Net baryon-number fluctuations

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CRC-TR 211
Strong-interaction matter under extreme conditions

DFG

EuroPLEx

H2020

HotQCD Collaboration, Phys.Rev.D 101 (2020) 7, 074502 and work in progress

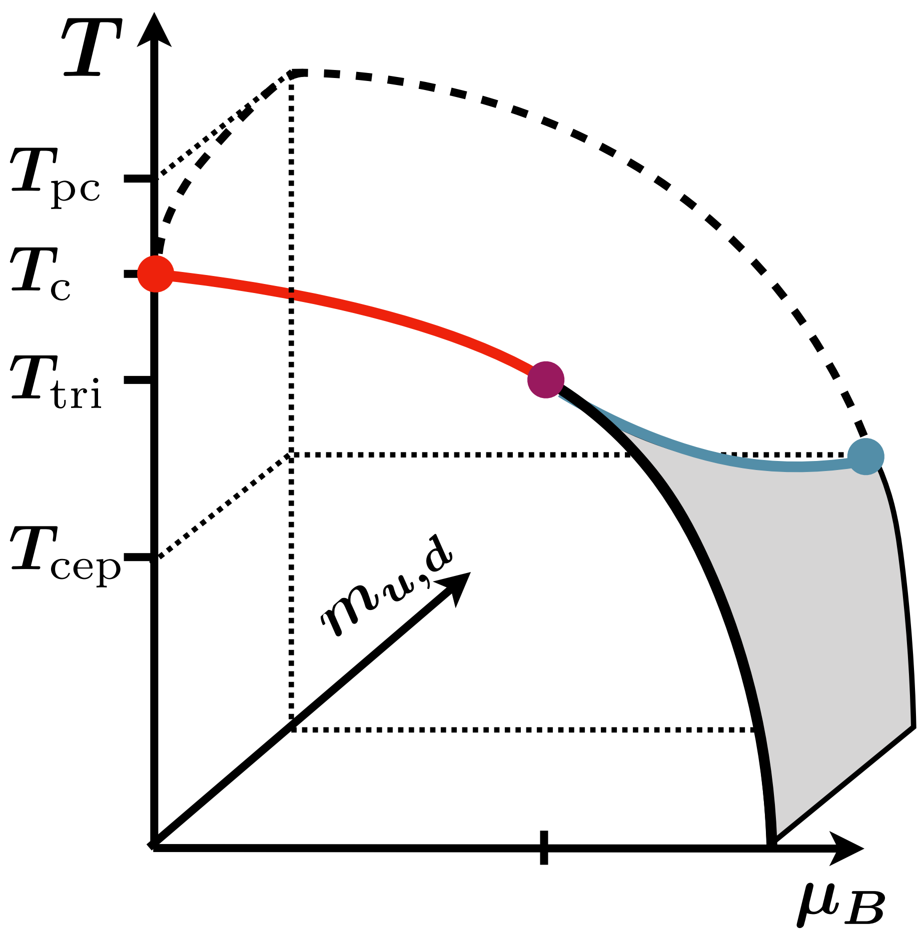
A. Bazavov, D. Bollweg, H.-T. Ding, P. Enns, J. Goswami, P. Hegde, O. Kaczmarek, F. Karsch, A. Lahiri, R. Larsen, S.-T. Li, Swagato Mukherjee, H. Ohno, P. Petreczky, C. Schmidt, S. Sharma, P. Steinbrecher

Bielefeld-Parma Collaboration, work in Progress

P. Dimopoulos, F. Di Renzo, J. Goswami, G. Nicotra, C. Schmidt, S. Singh, K. Zambello, F. Ziesché

Criticality in QCD and the Hadron Resonance Gas
29-31 Jul 2020

Motivation: The QCD Phase diagram



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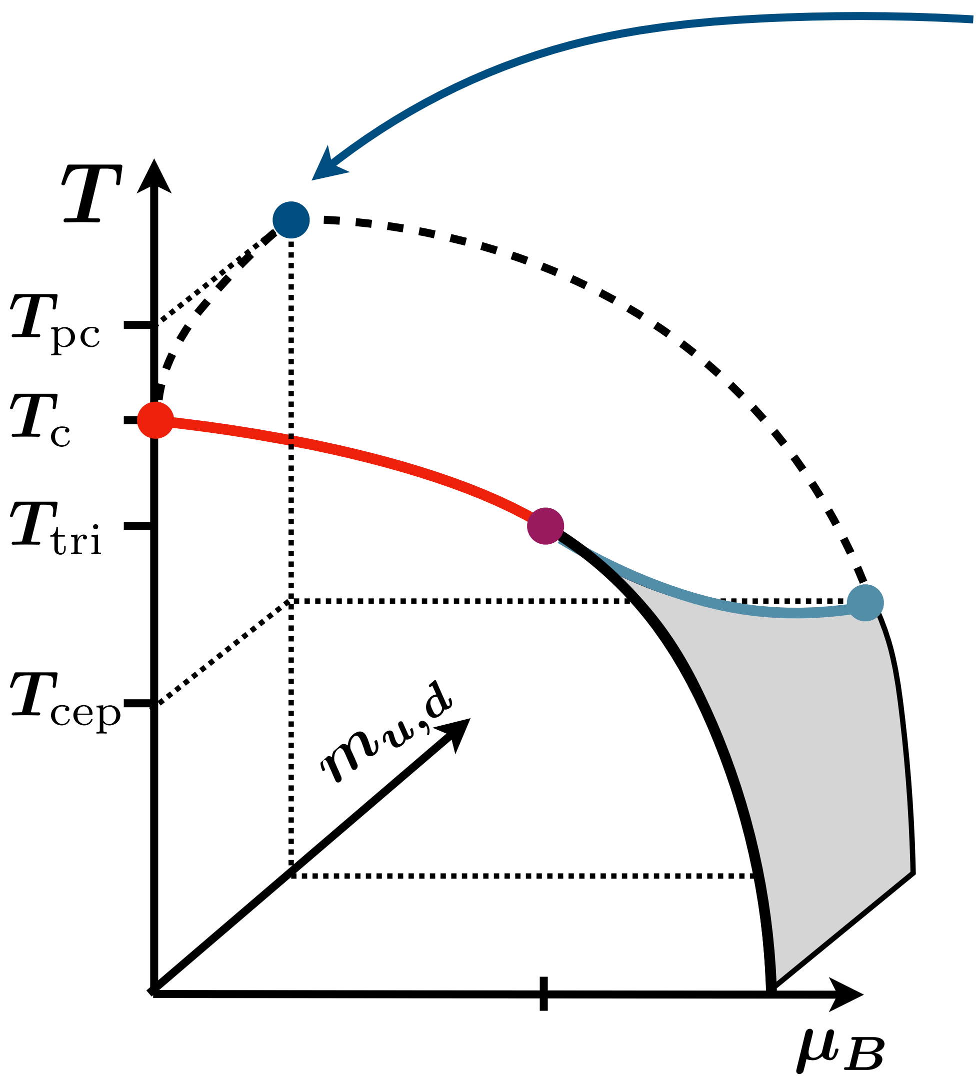
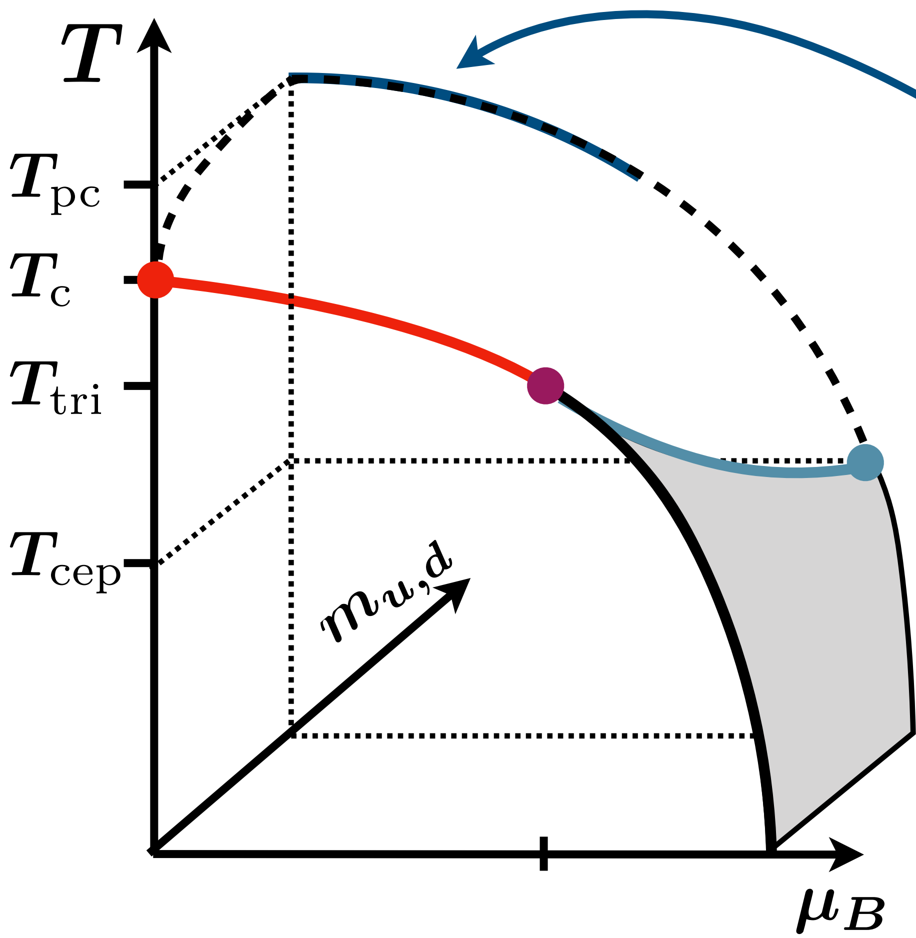


- New precise determination of the QCD transition temperature $T_{\rm pc} = 156.5 \pm 1.5$ MeV

HotQCD: PLB 795 (2019) 15



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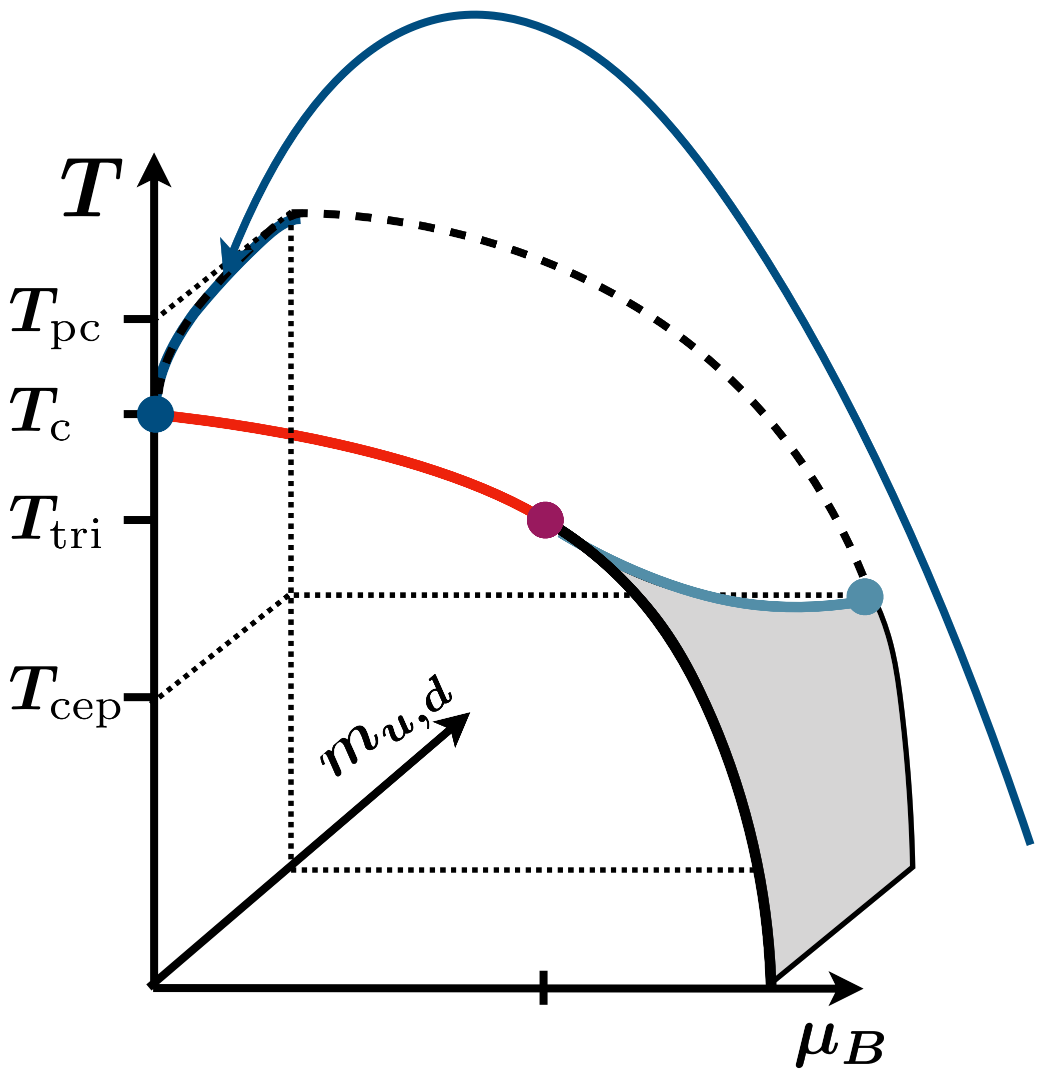
- The chiral crossover line with respect to μ_B

$$T_{\rm pc}(\mu_B) = T_{\rm pc}^0 \left(1 - \kappa_2^{B,f}\left(\frac{\mu_B}{T_{\rm pc}^0}\right)^2 - \kappa_4^{B,f}\left(\frac{\mu_B}{T_{\rm pc}^0}\right)^4\right)$$

