QCD in the heavy dense regime: Large N_c and quarkyonic matter

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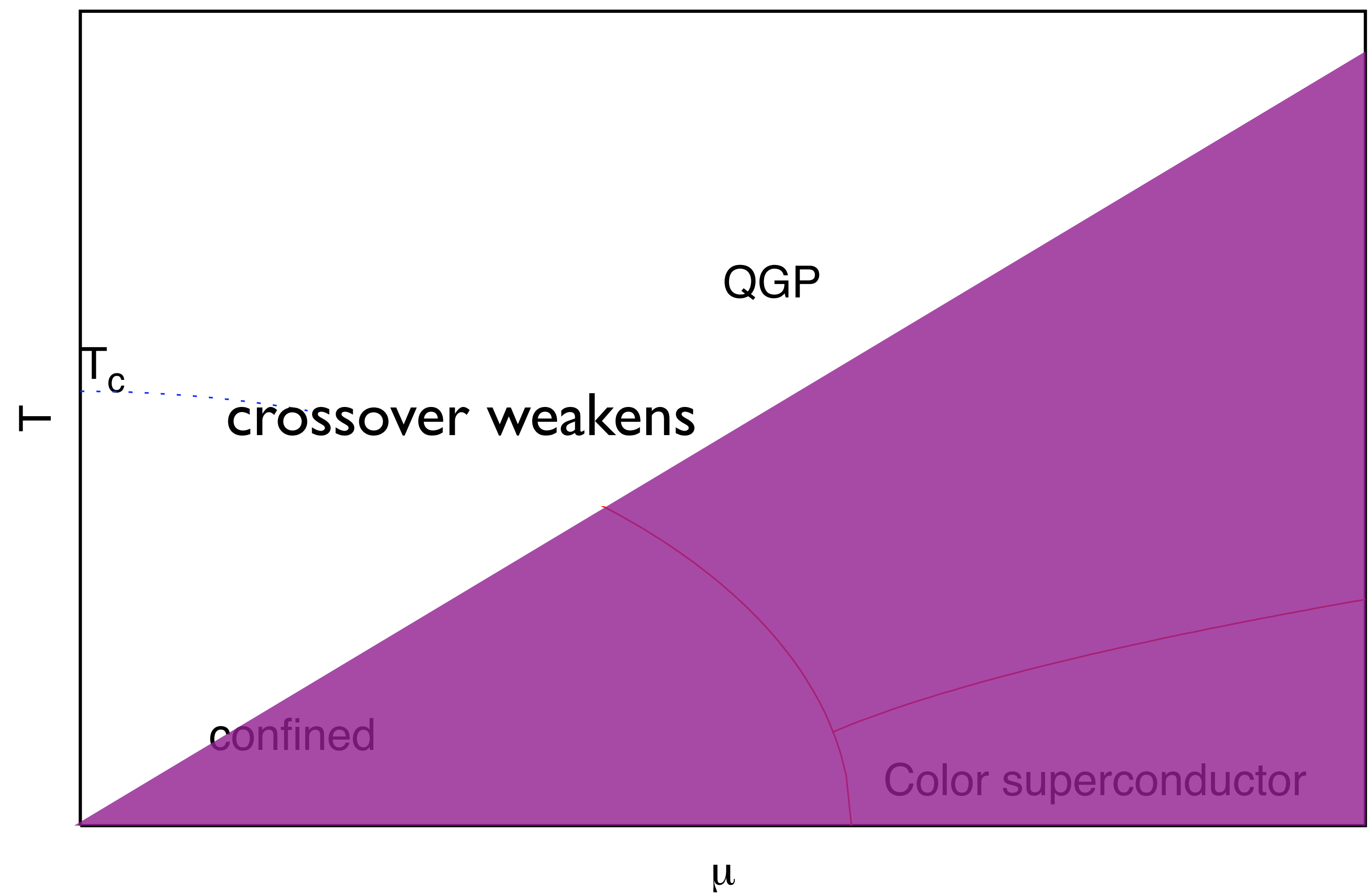
- Effective lattice theory for finite density heavy QCD
- The nuclear liquid gas transition
- What happens at large N_c

NIC

CRC-TR 211 Strong-interaction matter under extreme conditions

The lattice-calculable region of the phase diagram



- Sign problem prohibits direct simulation, circumvented by approximate methods: reweigthing, Taylor expansion, imaginary chem. pot., need $\mu/T \lesssim 1 \quad (\mu = \mu_B/3)$
- No critical point in the controllable region

Effective lattice theory for heavy dense QCD

- Two-step treatment: [Langelage, Lottini, O.P. JHEP (2011)]

  [Fromm, Langelage, Lottini, O.P. JHEP (2012)]

- Step I.: split temporal and spatial link integrations:

$$Z = \int DU_0 DU_i \, \det Q \, e^{S_g[U]} \equiv \int DU_0 e^{-S_{eff}[U_0]} = \int DL \, e^{-S_{eff}[L]}$$

Spatial integration after analytic strong coupling and hopping expansion $\sim \frac{1}{g^2}, \frac{1}{m_q}$

- Step II.: mild sign problem of effective theory, simulation

- Weak effective theory couplings: analytic expansion methods

Effective theory: start from Wilson's lattice action

Pure gauge part: character expansion

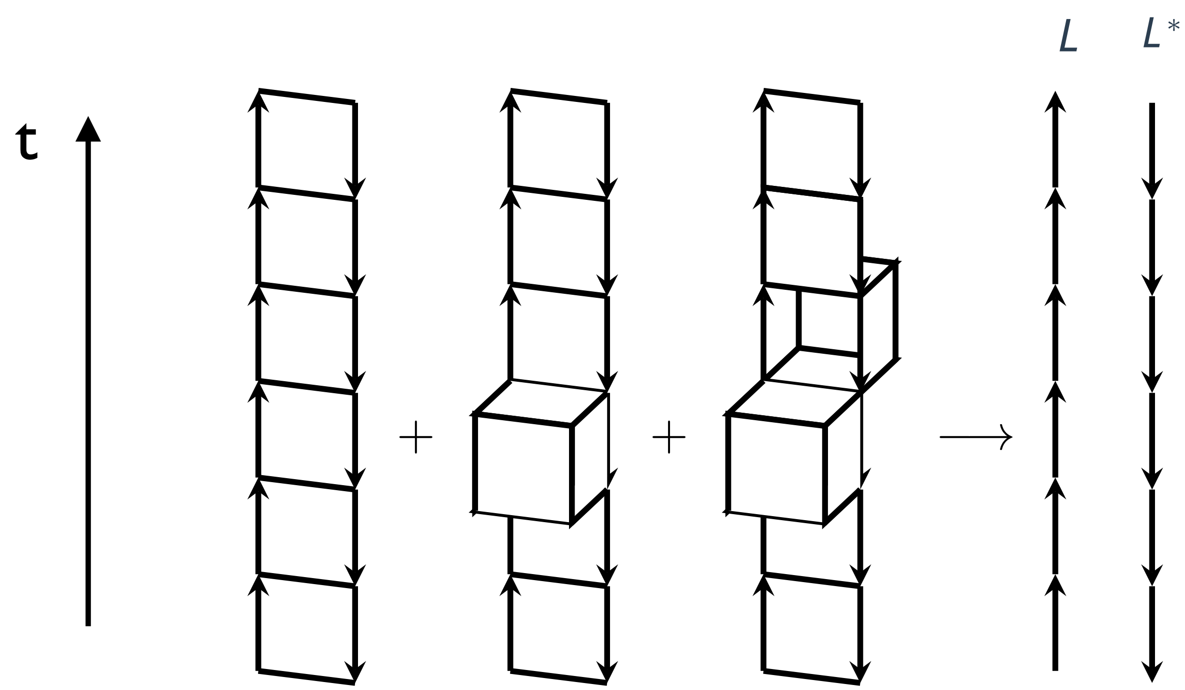
$$u(\beta) = \frac{\beta}{18} + \frac{\beta^2}{216} + \ldots < 1$$

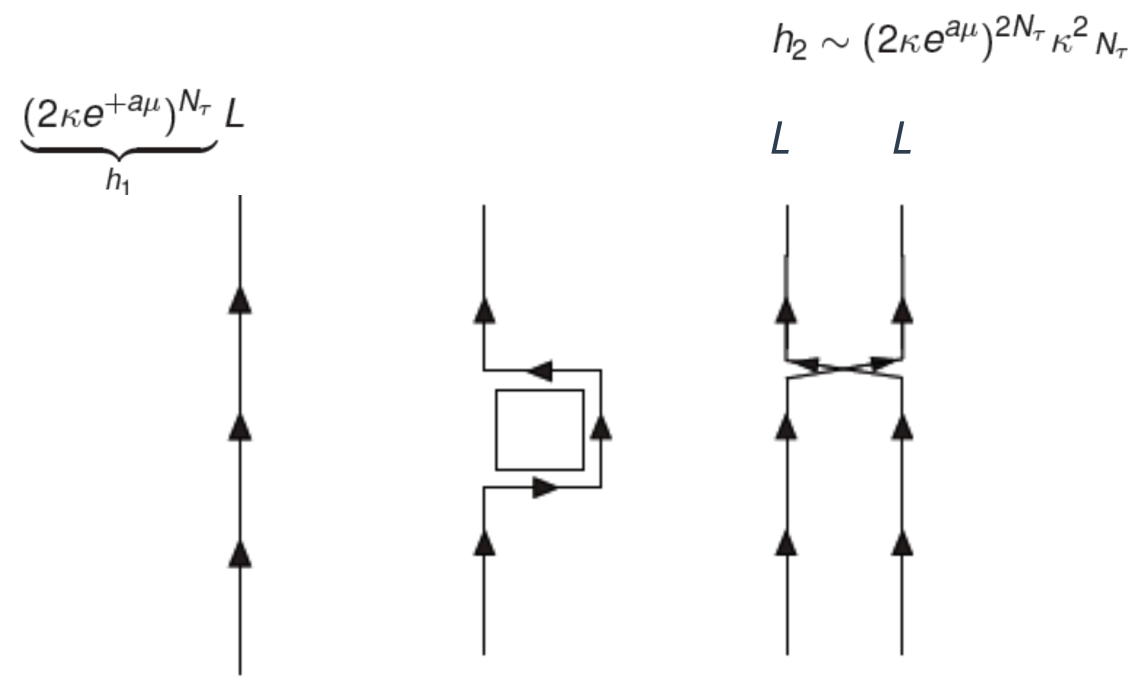
$$\beta = \frac{2N_c}{g^2} \qquad T = \frac{1}{aN_\tau}$$

Fermion determinant: hopping expansion

$$\kappa = \frac{1}{2am + 8}$$

Generates couplings over all distances, n-pt. couplings, higher reps.... :





$$\lambda(u, N_\tau \geq 5) = u^{N_\tau} \exp\left[N_\tau\left(4u^4 + 12u^5 - 14u^6 - 36u^7 + \frac{295}{2}u^8 + \frac{1851}{10}u^9 + \frac{1055797}{5120}u^{10}\right)\right]$$

The 3d effective lattice theory, leading interactions

$$Z = \int \mathcal{D}W \prod_{\langle \mathbf{x},\mathbf{y}\rangle} \left[1 + \lambda(L(\mathbf{x})L(\mathbf{y})^* + L(\mathbf{x})^* L(\mathbf{y}))\right]$$ pure gauge

$$\times \prod_{\mathbf{x}} \left[1 + h_1 L(\mathbf{x}) + h_1^2 L(\mathbf{x})^* + h_1^3\right]^{2N_f} \left[1 + \bar{h}_1 L(\mathbf{x}) + \bar{h}_1^2 L(\mathbf{x})^* + \bar{h}_1^3\right]^{2N_f}$$ stat. det.

