Fluctuation and conservation

- Fluctuations and the QCD phase diagram
  - bi-modal distributions

- Baryon number conservation 2.0:
  Correcting susceptibilities from lattice QCD for global B, Q, S conservation

A. Bzdak, D. Oliinychenko, J. Steinheimer, VK: arXiv:1804.04463

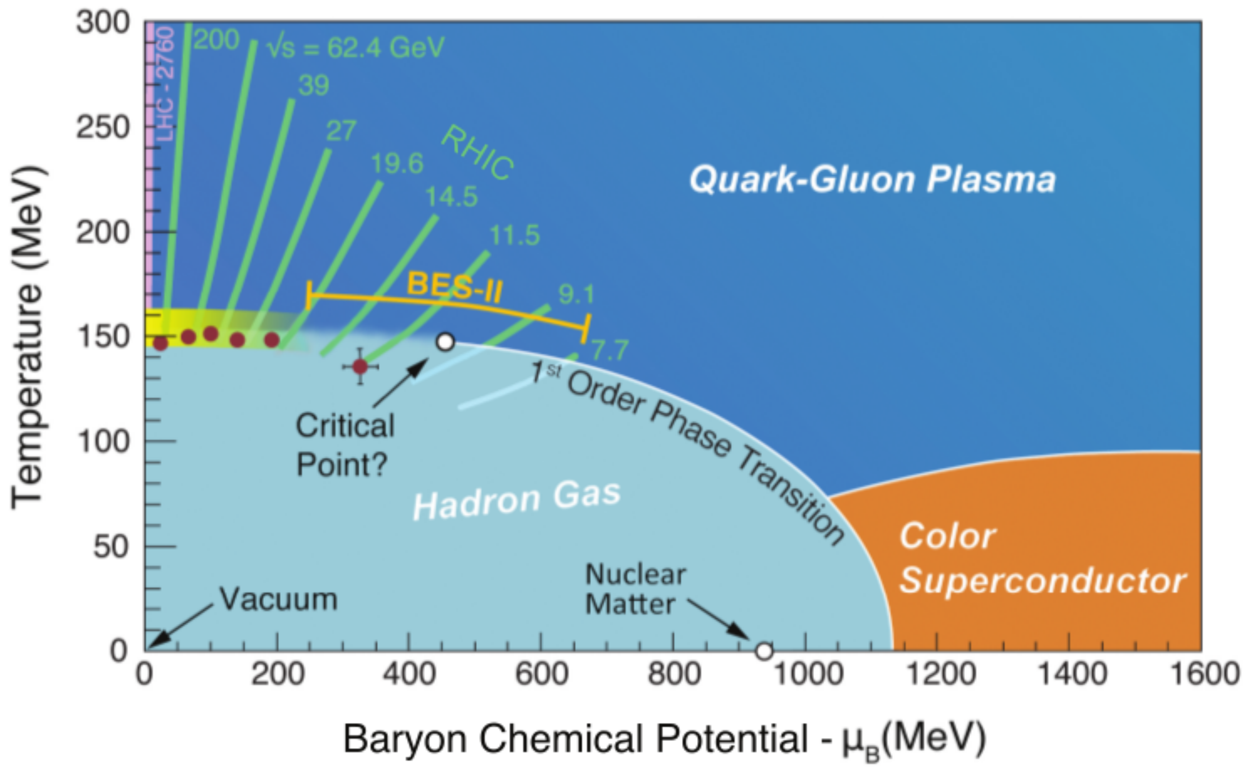
A. Bzdak, VK: arXiv:1811.04456

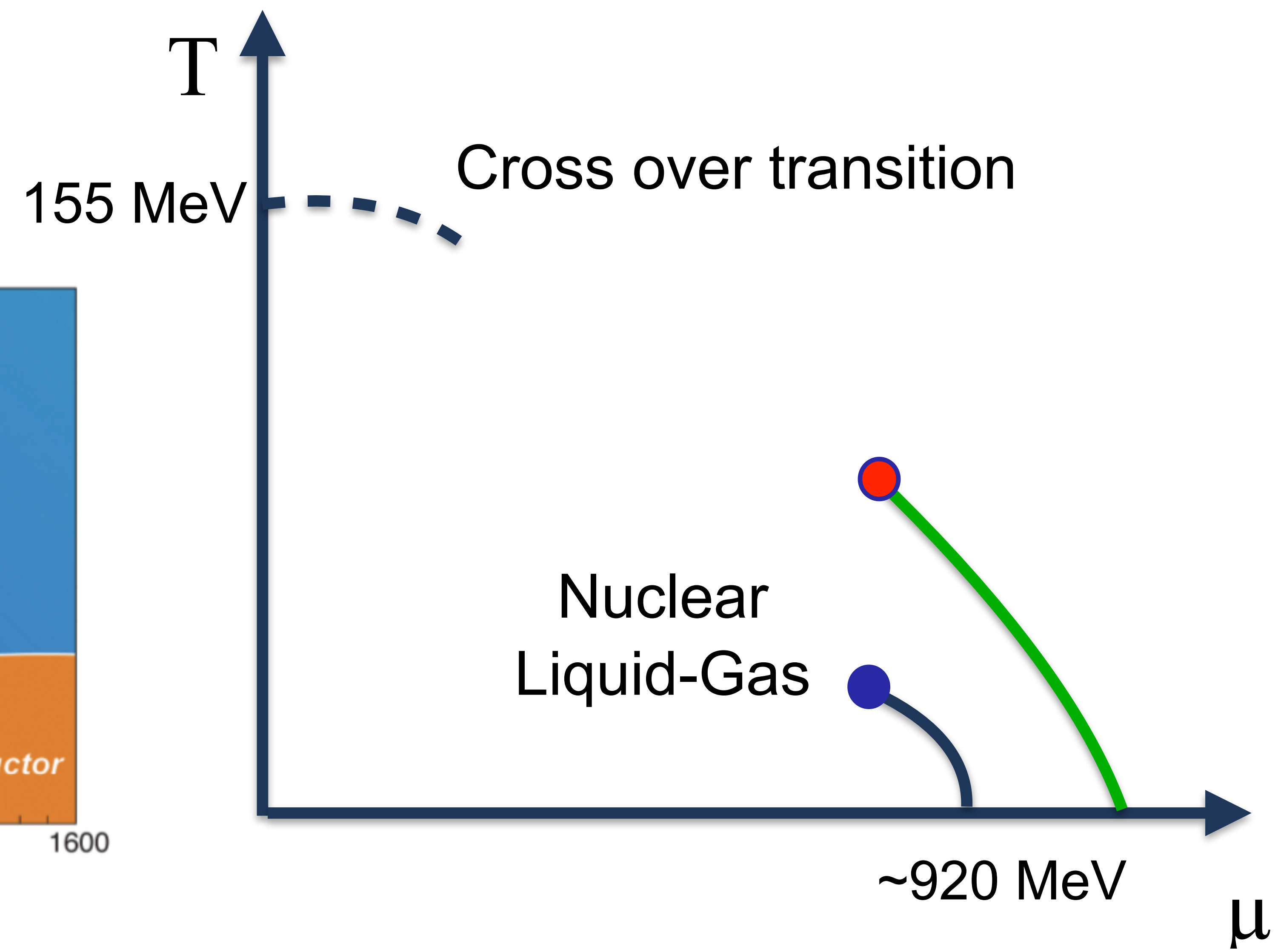
V. Vovchenko, O. Savchuk, R. Poberezhnyuk, M. Gorenstein, V.K., arXiv 2003.13905,