$$\kappa_2^{B,f} = 0.012(4), \quad \kappa_4^{B,f} = 0.00(4)$$

HotQCD: PLB 795 (2019) 15



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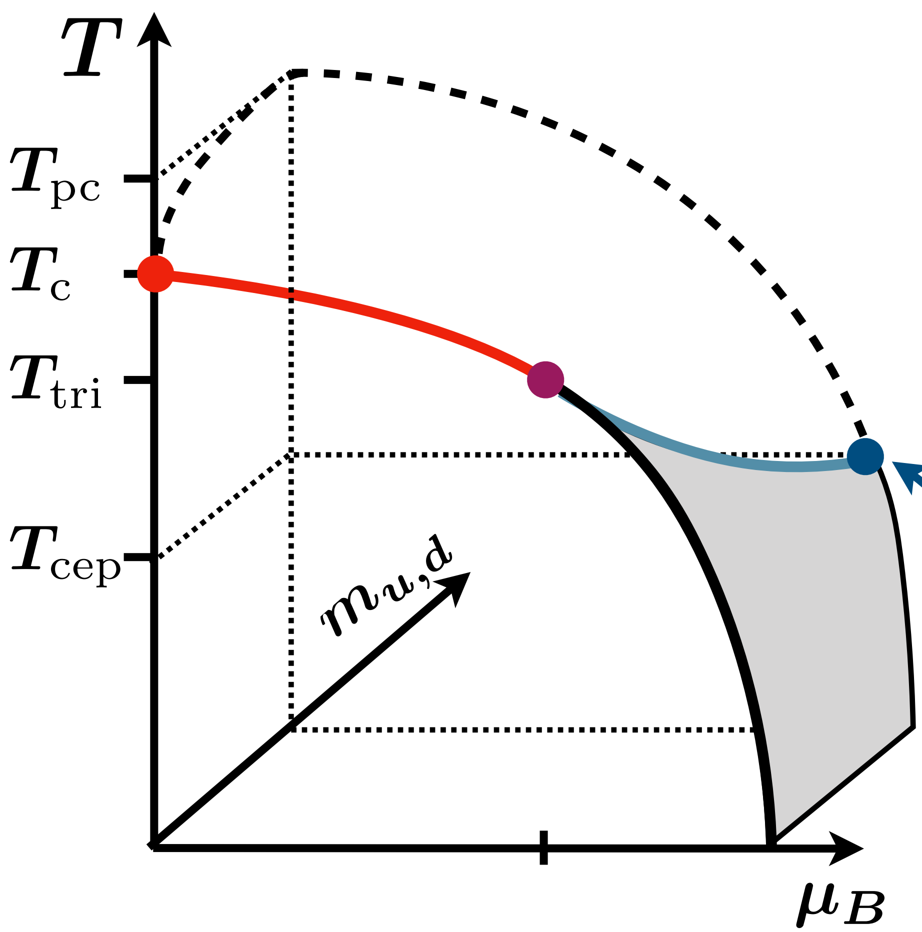
HotQCD: PLB 795 (2019) 15

- The chiral phase transition temperature and pseudo-critical line $T_c = 132^{+3}_{-6}$ MeV

HotQCD: PRL 123 (2019) 062002



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- The chiral phase transition temperature and pseudo-critical line $T_{\mathrm{c}} = \mathbf{132^{+3}_{-6}}\ \mathrm{MeV}$

HotQCD: PRL 123 (2019) 062002

- Expected bounds on the QCD critical end-point

$$T_{\mathrm{cep}} < T_{\mathrm{c}} = \mathbf{132^{+3}_{-6}}\ \mathrm{MeV}$$
$$\mu_B^{\mathrm{cep}} \gtrsim 3\, T_c$$

Plan

- **Methodology I: Taylor expansion**
  - Definition of cumulant ratios
  - Recent improvements
- **Methodology II: Simulations at imaginary chemical potential**
  - Symmetries of the partition function
- **Results: A comparison to HRG**
  - Using HRG to estimate difference of net-baryon and net-proton number fluctuations (only M/σ^2)
  - For a detailed comparison with different (interacting) HRG models see next talk by J. Goswami
- **Results: A comparison to STAR**
  - Thermodynamic consistency of the new $\sqrt{s_{NN}} = 54.4\ \mathrm{GeV}$ proton-number fluctuations
  - For a similar study of electric charge and strangeness fluctuations on the freeze-out line see talk by D. Bollweg
- **Singularities in the complex plane**
- **Summary and Outlook**

Methodology I: The Taylor expansion method

- Compute expansion coefficients of the pressure

$$\frac{p}{T^4} = \frac{\ln Z}{T^3 V} = \sum_{i,j,k=0}^{\infty} \frac{1}{i!j!k!} \boxed{\chi_{ijk,0}^{BQS}} \left(\frac{\mu_B}{T}\right)^i \left(\frac{\mu_Q}{T}\right)^j \left(\frac{\mu_S}{T}\right)^k$$

Gavai, Gupta (2001)
Bielefeld-Swansea (2002)

Cumulants of conserved charge fluctuations, can also be measured as event-by-event fluctuations in heavy ion collisions

$$\chi_{ijk,0}^{BQS} = \frac{\partial^i}{\partial(\mu_B/T)} \frac{\partial^j}{\partial(\mu_Q/T)} \frac{\partial^k}{\partial(\mu_S/T)} \frac{\ln Z}{T^3 V}$$

$\Rightarrow$ A comparison with experimental data constrain freeze-out parameter

Net baryon number fluctuations diverge at the critical point

$\Rightarrow$ Search for the critical point

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- **Notation:** relevant operators are derivatives of the determinant

$$D_i^f = \frac{\partial^i}{\partial \mu_f^i} \det \left[M_f(\mu_f)\right]^{1/4} = \frac{\partial^i}{\partial \mu_f^i} e^{\frac{1}{4} \mathrm{Tr} \ln M_f(\mu_f)}, \quad f \in \{u, d, s\}$$

up to 4th-order in μ:

exponential dependence:

$$D_1^f = \mathrm{Tr}\left[M_f^{-1} M_f^{(1)}\right]$$

$$D_2^f = -\mathrm{Tr}\left[M_f^{-1} M_f^{(1)} M_f^{-1} M_f^{(1)}\right] + \mathrm{Tr}\left[M_f^{-1} M_f^{(2)}\right]$$

$$\vdots$$

from 6th-order in μ onwards:

linear dependence:

$$D_1^f = \mathrm{Tr}\left[M_f^{-1} M_f^{(1)}\right]$$

$$D_2^f = -\mathrm{Tr}\left[M_f^{-1} M_f^{(1)} M_f^{-1} M_f^{(1)}\right]$$

all $M_f^{(k)} = 0$, for k>1

⇒ much less operators to measure!

Gavai, Sharma 2015

Methodology I: Cumulant ratios of conserved charge fluctuations

- Combine quark number fluctuations (χ_{ijk}^{uds}) to obtain hadronic fluctuations (χ_{ijk}^{BQS}).
- Determine strangeness (μ_S/T) and electric charge chemical (μ_Q/T) potentials by imposing strangeness neutrality $n_S = 0$ and $n_Q/n_B = 0.4$ (order by order in the expansion).
- From the pressure expansion we readily obtain the expansions for the n[th]-order cumulants:

$$\chi_n^B(T,\mu_B) = \sum_{k=0}^{k_{\max}} \tilde{\chi}_n^{B,k}(T)\hat{\mu}_B^k, \quad \text{with} \quad \hat{\mu}_B = \mu_B/T$$

- Define ratios to eliminate the leading order volume dependence

$$R_{nm}^B = \frac{\chi_n^B(T,\mu_B)}{\chi_m^B(T,mu_B)} = \frac{\sum_{k=0}^{k_{\max}} \tilde{\chi}_n^{B,k}(T)\hat{\mu}_B^k}{\sum_{l=0}^{l_{\max}} \tilde{\chi}_m^{B,l}(T)\hat{\mu}_B^l}$$

- In terms of the shape parameters of the distribution we find

$$R_{12} = M/\sigma^2,\ R_{31} = S\sigma^3/M,\ R_{32} = S\sigma,\ R_{42} = \kappa\sigma^2,\ \ldots$$

- Eventually we want calculate observables along the crossover (and freeze-out) line, we thus need spline interpolations of our data at discrete temperature values.

Methodology II: Calculations at imaginary chemical potential

- The fermion determinant stays real at imaginary chemical potential, the imaginary chemical potential is implemented as phases to the time linke link variables

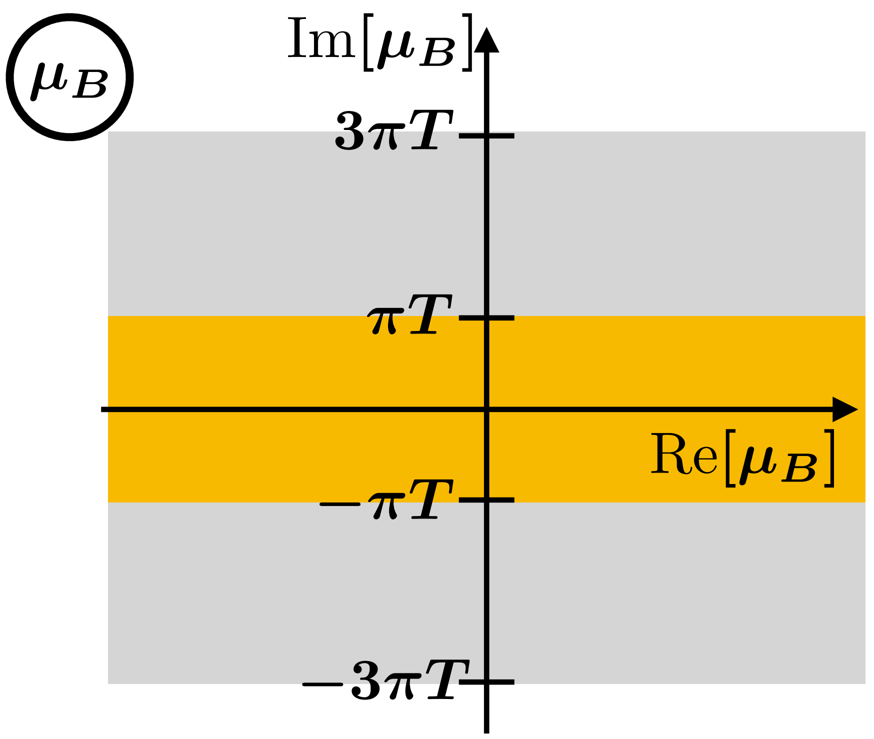
$$U_0(x) \to e^{i\hat{\mu}_I} U_0(x) \qquad U_0^\dagger(x) \to e^{-i\hat{\mu}_I} U_0^\dagger(x)$$

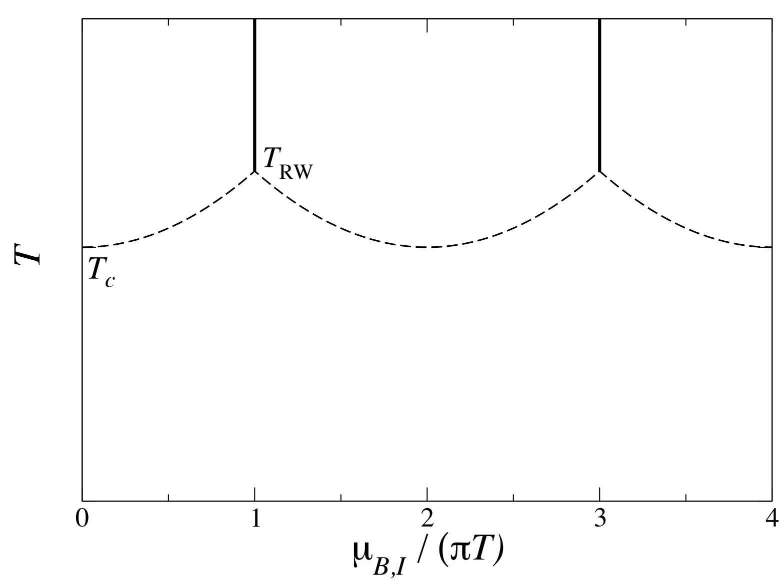
- Results can be analytically continued to real chemical potential

Taylor in $\mathrm{Im}[\mu_B]$ $\quad\to\quad$ Taylor in $\mathrm{Re}[\mu_B]$

[Borsanyi et al, JHEP 10 (2018) 205]

- The QCD partition function is periodic in $\mathrm{Im}[\mu_B]$ due to the Roberge-Weiss (RW) symmetry, with a periodicity $2\pi T$





[Bonati et al, Phys. Rev. D 99, 014502 (2019)]

