QCD transition line from the lattice

+ comparison to the HRG model

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How can lattice QCD support the experiments?

- Equation of state
  - Needed for hydrodynamic description of the QGP
- QCD phase diagram
  - Transition line at finite density
  - Constraints on the location of the critical point
- Fluctuations of conserved charges
  - Can be simulated on the lattice and measured in experiments
  - Can give information on the evolution of heavy-ion collisions
  - Can give information on the critical point

QCD transition line

$$\frac{T_c(\mu_B)}{T_c(\mu_B=0)} = 1 - \kappa_2 \left(\frac{\mu_B}{T_c(\mu_B)}\right)^2 - \kappa_4 \left(\frac{\mu_B}{T_c(\mu_B)}\right)^4$$

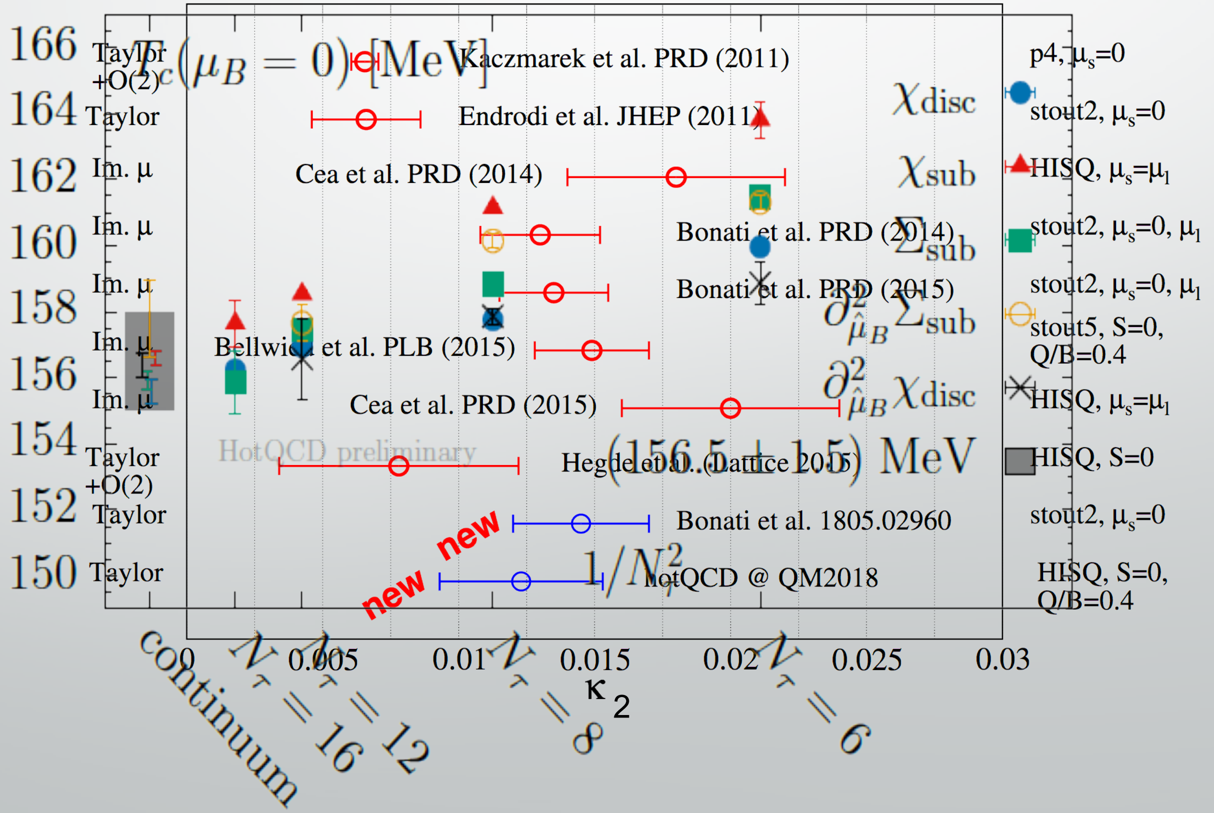
Collaborators: Szabolcs Borsanyi, Zoltan Fodor, Jana Guenther, Ruben Kara, Sandor Katz, Paolo Parotto, Attila Pasztor, Kalman Szabo

State of the art

- From direct simulations at μ_B=0:
  - $T_c(\mu_B=0)$=(156.5±1.5) MeV
  - K_2=0.012±0.004
  - K_4=0.000±0.004

HotQCD: PRL (2018)

Compilation by F. Negro



Observables

- We consider the following observables:

$$\langle\bar{\psi}\psi\rangle = -\left[\langle\bar{\psi}\psi\rangle_T - \langle\bar{\psi}\psi\rangle_0\right]\frac{m_{\rm ud}}{f_\pi^4}\,,$$

$$\chi = \left[\chi_T - \chi_0\right]\frac{m_{\rm ud}^2}{f_\pi^4}\,, \quad \text{with}$$

$$\langle\bar{\psi}\psi\rangle_{T,0} = \frac{T}{V}\frac{\partial \log Z}{\partial m_{\rm ud}} \quad \chi_{T,0} = \frac{T}{V}\frac{\partial^2 \log Z}{\partial m_{\rm ud}^2}$$

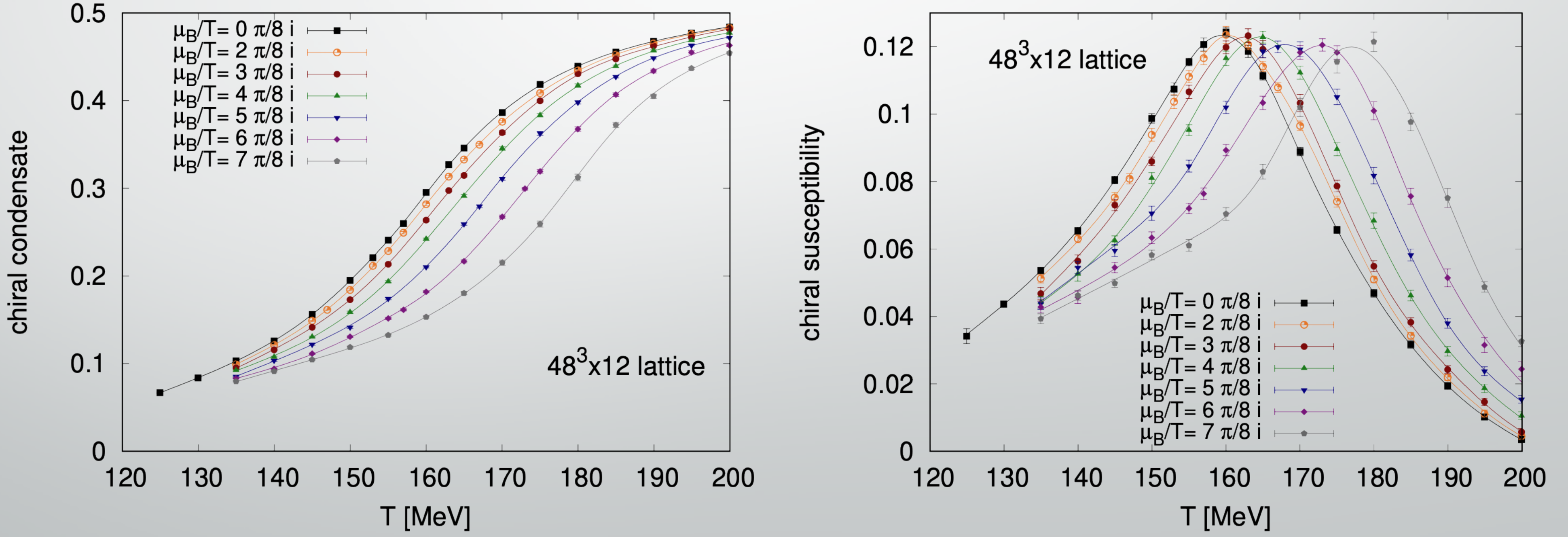
- The peak height of the susceptibility indicates the strength of the transition