V. Vovchenko, R. Poberezhnyuk, V.K., arXiv:2007.03850

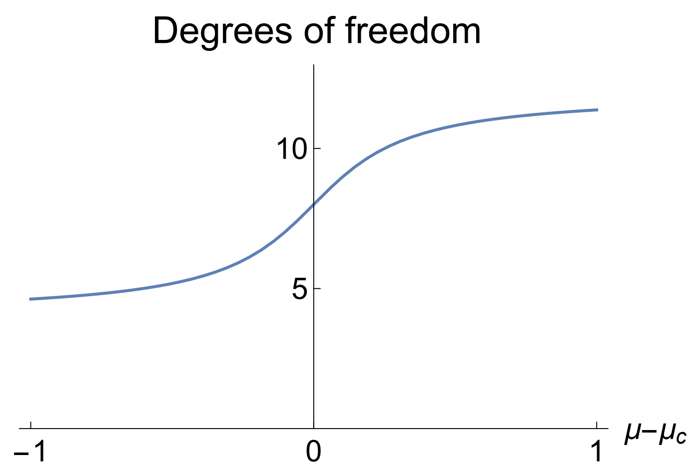
BEST COLLABORATION

The phase diagram





Cumulants of (baryon) number distribution



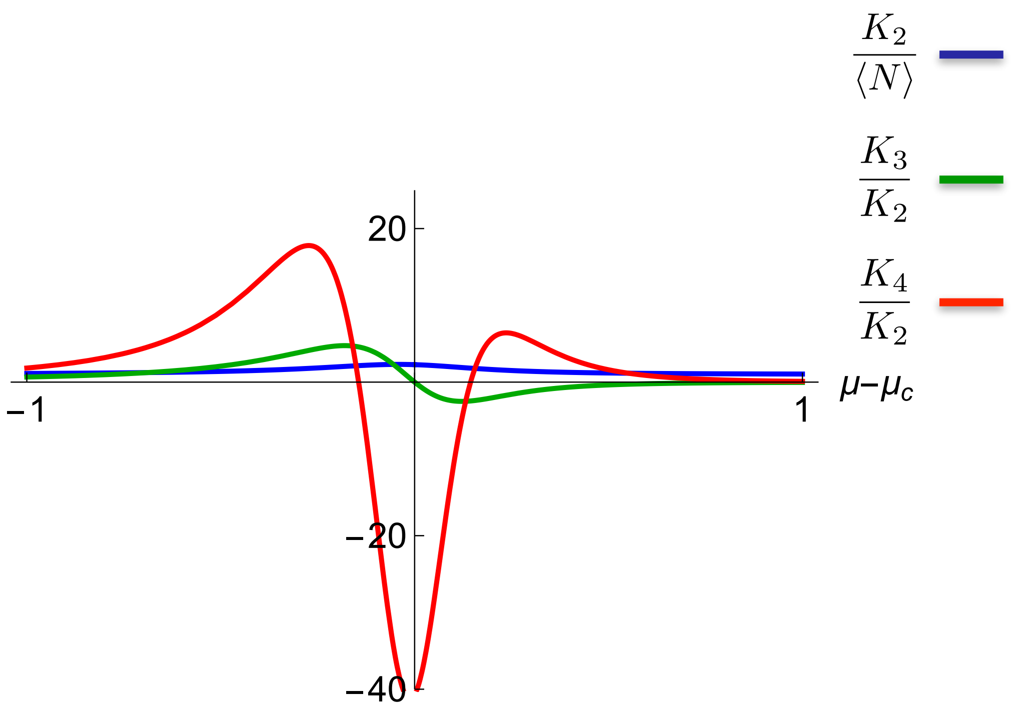
$$K_n = \frac{\partial^n}{\partial(\mu/T)^n} \ln Z = \frac{\partial^{n-1}}{\partial(\mu/T)^{n-1}} \langle N \rangle$$

$$K_1 = \langle N \rangle\,, \quad K_2 = \langle N - \langle N \rangle \rangle^2\,, \quad K_3 = \langle N - \langle N \rangle \rangle^3$$

