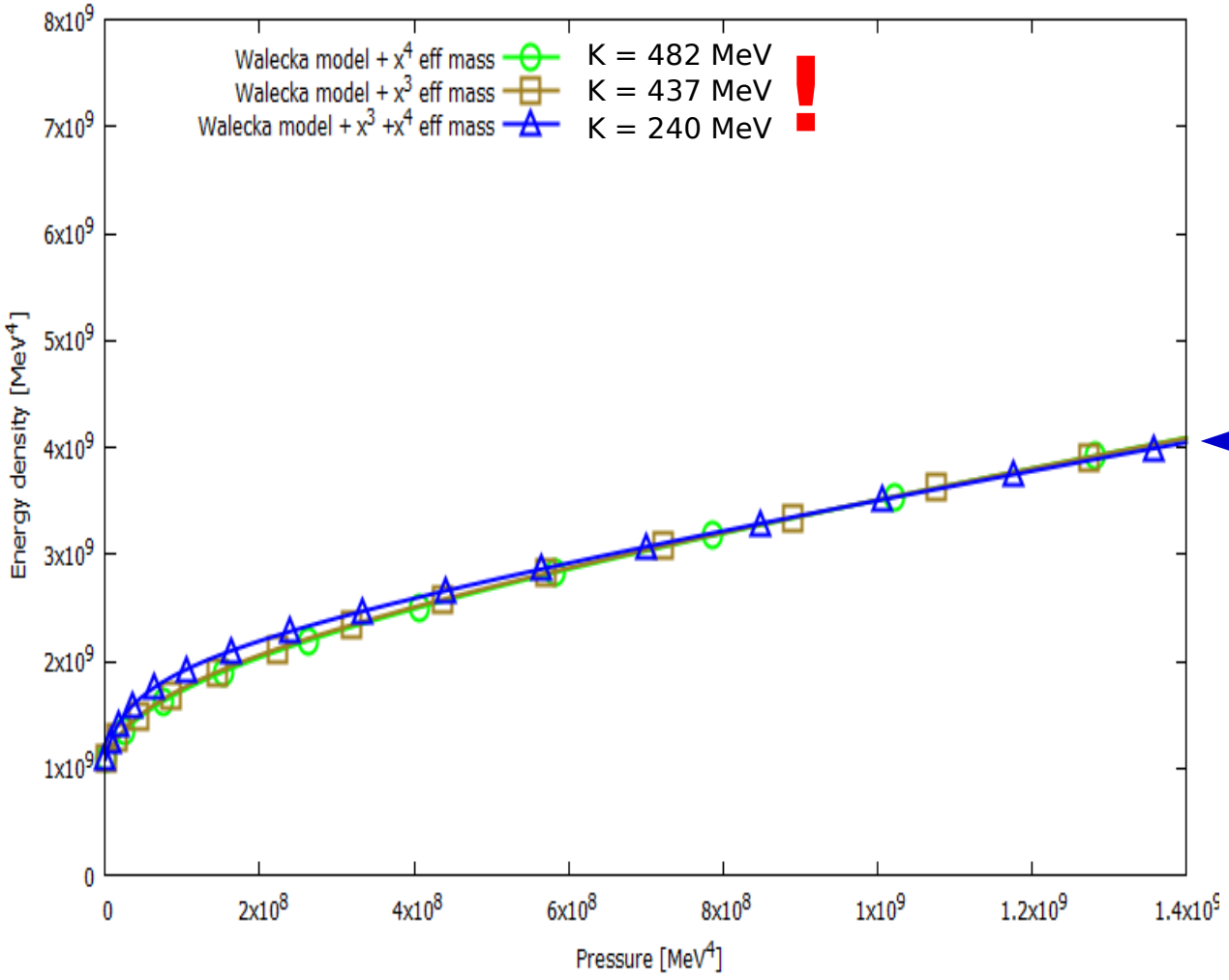
The effective mass and Landau mass are NOT independent!

The can not be fitted simultaneously

Incompressibility $$K = k_F^2 \frac{\partial^2(\epsilon/n)}{\partial k_F^2} = 9\,\frac{\partial p}{\partial n}$$