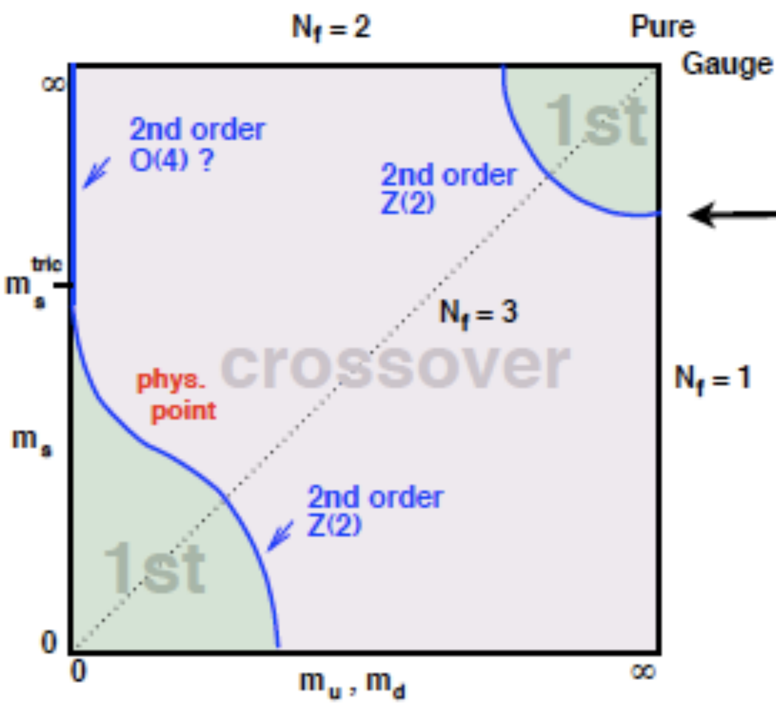
$$\times \prod_{\langle \mathbf{x},\mathbf{y}\rangle} \left[1 - h_2 N_f \,\mathrm{tr}\left(\frac{h_1 W(\mathbf{x})}{1 + h_1 W(\mathbf{x})}\right) \mathrm{tr}\left(\frac{h_1 W(\mathbf{y})}{1 + h_1 W(\mathbf{y})}\right)\right]$$ kinetic det.

$$\times \left[1 - h_2 N_f \,\mathrm{tr}\left(\frac{\bar{h}_1 W(\mathbf{x})^\dagger}{1 + \bar{h}_1 W(\mathbf{x})^\dagger}\right) \mathrm{tr}\left(\frac{\bar{h}_1 W(\mathbf{y})^\dagger}{1 + \bar{h}_1 W(\mathbf{y})^\dagger}\right)\right] \quad \ldots.$$

$$W(\mathbf{x}) = \prod_{\tau=0}^{N_\tau - 1} U_0(\tau, \mathbf{x}), \quad L(\mathbf{x}) = \mathrm{tr}(W(\mathbf{x})), \quad \mathcal{D}W = \prod_{\mathbf{x}\in\Lambda_s} \mathrm{d}W(x)\,.$$

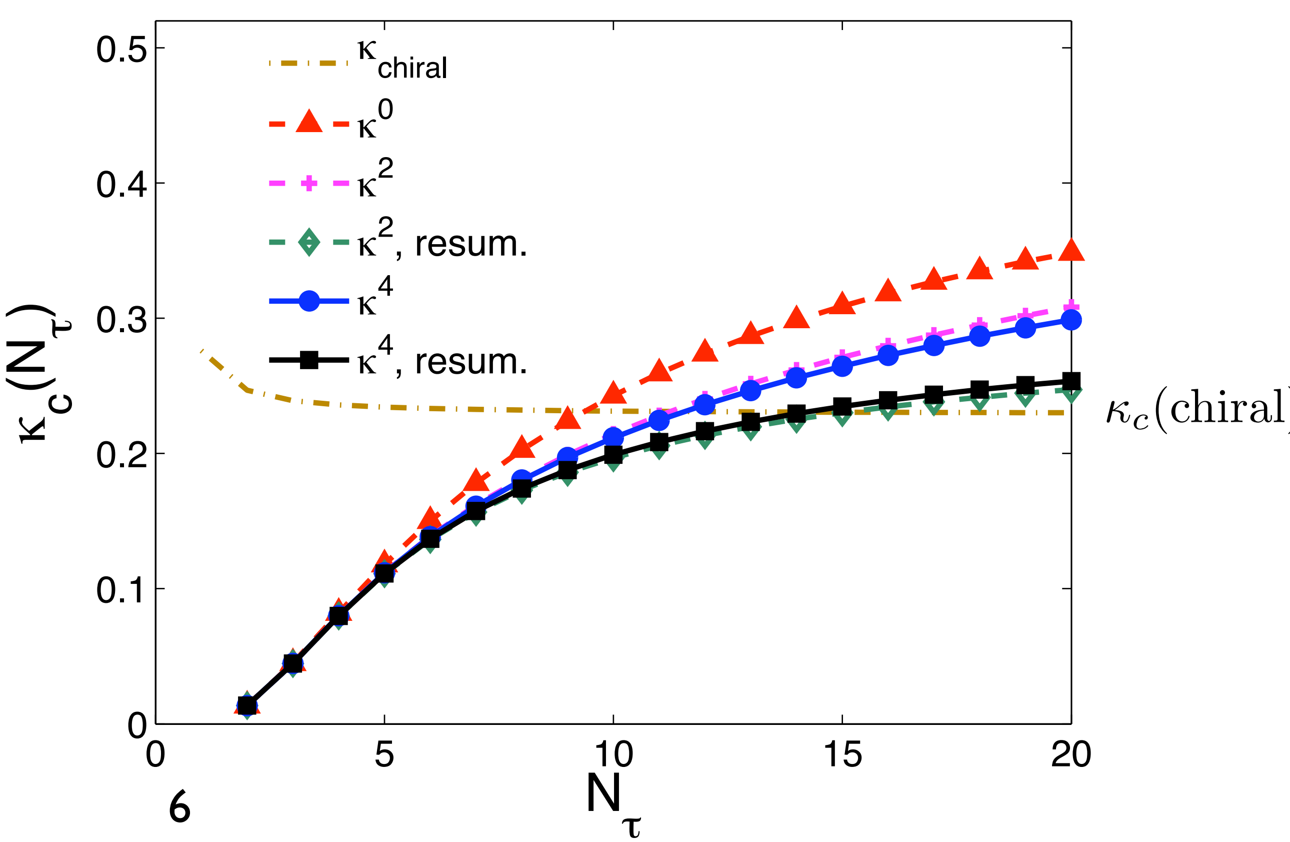
This is a 3d continuous spin model!

The deconfinement transition for heavy quarks



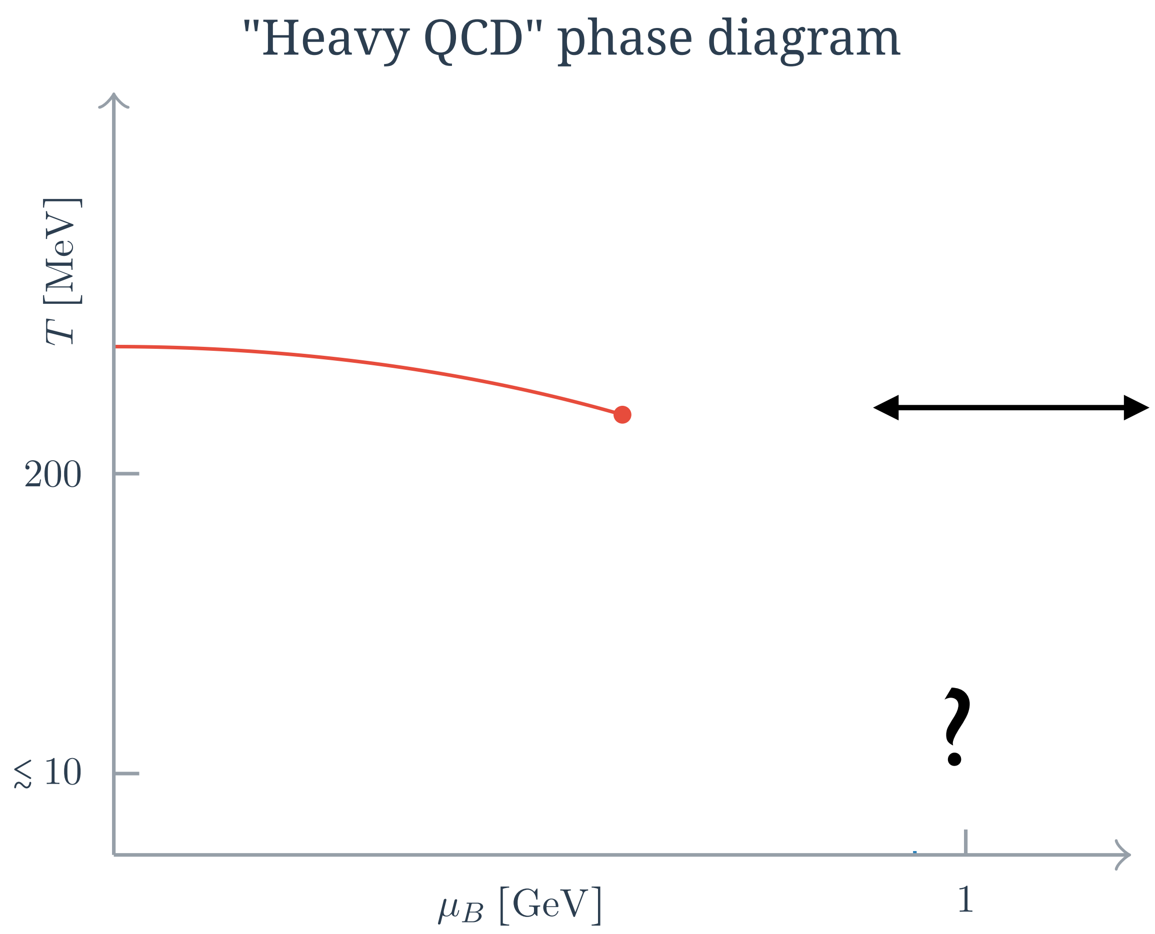
		eff. theory	4d MC, WHOT	4d MC, de Forcrand et al
N_f	M_c/T	$\kappa_c(N_\tau = 4)$	$\kappa_c(4)$, Ref. [23]	$\kappa_c(4)$, Ref. [22]
1	7.22(5)	0.0822(11)	0.0783(4)	~ 0.08
2	7.91(5)	0.0691(9)	0.0658(3)	–
3	8.32(5)	0.0625(9)	0.0595(3)	–

