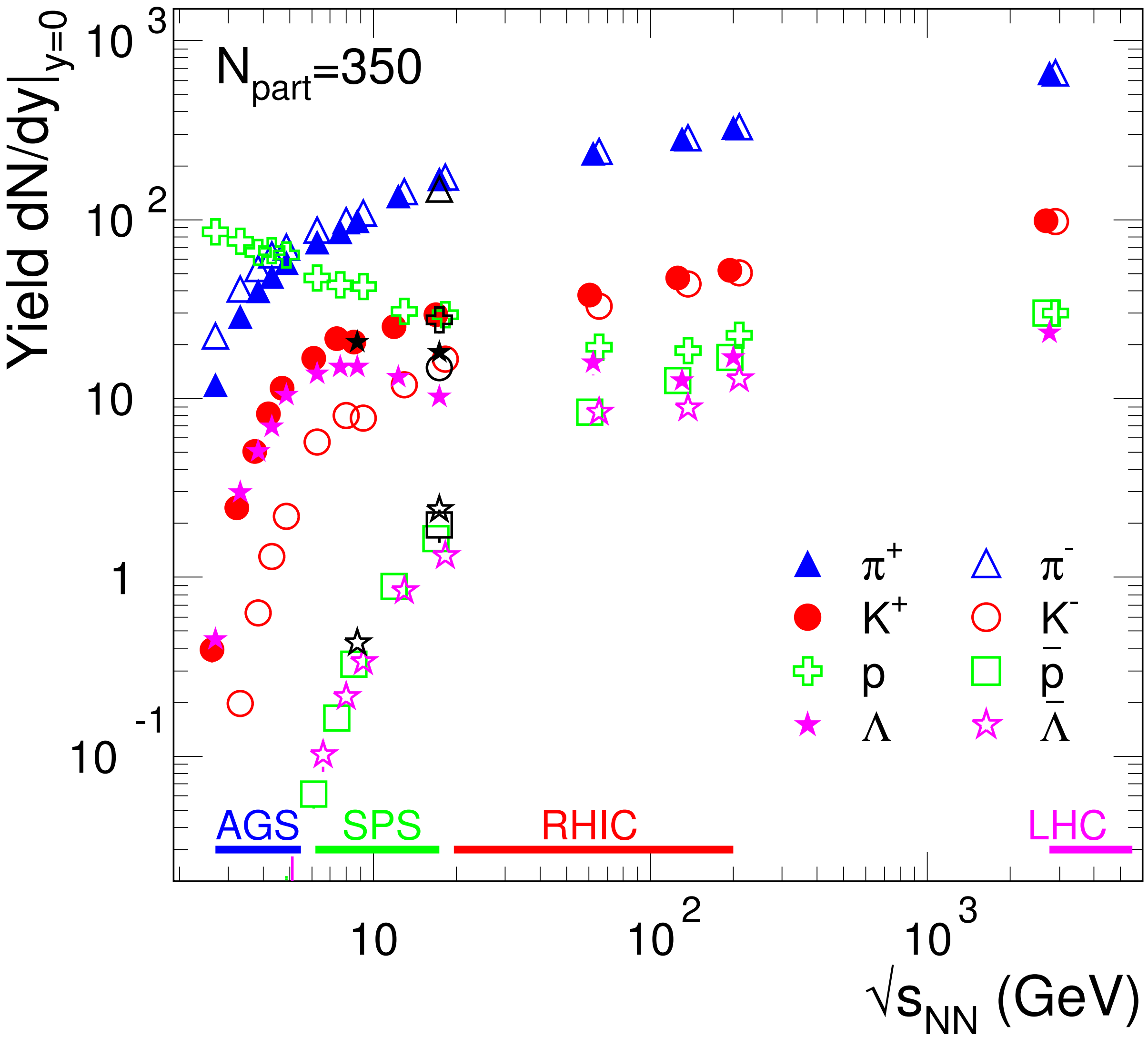
Hadron yields in central nucleus-nucleus collisions, the statistical hadronization model and the QCD phase diagram

A. Andronic - University of Münster

- Measurements of hadron yields
- The statistical (thermal) model and the thermal fits
- Thermal fits and the QCD phase diagram
- The charm quarks

Largely based on: Andronic, Braun-Munzinger, Redlich, Stachel, Nature 561 (2018) 321

Hadron yields at midrapidity (central collisions)



A.Andronic, arXiv:1407.5003

- lots of particles, mostly newly created ($m = E/c^2$)
- a great variety of species:

  $\pi^\pm$ ($u\bar{d}$, $d\bar{u}$), m=140 MeV
  $K^\pm$ ($u\bar{s}$, $\bar{u}s$), m=494 MeV
  p (uud), m=938 MeV
  Λ (uds), m=1116 MeV
  also: $\Xi(dss)$, $\Omega(sss)$...

- mass hierarchy in production (u, d quarks: remnants from the incoming nuclei)

...natural to think of the thermal (statistical) model ($\mathrm{e}^{-m/T}$)

also known as: statistical / hadron resonance gas / statistical hadronization model

...is in a way the simplest model

the analysis of hadron yields within the thermal model provides a "snapshot" of a nucleus-nucleus collision at chemical freeze-out
(the earliest in the collision timeline we can look with hadronic observables)
test hypothesis of hadron abundancies in equilibrium

...but the devil is in the details ...one needs:

- a complete hadron spectrum (all species of hadrons, see PDG, extra states?)
- canonical approach at low energies (and smaller systems)
- to understand the data well (control fractions from weak decays)

grand canonical partition function for specie (hadron) i :

$$\ln Z_i = \frac{V g_i}{2\pi^2}\int_0^\infty \pm p^2 \mathrm{d}p \ln[1 \pm \exp(-(E_i - \mu_i)/T)]$$

$g_i = (2J_i + 1)$ spin degeneracy factor; T temperature;
$E_i = \sqrt{p^2 + m_i^2}$ total energy; (+) for fermions (–) for bosons
$\mu_i = \mu_B B_i + \mu_{I_3} I_{3i} + \mu_S S_i + \mu_C C_i$ chemical potentials

μ ensure conservation (on average) of quantum numbers, fixed by "initial conditions"

i) isospin: $V_{cons} \sum_i n_i I_{3i} = I_3^{tot}$, with $V_{cons} = N_B^{tot} / \sum_i n_i B_i$

I_3^{tot}, N_B^{tot} isospin and baryon number of the system (=0 at high energies)

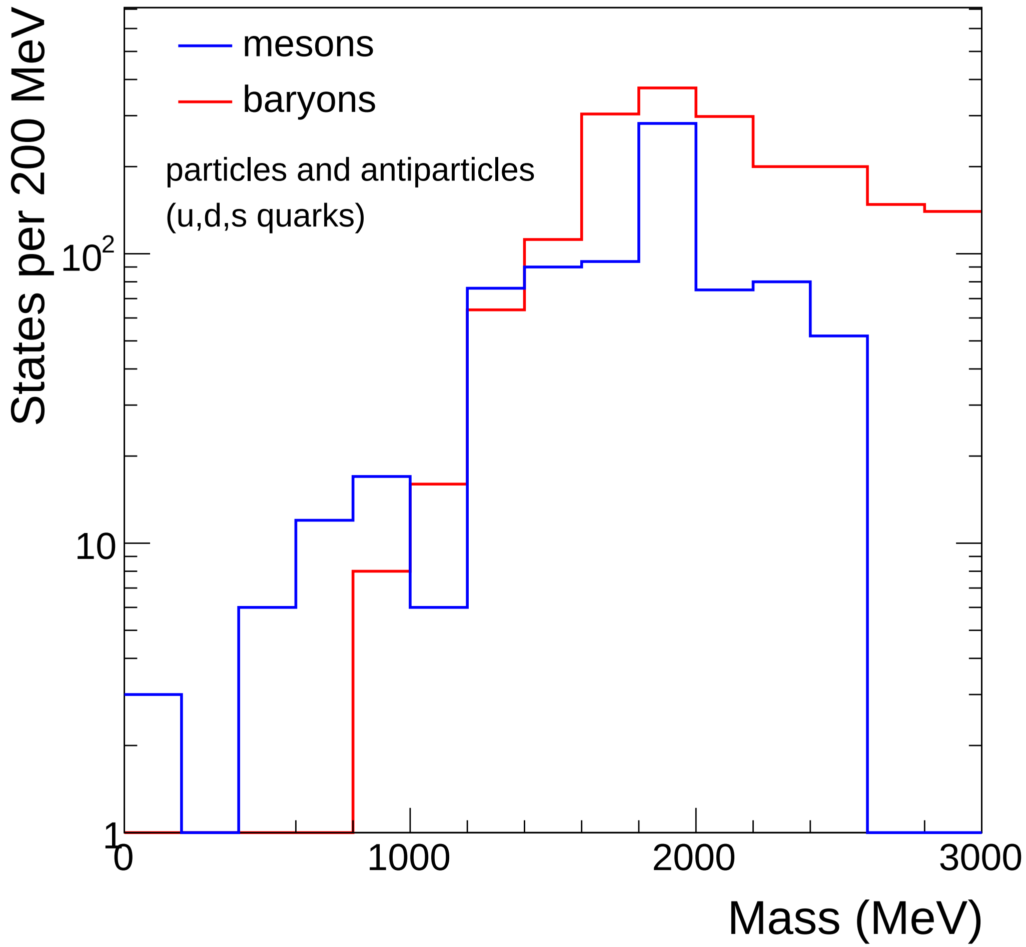
ii) strangeness: $\sum_i n_i S_i = 0$

iii) charm: $\sum_i n_i C_i = 0$.

Model input: hadron spectrum

...embodies low-energy QCD ...*vacuum masses*

well-known for $m < 2$ GeV; many confirmed states above 2 GeV, still incomplete



for high m, BR not well known, but can be reasonably guessed

T found to be robust in fits with spectrum truncated above 1.8 GeV

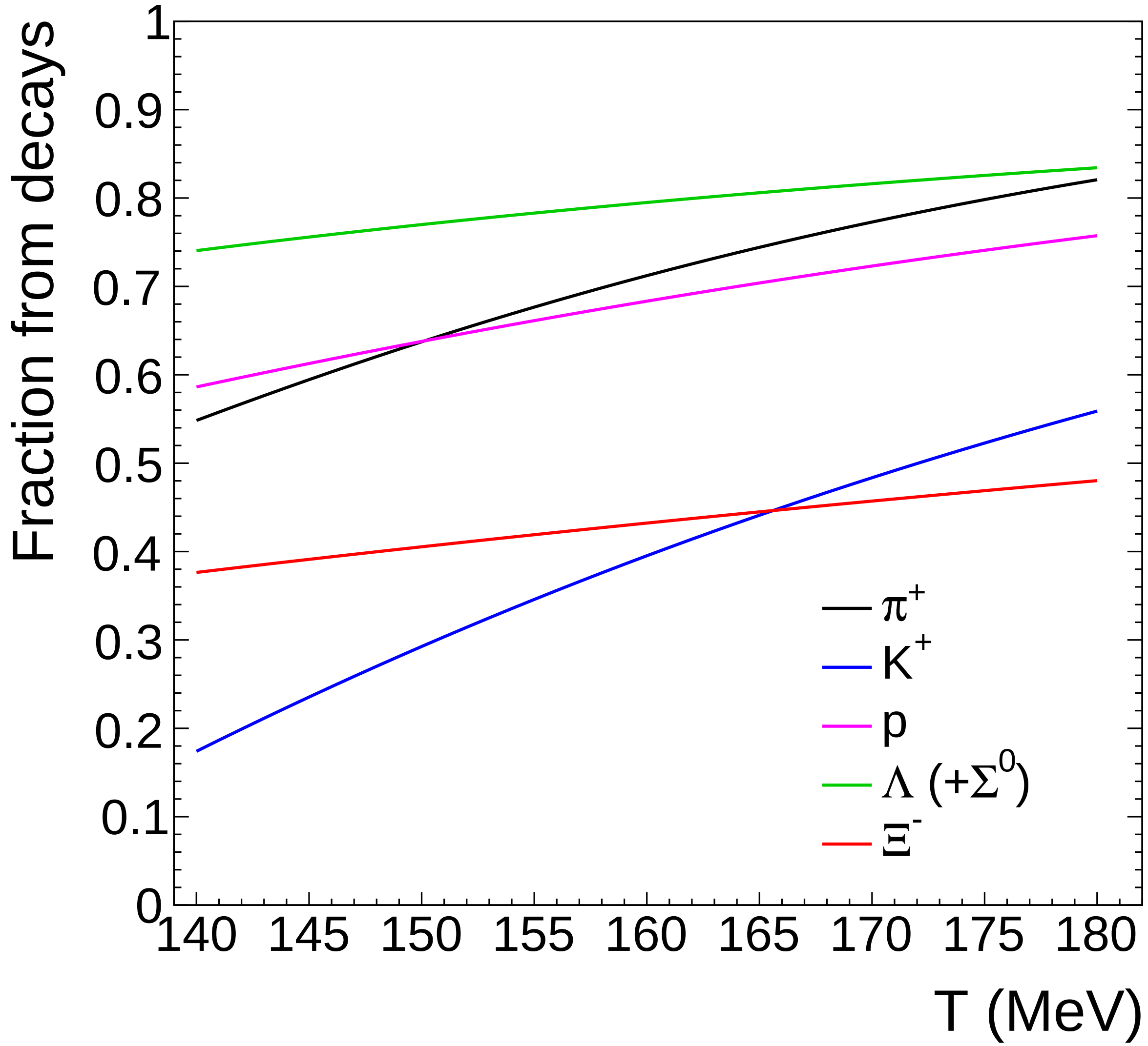
$$\rho(m) = c \cdot m^{-a} \exp(m/T_H)$$

