Chiral phase transition of (2+1)-flavor QCD

Anirban Lahiri (Bielefeld University)
for
HotQCD collaboration

BMBF project

ALICE Germany

Overview

UNIVERSITÄT BIELEFELD
Faculty of Physics

Overview

UNIVERSITÄT BIELEFELD
Faculty of Physics

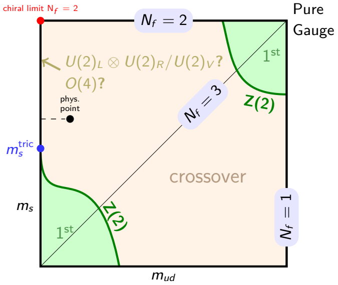
Introduction?

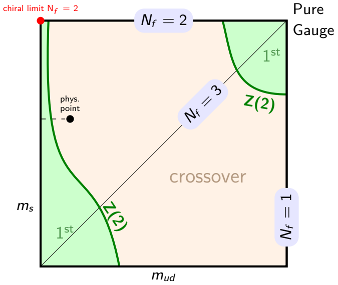
- In QCD, transition for physical quark masses and vanishing or small baryon density is a crossover.
- T_{pc} for physical mass QCD has been determined with very good accuracy from various observables : $T_{\mathrm{pc}}(\mu_B = 0) = 156.5 \pm 1.5$ MeV [HotQCD; PLB 795 15 (2019)].
- Important question : How do singular terms of the chiral phase transition at vanishing quark mass affect observables at physical values of the quark mass? $\Rightarrow$ How far the physical regime is from chiral limit w.r.t. scaling window?
- Key question : What is the chiral transition temperature, T_c^0?

UNIVERSITÄT BIELEFELD
Faculty of Physics

Introduction?

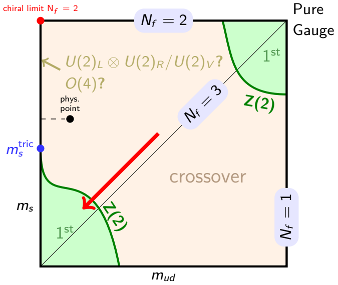
- Key question : What is the chiral transition temperature, T_c^0?
- Possibly another question : What is the nature of the thermal phase transition in the chiral limit?
- Two possible scenarios : [O. Philipsen and C. Pinke. Phys. Rev. D93, 114507, 2016.]

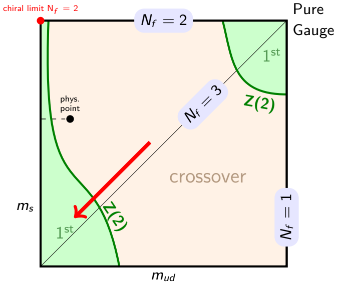




Introduction?

- Key question : What is the chiral transition temperature, T_c^0?
- Possibly another question : What is the nature of the thermal phase transition in the chiral limit?
- Two possible scenarios : [O. Philipsen and C. Pinke. Phys. Rev. D93, 114507, 2016.]

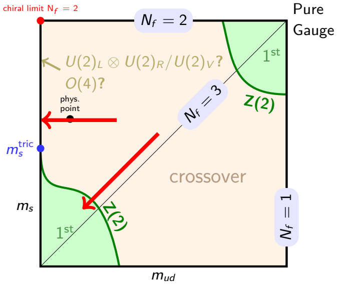


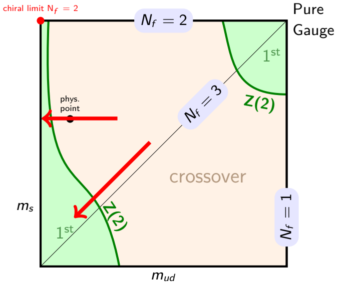


- $N_f = 3$: No direct evidence of 1^{st} order transition down to $m_\pi = 80$ MeV. Scaling argument pushes it further to $m_\pi = 50$ MeV. A. Bazavov *et. al.* Phys. Rev. D95, 074505 (2017).

Introduction?

- Key question : What is the chiral transition temperature, T_c^0?
- Possibly another question : What is the nature of the thermal phase transition in the chiral limit?
- Two possible scenarios : [O. Philipsen and C. Pinke. Phys. Rev. D93, 114507, 2016.]