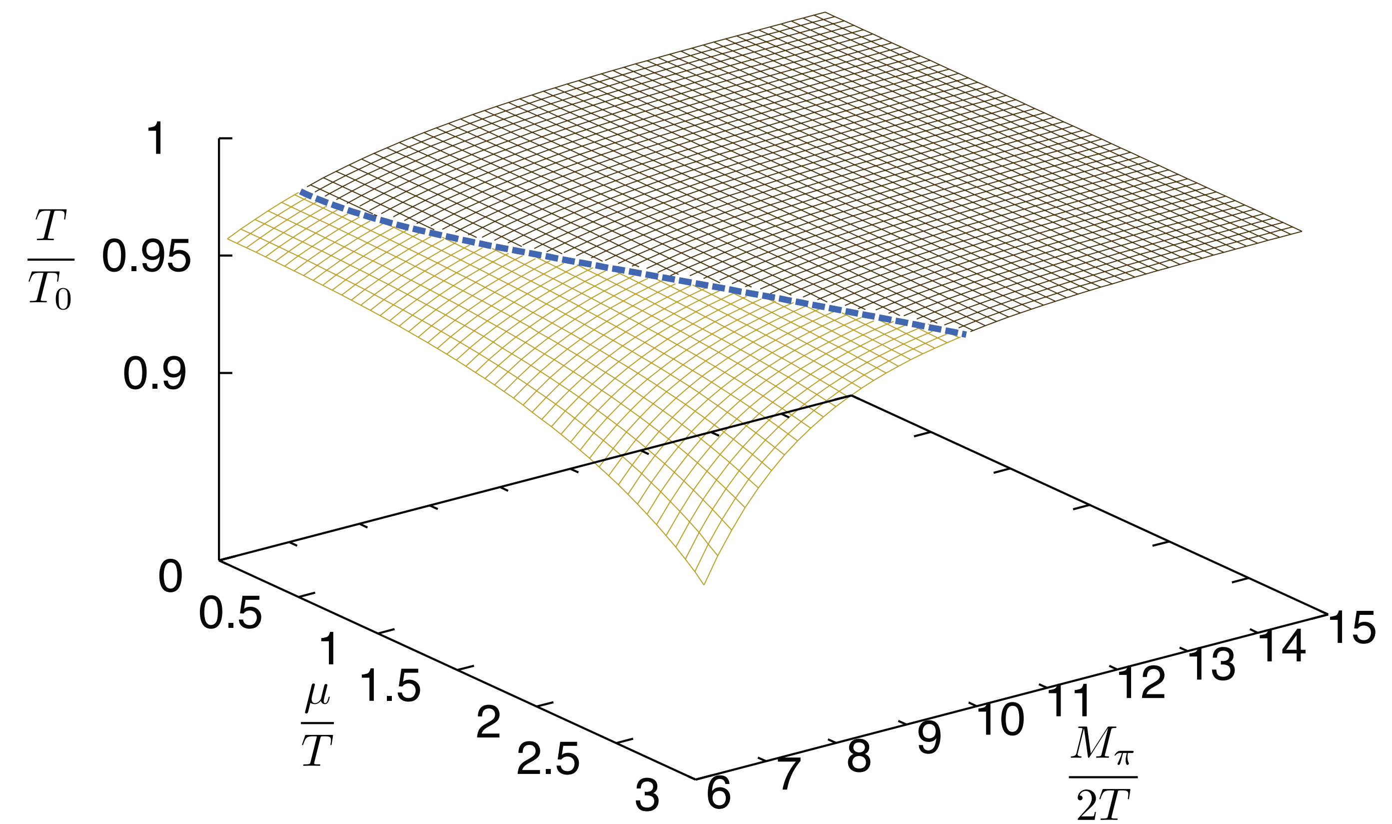
Accuracy ~5%, predictions for Nt=6,8,... available!



The fully calculated deconfinement transition

[Fromm, Langelage, Lottini, O.P. JHEP (2012)]





Same phase structure: continuum eff. PL theories

[Fischer, Lücker, Pawlowski PRD (2015); Lo, Friman, Redlich PRD (2014)]

Cold and dense: static strong coupling limit

[Fromm, Langelage, Lottini, Neuman, O.P. , PRL (2013)]

T=0: anti-fermions decouple:

$$\begin{aligned} h_1 &= (2\kappa e^{a\mu})^{N_\tau} = e^{\frac{\mu - m}{T}} \\ \bar{h}_1 &= (2\kappa e^{-a\mu})^{N_\tau} = e^{\frac{-\mu - m}{T}} \end{aligned}$$

$$Z(\beta = 0) \xrightarrow{T \to 0} \left[\prod_f \int dW (1 + h_1 L + h_1^2 L^* + h_1^3)^2 \right]^V = z_0^V$$

$$N_f = 1 : \; z_0 = 1 + 4h_1^3 + h_1^6$$

spin 3/2, 0

free baryon gas (HRG) emerges!

cf. finite T [Langelage, O.P. JHEP (2010)]

Silver blaze phenomenon + Pauli principle:

$$\lim_{T \to 0} a^3 n = \begin{cases} 0, & \mu < m \\ 2N_c, & \mu > m \end{cases}$$

1st order phase transition from vacuum to saturated quark crystal

$N_f = 2:$ The baryon gas (or liquid)

Δ^- $\quad 2n + 4\Delta^0$ $\quad 2p + 4\Delta^+$ $\quad \Delta^{++}$

$$z_0 = (1 + 4h_d^3 + h_d^6) + (6h_d^2 + 4h_d^5)h_u + (6h_d + 10h_d^4)h_u^2 + (4 + 20h_d^3 + 4h_d^6)h_u^3$$
$$+(10h_d^2 + 6h_d^5)h_u^4 + (4h_d + 6h_d^4)h_u^5 + (1 + 4h_d^3 + h_d^6)h_u^6$$

"Di-baryons": 3 spin 1 triplets, 1 spin 0 singlet, $\Delta^{++}\Delta^0$, $\quad pp$

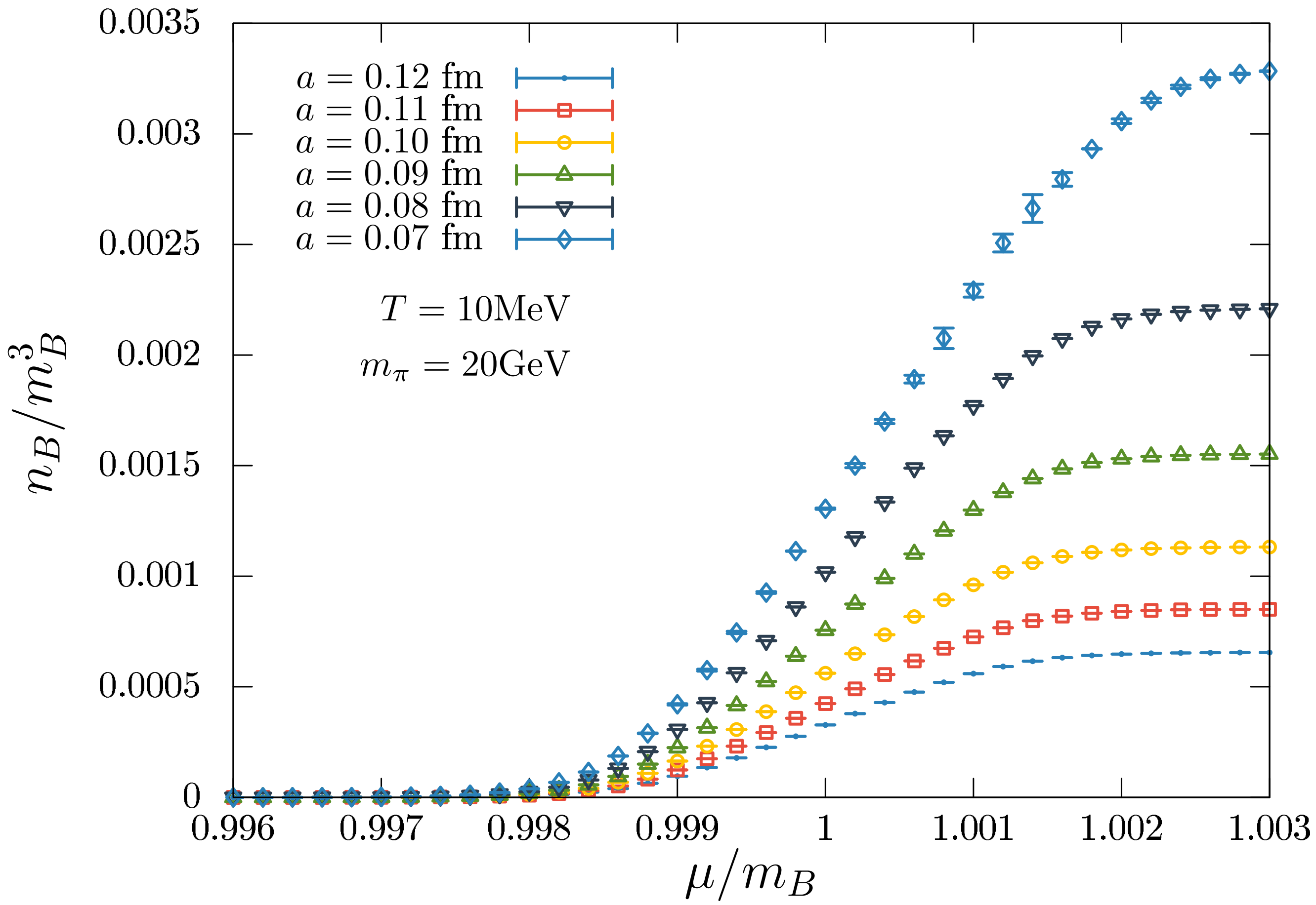
Complete spin-flavour structure of baryons (mesons for finite T or isospin chemical potential)

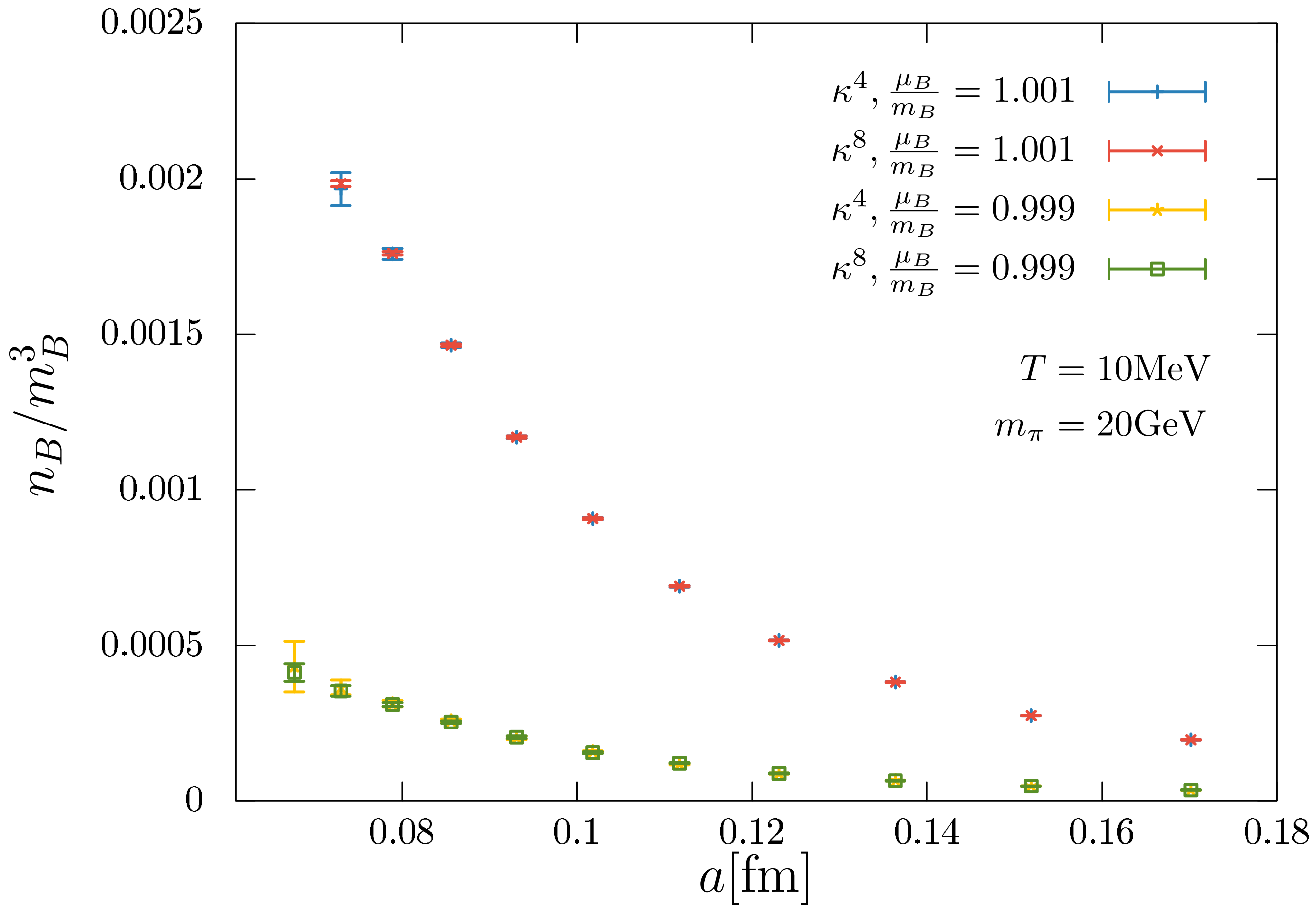
Gauge and Lorentz symmetries!

