ESTABLISHING THE NON-CRITICAL BASELINE FOR FLUCTUATION MEASUREMENTS

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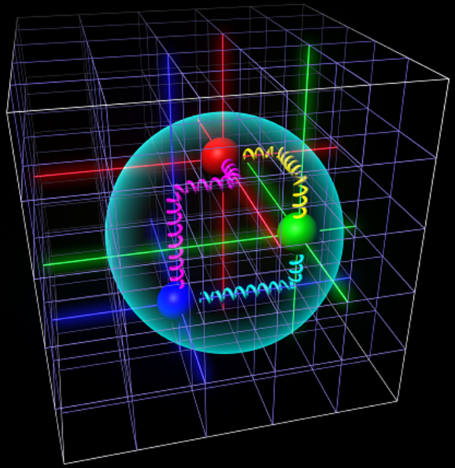
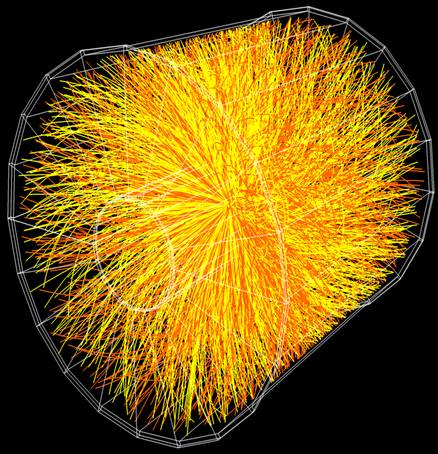


$$\hat{\chi}_2^B = \frac{\langle \Delta N_B^2 \rangle - \langle \Delta N_B \rangle^2}{VT^3}$$

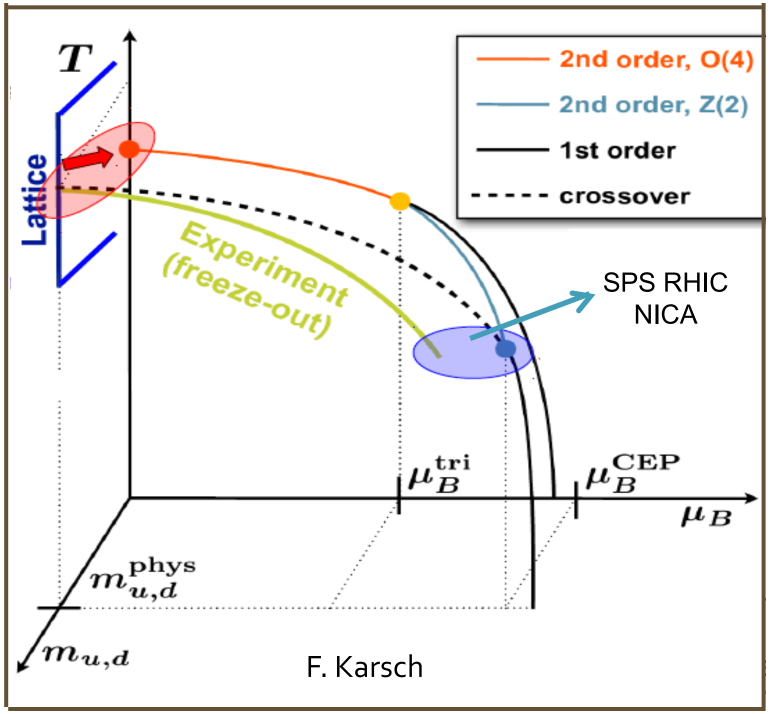


based largely on:
P. Braun-Munzinger, B. Friman, K. Redlich, A. Rustamov, J. Stachel , arXiv:2007.02463

Outline

- Why fluctuations
- The non-critical baseline
  - canonical formulation of higher order cumulants
  - introducing finite acceptances for baryons and anti-baryons
  - comparison to experimental data on net-protons
  - volume fluctuations
- Global vs. local conservation laws, multi-particle correlations
- Summary

Why Fluctuations?



A. Bazavov et al., Phys.Rev. D85 (2012) 054503

To probe the structure of strongly interacting matter

- Locate phase boundaries
- Search for critical phenomena
- …

E-by-E fluctuations are predicted within Grand Canonical Ensemble

$$\frac{\langle N^2\rangle - \langle N\rangle^2}{\langle N\rangle^2} = \frac{T\chi_T}{V} \qquad \chi_T = -\frac{1}{V}\left(\frac{\partial V}{\partial P}\right)_T$$

direct link to the EoS

$$\langle N^2\rangle - \langle N\rangle^2 = \kappa_2(N) = T^2\frac{\partial^2 lnZ}{\partial\mu^2}$$

probing the response of the system to external perturbations

Understanding the QCD phase transition

Freeze-out at the phase boundary

$$T_{fo}^{ALICE} = 156.5 \pm 1.5 \text{MeV} \pm 3 \text{ MeV(sys)}$$

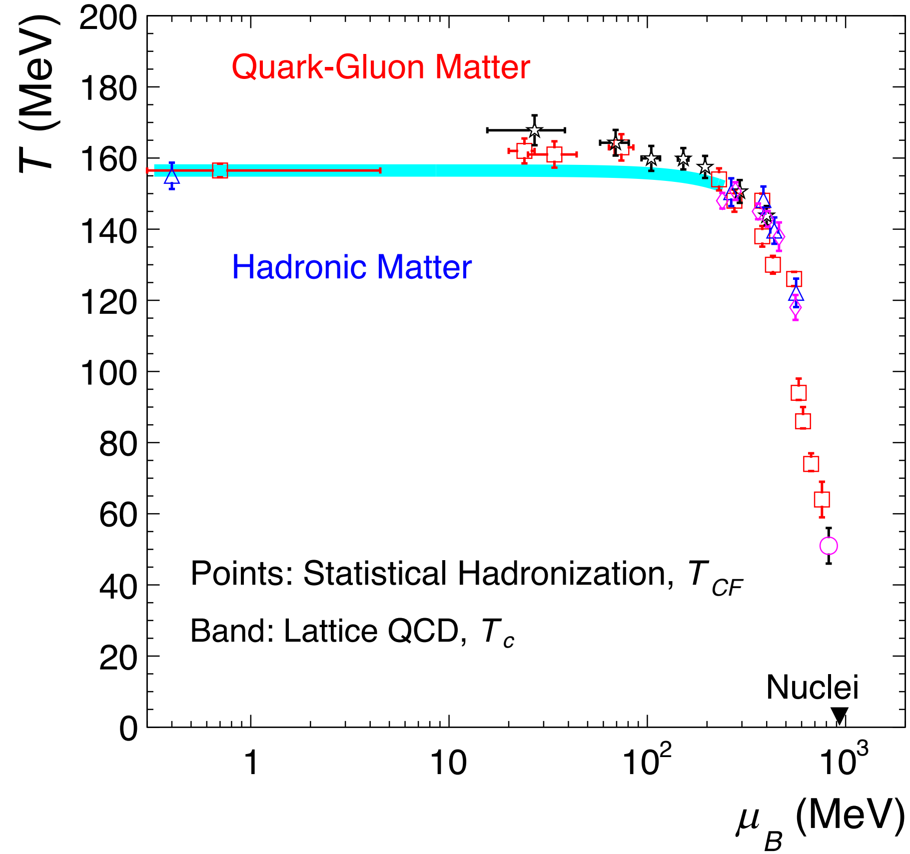
$$T_c^{LQCD} = 156.5 \pm 1.5 \ MeV$$

Experimental plan:

- measuring fluctuations of net-baryons along the QCD phase boundary

Open questions:

- the order of the phase transition
- existence of the critical endpoint
- ...



A. Andronic, P. Braun-Munzinger, K. Redlich and J. Stachel, Nature 561, 321–330 (2018)
A. Bazavov et al., Phys.Rev. D85 (2012) 054503

Theory vs. experiment

for a thermal system in a fixed volume V within the Grand Canonical Ensemble (CGE)

$$\hat{\chi}_2^B = \frac{\langle \Delta N_B^2 \rangle - \langle \Delta N_B \rangle^2}{VT^3} \equiv \frac{\kappa_2(\Delta N_B)}{VT^3}$$

$$\hat{\chi}_n^B = \frac{1}{VT^3} \frac{\partial^n lnZ(V, T, \mu_{B,Q,S})}{\partial(\mu_B/T)^n}$$

Assumptions in theory:

- *Volume is fixed in each event*
- ***Conservations are imposed on the averages***

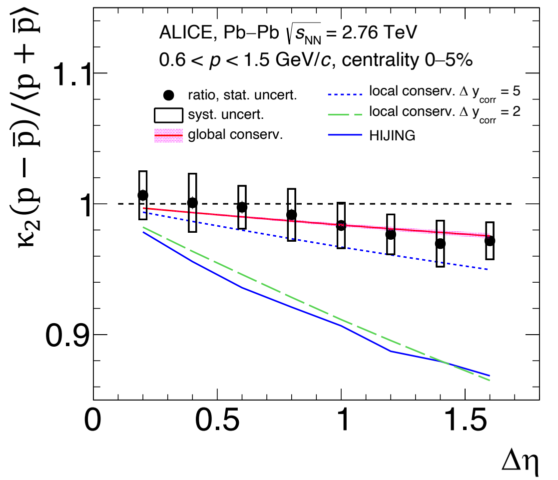
Reality in experiments:

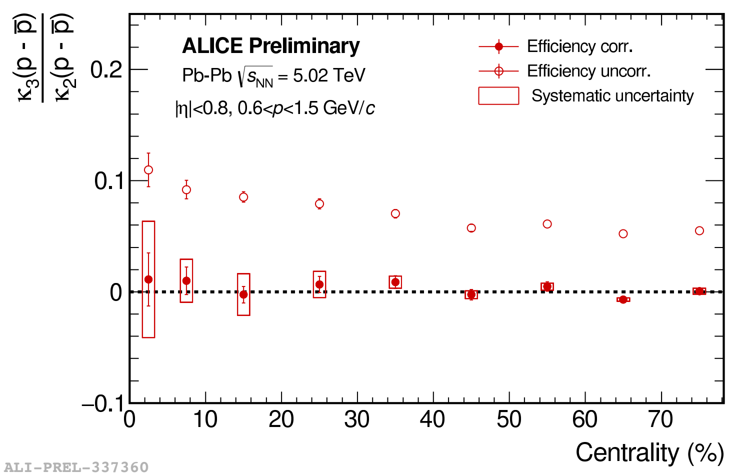
- *Volume fluctuates from E-to-E*
- ***Conservations depend on acceptance***