- $N_f = 3$: No direct evidence of 1^{st} order transition down to $m_\pi = 80$ MeV. Scaling argument pushes it further to $m_\pi = 50$ MeV. A. Bazavov *et. al.* Phys. Rev. D95, 074505 (2017).

UNIVERSITÄT BIELEFELD
Faculty of Physics

Overview

UNIVERSITÄT BIELEFELD
Faculty of Physics

Basic quantities

In terms of temperature T and symmetry breaking field $H = m_l/m_s$ the scaling variables are defined as :

$$t = \frac{1}{t_0}\frac{T - T_c^0}{T_c^0} \qquad \text{and} \qquad h = \frac{1}{h_0}\frac{m_l}{m_s} = \frac{1}{h_0}H$$

Scaling variable :

$$z = \frac{t}{h^{\frac{1}{\beta\delta}}} = z_0 \left(\frac{T - T_c^0}{T_c^0}\right)\left(\frac{1}{H^{1/\beta\delta}}\right); \quad z_0 = \frac{h_0^{\frac{1}{\beta\delta}}}{t_0}$$

Chiral condensate : $\langle\bar{\psi}\psi\rangle_f = \frac{T}{V}\frac{\partial \ln Z}{\partial m_f}$

Chiral susceptibility : $\chi_m^{fg} = \frac{\partial}{\partial m_g}\langle\bar{\psi}\psi\rangle_f$

UNIVERSITÄT BIELEFELD
Faculty of Physics

Scaling relations

Renormalization group invariant (RGI) definition of order parameter :

$$M = \frac{m_s}{f_K^4}\left((\langle\bar{\psi}\psi\rangle_u + \langle\bar{\psi}\psi\rangle_d) - \frac{m_u + m_d}{m_s}\langle\bar{\psi}\psi\rangle_s \right) \equiv \frac{\Sigma_{\mathsf{sub}}}{f_K^4}$$

RGI definition of order parameter susceptibility :

$$\chi_M = \frac{T}{V} m_s \left(\frac{\partial}{\partial m_u} + \frac{\partial}{\partial m_d} \right) M$$

Close to chiral limit, singular part behaves as :

$$\begin{aligned} M &= h^{1/\delta} f_G(z) \\ \chi_M &= \frac{1}{h_0} h^{1/\delta - 1} f_\chi(z) \end{aligned}$$

$f_G(z)$ and $f_\chi(z)$ are universal scaling functions which have been precisely determined from various spin models.

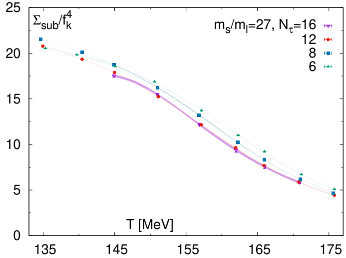
UNIVERSITÄT BIELEFELD
Faculty of Physics

Different estimators for pseudo-critical temperature

- For finite chemical potential : $t = \frac{1}{t_0}\left(\frac{T - T_c^0}{T_c^0} + \kappa_B \left(\frac{\mu_B}{T}\right)^2\right)$
- In scaling regime : $\frac{\partial}{\partial T} \sim \frac{\partial^2}{\partial \mu_B^2}$
- Different estimators for T_{pc} :
  1. Peak of chiral susceptibility : χ_M.
  2. Inflection point of chiral condensate : $\frac{\partial}{\partial T} M$.
  3. Minimum of $\frac{\partial^2}{\partial \mu_B^2} M$.
- At finite mass, different estimators of pseudo-critical temperature, in principle will give different results.
- In chiral limit, all pseudo-critical temperatures should merge to the chiral critical temperature, T_c^0.

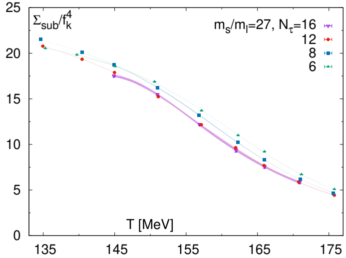
UNIVERSITÄT BIELEFELD
Faculty of Physics

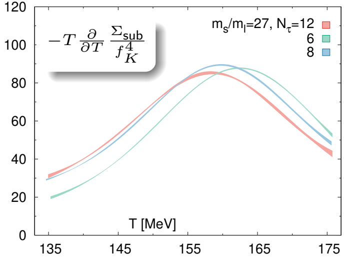
Scaling at physical Pion mass?



