Quark-Flavor Dependence of Transport Parameters in Hot QCD: the Quasiparticle Perspective

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Talk based on: V. M., M. Bluhm, C. Sasaki, K. Redlich, PRD 100 '19 (arxiv:1906.01697);

V. M., C. Sasaki, arxiv:2007.06846.

Motivation

Transport properties of QGP: ; ; ::: input for hydro simulations

- Lattice QCD
- Perturbative QCD
- AdS/CFT
- Effective models
- Green-Kubo formalism
- Kinetic theory
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Goal: impact of quark quasiparticles on transport parameters in hot QCD: $N_f = 0$ vs $N_f = 2 + 1$ at $= 0$.

Quasiparticle Model: Thermodynamics & Coupling G(T)

$$s = \sum_{i=l;l;s;s;g} \frac{d_i}{2} \int dp\ 2p^2 \frac{\frac{4}{3}p^2 + m_i^2(T)}{E_i T} f_i^0$$

$$f_i^0 = (\exp(E_i{=}T) \quad 1)^{\ 1}$$

$$E_i(T) = \sqrt{p^2 + m_i^2(T)}$$

w/ $m_i(G(T); T)$

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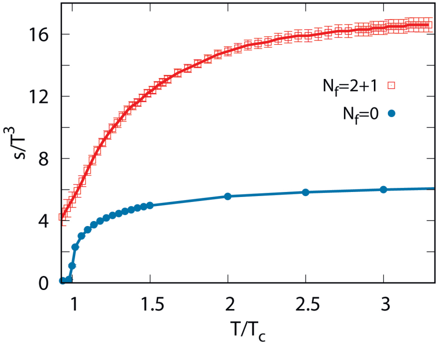

[lQCD: Wuppertal-Budapest Collaboration

Borsanyi et al., JHEP1207, 056 ’12; PLB730 ’14]

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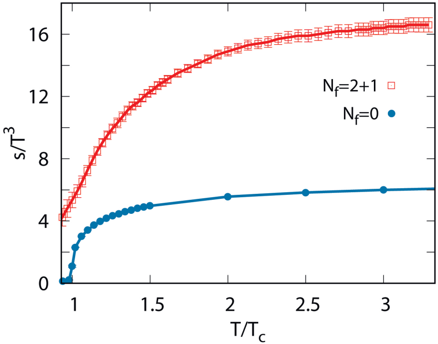

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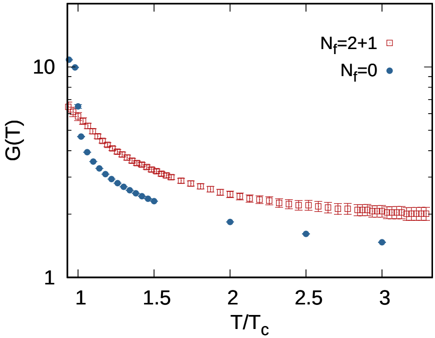

[V. M., M. Bluhm, K. Redlich, C. Sasaki, PRD 100 '19]

Quasiparticle Model: Effective Masses

Weakly-interacting particles with dynamical masses $m_i(G(T); T)$

$$m_i^2(T) = (m_i^0)^2 + \quad _i(T);$$

$$_{l;s}(T) = 2\, m^0_{l;s} \sqrt{\frac{G^2(T)T^2}{6}} + \frac{G^2T^2}{6};$$

$$_g(T) = \quad 3 + \frac{N_f}{2} \quad \frac{G^2(T)}{6} T^2;$$

$$m_l^0 = 5 \text{ MeV};\; m_s^0 = 95 \text{ MeV}; m_g^0 = 0 \text{ MeV}:$$

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[V. M., M. Bluhm, K. Redlich, C. Sasaki, PRD 100 ’19]

Quasiparticle Model: Thermodynamic Consistency

$$c_s^2 = \frac{\partial P}{\partial} = \frac{s}{T} \quad \frac{\partial s}{\partial T} \quad ^{1}$$

