CRC–TR 211
Strong-interaction matter under extreme conditions

UNIVERSITÄT BIELEFELD

Comparing conserved charge fluctuations from lattice QCD to HRG model calculations

Jishnu Goswami

With : F. Karsch, S. Mukherjee, P. Petreczky and C. Schmidt

HotQCD Collaboration

Acknowledgement: To all members of the collaboration

To venture an opinion is like moving a piece at chess: it may be taken, but it forms the beginning of a game that is won-----Johann Wolfgang von Goethe

If/when

Fluctuations in relativistic heavy ion collisions are described by the hadronic degrees of freedom in equilibrium

→ Consistent With HRG

- Deviation will suggest existence of Additional interaction/QCD critical point/ Non-equilibrium physics.....

And, since QCD(Lattice) is basically describing a equilibrium thermodynamical/ statistical mechanical system

→ Consistent With HRG

- Deviations will suggest changes of degrees of freedom.........

Probing freeze-out conditions in heavy ion collisions with moments of charge fluctuations

F. Karsch[1,2] and K. Redlich[3,4]

Of course, eventually thermodynamics at freeze-out should be described by thermal QCD, eg. through lattice calculations. A direct comparison of experimental and HRG model results on higher moments of charge fluctuations with lattice calculations in the hadronic phase is possible [13], but is still difficult as so far most lattice calculations are performed with staggered fermions on rather coarse lattices. They need to be performed closer to the continuum limit to reproduce the correct hadron spectrum [14, 15]. Fortunately the calculation of ratios of moments is less sensitive to such cut-off effects [5, 13, 16]. At present, it seems that lattice QCD calculations of ratios of moments of charge fluctuations, performed at non-vanishing chemical potential by using a Taylor expansion of thermodynamic observables [17, 18], are in good agreement with HRG model calculations for temperatures below the transition temperature. We will consider this issue in more

The Proposal

QGP is a transient state. If formed in HIC it will cool back to hadronic matter at low temperature. Combination of all possible hadrons and hadronic resonances is called HRG(Hadron Resonance Gas)

HRG: Particles listed in PDG booklet.

M. Tanabashi et al. (Particle Data Group), Phys. Rev. D 98, 030001 (2018)

QMHRG: Particles listed in PDG + resonances predicted by Quark Model.

S. Capstick and N. Isgur, Phys. Rev. D 34, 2809 (1986).

D. Ebert, R. N. Faustov and V. O. Galkin, Phys. Rev. D 79, 114029 (2009)

Naive expectation: Attractive and repulsive Interaction will be small at temperature, $T \lesssim T_{pc}(T_f)$

Pressure of hadron resonance gas(HRG),

$$P = \sum_H \frac{g}{2\pi^2} T^2 m_H^2 \sum_{n=1}^{\infty} \frac{(-\eta)^{n+1}}{n^2} K_2\left(\frac{n m_H}{T}\right) \exp[n\vec{\mu}\cdot\vec{c}/T]$$

$$\mu = (\mu_B, \mu_Q, \mu_S), c = (B, Q, S)$$

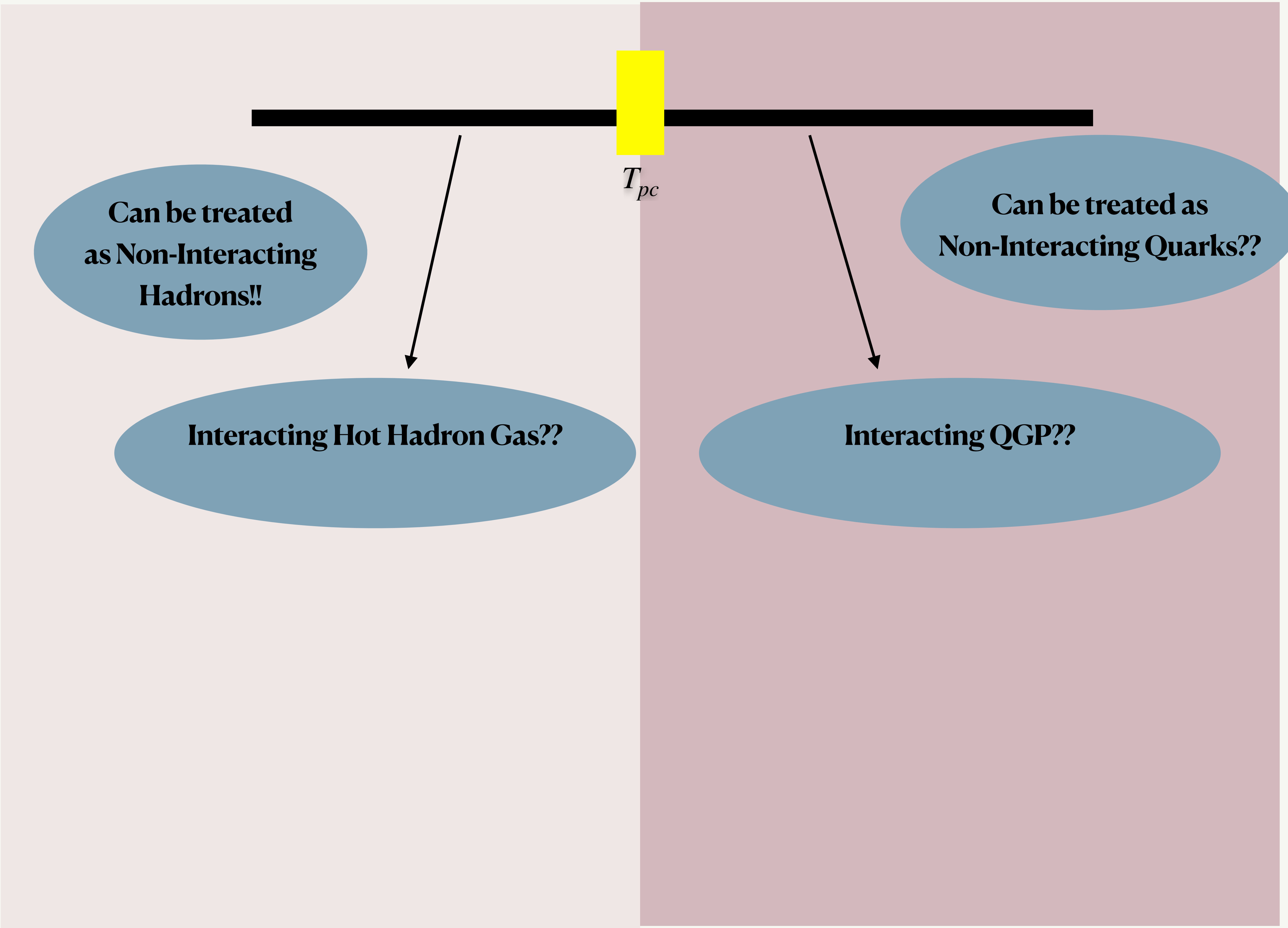
H is all possible (anti-)baryons and (anti-)mesons,

$$\chi_{lmn}^{BQS} = \left.\frac{\partial^{l+m+n}}{\partial\mu_B^l \mu_Q^m \mu_S^n} P\right|_{\vec{\mu}=0} \quad \text{HRG}$$

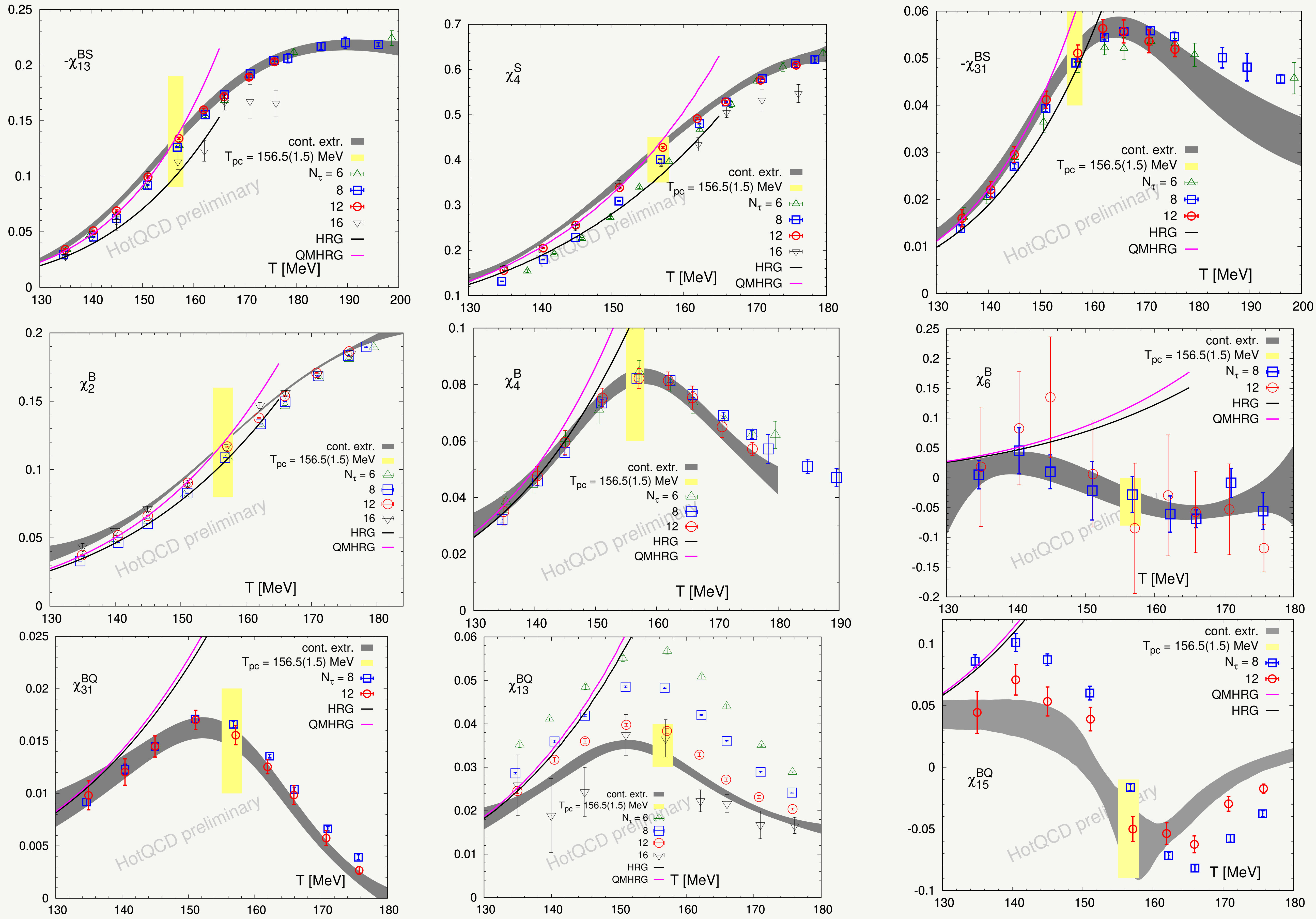
$$\sim B^l Q^m S^n \sum_n P_n$$

$$Z(T, V, \vec{\mu}) = \int DU \ \det M(\mu) \exp(-S_g)$$

$$\chi_{lmn}^{BQS} = \left.\frac{\partial^{l+m+n}}{\partial\mu_B^l \mu_Q^m \mu_S^n} \ln Z\right|_{\vec{\mu}=0} \quad \text{LQCD}$$


T_{pc}
Can be treated as Non-Interacting Hadrons!!
Can be treated as Non-Interacting Quarks??
Interacting Hot Hadron Gas??
Interacting QGP??