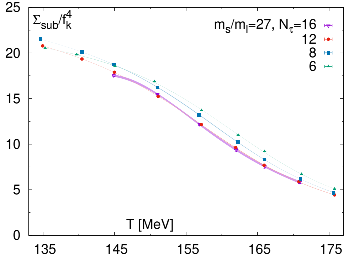
UNIVERSITÄT BIELEFELD
Faculty of Physics

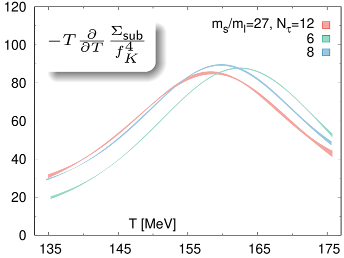
Scaling at physical Pion mass?

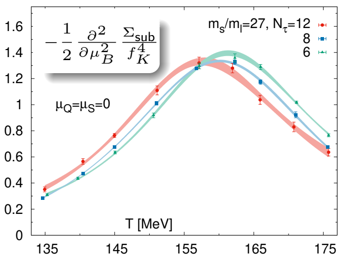




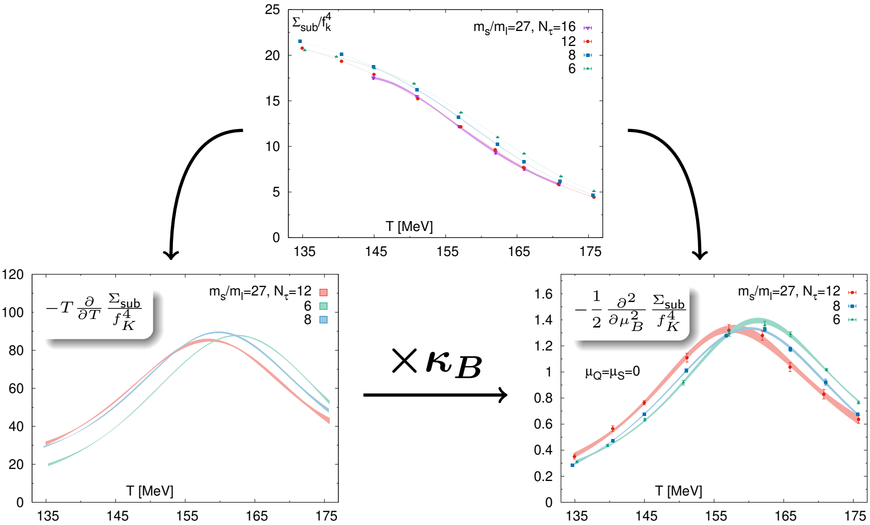
Scaling at physical Pion mass?







Scaling at physical Pion mass?



- Scaling expectation already holds for physical Pion.

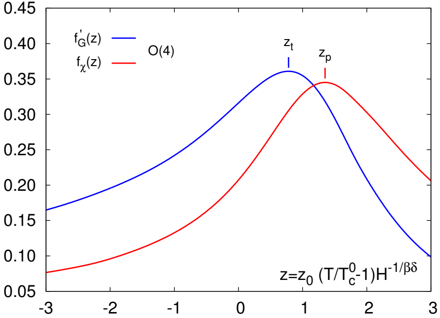
Overview

UNIVERSITÄT BIELEFELD
Faculty of Physics

Mass scaling of conventional estimators

Mass scaling of different estimators of $T_{\rm pc}$:

$$T_X(H) = T_c^0 \left(1 + \frac{z_X}{z_0} H^{1/\beta\delta}\right) \qquad X = t, p$$



In chiral limit :

$$\begin{aligned} M &= h^{1/\delta} f_G(z) \\ \chi_M &= \frac{1}{h_0} h^{1/\delta - 1} f_\chi(z) \\ \frac{\partial M}{\partial T} &= \frac{1}{t_0 T_c^0} h^{1/\delta - 1/\beta\delta} f_G'(z) \end{aligned}$$

$f_G(z)$ and $f_\chi(z)$ are universal scaling functions which have been precisely determined from various spin models.