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Pure Yang-Mills, $N_f = 0$

[lQCD: Borsanyi et al., JHEP1207, 056 '12;
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[lQCD: Borsanyi et al., Phys. Lett. B730 ’14;
Hadron resonance gas, m<2.5 GeV: Castorina et al., EPJ C66 ’10]

[V. M., C. Sasaki, arxiv:2007.06846]

Kinetic Theory: Relaxation Time Approximation

Shear viscosity:

$$= \frac{1}{15T} \sum_{i=l;l;s;s;g} d_i \int \frac{d^3p}{(2\quad)^3}\frac{p^4}{E_i^2} f_i^0(1 \quad f_i^0) \quad_i$$

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Bulk viscosity:

$$\zeta = \frac{1}{T} \sum_{i=l,\bar{l},s,\bar{s},g} d_i \int \frac{d^3p}{(2\pi)^3} f_i^0 (1 \pm f_i^0) \frac{1}{E_i^2} \left\{ \left(E_i^2 - T^2 \frac{\partial m_i^2(T)}{\partial T^2} \right) \frac{\partial P}{\partial \varepsilon} - \frac{p^2}{3} \right\}^2 \tau_i$$

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Electrical conductivity:

$$\sigma_{el} = \frac{1}{3T} \sum_{i=u,\bar{u},d,\bar{d},s,\bar{s}} q_i^2\, d_i \int \frac{d^3p}{(2\pi)^3} \frac{p^2}{E_i^2} f_i^0 (1 \pm f_i^0)\, \tau_i$$

common relaxation time τ_i for all coefficients

[V. M., M. Bluhm, K. Redlich, C. Sasaki, PRD 100 '19; V. M., C. Sasaki, arxiv:2007.06846]

Relaxation Times: Pure Yang-Mills vs QCD

$$ {}_{i}{}^{1} = n_i \quad {}_{i} = \int \frac{d^3p}{(2\;)^3} d_i f_i^0 $$

Relaxation Times: Pure Yang-Mills vs QCD

$$\tau_i^{-1} = n_i \Gamma_i = \int \frac{d^3p}{(2\pi)^3} d_i f_i^0$$

$$\sum_{j,i',j'} \int ds \int dt \frac{d\sigma_{ij \to i'j'}}{dt} (1 \pm f_{i'}^0)(1 \pm f_{j'}^0) P(s;T) \sin^2\theta(s;t;m_{i,j,i',j'}(T))$$

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Gluons in pure Yang-Mills:

$$\tau_g^{-1}(G(T);m_g(T)) = n_g \sigma_{gg\to gg}$$

Relaxation Times: Pure Yang-Mills vs QCD

$$ {}_{i}^{\;1} = n_i \;\; {}_i = \int \frac{d^3p}{(2\;)^3} d_i\, f_i^{\,0} $$

$$ \sum_{j;i^0;j^0} \int ds \int dt\, \frac{d\;\; {}_{ij!\;\; i^0 j^0}}{dt} (1 \quad f_{i^0}^{\,0})(1 \quad f_{j^0}^{\,0}) P(s;T) \sin^2 \;\; (s;t;m_{i;j;i^0;j^0}(T)) $$

Gluons in pure Yang-Mills:

$$ {}_g^{\;1}(G(T); m_g(T)) = n_g \;\; {}_{gg!\;\; gg} $$

Gluons in QCD:

$$ {}_g^{\;1}(G(T); m_{l;s;g}(T)) = n_g(\;\; {}_{gg!\;\; gg} + \;\; {}_{gg!\;\; ll} + \;\; {}_{gg!\;\; ss}) + $$

$$ n_l \;\; {}_{gl!\;\; gl} + n_l \;\; {}_{gl!\;\; gl} + n_s \;\; {}_{gs!\;\; gs} + n_s \;\; {}_{gs!\;\; gs} $$

Shear Viscosity: $N_f = 0$ vs $N_f = 2 + 1$

[V. M., Bluhm, Redlich, Sasaki, PRD 100 '19]

[FRG: Christiansen et al., PRL 115 '15; lQCD: Nakamura, Sakai, PRL 94 '05; Meyer, PRD 76 '07; Astrakhantsev et al., JHEP 1704 '17