- The peak position in temperature serves as a definition for the chiral cross-over temperature

Observables

- Plan:
  - Calculate these two observables at finite imaginary μ_B and finite temperature T
  - Use the shift of these observables as a function of imaginary μ_B to determine T_c, K_2 and K_4

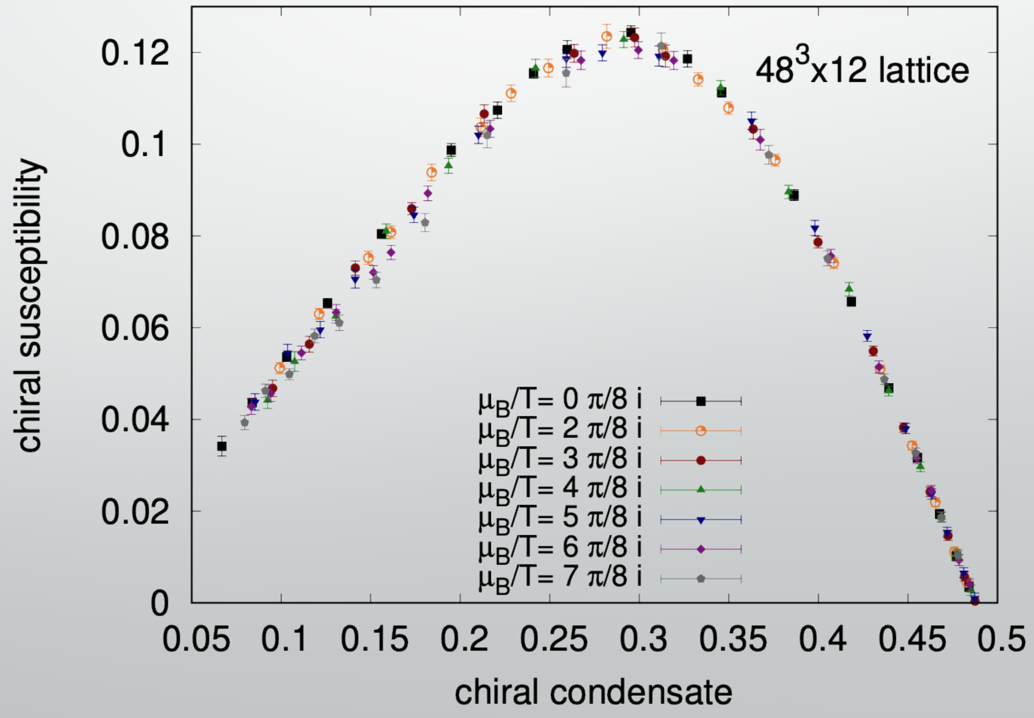
S. Borsanyi, C. R. et al., PRL (2020)



Observables

- Observation
  - When we plot the chiral susceptibility as a function of the chiral condensate, we observe a very weak chemical potential dependence

S. Borsanyi, C. R. et al., PRL (2020)

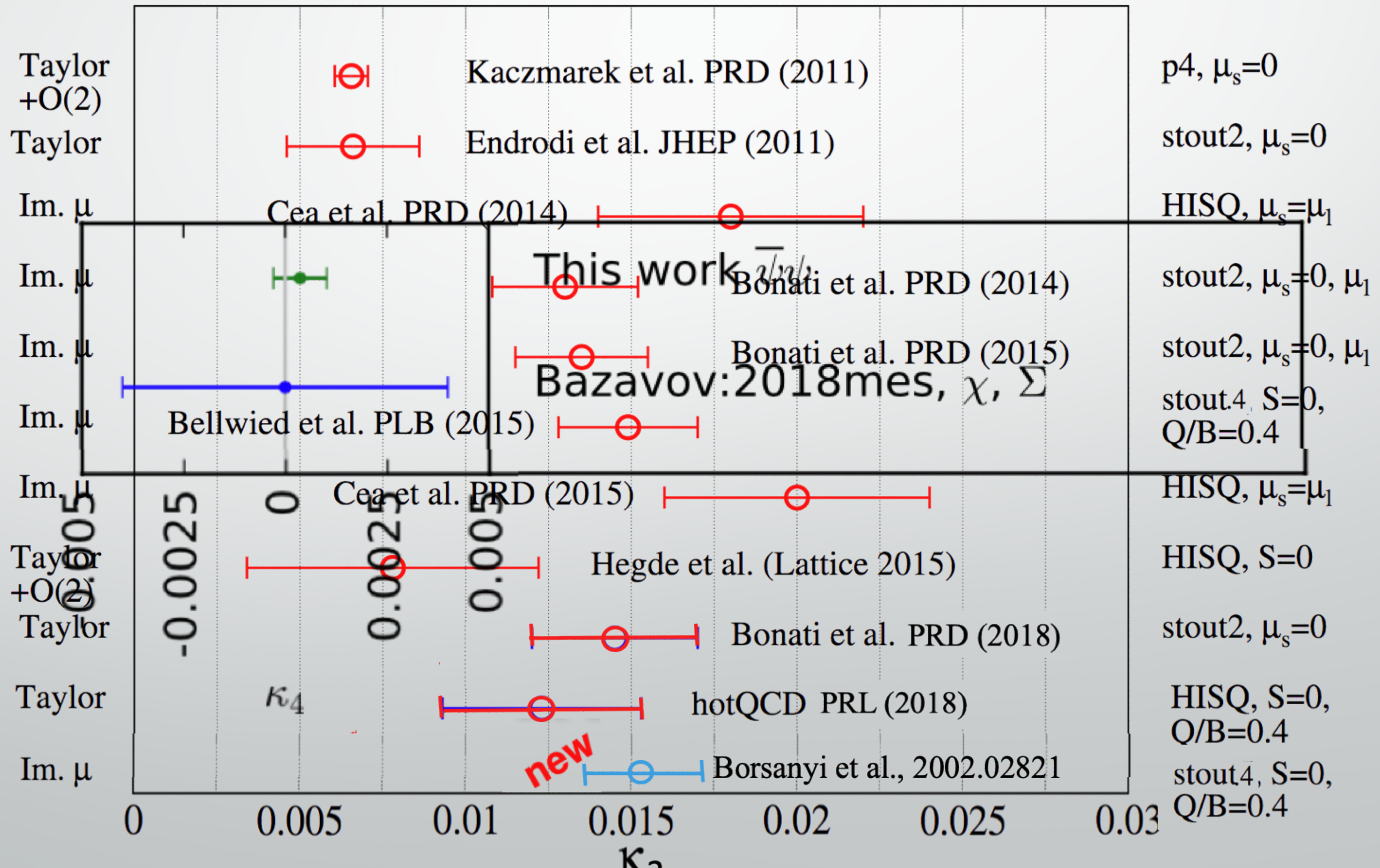


Procedure

- Find the peak in the curve $\chi(\langle\bar{\psi}\psi\rangle)$ through a low-order polynomial fit for each N_t and imaginary μ_B. This yields $\langle\bar{\psi}\psi\rangle_c$
- Use an interpolation of $\langle\bar{\psi}\psi\rangle(T)$ to convert $\langle\bar{\psi}\psi\rangle_c$ to T_c for each N_t and imaginary μ_B.
- Perform a fit of $T_c(N_t, \mathrm{Im}\mu_B/T_c)$ to determine the coefficients K_2 and K_4
- This leads to 2^8=256 independent analyses