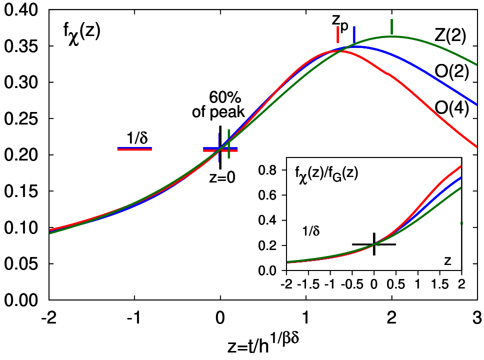
UNIVERSITÄT BIELEFELD
Faculty of Physics

Scaling functions : Some intriguing facts

$$T_X(H) = T_c^0 \left(1 + \frac{z_X}{z_0} H^{1/\beta\delta}\right)$$

Our approach : Use z_X at or close to 0. We choose to work with $X = \delta$ and 60 :

$$\frac{H\chi_M(T_\delta, H)}{M(T_\delta, H)} = \frac{1}{\delta}$$
$$\chi_M(T_{60}, H) = 0.6\chi_M^{max}$$

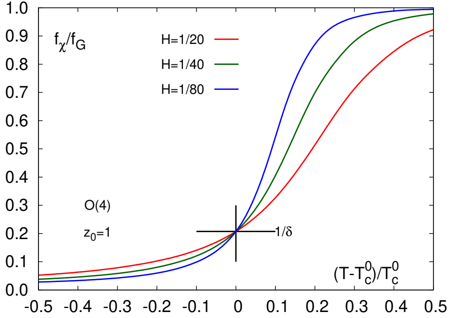


Dependence on quark mass (H) reduced by two orders of magnitude $\Rightarrow$

	z_p	$z^-_{60\%}$
$O(2)$	1.56	-0.005
$O(4)$	1.37	-0.013
$Z(2)$	2.00	0.10

Scaling functions : Some intriguing facts

$$\frac{f_\chi(z)}{f_G(z)} = \left\{ \begin{array}{ccc} 0 & , & z \to -\infty \\ 1/\delta & , & z = 0 \\ 1 & , & z \to +\infty \end{array} \right\}$$



- Behavior of $H\chi_M/M$ is like Binder cumulant at critical point. [F. Karsch and E. Laermann. Phys. Rev. D50, 6954, 1994.]
- Ratio is expected to have a constant value at the crossing point, $z = 0$, *i.e.* in chiral limit at T_c^0.
- Determine temperature $T_\delta(H)$ which satisfies :

$$\frac{H\chi_M\left(T_\delta, H\right)}{M\left(T_\delta, H\right)} = \frac{1}{\delta} \Rightarrow T_c^0 = \lim_{H\to 0} T_\delta(H)$$

.

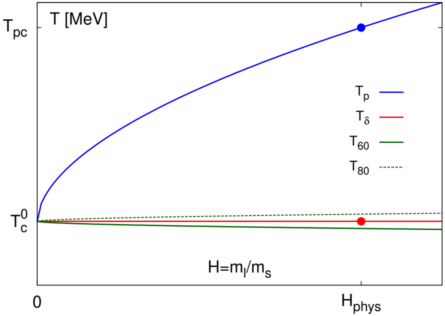
- Uniqueness of the crossing point gets spoiled in presence of regular terms.

UNIVERSITÄT BIELEFELD
Faculty of Physics

Improved estimators : basic philosophy

$$T_X(H) = T_c^0 \left(1 + \frac{z_X}{z_0} H^{1/\beta\delta}\right)$$

- Our approach : Use z_X at or close to 0.
- We choose to work with $X = \delta$ and 60.