Bulk Viscosity: $N_f = 0$ vs $N_f = 2 + 1$

[V. M. and C. Sasaki, arxiv:2007.06846]

[AdS/CFT: Li et al., JHEP 06 '15; lQCD: Meyer, PRL 100 '08; Sakai, Nakamura, PoS LAT2007 '07; Astrakhantsev et al., JHEP 101 '17]

Non-perturbative vs Perturbative QCD Regimes

Linear: $- \; / \quad \frac{1}{3} \quad c_s^2$ AdS/CFT [Buchel, PRD 72 '05]

Quadratic: $- \; / \quad \frac{1}{3} \quad c_s^2 \quad {}^2$ pQCD [Weinberg, Astrophys. J. 168 '71]

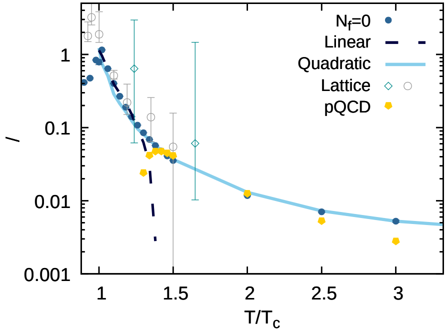
[V. M. and C. Sasaki, arxiv:2007.06846]

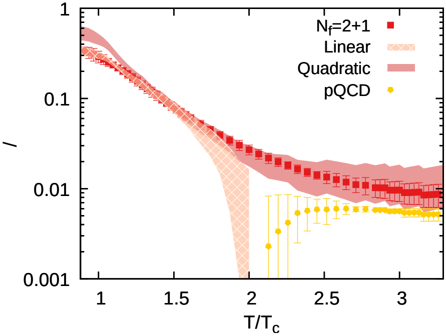
pQCD Next-to-Leading-Log Approximation

$$\text{NLL} = \frac{T^3}{g^4}\frac{1}{\ln(\ _1 = m_D)}\ ; \qquad \text{NLL} = \frac{A g^4 T^3}{16\ ^2 \ln(\ _2 = m_D)}$$

$$m_D^2 = (1 + N_f = 6) g^2 T^2,\ g\ \ !\ \ G(T)$$

[Arnold, Moore, Yaffe, JHEP 05 '03; Arnold, Dogan, Moore, PRD 74 '06]





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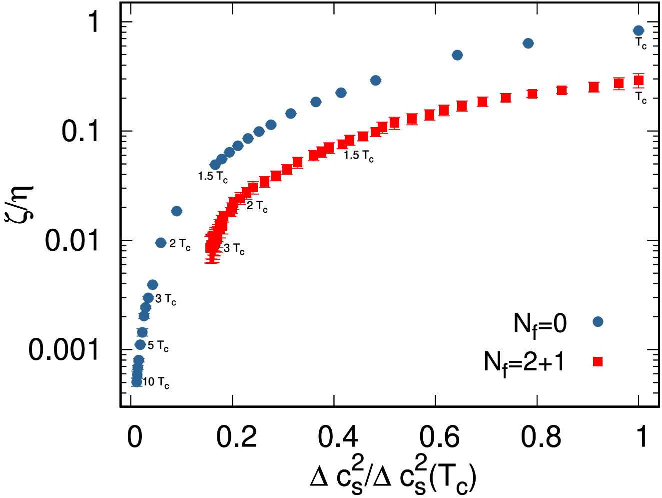
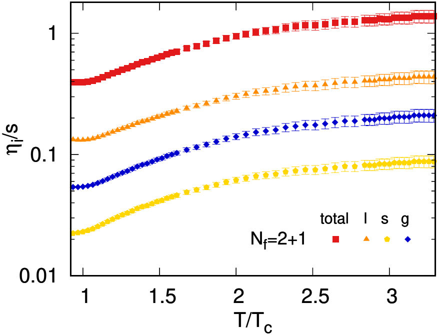
Conformal Limit: $N_f = 0$ vs $N_f = 2+1$