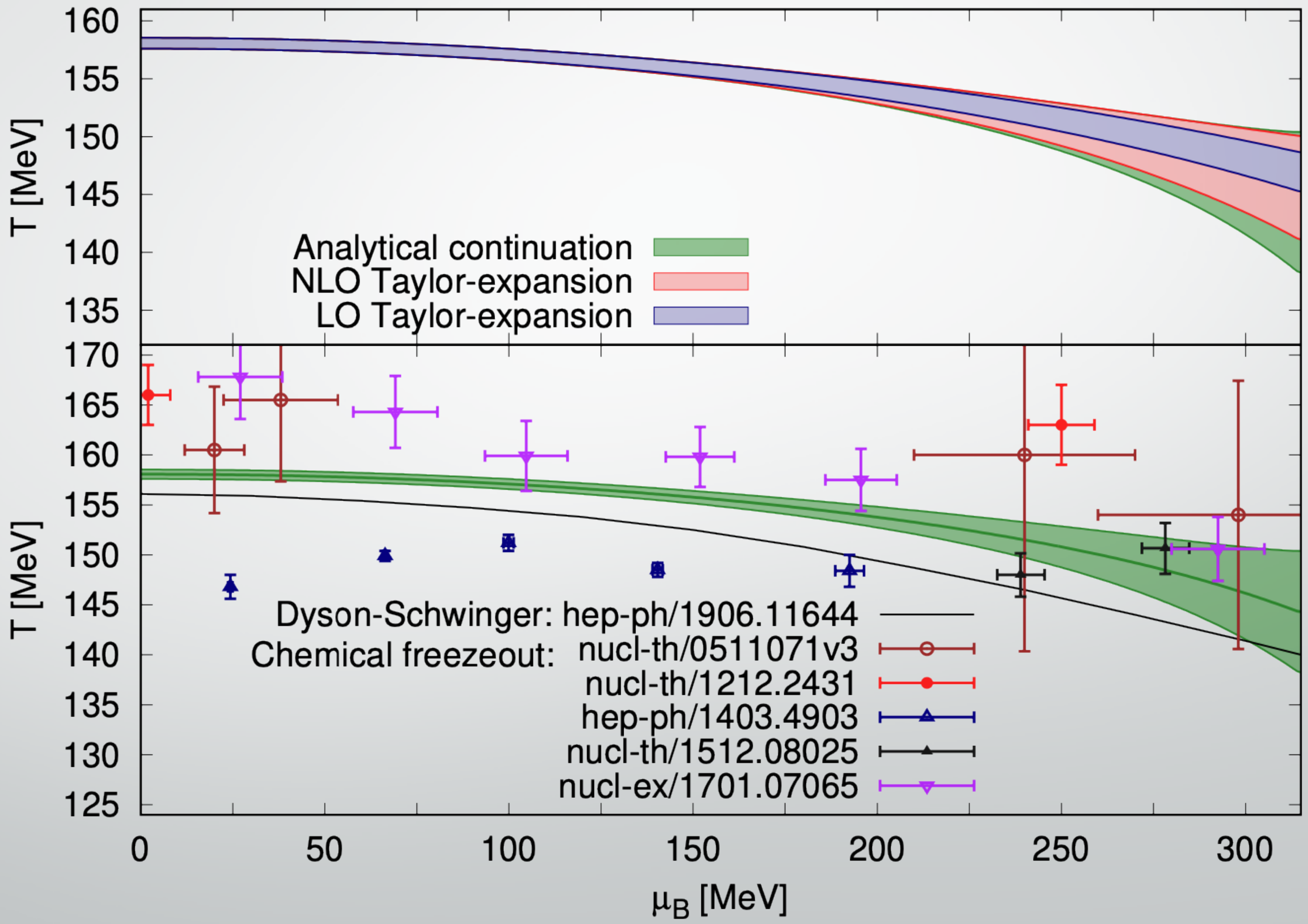
Results

$$T_c(LT=4,\mu_B=0) = 158.0 \pm 0.6 \text{ MeV}$$
$$\kappa_2 = 0.0153 \pm 0.0018\ ,$$
$$\kappa_4 = 0.00032 \pm 0.00067$$



Results

S. Borsanyi, C. R. et al., PRL (2020)



Width of the transition

- Natural definition: second derivative of the susceptibility at T_c

$$(\Delta T)^2 = -\chi(T_c)\left[\frac{d^2}{dT^2}\chi\right]^{-1}_{T=T_c}$$

- This turns out to be noisy, so we replace it by σ, a proxy for ΔT defined as:

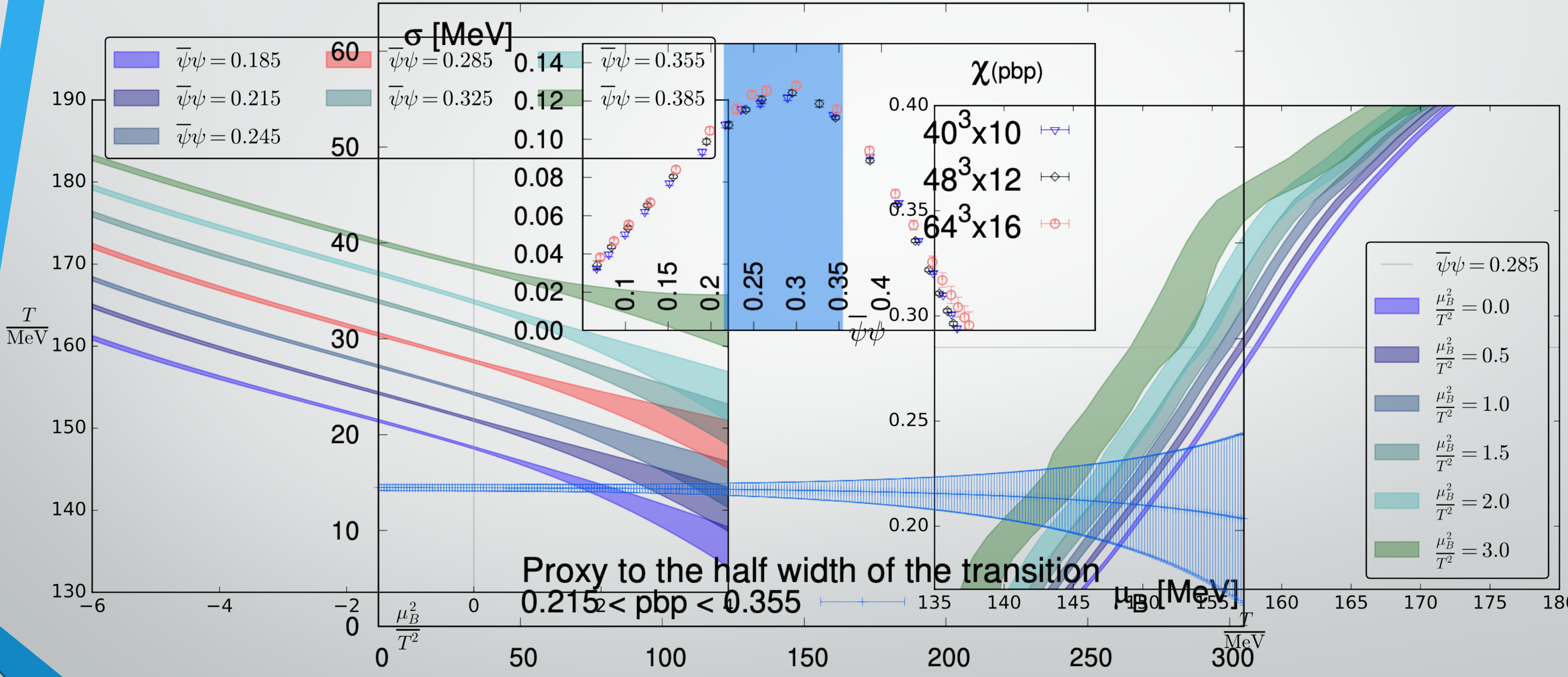
$$\langle\bar{\psi}\psi\rangle(T_c \pm \sigma/2) = \langle\bar{\psi}\psi\rangle_c \pm \Delta\langle\bar{\psi}\psi\rangle/2$$

with $\langle\bar{\psi}\psi\rangle_c = 0.285$ and $\Delta\langle\bar{\psi}\psi\rangle = 0.14$

- The exact range is chosen such that σ coincides with ΔT at zero and imaginary μ_B.