Cumulants and correlations



Eigen Volume HRG(EV-HRG)

Total pressure of our system,

$$P = \sum_{M} P_M + \sum_{B} P_B + \sum_{\bar{B}} P_{\bar{B}}$$

b is the excluded volume parameter.

Interaction between MM, $MB(\bar{B})$, $B\bar{B}$ is neglected

$$P_R = \sum_{i \in B(\bar{B})} P_{id}(T,0) \ \exp[(\mu - bP_R)/T]$$

$$P_R = \sum_{i \in B(\bar{B})} \frac{T}{b} \mathrm{W}\left(\frac{\mathrm{b}}{\mathrm{T}} \mathrm{P}_{\mathrm{id}}(\mathrm{T}, \mu)\right)$$ for, $b \neq 0$

W is the LambertW function

$$= P_{id} - (b/T)P_{id}^2(T) + (3b^2/2T^2)P_{id}^3(T) + \ldots\ldots$$

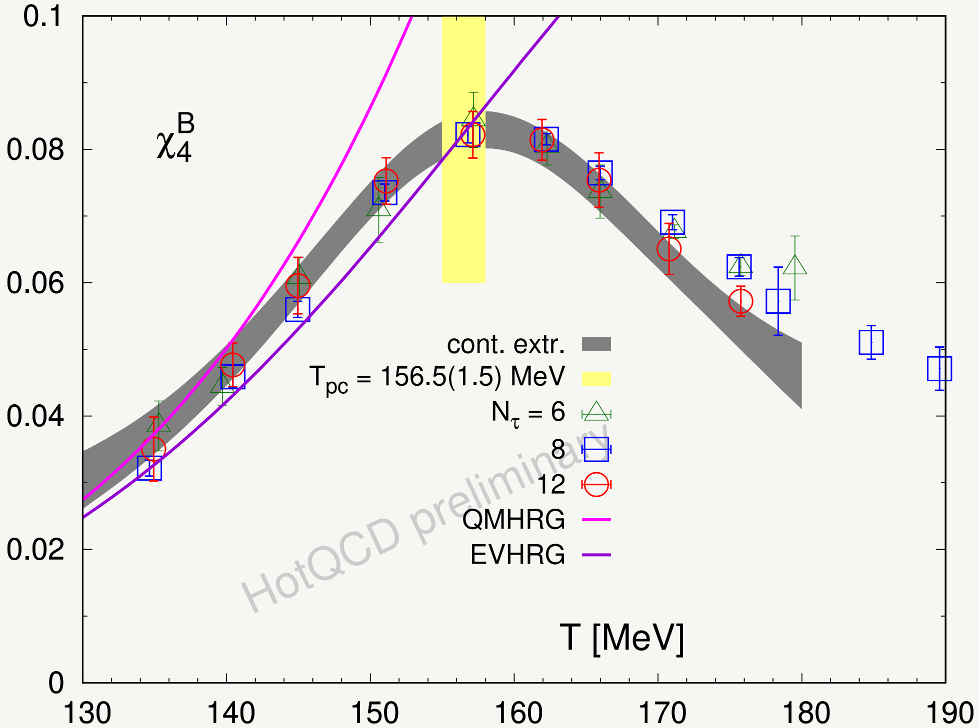
$$\chi_{lmn}^{BQS} = [\chi_{lmn}^{BQS}]_{id} - (2b/T)F\big(B, Q, S, P_{id}(T)\big) + (9b^2/T^2)G\big(B, Q, S, P_{id}(T)\big)$$
$$-(32b^3/3T^3)H\big(B, Q, S, P_{id}(T)\big)\ldots\ldots$$

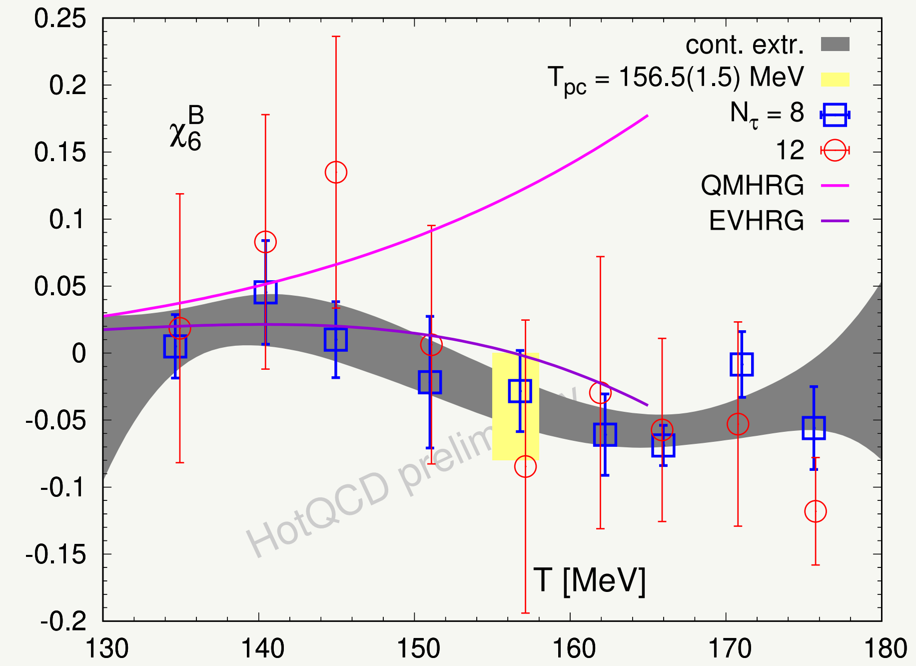
At, $T \ll T_{pc}$, $P_{id} \to 0$ **hence,** $P_R \to P_{id}$, $\chi_{lmn}^{BQS} \to [\chi_{lmn}^{BQS}]_{id}$

Ref: Excluded volume hadron gas model for particle number ratios in *AA* collisions, Mark I. Gorenstein et. al, arXiv: nucl-th/9711062.[More papers by their group]