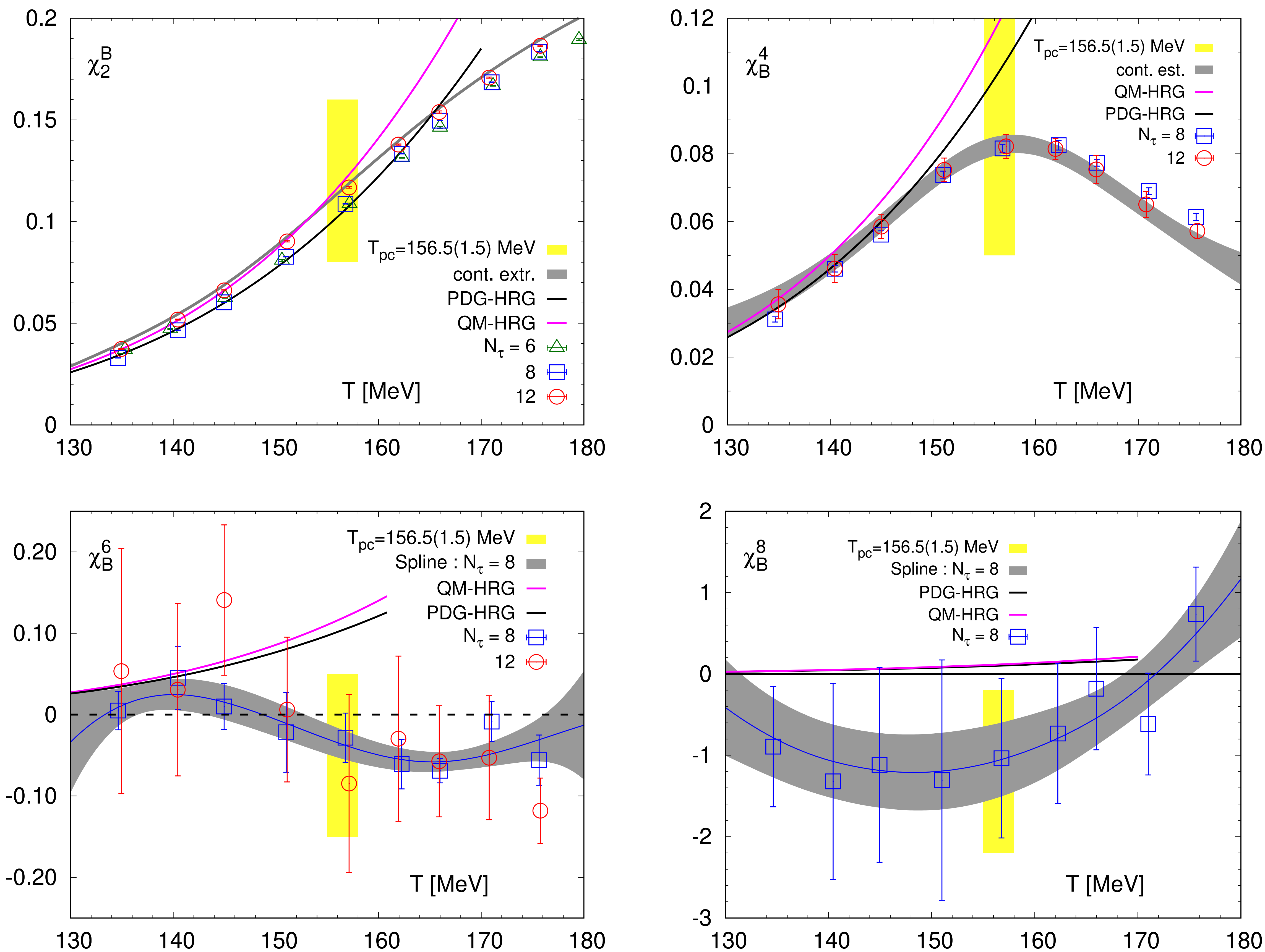
Methodology: Lattice setup and statistics

- Use (2+1)-flavor of HISQ-fermions, with physical strange and light quark masses.
- **Lattices sizes** are $32^3 \times 8,\ 48^3 \times 12,\ 64^3 \times 16$, at 9 different temperature values.
- **Statistics:** Compared to our previous analysis of skewness and kurtosis [HotQCD, PRD 96 (2017) 074510] we increased the statistics on ($N_\tau = 8$)-lattices by a factor 3-4 and on ($N_\tau = 12$)-lattices by a factor 6-8. I.e. we have now

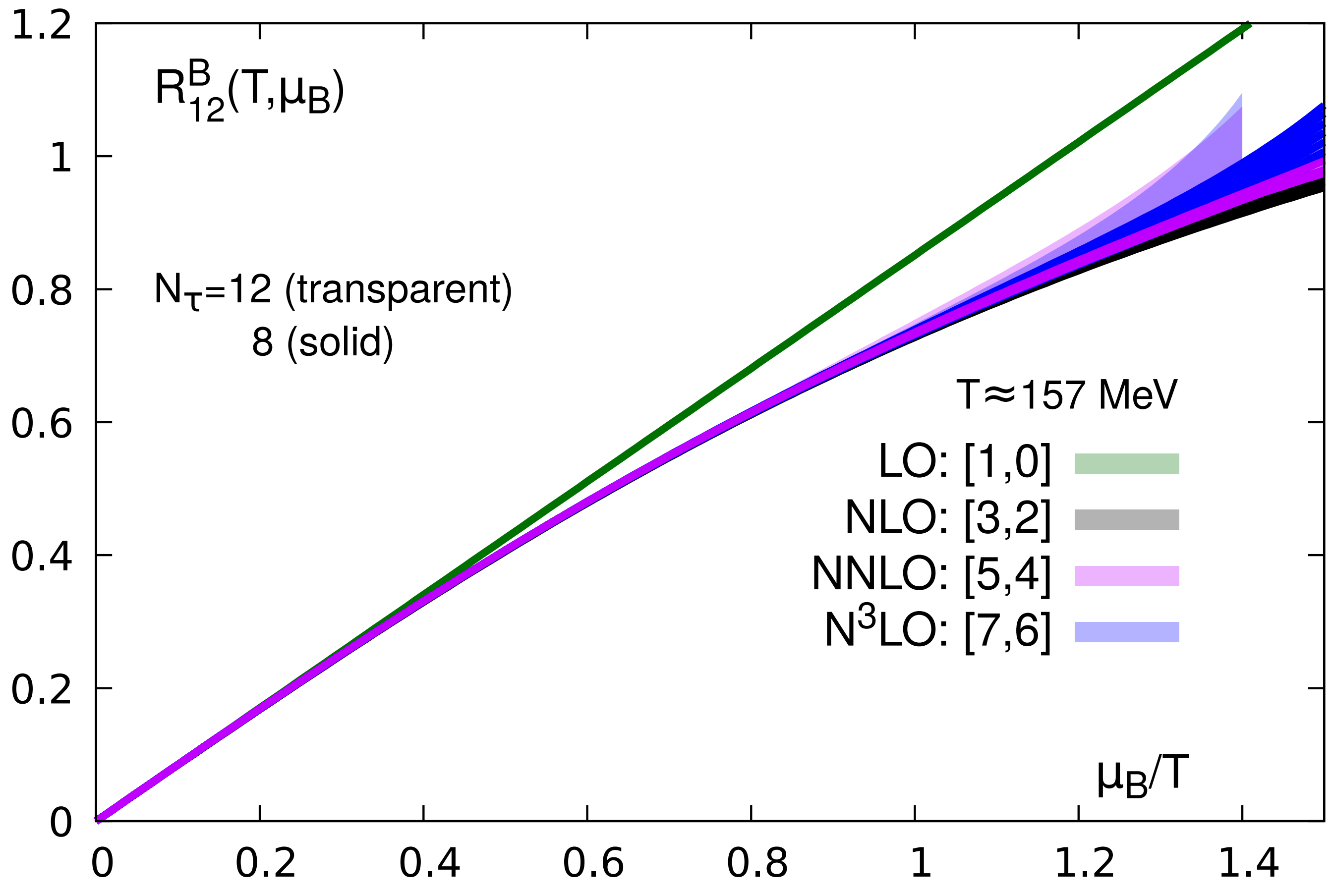
N_τ	8	12	16
#conf.	$1.2 \cdot 10^6$	$(2-3) \cdot 10^5$	10^4

- **Order of the expansion:** We can now go to N^{3}LO, compared to NLO in our previous study. I.e., we include 8-th order expansion coefficients of the pressure.
- Recent Calculations were performed on Summit, using Nvidia's V100 GPU's.

Results: The expansion coefficients of the pressure



Results: The net baryon-number density R_{12}^B



- Cut-off effects are negligible for $\mu_B/T \leq 1$ and of the same order as the statistical error at $N_\tau = 12$ for $\mu_B/T \leq 1.2$.

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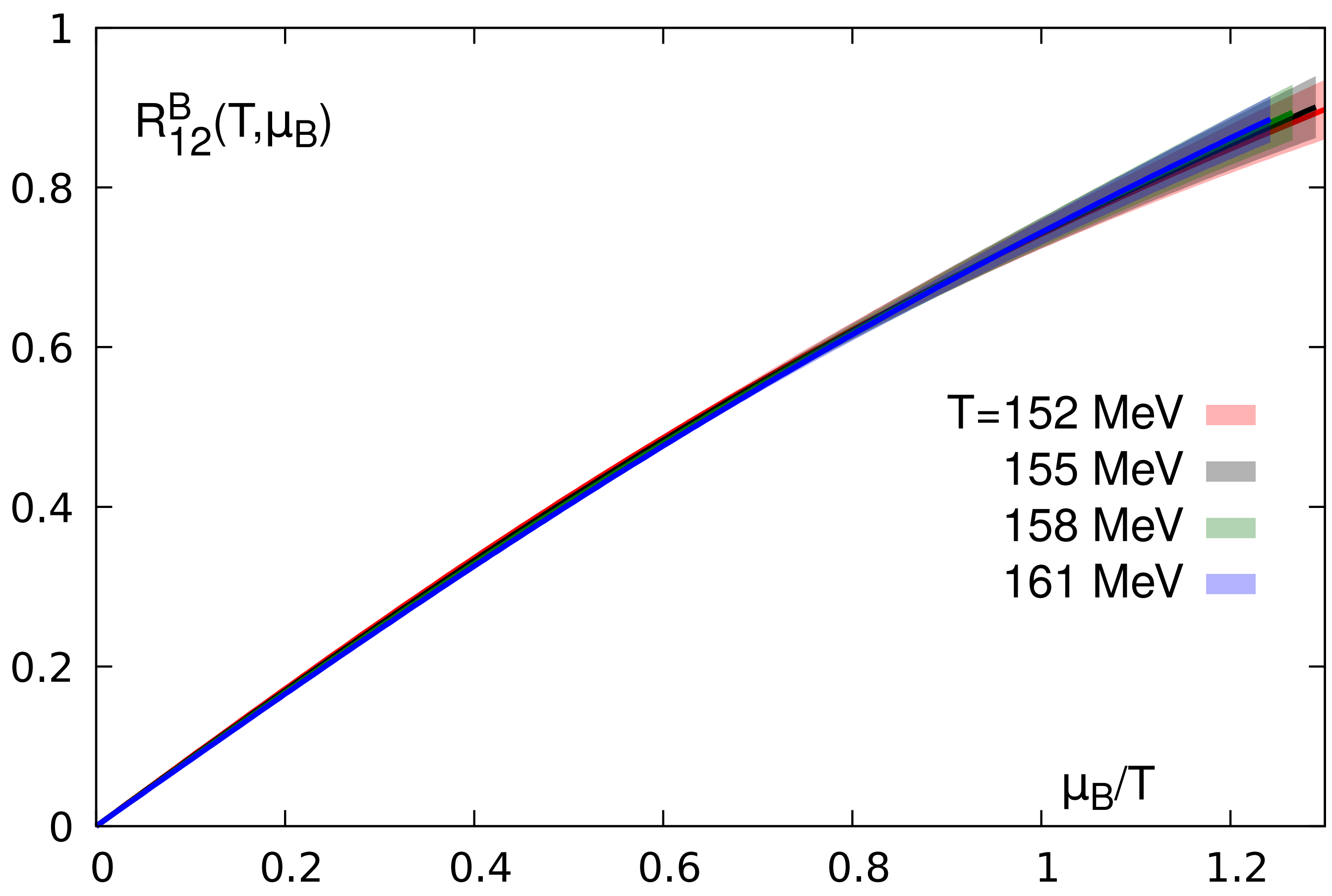

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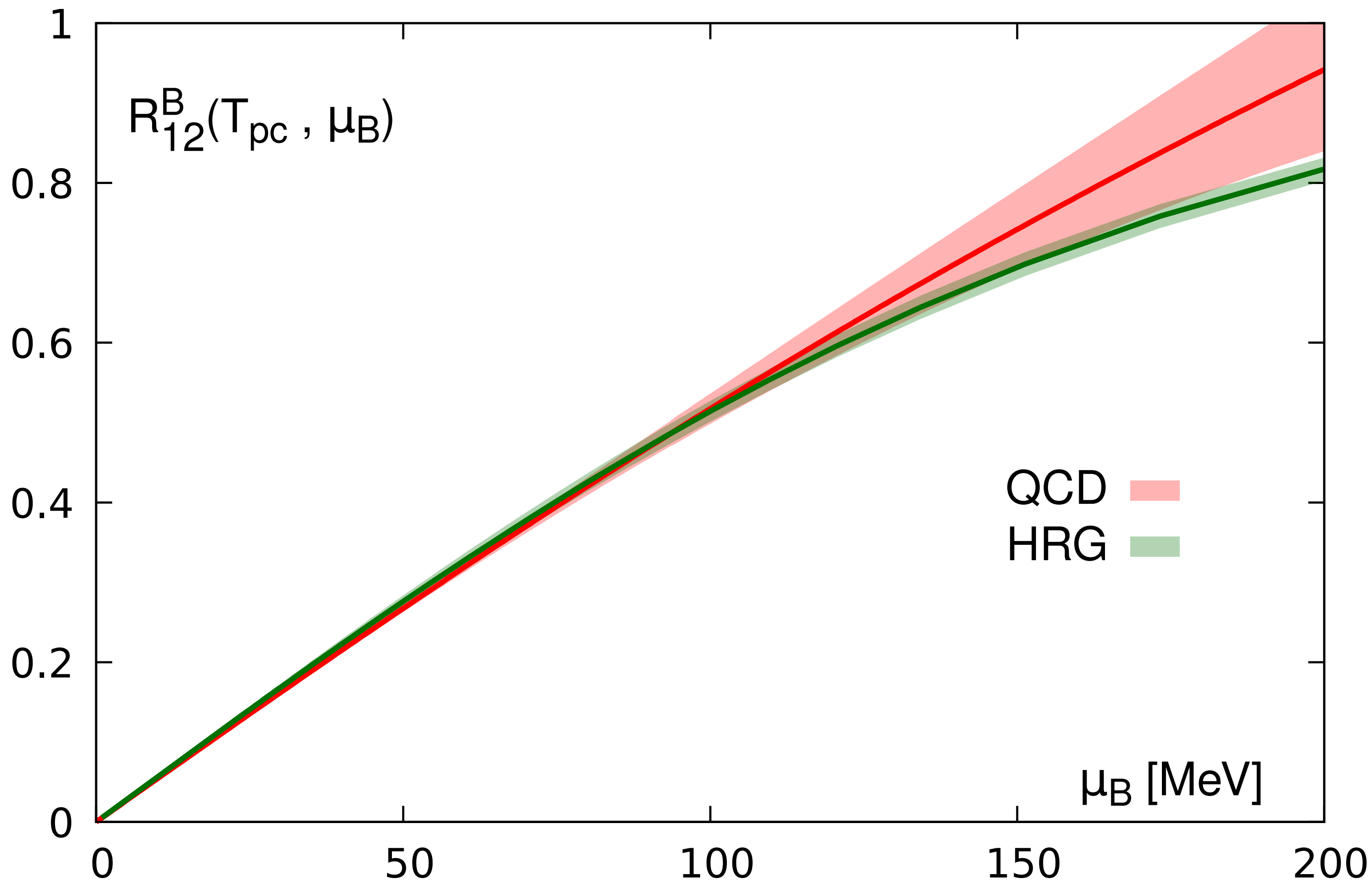