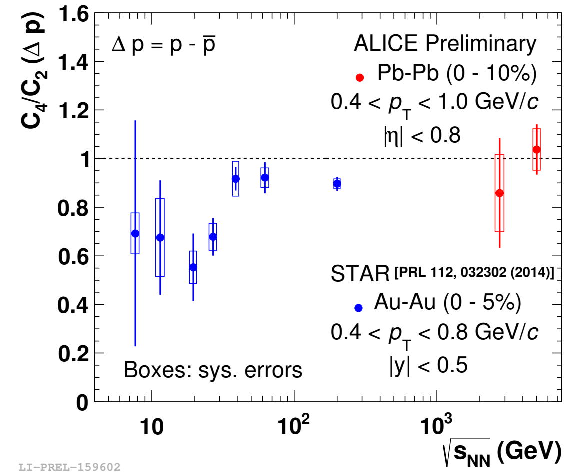
$$\frac{\kappa_4^{exp}(\Delta N_B)}{\kappa_2^{exp}(\Delta N_B)} \neq \frac{\hat{\chi}_4^B}{\hat{\chi}_2^B} \qquad \frac{\kappa_3^{exp}(\Delta N_B)}{\kappa_2^{exp}(\Delta N_B)} \neq \frac{\hat{\chi}_3^B}{\hat{\chi}_2^B}$$

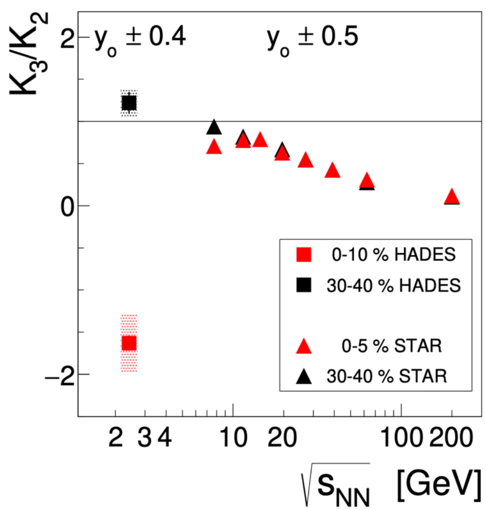
P. Braun-Munzinger, A. Rustamov, J. Stachel, NPA 960 (2017) 114
V. Skokov, B. Friman, and K. Redlich, Phys.Rev. C88 (2013) 034911

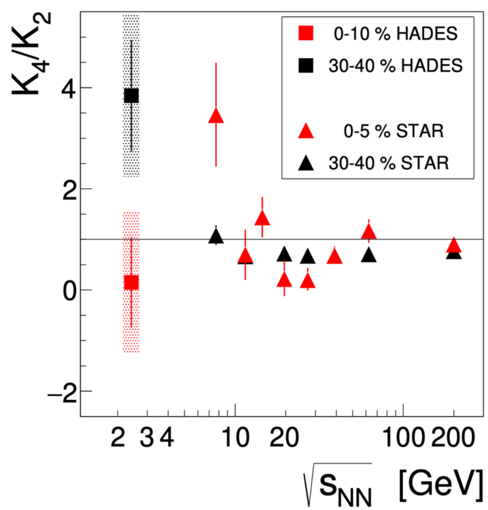
Experimental results on net-protons

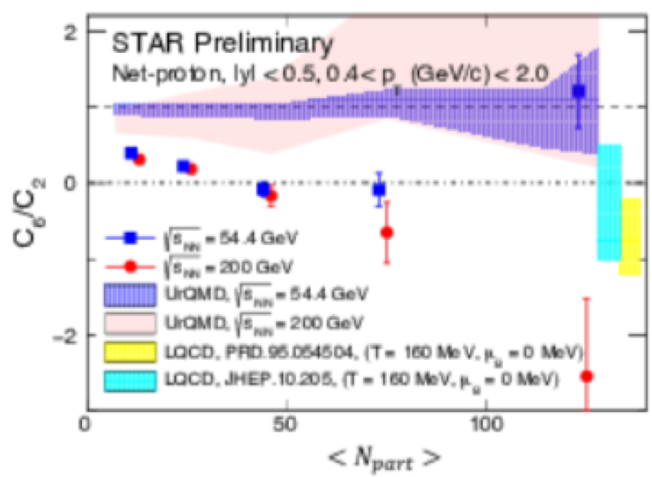








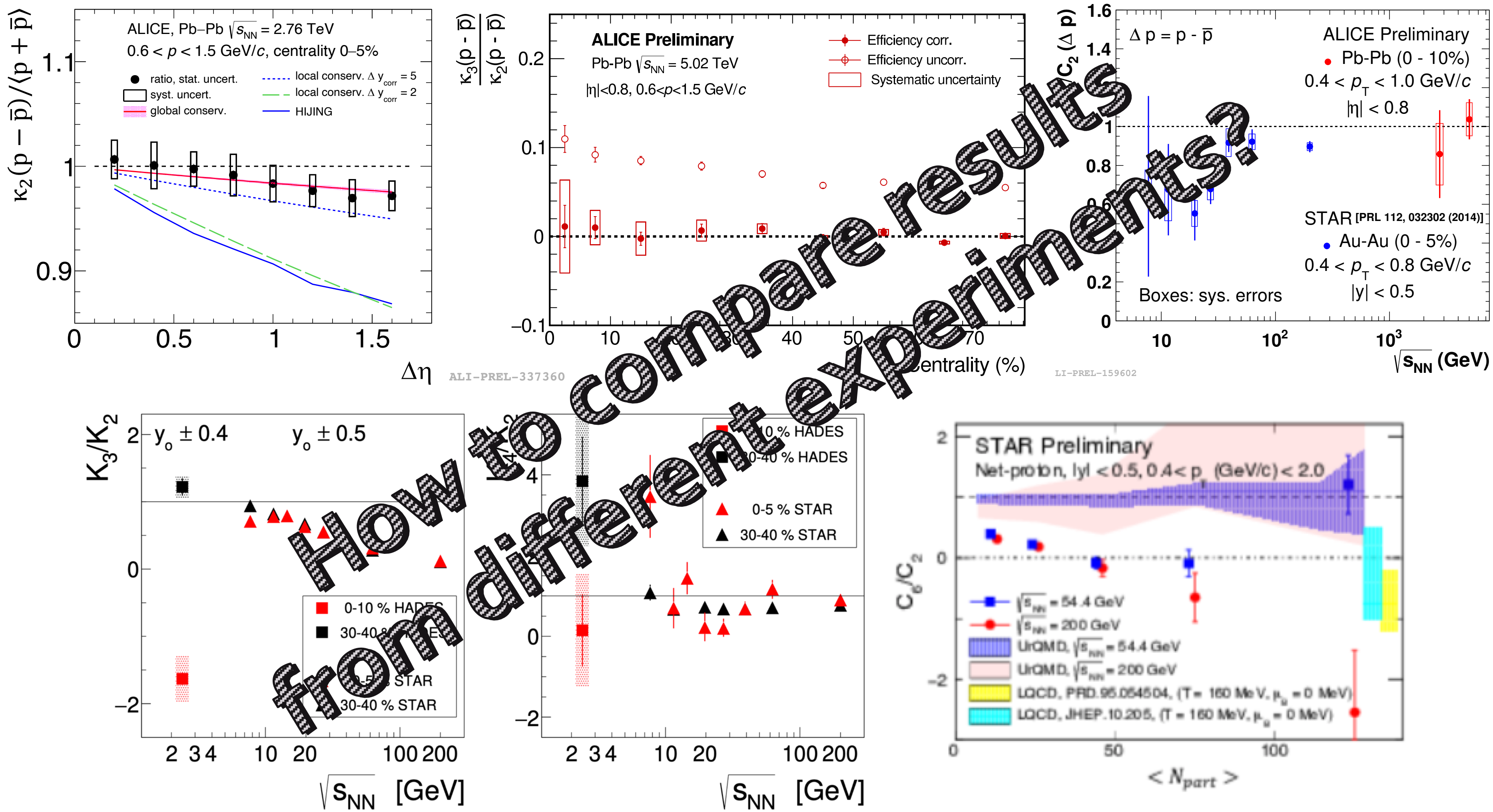




Note different notations: $\kappa_n \equiv C_n \equiv K_n$

ALICE: Phys. Lett. B 807 (2020) 135564, M. Arslandok, QM19, N. Behera, QM18
HADES: arXiv:2002.08701, PRC in print
STAR: arXiv:2001.02852, A. Pandav, QM19

Experimental results on net-protons



How to compare results from different experiments?

Note different notations: $\kappa_n \equiv C_n \equiv K_n$

ALICE: Phys. Lett. B 807 (2020) 135564, M. Arslandok, QM19, N. Behera, QM18
HADES: arXiv:2002.08701, PRC in print
STAR: arXiv:2001.02852, A. Pandav, QM19

establishing the non-critical baseline

- canonical formulation of higher order cumulants
- introducing finite acceptance for baryons and anti-baryons
- comparison to experimental data

P. Braun-Munzinger, B. Friman, K. Redlich, A. Rustamov, J. Stachel , arXiv:2007.02463

The formalism

Canonical partition function in a finite volume V at temperature T

$$Z_B(V,T) = \sum_{N_B=0}^{\infty}\sum_{N_{\bar{B}}=0}^{\infty} \frac{(\lambda_B z_B)^{N_B}}{N_B!}\frac{(\lambda_{\bar{B}} z_{\bar{B}})^{N_{\bar{B}}}}{N_{\bar{B}}!}\delta(N_B - N_{\bar{B}} - B) = \left(\frac{\lambda_B z_B}{\lambda_{\bar{B}} z_{\bar{B}}}\right)^{\frac{B}{2}} I_B\left(2z\sqrt{\lambda_B\lambda_{\bar{B}}}\right)$$

- B net-baryon number, conserved in each event
- I_B modified Bessel function of the first kind
- $z_B, z_{\bar{B}}$ single particle partition functions for baryons, antibaryons
- $\lambda_B, \lambda_{\bar{B}}$ auxiliary parameters for calculating mean number of baryons, antibaryons

$$z = \sqrt{z_B z_{\bar{B}}} = \sqrt{\langle N_B\rangle_{GCE}\langle N_{\bar{B}}\rangle_{GCE}}$$

$\langle N_B\rangle_{GCE}, \langle N_{\bar{B}}\rangle_{GCE}$ are in GCE, **experiments measure canonical multiplicities $\langle N_B\rangle$, $\langle N_{\bar{B}}\rangle$**

$$\langle N_B\rangle = \lambda_B \frac{\partial lnZ_B}{\partial\lambda_B}\bigg|_{\lambda_B=\lambda_{\bar{B}}=1} = z\frac{I_{B-1}(2z)}{I_B(2z)}$$