Cold and dense regime: onset of baryon matter

$\sim u^5 \kappa^8$

[Glesaaen, Neuman, O.P., JHEP (2016)]





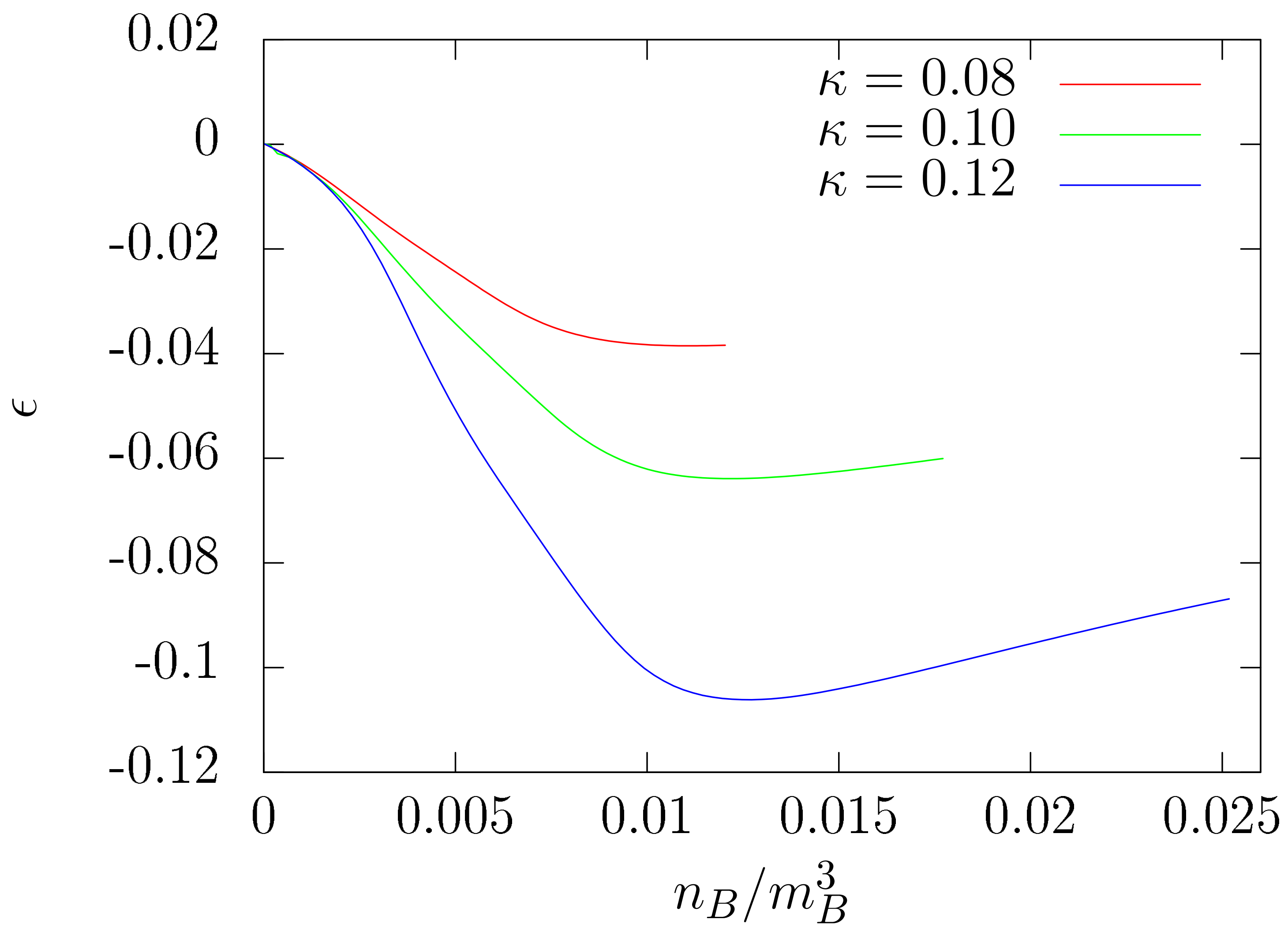
- Continuum approach ~a as expected for Wilson fermions
- Cut-off effects grow rapidly beyond onset transition: lattice saturation!
- Finer lattice necessary for larger density!

Binding energy per nucleon

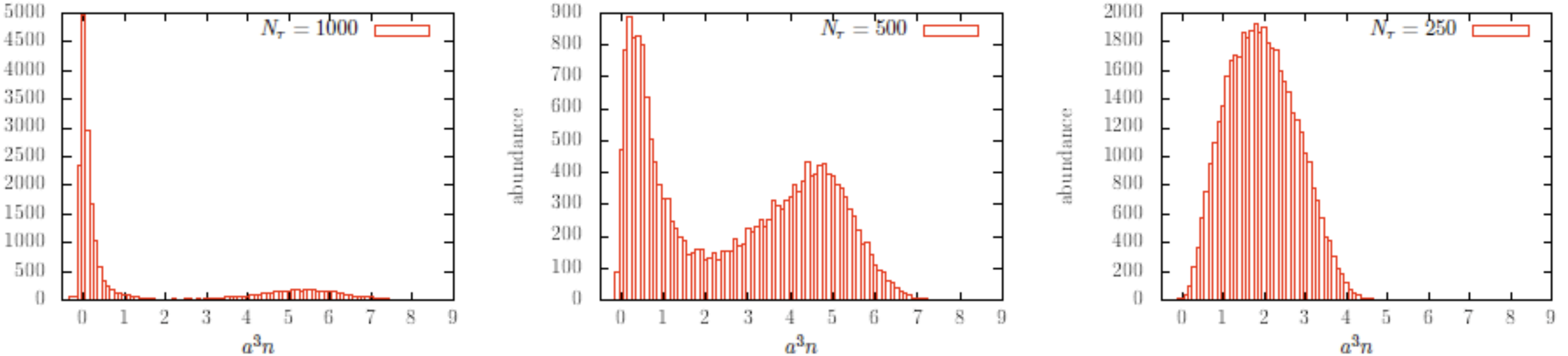
[Langelage, Neuman, O.P., JHEP (2014)]

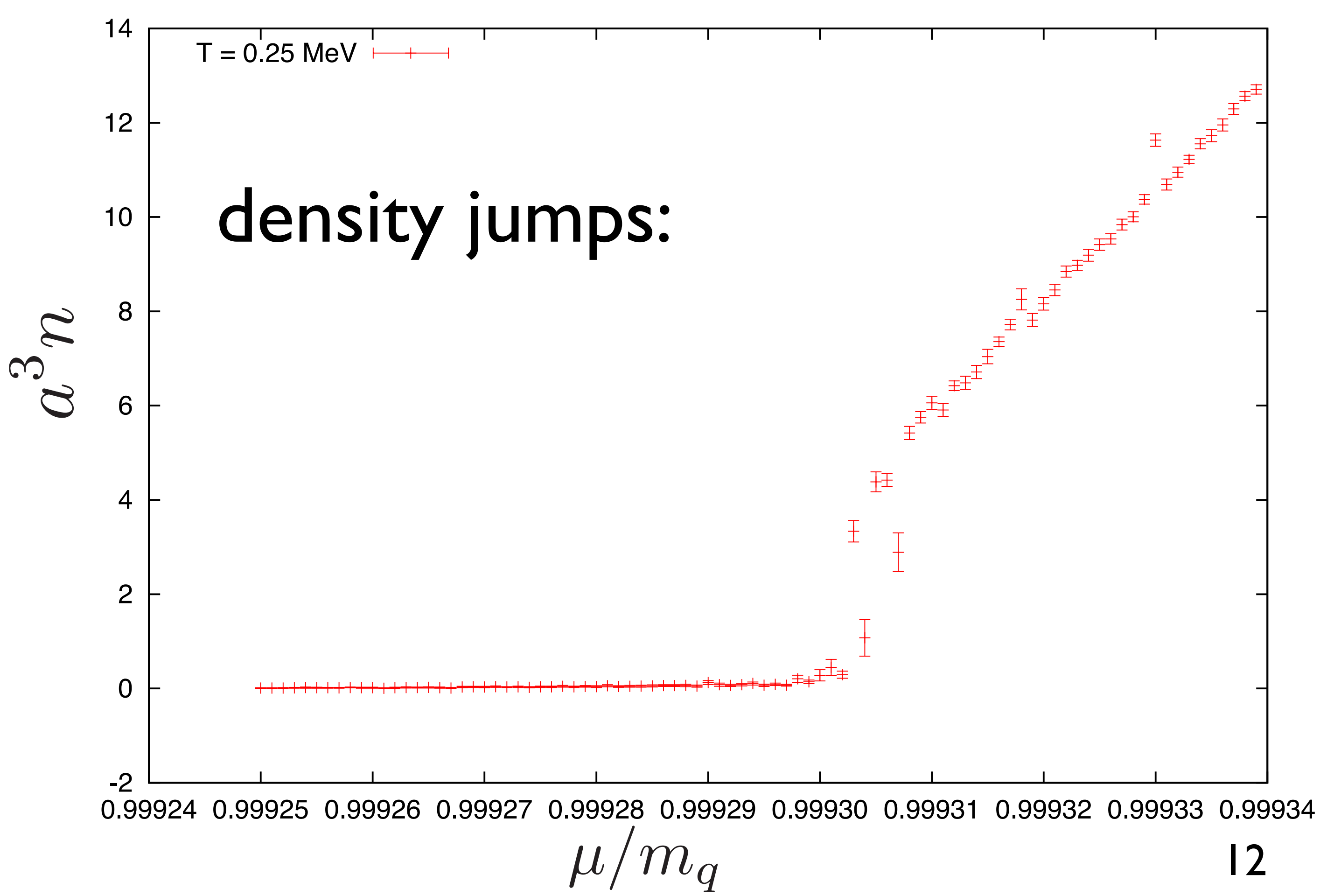
$$LO$$

$$\epsilon \equiv \frac{e - n_B m_B}{n_B m_B} = -\frac{4}{3}\frac{1}{a^3 n_B}\left(\frac{z_3}{z_0}\right)^2 \kappa^2$$



