Shear viscosity in QCD

And why it's hard to calculate

Guy Moore

TU Darmstadt

Outline

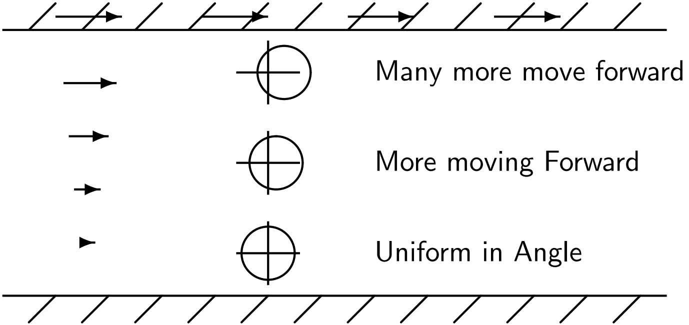
This talk is not about thermodynamics, but the organizers wanted something “thermodynamics adjacent.”

- What is shear viscosity?
- Why is it hard in general?
- What happens in QCD at weak coupling?
- What about physical coupling?
- What about the lattice?

What is shear viscosity

Definition of shear viscosity

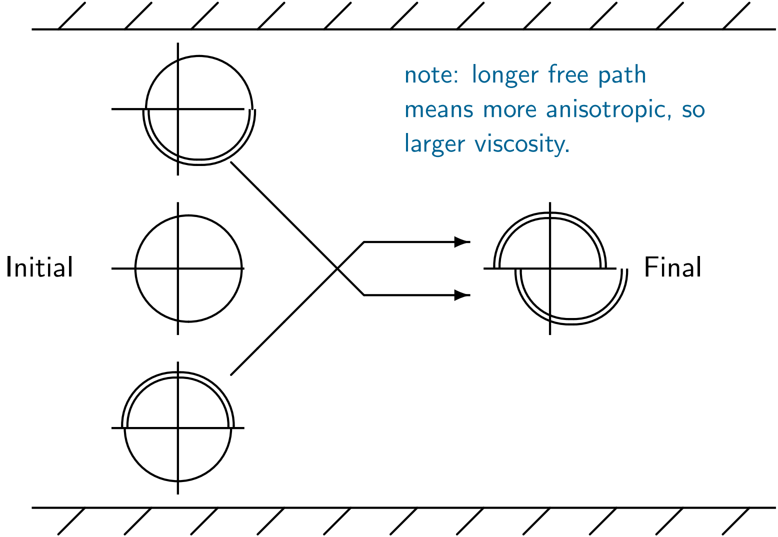
Fluid separates two plates. The top plate moves horizontally, the bottom plate is at rest



Fluid between the plates has varying horizontal velocity $v_x(z)$

Definition, page 2

Propagation leads to momentum anisotropy:



note: longer free path means more anisotropic, so larger viscosity.

Skewed momentum distribution.

Definition of shear

The force-per-area on the bottom surface is linear in Δv:

$$\frac{F}{A} = \eta \frac{\Delta v}{L} \qquad \text{or} \qquad T_{xy} = \eta \partial_y v_x$$

with Δv the velocity difference,
L the distance between the plates, and A the area.
The coefficient η is the shear viscosity.

By definition η is the linearization about equilibrium, and is only defined when L is large and $\Delta v / L$ is small. There is a fluctuation-dissipation (Kubo) formula,

$$\eta = i\partial_\omega \int d^3x \int_0^\infty dt\, e^{i\omega t} \langle [T^{xy}(x,t), T^{xy}(0,0)] \rangle \Big|_{\omega=0}$$

Why is shear viscosity hard?

Shear viscosity of water

Ask a soft condensed-matter or molecular physicist to compute the shear viscosity of water at room temperature, from first principles.

"We have enough trouble predicting the freezing and boiling points to better than 30%. Forget about η!"

But this is a simple, "weakly coupled" system!

- QED with 4 particle types: $^{16}O^{+8}$, p^+, e^-, γ
- Weak coupling $\alpha_{\rm EM} = 1/137$
- Nonrelativistic

What's the problem?