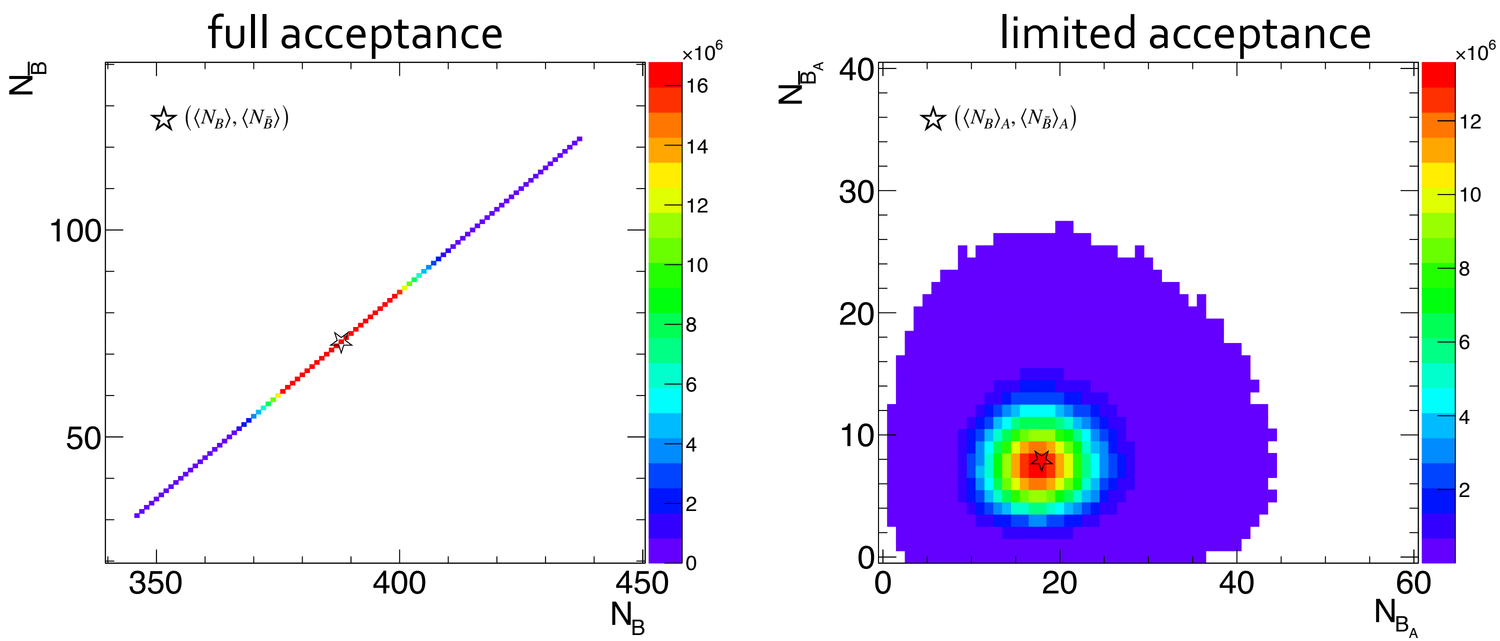
$$\langle N_{\bar{B}}\rangle = \lambda_{\bar{B}} \frac{\partial lnZ_B}{\partial\lambda_{\bar{B}}}\bigg|_{\lambda_B=\lambda_{\bar{B}}=1} = z\frac{I_{B+1}(2z)}{I_B(2z)}$$

we recalculate z by solving Eq. for $\langle N_B\rangle$ or $\langle N_{\bar{B}}\rangle$

Full vs. limited acceptance

the underlying PDF for anti-baryons

$$P_B(N_{\bar{B}}) = \frac{1}{I_B(2z)} \frac{z^B z^{2N_{\bar{B}}}}{(N_{\bar{B}} + B)!\, N_{\bar{B}}!}, \qquad N_B = N_{\bar{B}} + B$$



$$\alpha_B = \frac{\langle N_B\rangle_A}{\langle N_B\rangle}$$

$$\alpha_{\bar{B}} = \frac{\langle N_{\bar{B}}\rangle_A}{\langle N_{\bar{B}}\rangle}$$

fluctuations of net-baryons appear only inside limited acceptance

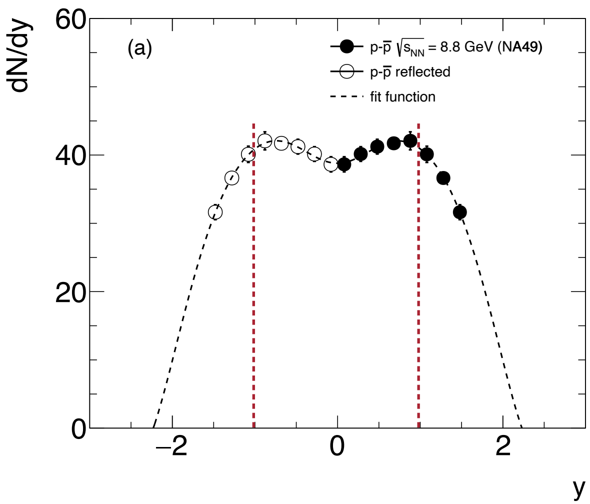
net-proton is a proxy of net-baryon; if and only if the isospin correlations are negligible

net-protons fluctuate event in full acceptance

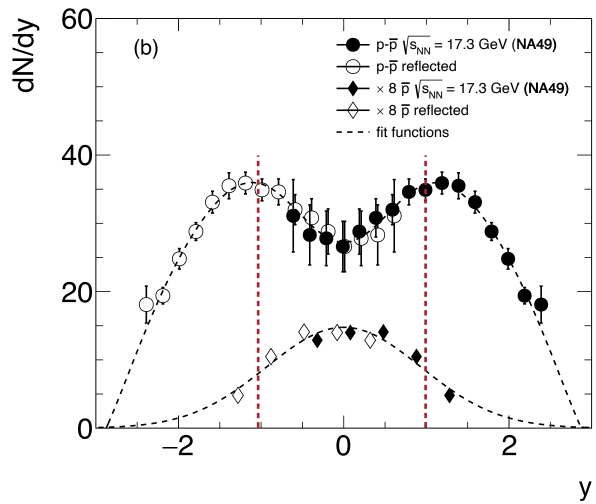
Introducing finite acceptances

in experiments the finite acceptance is introduced by applying cuts on y and/or $p_\perp$

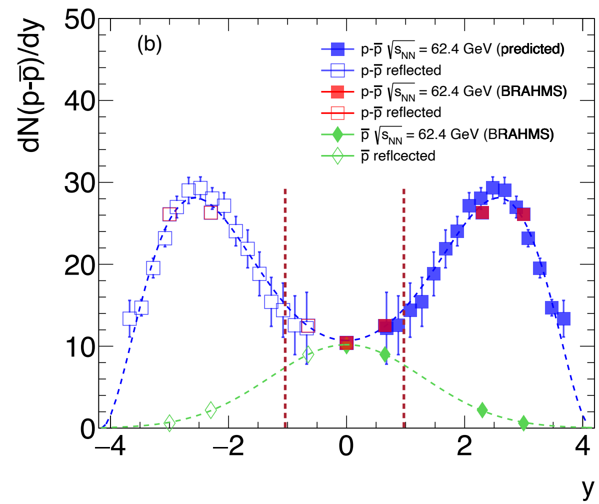
NA49 Pb-Pb, $\sqrt{s_{NN}}$=8.8 GeV



NA49 Pb-Pb, $\sqrt{s_{NN}}$=17.3 GeV



BRAHMS Au-Au, $\sqrt{s_{NN}}$=62.4 GeV



we take into account acceptances for both baryons and anti-baryons

Note

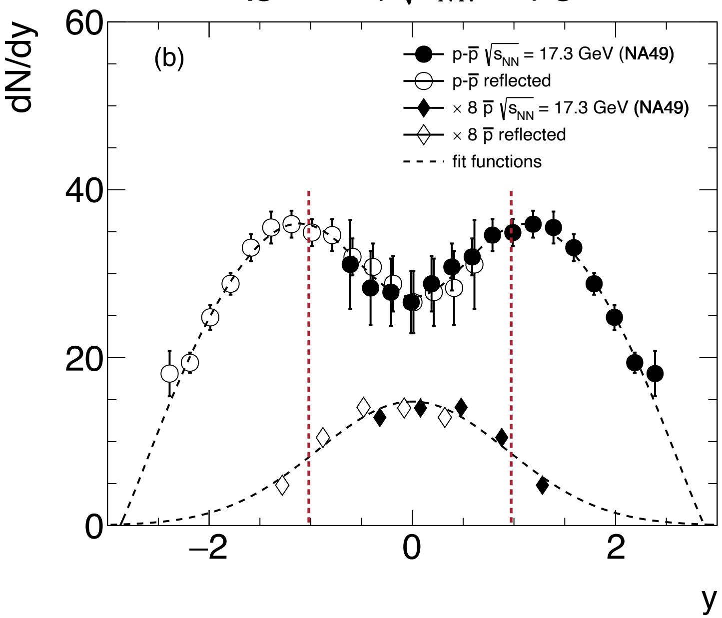
there are only 4 measurements from BRAHMS (■, ♦)
the full distribution for BRAHMS (blue dashed curve) is our prediction

NA49: PRL. 82 (1999) 2471-2475, PRC 83 (2011) 014901
BRAHMS: Phys.Lett.B 677 (2009) 267-271

Important caveat

- The **STAR** data contain $p_\perp$ cuts (in addition to y cuts) and contributions from feed-down
- The **NA49** y distributions are $p_\perp$ integrated and are corrected for feed-down effects

NA49 Pb-Pb, $\sqrt{s_{NN}}$ = 17.3 GeV



NA49: PRL. 82 (1999) 2471-2475, PRC 83 (2011) 014901

empirical solution

$$\gamma_p \int_{-0.5}^{0.5} \frac{dN_p}{dy} dy = \langle n_p \rangle \qquad \gamma_{\bar{p}} \int_{-0.5}^{0.5} \frac{dN_{\bar{p}}}{dy} dy = \langle n_{\bar{p}} \rangle$$