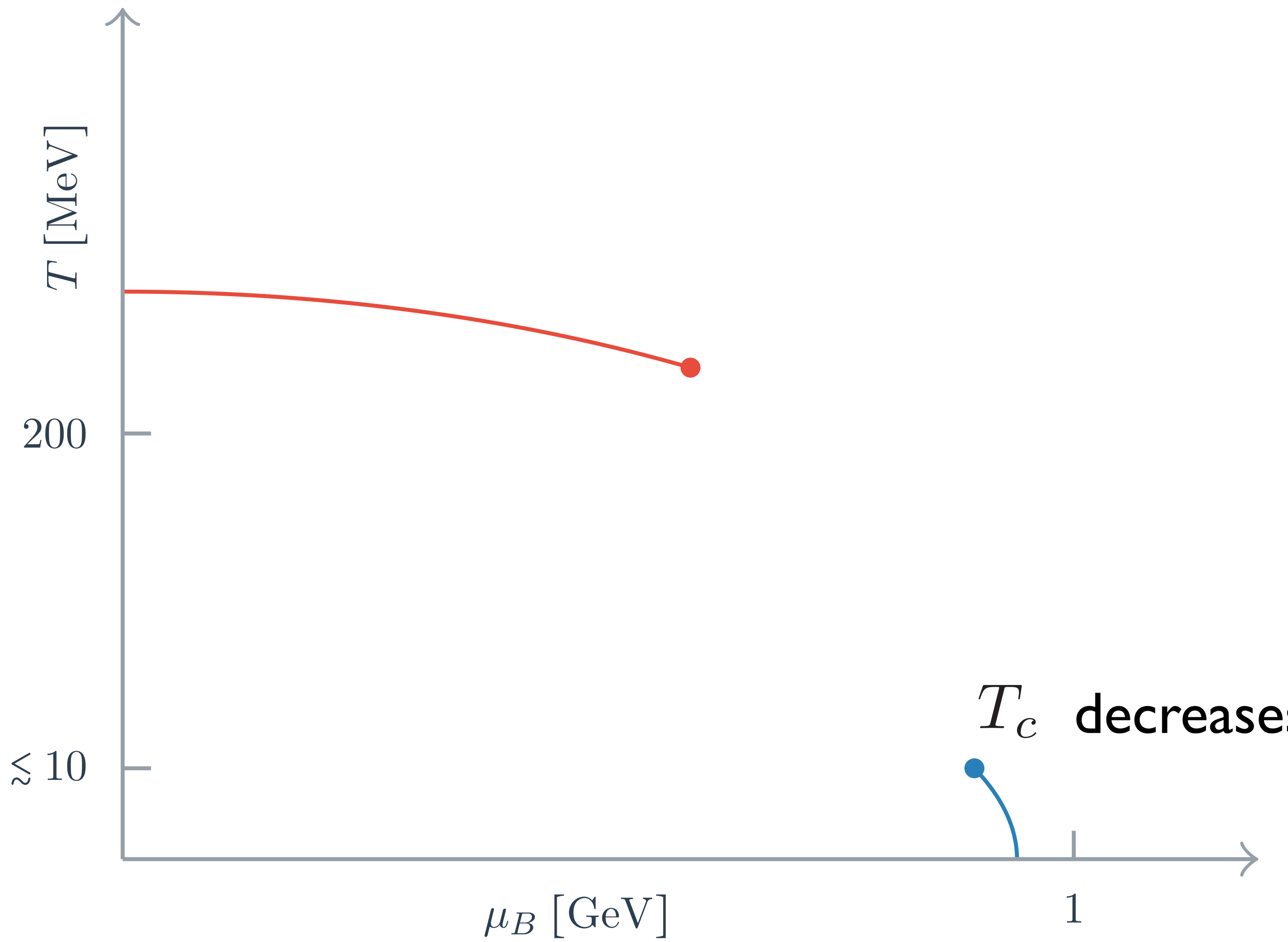
Light quarks: first order transition + endpoint





- phase coexistence: first order
- for higher $T = \frac{1}{aN_\tau}$ crossover
- nuclear liquid gas transition!

"Heavy QCD" phase diagram



T_c decreases with growing quark mass

QCD at large N_c

Definition, ['t Hooft NPB (1974)]: $N_c \longrightarrow \infty, \quad g^2 N_c = const.$

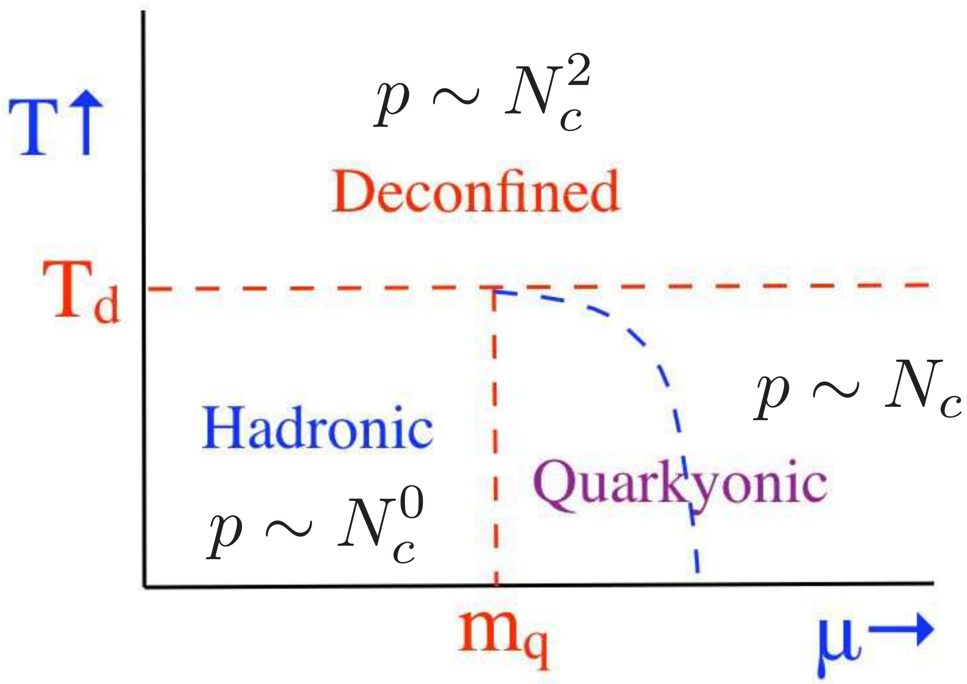
- suppresses quark loops in Feynman diagrams
- mesons are free;
  corrections: cubic interactions $\sim 1/\sqrt{N_c}$, quartic int. $\sim 1/N_c$
- meson masses $\sim \Lambda_{QCD}$
- baryons: N_c quarks, baryon masses $\sim N_c \Lambda_{QCD}$
- baryon interactions: $\sim N_c$

[Witten NPB (1979)]

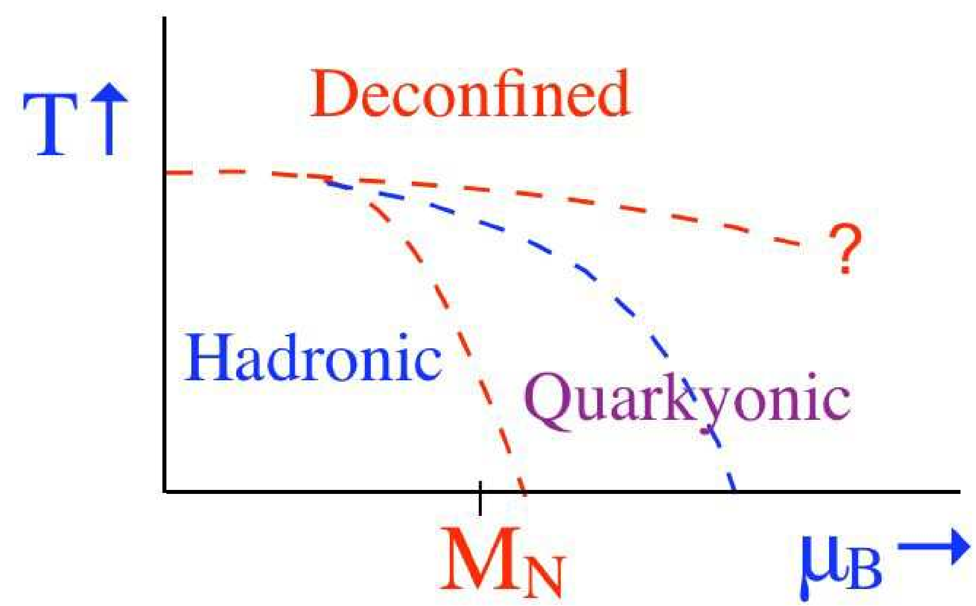
Conjectures on the QCD phase diagram

[McLerran, Pisarski NPA (2007), …]

large N_c



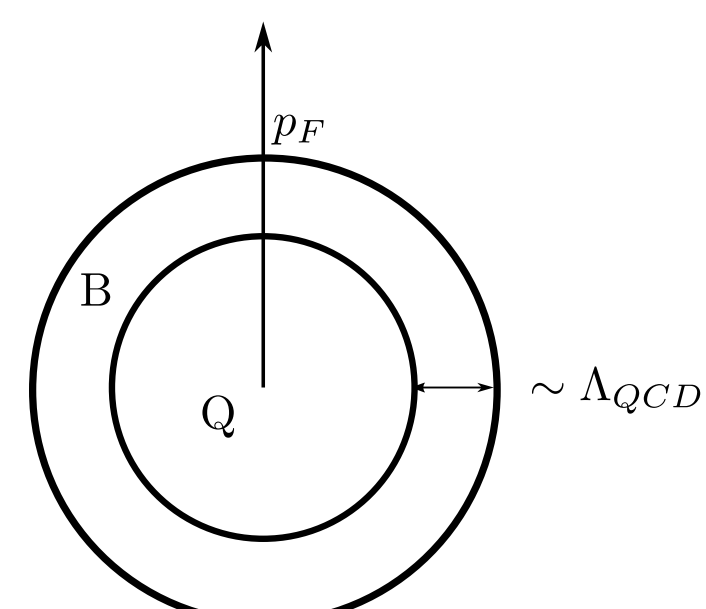
$N_c = 3$



Quarkyonic matter in momentum space:

Fermi sea of quarks, surrounded by Fermi shell of baryons;

$p_F \sim \mu$ interpolates from purely baryonic to quark matter



From conjecture to calculation: eff. theory for general N_c

[O.P., Scheunert JHEP (2019)]

"In principle" straight-forward: use character expansion for general N_c and recompute all integrals for general N_c.

For example, static strong coupling limit

$$Z(\beta = 0) = \left(1 + (N_c + 1)h_1^{N_c} + h_1^{2N_c}\right)^V$$

Baryonic spin degeneracy depends on number of colours!