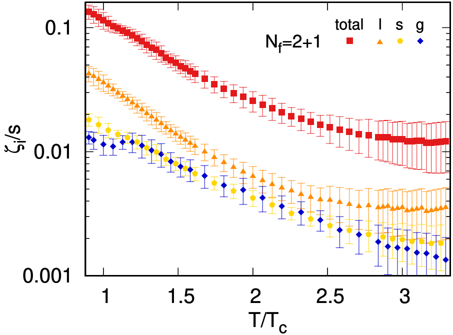


$c_s^2 = 1{=}3 \quad c_s^2 \quad$ conformality measure

[V. M. and C. Sasaki, arxiv:2007.06846]

Individual Contributions to Shear and Bulk Viscosity





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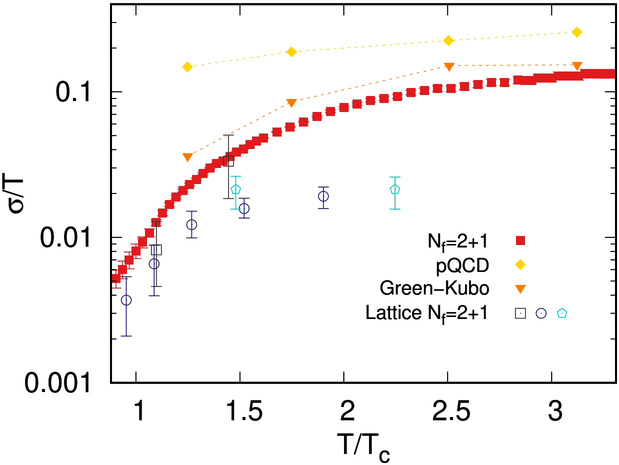
Electrical Conductivity: $N_f = 2 + 1$



[pQCD, Green-Kubo: Puglisi et al., PRD 90 '14; lQCD: Ding et al., PoS 185 '11; Amato et al., PRL 111 '13; Aarts et al., JHEP 02 '15]

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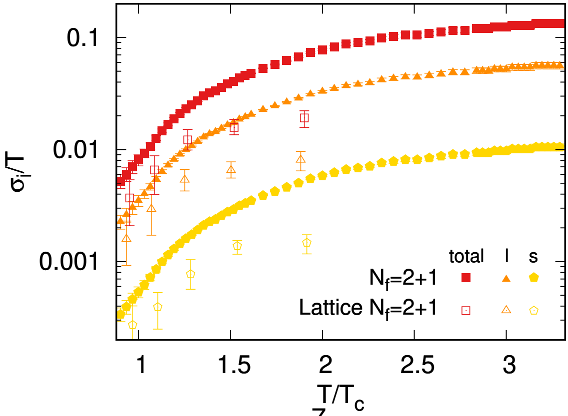

$$\sigma = \frac{1}{3T} \sum_{i=u,\bar{u},d,\bar{d},s,\bar{s}} q_i^2\, d_i \int \frac{d^3p}{(2\pi)^3} \frac{p^2}{E_i^2} f_i^0 (1 - f_i^0)\, \tau_i$$

lQCD: $M \approx 384$ MeV $\Rightarrow$ larger quark masses

[V. M. and C. Sasaki, arxiv:2007.06846; lQCD: G. Aarts et al., JHEP 02 '15]

Summary. Quasiparticle Model:

- consistent with lattice EoS;
- accommodates perturbative and non-perturbative effects;
- pure Yang-Mills:
  - $=s$, $=s$ exhibit non-trivial behavior around T_c;
  - $=s$, $=s$, $=$ agree with first-principle calculations.
- QCD:
  - $=s$, $=s;$ $=T$ change smoothly with T;
  - $=$ close to pQCD expansions at high T;
  - $=T$ consistent with lQCD around T_c;
  - quasi-quarks increase values of transport parameters;
  - quasi-quarks delay restoration of conformal invariance

Perspective: $;\ ;\ ;\ \neq 0$, additional flavors, QPM for hadrons...