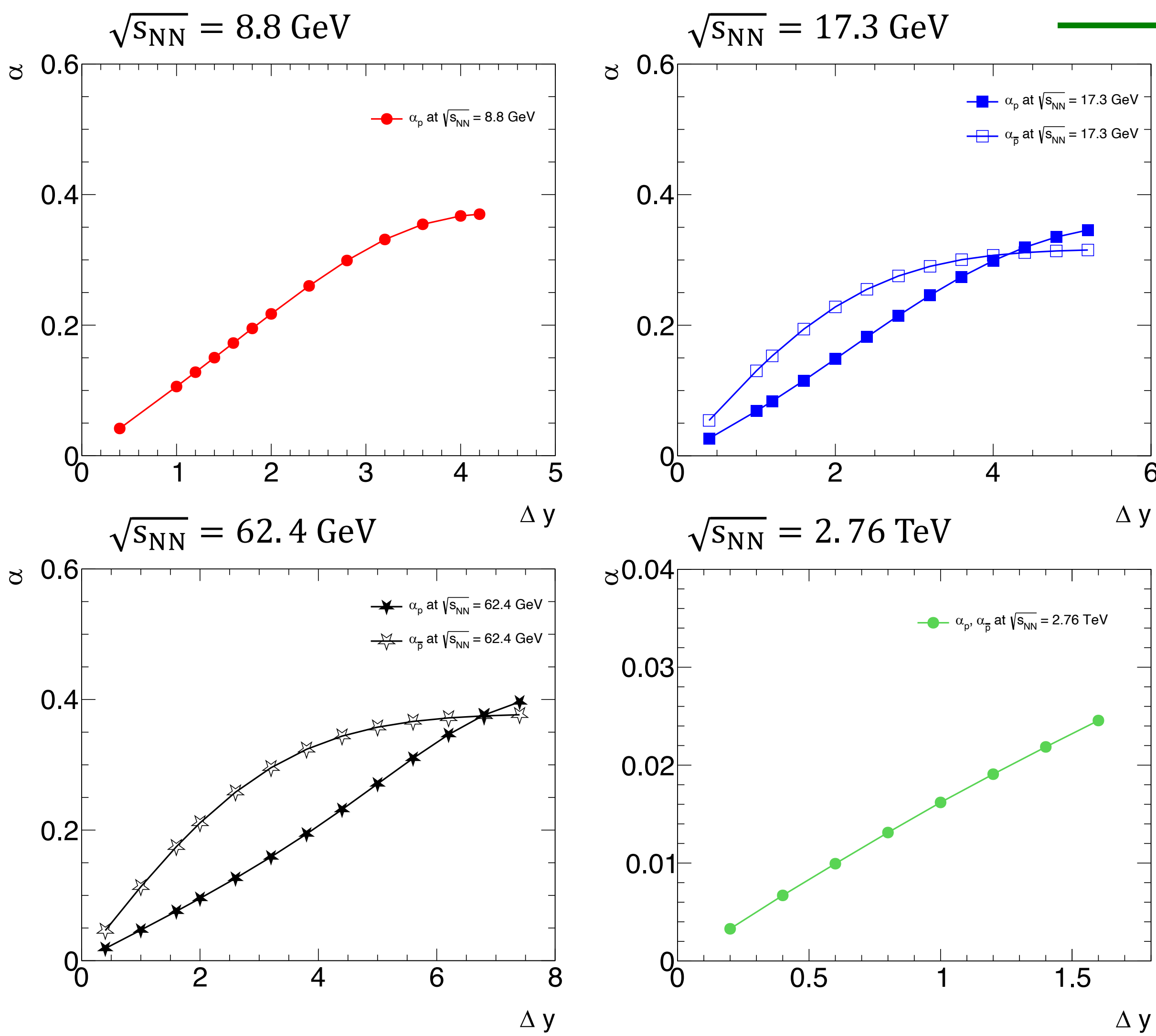
$\frac{dN_p}{dy}, \frac{dN_{\bar{p}}}{dy}$ rapidity distributions from **NA49, BRAHMS**

$\langle n_p \rangle, \langle n_{\bar{p}} \rangle$ mean numbers from STAR cumulant analysis

$$\alpha_p = \frac{\gamma_p \int_{y_{min}}^{y_{max}} \frac{dN_p}{dy} dy}{\langle N_B \rangle} \qquad \alpha_{\bar{p}} = \frac{\gamma_{\bar{p}} \int_{y_{min}}^{y_{max}} \frac{dN_{\bar{p}}}{dy} dy}{\langle N_{\bar{B}} \rangle}$$

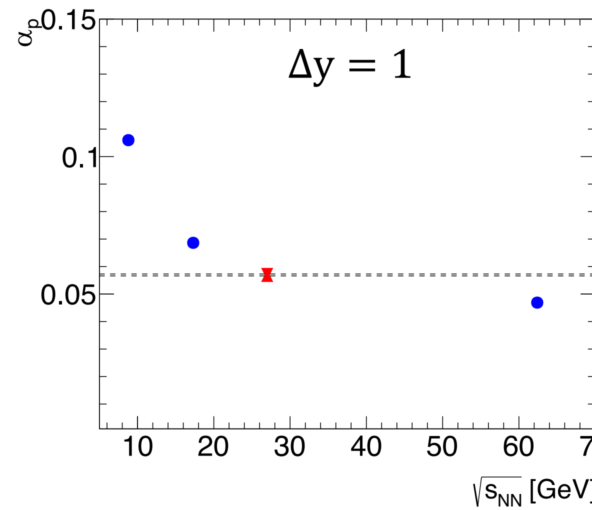
$$|y_{max} - y_{min}| = \Delta y$$

Accepted protons and anti-protons

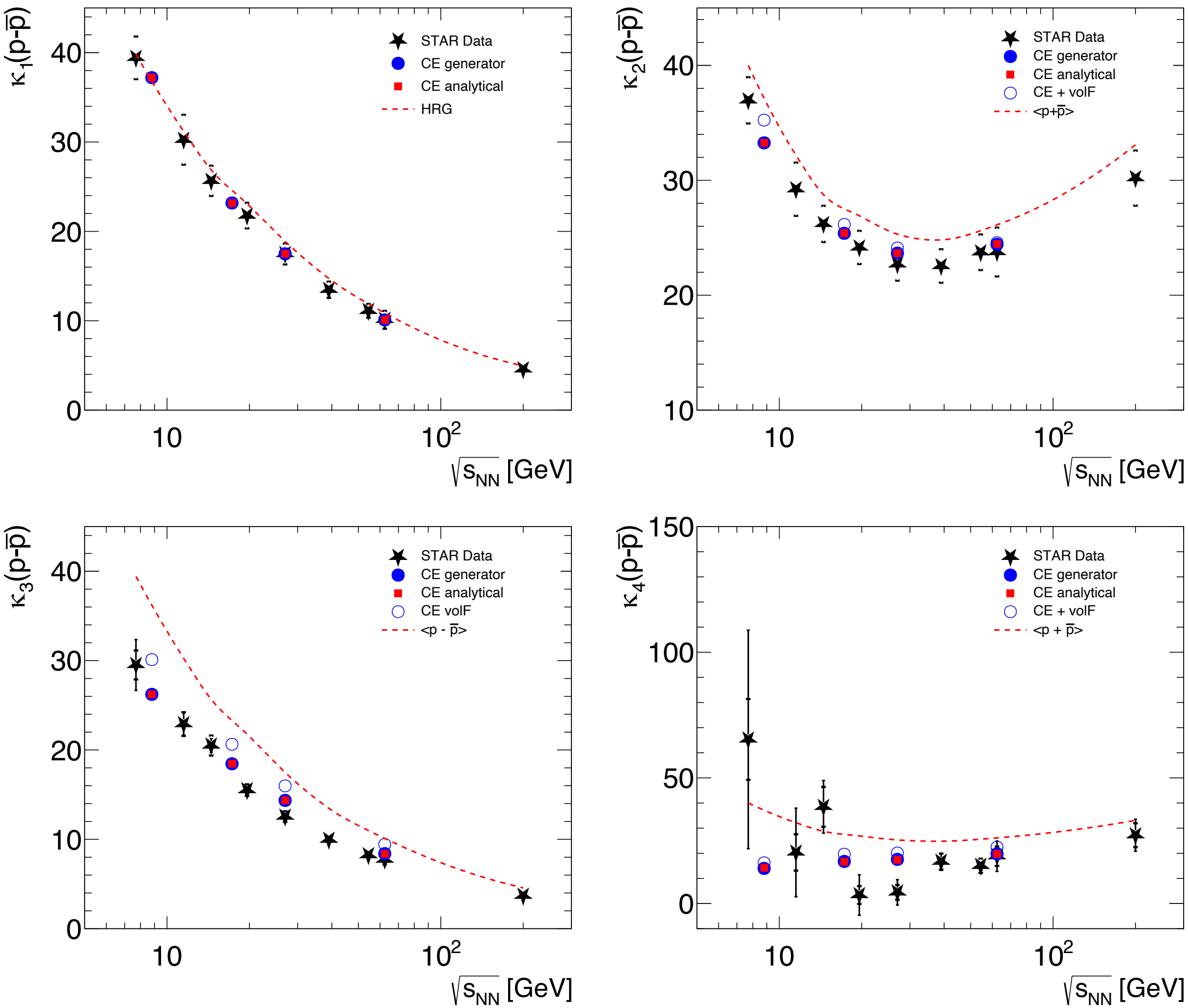


full symbols: α_p
open symbols: $\alpha_{\bar{p}}$

a constant cut in ***y*** introduces energy dependent acceptances



Predicted cumulants vs. STAR data



κ_1 values from STAR are used as input to our calculations, presented for consistency only

remarkable agreement between analytical/generated values and the STAR data.