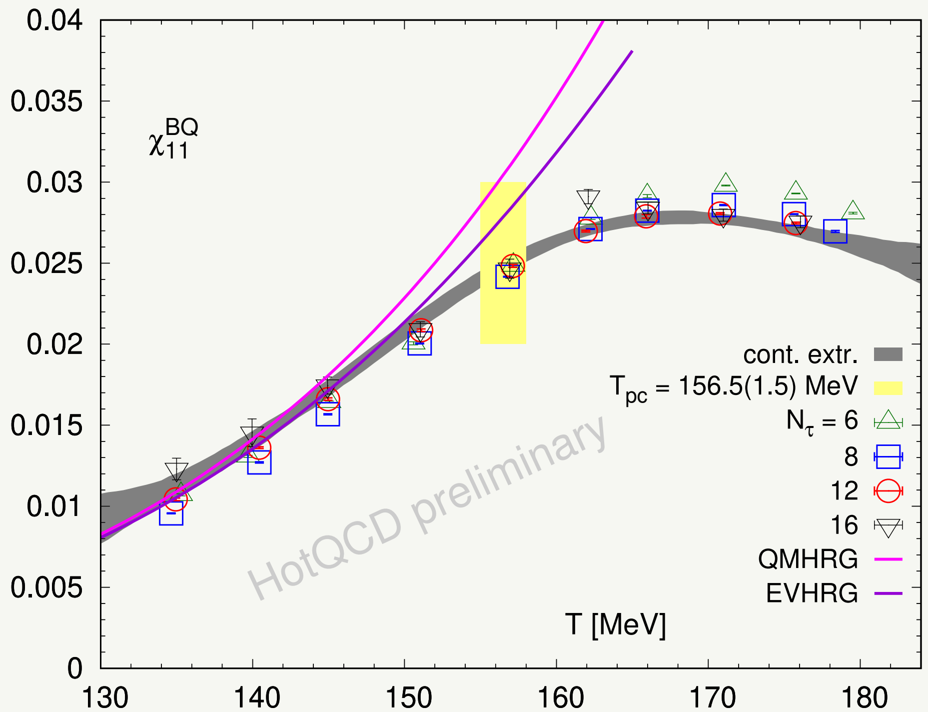
Conserved charge cumulants vs EVHRG

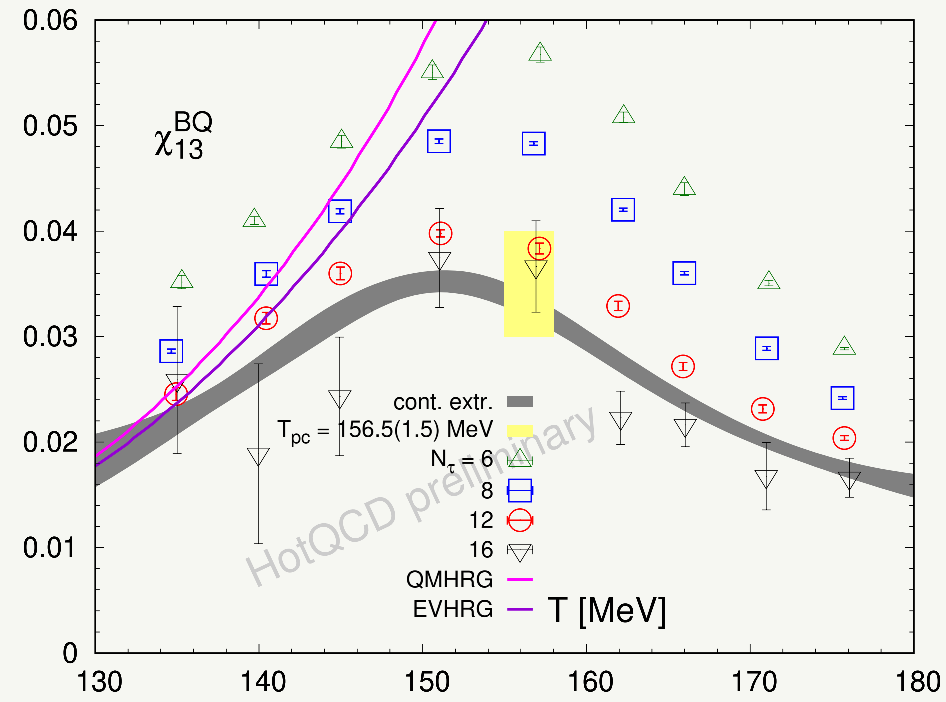
$b = 4(4\pi r^3/3), r \sim 0.5$ fm



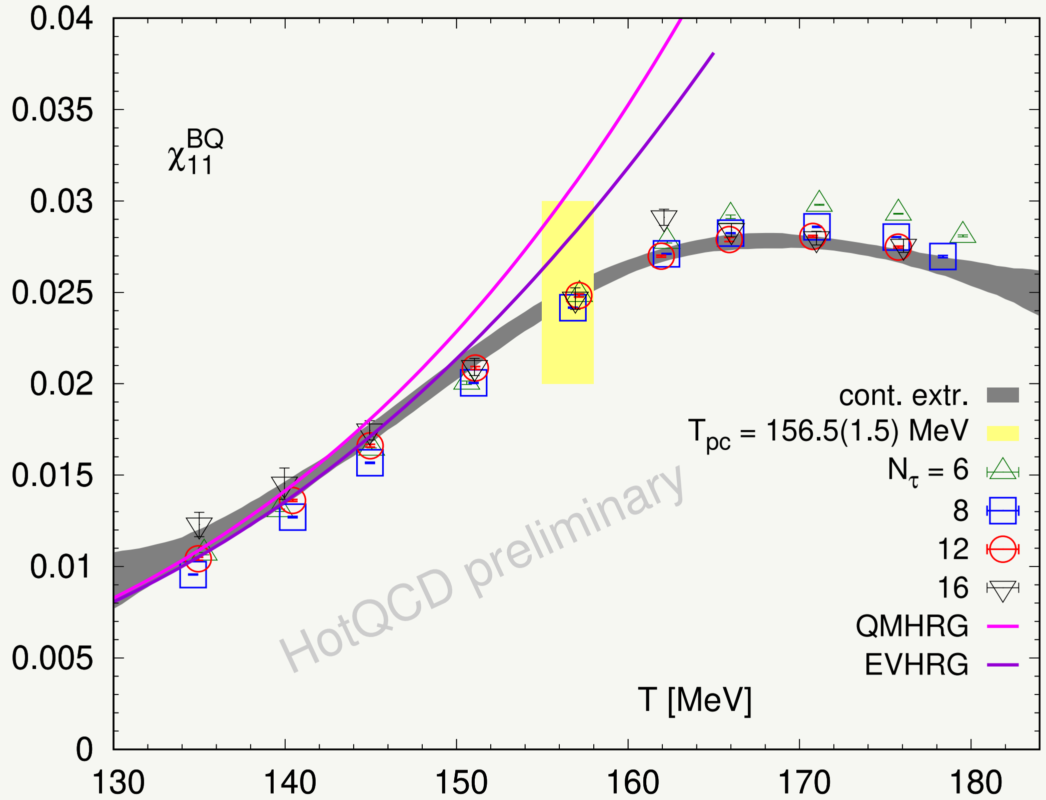


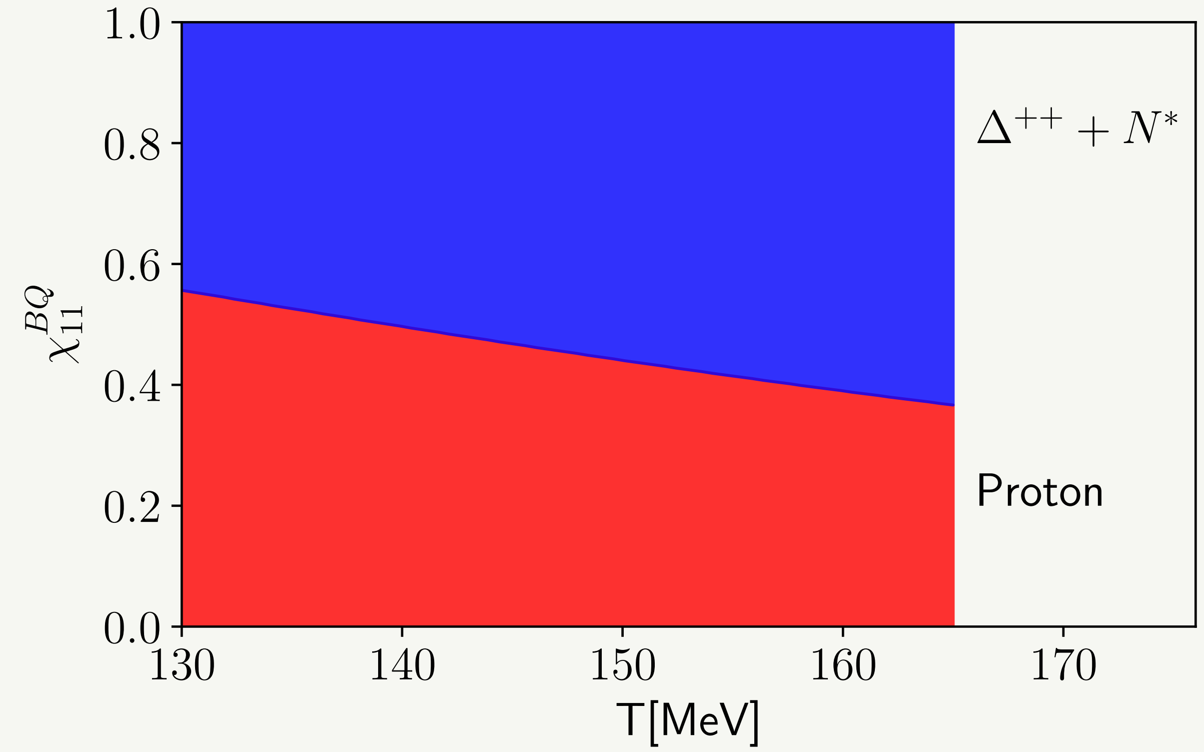
Since b is a free parameter consistent tuning across all sector probably not possible!!





