Quark-Flavor Dependence of Transport Parameters in Hot QCD: the Quasiparticle Perspective

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In collaboration with: C. Sasaki, K. Redlich, M. Bluhm

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Talk based on: V. M., M. Bluhm, C. Sasaki, K. Redlich, PRD 100 '19 (arxiv:1906.01697);

V. M., C. Sasaki, arxiv:2007.06846.

Motivation

Transport properties of QGP: $\eta,\ \zeta,\ \sigma$... – input for hydro simulations

- Lattice QCD
- Perturbative QCD
- AdS/CFT
- Effective models
- Green-Kubo formalism
- Kinetic theory
- $\cdots$

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Goal: impact of quark quasiparticles on transport parameters in hot QCD: $N_f = 0$ vs $N_f = 2+1$ at $\mu = 0$.

Quasiparticle Model: Thermodynamics & Coupling G(T)

$$s = \sum_{i=l,\bar{l},s,\bar{s},g} \frac{d_i}{\pi^2} \int dp \, 2p^2 \, \frac{\frac{4}{3}p^2 + m_i^2(T)}{E_i T} f_i^0$$

$$f_i^0 = (\exp(E_i/T) \pm 1)^{-1}$$

$$E_i(T) = \sqrt{p^2 + m_i^2(T)}$$

w/ $m_i(G(T), T)$

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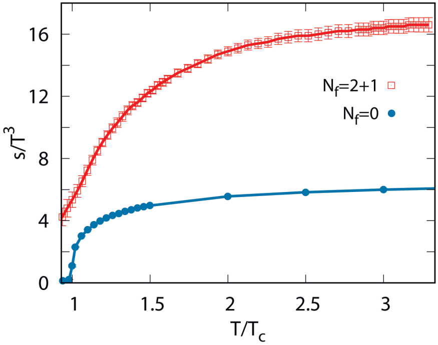

[lQCD: Wuppertal-Budapest Collaboration

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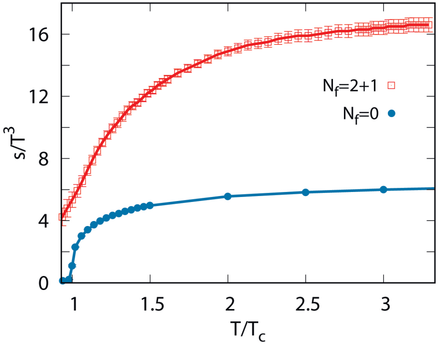

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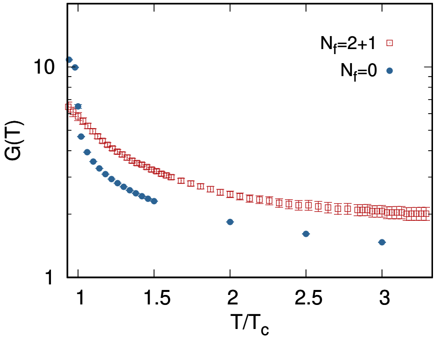

[V. M., M. Bluhm, K. Redlich, C. Sasaki, PRD 100 ’19]

Quasiparticle Model: Effective Masses

Weakly-interacting particles with dynamical masses $m_i(G(T), T)$

$$m_i^2(T) = (m_i^0)^2 + \Pi_i(T),$$

$$\Pi_{l,s}(T) = 2\left[m_{l,s}^0\sqrt{\frac{G^2(T)T^2}{6}} + \frac{G^2T^2}{6}\right],$$

$$\Pi_g(T) = \left(3 + \frac{N_f}{2}\right)\frac{G^2(T)}{6}T^2,$$

$$m_l^0 = 5\ \mathrm{MeV},\ m_s^0 = 95\ \mathrm{MeV}, m_g^0 = 0\ \mathrm{MeV}.$$

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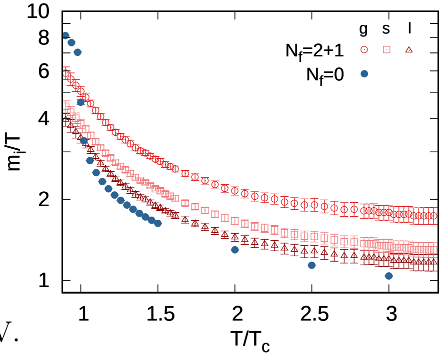