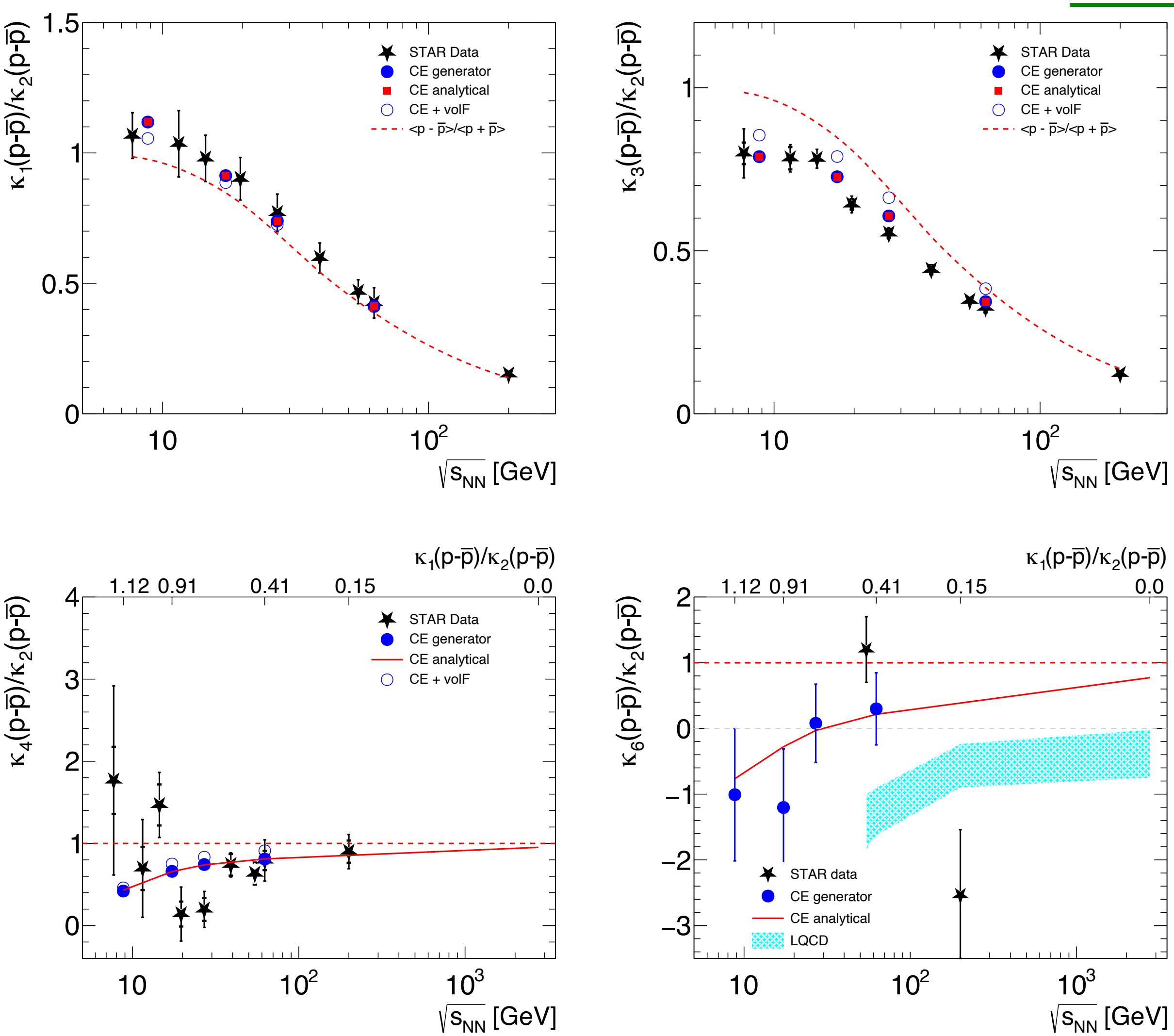
$\kappa_2, \kappa_3, \kappa_4$ values are suppressed compared to the GCE baseline

the amount of suppression in data is consistent with canonical effects

the data points for κ_4 fluctuate around the canonical baseline

STAR data: B. Mohanty, Nu Xu, this workshop

Cumulant ratios vs. STAR data



as already observed for pure cumulants, remarkable agreement between calculations and the STAR data is obvious

artefact of fixed acceptance in rapidity:

for higher energies the ratios approach the HRG baseline

significant reduction of κ_6/κ_2 going from positive values at LHC to negative values at lower energies

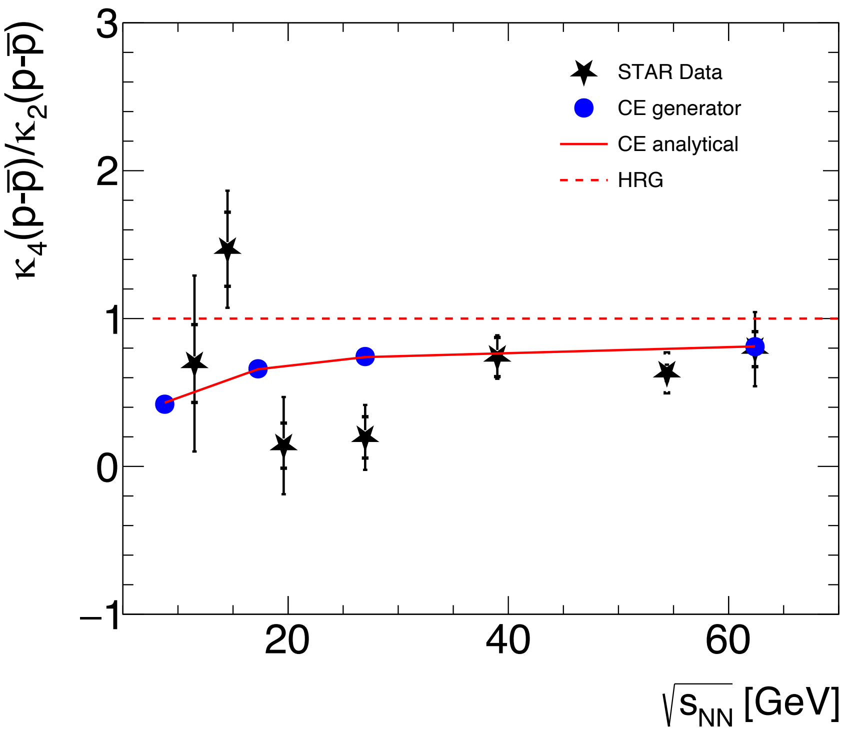
LQCD results for κ_6/κ_2 are negative for all energies

STAR data: B. Mohanty, Nu Xu, this workshop

LQCD: A. Bazavov et al., Phys.Rev.D 101 (2020) 7, 074502

Hypothesis test for κ_4/κ_2

$\sqrt{s_{NN}} = 8.8 - 62.4$ GeV



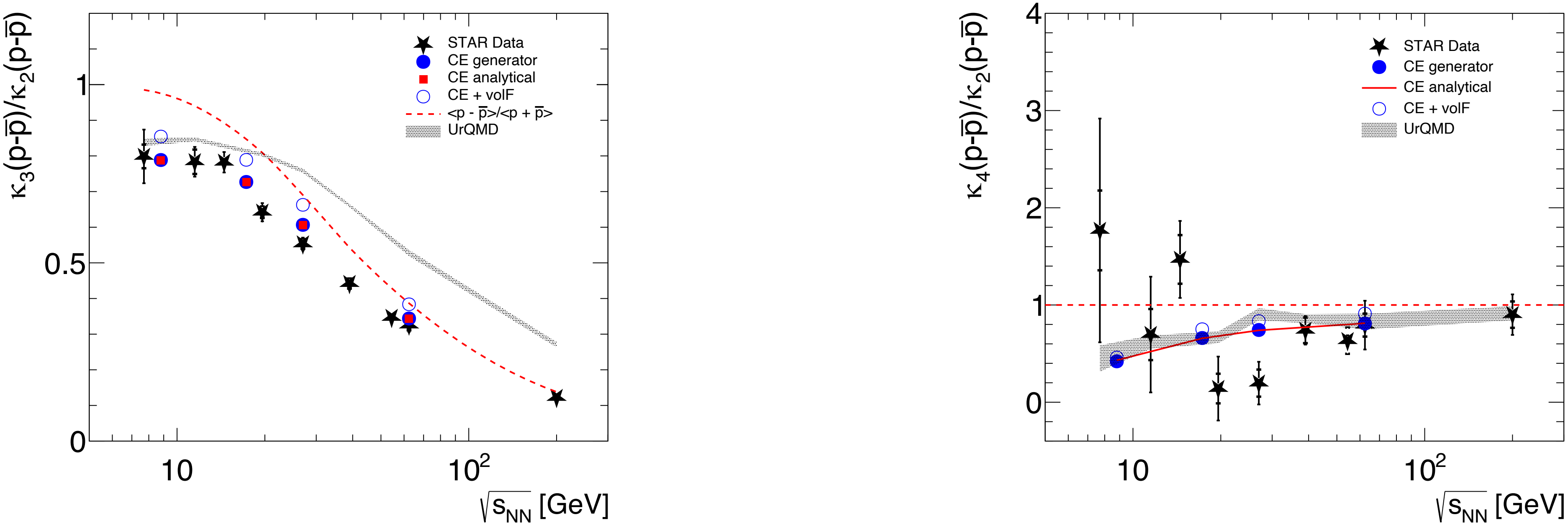
Kolmogorov-Smirnov test

- null-hypothesis
  the data and CE baseline are consistent,
  rejected when p-value is < 0.1
- obtained p-value: > 0.3

the observed deviations between the STAR data and the canonical baseline are not statistically significant

STAR data: B. Mohanty, Nu Xu, this workshop

Note on comparison to UrQMD



conflicting behavior in UrQMD

κ_3/κ_2 : **above 20 GeV the UrQMD results are significantly above the STAR data and the HRG baseline**

κ_4/κ_2 : **the UrQMD results are in agreement with the canonical suppression**

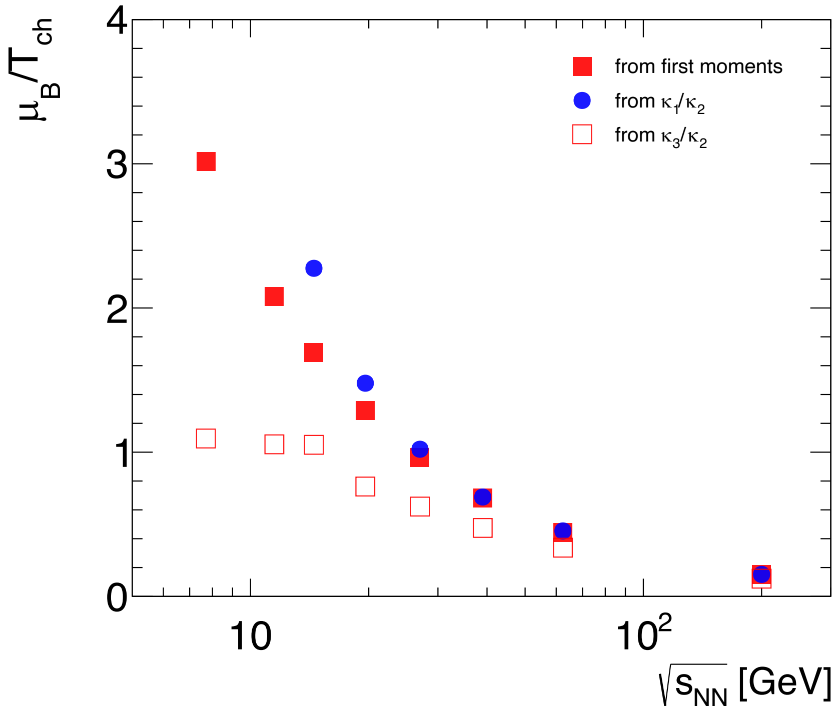
STAR data, UrQMD: B. Mohanty, Nu Xu, this Workshop