- Not "weakly coupled" in a useful way, $V \sim T$
- Many-body
- long distance and low frequency property

The things which make the problem hard with water are all there in QCD:

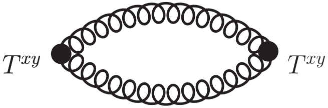
- Large coupling $\alpha_s \sim 0.3$ means $T \sim V$, perturbation theory is limited
- Many-body but relativistic so the bodies constantly annihilate and even basics like particle number are not well defined

So temperatures near T_c are not under perturbative control.
(But maybe the lattice? We'll come back to that.)

But what about much higher temperatures?
Coupling is weak, $T \gg V$ and we have perturbation theory.
Kinetic theory, similar to η for a gas.
Shear for a gas is also hard, but that's because of hard 2-molecule physics. Here we have simple elementary particles.

Weak-coupling QCD

Naive perturbation theory or Kinetic Theory?



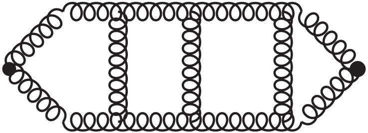
We want

$$i\partial_\omega \int d^3x \int_0^\infty dt e^{i\omega t} \langle [T^{xy}(x,t), T^{xy}(0,0)] \rangle |_{\omega=0}$$

At leading order, that's just one loop, right? Right?

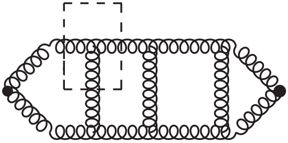
No, because we want the $\vec{p} = 0$, $\omega \to 0$ limit!

The lines each go on-shell at the same time, giving a $1/\alpha$ enhancement.
"Ladder" graphs have one enhancement per extra loop, and are leading-order!

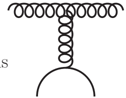


You also need self-energies including imaginary (dissipative) parts.
This resums into a *kinetic theory* (Boltzmann equations).

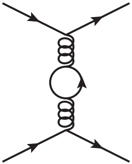
Subcomponents represent scattering matrix elements

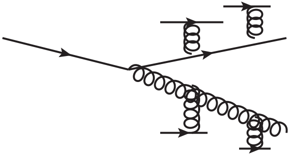


the box is



But we need more scattering processes even at Leading Order!

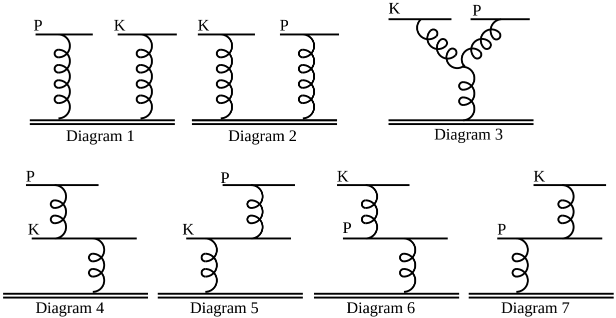




The so-called LPM resummation is required even within the individual objects, which must be resummed using kinetic theory....

Next to leading order?

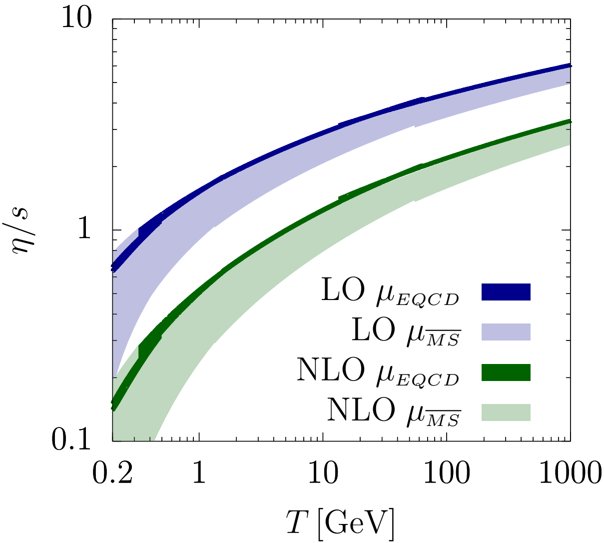
One needs a next-to-leading order calculation to make a valid error analysis.



Complex interference effects arise at NLO.

Remarkably, these can also be resummed.

NLO vs LO results



Dimensionless ratio η/s, LO vs NLO, as a function of temperature

Even at $T = 100$GeV the NLO correction is a factor of 2.

Lattice approaches?

Correlator on the lattice?