[V. M., M. Bluhm, K. Redlich, C. Sasaki, PRD 100 '19]

Quasiparticle Model: Thermodynamic Consistency

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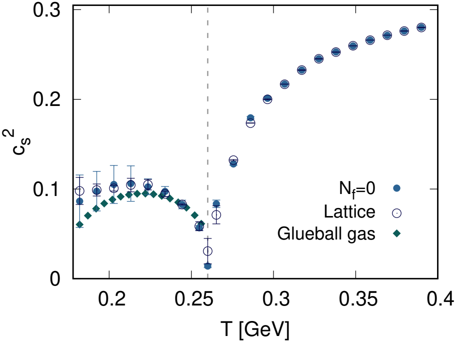
Pure Yang-Mills, $N_f = 0$



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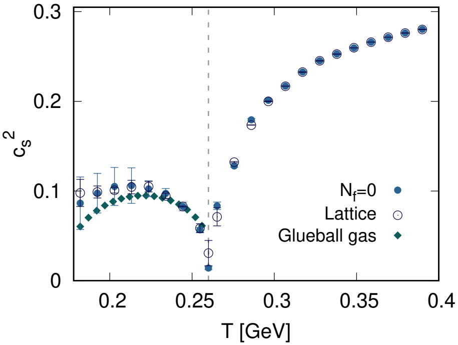

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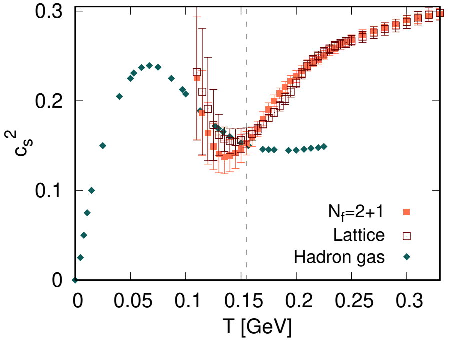
QCD, $N_f = 2 + 1$



[lQCD: Borsanyi et al., Phys. Lett. B730 '14;
Hadron resonance gas, m<2.5 GeV: Castorina et al., EPJ C66 '10]

[V. M., C. Sasaki, arxiv:2007.06846]

Kinetic Theory: Relaxation Time Approximation

Shear viscosity:

$$\eta = \frac{1}{15\,T} \sum_{i=l,\bar{l},s,\bar{s},g} d_i \int \frac{d^3p}{(2\pi)^3} \frac{p^4}{E_i^2} f_i^0 (1 \pm f_i^0) \tau_i$$

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Bulk viscosity:

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Electrical conductivity:

$$\sigma = \frac{1}{3T} \sum_{i=u,\bar{u},d,\bar{d},s,\bar{s}} q_i^2 \, d_i \int \frac{d^3p}{(2\pi)^3} \frac{p^2}{E_i^2} f_i^0 (1 - f_i^0) \tau_i$$

$*$ common relaxation time τ_i for all coefficients

[V. M., M. Bluhm, K. Redlich, C. Sasaki, PRD 100 '19; V. M., C. Sasaki, arxiv:2007.06846]

Relaxation Times: Pure Yang-Mills vs QCD

$$\tau_i^{-1} = n_i\,\bar{\sigma}_i = \int \frac{d^3p}{(2\pi)^3} d_i\, f_i^0 \times$$