Landau mass $$m_L = \frac{k_F}{v_F} = \sqrt{k_F^2 + m_{N,eff}^2}$$

The Equation of State of different model fits



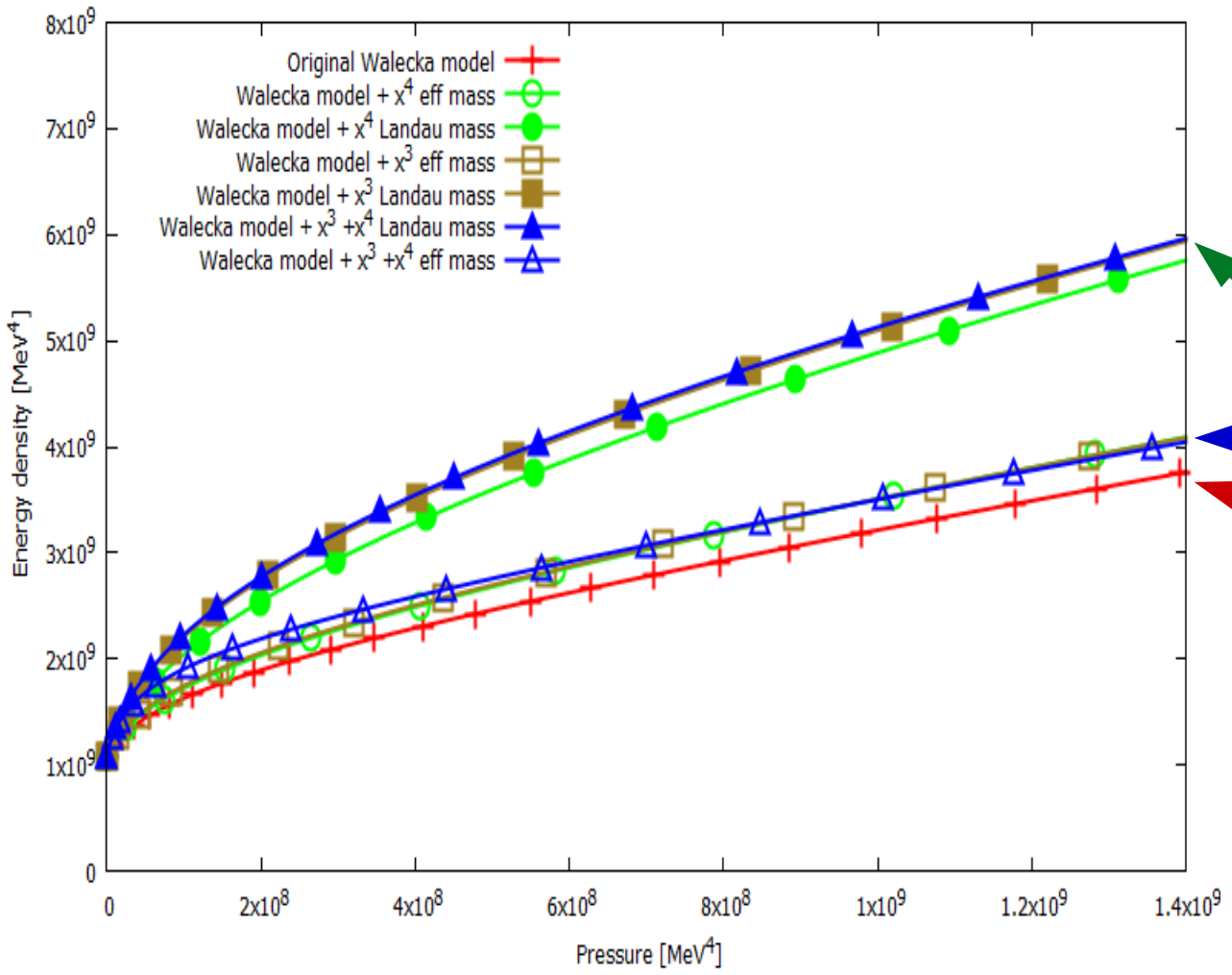
Adding higher-order terms

- helps, at lower pressure
- more parameter more constraints:

← Effective mass fit $m_{Eff} = 0.6\ m_N$

Different models give similar EoS

The Equation of State of different model fits



Adding higher-order terms

- helps, at lower pressure
- more parameter more constraints:

Landau mass fit $m_{Eff} = 0.83\ m_N$

Effective mass fit $m_{Eff} = 0.6\ m_N$

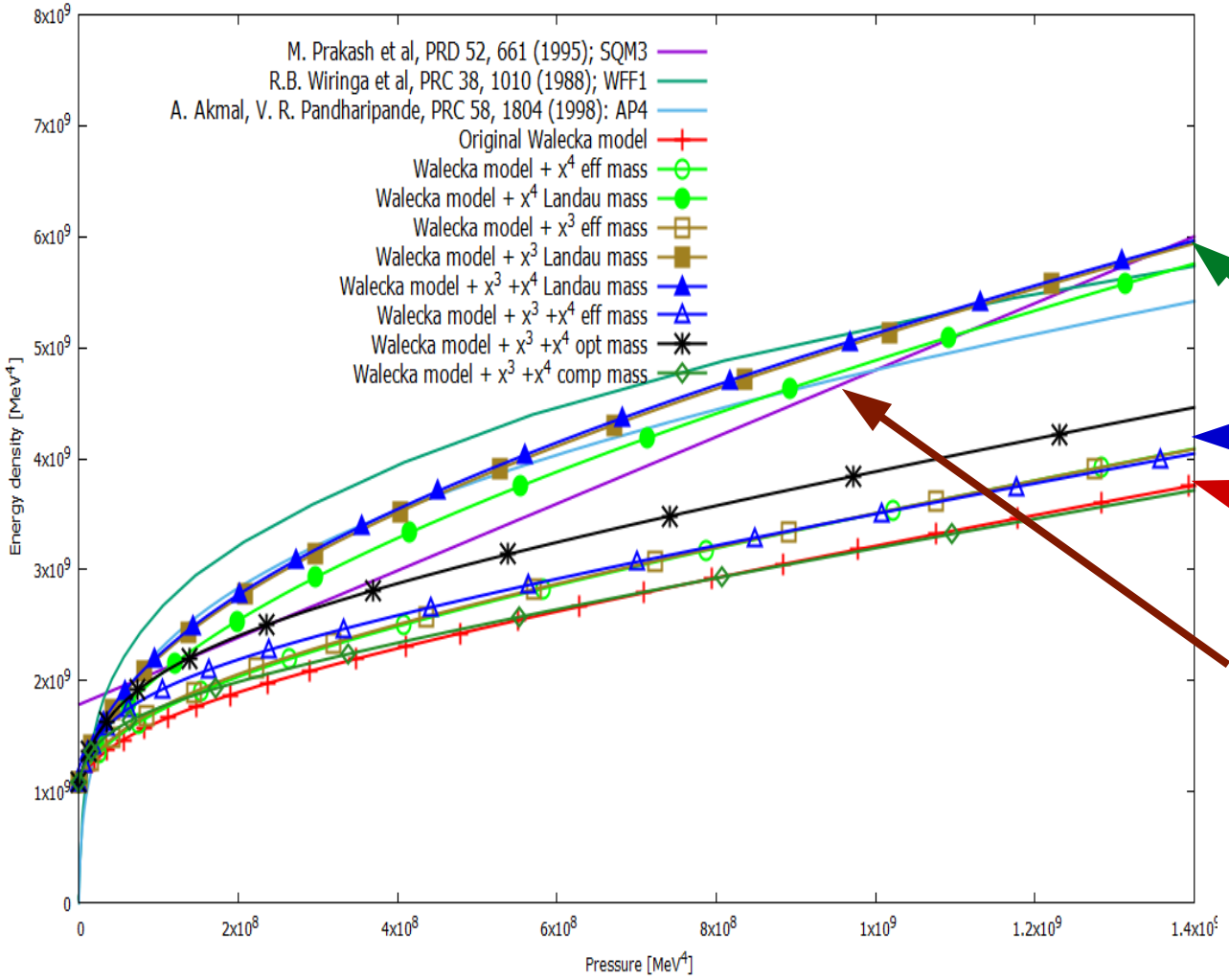
Original Walecka model

Different models give similar EoS

Depending on the 'fit-type'