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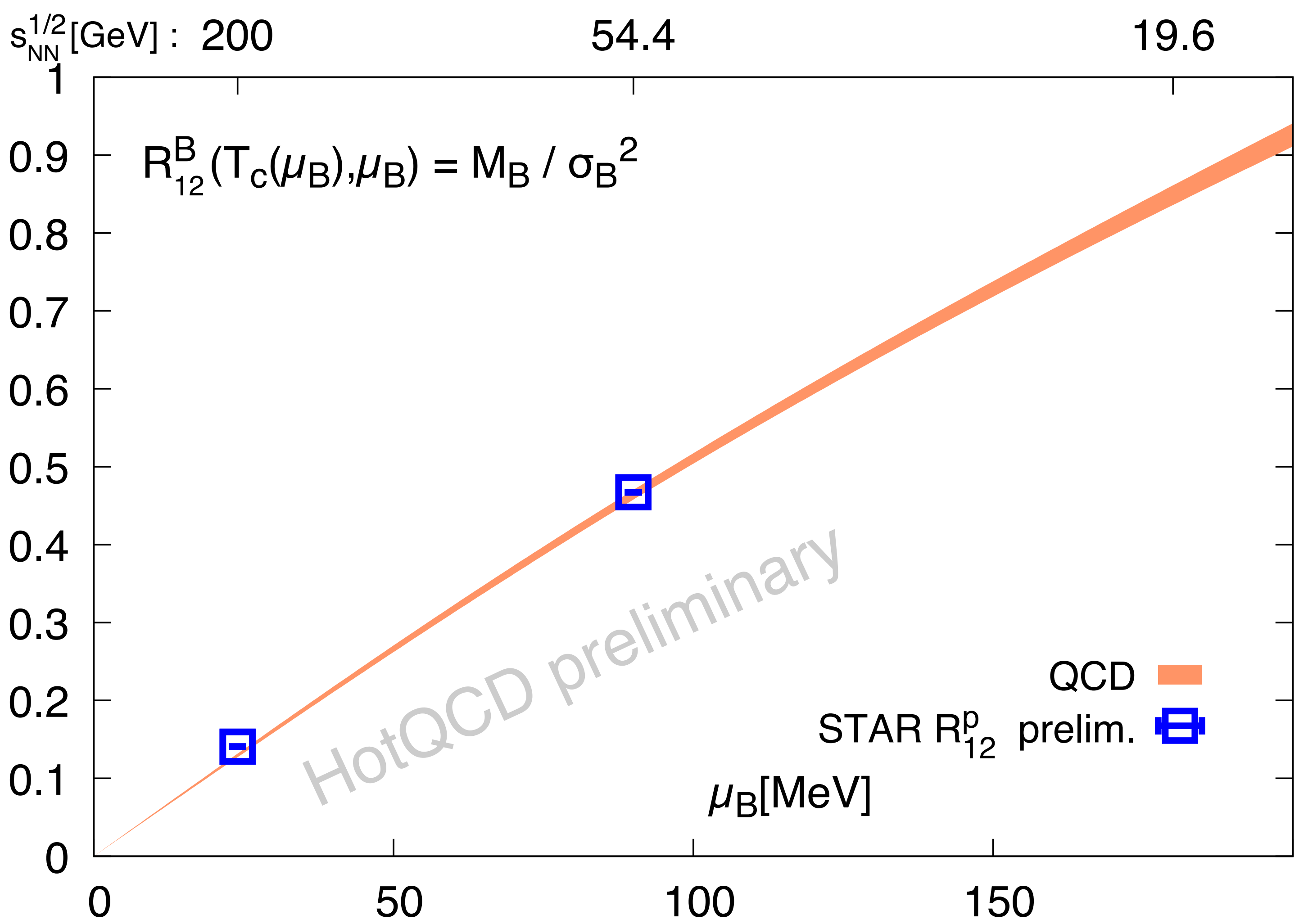

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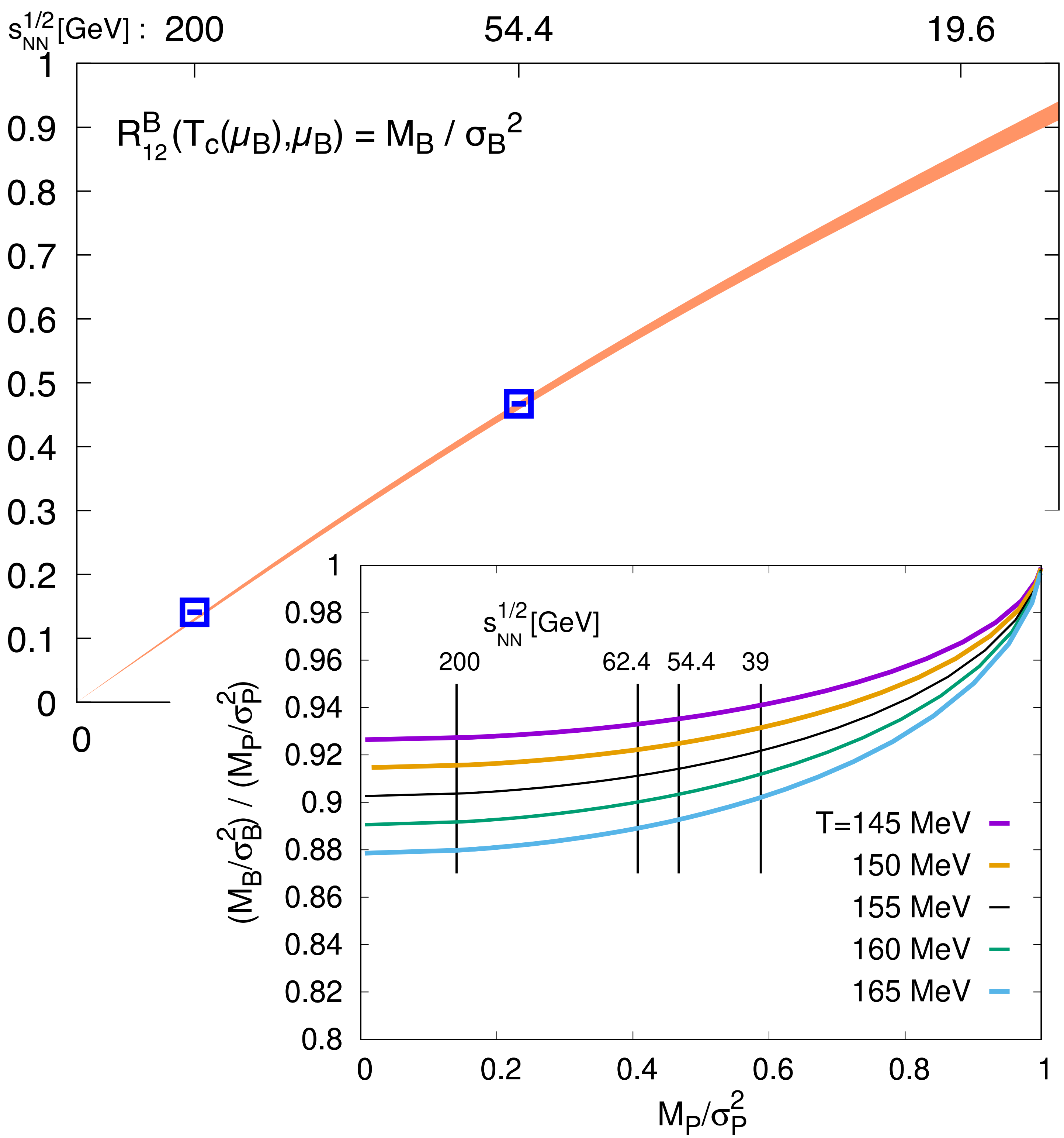

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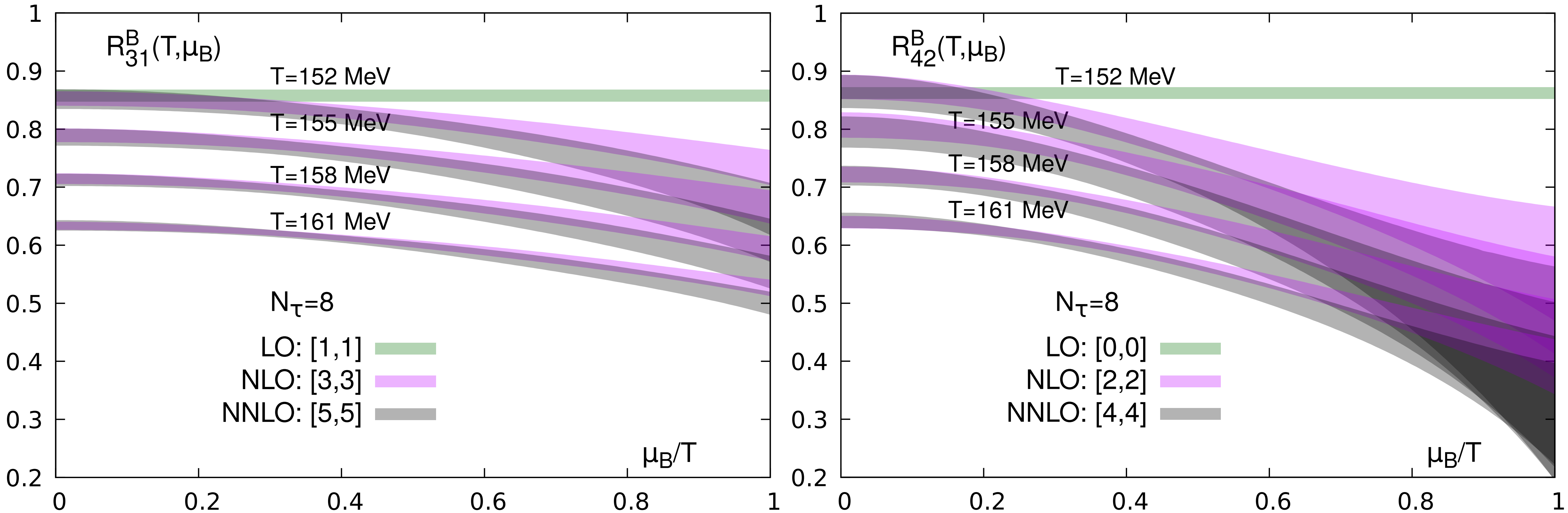
- R_{12}^B may be used to eliminate μ_B in studies of higher order cumulants
- The difference of R_{12}^B and R_{12}^P is less than 10% in the HRG, one may or may not account for this difference in the determination of μ_B .

Results: Skewness and kurtosis

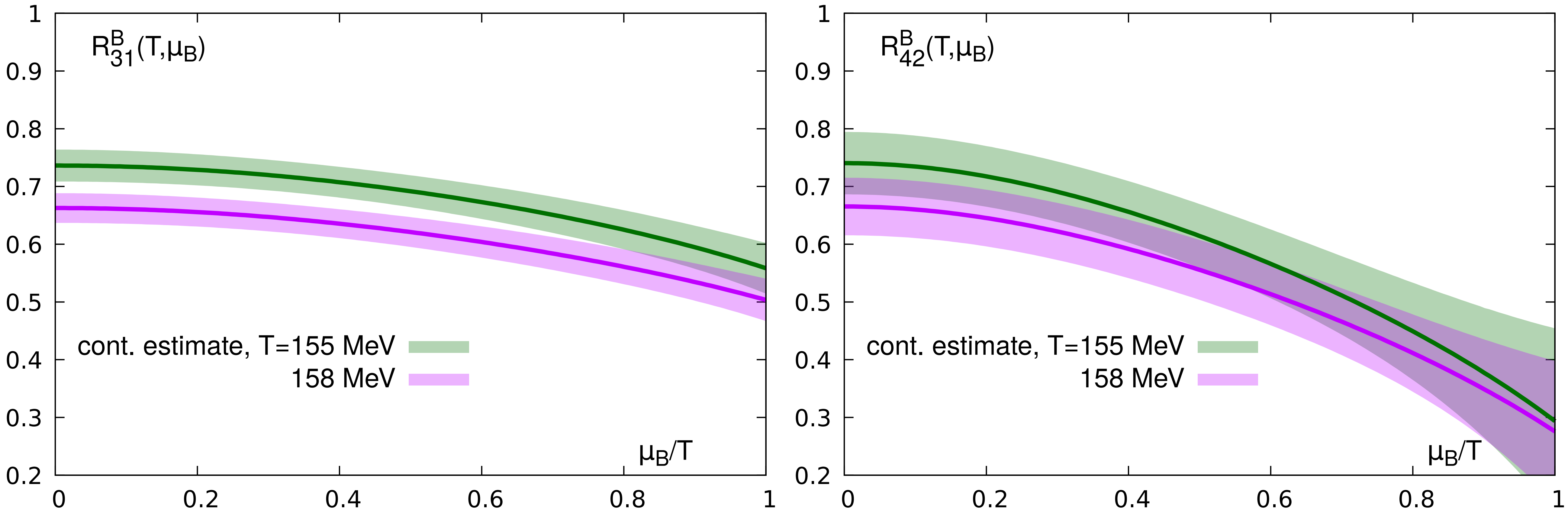
- Skewness and kurtosis ratios R_{31}^B and R_{42}^B on $(N_\tau = 8)$-lattices



- Convergence gets worth with increasing order of the cumulant and with decreasing temperature.
- NLO and NNLO corrections are negative.