Width of the transition

S. Borsanyi, C. R. et al., PRL (2020)

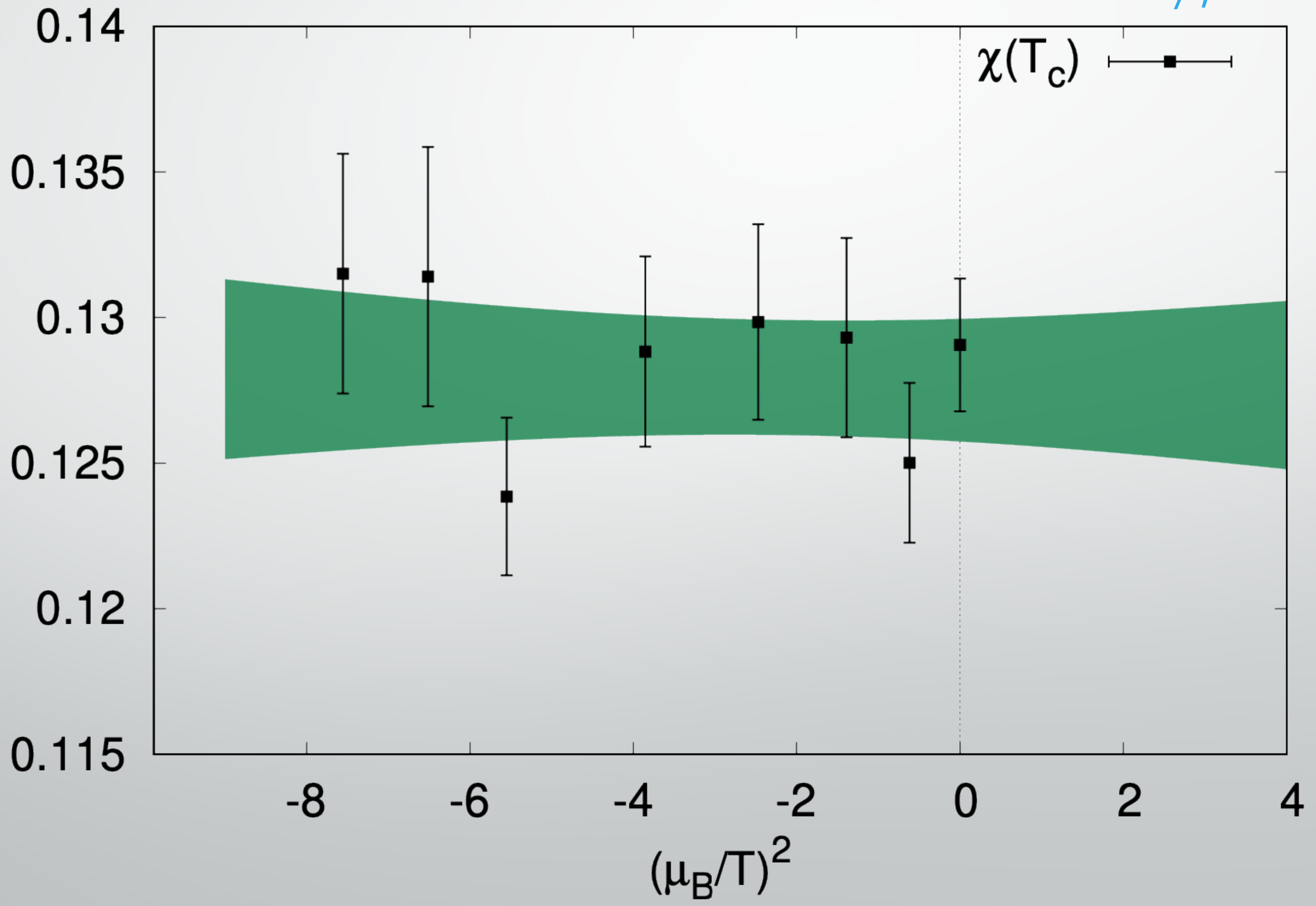


- The width of the transition is constant up to μ_B~300 MeV

Strength of the transition

- Height of the peak of the chiral susceptibility at the crossover temperature: proxy for the strength of the crossover

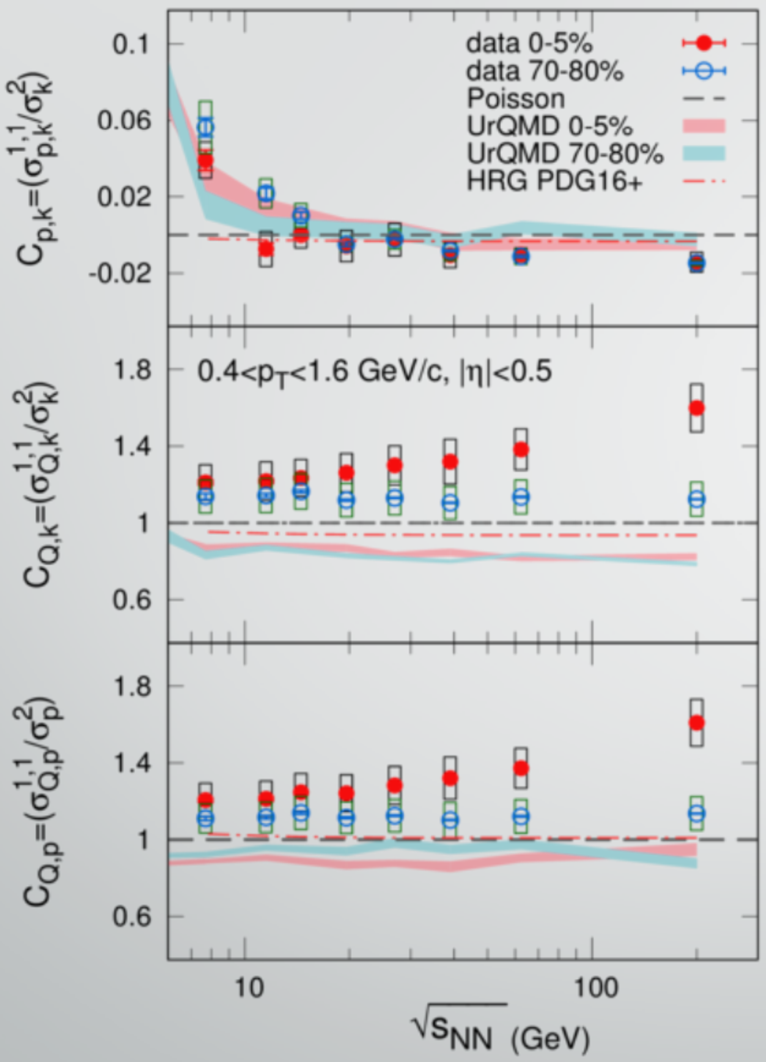
S. Borsanyi, C. R. et al., PRL (2020)



Comparison to the HRG model: off-diagonal correlators

Collaborators: Rene Bellwied, Szabolcs Borsanyi, Zoltan Fodor, Jana Guenther, Jacquelyn Noronha-Hostler, Paolo Parotto, Attila Pasztor, Claudia Ratti, Jamie M. Stafford

Off-diagonal fluctuations of conserved charges



STAR: Phys.Rev.C 100 014902 (2019)

- The measurable species in HIC are only a handful. How much do they tell us about the correlation between conserved charges?
- Historically, the proxies for B, Q and S have been p, p,π,K and K themselves → what about off-diagonal correlators?
- We want to find:
  - The main contributions to off-diagonal correlators
  - A way to compare lattice to experiment