→ bands appear

The Equation of State of different model fits



Adding higher-order terms

- helps, at lower pressure
- more parameter more constraints:

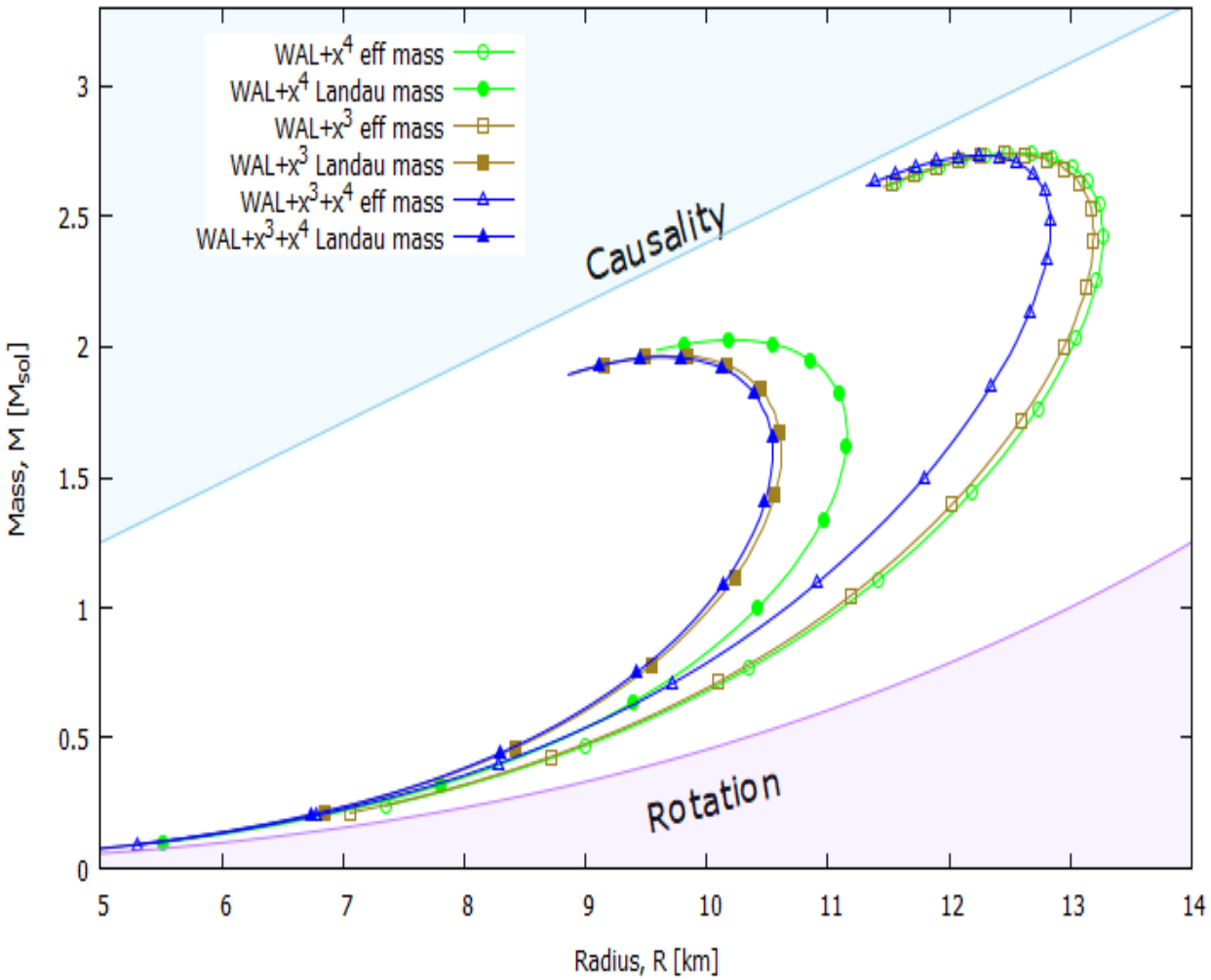
Landau mass fit $m_{Eff} = 0.83\ m_N$

Effective mass fit $m_{Eff} = 0.6\ m_N$

Original Walecka model

Realistic nuclear matter EoSs, like WFF1, AP4 (SQM) support the Landau mass fits well.