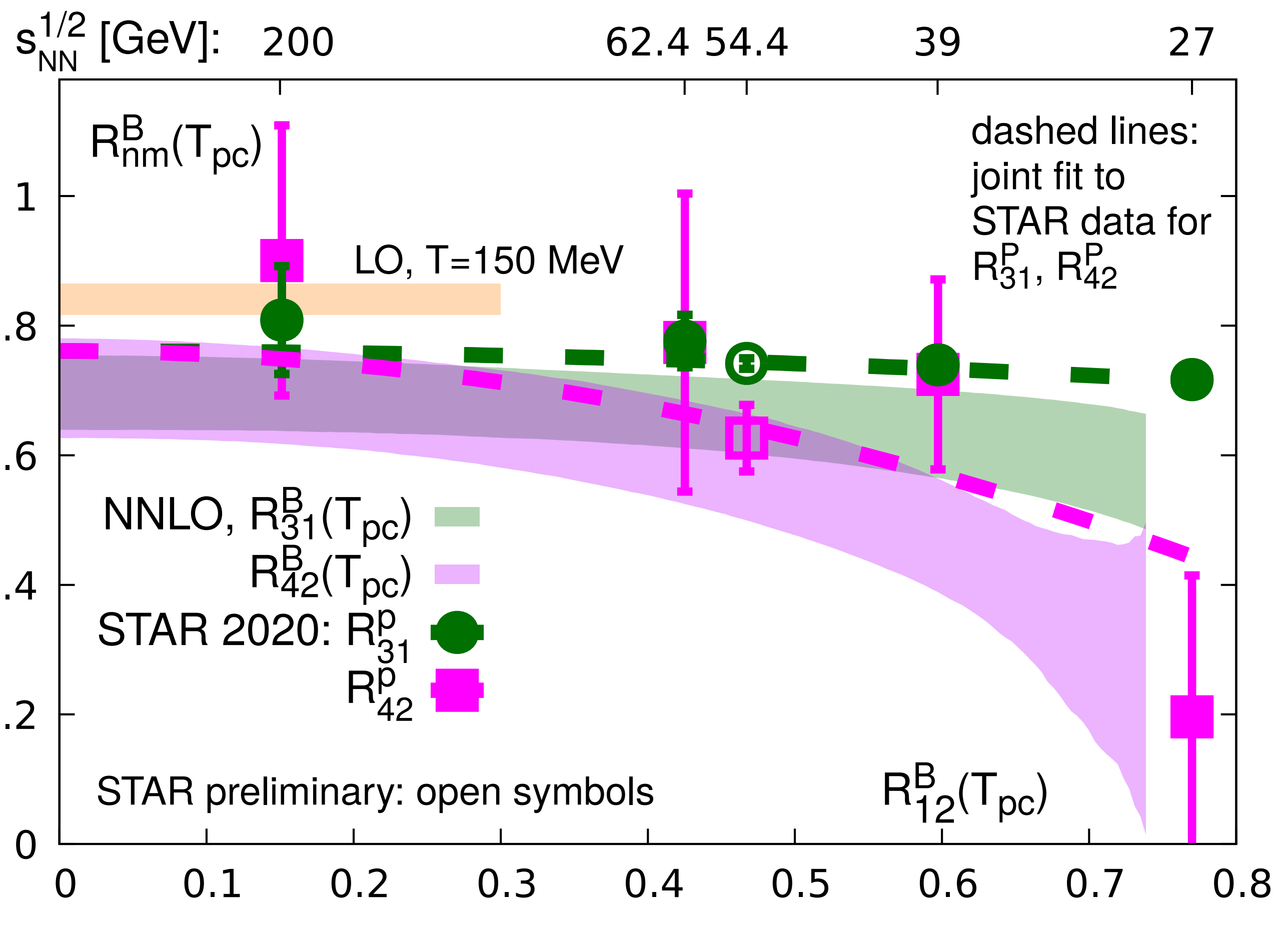
Results: Skewness and kurtosis

- Continuum estimates of R_{31}^B and R_{42}^B as function of μ_B/T for various temperatures.



- Ratios drop with increasing μ_B/T and with increasing temperature.

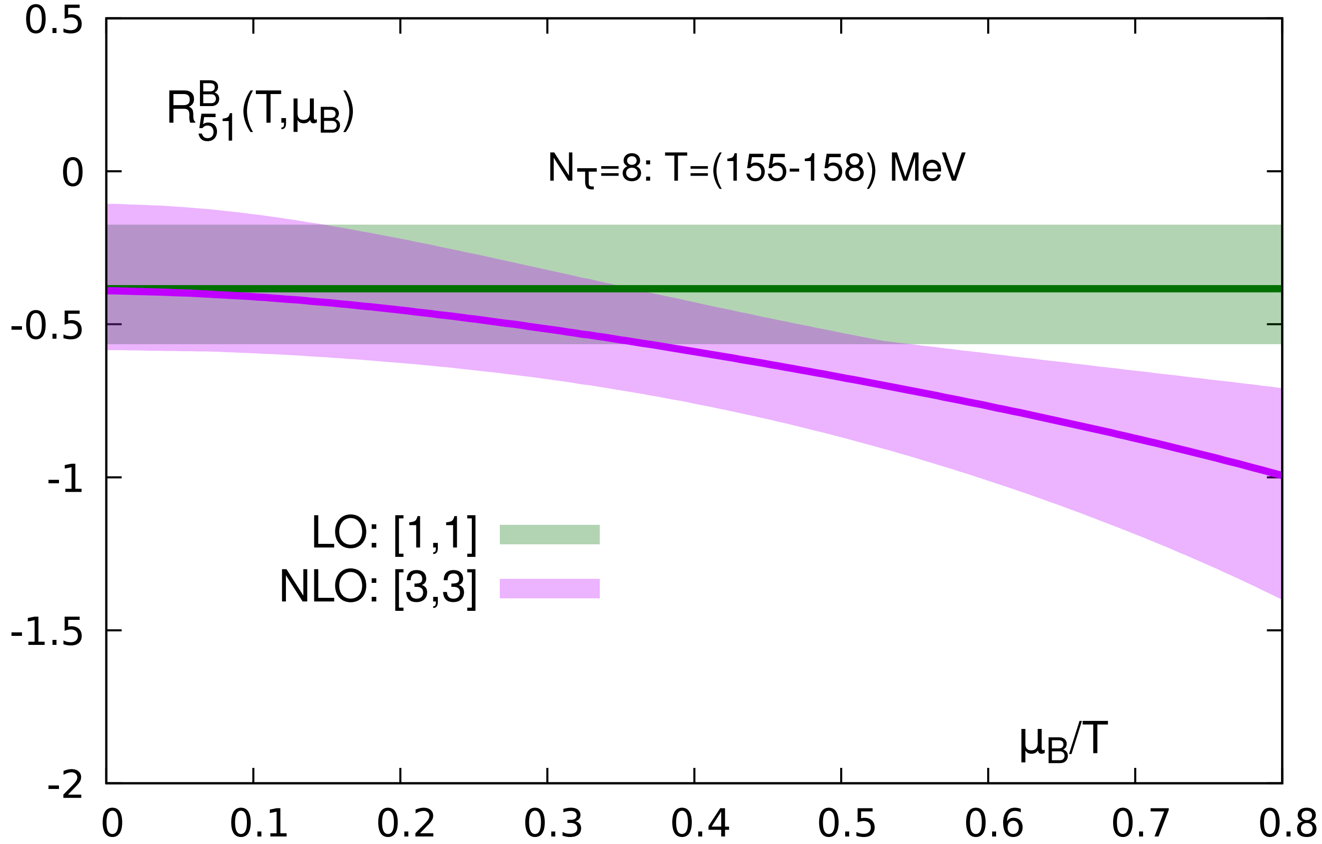
Results: Skewness and kurtosis

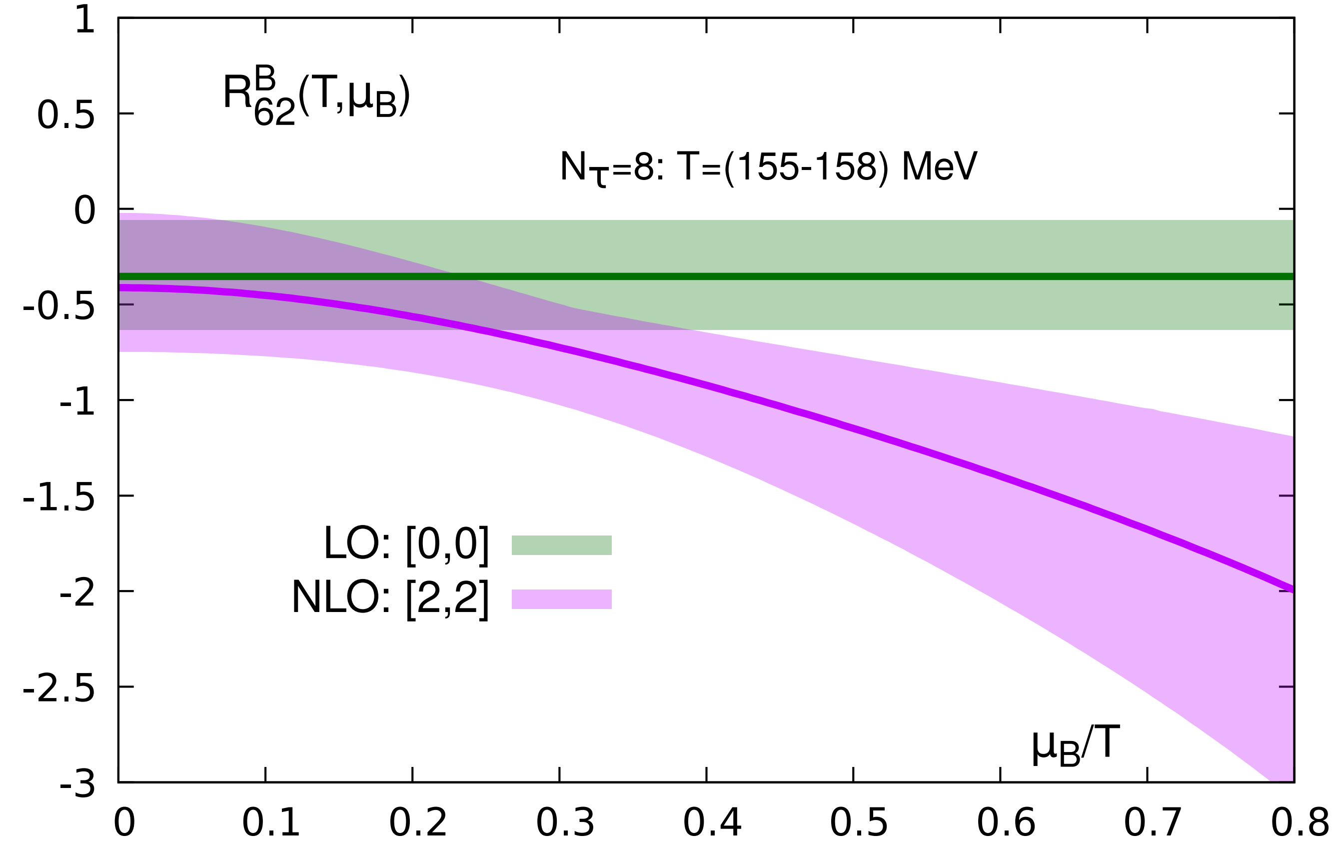


- Continuum estimates of R^B_{31} and R^B_{42} as function of R^B_{12} on the crossover line.
- Star data at $\sqrt{s_{NN}} = 54.4$ GeV favors a freeze-out temperature slightly below the crossover.
- The estimate of the freeze-out temperature $T_f = 165$ MeV for $\sqrt{s_{NN}} = 200$ GeV (from a statistical model analysis) is not consistent with a determination of T_f from the skewness and kurtosis data by STAR.