Extraction of freeze-out parameters

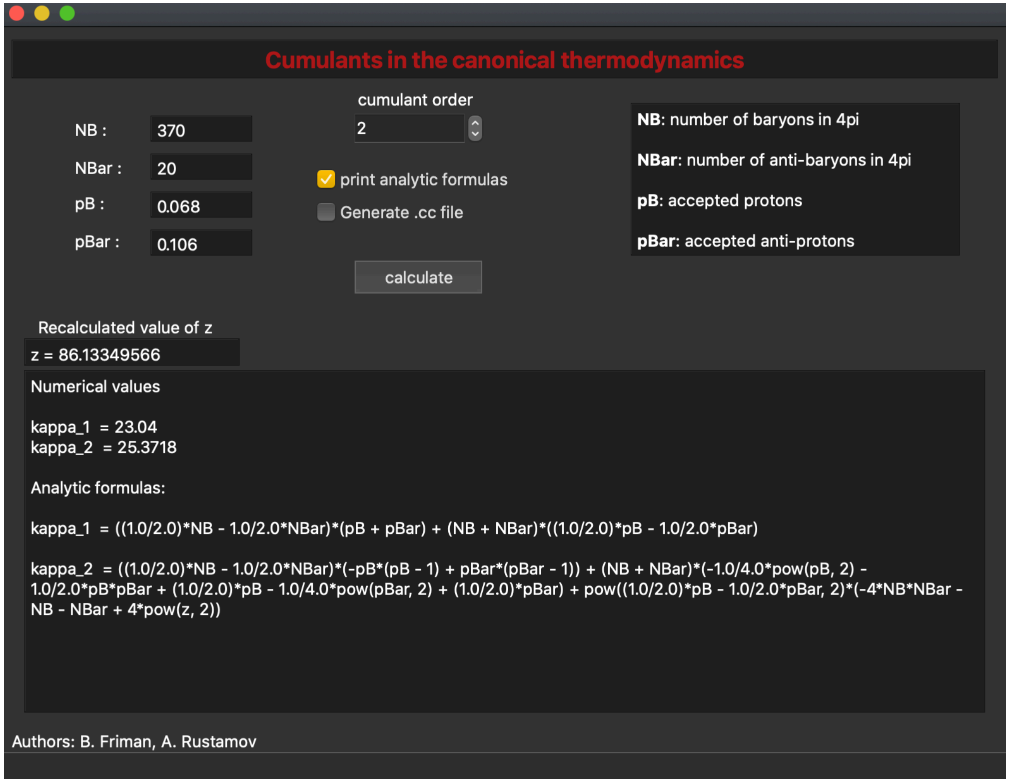
freeze-out parameters from higher cumulants using **G**rand **C**anonical **E**nsemble formulation

$$\frac{\mu_B}{T_{ch}} = \text{atanh}\left(\frac{\kappa_1}{\kappa_2}\right) = \text{atanh}\left(\frac{\kappa_3}{\kappa_2}\right)$$

if canonical effects alter cumulant ratios the extracted freeze-out parameters will give spurious results



A dedicated Python package



a Python package for calculating both analytic formulas and numerical values for net-baryon cumulants of any order in the finite acceptance is available for download

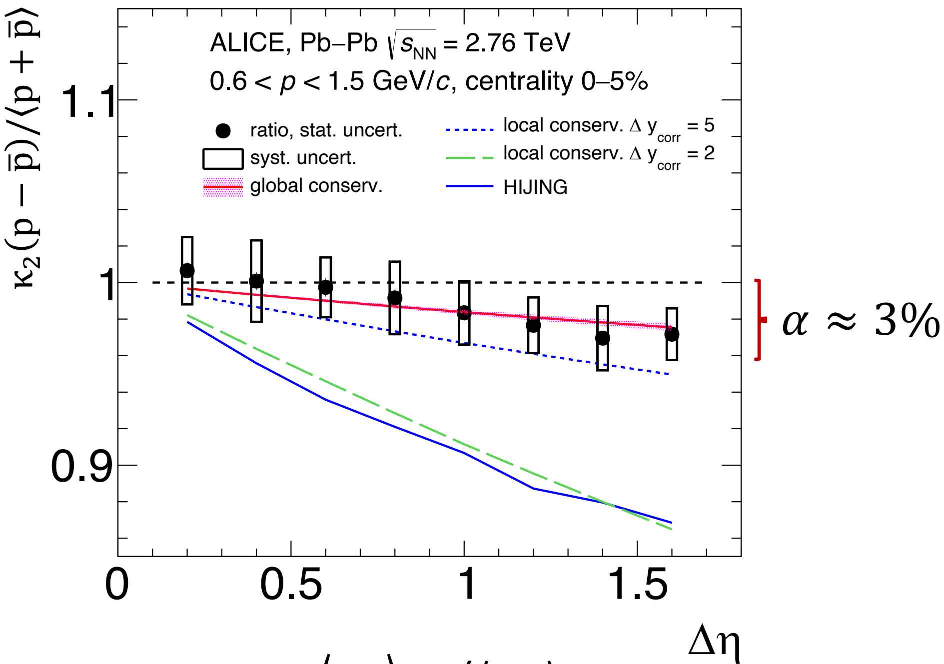
git clone https://github.com/e-by-e/Cumulants-CE.git

global vs. local conservations

multi-particle correlations

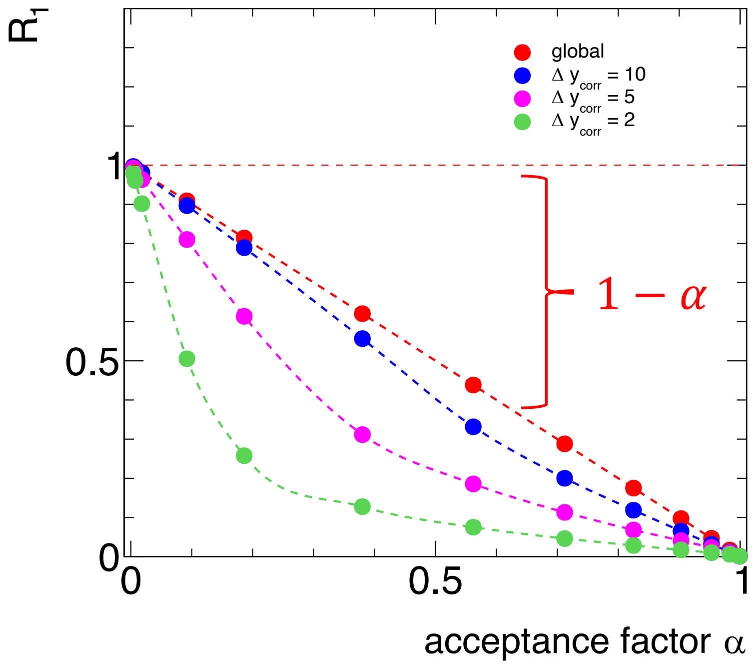
P. Braun-Munzinger, A. Rustamov, J. Stachel, arXiv:1907.03032
B. Ling and M. A. Stephanov, Phys. Rev. C 93, 034915 (2016).
A. Bzdak, V. Koch, and N. Strodthoff, Phys. Rev. C 95, 054906 (2017)

Global vs. local correlations



$$\alpha = \langle n_p \rangle_{\Delta\eta} / \langle n_B \rangle_{4\pi}$$

ALICE: Phys. Lett. B 807 (2020) 135564



P. Braun-Munzinger, A. Rustamov, J. Stachel, arXiv:1907.03032

- The data are best described by global baryon number conservation: $\approx 1-\alpha$
- HIJING corresponds to $\Delta y_{corr} = 2$, not consistent with the data

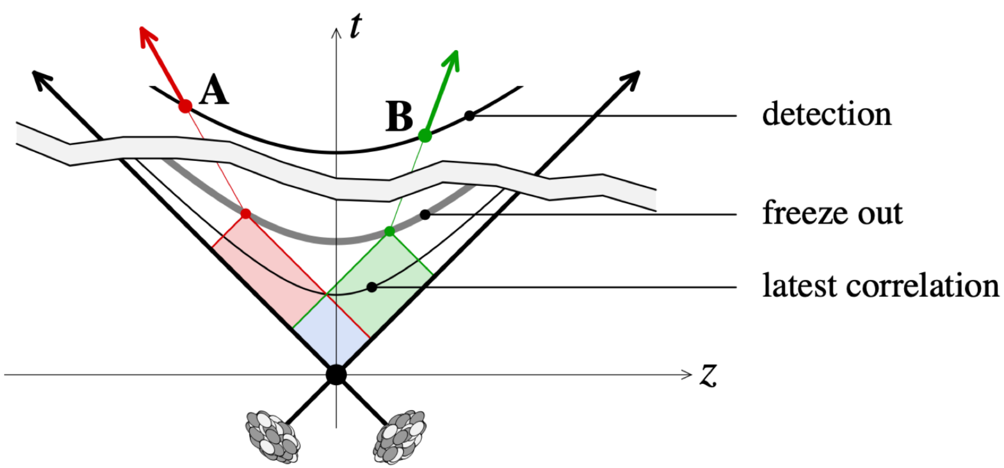
baryon production in string models is not consistent with data

the ALICE data indicate long range correlations

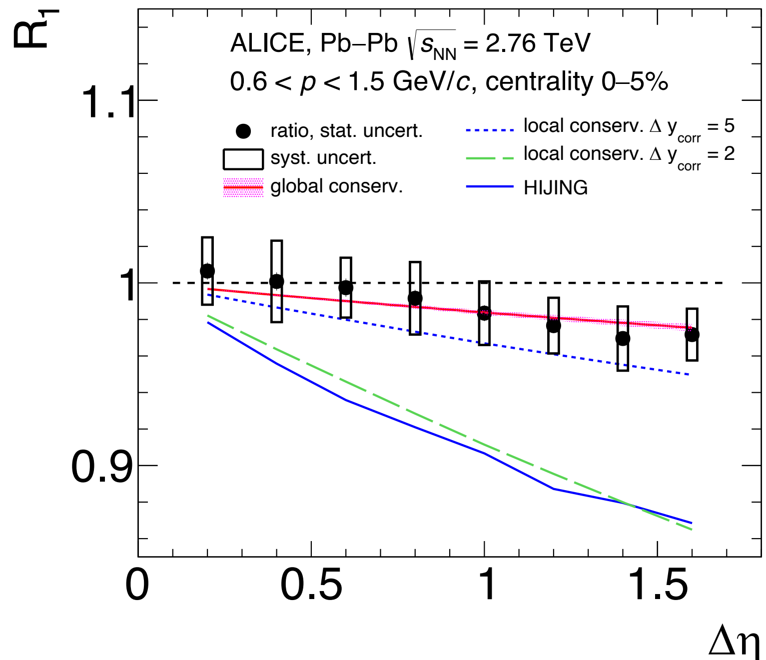
cf. also J. Adolfsson et al., 2003.10997 [hep-ph]