$T_H \simeq 180$ MeV (max T for hadrons)

$(2J+1)$ counted in

Decays (feed-down)

(almost all) hadrons are subject to strong and electromagnetic decays



$\Delta \rightarrow p(n) + \pi$, $\rho \rightarrow \pi + \pi$

$\Sigma^0 \rightarrow \Lambda + \gamma$

weak decays can be treated as well ...to account for the exact experimental situation

contribution of resonances is significant (and particle-dependent)

(plot for μ_B=0)

Considering widths of resonances

$$n_i = N_i/V = -\frac{T}{V}\frac{\partial \ln Z_i}{\partial \mu} = \frac{g_i}{2\pi^2}\int_0^\infty \frac{p^2 \mathrm{d}p}{\exp[(E_i-\mu_i)/T]\pm 1}$$

$$n_i = \frac{g_i}{2\pi^2}\frac{1}{N_{BW}}\int_{M_{thr}}^\infty \mathrm{d}m \int_0^\infty \frac{\Gamma_i^2}{(m-m_i)^2+\Gamma_i^2/4}\cdot\frac{p^2 \mathrm{d}p}{\exp[(E_i^m-\mu_i)/T]\pm 1}$$

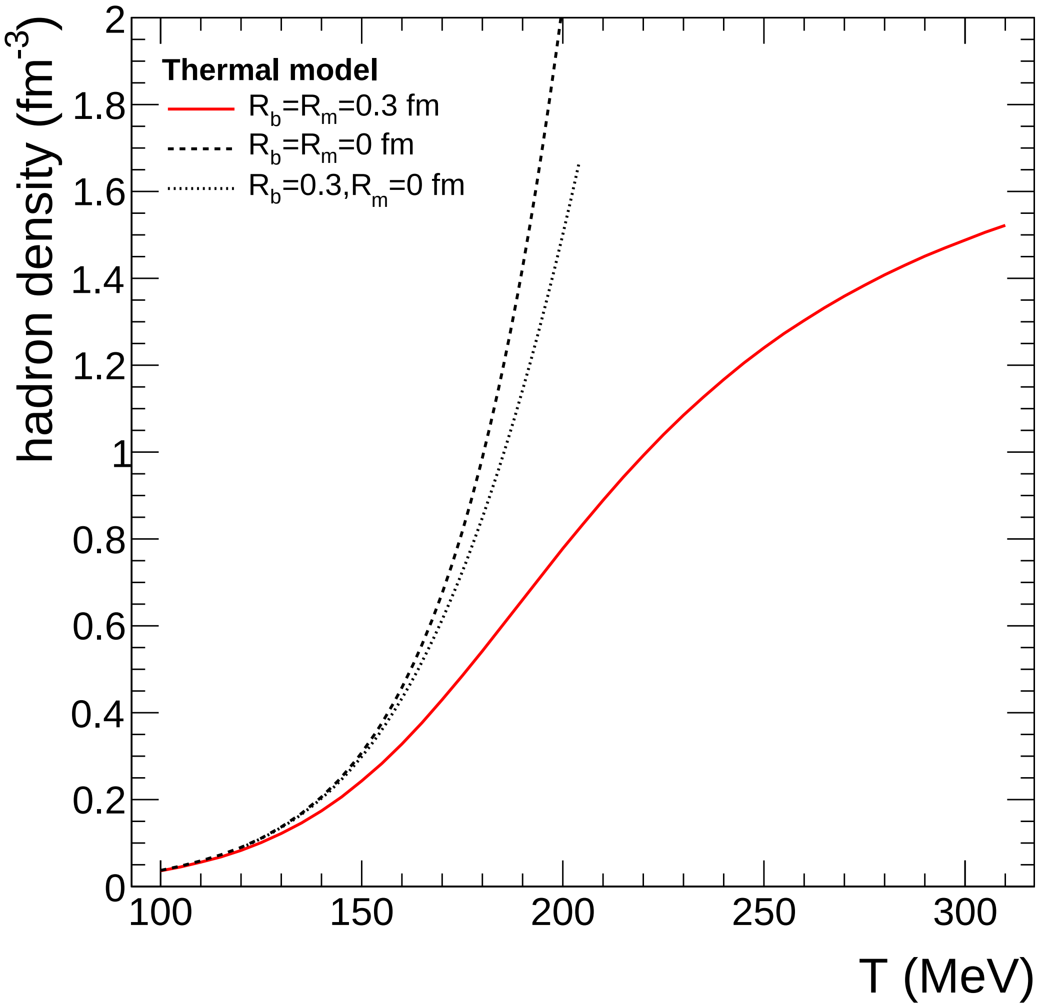
M_{thr} threshold mass for the decay channel.

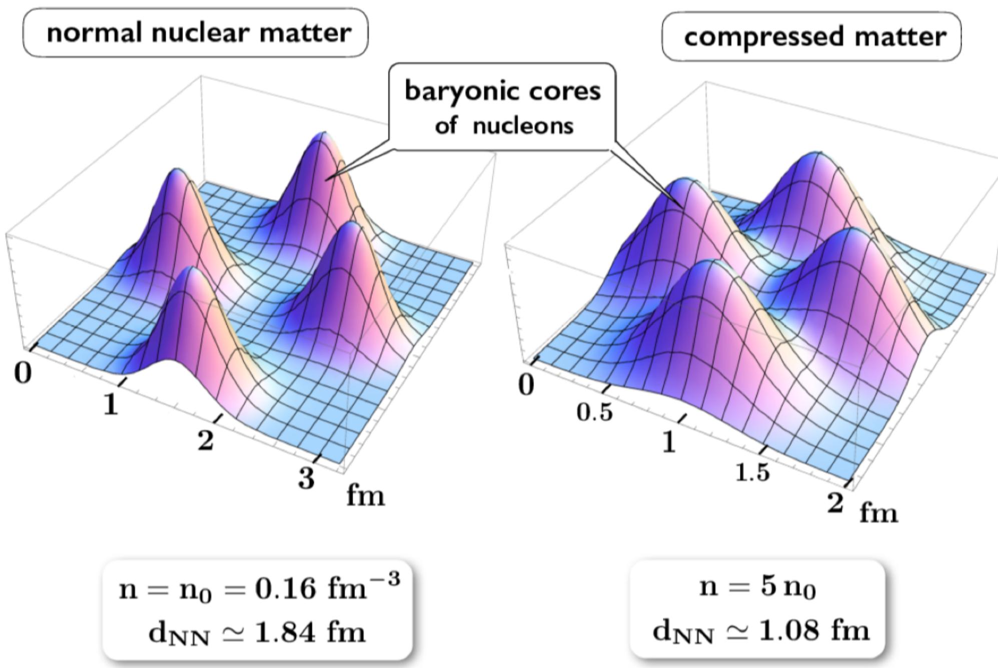
Example: for $\Delta^{++} \rightarrow p + \pi^+$, M_{thr}=1.068 GeV ($m_{\Delta^{++}}$=1.232 GeV)

Important mainly at "low" temperatures ($T \lesssim$150 MeV)

Hadron densities

A. Andronic





Weise, arXiv:1811.09682

(baryons: gaussians, r=0.5 fm)

"hadron gas": a dense system (also nuclear matter is rather a liquid than a gas)

(the usual case is $R_{baryon} = R_{meson} = 0.3$ fm ...hard-sphere repulsion)

Air at NTP: intermolecule distance $\simeq$50 $\times$ molecule size

Canonical correction ("canonical suppression")

needed whenever the abundance of hadrons with a given quantum number is very small ...so that one needs to enforce exact quantum-number conservation

in AA collisions:

strangeness at low energies

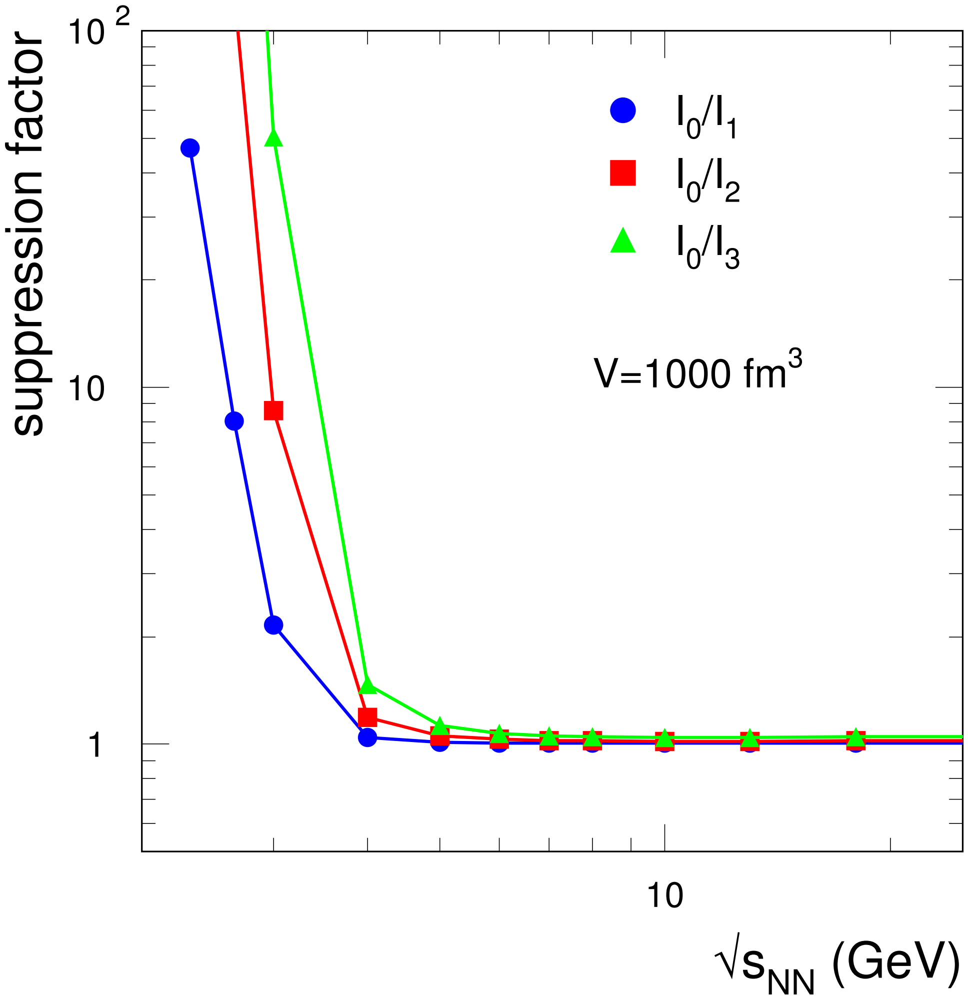
$$n_{i,S}^{C} = n_{i,S}^{GC} \cdot \frac{I_s(N_S)}{I_0(N_S)}$$

$N_S = V_c \cdot \sum S \cdot n_{i,S}$, total amount of strangeness-carrying hadrons (part.+antipart.)

$$n_{K,\Lambda}^{C} = n_{K,\Lambda}^{GC} \cdot \frac{I_1(N_S)}{I_0(N_S)},$$

$$n_{\phi}^{C} = n_{\phi}^{GC}$$

...negligible for $\sqrt{s_{NN}}$ >5 GeV



relevant for small volumes (peripheral AA, pp, p–Pb collisions)

Strangeness suppression factor, γ_s

...a non-thermal fit parameter, to check possible non-thermal production of strangeness

for a hadron carrying "absolute" strangeness $s = |S - \bar{S}|$: $n_i \rightarrow n_i \gamma_s^s$