Results: Fifth and sixth order cumulant ratios R_{51}^B and R_{62}^B

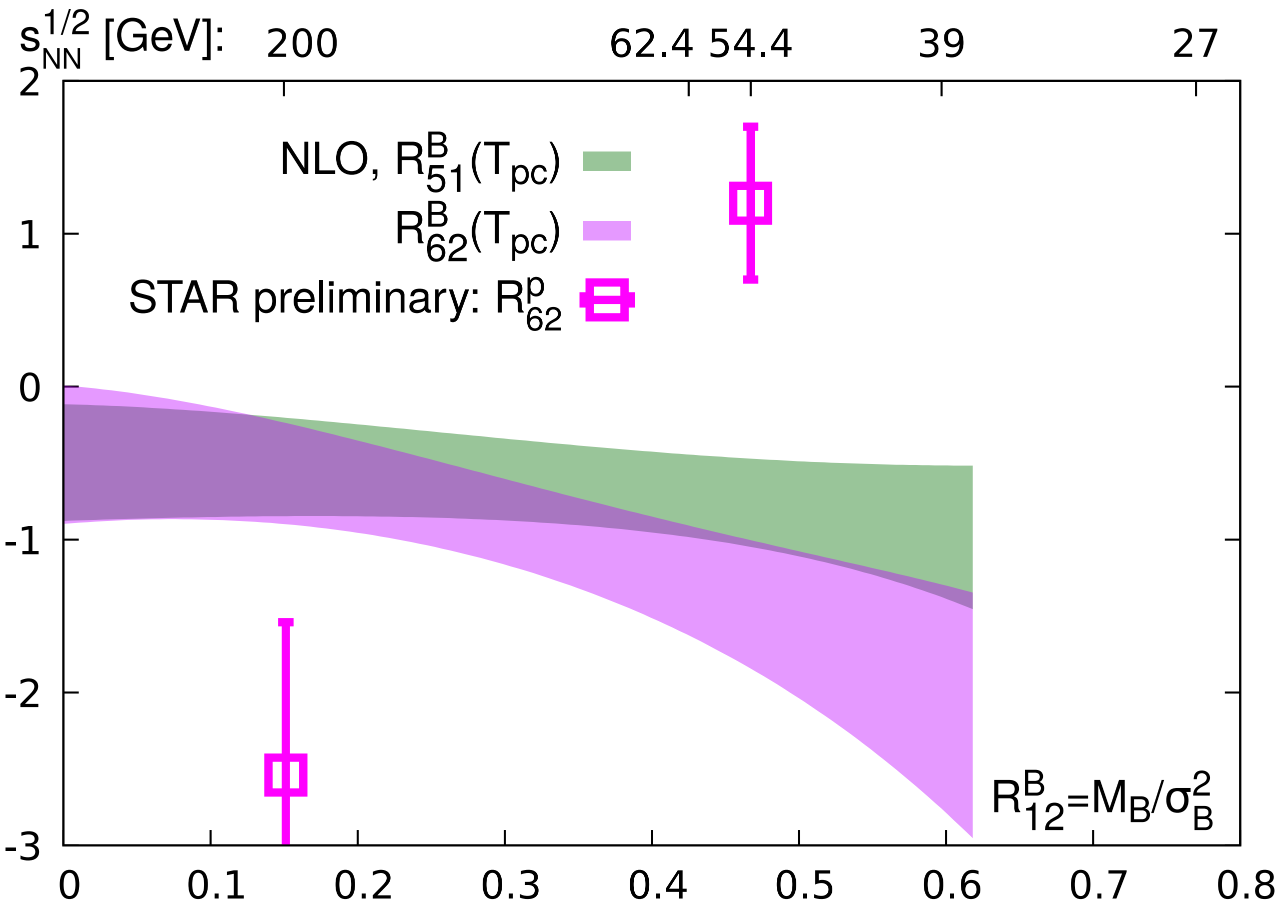
- R_{51}^B and R_{62}^B on $(N_\tau = 8)$-lattices





- Large statistical uncertainties
- NLO corrections are negative

Results: Fifth and sixth order cumulant ratios R^B_{51} and R^B_{62}



- R^B_{51} and R^B_{62} on $(N_\tau = 8)$-lattices
- Not consistent with STAR data:
  - ➡ A. Pandav@SQM19

$$\sqrt{s_{NN}} = 200 \text{ GeV}: R^P_{62} < 0$$

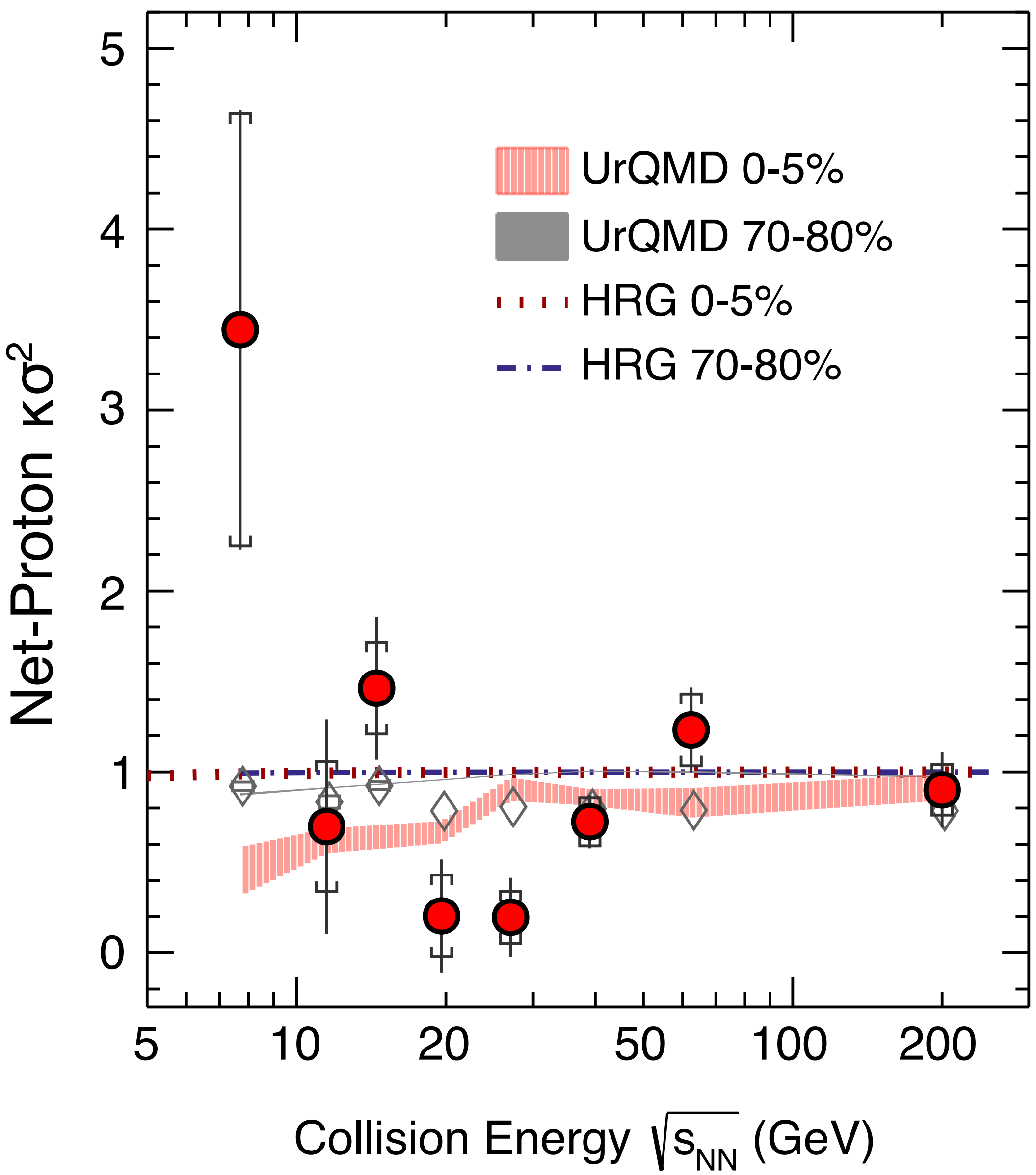
$$\sqrt{s_{NN}} = 54.4 \text{ GeV}: R^P_{62} > 0$$

  - ➡ Lattice QCD predictions

$\sqrt{s_{NN}}$	R^B_{51}	R^B_{62}
200	$-0.5(3)$	$-0.7(3)$
54.4	$-0.7(4)$	$-2(1)$

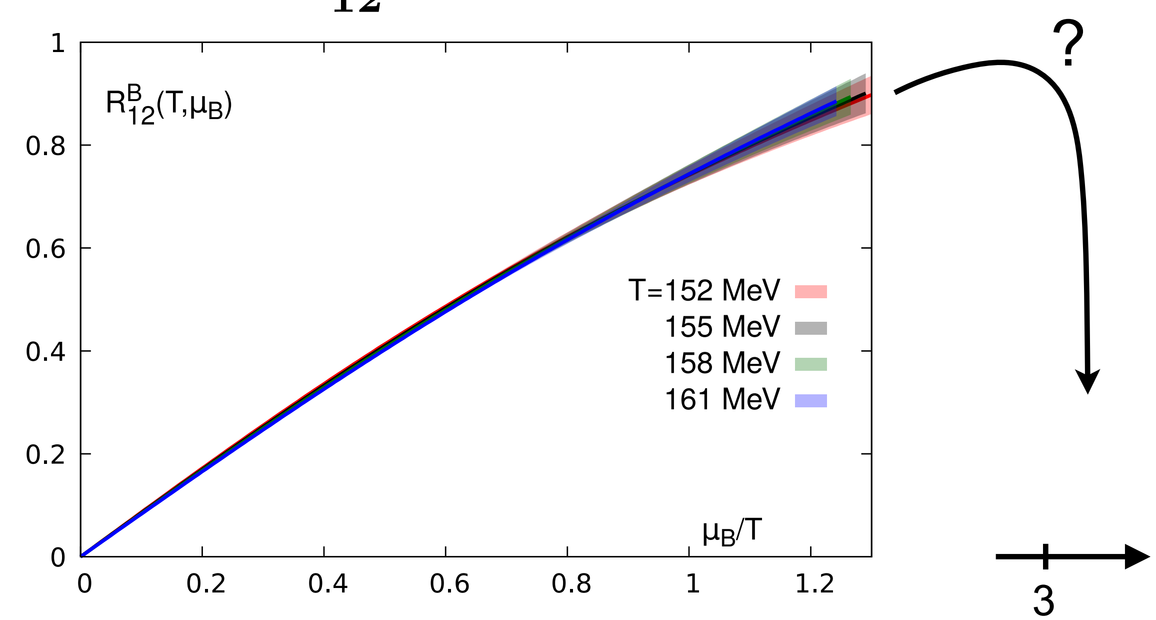
What about the critical point?

STAR, arXiv:2001.02852



- STAR: indication for non-monotonic behavior in R^P_{42} for $s^{1/2}_{NN} < 27$ GeV with 3.0σ significance [2001.02852].
  $\Rightarrow$ Hint at the critical point?

- Lattice: would need a reliable10th-order calculation to see non-monotonic behavior in R^B_{42}.
  $\Rightarrow$ Look for a divergence of χ^B_2, a zero of R^B_{12}



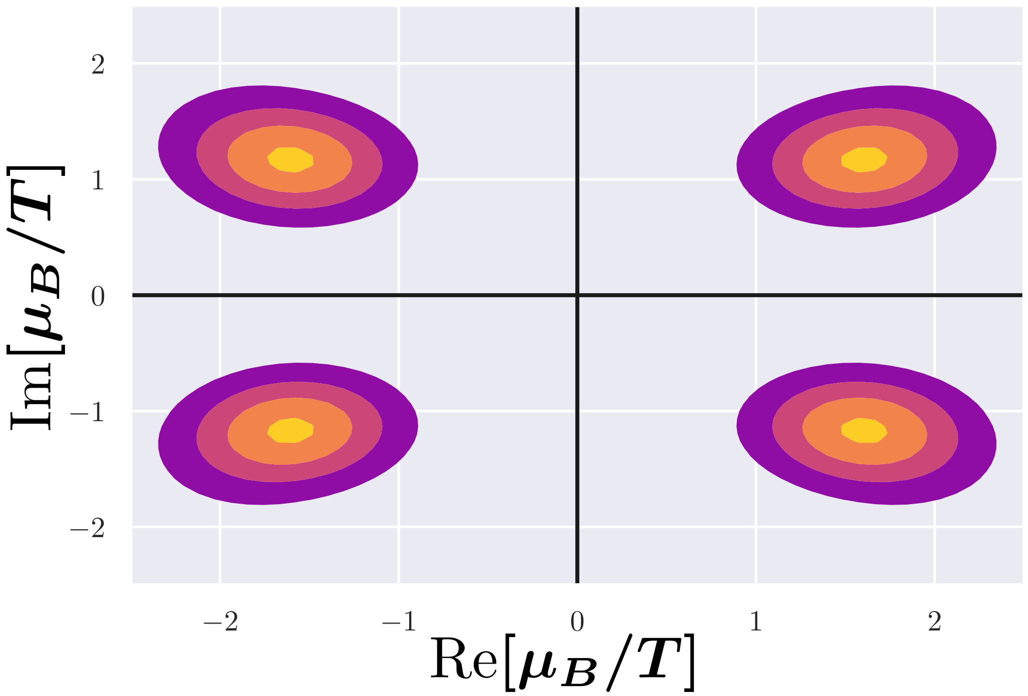
$\Rightarrow$ Need smaller temperatures

$\Rightarrow$ Look for poles of χ^B_2 in the complex plane

Singularities in the complex plane

$T = 145$ MeV

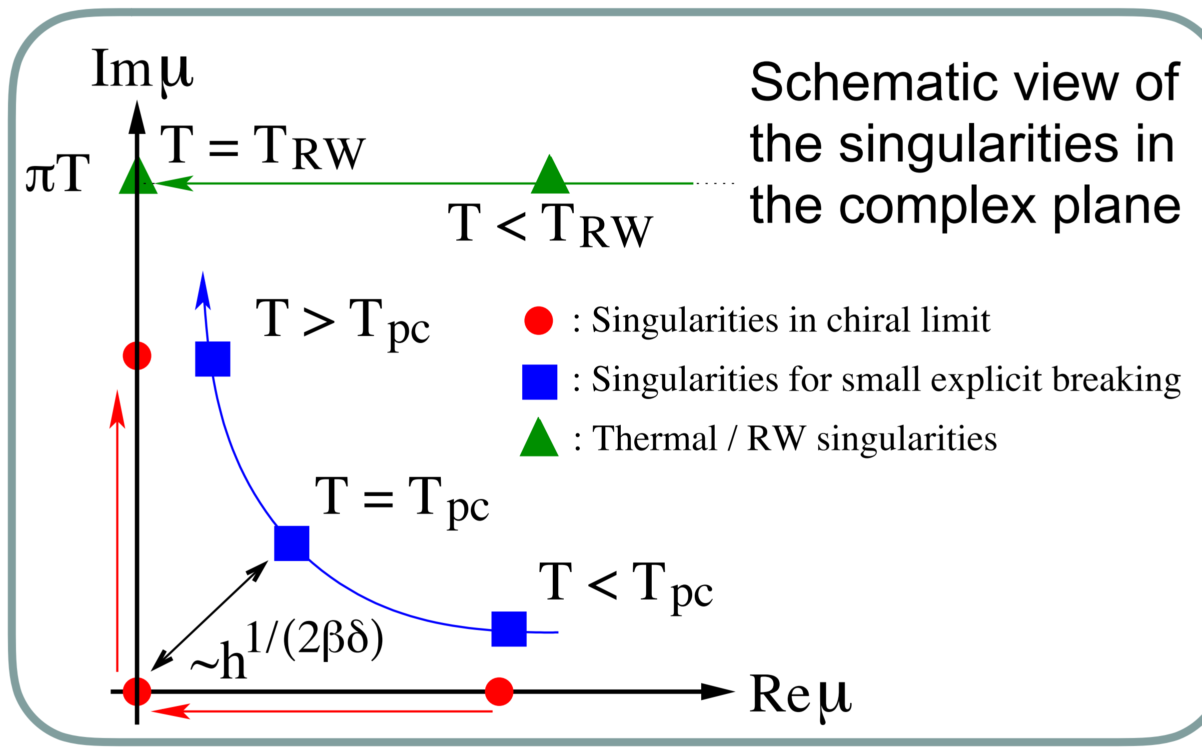
Poles of the [2,4]-Padé



- Convert the Taylor series of χ_2^B into a [n,m]-Padé, with the constraint $n+m \leq 6$.
- Estimate of the radius of convergence of the Taylor series $\rho \sim (1.8 - 2.0)$

  $\Rightarrow$ Need to find resummation schemes to extend range of validity

- The poles may approach the real axis for $T \to T_{cep}$.