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Gluons in pure Yang-Mills:

$$\tau_g^{-1}(G(T), m_g(T)) = n_g \bar{\sigma}_{gg\to gg}$$

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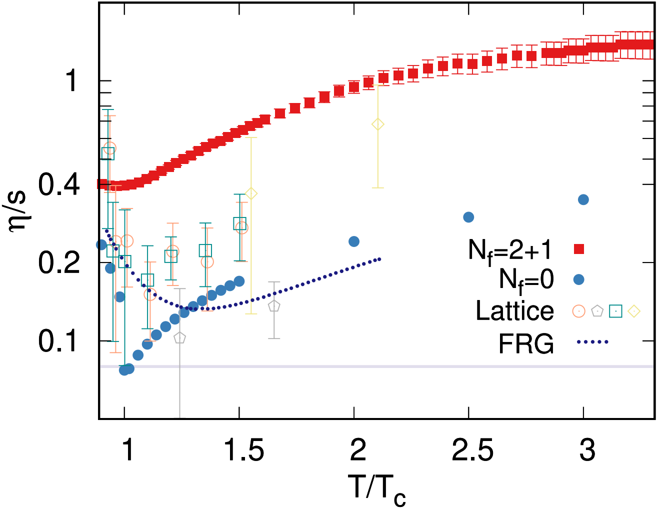
Gluons in QCD:

$$\tau_g^{-1}(G(T), m_{l,s,g}(T)) = n_g(\bar{\sigma}_{gg\to gg} + \bar{\sigma}_{gg\to l\bar{l}} + \bar{\sigma}_{gg\to s\bar{s}}) +$$

$$n_l \bar{\sigma}_{gl\to gl} + n_{\bar{l}} \bar{\sigma}_{g\bar{l}\to g\bar{l}} + n_s \bar{\sigma}_{gs\to gs} + n_{\bar{s}} \bar{\sigma}_{g\bar{s}\to g\bar{s}}$$

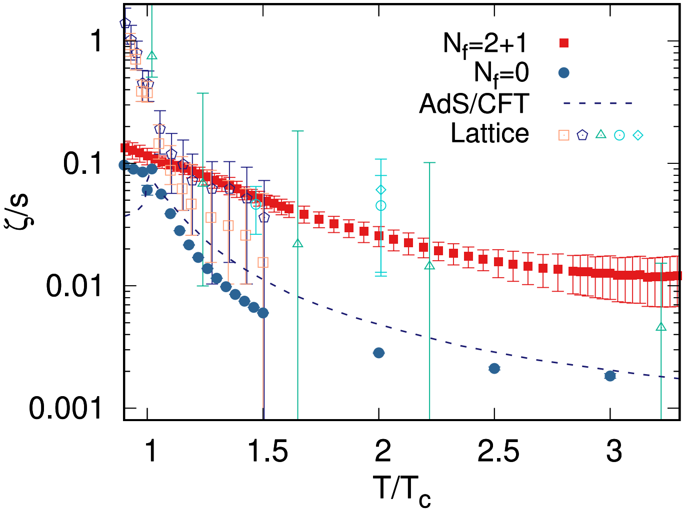
Shear Viscosity: $N_f = 0$ vs $N_f = 2 + 1$



[V. M., Bluhm, Redlich, Sasaki, PRD 100 '19]

[FRG: Christiansen et al., PRL 115 '15; lQCD: Nakamura, Sakai, PRL 94 '05; Meyer, PRD 76 '07; Astrakhantsev et al., JHEP 1704 '17]

Bulk Viscosity: $N_f = 0$ vs $N_f = 2+1$



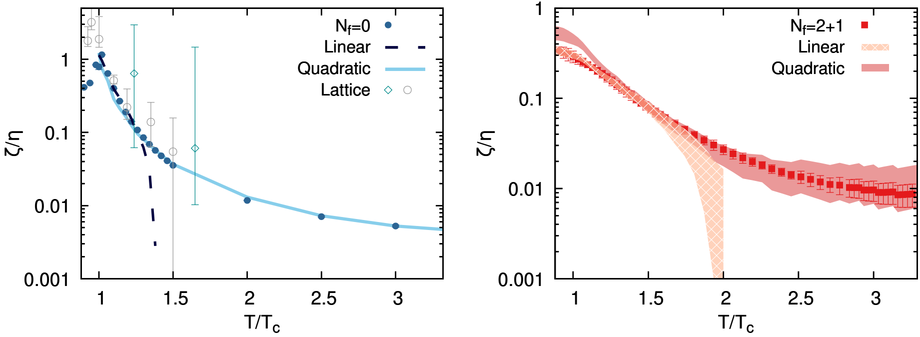
[V. M. and C. Sasaki, arxiv:2007.06846]