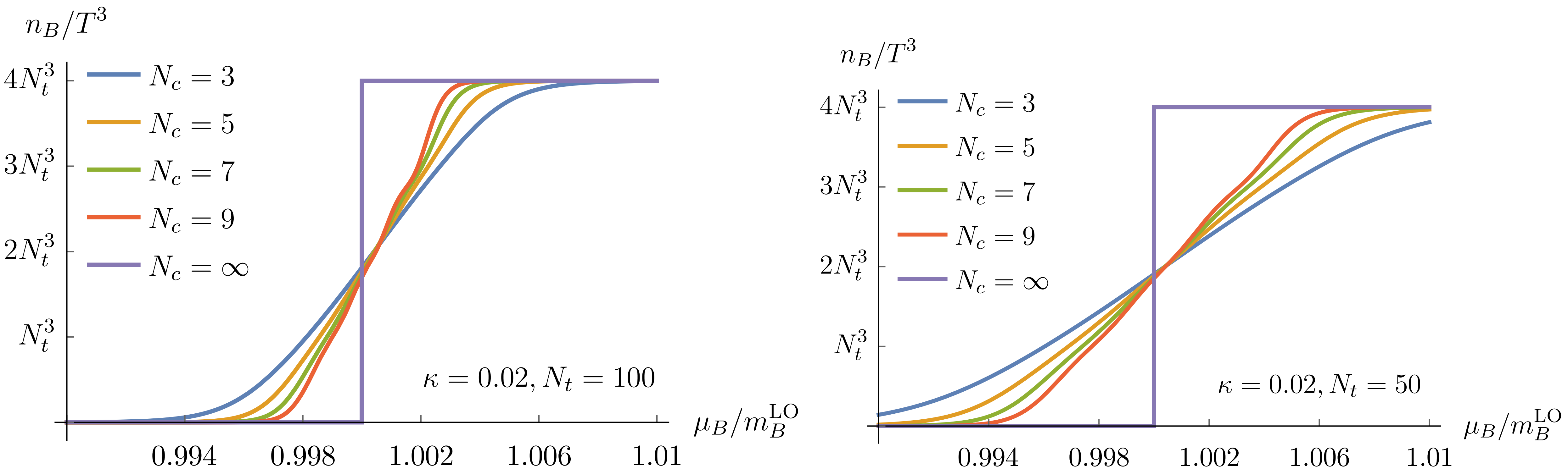
Thermodynamic functions for large N_c

Strong coupling limit

Order hopping expansion		κ^0	κ^2	κ^4
$h_1 < 1$ $(\mu_B < m_B)$	$a^4 p$	$\sim \frac{1}{6N_\tau} N_c^3 h_1^{N_c}$	$\sim -\frac{1}{48} N_c^7 h_1^{2N_c}$	$\sim \frac{3N_\tau \kappa^4}{800} N_c^8 h_1^{2N_c}$
	$a^3 n_B$	$\sim \frac{1}{6} N_c^3 h_1^{N_c}$	$\sim -\frac{N_\tau}{24} N_c^7 h_1^{2N_c}$	$\sim \frac{(9N_\tau+1)N_\tau}{1200} N_c^8 h_1^{2N_c}$
	$a^4 e$	$\sim -\frac{\ln(2\kappa)}{6} N_c^4 h_1^{N_c}$	$\sim \frac{N_\tau \ln(2\kappa)}{48} N_c^8 h_1^{2N_c}$	
	ϵ	0	$\sim -\frac{1}{4} N_c^3 h_1^{N_c}$	
$h_1 > 1$ $(\mu_B > m_B)$	$a^4 p$	$\sim \frac{4\ln(h_1)}{N_\tau} N_c$	$\sim -12 N_c$	$\sim 198 N_c$
	$a^3 n_B$	~ 4	$\sim -N_\tau \frac{N_c^4}{h_1^{N_c}}$	$\sim -\frac{(59N_\tau - 19)N_\tau}{20} \frac{N_c^5}{h_1^{N_c}}$
	$a^4 e$	$\sim -4\ln(2\kappa) N_c$	$\sim 24 \ln(2\kappa) N_c$	
	ϵ	0	~ -6	

Beyond the onset transition: $p \sim N_c$ definition of quarkyonic matter!

The baryon onset transition for growing N_c



Transition becomes more strongly first-order for every T!

Gauge corrections

So far strong coupling limit, not consistent with 't Hooft scaling

$$u(\beta) = \frac{1}{\lambda_H} = \frac{1}{g^2 N_c} < 1$$

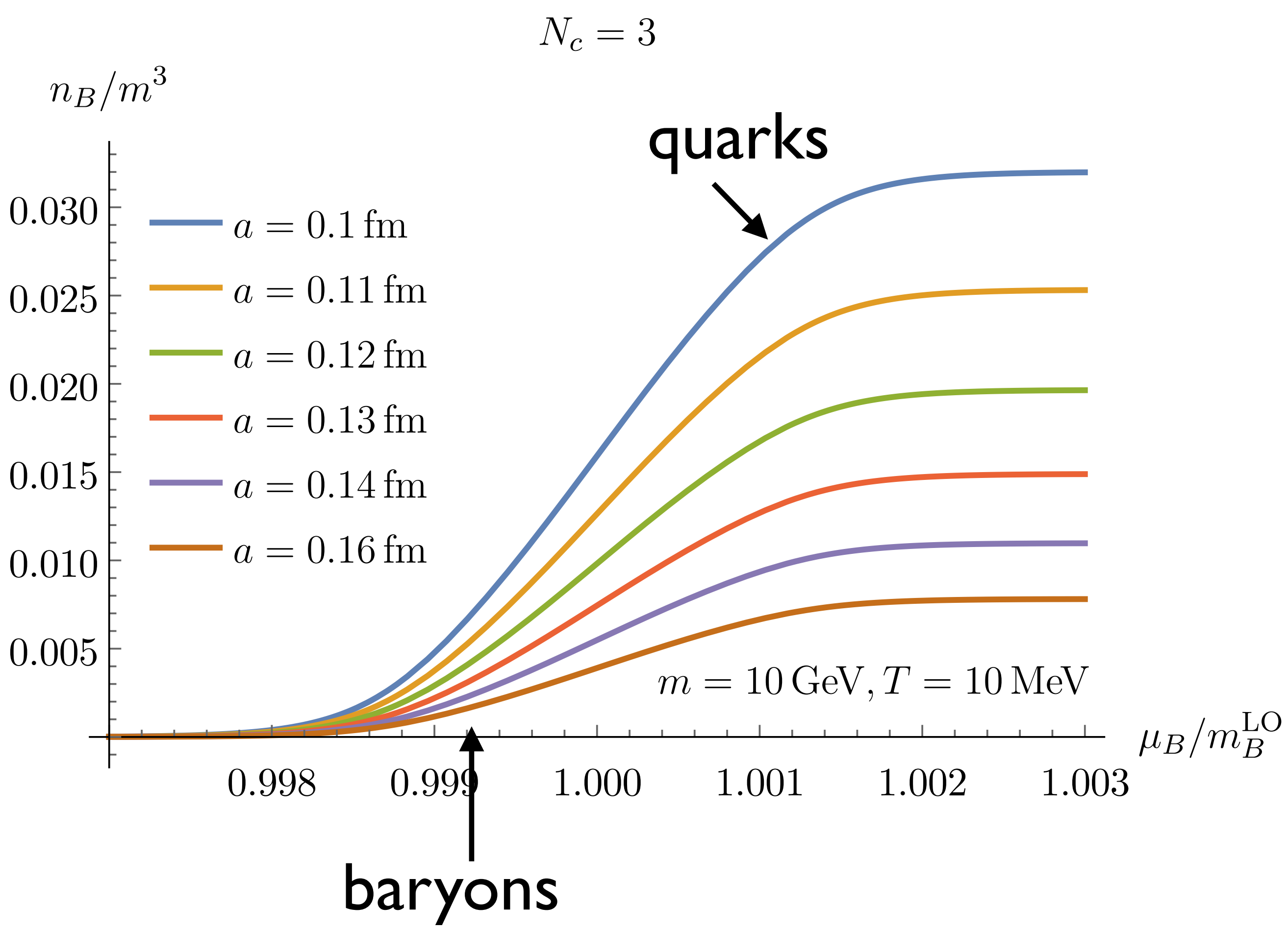
[Gross, Witten PRD (1980)]

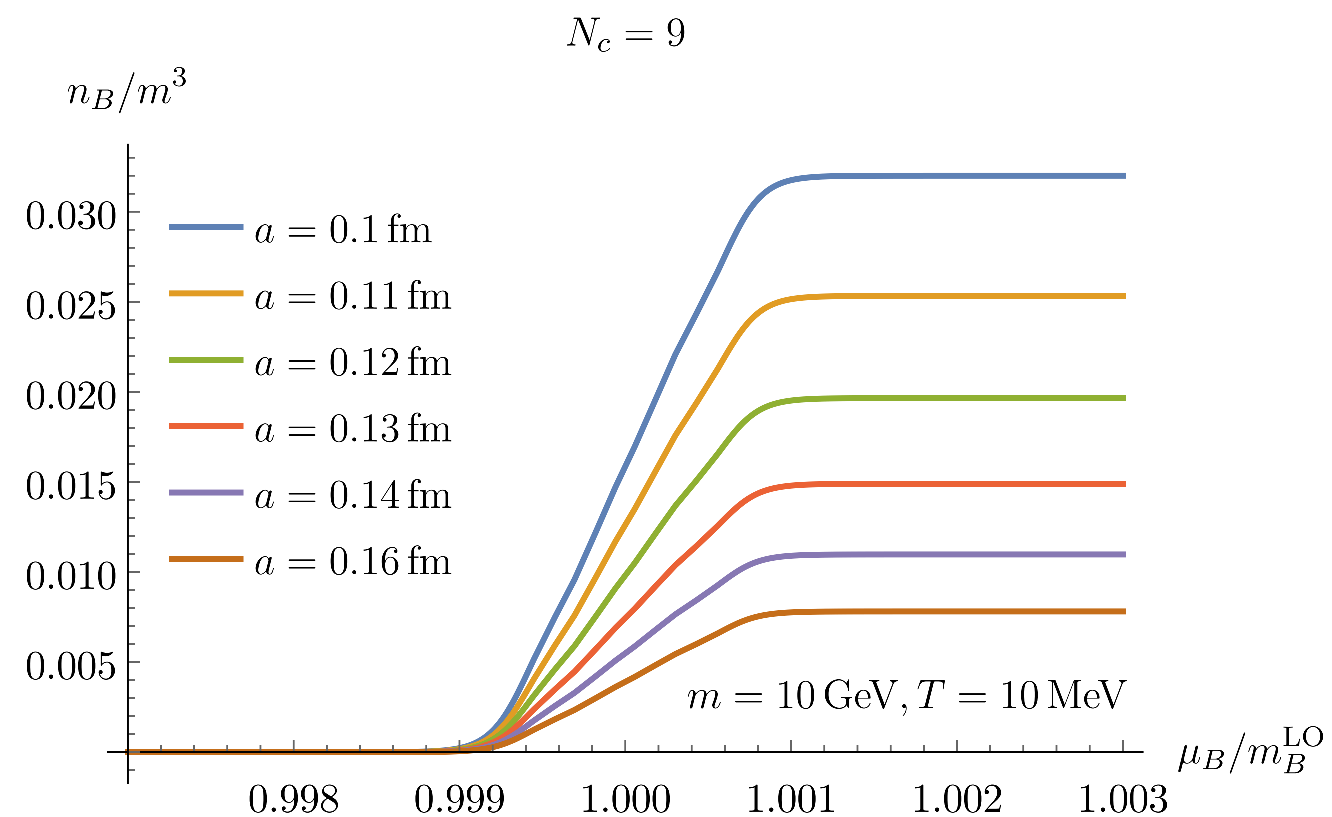
But: interchange of strong coupling and large N_c -limit "highly suspicious" in 1+1d

Here: jump to lattice saturation for large N_c, unphysical

Take continuum limit first!