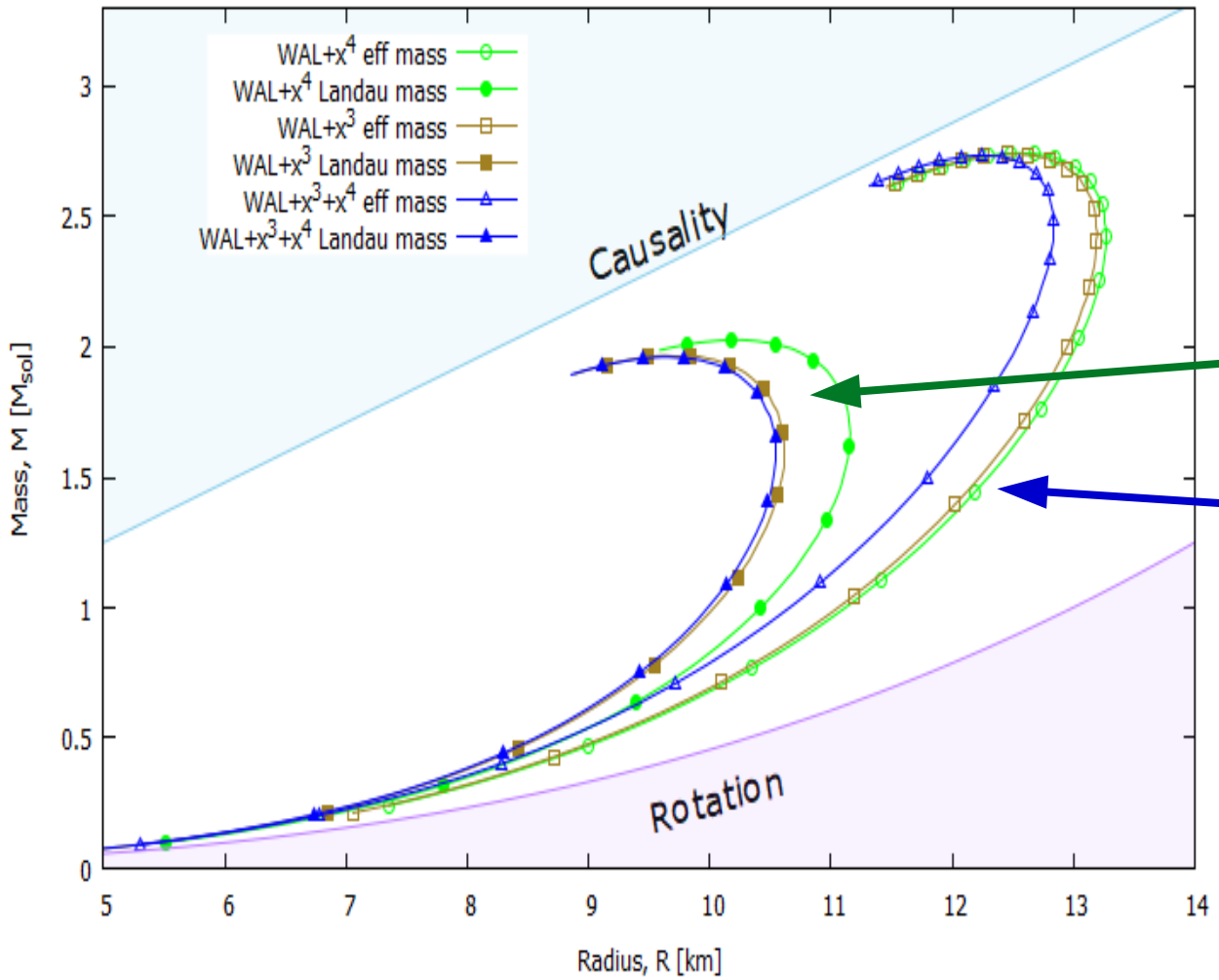
The M-R diagrams: EoS & effective mass fit



SYMMETRIC nuclear matter EoS

- Cases with extra x^3 and/or x^4 terms provide similar band structures in the M-R diagram

The M-R diagrams: EoS & effective mass fit



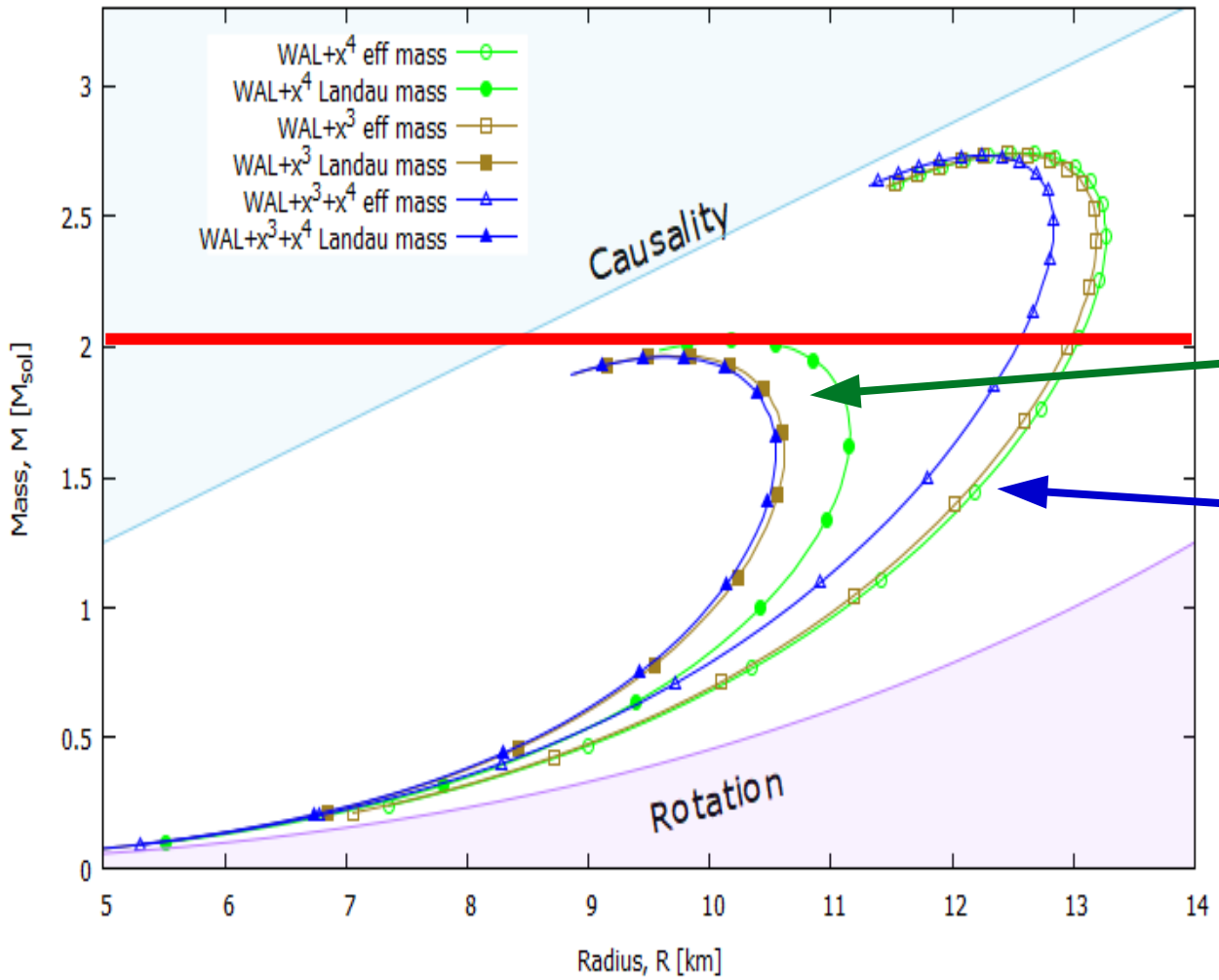
SYMMETRIC nuclear matter EoS

- Cases with extra x^3 and/or x^4 terms provide similar band structures in the M-R diagram