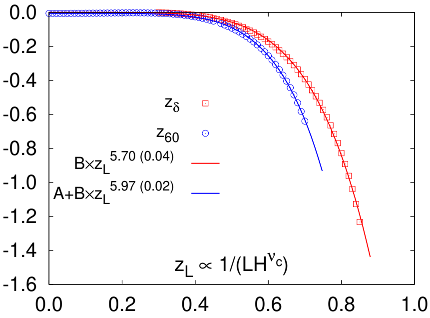
Because of the the reduced variation w.r.t. H, up to the regular contributions, the pseudo-critical temperatures defined by the improved estimators at any finite value of H, *e.g.* H_{phys}, already gives a close estimate of T_c^0.

Improved estimators on finite volumes

- System size (L) is also a scaling field resulting into additional scaling variable $z_L \propto 1/\left(LH^{\nu_c}\right)$.
- We have used O(4) finite size scaling function for our calculations.

[J. Engels and F. Karsch. Phys. Rev. D90, 014501, 2014.]

$$T_X(H,L) = T_c^0\left(1 + \frac{z_X(z_L)}{z_0} H^{1/\beta\delta}\right)$$



Improved estimators on finite volumes

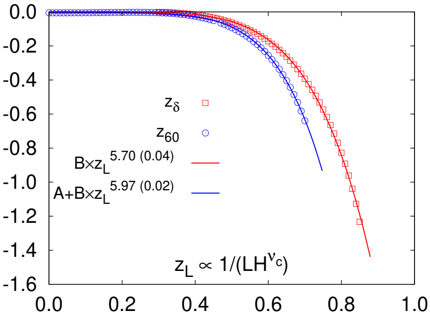
- System size (L) is also a scaling field resulting into additional scaling variable $z_L \propto 1/\left(LH^{\nu_c}\right)$.
- We have used O(4) finite size scaling function for our calculations.

[J. Engels and F. Karsch. Phys. Rev. D90, 014501, 2014.]

- Both the estimators seems to approach thermodynamic limit faster than $1/V$. Determine temperature $T_\delta(H, L)$ which satisfies :

$$\frac{H\chi_M\left(T_\delta, H, L\right)}{M\left(T_\delta, H, L\right)} = \frac{1}{\delta} \Rightarrow T_c^0 = \lim_{H\to 0} \lim_{L\to\infty} T_\delta\left(H, L\right).$$

$$T_X(H, L) = T_c^0\left(1 + \frac{z_X(z_L)}{z_0} H^{1/\beta\delta}\right)$$



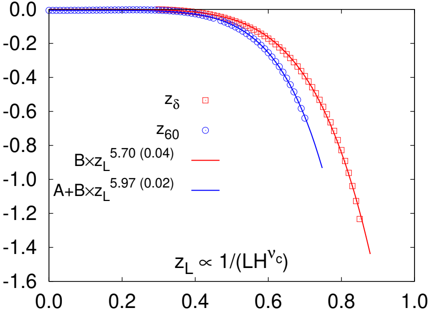
Improved estimators on finite volumes

- System size (L) is also a scaling field resulting into additional scaling variable $z_L \propto 1/\left(LH^{\nu_c}\right)$.
- We have used O(4) finite size scaling function for our calculations.

[J. Engels and F. Karsch. Phys. Rev. D90, 014501, 2014.]

- Both the estimators seems to approach thermodynamic limit faster than $1/V$. Determine temperature $T_\delta(H,L)$ which satisfies :

$$T_X(H,L) = T_c^0 \left(1 + \frac{z_X(z_L)}{z_0} H^{1/\beta\delta}\right)$$



$$\frac{H\chi_M\left(T_\delta,H,L\right)}{M\left(T_\delta,H,L\right)} = \frac{1}{\delta} \Rightarrow T_c^0 = \lim_{H\to 0} \lim_{a\to 0} \lim_{L\to\infty} T_\delta\left(H,L\right).$$

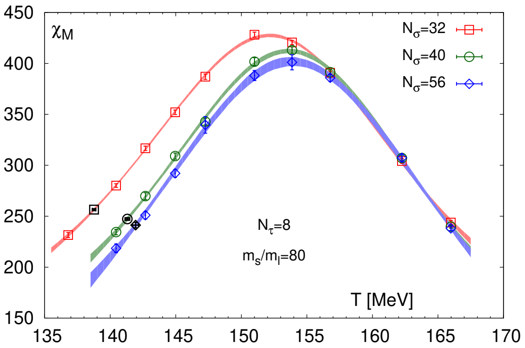
UNIVERSITÄT BIELEFELD
Faculty of Physics

Overview

UNIVERSITÄT BIELEFELD
Faculty of Physics

No evidence for 1^{st} order transition

- Volume dependence of χ_M is studied for $H = 1/80$ which corresponds to $m_\pi = 80$ MeV.
- $\chi_M^{\max}$ is NOT proportional to volume.
- $\chi_M^{\max}$ seems to saturate towards thermodynamic limit.
- T_{pc} and T_{60} increase towards thermodynamic limit.