Probing the early collision times

A. Dumitru, F. Gelis, L. McLerran, R.Venugopalan , Phys. A810 (2008) 91–108



$$\tau_i \leq \tau_f e^{-|y_A - y_B|/2}$$

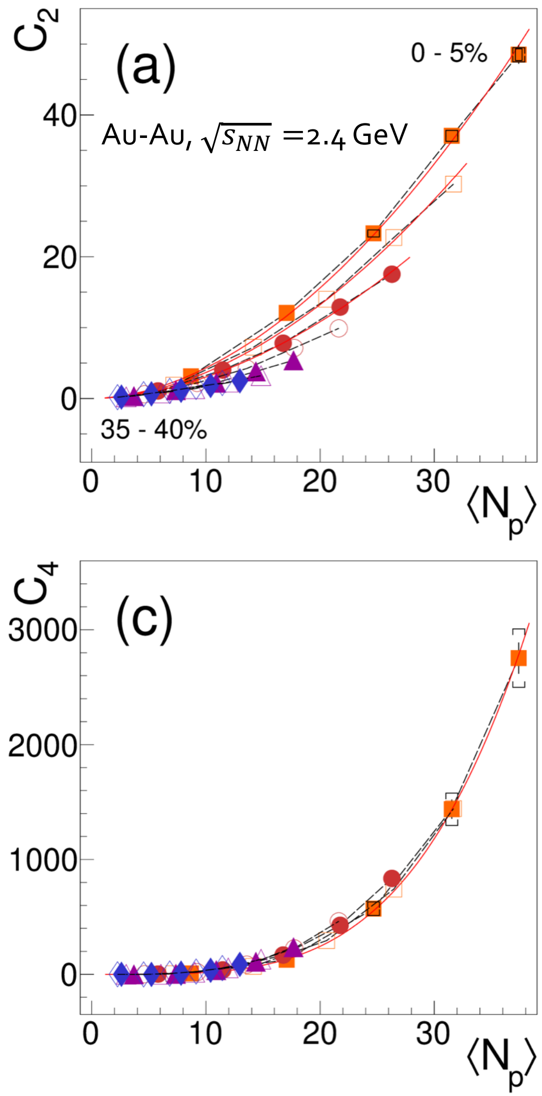


ALICE: Phys. Lett. B 807 (2020) 135564

long range rapidity correlations can only be created at early times–shortly after the collision or even in the wavefunctions of the incoming projectiles, that form sheets of Color Glass Condensate

A. Dumitru, F. Gelis, L. McLerran, R.Venugopalan , Phys. A810 (2008) 91–108

Multi-particle correlations at HADES



$\kappa_2 = \kappa_1 + C_2$ $\kappa_3 = \kappa_1 + 3C_2 + C_3$ $\kappa_4 = \kappa_1 + 7C_2 + 6C_3 + C_4$

B. Ling and M. A. Stephanov, Phys. Rev. C 93, 034915 (2016).

A. Bzdak, V. Koch, and N. Strodthoff, Phys. Rev. C 95, 054906 (2017)

long range correlations $(\Delta y_{corr} \gg \Delta y)$: $C_n \sim \langle N_p \rangle^n$

short range correlations $(\Delta y_{corr} \ll \Delta y)$: $C_n \sim \langle N_p \rangle$

Analyzed rapidity intervals:

- $\Delta y = 0.2, 0.4, 0.6, 0.8, 1$
- $\langle N_p \rangle$ - mean number of protons in selected Δy
- fit function: $C_0 \langle N_p \rangle^{\alpha}$

HADES: arXiv:2002.08701, PRC in print

centrality	$\alpha[C_2]$	$\alpha[C_3]$	$\alpha[C_4]$
0-5%	1.86±0.04	2.84±0.05	3.89±0.14

$\alpha \approx n \rightarrow \Delta y_{corr} \geq 1$

NOTE: canonical effects are not taken into account

the data indicates strong multi-particle correlations

The high μ_B corner of the phase diagram

Near future plans at HADES:

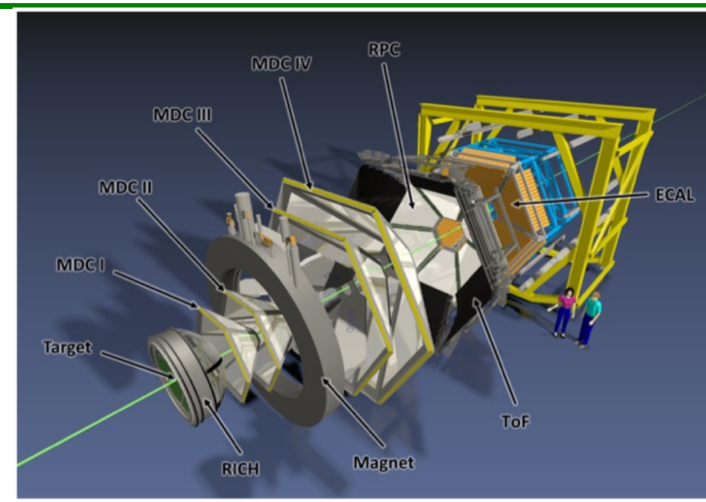
Au-Au collisions, with projectile kinetic energies of 0.8*A*, 0.6*A*, 0.4*A*, 0.2*A* GeV

$\sim 10^9$ events for each energy

systematic study of fluctuations and correlation functions and the high values of μ_B

probing the artefacts of nuclear liquid-gas phase transition

studying contributions of stopped protons to multi-particle correlations



SEARCHING FOR CRITICAL BEHAVIOR AND LIMITATIONS OF THE UNIVERSAL FREEZE-OUT LINE

Au+Au collisions at 0.2A–0.8A GeV

The HADES Collaboration

HADES

Spokespersons: J. Stroth (j.stroth@gsi.de), P. Tlusty (tlusty@ujf.cas.cz)
GSI contact: J. Pietraszko (j.pietraszko@gsi.de)

Infrastructure: SIS18 and HADES cave

Beam: slow extraction
Au at 0.8*A*–0.6*A*–0.4*A*–0.2*A* GeV, 1.2×10^6 ions/s (flat top)
C at 0.8*A*–0.6*A* GeV, 3×10^6 ions/s (flat top)

Abstract

We will extend our exploration of the QCD phase diagram towards the location of the nuclear liquid-gas phase transition. Two longer Au+Au runs (30 shifts each) are dedicated to low-mass dielectron and strangeness production while two shorter Au+Au runs (9 shifts each) will focus on the most abundant (non-strange) particles only, suitable for event-by-event analysis of particle correlations and fluctuations as well as to extract temperature of the system at freeze-out. We aim at high statistics to enable (i) laboratory studies of the matter properties (Equation-of-State) in compact stellar objects and (ii) detection of measurable consequences of phase transition and critical point in the QCD phase diagram. Moreover, C+C collisions (6 shifts each) will be investigated to provide reference data.
In the following we elucidate the proposed studies using the HADES spectrometer.

This is a proposal for a new experiment

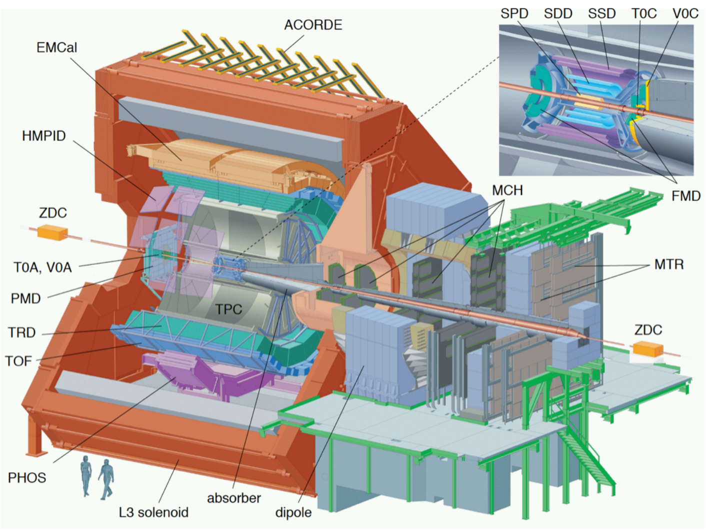
In total we request 94 shifts

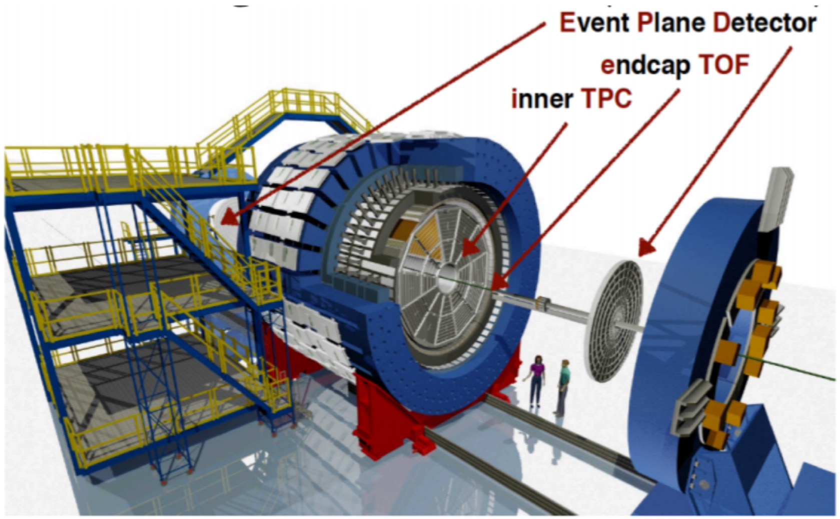
the proposal is submitted

Near Future Experiments

- **ALICE upgrade**
  - new ITS: better vertexing
  - faster TPC: MWPC → GEMs
  - record minimum-bias Pb-Pb data at 50 kHz (currently < 1 kHz)
    - order of magnitude more events
    - measuring κ_6, may be beyond

- **STAR upgrade, BES - II**
  - iTPC: $|\eta| < 1.5$
    - better dE/dx resolution
    - lower momentum acceptance
  - EPD: $2.1 < |\eta| < 5.1$
    - centrality determination
    - ~ factor 20 more statistics





Zhangbu Xu: QM19

Summary

- The non-critical baseline for net-baryon cumulants is developed
- Overall the experimental results from **STAR** and **ALICE** follow the non-critical baseline predictions
- Contributions due to local baryon number conservation at LHC energies are negligible
  - The **ALICE** data strongly indicate long range correlations, implying sensitivity to early stages of collisions
- The data from **HADES** indicate strong multi-particle correlations
  - For firm conclusions canonical effects are to be accounted for
  - Proposed experiments in **HADES** will shed light on the nature of multi-particle correlations
- Near future experiments at **ALICE, HADES, STAR** will allow for high precision measurements of cumulants beyond fourth order

Thank you for your attention!