Why not compute the desired correlator on the lattice?

$$\eta = i\partial_\omega \int d^3x \int_0^\infty dt\, e^{i\omega t} \langle [T^{xy}(x,t), T^{xy}(0,0)] \rangle \Big|_{\omega=0}$$

- This is a commutator!
- This is at real time, integrated over time.

The lattice can only handle Euclidean time:

$$Z = \int \mathcal{D}(A^\mu, \bar\psi, \psi) e^{-S_E[A,\bar\psi,\psi]} \qquad \text{converges absolutely,}$$

$$Z = \int \mathcal{D}(A^\mu, \bar\psi, \psi) e^{iS_M[A,\bar\psi,\psi]} \qquad \text{does not}$$

$$\text{but} \quad G(\tau) = \int d^3x \langle T^{xy}(x,\tau) T^{xy}(0,0) \rangle_\beta \quad = \langle T^{xy}(x, i\tau) T^{xy}(0,0) \rangle_\beta$$

is at least related to the correlator we need!

Spectral function

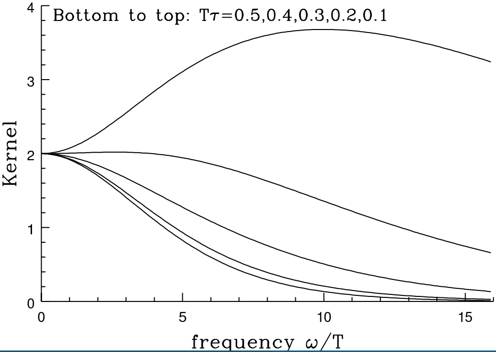
Shear viscosity is $\lim_{\omega\to 0}\sigma(\omega)/\omega$,

$$\sigma(\omega) = i\int d^3x \int_{-\infty}^{\infty} dt\, e^{i\omega t}\langle [T^{xy}(x,t), T^{xy}(0,0)]\rangle$$

Relation to $G(\tau)$: an "inverse problem"

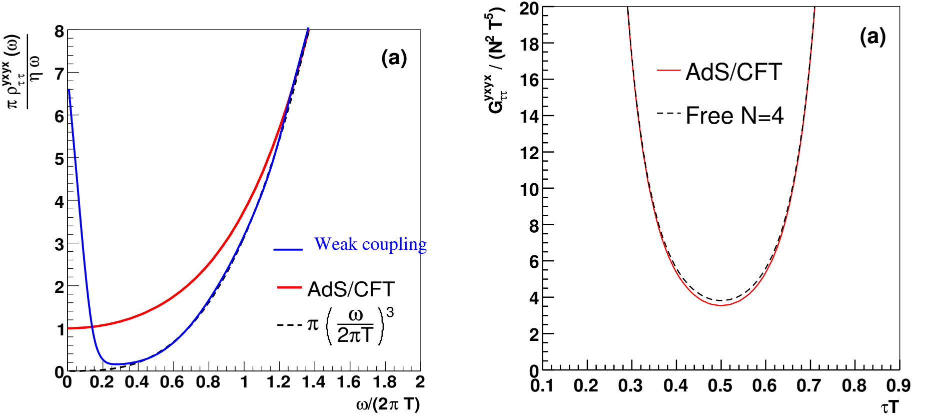
$$G(\tau) = \int \frac{d\omega}{2\pi}\frac{\sigma(\omega)}{\omega}K(\omega,\tau)\,, \qquad K(\omega,\tau) = \frac{\omega\cosh(\omega(\tau - 1/2T))}{\sinh(\omega/2T)}$$

Here is $K(\omega,\tau)$ as function of ω for several τ.



Challenges to reconstruction

Weak-coupling vs Strong: ρ left, $G(\tau)$ right



Huge changes near $\omega = 0$ (viscosity) from tiny changes in $G(\tau)$.

Teaney hep-ph/0602044

Conclusions

Theoretically predicting shear viscosity in QCD is an important but challenging problem.

- Shear viscosity is an important fluid property
- Calculable in field theory with Kubo relation
- Challenge: Minkowski, and small frequency and momentum limit
- Challenge: lack of weak coupling, $T \sim V$
- Perturbative regime: resummations needed, only works for $T > 1000$GeV
- Lattice: relation between Euclidean and Minkowski
  Spectral reconstruction is an inverse problem

Of course we should keep trying, because this is important physics!