Continuum approach

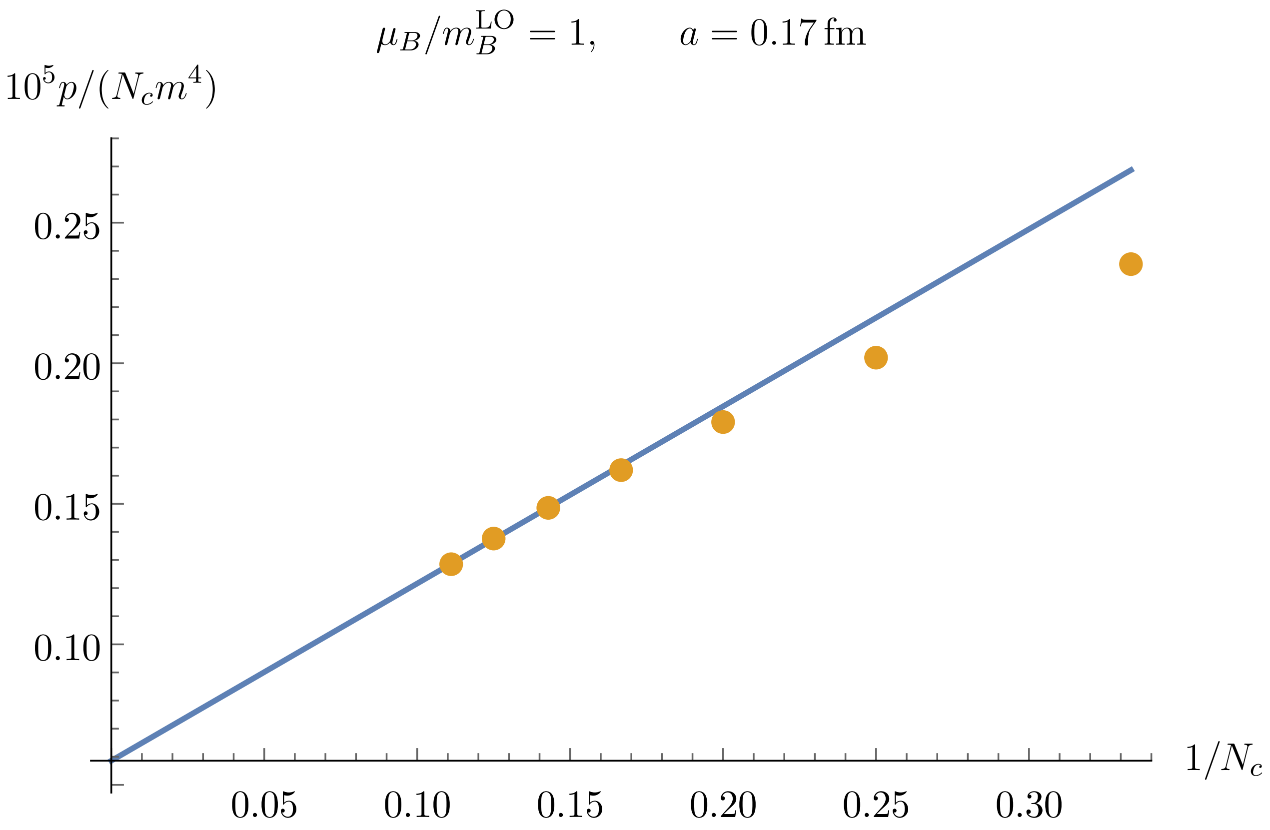


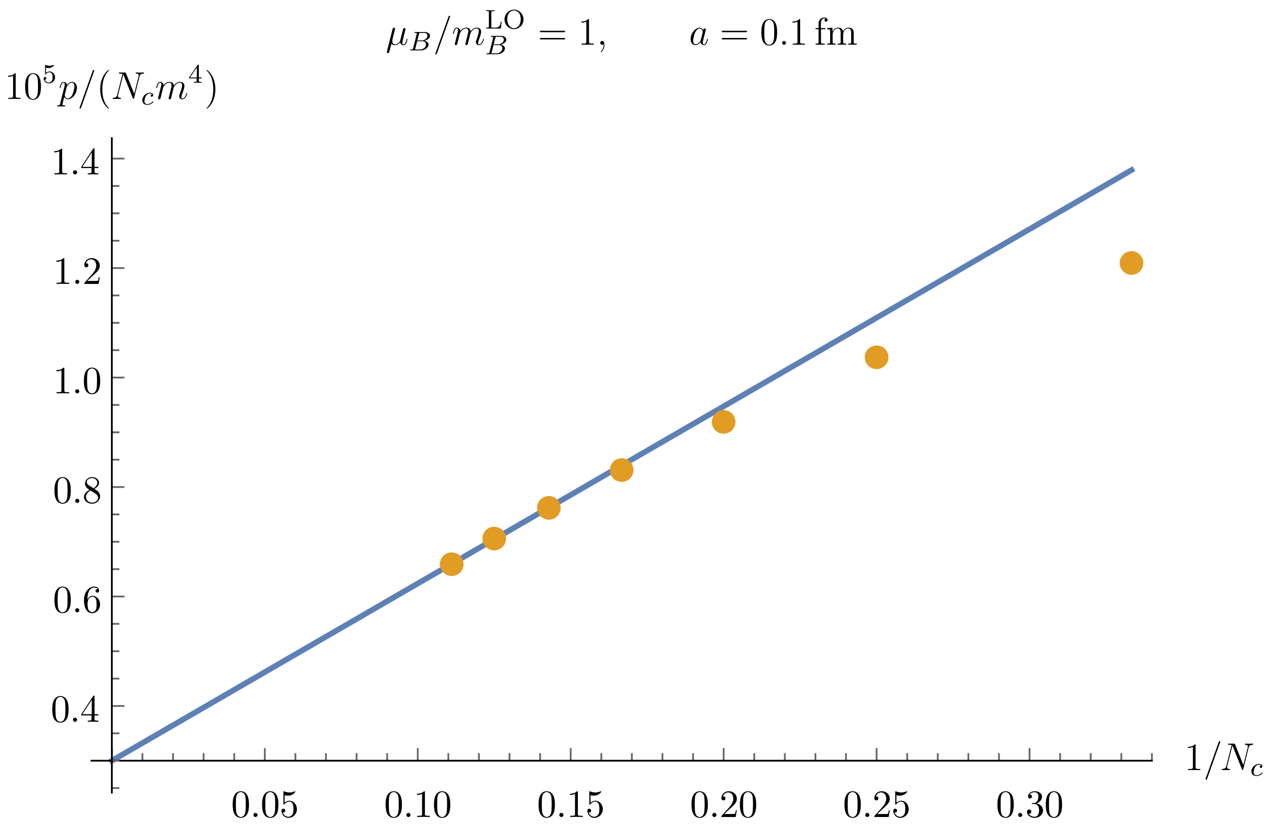


Steepening of transition still holds!

Smooth transition from baryons to quarks: quarkyonic matter on the lattice?