Cumulants scale with volume (extensive): $K_n \sim V$

Volume not well controlled in heavy ion collisions

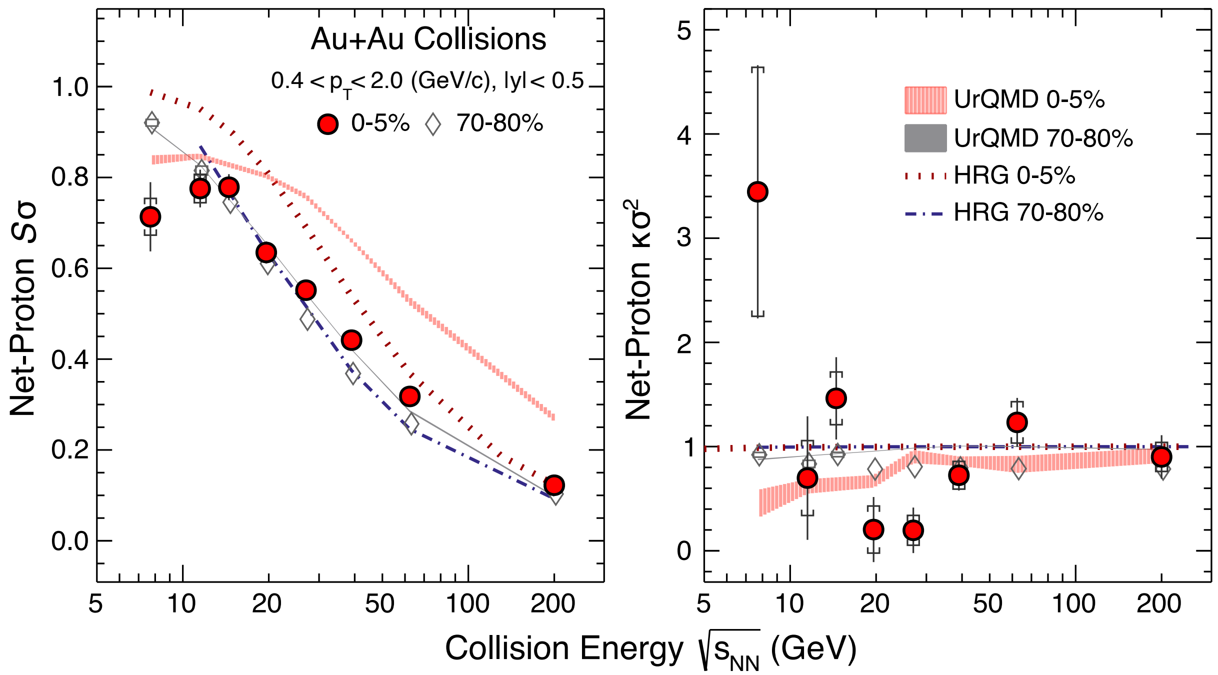
Cumulant Ratios: $\frac{K_2}{\langle N \rangle}, \frac{K_3}{K_2}, \frac{K_4}{K_2}$



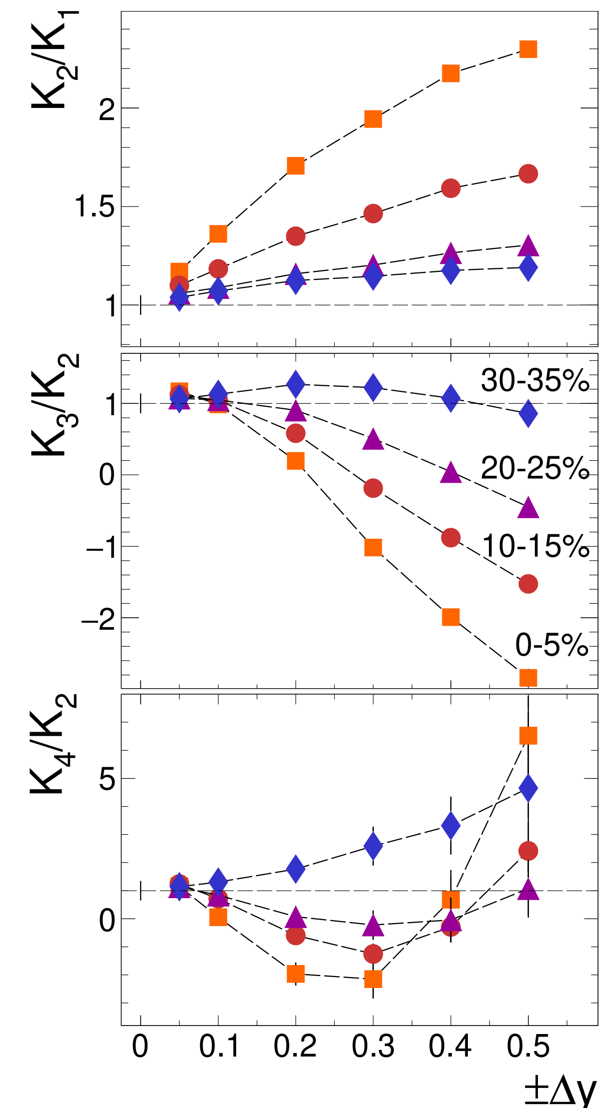
Baryon number cumulants measure derivatives of the EOS w.r.t chemical potential