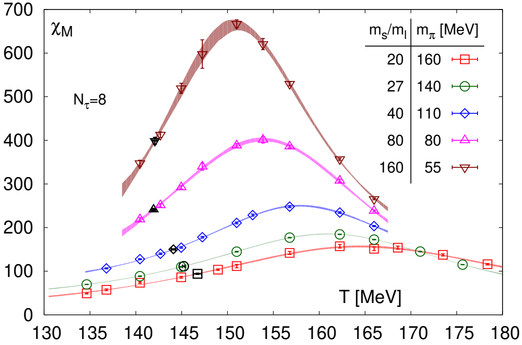


- Possibility for 1^{st} order phase transition can be ruled out at $m_\pi = 80$ MeV for $N_\tau = 8$.
- Similar results are also obtained for $N_\tau = 6$ and 12.

UNIVERSITÄT BIELEFELD
Faculty of Physics

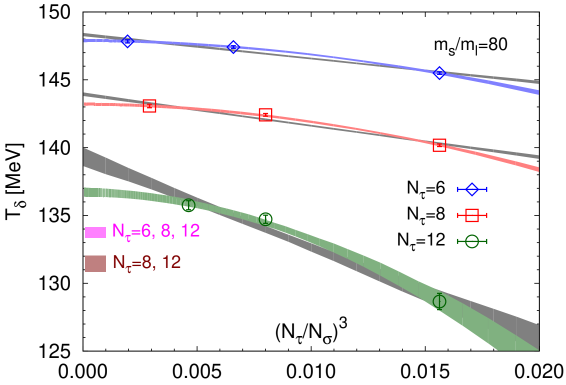
Chiral susceptibility

- The increase of $\chi_M^{\max}$ is apparently consistent with $H^{1/\delta-1}$ with $\delta \approx 4.8$.
- Precise determination of δ is not possible with the present data.
- Preliminary analyses with H_c being a free parameter gives a quite uncertain estimate of H_c with 0 within the range.



- Saturating trend of T_{60} towards chiral limit even at $N_\tau = 8$ already puts this as an improved estimator.
- There is no strong evidence for H_c being non-zero.

T_c^0 in continuum : 'Proper' limits

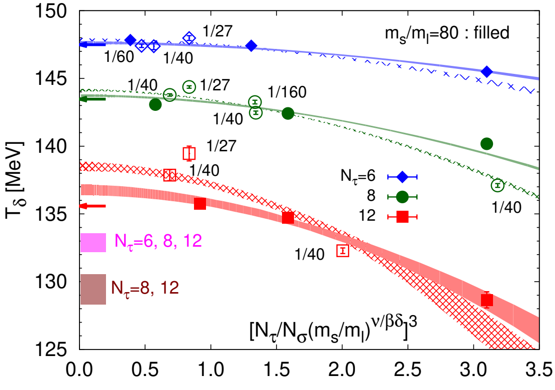


- Results for fixed H have been extrapolated to thermodynamic limit.
- Systematic uncertainty comes in form of difference between $O(4)$ and $1/V$ extrapolations.
- Continuum extrapolation are performed with(out) $N_\tau = 6$ results which is another source of systematic uncertainty.