Off-diagonal correlators: HRG model

Simple formulation, ideal gas of *all* hadronic resonances[1]. The pressure reads:

$$\frac{P\left(T,\mu_B,\mu_Q,\mu_S\right)}{T^4}=\sum_R \frac{(-1)^{B_R+1}\,d_R}{2\pi^2T^3}\int_0^\infty dp\,p^2\log\left[1+(-1)^{B_R+1}\exp\left(-\sqrt{p^2+m_R^2}/T+\mu_R/T\right)\right]$$

with:

$$\mu_R=\mu_B B_R+\mu_Q Q_R+\mu_S S_R$$

Susceptibilities in the HRG simply read:

$$\chi_{ijk}^{BQS}(T,\mu_B,\mu_Q,\mu_S)=\sum_R B_R^i\,Q_R^j\,S_R^k\,I_{i+j+k}^R(T,\mu_B,\mu_Q,\mu_S)$$

where:

$$I_{i+j+k}^R(T,\mu_B,\mu_Q,\mu_S)=\frac{\partial^{i+j+k}P_R/T^4}{\partial\left(\mu_R/T\right)^{i+j+k}}$$

[1] we use the list PDG2016+ from **P. Alba, PP *et al.*, Phys.Rev.D 96 034517 (2017)**

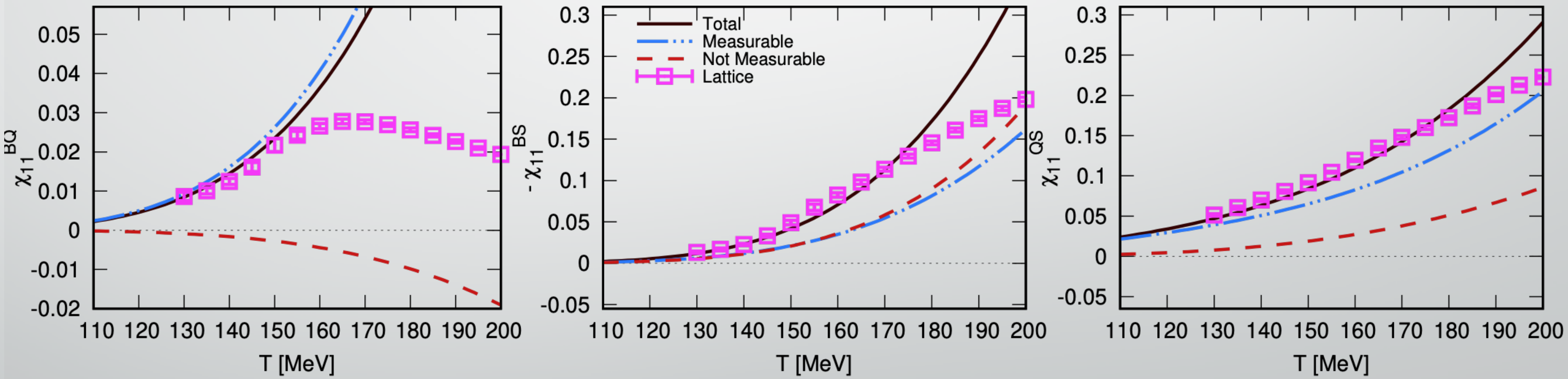
Off-diagonal correlators: HRG model

◇ The species that are stable under strong interactions, AND are **measurable**

$$\pi^{\pm},\ K^{\pm},\ p\,(\overline{p})\,,\ \Lambda(\overline{\Lambda}),\ \Xi^{-}(\overline{\Xi}^{+}),\ \Omega^{-}(\overline{\Omega}^{+})$$

⟶ we inevitably lose a good chunk of conserved charges!

- Thanks to the **separation between observable and non-observables species**, one can pinpoint what can be measured and what cannot of χ_{ijk}^{BQS}



See also PBM et al., PLB (2015)

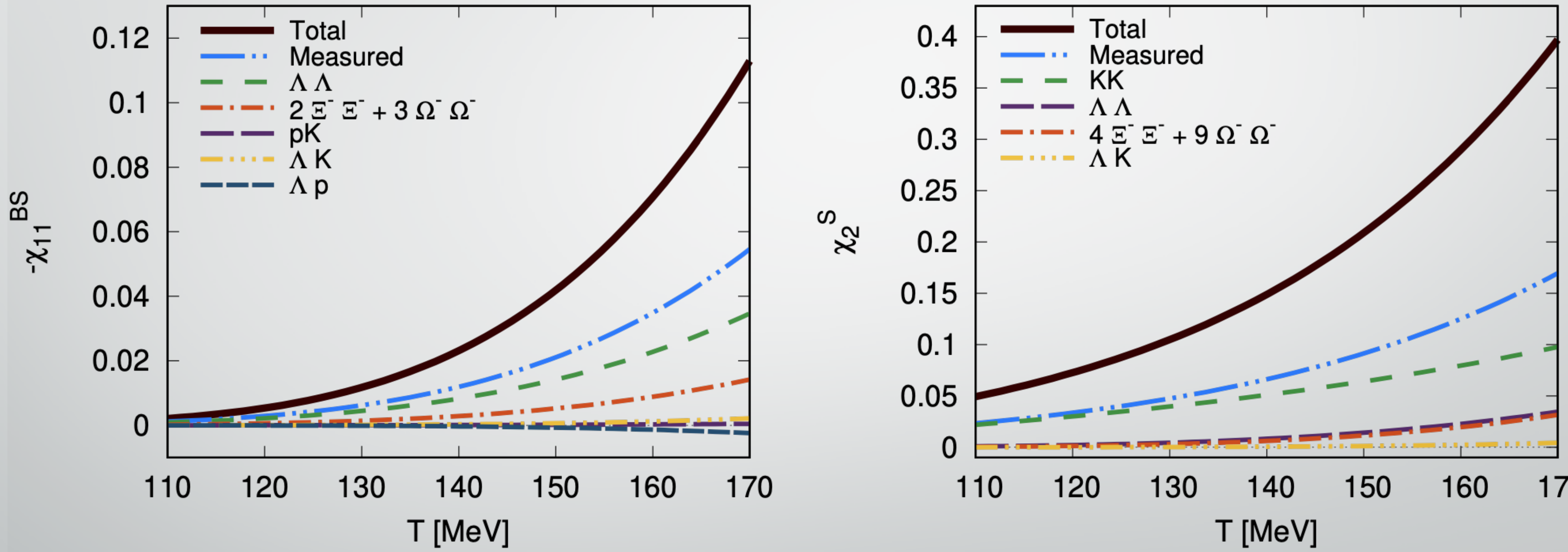
R. Bellwied, C. R. et al., PRD (2020)

- For the **proton- and kaon-dominated** χ_{BQ} **and** χ_{QS}, a large part of the full correlator is carried by measurable particles
- χ_{BS} is less transparent, and requires careful analysis of its contributions

Measurable contribution breakdown

- Each 2-particle correlation can be isolated in the HRG model
- In light of studying the ratio χ_{11}^{BS}/χ_2^S, we consider χ_{11}^{BS} and χ_2^S