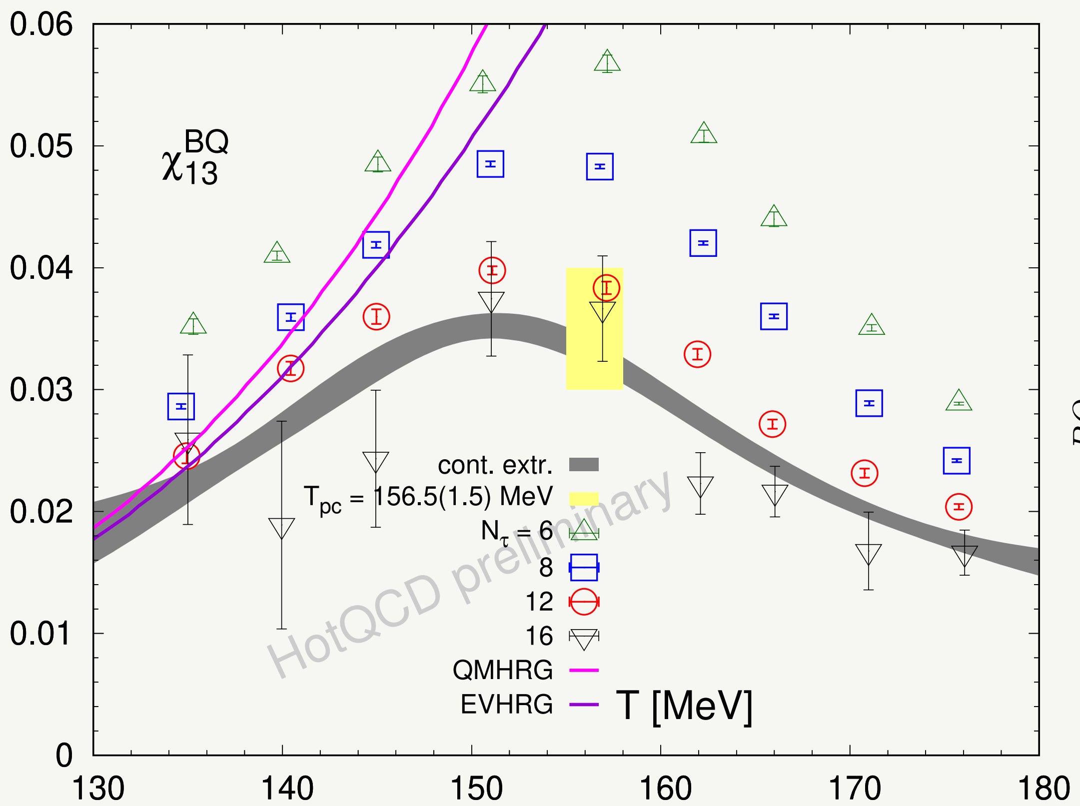
Importance of resonances

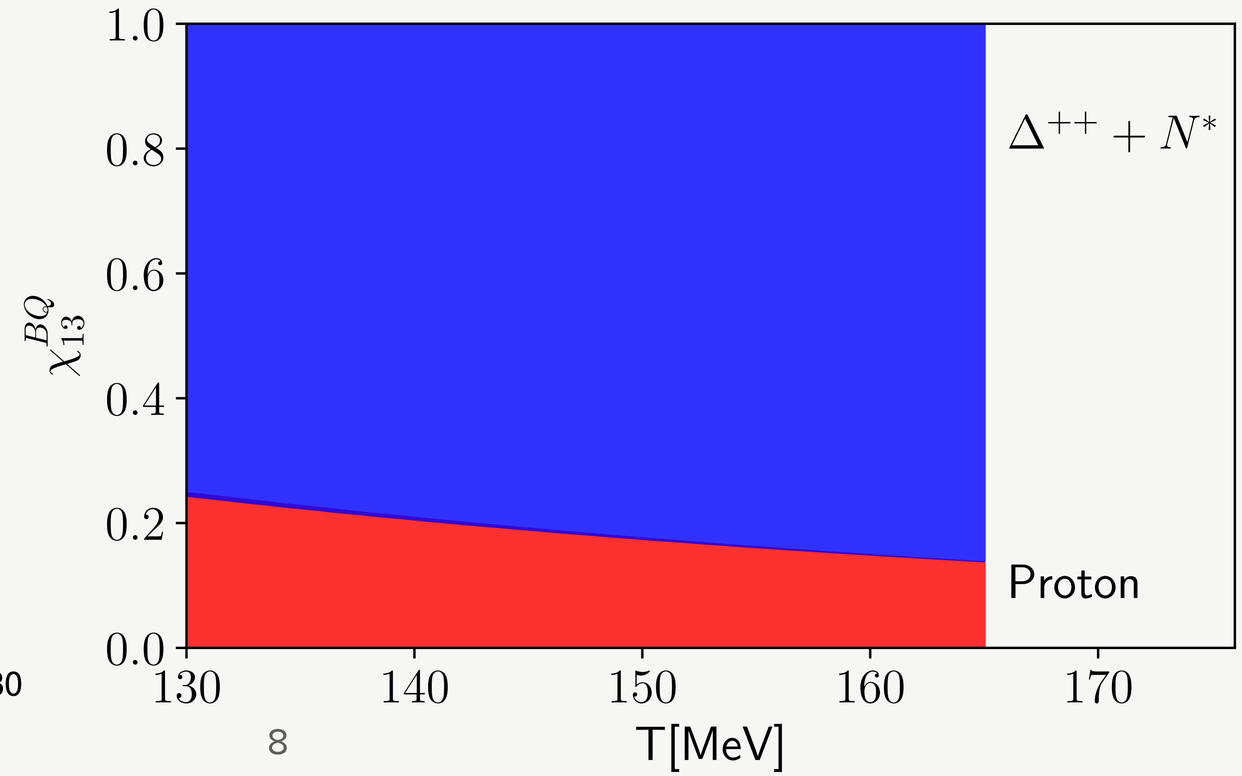




Ignoring Contribution of other Baryons $\leq 10\ \%$

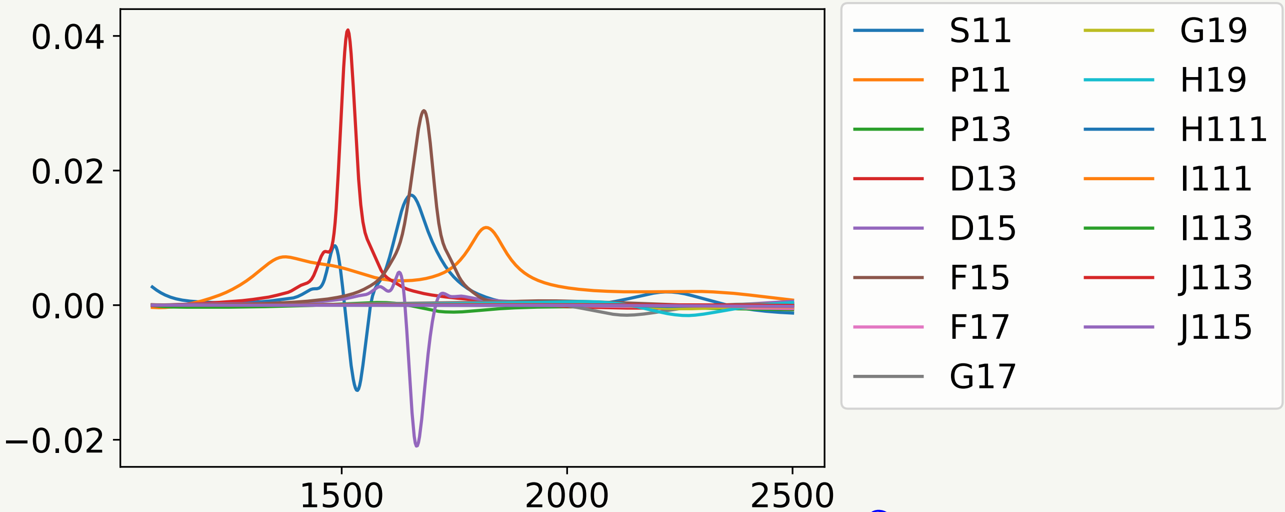
Contribution in a non interacting QMHRG

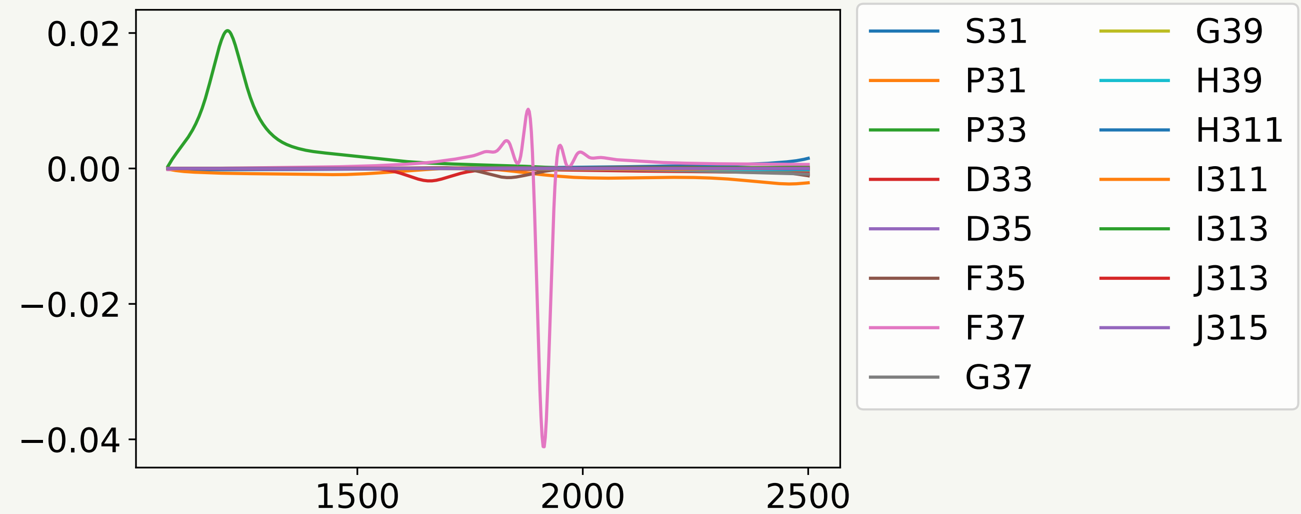




Ignoring Contribution of other Baryons $\leq 2\ \%$

Hadron interactions with Smatrix





$$P_{id} = \sum_{B(\bar{B})} \frac{g}{2\pi^2} \frac{m_H^2}{T^2} K_2\left(\frac{m_H}{T}\right)$$

http://gwdac.phys.gwu.edu/analysis/pin_analysis.html

$$P_{int} = \frac{g}{2\pi^2} \int_{m_{th}}^{\infty} d\epsilon\ K_2(\epsilon/T)\ (\epsilon/T)^2\ \frac{d\delta_{IJ}}{\pi d\epsilon}$$

$$P = P_{id} + \sum_{I_z=3/2,1/2} P_{int}$$

$$\pi + N \ \rightarrow \ N^* \rightarrow \pi + N$$

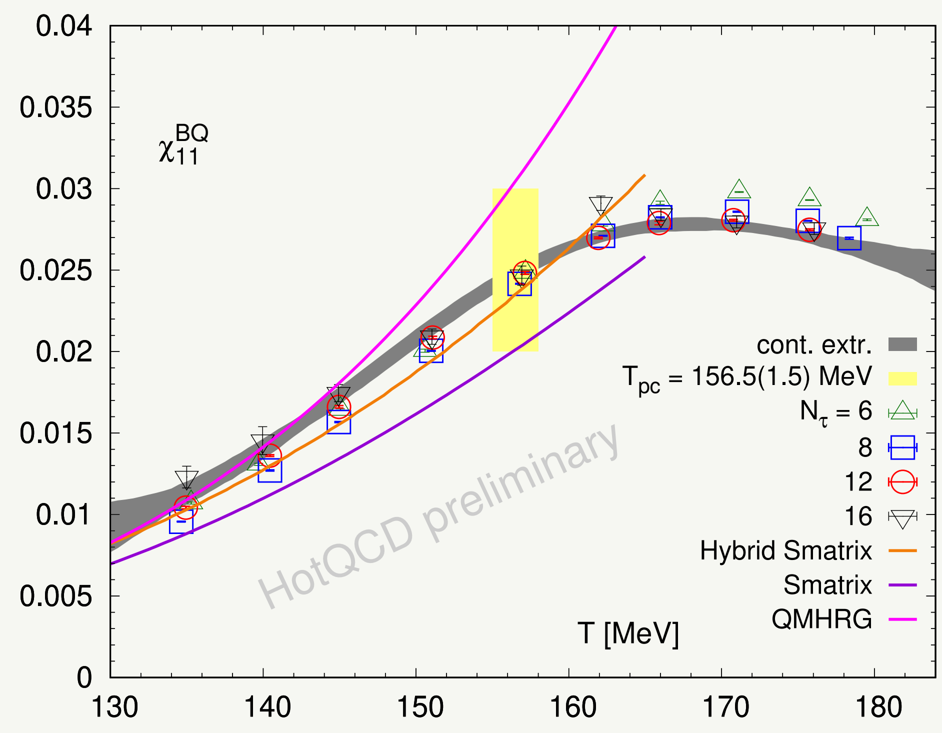
$$\pi + N \ \rightarrow \ \Delta^{++} \rightarrow \pi + N$$

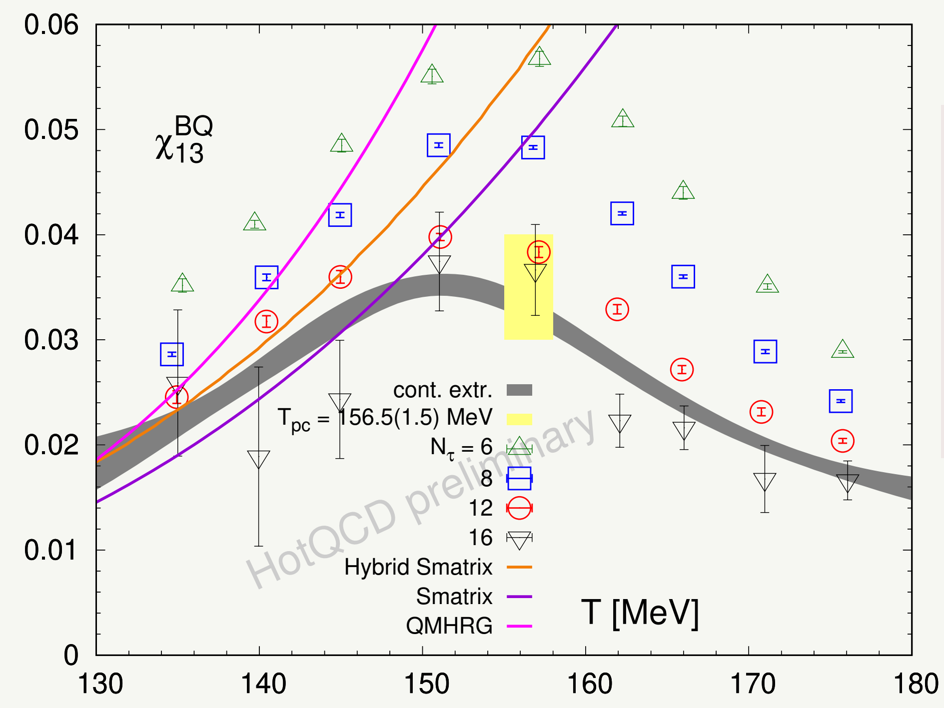
Other channels are missing!!

For Delta's, $Q = [2,1,-1,0]$ For N*'s, $Q = [1,-1]$

Note that, $d\delta_{IJ}/d\epsilon \sim \pi\delta(\epsilon - m_H)$ $P_{int} \Rightarrow P_{id}$, stable particles. e. g. nucleon gas.

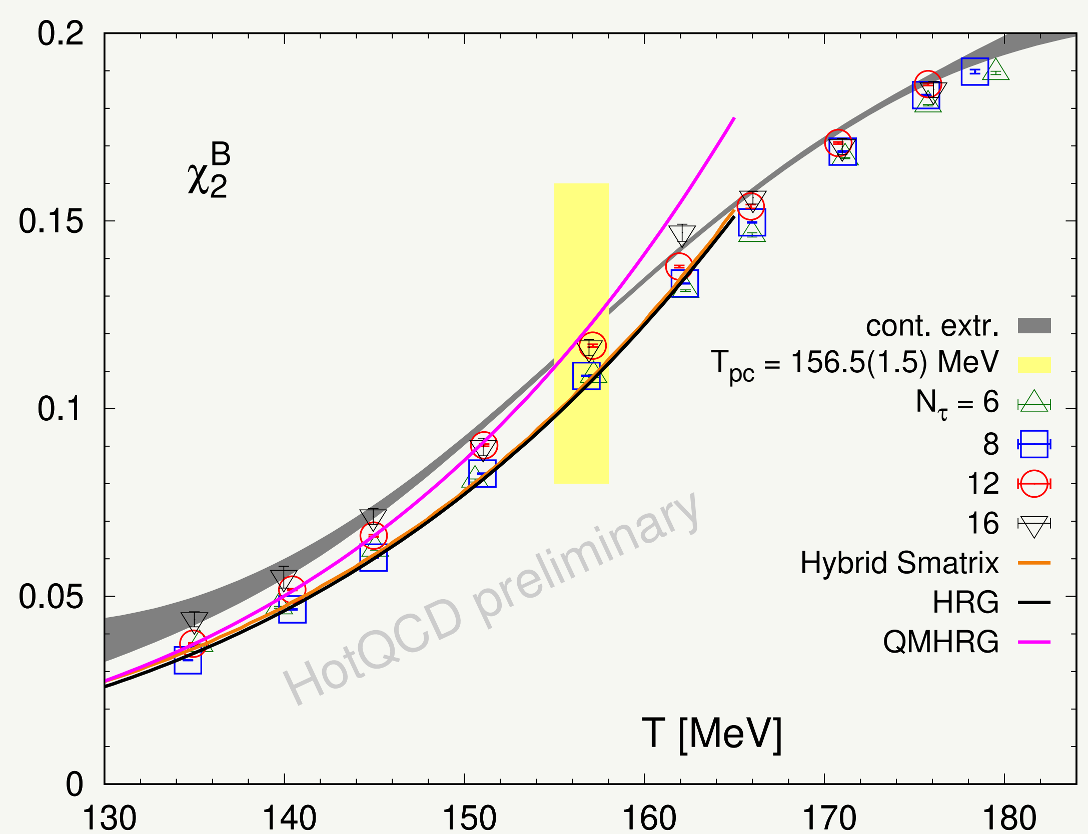
Ref: Phys.Lett.B 778 (2018) 454-458, P Man Lo, B Friman, K Redlich, C Sasaki , Phys. Rev. C 99, 044919 (2019), A. Dash, S. Samanta, B. Mohanty.