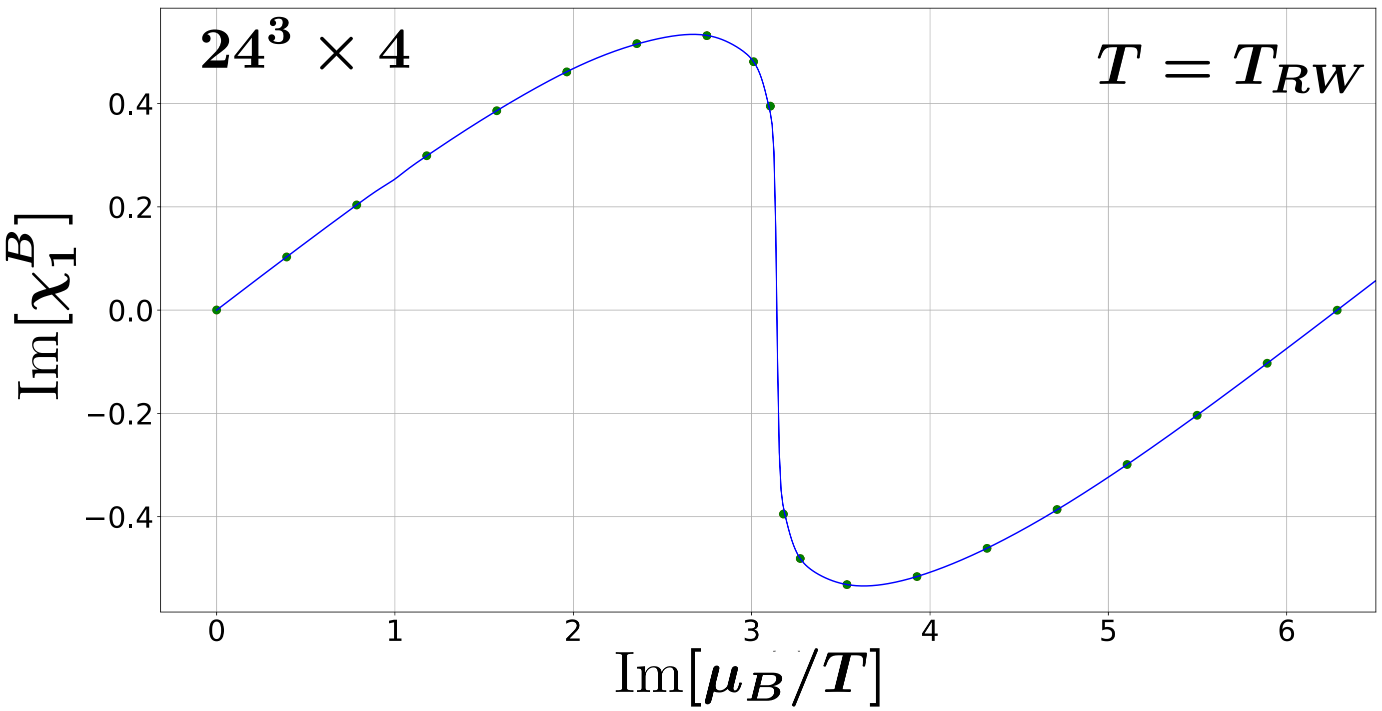


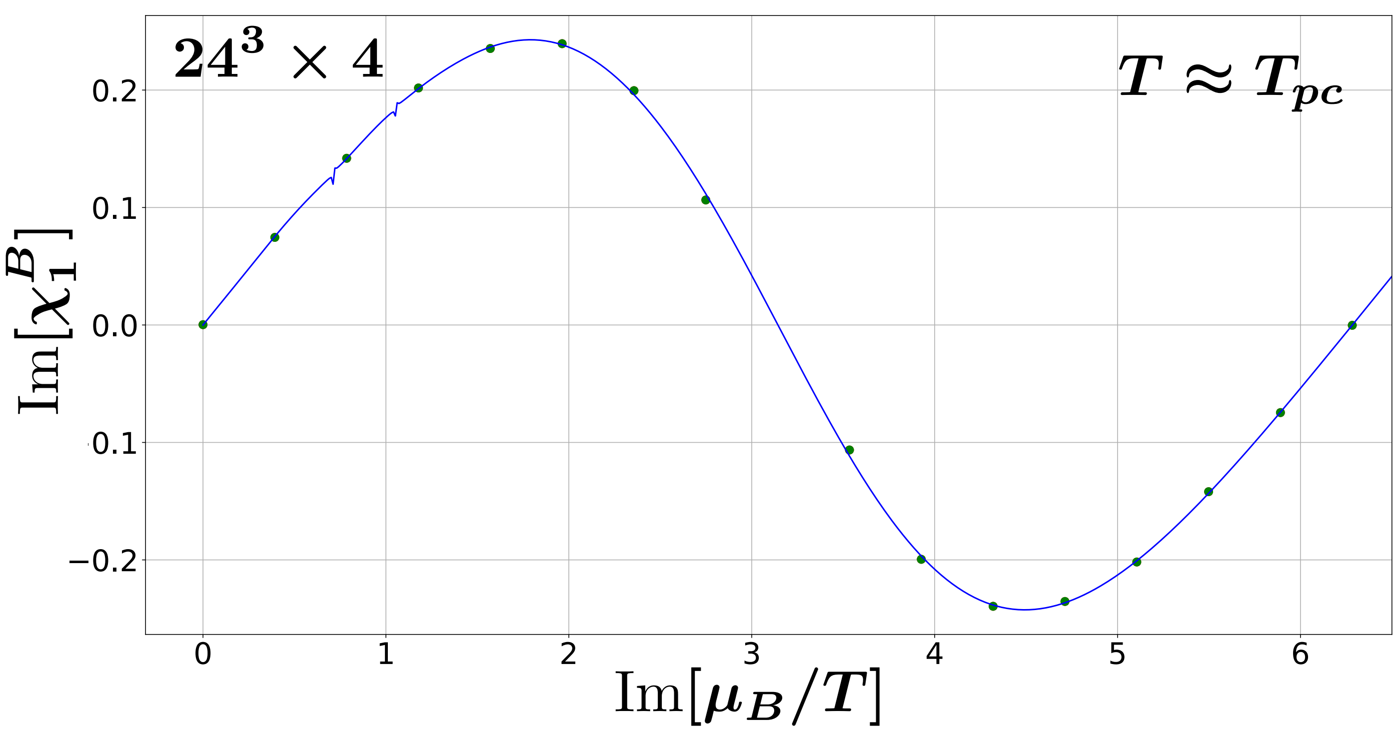
[Almási et al, PRD 100, 016016 (2019)]

$\Rightarrow$ See also: V. Skokov, INT 2020; M. Stephanov, hep-lat/0603014; C. Itzykson, et al Nucl.Phys. B220 (1983) 415.

Singularities in the complex plane

Bielefeld-Parma (preliminary)





- Add lattice calculations at imaginary chemical potential to constrain the singularities further
- Taylor expansion to $\mathcal{O}((\mu_B/T)^4)$ at each point
- Fit a rational function of high order to the data points and derivatives
- Investigate poles of the function

Singularities in the complex plane

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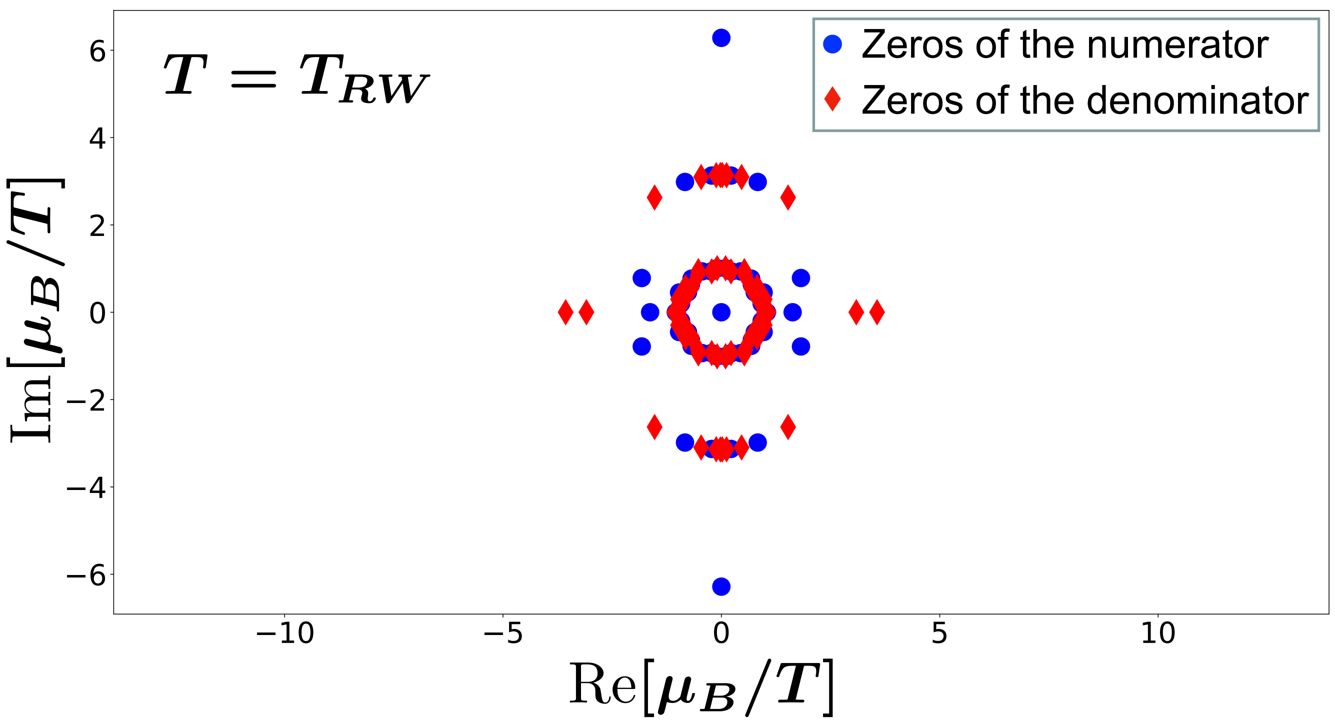

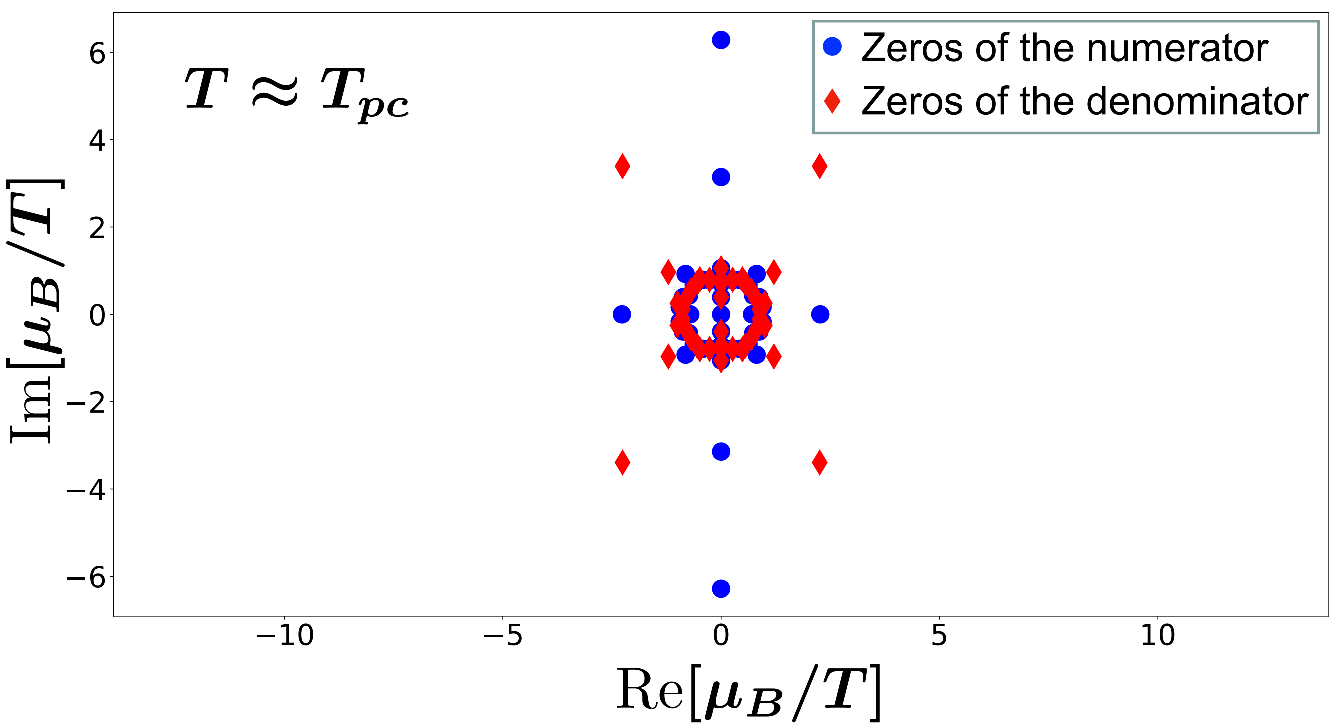

- Obtain zeros of the numerator and denominator
- Some zeros match to high precision, for others its not so clear…