R. Bellwied, C. R. et al., PRD (2020)

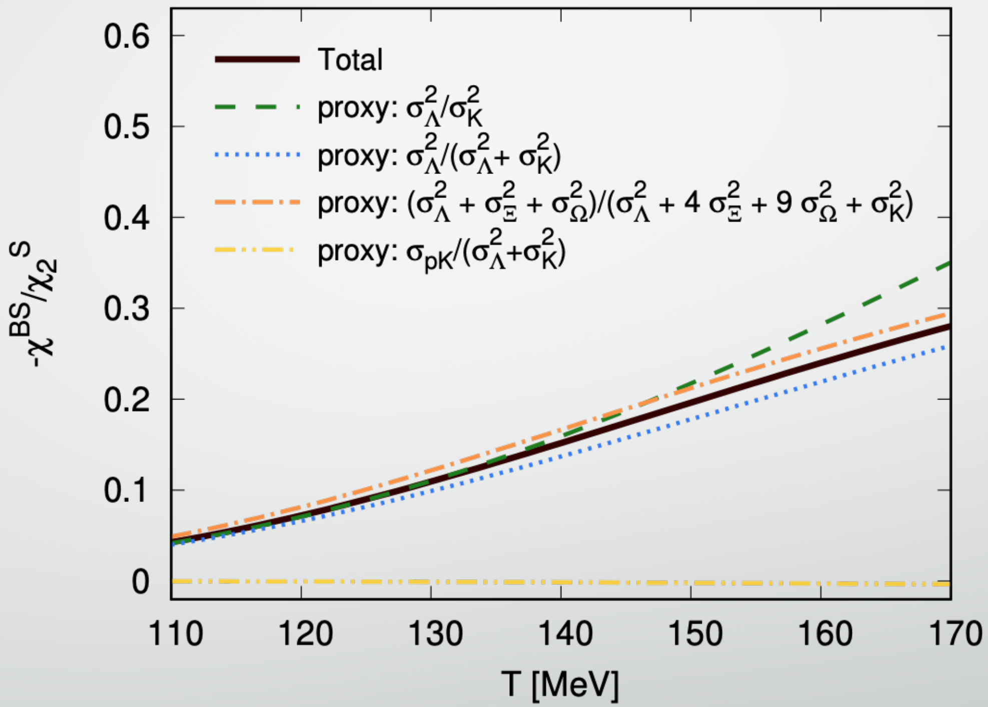


- Different-particle correlations are negligible throughout, while the contribution from multi-strange baryons is sizable

Hadronic proxies

Constructing a proxy not a trivial task: consider main contributions to numerator and denominator

R. Bellwied, C. R. et al., PRD (2020)

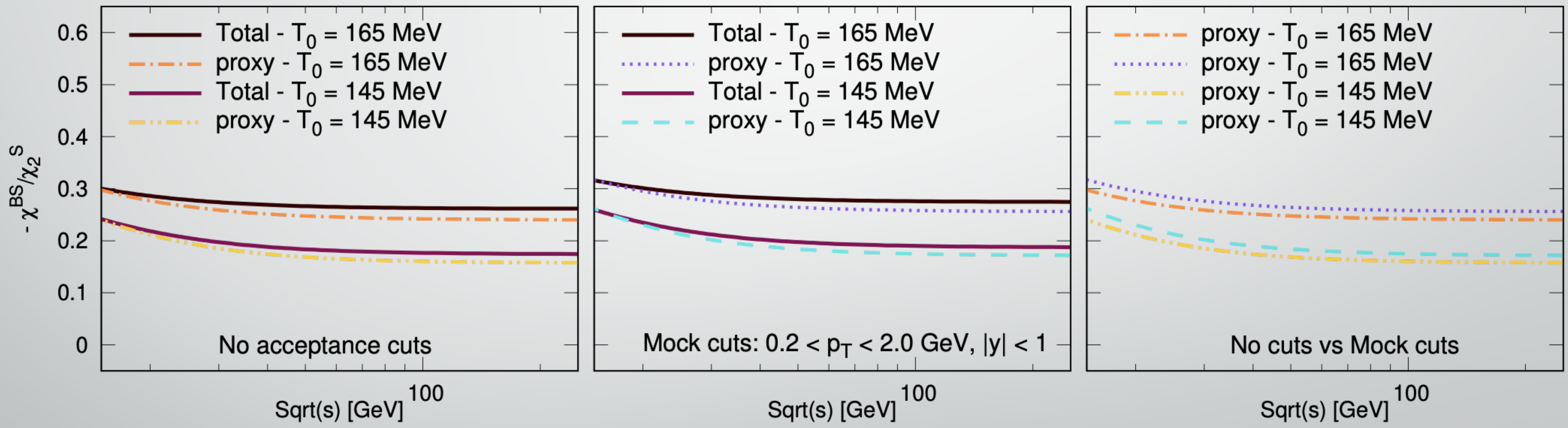


- Good proxy for χ^{BS}_{11}/χ^S_2: $\widetilde{C}^{\Lambda,\Lambda K}_{BS,SS} = \sigma^2_\Lambda/(\sigma^2_K+\sigma^2_\Lambda)$

Hadronic proxies: finite μ_B and kinematic cuts

- Consider our proxy along parametrized freeze-out lines with different $T(\mu_B = 0)$
- We look the ratio χ_{11}^{BS}/χ_2^S, in the case:
  - With no acceptance cuts
  - With "mock" cuts: $0.2 \leq p_T \leq 2.0\,\mathrm{GeV},\ |y| \leq 1.0$

R. Bellwied, C. R. et al., PRD (2020)



- The proxy works well at finite μ_B, and **the effect of cuts is minimal!**

Note: when taking these ratios, **the same cuts** were applied to all species involved

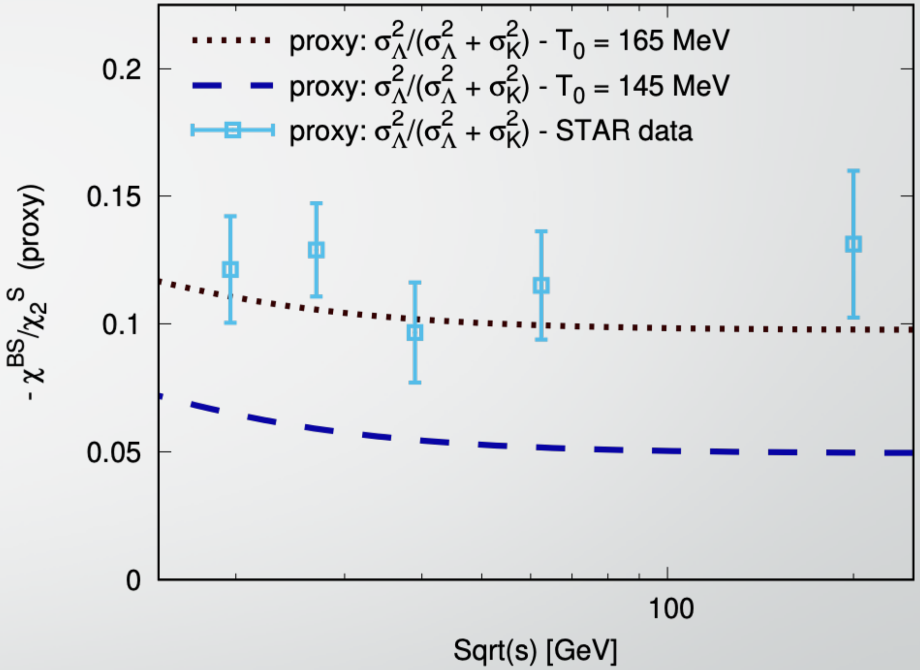
Hadronic proxies: a comparison to experiment

R. Bellwied, C. R. et al., PRD (2020)

- Compare to STAR data with the same cuts as in the experiment:

$$\Lambda: \quad 0.9 < p_T < 2.0\,\text{GeV} \quad |y| < 0.5$$
$$K: \quad 0.4 < p_T < 1.6\,\text{GeV} \quad |y| < 0.5$$

- A comparison along the same freeze-out lines as before shows a preferred $T(\mu_B = 0) \sim 165\,\text{MeV}$
- **Note:** a factor ~ 3 separates the case with same and different cuts! (see previous slide)



STAR data: PLB (2018), 2001.06419

Crucial to have same cuts if comparing with lattice results

Conclusions

- We obtained the most accurate results for the QCD transition line so far
- The curvature at μ_B=0 is very small. Its NLO correction is compatible with zero
- The width of the phase transition remains constant up to μ_B~300 MeV
- The strength of the phase transition remains constant up to μ_B~300 MeV
- We see no sign of criticality in the explored range
- We found good proxies for off-diagonal correlators
- Their dependence on kinematic cuts is mild