Cumulants have been measured

STAR
arXiv:2001.02852



HADES
arXiv:2002.08701

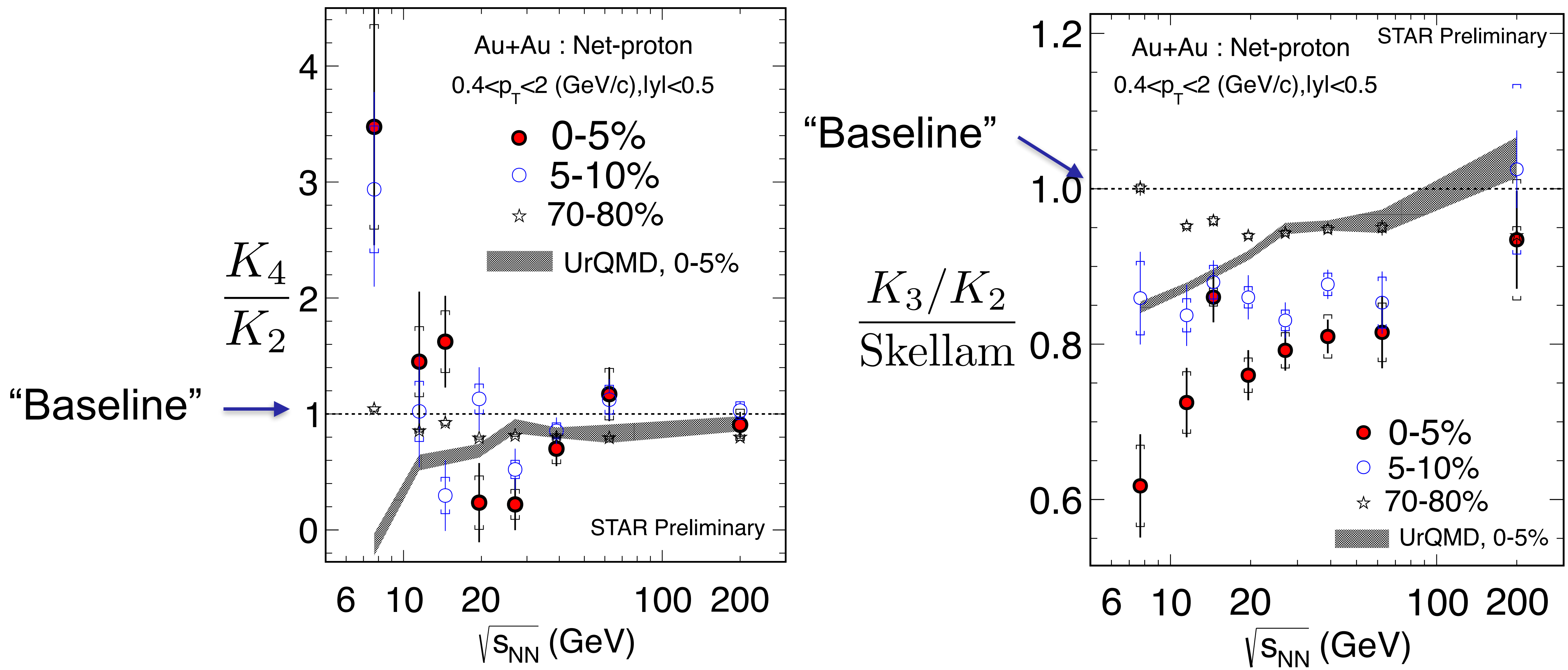


Alternative approach:

Look at integrated correlation functions a.k.a factorial cumulants

Latest STAR result on net-proton cumulants

X. Luo, NPA 956 (2016) 75

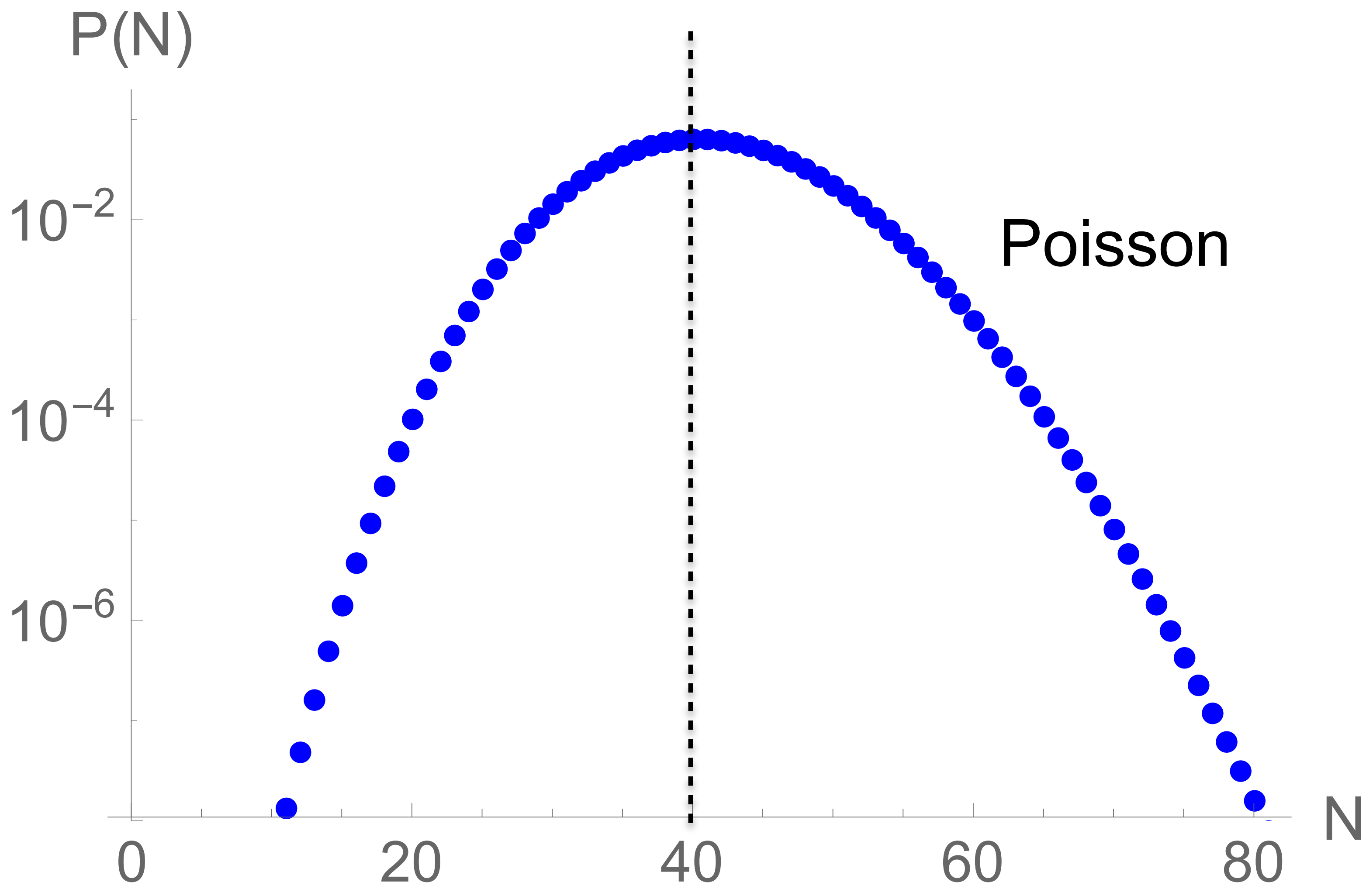


K_4/K_2 above baseline K_3/K_2 below baseline

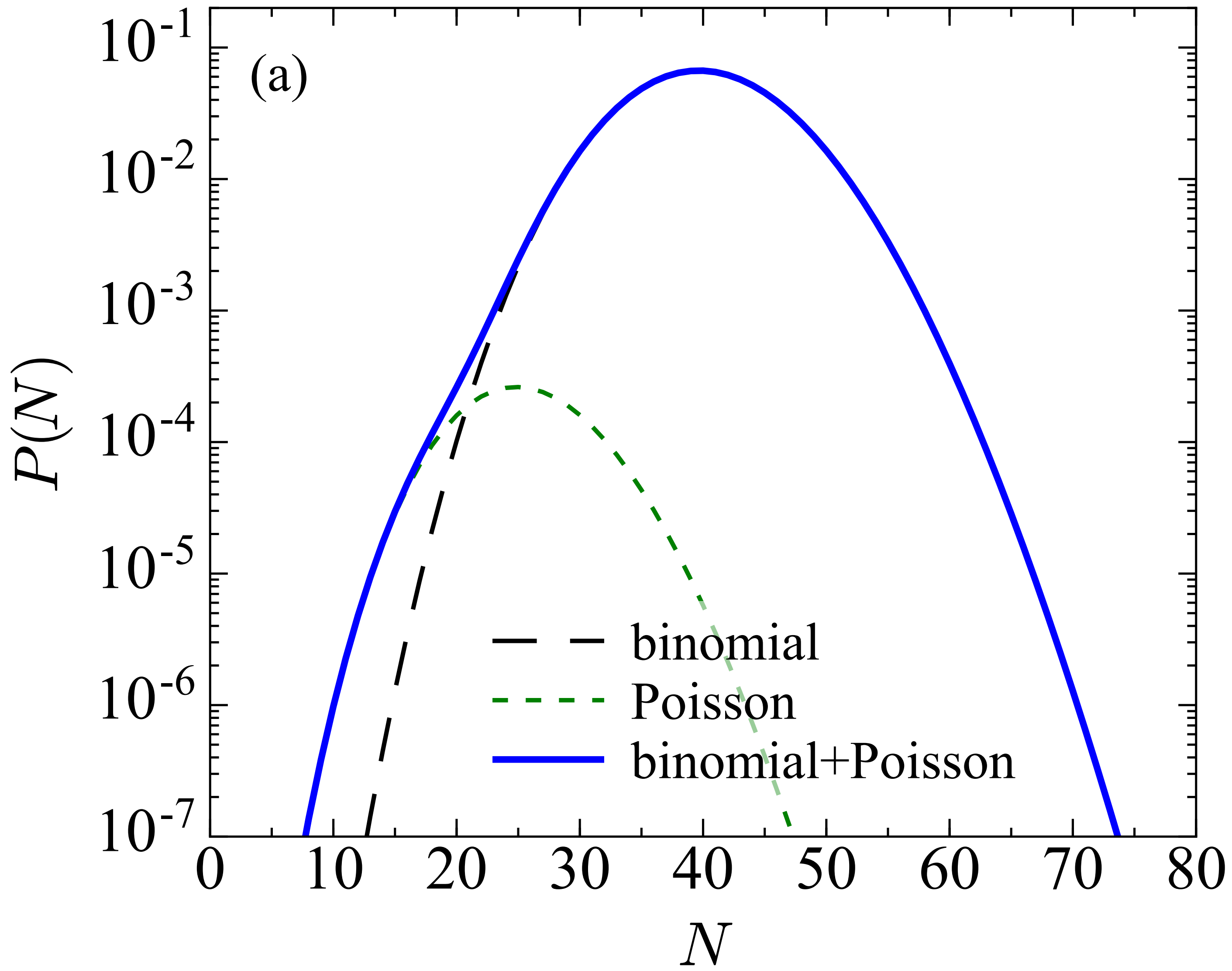
Shape of probability distribution

$$K_3 < \langle N \rangle \qquad K_3 = \langle N - \langle N \rangle \rangle^3$$

$$K_4 > \langle N \rangle \qquad K_4 = \langle N - \langle N \rangle \rangle^4 - 3 \langle N - \langle N \rangle \rangle^2$$



Simple two component model



Weight of small component: ~0.3%

Two component model

$$P(N) = (1-\alpha)P_{(a)}(N) + \alpha P_{(b)}(N)$$

$$\bar{N} = \langle N_{(a)} \rangle - \langle N_{(b)} \rangle > 0$$

For $P_{(a)}$, $P_{(b)}$ Poisson, or (to good approximation) Binomial

$$C_n = (-1)^n K_n^B \bar{N}^n \quad n \geq 2$$ C_n : Factoral cumulant

$$K_n^B \ : \ \text{Cumulant of Bernoulli distribution}$$

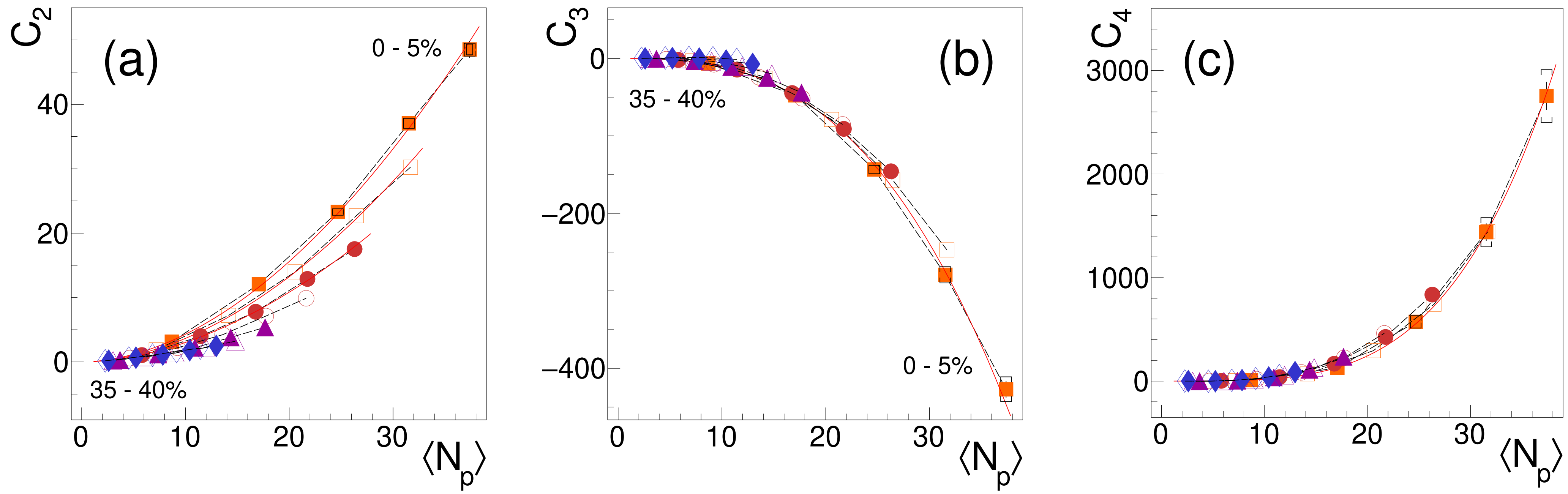
$$\alpha \ll 1, K_n^B = \alpha \ \Rightarrow \ C_n \simeq \alpha(-1)^n \bar{N}^n$$