Landau mass fit $m_{Eff} = 0.83\ m_N$

Effective mass fit $m_{Eff} = 0.6\ m_N$

The M-R diagrams: EoS & effective mass fit



SYMMETRIC nuclear matter EoS

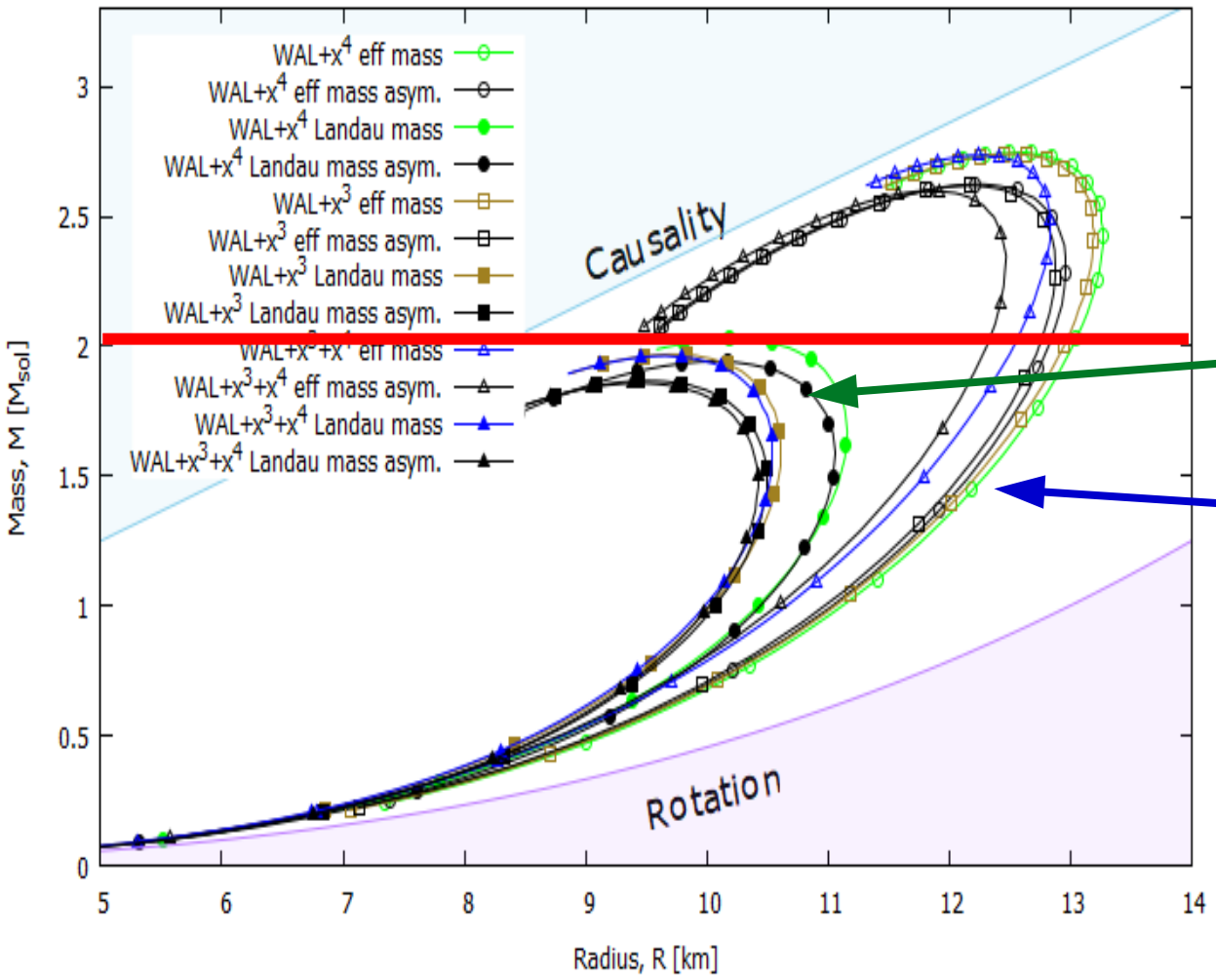
- Cases with extra x^3 and/or x^4 terms provide similar band structures in the M-R diagram

Landau mass fit $m_{Eff} = 0.83\ m_N$

Effective mass fit $m_{Eff} = 0.6\ m_N$

→ Landau mass fits provide compact star with lower M_{max} but closer to the observations