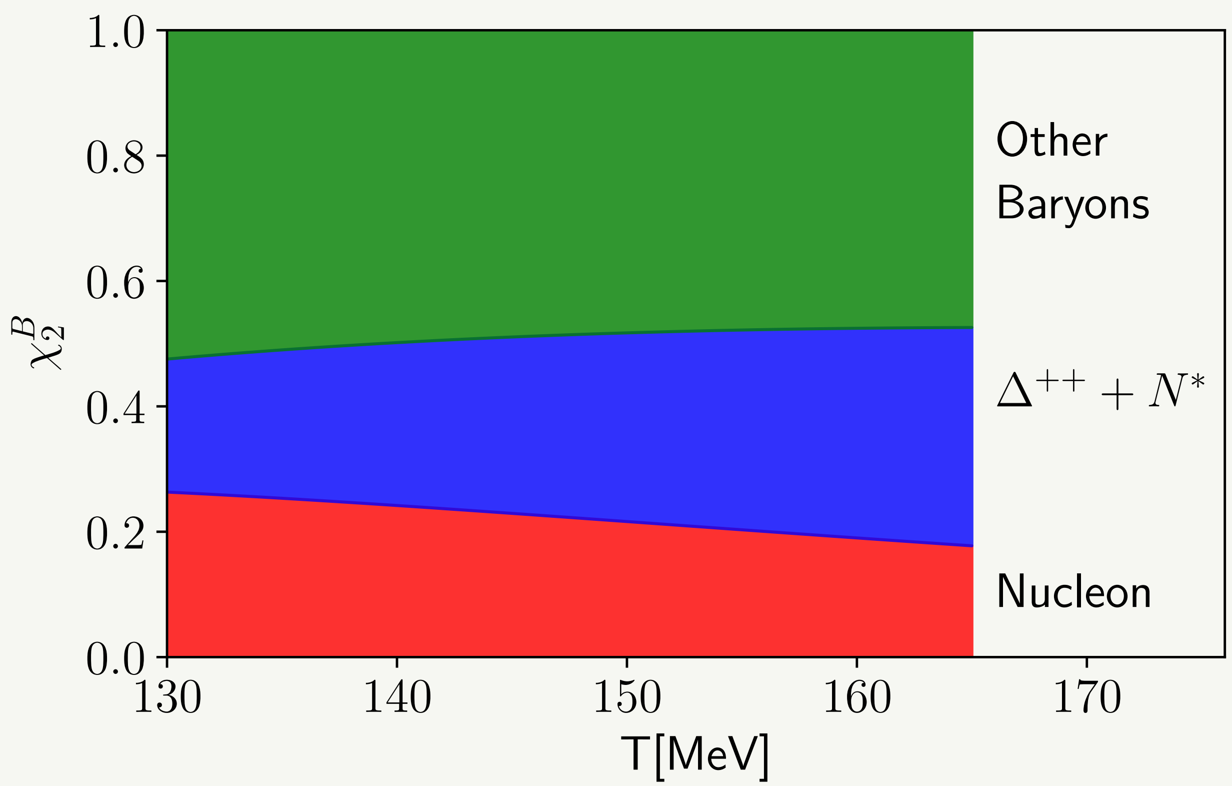




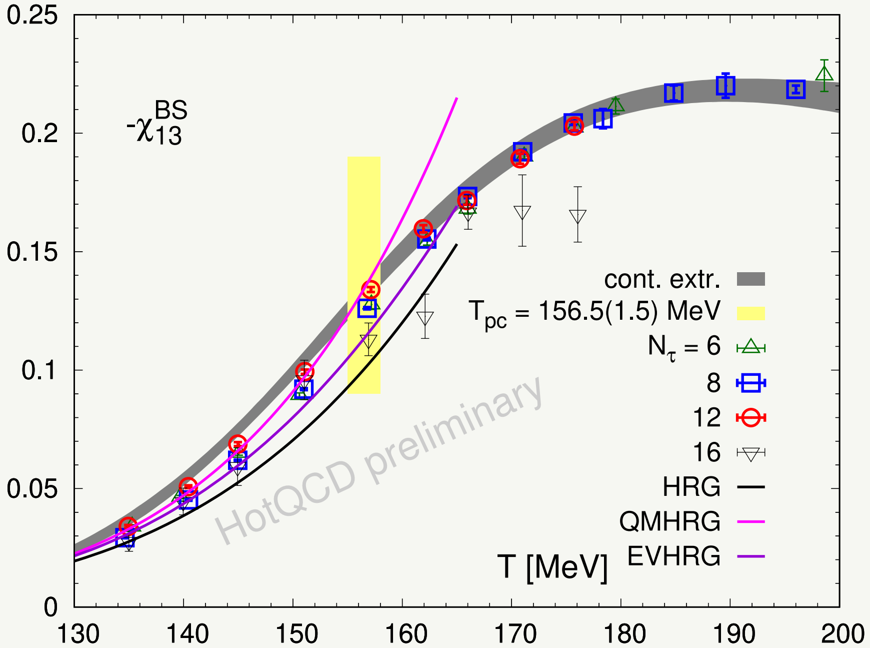
Smatrix with only π, N interactions is not sufficient.

Hybrid Smatrix: The resonances that do not contribute to Smatrix analysis treated as stable particle

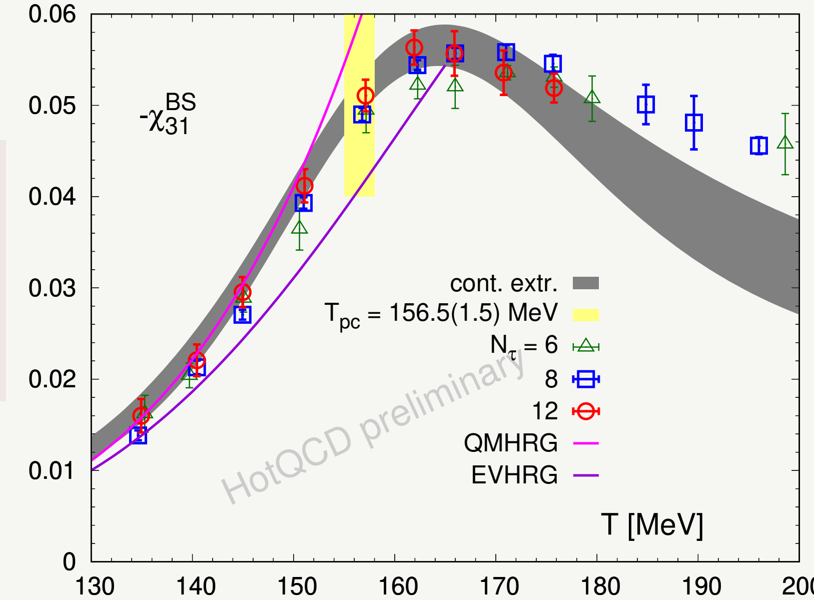


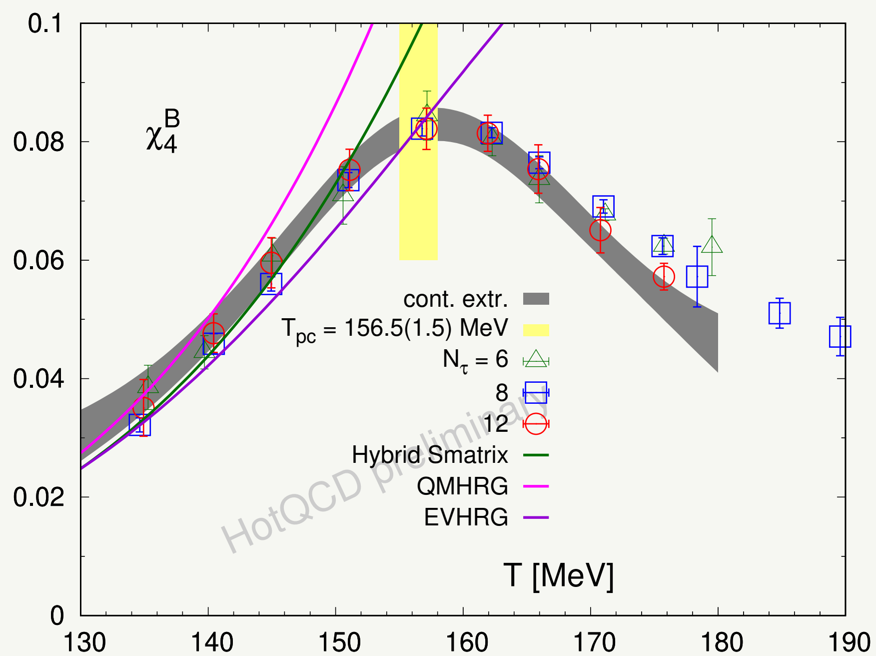


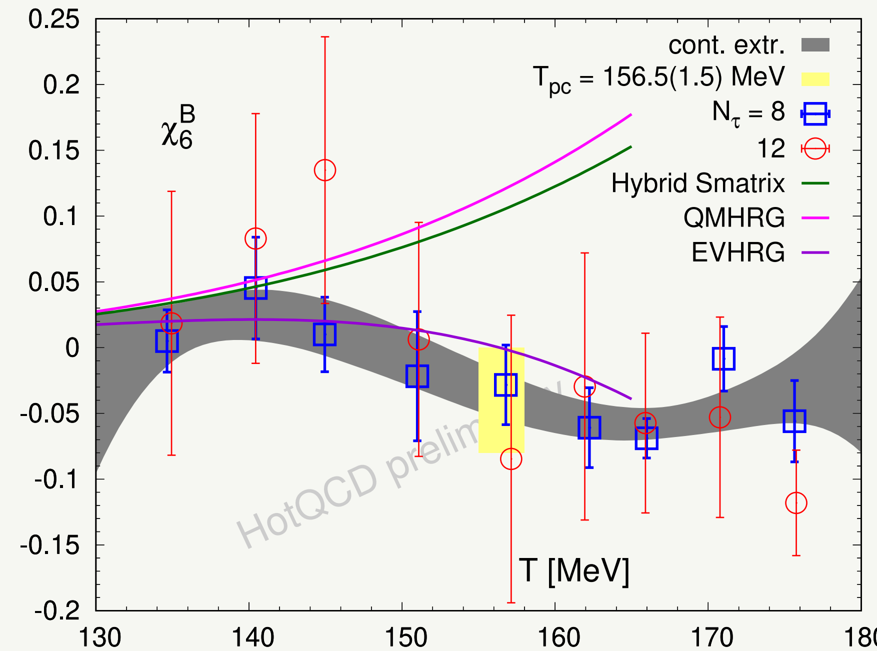
Missing Baryons??