$\Rightarrow |C_n| \sim \langle N \rangle^n$ as seen by STAR (i.e. “infinite” correlation length)

predict: $$\frac{C_4}{C_3} = \frac{C_5}{C_4} = \frac{C_{n+1}}{C_n} = -\bar{N}$$ $\bar{N} \simeq 15$

Clear and falsifiable prediction: $C_5 \approx -2650 \quad C_6 \approx 41000$

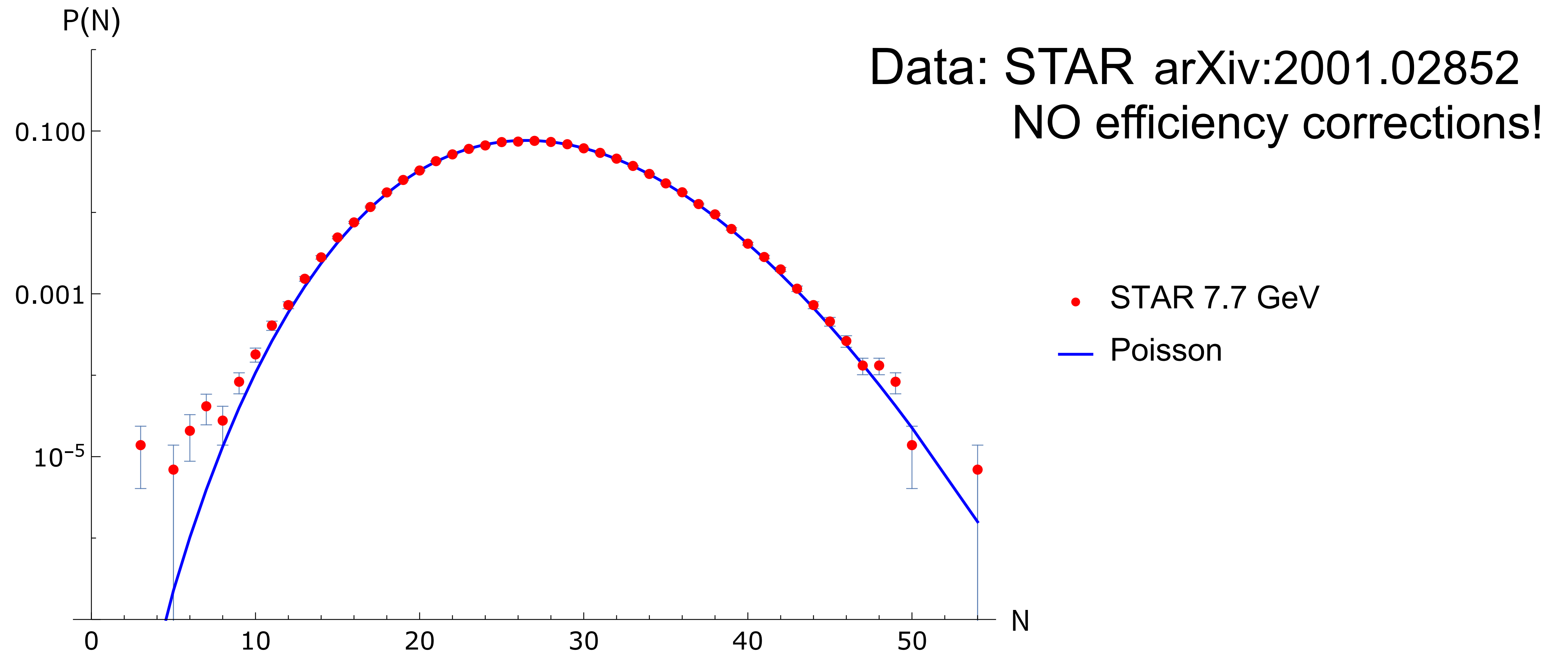
Hades see similar trend
(arXiv:2002.08701)