Examples: $K^{\pm}$ $(u\bar{s}, \bar{u}s)$: $n_K \gamma_s$, Λ (uds): $n_\Lambda \gamma_s$,

$\Xi(dss)$: $n_\Xi \gamma_s^2$, $\Omega(sss)$: $n_\Omega \gamma_s^3$, $\phi(s\bar{s})$: $n_\phi \gamma_s^2$

in principle, usage of γ_s is to be avoided if one tests the basic thermal model

even as some models employ it ($\Rightarrow \gamma_s = 0.6 - 0.8$), all agree that it is not needed at RHIC, LHC energies (for central collisions)

here (central AA collisions) we fix γ_s=1

Thermal fits of hadron abundances

$$n_i = N_i/V = -\frac{T}{V}\frac{\partial \ln Z_i}{\partial \mu} = \frac{g_i}{2\pi^2}\int_0^\infty \frac{p^2 \mathrm{d}p}{\exp[(E_i-\mu_i)/T] \pm 1}$$

Latest PDG hadron mass spectrum ...quasi-complete up to m=2 GeV;
our code: 555 species (including fragments, charm and bottom hadrons)

for resonances, the width is considered in calculations

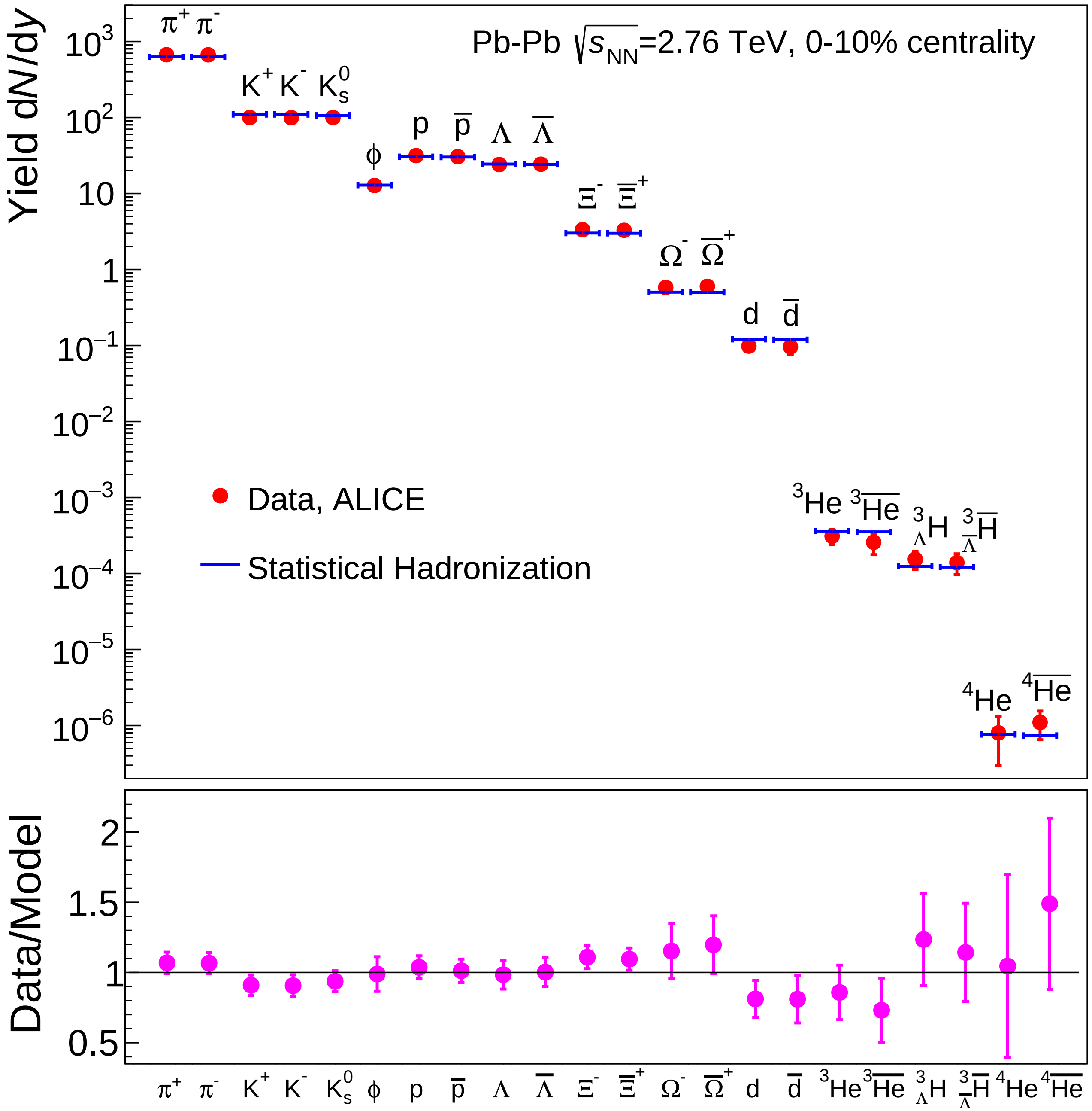
canonical treatment whenever needed (small abundances)

Minimize: $\chi^2 = \sum_i \frac{(N_i^{exp} - N_i^{therm})^2}{\sigma_i^2}$

N_i hadron yield, σ_i experimental uncertainty (stat.+syst.)

$\Rightarrow (T, \mu_B, V)$ *...tests chemical freeze-out* (chemical equilibrium)

Thermal fit – LHC, Pb–Pb, 0-10%



matter and antimatter produced in equal amounts

$T_{CF} = 156.6 \pm 1.7$ MeV
$\mu_B = 0.7 \pm 3.8$ MeV
$V_{\Delta y=1} = 4175 \pm 380$ fm^3

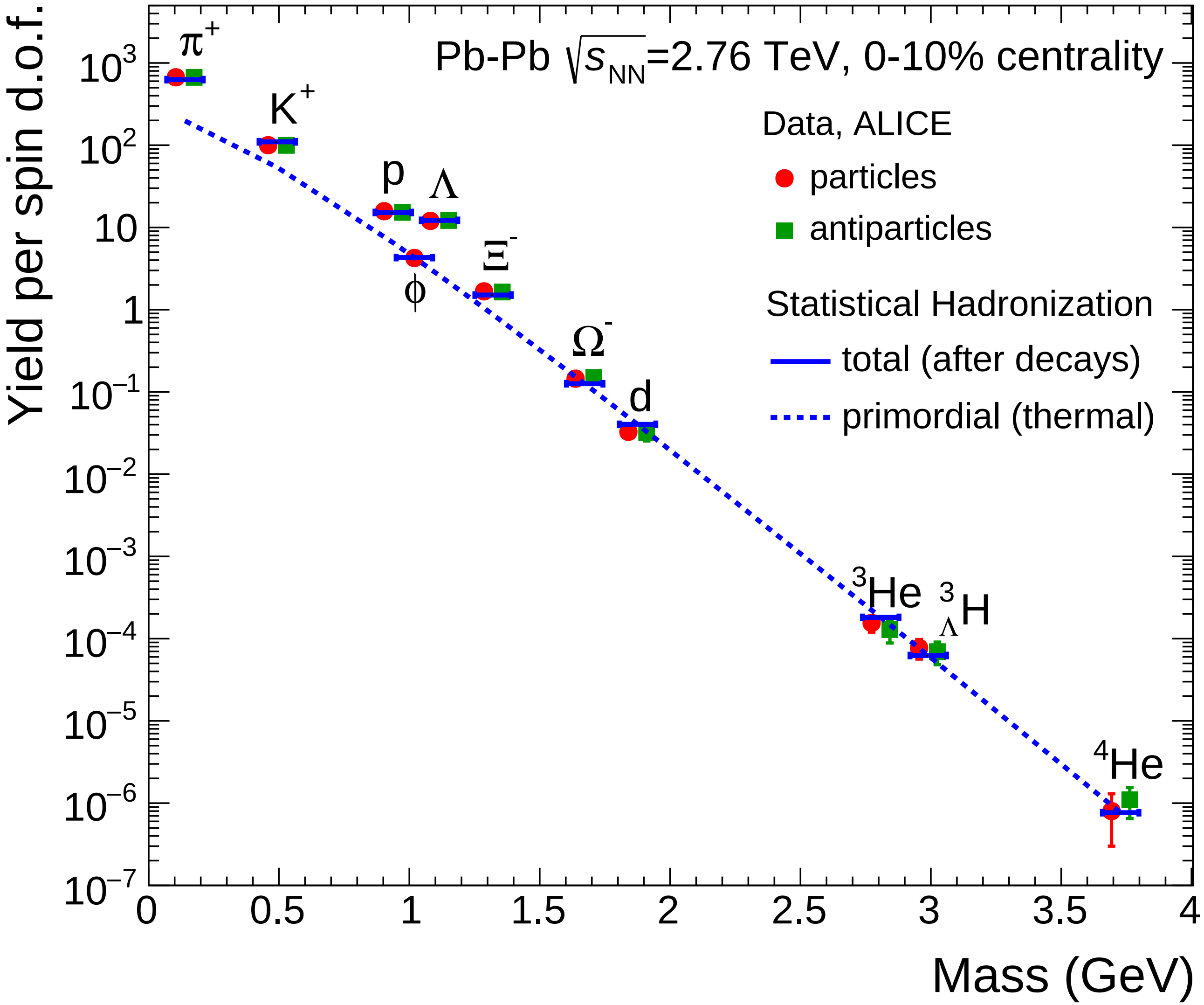
$\chi^2/N_{df} = 16.7/19$

S-matrix treatment

remarkably, loosely-bound objects are also well described
($^3_\Lambda$H with 25% B.R.)

hadronization as bags of quarks and gluons?

Model uncertainties 1. Hadron spectrum



contribution of resonances is significant (and particle-dependent)

Fit of ϕ, Ω, d, ^{3}He, $^3_\Lambda$H, ^{4}He:

$T_{CF} = 156 \pm 2.5$ MeV

$(\chi^2/N_{df} = 7.4/8)$

Fit of nuclei (d, ^{3}He, ^{4}He):

$T_{CF} = 159 \pm 5$ MeV

3-4 MeV upper bound of systematic uncertainty due to hadron spectrum

hadron eigenvolumes ...to mimick interactions (beyond low-density, Dashen-Ma)

we consider that $R_{meson} = 0.3, R_{baryon} = 0.3$ *fm is a reasonable case*

point-like hadrons lead to same T, but volume larger by 20-25%

an extreme case, $R_{meson} = 0, R_{baryon} = 0.3$ fm leads to
$T = 161.0 \pm 2.0$ MeV, $\mu_B = 0$ *fixed*, $V = 3470 \pm 280$ fm^3

NB: in this case, the result is rather sensitive on the set of hadrons in the fit
for instance, using hadrons up to Ω, cannot constrain T (unphysically large)
Vovchenko, Stöcker (et al.), JPG 44 (2017) 055103, arXiv:1606.06350

...and anything else can be imagined, see (R dependent on mass & strangeness)
Alba, Vovchenko, Gorenstein, Stöcker, NPA 974 (2018) 22, etc.