Backup slides

Width of the transition

- Natural definition: second derivative of the susceptibility at T_c

$$(\Delta T)^2 = -\chi(T_c)\left[\frac{d^2}{dT^2}\chi\right]^{-1}_{T=T_c}$$

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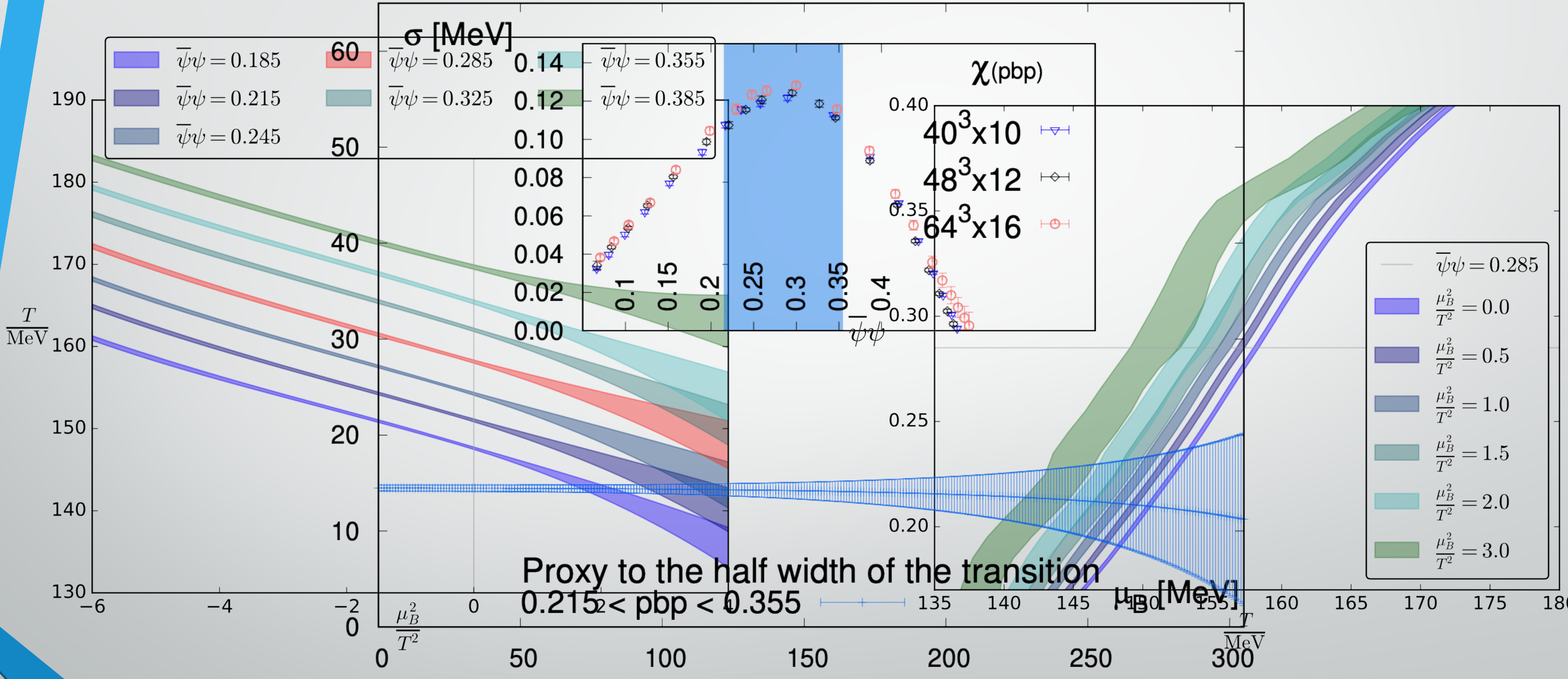
- The exact range is chosen such that σ coincides with ΔT at zero and imaginary μ_B.

Width of the transition

S. Borsanyi et al., 2002.02821



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Strength of the transition

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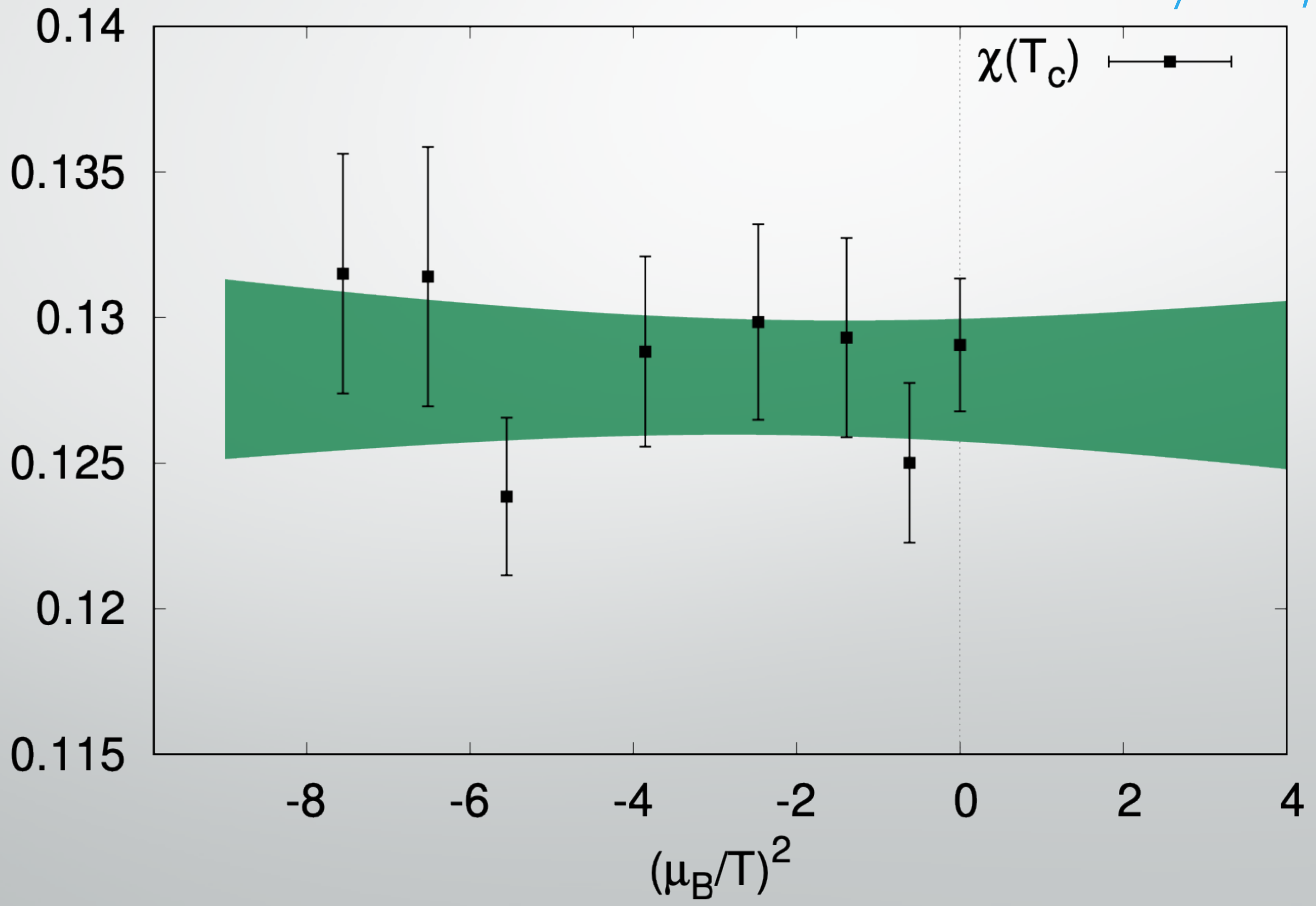
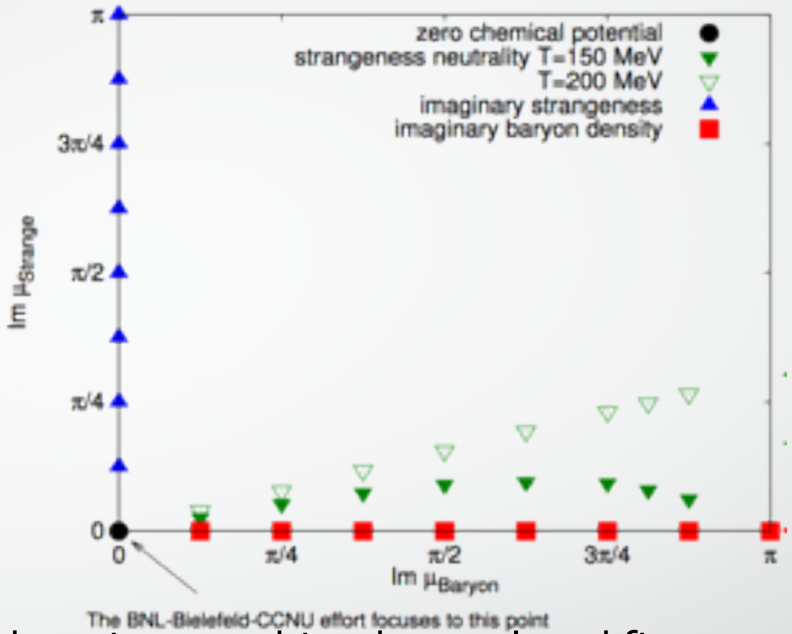