[AdS/CFT: Li et al., JHEP 06 '15; lQCD: Meyer, PRL 100 '08; Sakai, Nakamura, PoS LAT2007 '07; Astrakhantsev et al., JHEP 101 '17]

Non-perturbative vs Perturbative QCD Regimes

$$\text{Linear:} \quad \frac{\zeta}{\eta} \propto \left(\frac{1}{3} - c_s^2\right) \quad - \quad \text{AdS/CFT} \text{ [Buchel, PRD 72 '05]}$$

$$\text{Quadratic:} \quad \frac{\zeta}{\eta} \propto \left(\frac{1}{3} - c_s^2\right)^2 \quad - \quad \text{pQCD} \text{ [Weinberg, Astrophys. J. 168 '71]}$$



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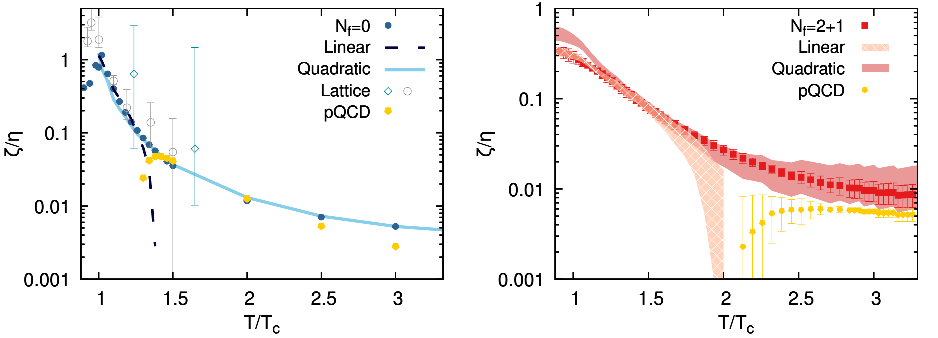
pQCD Next-to-Leading-Log Approximation

$$\eta_{\rm NLL} = \frac{T^3}{g^4}\frac{\eta_1}{\ln(\mu_1^*/m_D)}\,, \qquad \zeta_{\rm NLL} = \frac{Ag^4T^3}{16\pi^2\ln(\mu_2^*/m_D)}$$

$$m_D^2 = (1+N_f/6)g^2T^2,\; g\to G(T)$$

[Arnold, Moore, Yaffe, JHEP 05 ’03; Arnold, Dogan, Moore, PRD 74 ’06]



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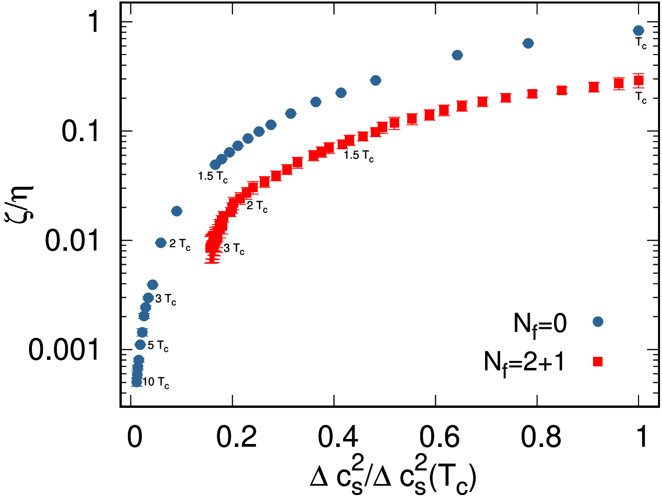
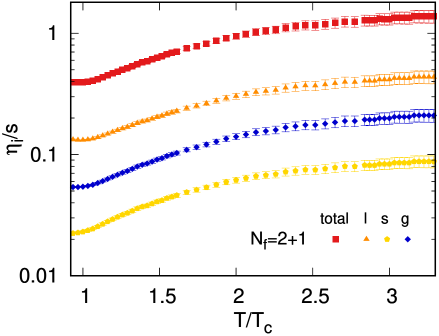
Conformal Limit: $N_f = 0$ vs $N_f = 2 + 1$