T-dependent Breit-Wigner resonance widths:
Vovchenko, Gorenstein, Stöcker, PRC 98 (2018) 034906

Interactions, rightly done

...for now only at the LHC ($\mu_B \simeq$0)

non-strange baryon sector treated in S-matrix formalism
(πN scattering phase shifts, *including non-resonant contributions*)

PLB 792 (2019) 304

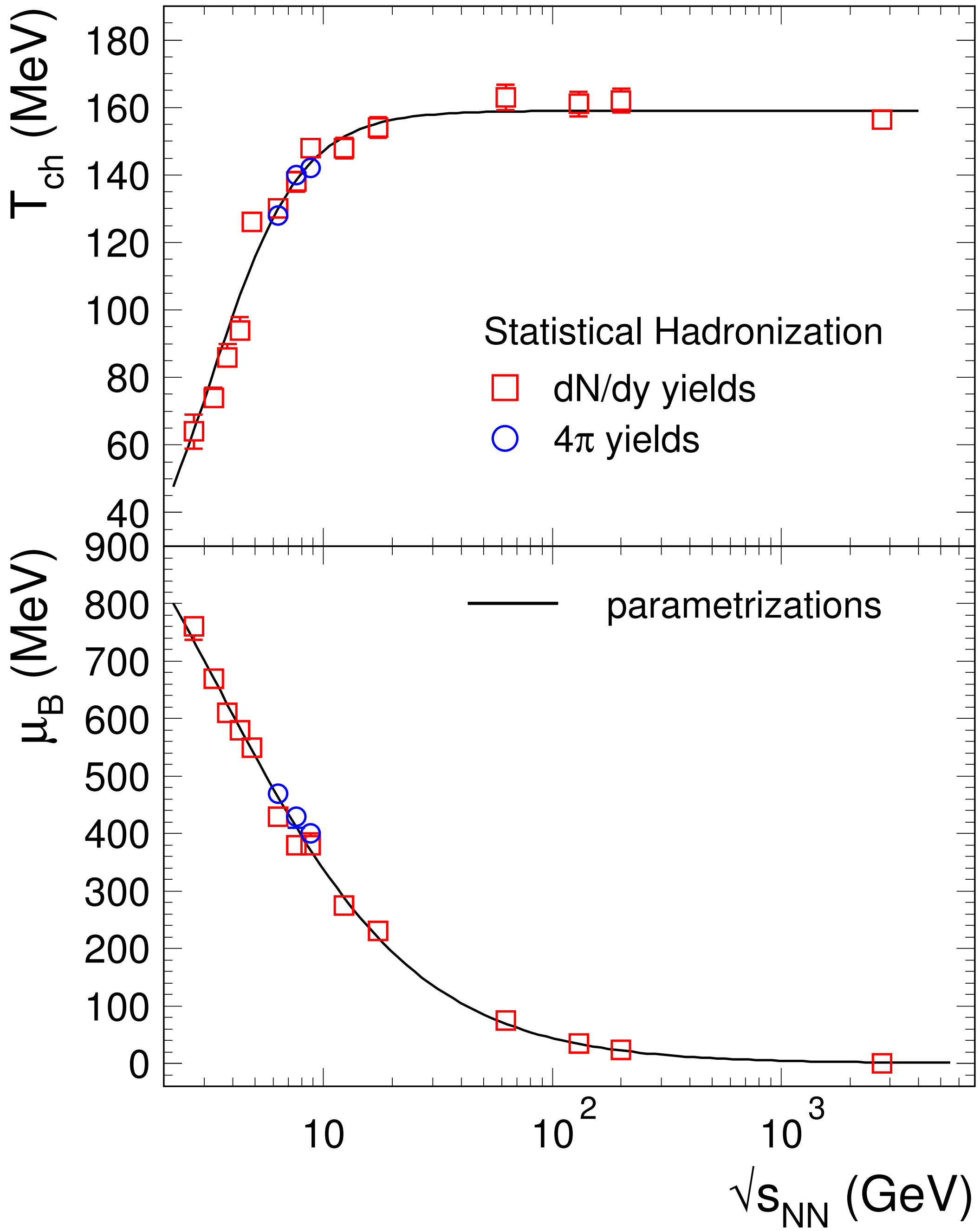
solved the so-called "proton puzzle" (too many protons in the statistical model)

for T=156 MeV, proton yield decreased by 17% compared to point-like

still missing: strange baryon sector (∼7% of protons from Λ^*, Σ^*, T-dep.)
PRC 98 (2018) 044910

NB: presence of resonances implies interaction
(this is why moderate $R = 0.3$ fm is a reasonable choice)

Energy dependence of T, μ_B (central collisions)



thermal fits exhibit a limiting temperature:

$$T_{lim} = 158.4 \pm 1.4 \text{ MeV}$$

$$T_{CF} = T_{lim} \frac{1}{1+\exp(2.60-\ln(\sqrt{s_{NN}}(\text{GeV}))/0.45)}$$

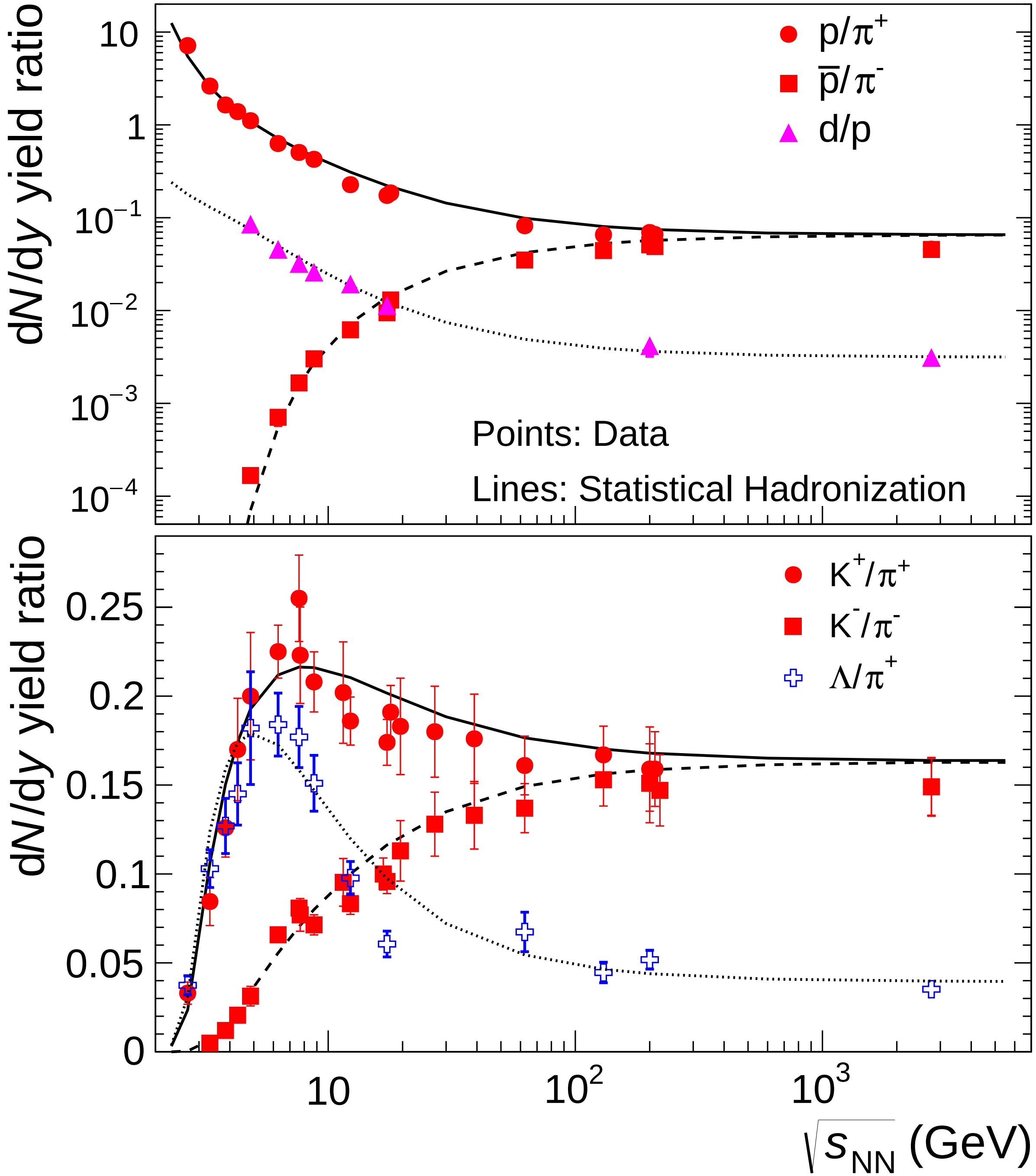
$$\mu_B[\text{MeV}] = \frac{1307.5}{1+0.288\sqrt{s_{NN}}(\text{GeV})}$$

NPA 772 (2006) 167, PLB 673 (2009) 142

μ_B is a measure of the net-baryon density, or matter-antimatter asymmetry

determined by the "stopping" of the colliding nuclei

The grand (albeit partial) view



Data:

AGS: E895, E864, E866, E917, E877

SPS: NA49, NA44

RHIC: STAR, BRAHMS

LHC: ALICE

NB: no contribution from weak decays

d/p ratio is well described for all energies

"structures" described by SHM

...determined by strangeness conservation

Λ/π peak reflects increasing T and decreasing μ_B

The statistical (thermal) model and light nuclei

...is a topic hotly debated, currently

...see dedicated reviews:

Dönigus, IJMPE 29 (2020) 20040001

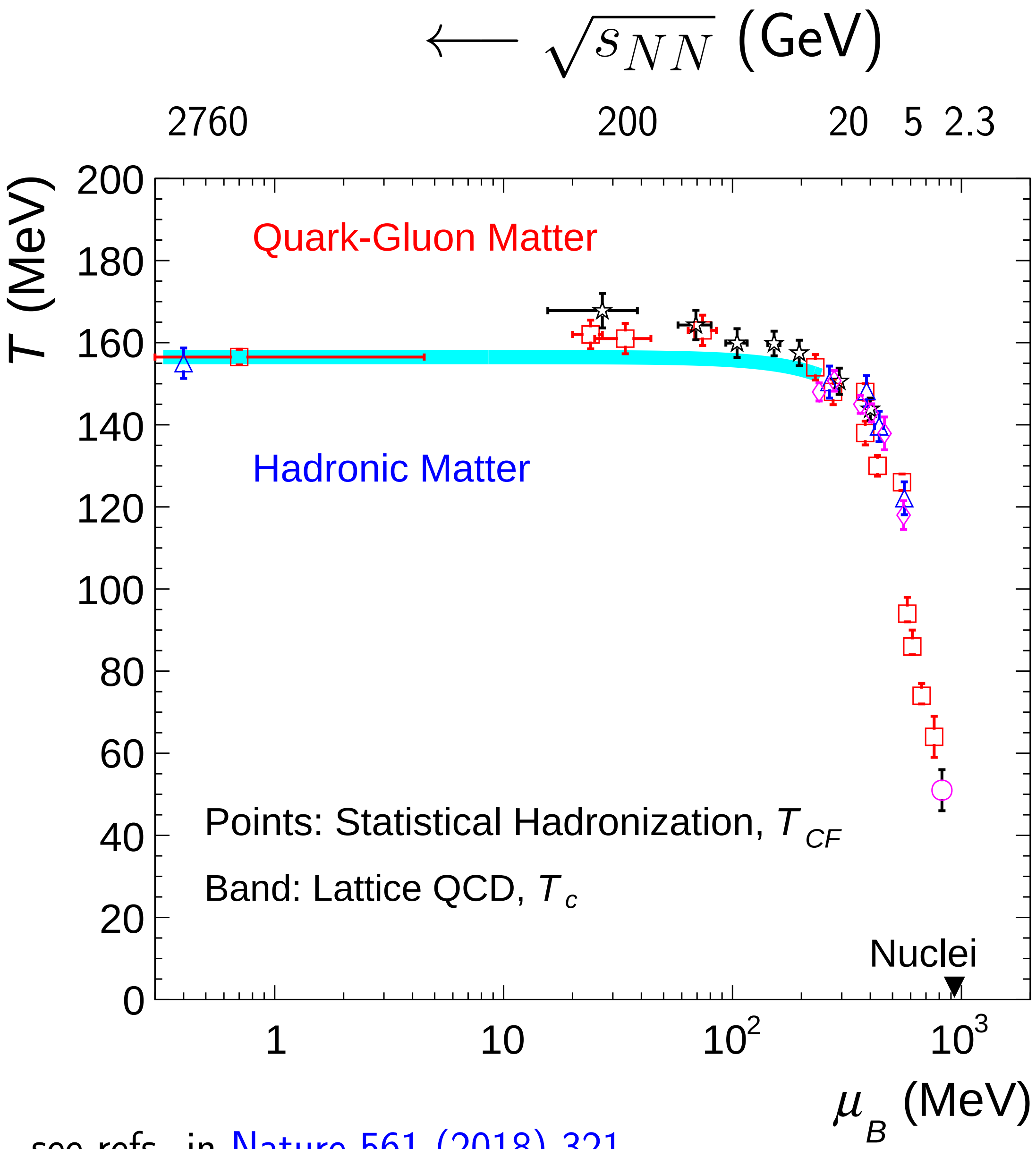
Light nuclei in the hadron resonance gas

Braun-Munzinger, Dönigus, NPA 987 (2019) 144

Loosely-bound objects produced in nuclear collisions at the LHC

Chen, Keane, Ma, Tang, Xu, Phys.Rep. 760 (2018) 1

Antinuclei in Heavy-Ion Collisions



see refs. in Nature 561 (2018) 321

at LHC, remarkable "coincidence" with Lattice QCD results

at LHC ($\mu_B \simeq 0$): purely-produced (anti)matter ($m = E/c^2$), as in the Early Universe