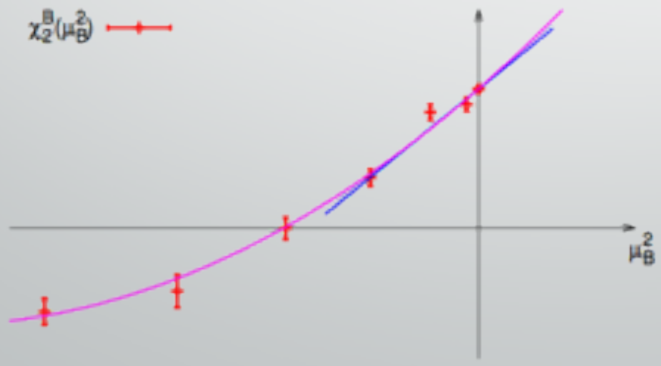
Pressure coefficients: simulations at imaginary μ_B

Simulations at imaginary μ_B:

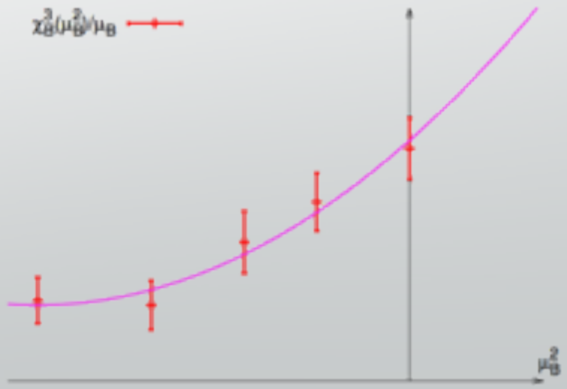


Common technique: [de Forcrand, Philipsen (2002)], [D'Elia and Lombardo, (2002)], [Bonati et al., (2015), (2018)], [Cea et al., (2015)]

Strategy: simulate lower-order fluctuations and use them in a combined, correlated fit



$$\chi_2^B(\mu_B^2) \approx \chi_2^B(0) + \frac{1}{2}\mu_B^2\chi_4^B(0) + \frac{1}{24}\mu_B^4\chi_6^B(0) + \dots$$

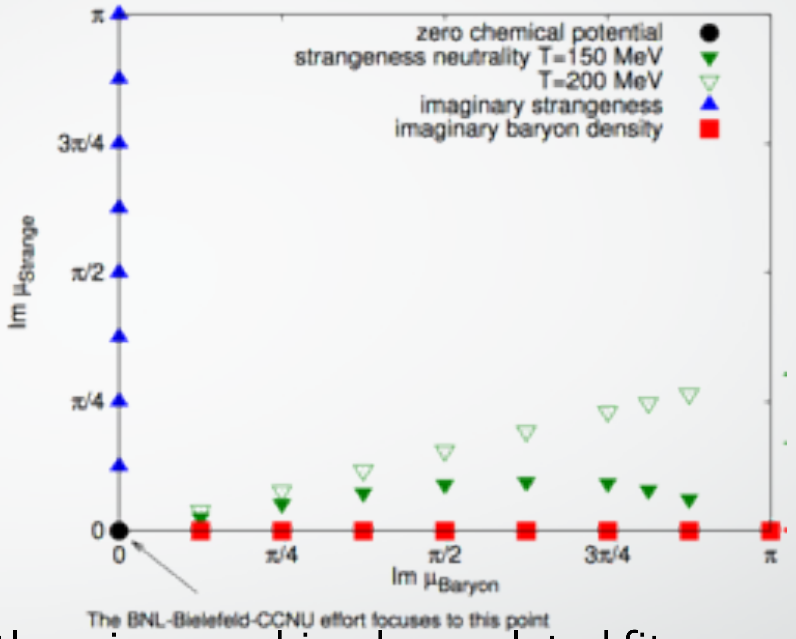


$$\frac{\chi_3^B(\mu_B^2)}{\mu_B} \approx \chi_4^B(0) + \frac{1}{6}\mu_B^2\chi_6^B(0) + \frac{1}{120}\mu_B^4\chi_8^B(0)$$

See also M. D'Elia et al., PRD (2017)

Pressure coefficients: simulations at imaginary μ_B

Simulations at imaginary μ_B:



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Strategy: simulate lower-order fluctuations and use them in a combined, correlated fit

$$\chi_1^B(\hat\mu_B) = 2c_2\hat\mu_B + 4c_4\hat\mu_B^3 + 6c_6\hat\mu_B^5 + \frac{4!}{7!}c_4\epsilon_1\hat\mu_B^7 + \frac{4!}{9!}c_4\epsilon_2\hat\mu_B^9$$

$$\chi_2^B(\hat\mu_B) = 2c_2 + 12c_4\hat\mu_B^2 + 30c_6\hat\mu_B^4 + \frac{4!}{6!}c_4\epsilon_1\hat\mu_B^6 + \frac{4!}{8!}c_4\epsilon_2\hat\mu_B^8$$

$$\chi_3^B(\hat\mu_B) = 24c_4\hat\mu_B + 120c_6\hat\mu_B^3 + \frac{4!}{5!}c_4\epsilon_1\hat\mu_B^5 + \frac{4!}{7!}c_4\epsilon_2\hat\mu_B^7$$

$$\chi_4^B(\hat\mu_B) = 24c_4 + 360c_6\hat\mu_B^2 + c_4\epsilon_1\hat\mu_B^4 + \frac{4!}{6!}c_4\epsilon_2\hat\mu_B^6.$$

See also M. D'Elia et al., PRD (2017)

Merging with HRG model at low T

$\Rightarrow$ **Smooth merging with Hadron Resonance Gas (HRG) model through:**

$$\frac{P_{\text{Final}}(T,\mu_B)}{T^4} = \frac{P(T,\mu_B)}{T^4}\frac{1}{2}\left[1+\tanh\left(\frac{T-T'(\mu_B)}{\Delta T}\right)\right] + \frac{P_{\text{HRG}}(T,\mu_B)}{T^4}\frac{1}{2}\left[1-\tanh\left(\frac{T-T'(\mu_B)}{\Delta T}\right)\right]$$

where:

- $T'(\mu_B)$ is the "transition" temperature, depending on μ_B:

$$T'(\mu_B) = T_0 + \frac{\kappa}{T_0}\mu_B^2 - T^*$$

- ΔT is a measure of the overlap region size

$\Rightarrow$ **In the following:** $T^* = 23\,\text{MeV}$, $\Delta T = 17\,\text{MeV}$