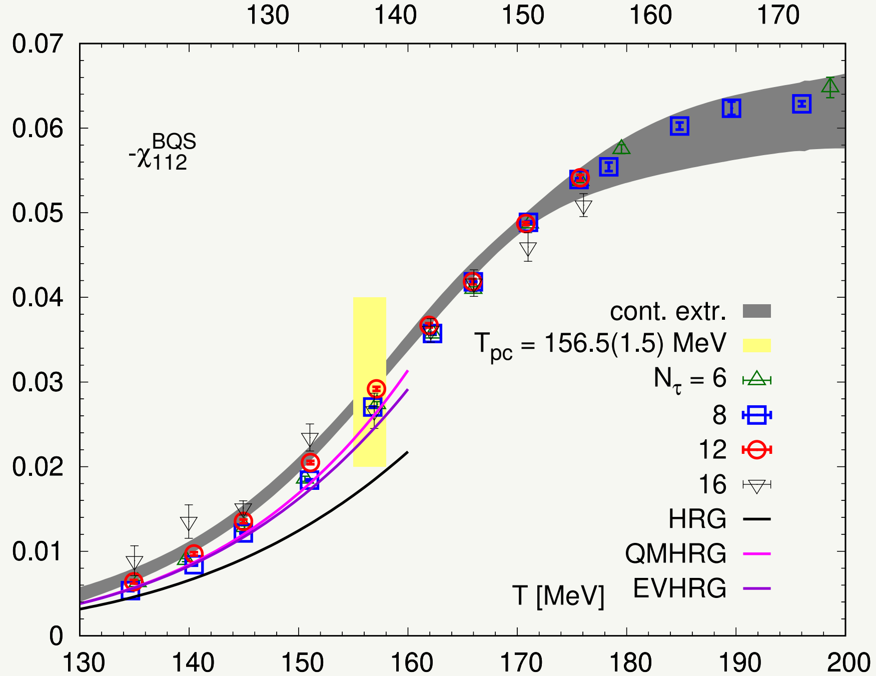


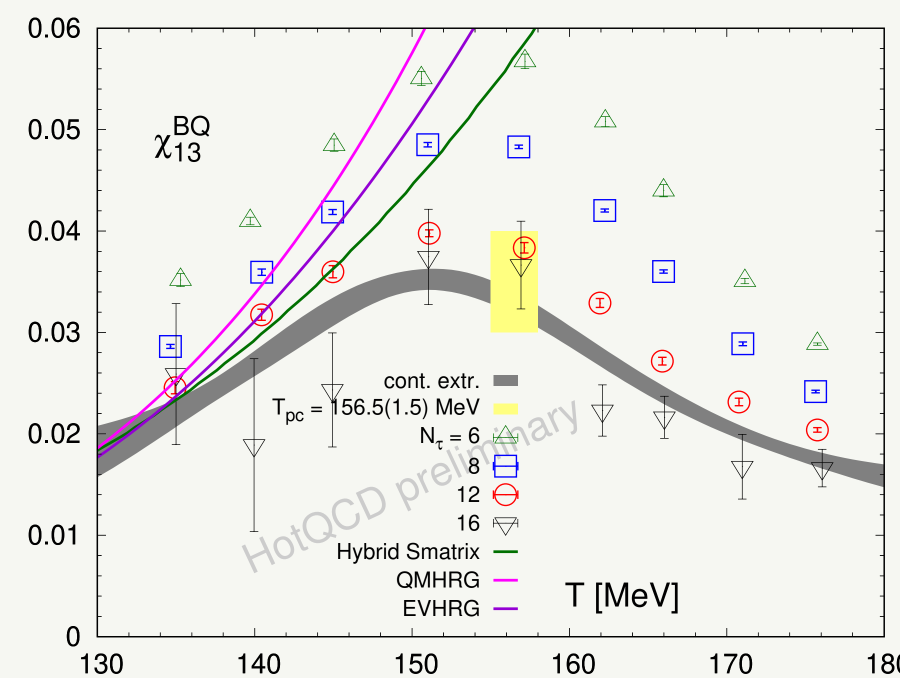
Assessment: In Most cases the QMHRG model works better than other models.



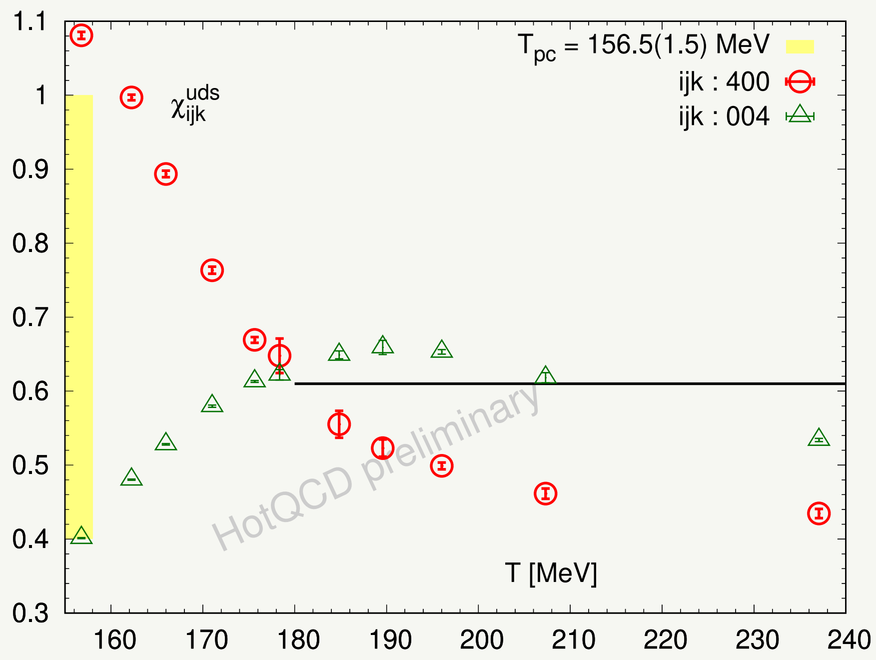




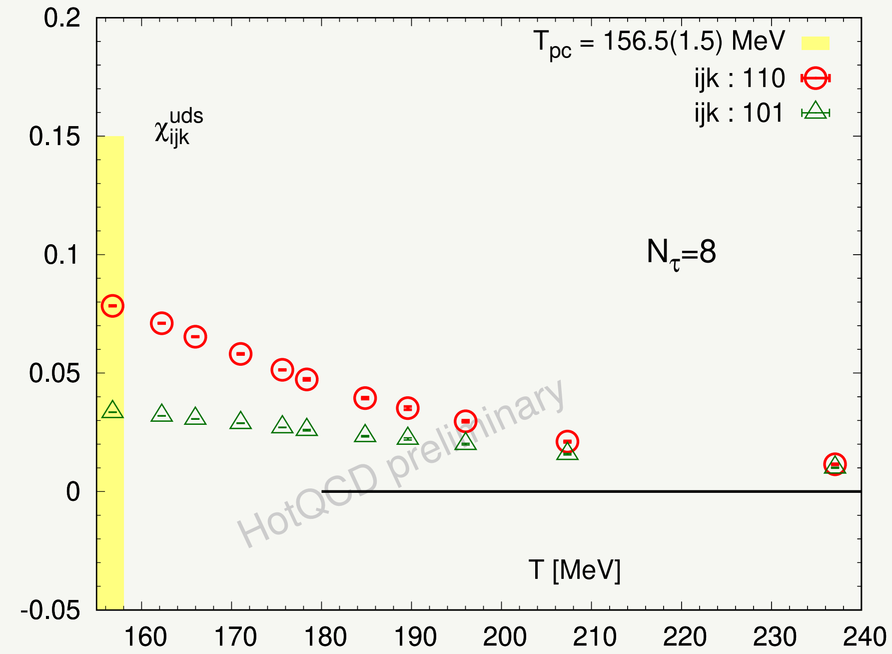




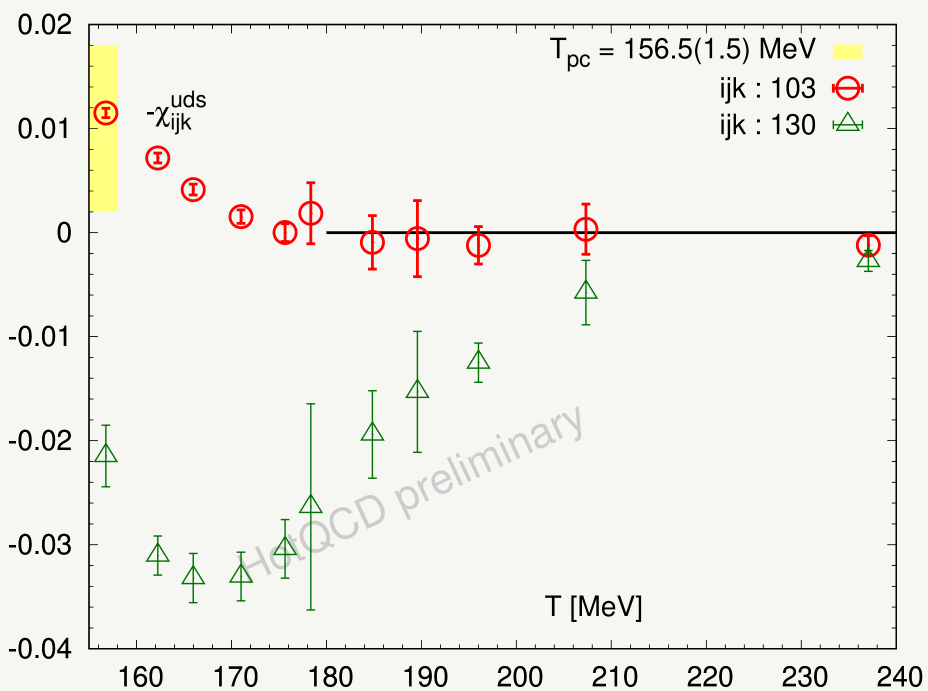
Quark Number susceptibility at $T \geq T_{pc}$