The M-R diagrams: EoS & effective mass fit



ASYMMETRIC nuclear matter EoS

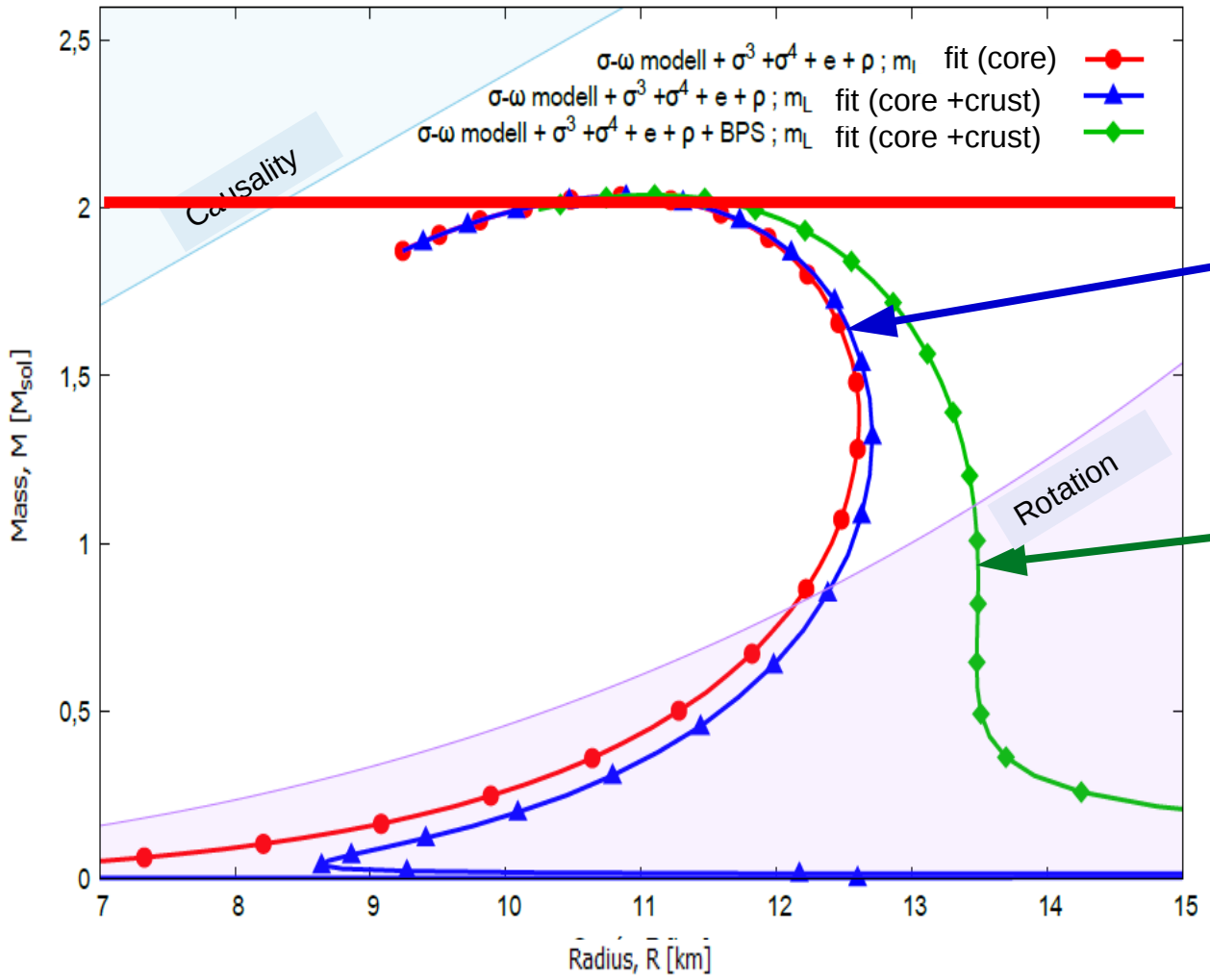
- Cases with extra x^3 and/or x^4 terms provide similar band structures in the M-R diagram