$$\frac{C_{n+1}}{C_n} \simeq -10$$

Caveat: rather significant N_{part} fluctuations to be corrected for

Multiplicity distribution @ 7.7 GeV

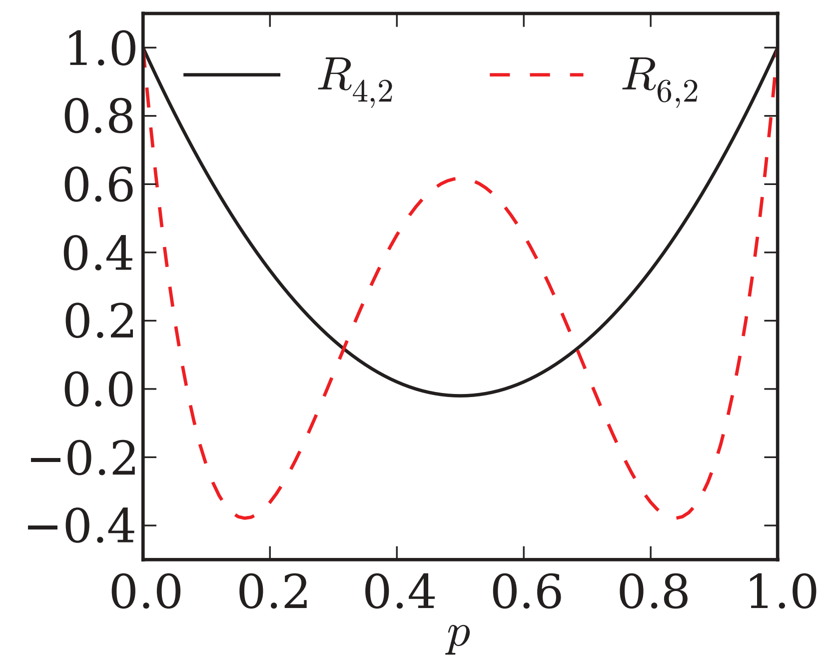


Now we need to figure out what this means….

First question: How does it look in the revised data?

Baryon number conservation and lattice susceptibilities

- Charges (baryon number, strangeness, electric charge) are conserved globally in HI collisions
- Lattice (and most other calculations) work in the grand canonical ensemble: charges may fluctuate
- Effect of charge conservation have been calculated in the **ideal gas/HRG** limit. NON-neglibile corrections especially for higher order cumulants (Bzdak et al 2013, Rustamov et al. 2017,…)
- Wouldn't it be nice to know what the effect of charge conservation on **real QCD** (aka lattice) susceptibilities is?



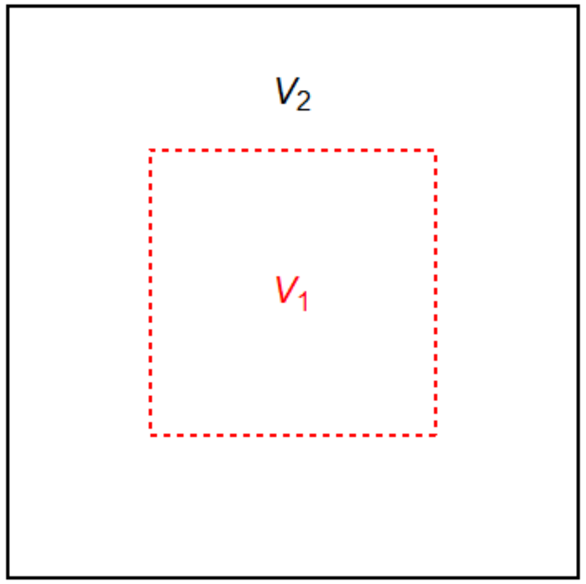
This can actually be done!

V. Vovchenko, O. Savchuk, R. Poberezhnyuk, M. Gorenstein, V.K., arXiv 2003.13905,

V. Vovchenko, R. Poberezhnyuk, V.K., arXiv:2007.03850

Subensemble acceptance method (SAM)

Partition a thermal system with a globally conserved charge *B (canonical ensemble)* into two subsystems which can exchange the charge



$$V = V_1 + V_2$$

Assume thermodynamic limit:

$$V, V_1, V_2 \to \infty; \quad \frac{V_1}{V} = \alpha = const; \quad \frac{V_2}{V} = (1-\alpha) = const;$$

$$V_1, V_2 \gg \xi^3 \qquad \xi = correlation\ length$$

The canonical partition function then reads:

$$Z^{ce}(T,V,B) = \sum_{B_1} Z^{ce}(T,V_1,B_1) Z^{ce}(T,V-V_1,B-B_1)$$

The probability to have charge B_1 *in* V_1 is:

$$P(B_1) \sim Z^{ce}(T,\alpha V,B_1) Z^{ce}(T,(1-\alpha)V,B-B_1), \qquad \alpha \equiv V_1/V$$

Subensemble acceptance method (SAM)

In the thermodynamic limit, $V \to \infty$, Z^{ce} expressed through free energy *density*

$$Z^{\text{ce}}(T, V, B) \overset{V\to\infty}{\simeq} \exp\left[-\frac{V}{T} f(T, \rho_B)\right]$$

Cumulant generating function for B_1:

$$G_{B_1}(t) \equiv \ln\langle e^{t\,B_1}\rangle = \ln\left\{\sum_{B_1} e^{t\,B_1} \exp\left[-\frac{\alpha V}{T} f(T, \rho_{B_1})\right] \exp\left[-\frac{\beta V}{T} f(T, \rho_{B_2})\right]\right\} + \tilde{C}$$

Cumulants of B_1:

$$\kappa_n[B_1] = \left.\frac{\partial^n G_{B_1}(t)}{\partial t^n}\right|_{t=0} \equiv \tilde{\kappa}_n[B_1(t)]|_{t=0}$$

or

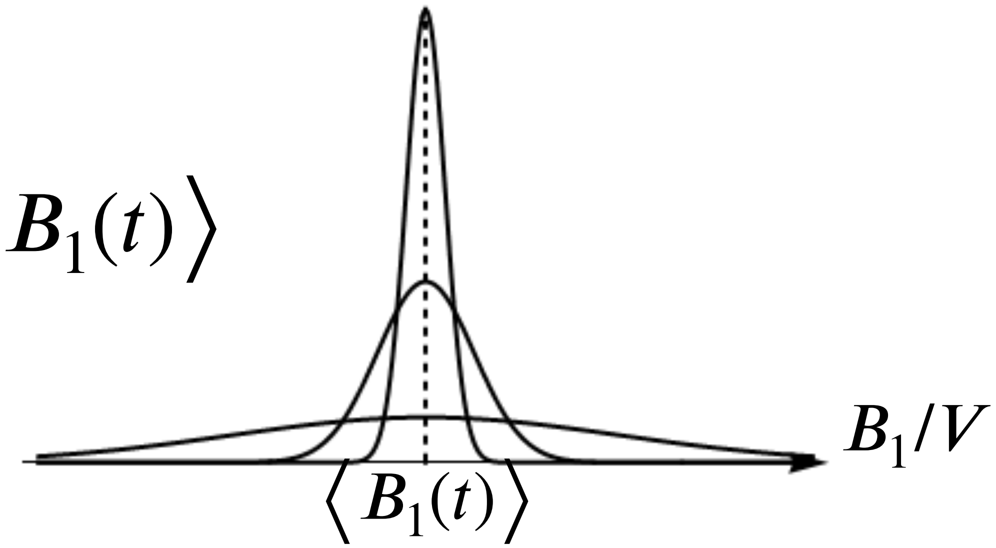
$$\kappa_n[B_1] = \left.\frac{\partial^{n-1} \tilde{\kappa}_1[B_1(t)]}{\partial t^{n-1}}\right|_{t=0}$$