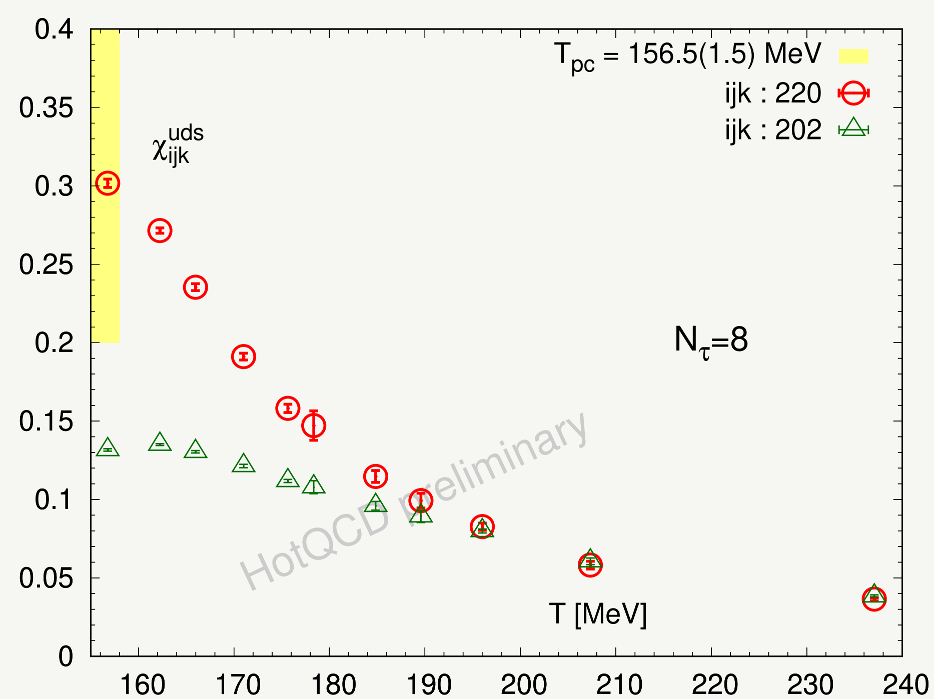
$O(g^2)$



$O(g^6 \log g)$



$O(g^6 \log g)$

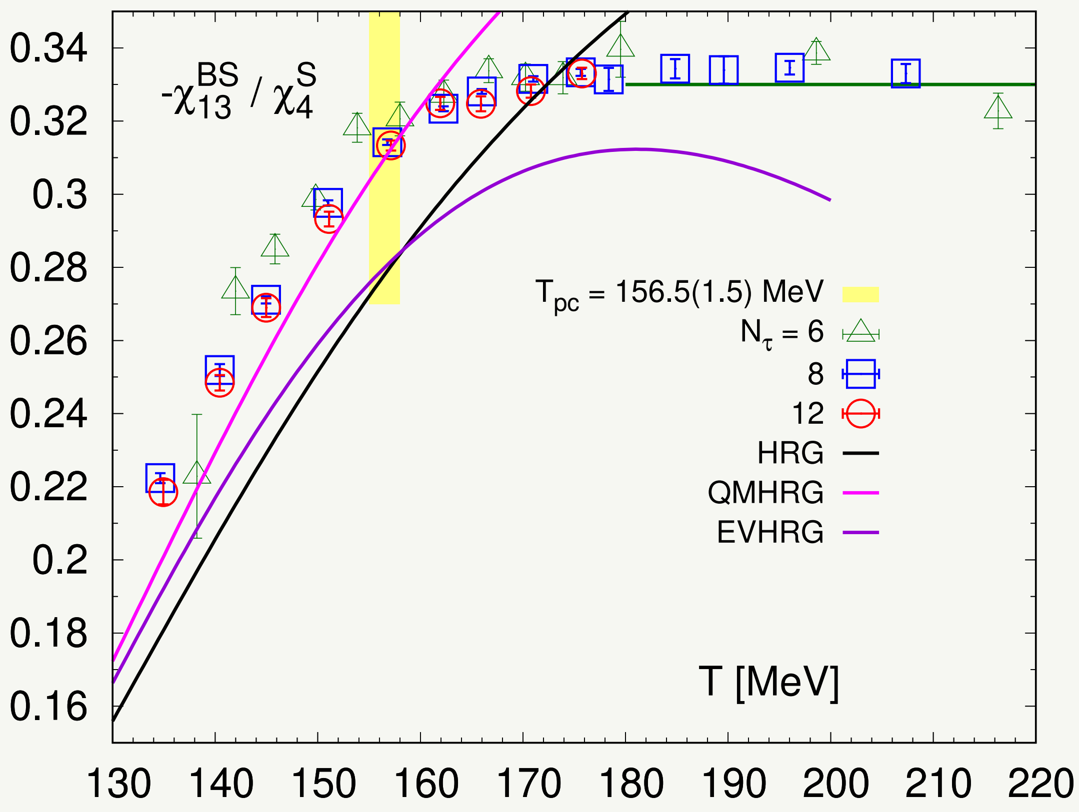


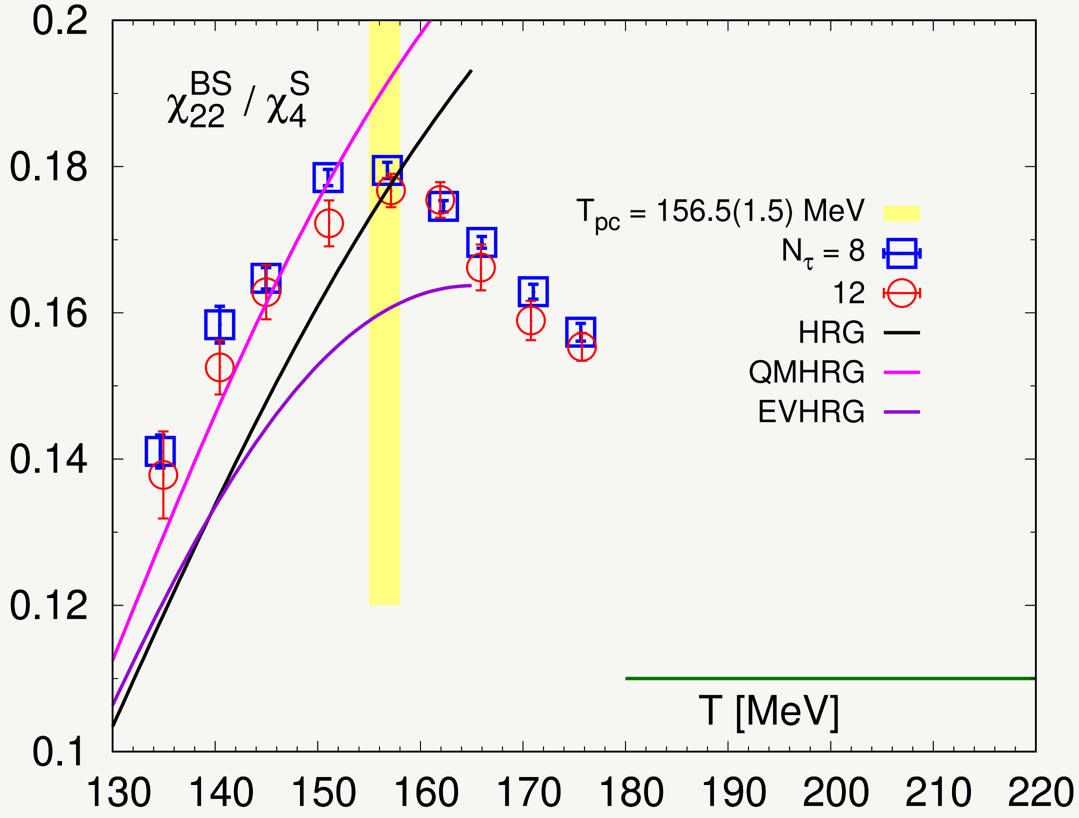
$O(g^3)$

Deviations from ideal gas follow the expectation predicted by weak coupling calculations!!

H.-T. Ding, Swagato Mukherjee, H. Ohno, P. Petreczky, H.-P. Schadler, Phys. Rev. D 92, 074043 (2015)

Quark Number susceptibility at $T \geq T_{pc}$





$$\frac{\chi_{13}^{BS}}{\chi_4^S} = \frac{1}{3} + \frac{2}{3}\frac{\chi_{13}^{us}}{\chi_4^s}$$

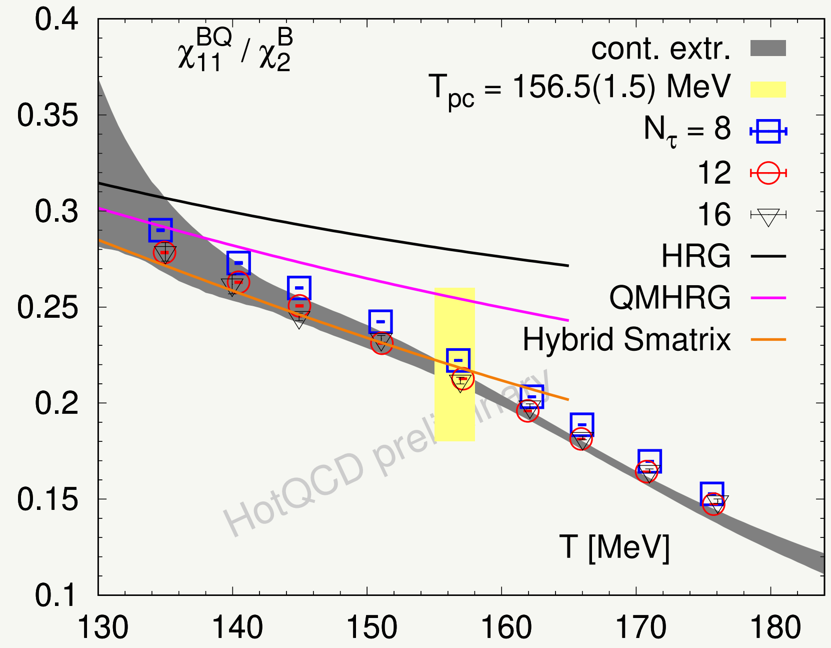
$O(g^6 \log g)$

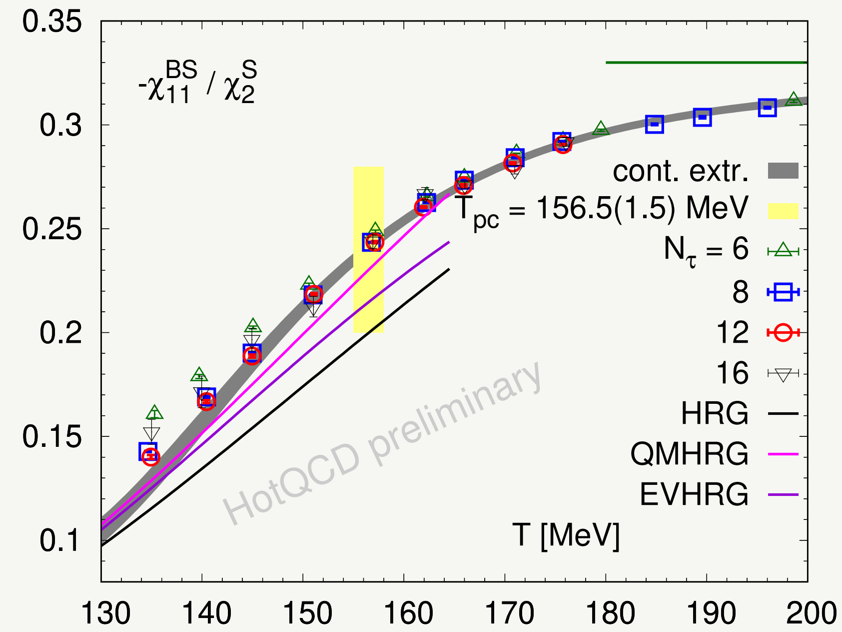
Effects of non-diagonal quark flavour correlations!!

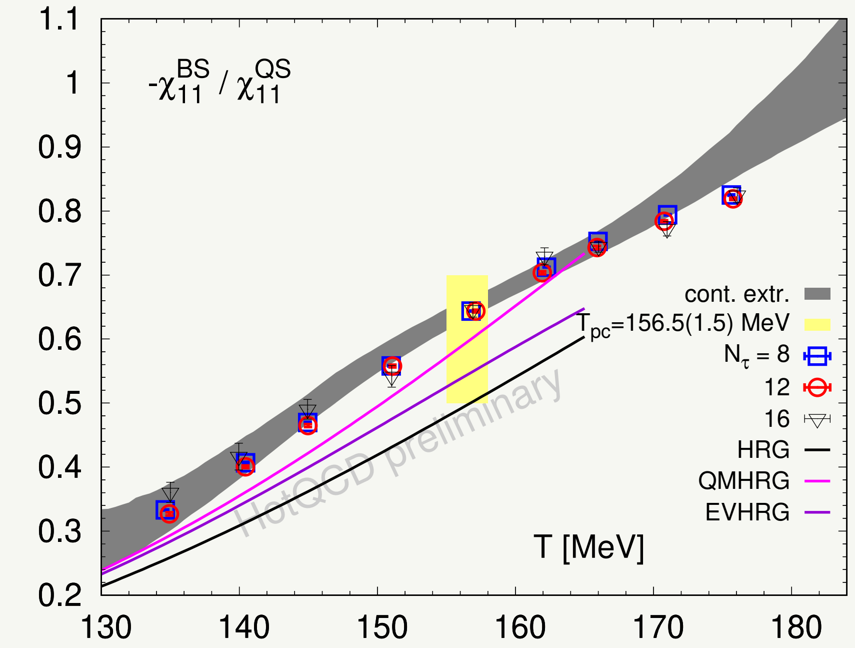
$$\frac{\chi_{22}^{BS}}{\chi_4^S} = \frac{1}{9} + \frac{4}{9}\frac{\chi_{13}^{us}}{\chi_4^s} + \frac{2}{9}\frac{\chi_{22}^{us}}{\chi_4^s} + \frac{2}{9}\frac{\chi_{112}^{uds}}{\chi_4^s}$$