$\mu_B > 0$: more matter, from "remnants" of the colliding nuclei

$\mu_B \gtrsim 400$ MeV: *the critical point awaiting discovery*

(RHIC BES / FAIR)

points: independent analyses of same data → "model/code uncert." are small

Matching HRG and LQCD

a subject of (still) some debate

hadrons are not necessarily point-particles

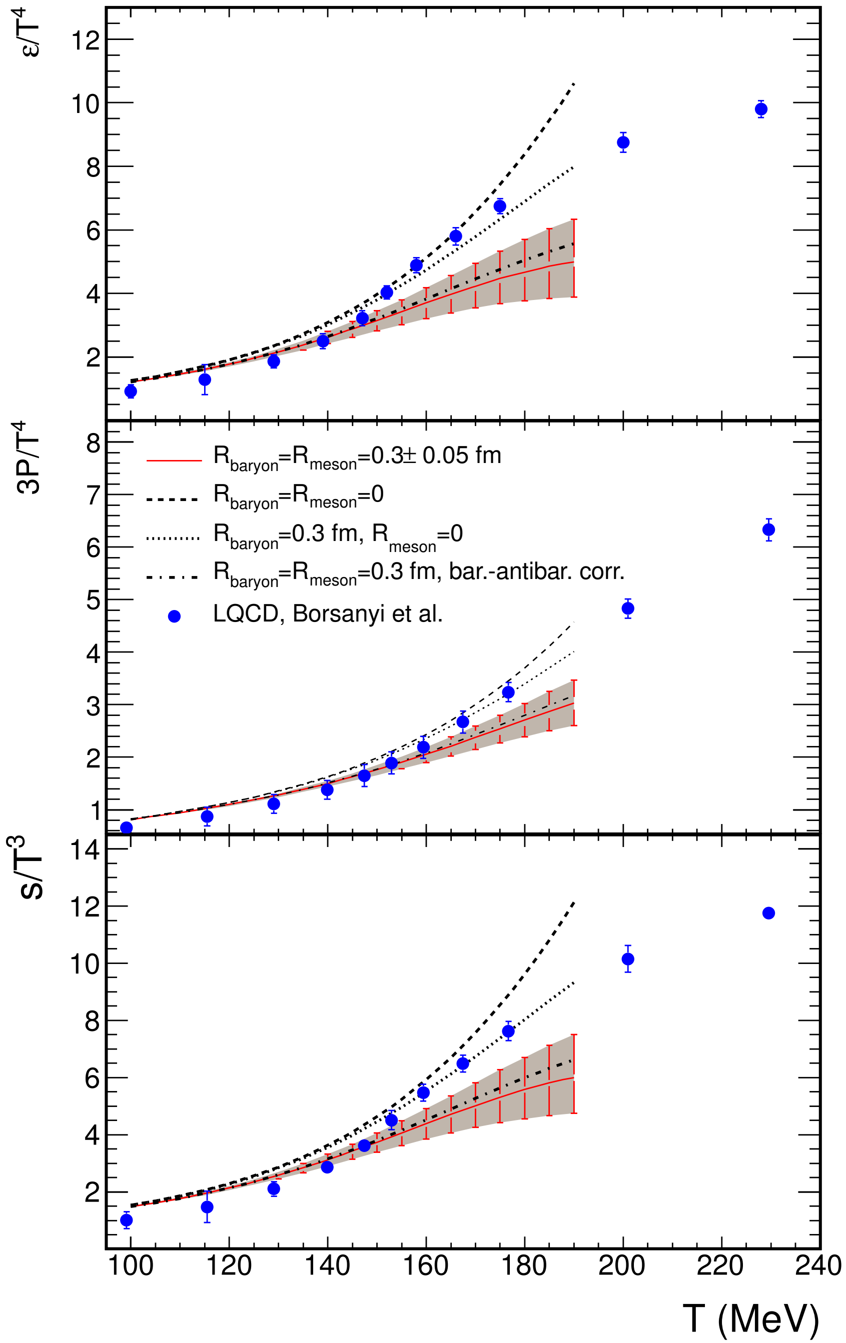
an "excluded-volume" (hard sphere repulsion) is often employed

AA et al., PLB 718 (2012) 80

Several versions exist
for instance via van der Waals int.

Vovchenko, Gorenstein, Stöcker, PRL 118 (2017) 182301

up to T=200 MeV
(does it make sense to extend HRG to 200 MeV?)



Summary on the ligh-quark sector

- abundance of hadrons with light quarks consistent with chemical equilibration
- there is a variety of approaches ... *a personal bias: the "minimal model"*

  a minimal set of parameters, means a well-constrained model
- the thermal model provides a simple way to access the QCD phase boundary

  ...at high energies (at low energies canonical suppression needs more care)

  ...but is it more than a 1st order description (of loosely-bound objects)?

  ...and what fundamental point does it make about hadronization?

  (statistical features dominate, but understanding still missing as a dynamical process)
- more insights from higher moments and from heavier (charm) quarks

"throw in" (pQCD production): $N_{c\bar{c}} = 9.6 \rightarrow g_c = 30.1$ $(I_1/I_0 = 0.974)$

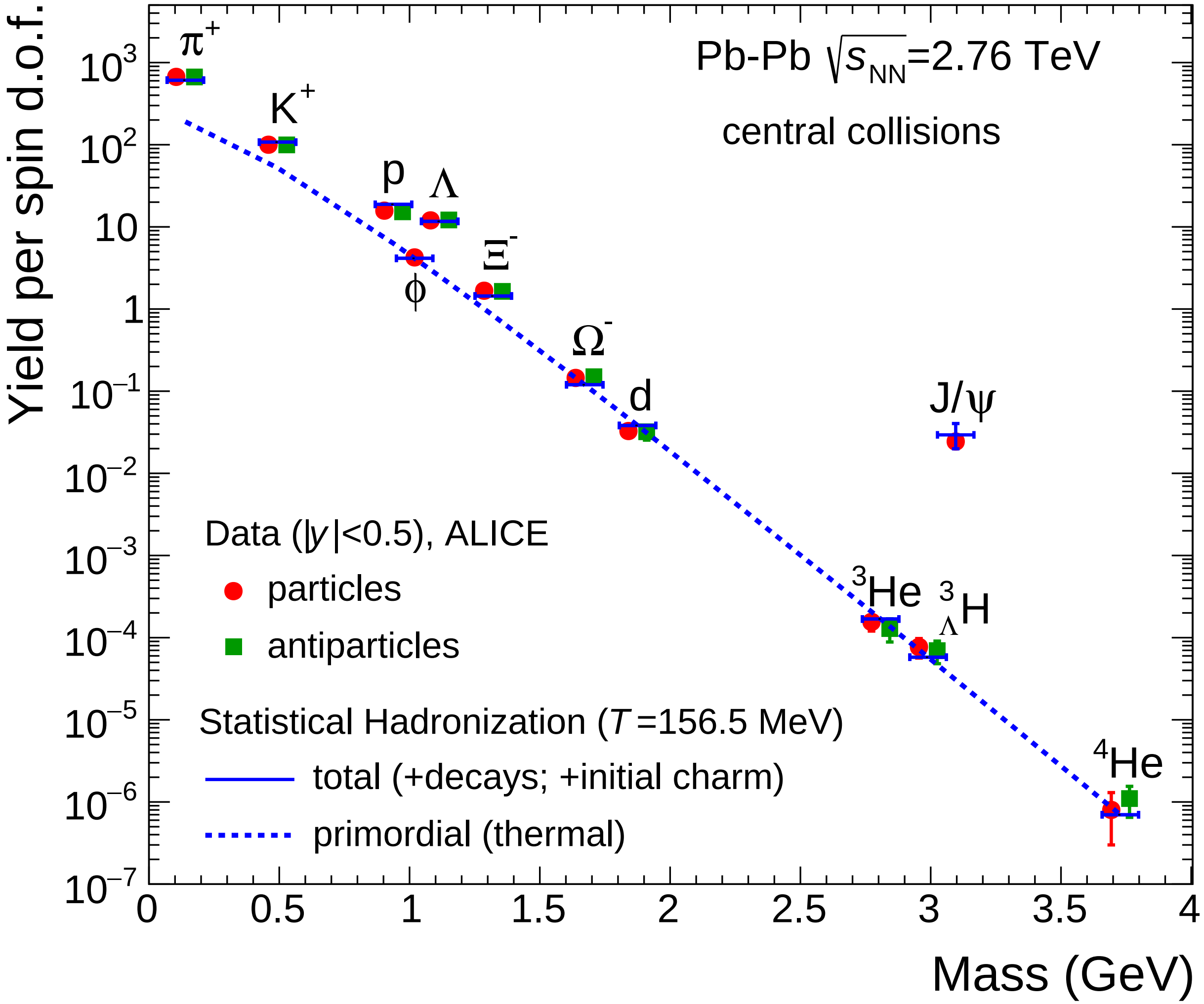
central collisions

assume:

- full thermalization of $c, \bar{c}$ ("mobility" in V≃4000 fm^3)

- full color screening (Matsui-Satz)

Model predicts all charm chemistry ($\psi(2S)$, $X(3872)$)



π, $K^{\pm}$, K^0 from charm included in the thermal fit (0.7%, 2.9%, 3.1% for T=156.5 MeV)

PLB 797 (2019) 134836

SHM for charm: method and inputs

- Thermal model calculation (grand canonical) T,μ_B: $\rightarrow n_X^{th}$

- $N_{c\bar{c}}^{dir} = \frac{1}{2} g_c V (\sum_i n_{D_i}^{th} + n_{\Lambda_i}^{th}) + g_c^2 V (\sum_i n_{\psi_i}^{th} + n_{\chi_i}^{th})$

- $N_{c\bar{c}} << 1 \rightarrow$ Canonical (J.Cleymans, K.Redlich, E.Suhonen, Z. Phys. C51 (1991) 137):

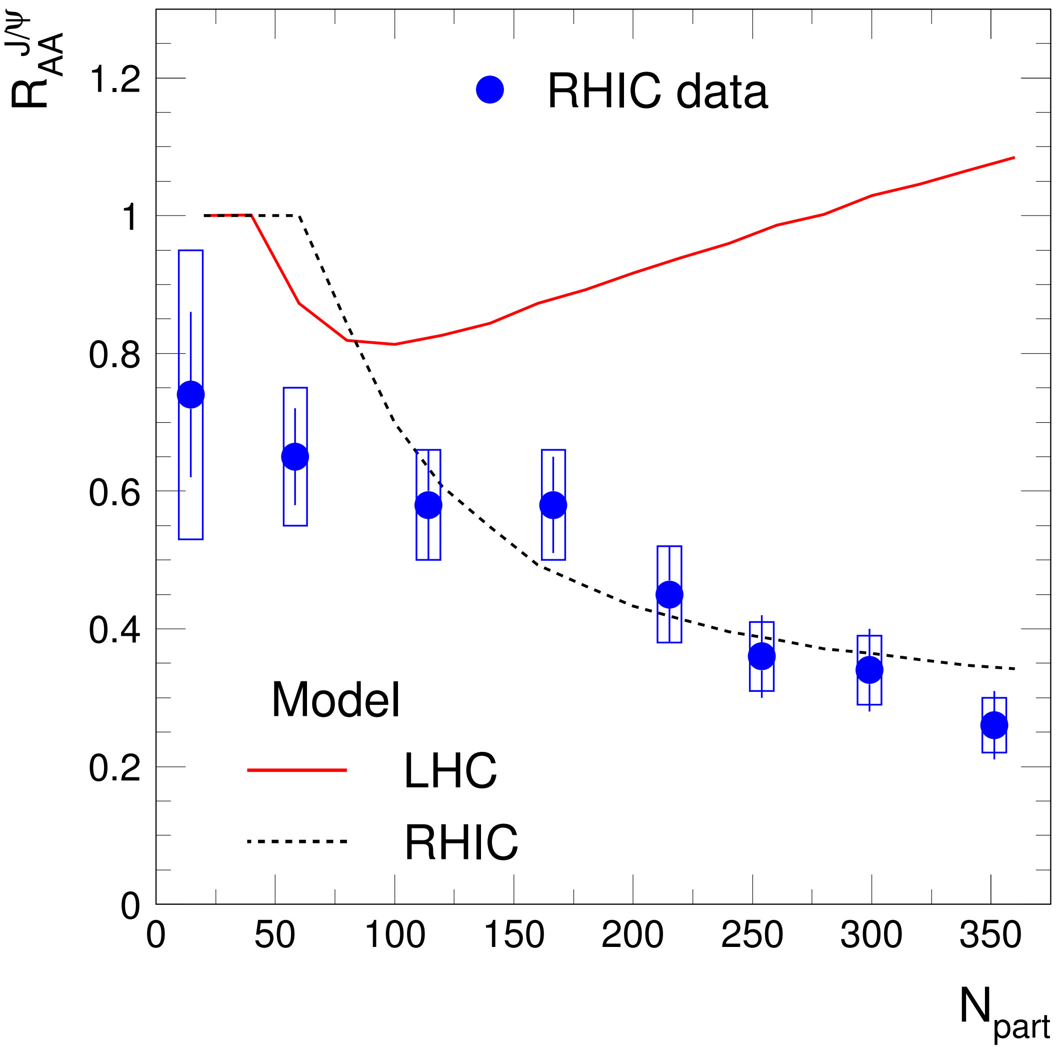
$$N_{c\bar{c}}^{dir} = \frac{1}{2} g_c N_{oc}^{th} \frac{I_1(g_c N_{oc}^{th})}{I_0(g_c N_{oc}^{th})} + g_c^2 N_{c\bar{c}}^{th} \quad \rightarrow g_c \text{ (charm fugacity)}$$