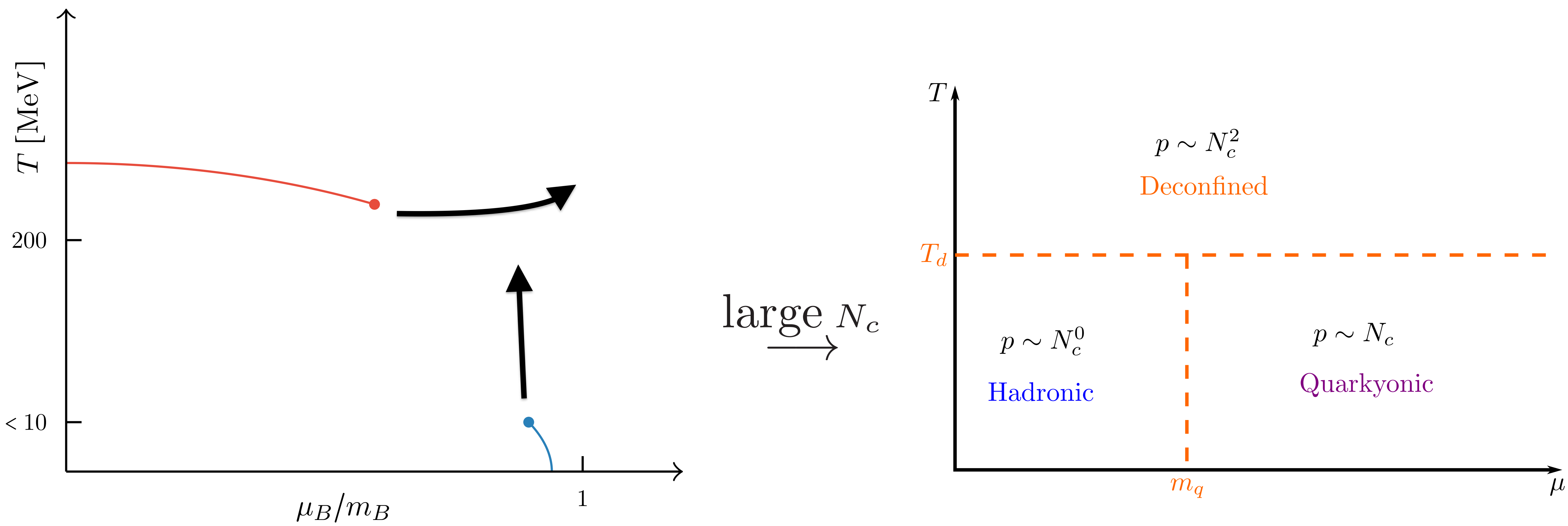
Continuum approach, pressure





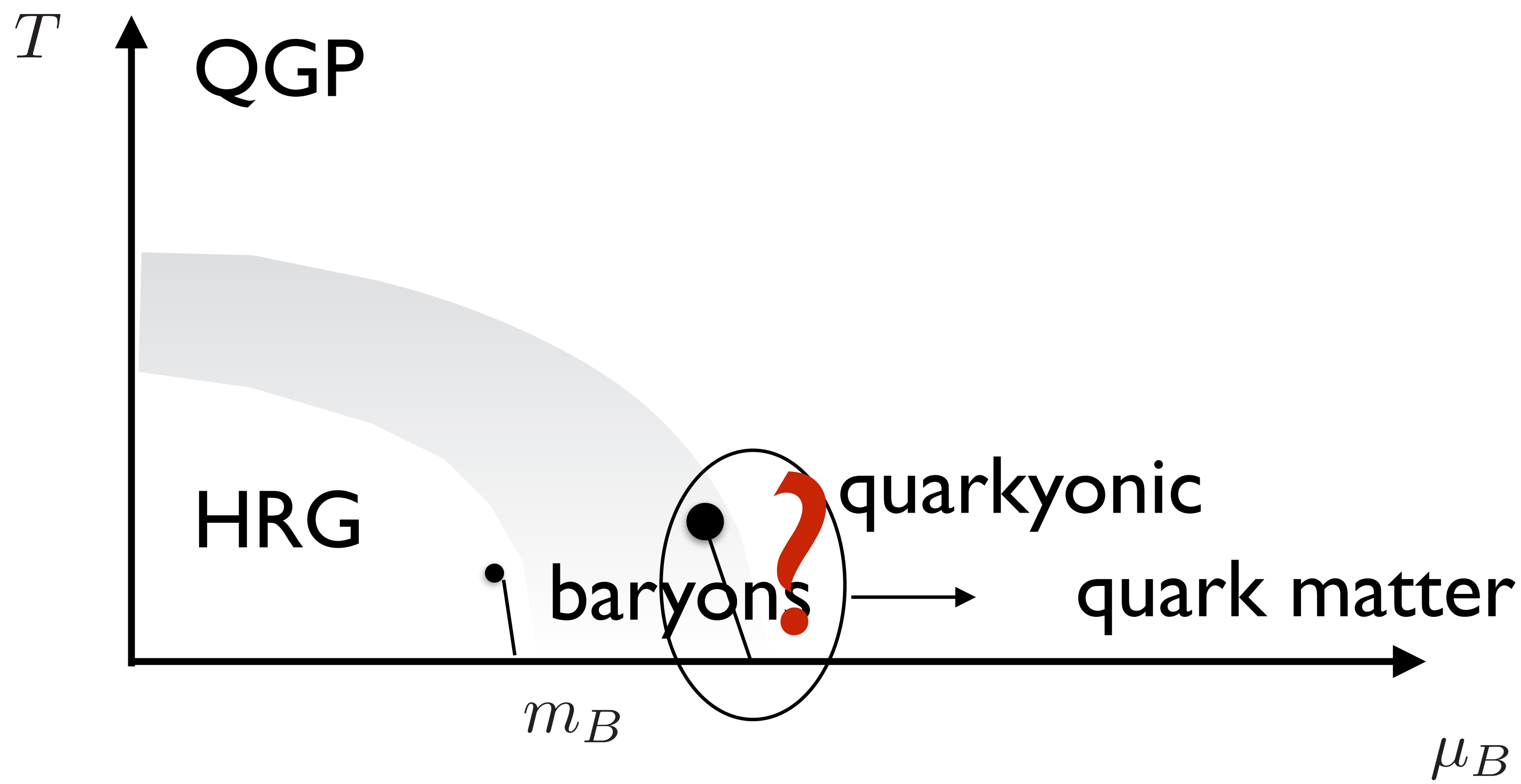
$p \sim N_c(1 + \text{const.} N_c^{-1})$ when varying lattice spacing, before saturation.

Altogether:



- Conjectured large N_c phase diagram emerges smoothly in heavy QCD
- Varying N_c : dense QCD is consistent with quarkyonic matter
- Should also hold for light quarks!

Implications for physical QCD?



Conclusions

- Nuclear liquid gas transition and equation of state calculable in the heavy mass region
- Varying N_c: dense QCD is consistent with quarkyonic matter;

  Physical meaning? At least initially equivalent to baryonic matter
- $N_c = 3$: no new phase transition

  liquid gas transition = quarkyonic transition
- There may or may not be an additional chiral transition for light quarks

Backup slides

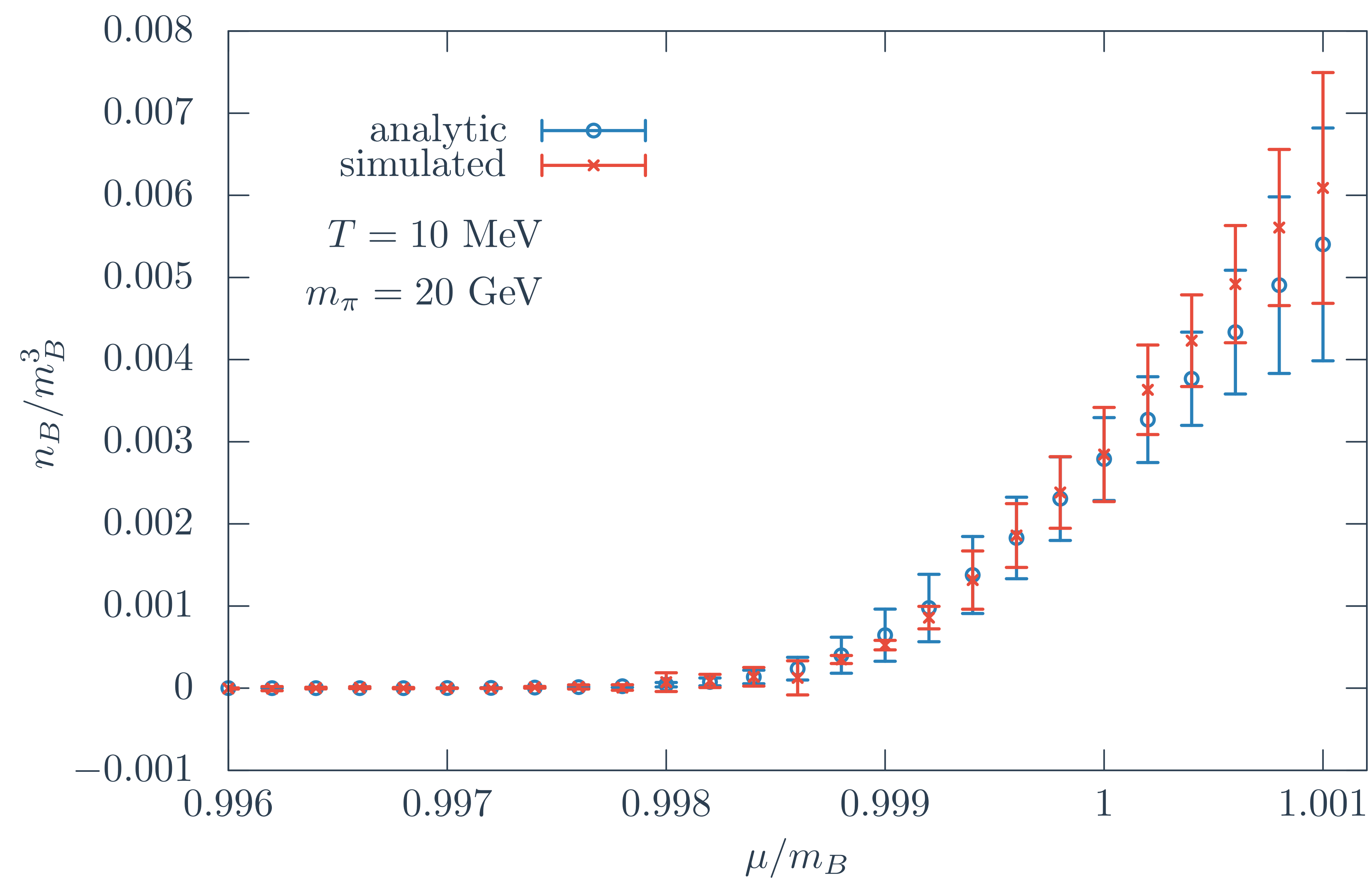
Linked cluster expansion of effective theory

$$\mathcal{Z} = \int \mathcal{D}\phi \, e^{-S_0[\phi]+\frac{1}{2}\sum v_{ij}(x,y)\phi_i(x)\phi_j(y)+\frac{1}{3!}\sum u_{ijk}(x,y,z)\phi_i(x)\phi_j(y)\phi_k(z)+\ldots}$$

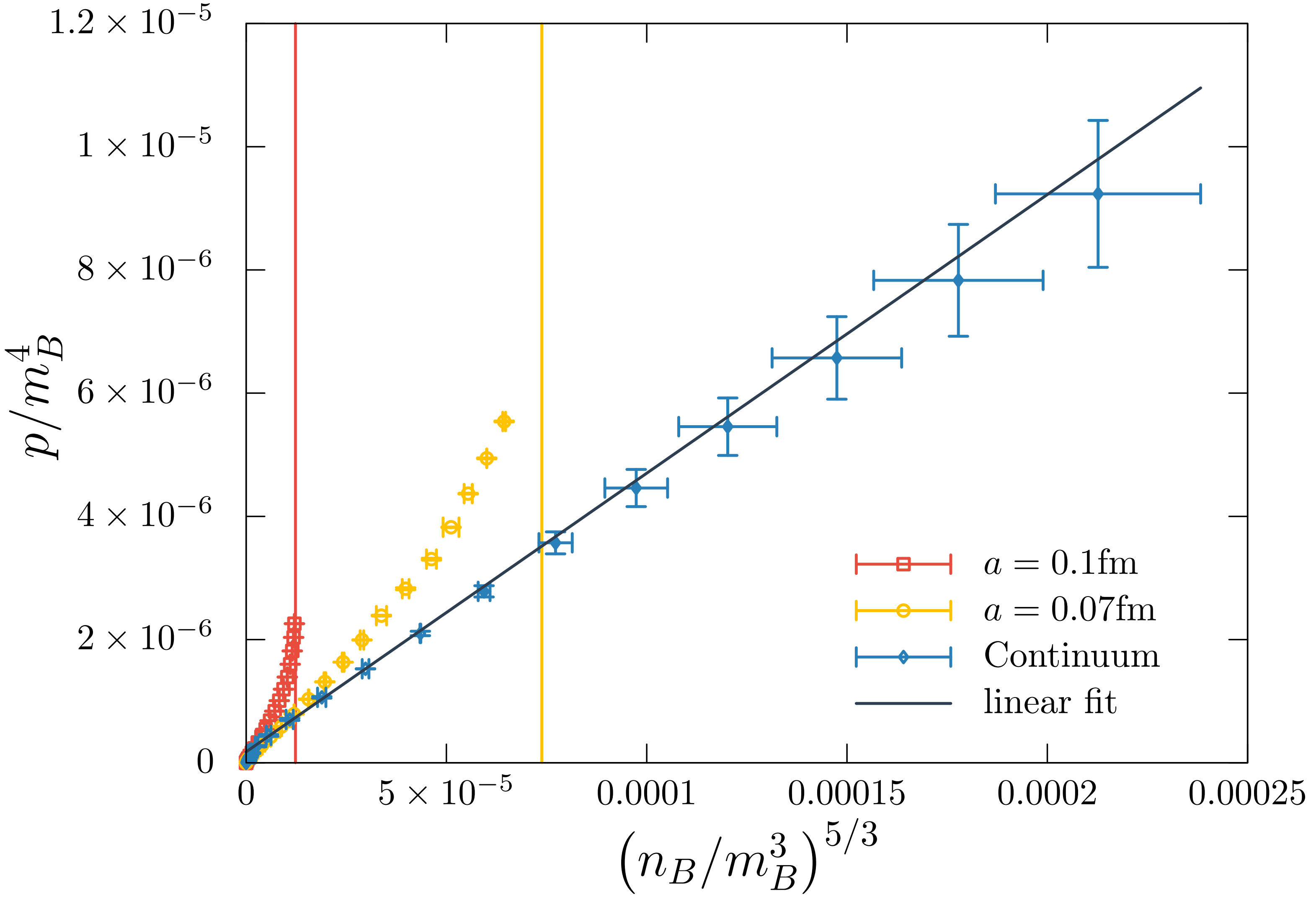
"perturbation theory" in effective couplings

Glesaaen, Neuman, O.P. 15

through $u^5\kappa^8$



Equation of state of heavy nuclear matter, continuum



- EoS fitted by polytrope, non-relativistic fermions!
- Can we understand the pre-factor? Interactions, mass-dependence…