Chiral extrapolation : $T_X(H) = T_c^0 \left(1 + \frac{z_X}{z_0} H^{1/\beta\delta}\right) + c_X H^{1-1/\delta+1/\beta\delta}$

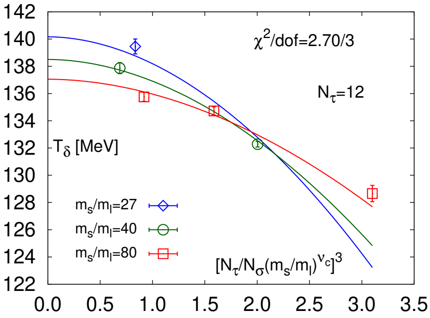
UNIVERSITÄT BIELEFELD
Faculty of Physics

T_c^0 in continuum : 'Improper' limits



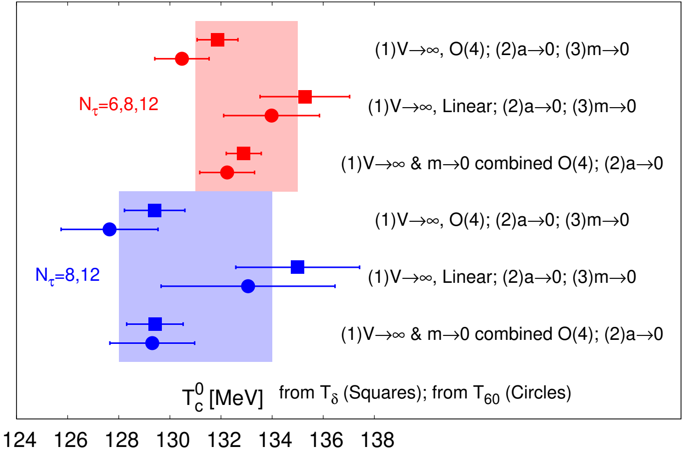
- Continuum extrapolation are performed with(out) $N_\tau = 6$ results which is another source of systematic uncertainty.

- Results for fixed N_τ have been extrapolated to thermodynamic limit and chiral limit simultaneously using $O(4)$ scaling functions.



UNIVERSITÄT BIELEFELD
Faculty of Physics

T_c^0 : A single number



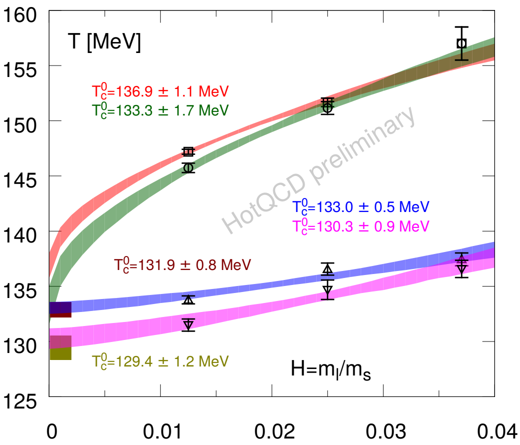
Final number we have quoted : $T_c^0 = 132^{+3}_{-6}$ MeV.

HotQCD; PRL 123, 062002 (2019).

UNIVERSITÄT BIELEFELD
Faculty of Physics

Preliminary comparison with conventional estimator

- Disclaimer : All $T_{\rm pc}$ numbers and T_δ for $H = 1/27$ are not infinite volume extrapolated.
- A little tension can be seen for $T_{\rm pc}$ calculation for $H = 1/40$.
- Still compares well.
- In thermodynamic limit, as we have seen earlier, $T_{\rm pc}$ will presumably increase which may pull down T_c^0, more closer to the current estimate.

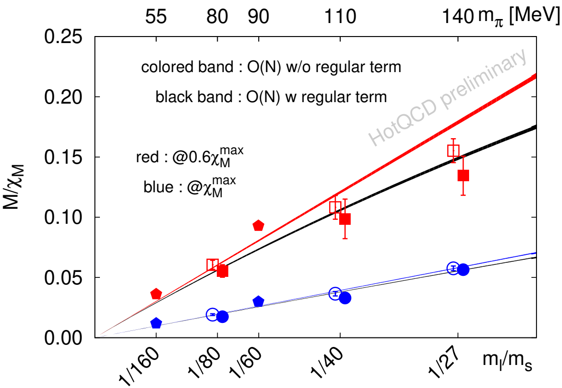


- Stability of new estimators are vivid.