Outcome: $N_D = g_c V n_D^{th} I_1/I_0 \quad N_{J/\psi} = g_c^2 V n_{J/\psi}^{th}$

Inputs: T, μ_B, $V_{\Delta y=1} (= (\mathrm{d}N_{ch}^{exp}/\mathrm{d}y)/n_{ch}^{th})$, $N_{c\bar{c}}^{dir}$ (exp. or pQCD)

Assumed minimal volume for QGP: V_{QGP}^{min}=200 fm^3

High hopes for charmonium at the LHC



$$R_{AA}^{J/\psi} = \frac{dN_{J/\psi}^{AuAu}/dy}{N_{coll} \cdot dN_{J/\psi}^{pp}/dy}$$

- "suppression" at RHIC
- "enhancement" at LHC

$N_{J/\psi} \sim (N_{c\bar{c}}^{dir})^2$

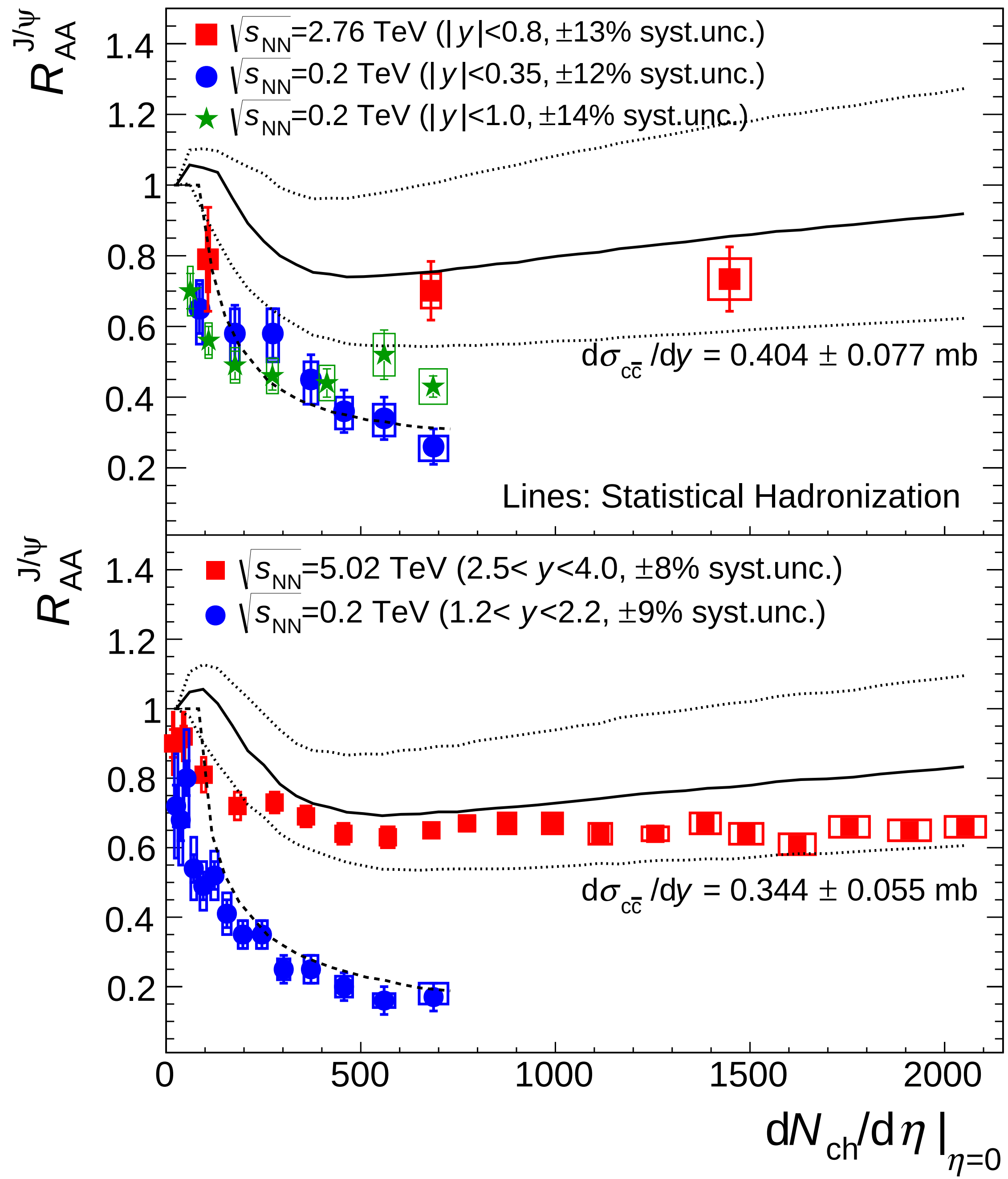
What is so different at LHC?

(compared to RHIC)

$\sigma_{c\bar{c}}$: ∼10x, Volume: 2-3x

PLB 652 (2007) 259

this is a generic prediction of SHM ...was confirmed by data



- "suppression" at RHIC (PHENIX)
- dramatically different at the LHC

J/ψ is another observable (charm) for the phase boundary

calculations are for T=156 MeV

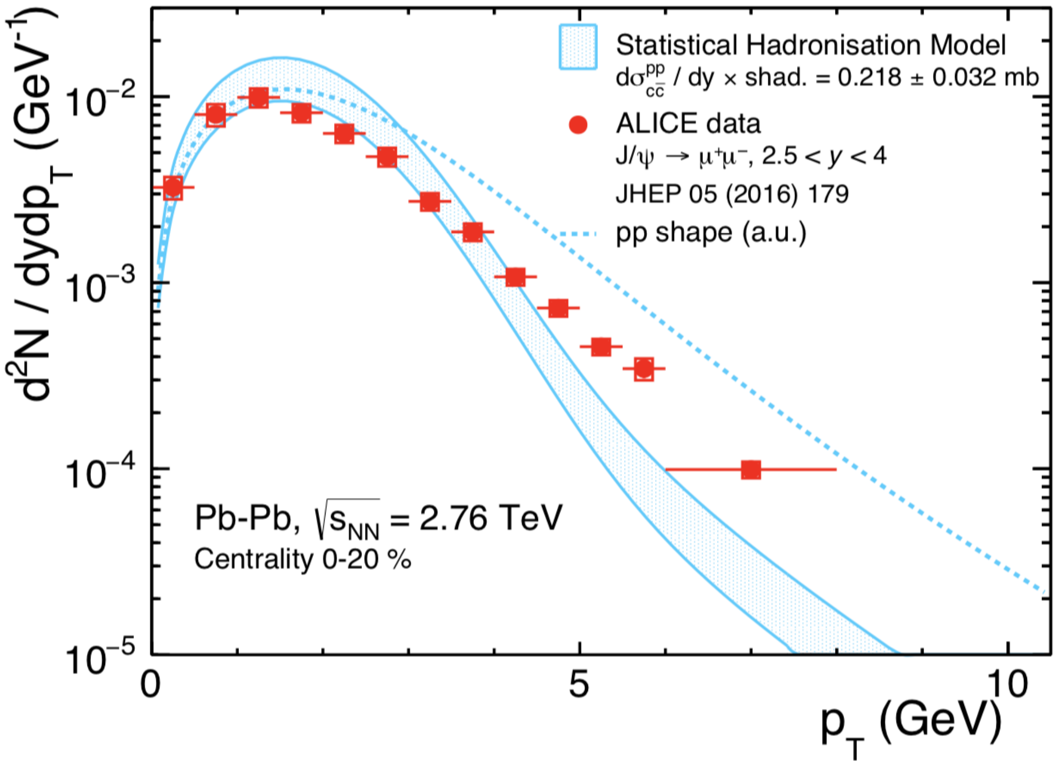
$\sigma_{c\bar{c}}$: current knowledge at LHC (data in pp, p–Pb; ALICE, LHCb)

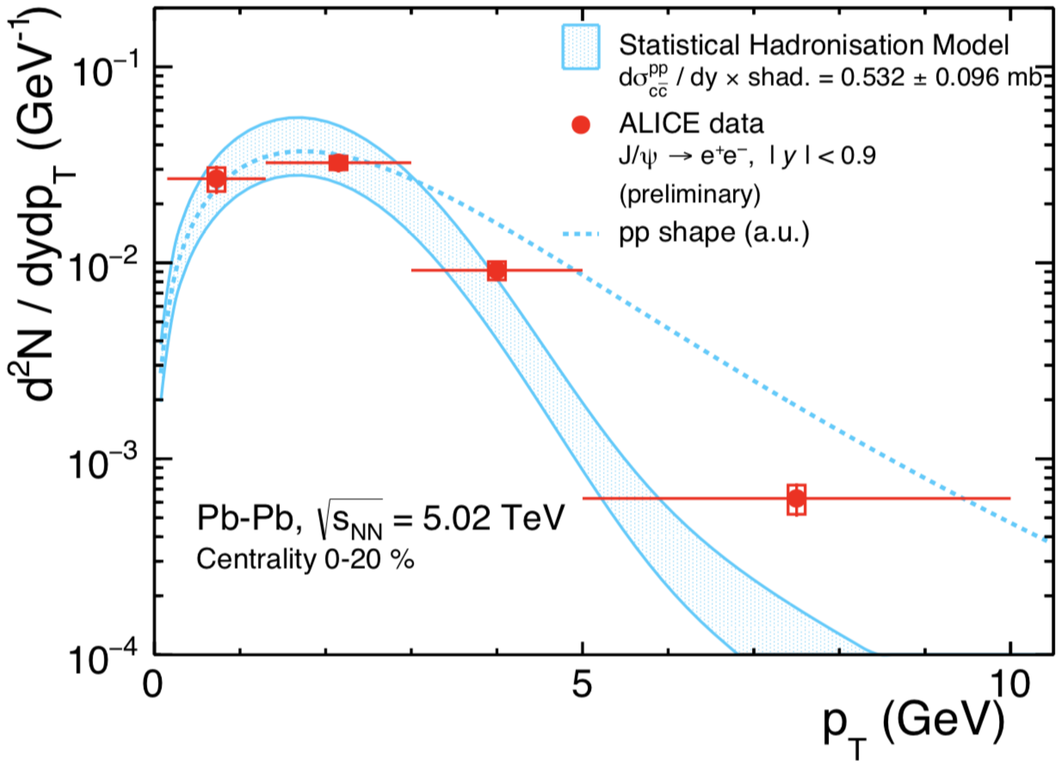
$\mathrm{d}N_{ch}/\mathrm{d}\eta \sim \varepsilon$

(>20 GeV/fm^3, for $\mathrm{d}N_{ch}/\mathrm{d}\eta \simeq 2000$)

Transverse momentum dependence in SHM: J/ψ

full hydrodynamic flow (MUSIC(3+1)D, IP-Glasma; parametrized via blast-wave)