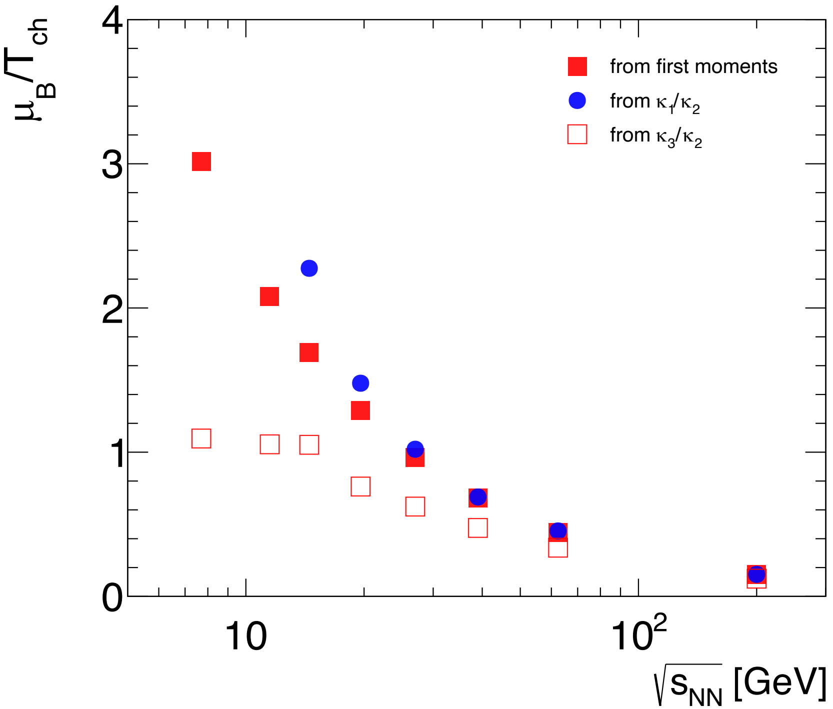
BACKUP SLIDES

Extraction of freeze-out parameters

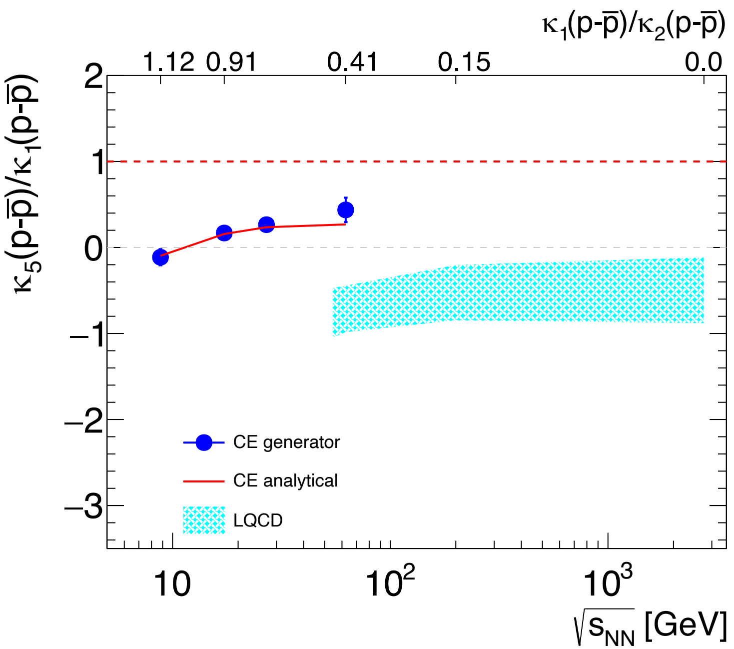
freeze-out parameters from higher cumulants using **Grand Canonical Ensemble** formulation

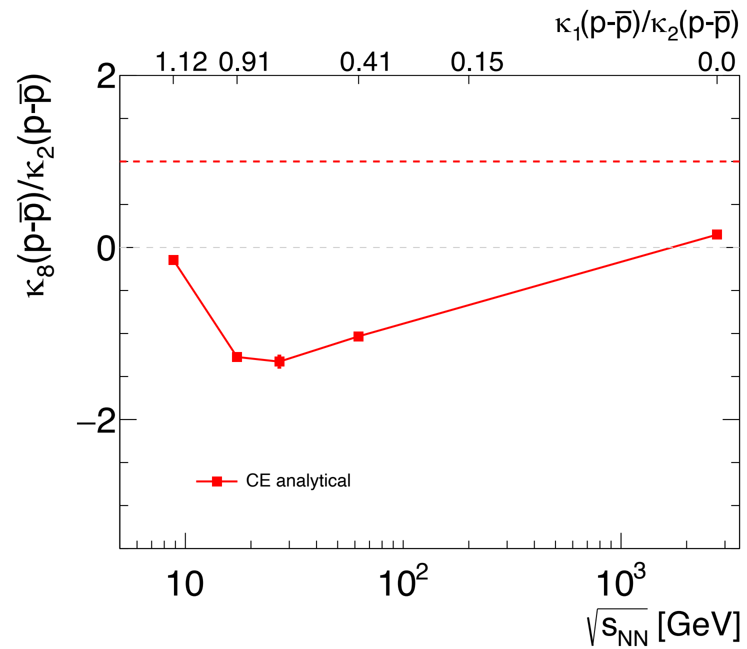
$$\frac{\mu_B}{T_{ch}} = \text{atanh}\left(\frac{\kappa_1}{\kappa_2}\right) = \text{atanh}\left(\frac{\kappa_3}{\kappa_2}\right)$$

if canonical effects alter cumulant ratios the extracted freeze-out parameters will give spurious results



5^{th} and 8^{th} order cumulants



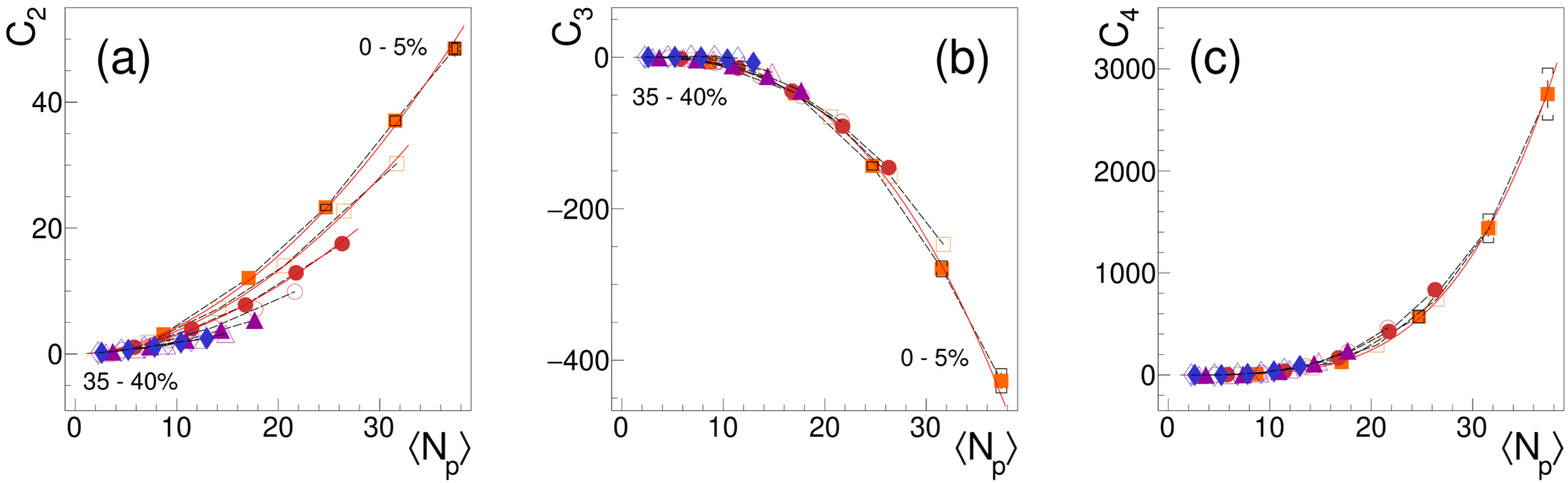


Input to our model

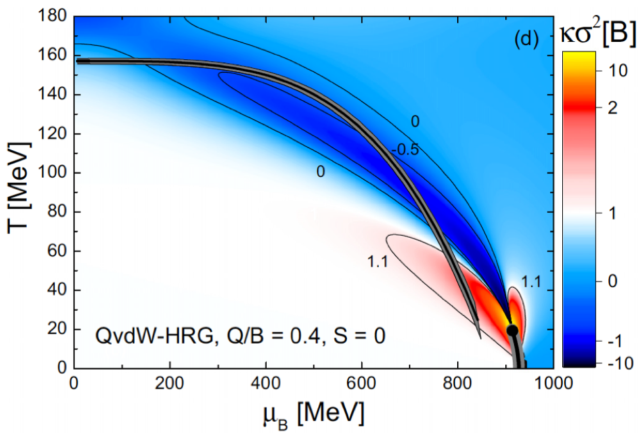
$\sqrt{s_{NN}}$ [GeV]	$\langle N_B \rangle$	$\langle N_{\bar{B}} \rangle$	$\langle N_p \rangle$	$\langle N_{\bar{p}} \rangle$	z
8.8	353	2	130	0.51	26.608
17.3	368	16	154.6	4.36	76.833
27	373 (377)	30 (34)	–	–	105.914 (113.354)
62.4	384	70	181.5	33.23	164.132

P. Braun-Munzinger, B. Friman, K. Redlich, A. Rustamov, J. Stachel , arXiv:2007.02463

The high μ_B corner of the phase diagram



HADES: arXiv:2002.08701, PRC in print



R. Poberezhnyuk, V. Vovchenko, A. Motornenko, M.I. Gorenstein,
H. Stoecker, PRC Phys.Rev. C100 (2019) no.5, 054904