Landau mass fit $m_{Eff} = 0.83\ m_N$

Effective mass fit $m_{Eff} = 0.6\ m_N$

→ Nuclear ASYMMETRY has weak decreasing effect on the M_{max}

The M-R diagrams: Best fit with crust+core EoS



ASYMMETRIC nuclear matter EoS

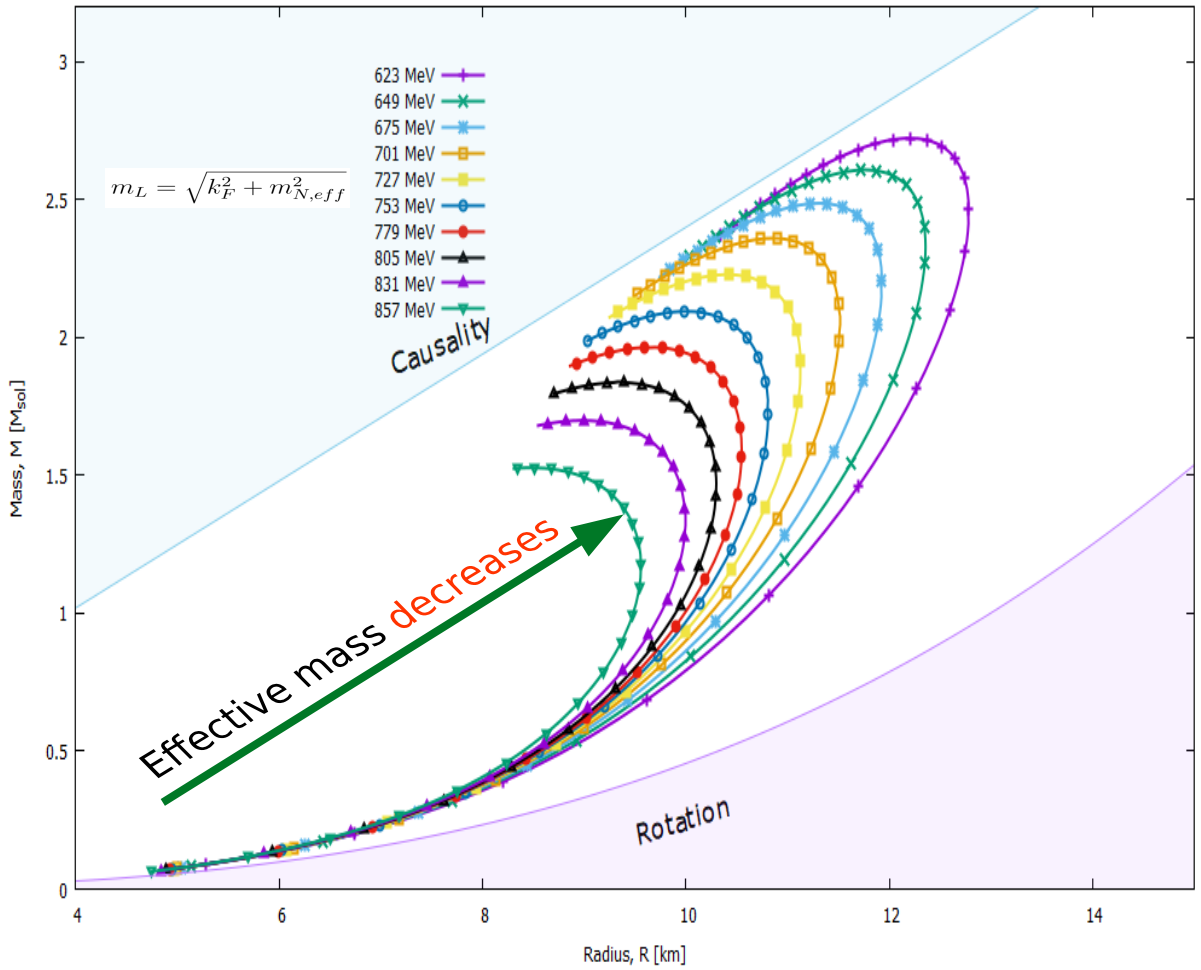
- Cases with extra x^3 and x^4

Added electron and proton with crust, and Landau mass parameter fit m_L = 0.83 m_N

Added electrons and protons with BPS for the crust and Landau mass parameter fit m_L = 0.83 m_N

→ Core + crust model is more realistic, but no effect on the M_{max}

The M-R diagrams: EoS & effective mass fit

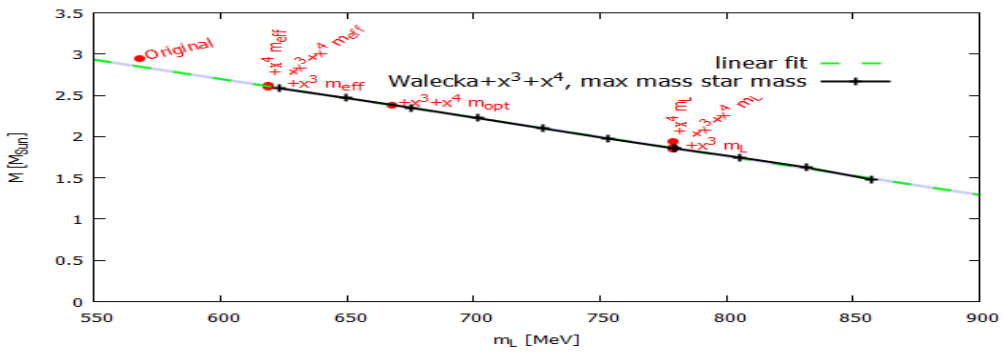


Evolution/scaling in M_{max} appears

- The M_{max} is increasing as the Landau (effective) mass is decreasing

→ Scaling by nuclear parameters

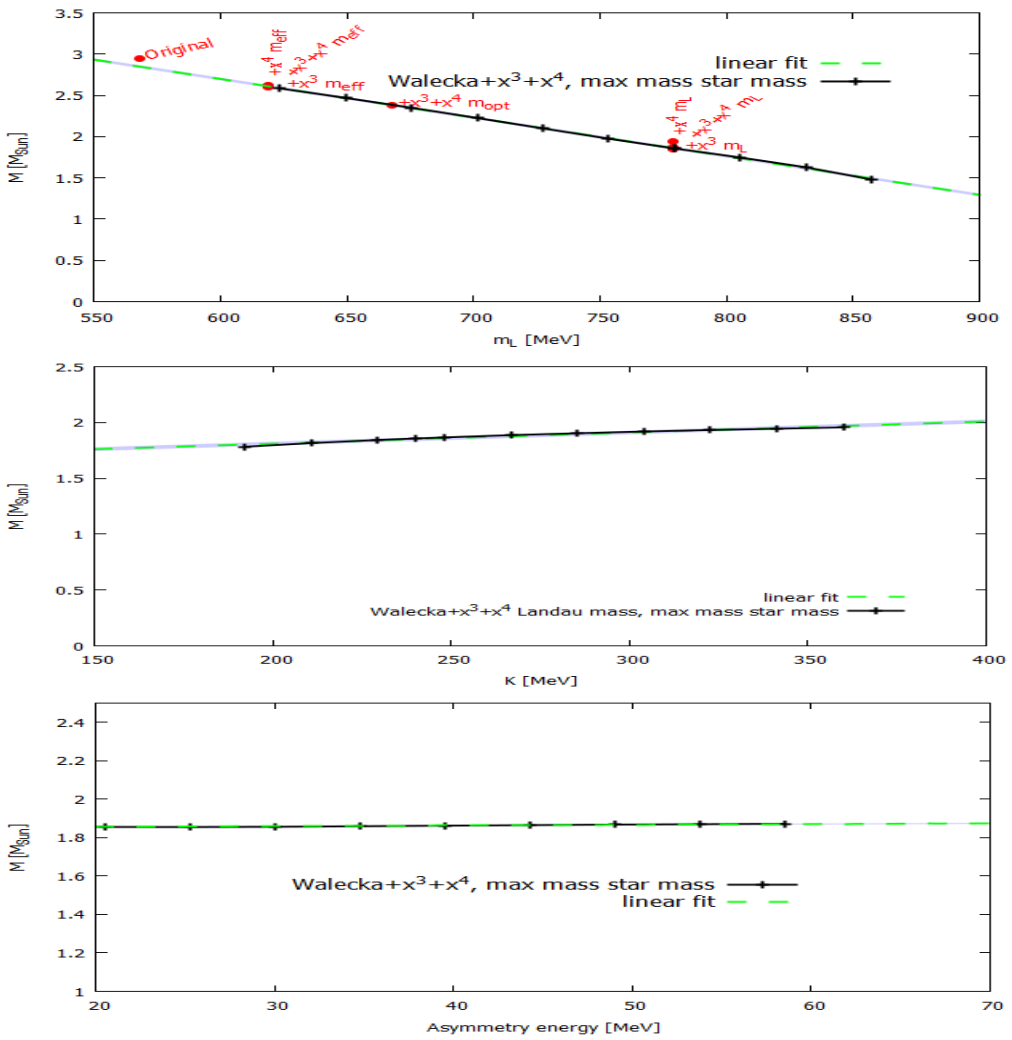
Scaling: maximum star mass vs. nuclear parameters