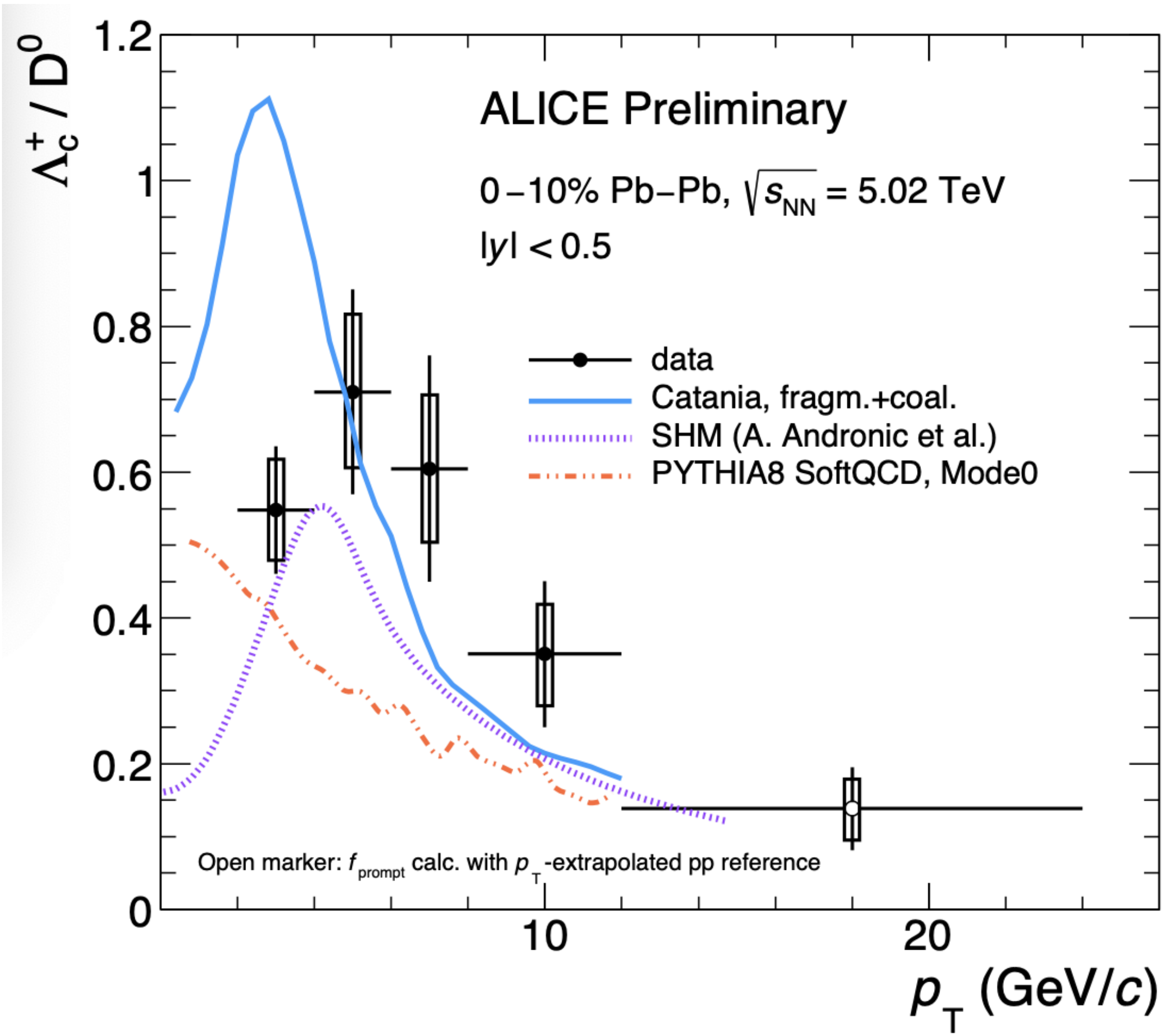




PLB 797 (2019) 134836 (M.Köhler)

Very good agreement with the data for low p_T (the relevant domain for SHM)

Ratio Λ_c/D^0



Good agreement with the data for low p_T

There is significant charm-baryon enhancement in pp collisions at the LHC

In the statistical hadronization model:

- the hadronization is a rapid process in which all quark flavors take part concurrently
- all charmonium and open charm states are generated exclusively at hadronization (chemical freeze-out) ...full color screening

The model is very successful in reproducting the J/ψ and open charm data

A handle for hadronization T with a mass scale well above T

"The competition":

the kinetic model, continuous J/ψ destruction and (re)generation in QGP

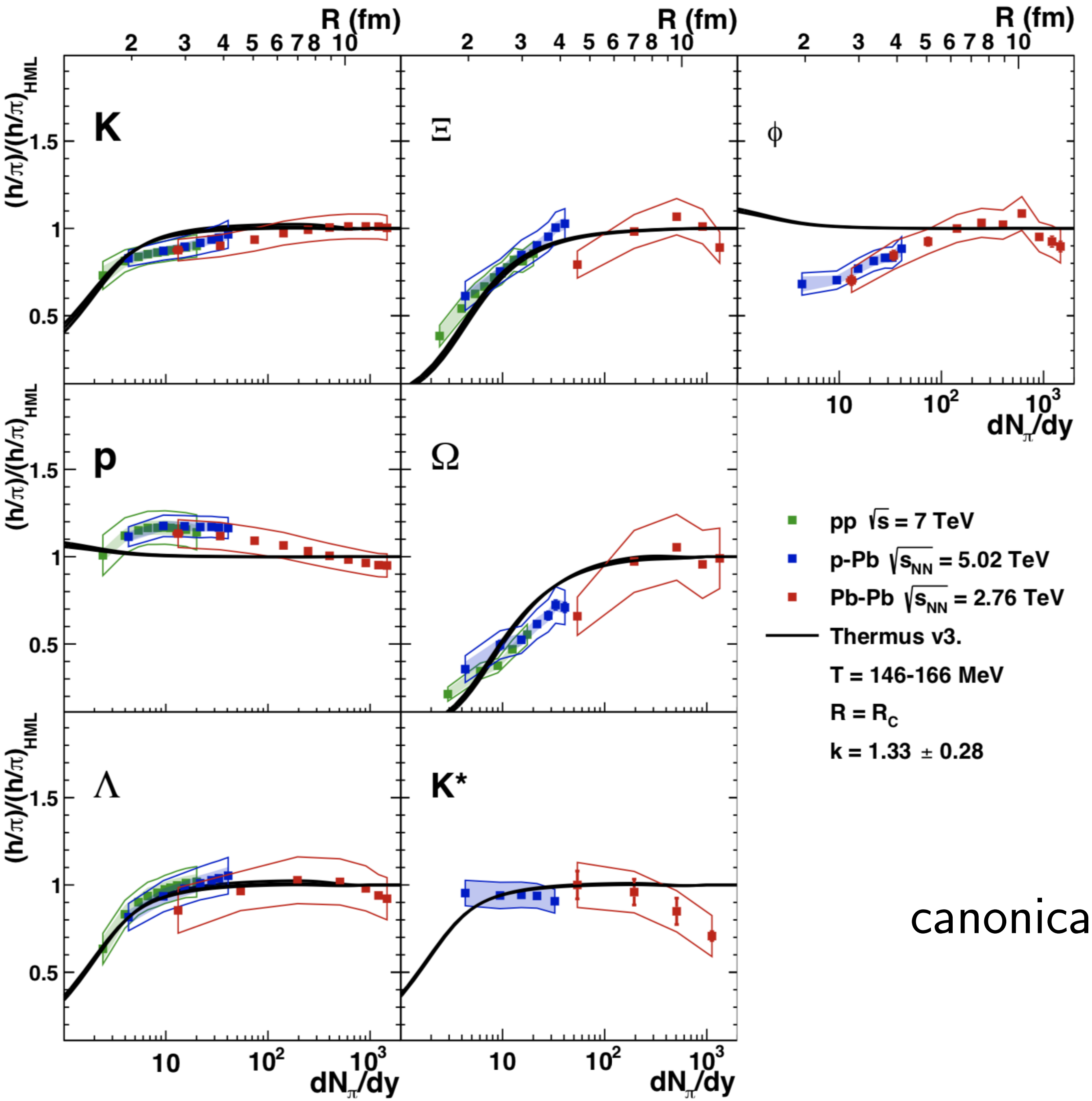
(only up to 2/3 of the J/ψ yield (LHC, central collisions) originates from deconfined c and $\bar{c}$ quarks)

Discriminating the two pictures implies providing an answer to fundamental questions related to the fate of hadrons in a hot deconfined medium.

A precision ($\pm$10%) measurement of $\mathrm{d}\sigma_{c\bar{c}}/\mathrm{d}y$ in Pb-Pb (Au-Au) collisions needed for a stringent test

(within reach with the upgraded detectors at the LHC and RHIC)

Extra slides



Vislavicius, Kalweit, arXiv:1610.03001

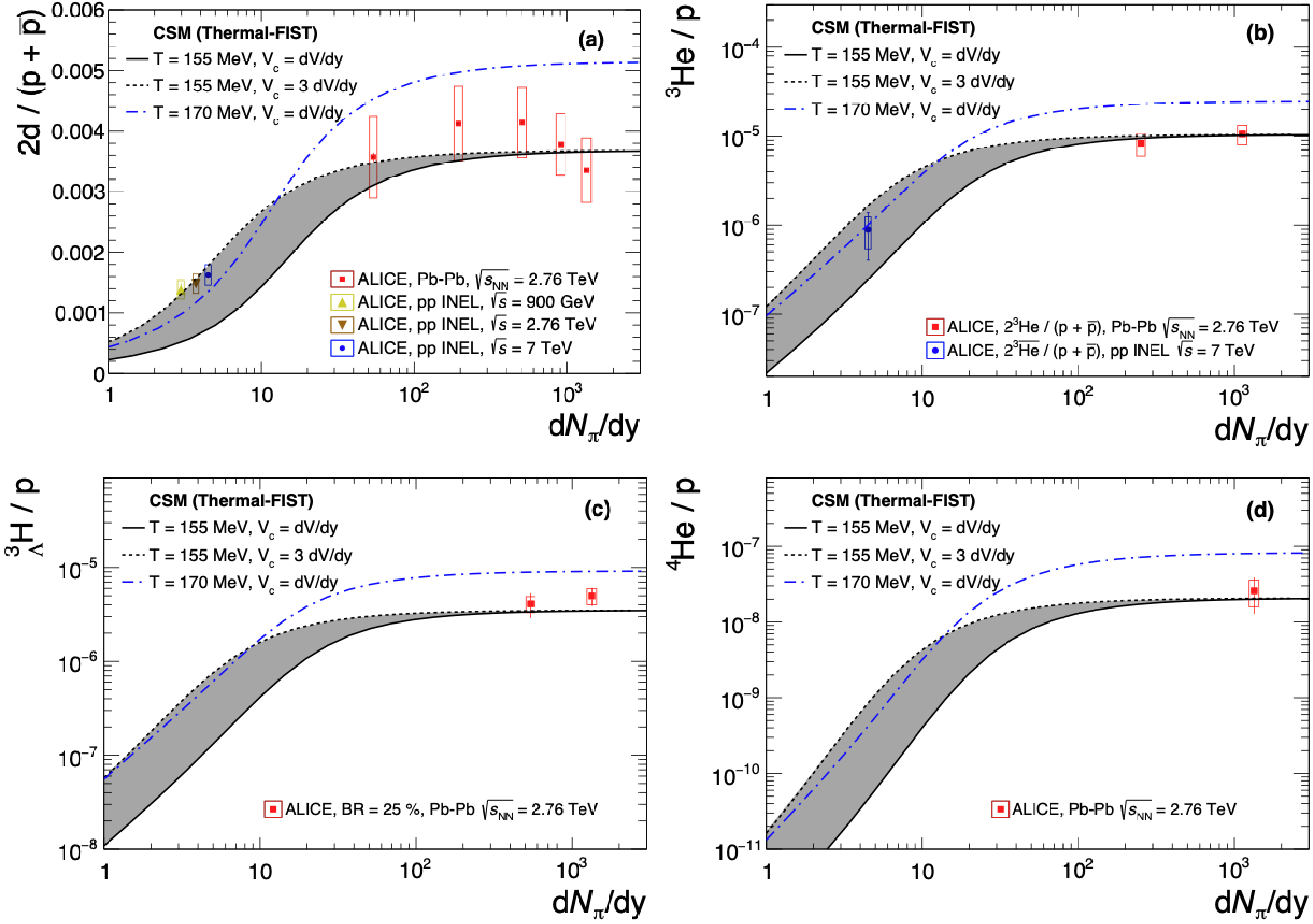
ratios to high multiplicity limit (HML)