Order of chiral transition : work in progress....

$$\frac{M}{\chi_M} = H\frac{f_G(z)}{f_\chi(z)}$$

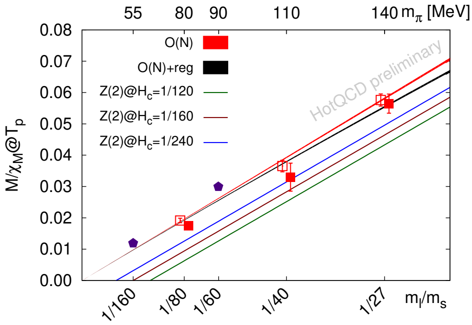
- For small H the data seems to be linear.
- Lines are NOT fitted curves rather expectations for $O(2)$ and $O(4)$.
- Regular term $\propto H^{2-1/\delta}$.
- Coefficient of the regular term is NOT fitted, rather taken from MEoS fits.

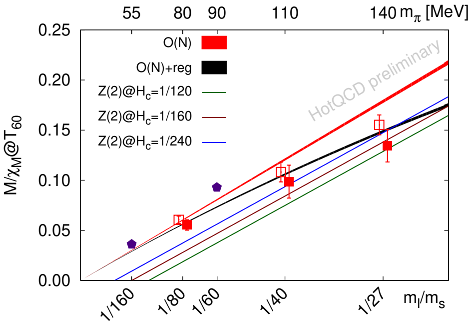


- $Z(2)$ transition, at some finite H_c, will results into a sudden drop in the ratio $\Rightarrow$ 1^{st} order transition is unlikely for $m_\pi > 55$ MeV.
- Additional low H measurements : slope can be directly determined as a fit parameter.

UNIVERSITÄT BIELEFELD
Faculty of Physics

Order of chiral transition : work in progress....





- $Z(2)$ lines are schematic : $\dfrac{M}{\chi_M} = (H - H_c)\,\dfrac{f_G(z)}{f_\chi(z)}$
- If M is not exactly order parameter then the $Z(2)$ lines will have a curvature.
- Mixing becomes weak as H_c becomes small.
- Data seems to favor $O(N)$ compared to $Z(2)$.

UNIVERSITÄT BIELEFELD
Faculty of Physics

Overview

UNIVERSITÄT BIELEFELD
Faculty of Physics

Summary and Outlook

- Summary :
  1. Scaling fits with $O(4)$ exponents worked reasonably.
  2. Within error same results obtained taking chiral and continuum extrapolations in different order.
  3. Current estimate of T_c^0, in continuum, is 132^{+3}_{-6} MeV.
- Outlook :
  1. Comparison with conventional estimators will be done with finite volume effects taken into account.
  2. Additional low H measurements will help us to be confident about the scaling behavior.
  3. Precise calculation of disconnected part of chiral susceptibilities at high temperatures can throw some light into $U_A(1)$ restoration in chiral limit.

UNIVERSITÄT BIELEFELD
Faculty of Physics

Summary and Outlook

- Summary :
  1. Scaling fits with $O(4)$ exponents worked reasonably.
  2. Within error same results obtained taking chiral and continuum extrapolations in different order.
  3. Current estimate of T_c^0, in continuum, is 132^{+3}_{-6} MeV.
- Outlook :
  1. Comparison with conventional estimators will be done with finite volume effects taken into account.
  2. Additional low H measurements will help us to be confident about the scaling behavior.
  3. Precise calculation of disconnected part of chiral susceptibilities at high temperatures can throw some light into $U_A(1)$ restoration in chiral limit.

Symmetry transformations

$$
\begin{tikzcd}[column sep=large]
\chi_{5,con} & \pi : \bar{q}\gamma_5 \frac{\tau}{2} q \arrow[r, leftrightarrow, "SU(2)_L \times SU(2)_R"] \arrow[d, leftrightarrow, "U(1)_A"'] & \sigma : \bar{q} q \arrow[d, leftrightarrow, "U(1)_A"] & \chi_{con} + \chi_{disc} \\
\chi_{con} & \delta : \bar{q} \frac{\tau}{2} q \arrow[r, leftrightarrow, "SU(2)_L \times SU(2)_R"'] & \eta : \bar{q}\gamma_5 q & \chi_{5,con} - \chi_{5,disc}
\end{tikzcd}
$$

UNIVERSITÄT BIELEFELD
Faculty of Physics