Evolution/scaling of the maximum mass/radius of the compact star

- The M_{max} is increasing as the Landau (effective) mass is decreasing

Scaling: maximum star mass vs. nuclear parameters



Evolution/scaling of the maximum mass/radius of the compact star

- The M_{max} is increasing as the Landau (effective) mass is decreasing

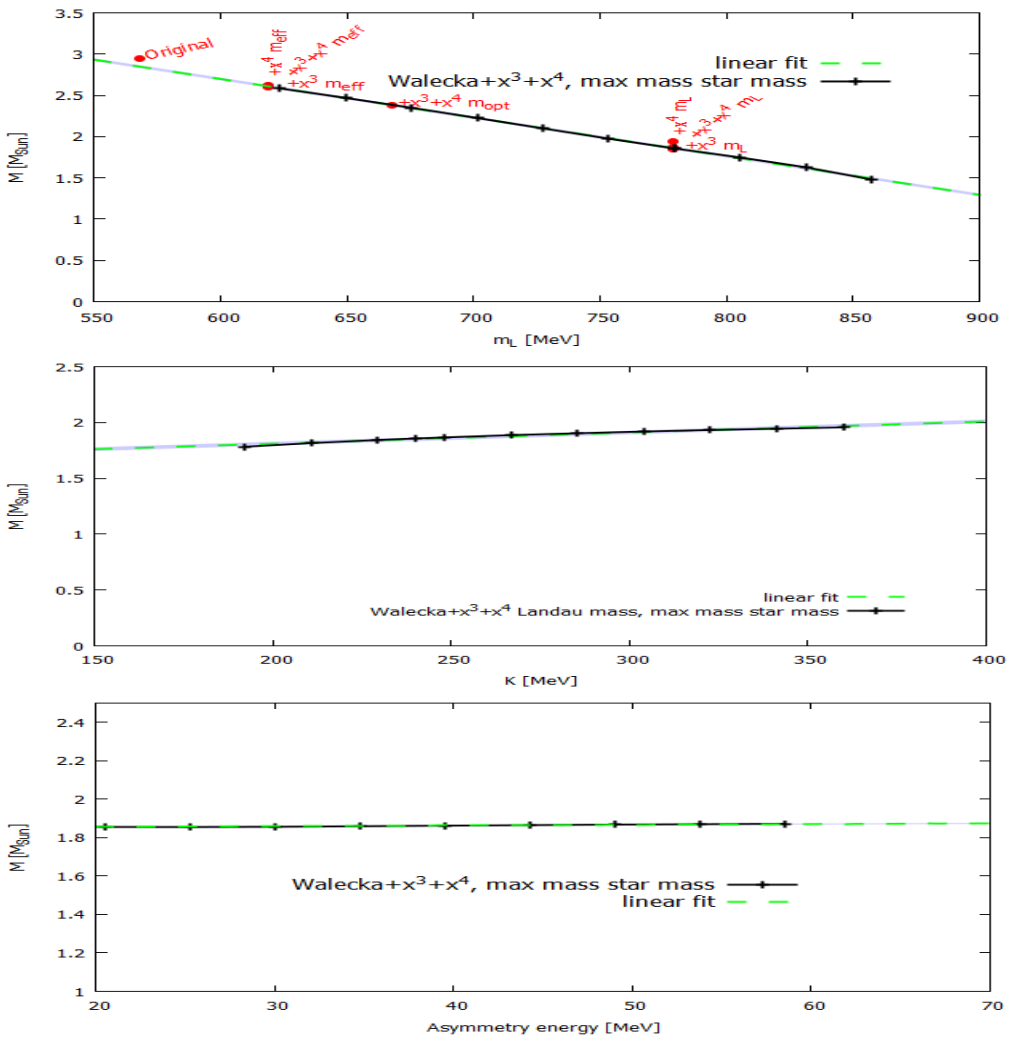
→ Scaling by nuclear parameters

- Fit errors are small < 1%
- M_{max} depends linearly by parameters

$$m_L,\ m_{Eff} >_{10x} K >_{10x} a_{sym}$$

- Good approximation using effective mass, independently of the scalar interaction term
- Similar scaling for R_{max}

Scaling: maximum star mass vs. nuclear parameters



SYMMETRIC nuclear matter

Maximal mass (in $M_\odot$)

$$M_{maxM} = 5.51 - 0.005\ m_L$$

$$M_{maxM} = 1.79 + 0.001\ K$$

ASYMMETRIC nuclear matter

Maximal mass (in $M_\odot$)

$$M_{maxM} = 5.50 - 3.64\ m_L$$

$$M_{maxM} = 1.61 + 0.24\ K$$

$$M_{maxM} = 1.85 + 0.01\ a_{sym}$$

To take away...

- **Theoretical (maximal) uncertainties were tested in FRG**
  - Microscopical level (EoS, phases, compressibility): 10-25%
  - Macroscopical astrophysical level (M,R,compactness): 5-10%

- **Uncertainties by the realistic nuclear matter parameters**
  - Linear dependence on the m_L, $m_{Eff} >_{10x} K >_{10x} a_{sym}$
  - Varying m_L, m_{Eff} cause ~10% uncertainty on M and R
  - Differences on symmetric/asymmetric matter is ~1-3%

... and now, something
completely different...

Unique things with David

Unique things with David

Happy Birthday David!