s correlation volume: Δy=1.33±0.28

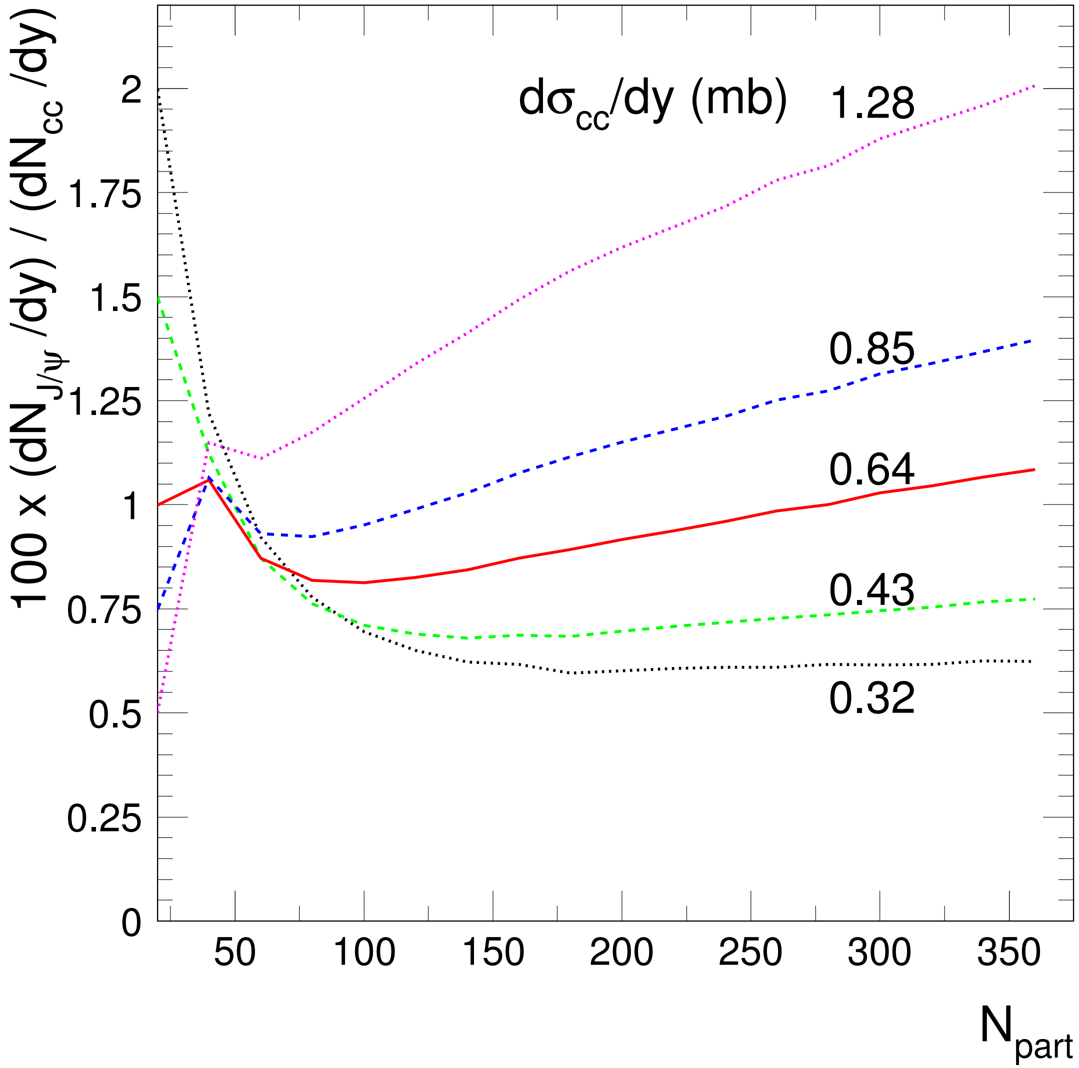
canonical statistical hadronization describes data well

Canonical treatment for nuclei

A. Andronic



Vovchenko, Dönigus, Stöcker, PLB 785 (2018) 171



$$\frac{\mathrm{d}N^{AA}_{J/\psi}/\mathrm{d}y}{\mathrm{d}N^{AA}_{c\bar{c}}/\mathrm{d}y}$$

("proxy" for R_{AA})

- "enhancement" at the LHC

  $N_{J/\psi} \sim (N^{dir}_{c\bar{c}})^2$

  canonical suppression (mostly) lifted, quadratic term dominant

  it can be more dramatic at FCC

AA et al., in N. Armesto et al., "Last Call...", JPG 35 (2008) 054001

this was for $\sqrt{s_{NN}} = 5.5$ TeV ... but is a generic prediction of the model

Transverse momentum dependence in SHM

$$\frac{\mathrm{d}N}{p_T \mathrm{d}p_T} \sim \int_0^R r\mathrm{d}r \left[m_T \cosh\rho K_1\left(\frac{m_T\cosh\rho}{T}\right) I_0\left(\frac{p_T\sinh\rho}{T}\right) - p_T\sinh\rho K_0\left(\frac{m_T\cosh\rho}{T}\right) I_1\left(\frac{p_T\sinh\rho}{T}\right)\right]$$

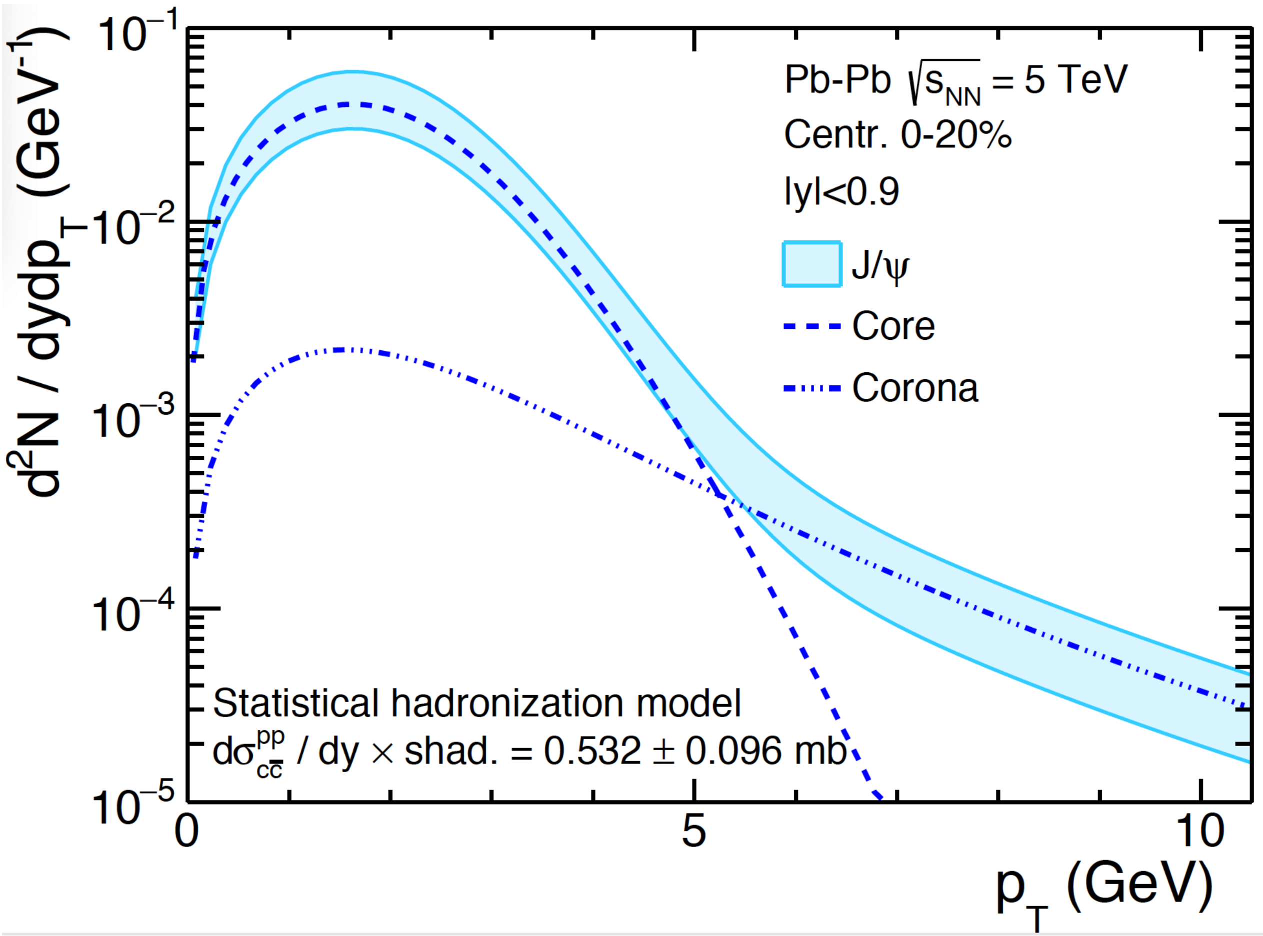
$\rho = \tanh^{-1}\beta$,

$\beta = \beta_T^s (r/R)^n$

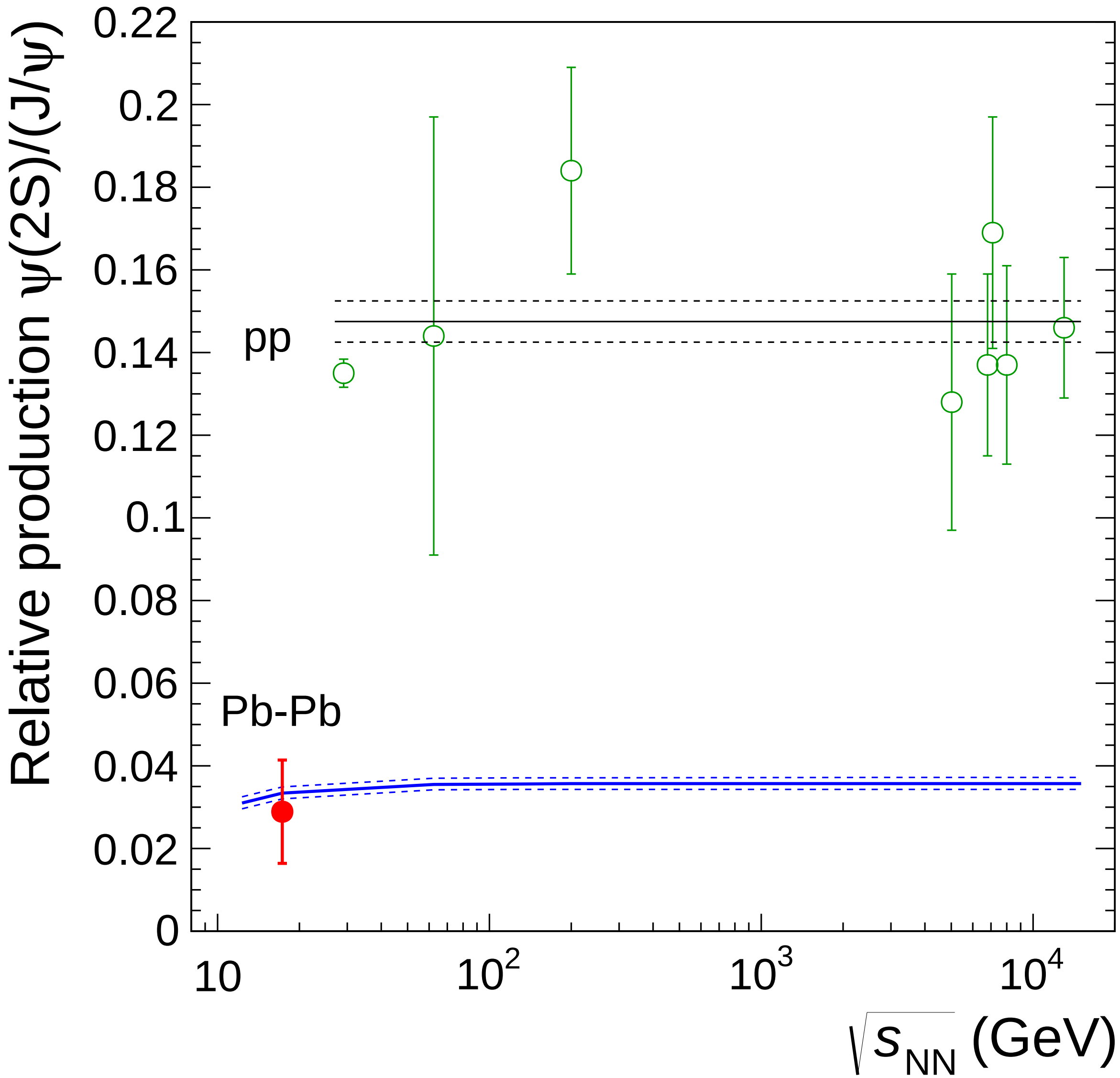
β_T^s at freeze-out (T_{CF})

hypersurface from hydro

MUSIC(3+1)D (IP-Glasma)



PLB 797 (2019) 134836 (M.Köhler)



for D, stat. hadronization is a simpler act, may be at work in pp and in e^+e^-

PLB 678 (2009) 350

..litmus test: $\psi(2S)$

The measurement in Pb–Pb at LHC is a central goal for Run 3,4

see Yellow report WG5 HL-LHC