Singularities in the complex plane

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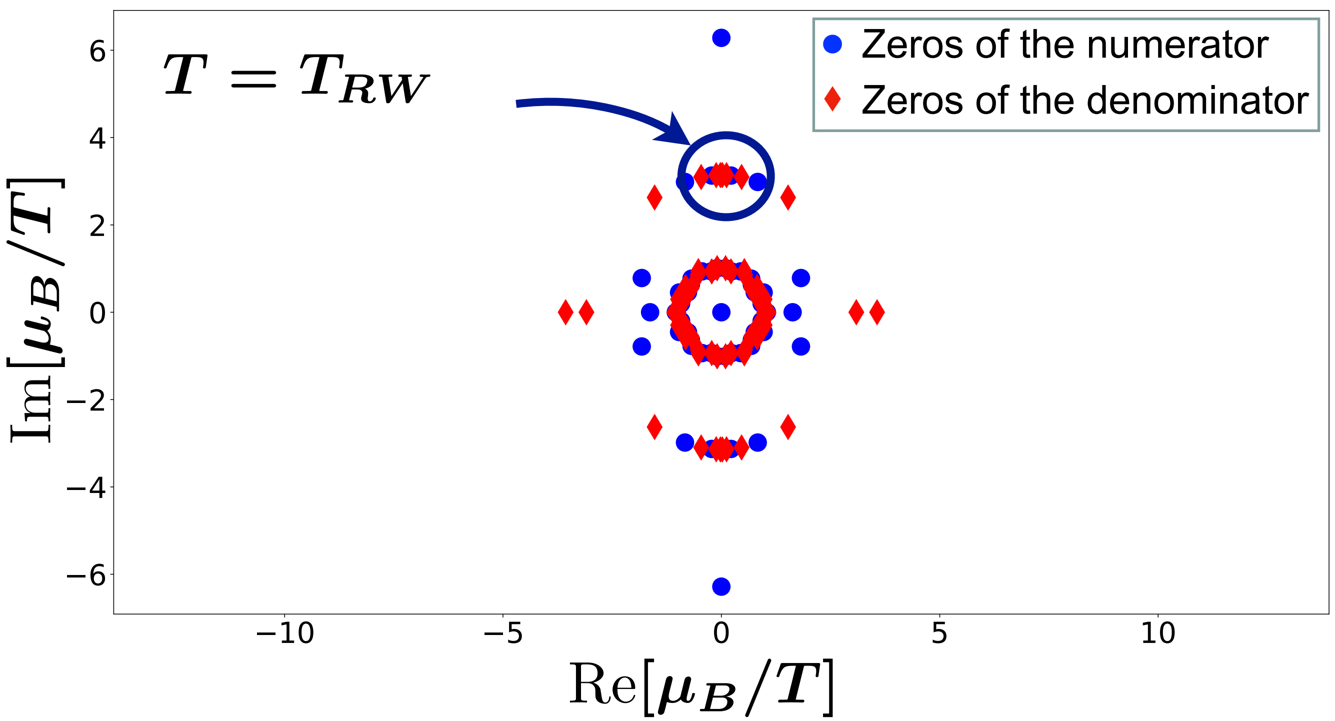

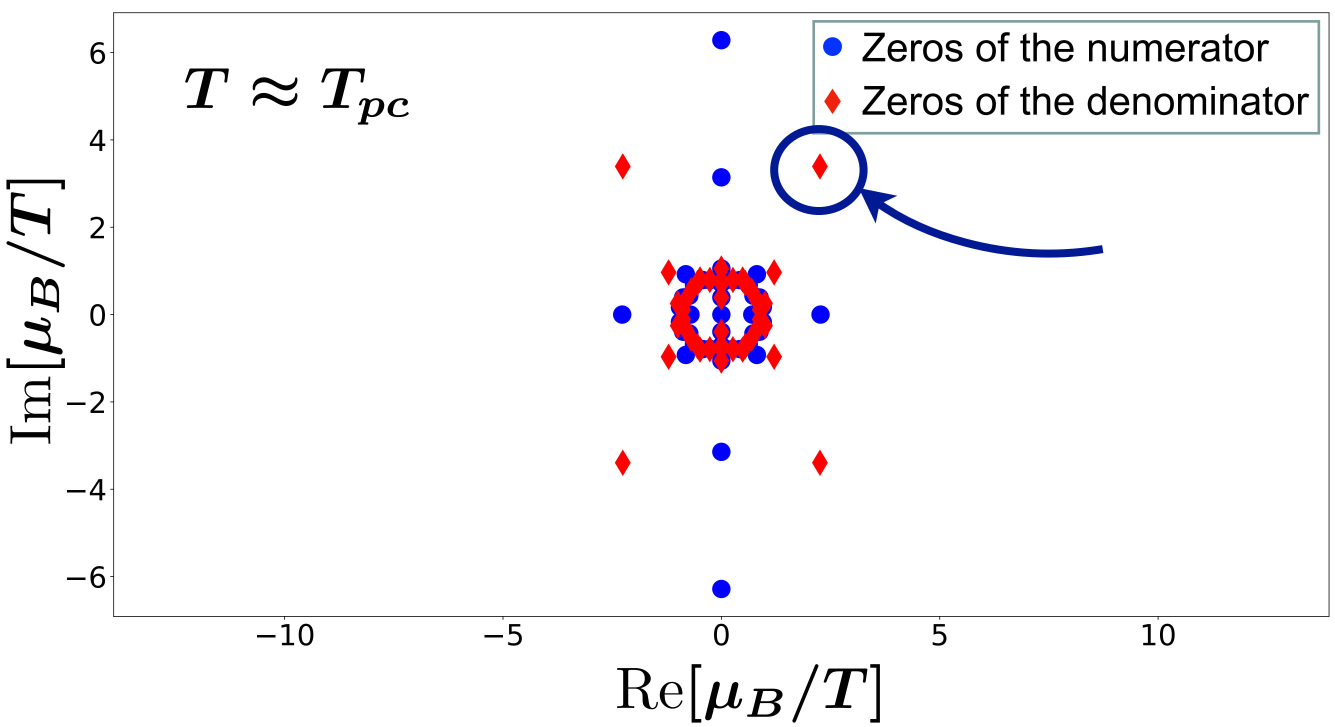

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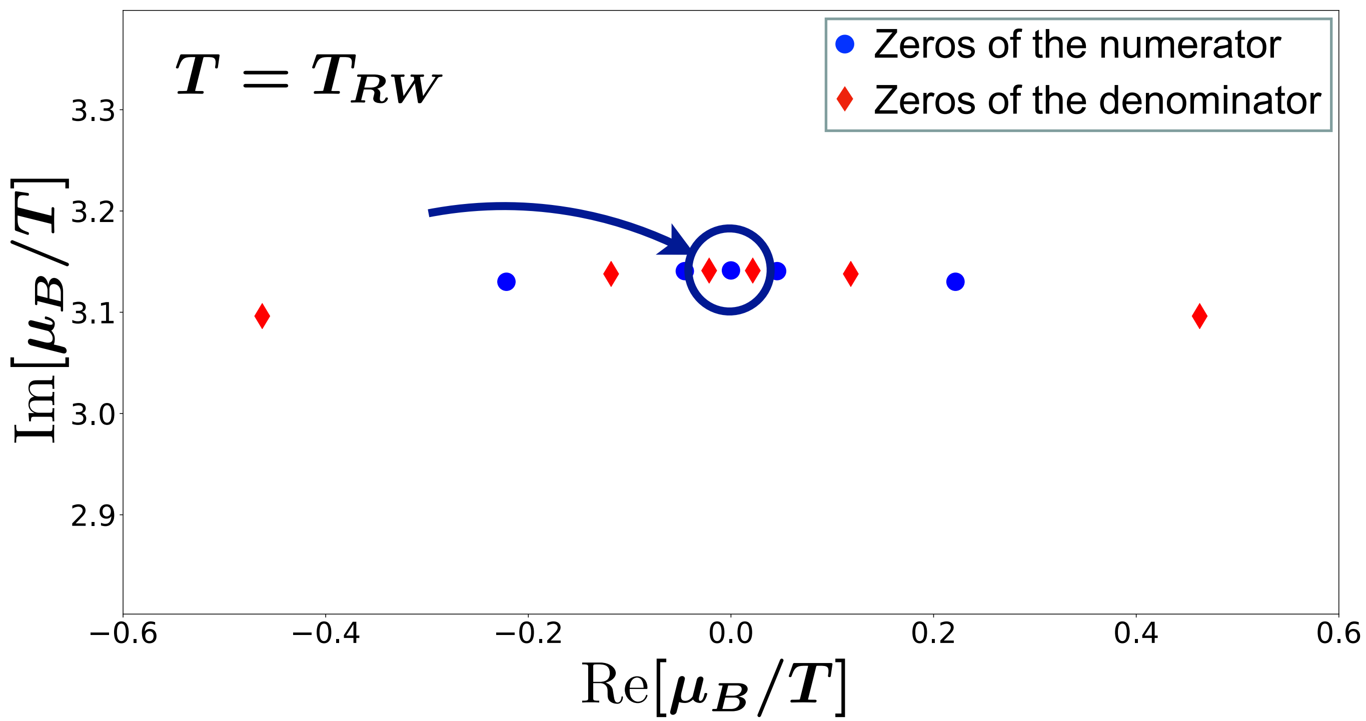

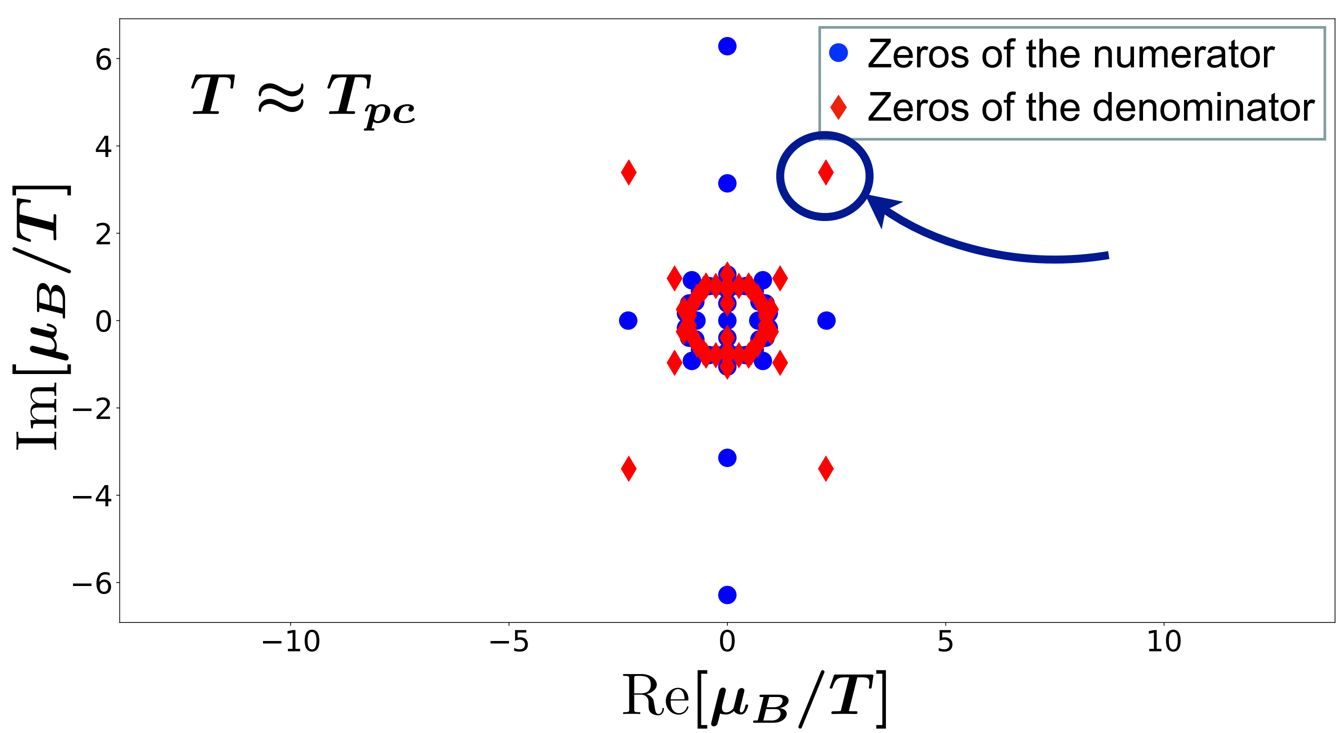

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- Some zeros match to high precision, for others its not so clear…
- The RW-singularity seems to be stable
- Other singularities need further investigations…work in progress.
- Singularities may also be understood in terms of the Fourier coefficients of the $\mathrm{Im}[\chi_1^B]$

[Vovchenko et al, PRD 97, 114030 (2018)]

[Almási et al, PRD 100, 016016 (2019)]

[Almási et al, PLB 793 (2019)]

Summary and outlook

- Lattice QCD calculations show a significant increase in precision for cumulant ratios along the crossover line in the QCD phase diagram for $\mu_B/T \leq 1.2$ due to increase in the statistics and thus also the order of the expansion.
- Presented first calculations of R_{51}^B and R_{62}^B along the crossover line.
- Freeze-out temperature at $\sqrt{s_{NN}} = 200\ \mathrm{GeV}$ seems not consistent with higher order cumulants.
- At $\sqrt{s_{NN}} = 54.4\ \mathrm{GeV}$ we find thermodynamic consistency of the STAR data except for the R_{62}^P data.
- From preliminary data at imaginary chemical potential we could identify the RW-singularity in the complex chemical potential plane, other singularities need further investigations.
- **Outlook:**
  - Need to increase statistics for ($N_\tau = 12$)- and ($N_\tau = 16$)-lattices further.
  - Investigate resummation schemes to push Taylor expansion results to larger μ_B/T.
  - Evaluate Fourier coefficients of the net-baryon density.