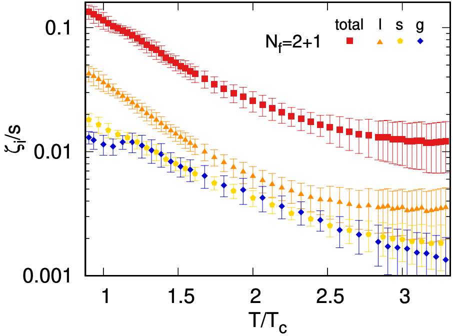


$\Delta c_s^2 = 1/3 - c_s^2$ – conformality measure

[V. M. and C. Sasaki, arxiv:2007.06846]

Individual Contributions to Shear and Bulk Viscosity





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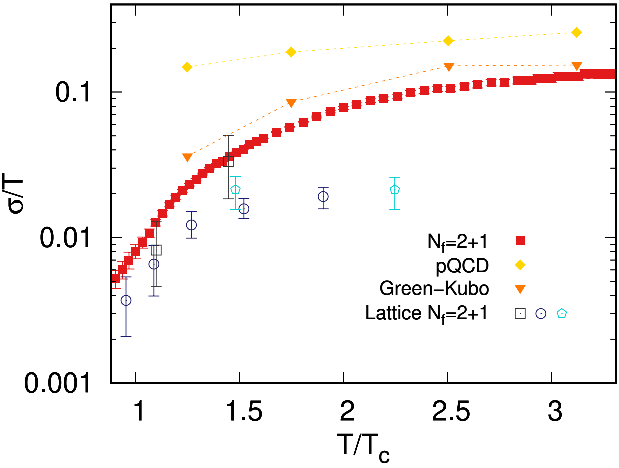
Electrical Conductivity: $N_f = 2 + 1$



[pQCD, Green-Kubo: Puglisi et al., PRD 90 '14; lQCD: Ding et al., PoS 185 '11; Amato et al., PRL 111 '13; Aarts et al., JHEP 02 '15]

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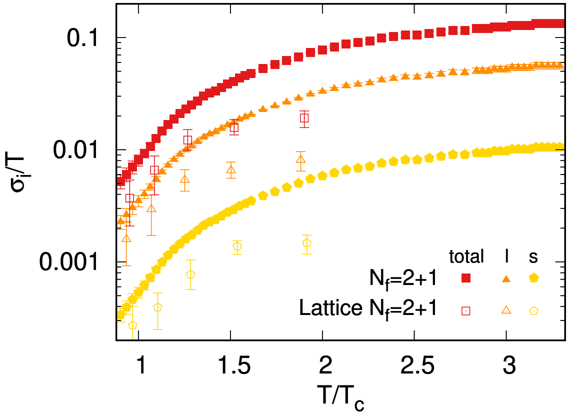

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lQCD: $M_\pi \approx 384$ MeV $\Longrightarrow$ larger quark masses

[V. M. and C. Sasaki, arxiv:2007.06846; lQCD: G. Aarts et al., JHEP 02 '15]

Summary. Quasiparticle Model:

- consistent with lattice EoS;
- accommodates perturbative and non-perturbative effects;
- pure Yang-Mills:
  - η/s, ζ/s exhibit non-trivial behavior around T_c;
  - η/s, ζ/s , ζ/η agree with first-principle calculations.
- QCD:
  - η/s, ζ/s, σ/T change smoothly with T;
  - ζ/η close to pQCD expansions at high T;
  - σ/T consistent with lQCD around T_c;
  - quasi-quarks increase values of transport parameters;
  - quasi-quarks delay restoration of conformal invariance

Perspective: $\tau_{\eta}, \zeta, \sigma$, $\mu \neq 0$, additional flavors, QPM for hadrons...