$O(g^3)$

Lattice QCD to Phenomenology







$T_f \sim 156.5(1.5)$ MeV

$$\chi_{11}^{BQ}/\chi_2^B = 0.214(3)$$

$$-\chi_{11}^{BS}/\chi_2^S = 0.239(6)$$

HotQCD preliminary

$$-\chi_{11}^{BS}/\chi_{11}^{QS} = 0.646(5)$$

Extracted from ALICE data at freeze-out,

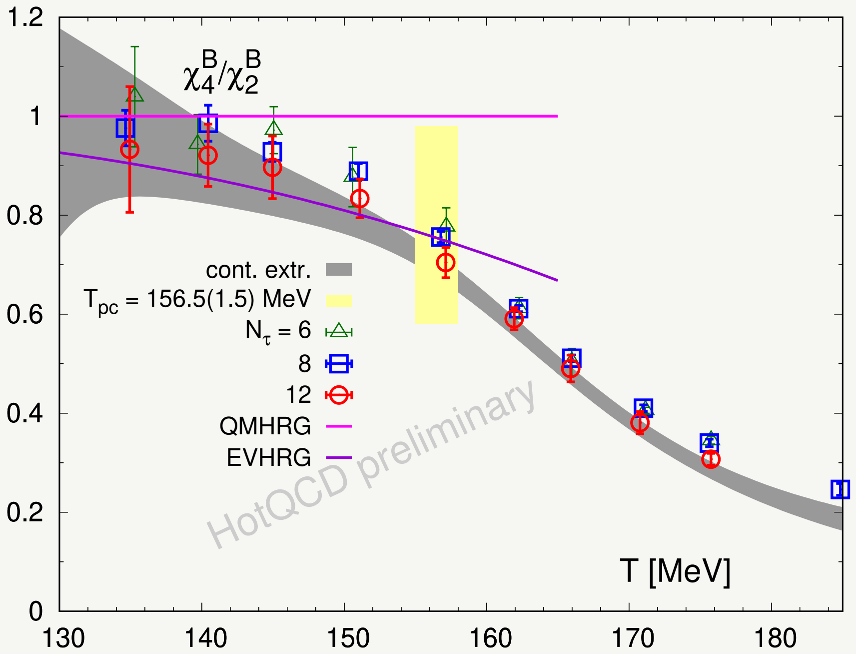
$$-\chi_{11}^{BS}/\chi_2^S = 1 - 2\chi_{11}^{QS}/\chi_2^S$$

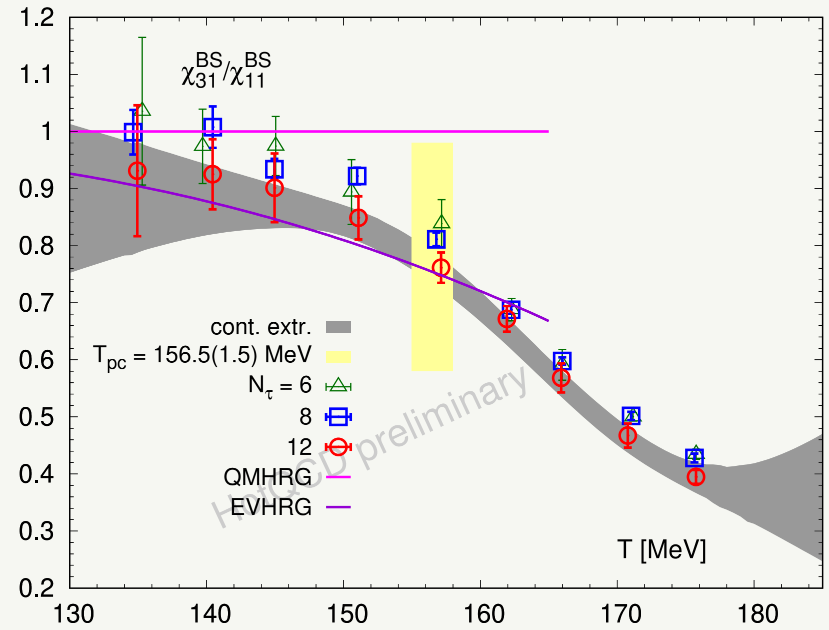
Braun-Munzinger et al.

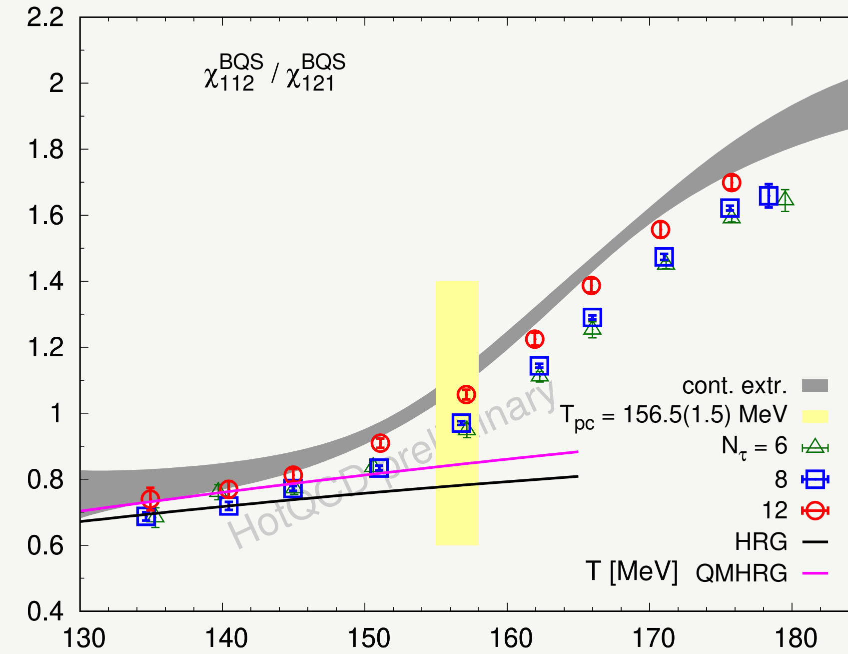
$$-\chi_{11}^{BS}/\chi_2^S > 0.193 \pm 0.0127$$

Ref: P. Braun-Munzinger et al. Phys. Lett. B747 (2015) 292, arXiv:1412.8614

Lattice QCD to Phenomenology







$T_f \sim 156.5(1.5)$ MeV

$$\chi_4^B/\chi_2^B = 0.63(6)$$

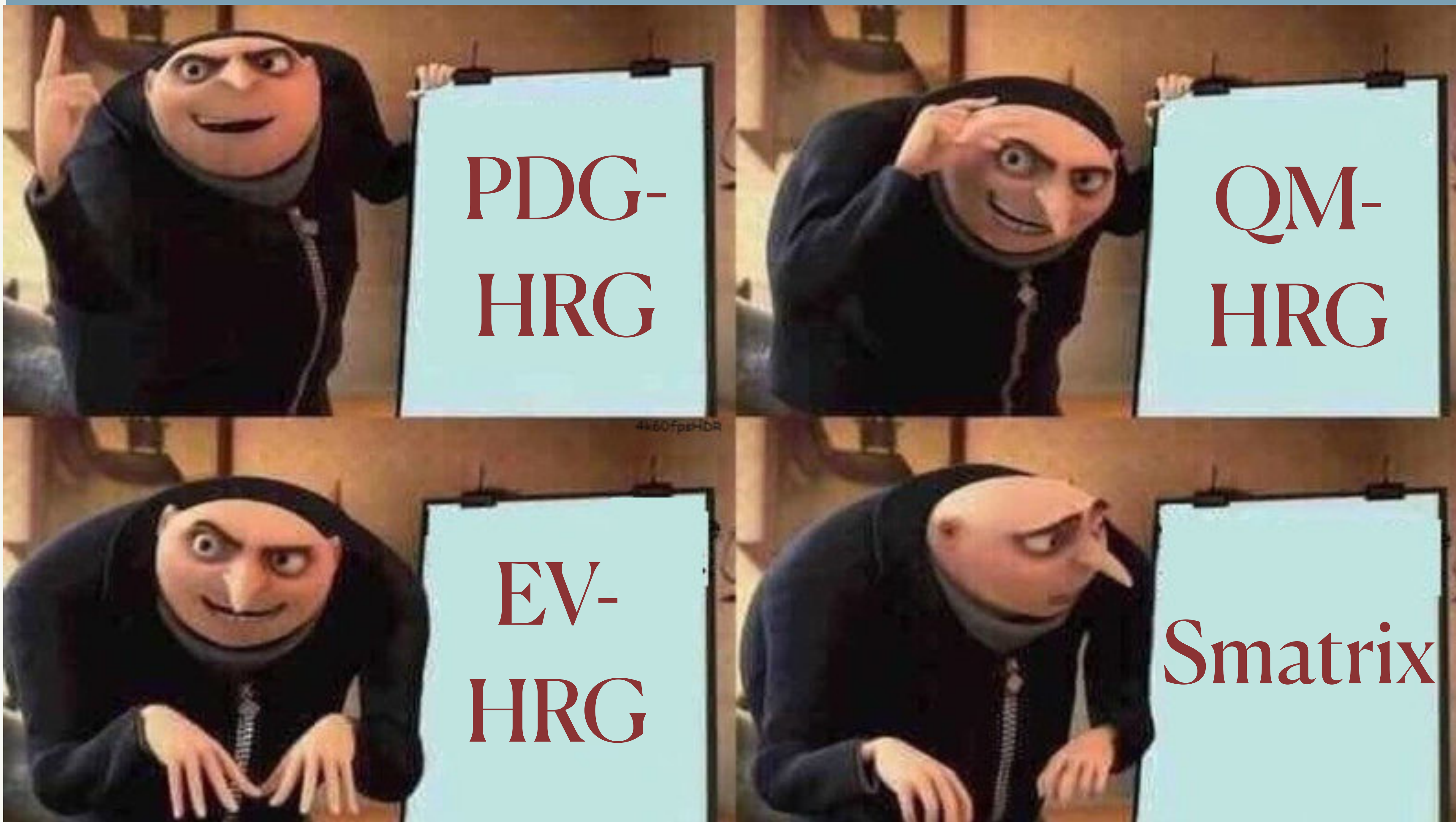
$$\chi_{31}^{BS}/\chi_{11}^{BS} = 0.76(5)$$

$$\chi_{112}^{BQS}/\chi_{121}^{BQS} = 1.08(3)$$

HotQCD preliminary

Conclusions

- **The *cumulants* and *correlations* of conserved charge fluctuations calculated in Lattice QCD agrees well with QMHRG model in most cases (especially in the strangeness sector) at $T \leq T_{pc}$.**
- **We have provided (preliminary) continuum extrapolated results of the ratio of some *cumulants* that can be compared at freeze-out with the fluctuations and correlations currently being measured by the ALICE collaboration.**
- **The *cumulants* and *correlations* at $\mu_{B,Q,S} = 0$ provide the basis for Taylor expansions of various thermodynamic quantities for non-zero chemical potentials [see talks by C. Schmidt and D. Bollweg].**
- **The deviations between higher order *cumulants* in QCD (Lattice) from HRG will influence the determination of freeze-out parameters.**



Thank you for your attention!!