All κ_n can be calculated by determining the *t*-dependent first cumulant $\tilde{\kappa}_1[B_1(t)]$

Making the connection…

$$\tilde{\kappa}_1[B_1(t)] = \frac{\sum_{B_1} B_1 \tilde{P}(B_1;t)}{\sum_{B_1} \tilde{P}(B_1;t)} \equiv \langle B_1(t) \rangle \qquad \text{with} \qquad \tilde{P}(B_1;t) = \exp\left\{ tB_1 - V\,\frac{\alpha f(T,\rho_{B_1}) + \beta f(T,\rho_{B_2})}{T} \right\}.$$

Thermodynamic limit: $\tilde{P}(B_1;t)$ *highly peaked* at $\langle B_1(t) \rangle$



$\langle B_1(t) \rangle$ is a solution to equation $d\tilde{P}$ / dB_1 = 0:

$$t = \hat{\mu}_B[T,\rho_{B_1}(t)] - \hat{\mu}_B[T,\rho_{B_2}(t)]$$

where $\hat{\mu}_B \equiv \mu_B/T, \qquad \mu_B(T,\rho_B) = \partial f(T,\rho_B)/\partial \rho_B$

***t* = 0:** $\rho_{B_1} = \rho_{B_2} = B/V,\ B_1 = \alpha B$, i.e. conserved charge uniformly distributed between the two subsystems

Second order cumulant

$$t = \hat{\mu}_B[T, \rho_{B_1}(t)] - \hat{\mu}_B[T, \rho_{B_2}(t)] \qquad (*)$$

$$\frac{\partial(*)}{\partial t}: \qquad 1 = \left(\frac{\partial \hat{\mu}_B}{\partial \rho_{B1}}\right)_T \left(\frac{\partial \rho_{B1}}{\partial \langle B_1 \rangle}\right)_V \frac{\partial \langle B_1 \rangle}{\partial t} - \left(\frac{\partial \hat{\mu}_B}{\partial \rho_{B2}}\right)_T \left(\frac{\partial \rho_{B2}}{\partial \langle B_2 \rangle}\right)_V \frac{\partial \langle B_2 \rangle}{\partial \langle B_1 \rangle} \frac{\partial \langle B_1 \rangle}{\partial t}$$

$$\left(\frac{\partial \hat{\mu}_B}{\partial \rho_{B1,2}}\right)_T \equiv \left[\chi_2^B \left(T, \rho_{B_{1,2}}\right) T^3\right]^{-1}, \qquad \rho_{B_1} \equiv \frac{\langle B_1 \rangle}{\alpha V}, \qquad \rho_{B_2} \equiv \frac{\langle B_2 \rangle}{(1-\alpha)V}, \qquad \langle B_2 \rangle = B - \langle B_1 \rangle, \qquad \frac{\partial \langle B_1 \rangle}{\partial t} \equiv \tilde{\kappa}_2[B_1(t)]$$

Solve the equation for $\tilde{\kappa}_2$:

$$\tilde{\kappa}_2[B_1(t)] = \frac{V\,T^3}{[\alpha\, \chi_2^B(T, \rho_{B_1})]^{-1} + [(1-\alpha)\, \chi_2^B(T, \rho_{B_2})]^{-1}}$$

***t* = 0:**

$$\kappa_2[B_1] = \alpha\,(1-\alpha)\, V\, T^3\, \chi_2^B$$

Higher-order cumulants: iteratively differentiate $\tilde{\kappa}_2$ w.r.t. t

Full result up to sixth order

$$\beta = 1 - \alpha$$

$$\kappa_1[B_1] = \alpha\, VT^3\, \chi_1^B$$

$$\kappa_2[B_1] = \alpha\, VT^3\, \beta\, \chi_2^B$$

$$\kappa_3[B_1] = \alpha\, VT^3\, \beta\, (1-2\alpha)\, \chi_3^B$$

$$\kappa_4[B_1] = \alpha\, VT^3\, \beta\, \left[\chi_4^B \; - 3\alpha\beta \frac{(\chi_3^B)^2 + \chi_2^B\, \chi_4^B}{\chi_2^B}\right]$$

$$\kappa_5[B_1] = \alpha\, VT^3\, \beta\, (1-2\alpha) \left\{[1-2\beta\alpha]\chi_5^B \; - 10\alpha\beta \frac{\chi_3^B \chi_4^B}{\chi_2^B}\right\}$$

$$\kappa_6[B_1] = \alpha\, VT^3\, \beta\, [1 - 5\alpha\beta(1-\alpha\beta)]\, \chi_6^B + 5\, VT^3\, \alpha^2\, \beta^2 \left\{9\alpha\beta \frac{(\chi_3^B)^2\, \chi_4^B}{(\chi_2^B)^2} - 3\alpha\beta \frac{(\chi_3^B)^4}{(\chi_2^B)^3} - 2(1-2\alpha)^2 \frac{(\chi_4^B)^2}{\chi_2^B} - 3[1-3\beta\alpha] \frac{\chi_3^B\, \chi_5^B}{\chi_2^B}\right\}$$

$$\chi_n^B = \frac{\partial^n (p/T^4)}{\partial(\mu_B/T)^n}$$ – grand-canonical susceptibilities e.g from Lattice QCD!!

Details: Vovchenko, et al. arXiv:2003.13905

Cumulant ratios

Some common cumulant ratios:

scaled variance
$$\frac{\kappa_2[B_1]}{\kappa_1[B_1]} = (1-\alpha)\,\frac{\chi_2^B}{\chi_1^B},$$

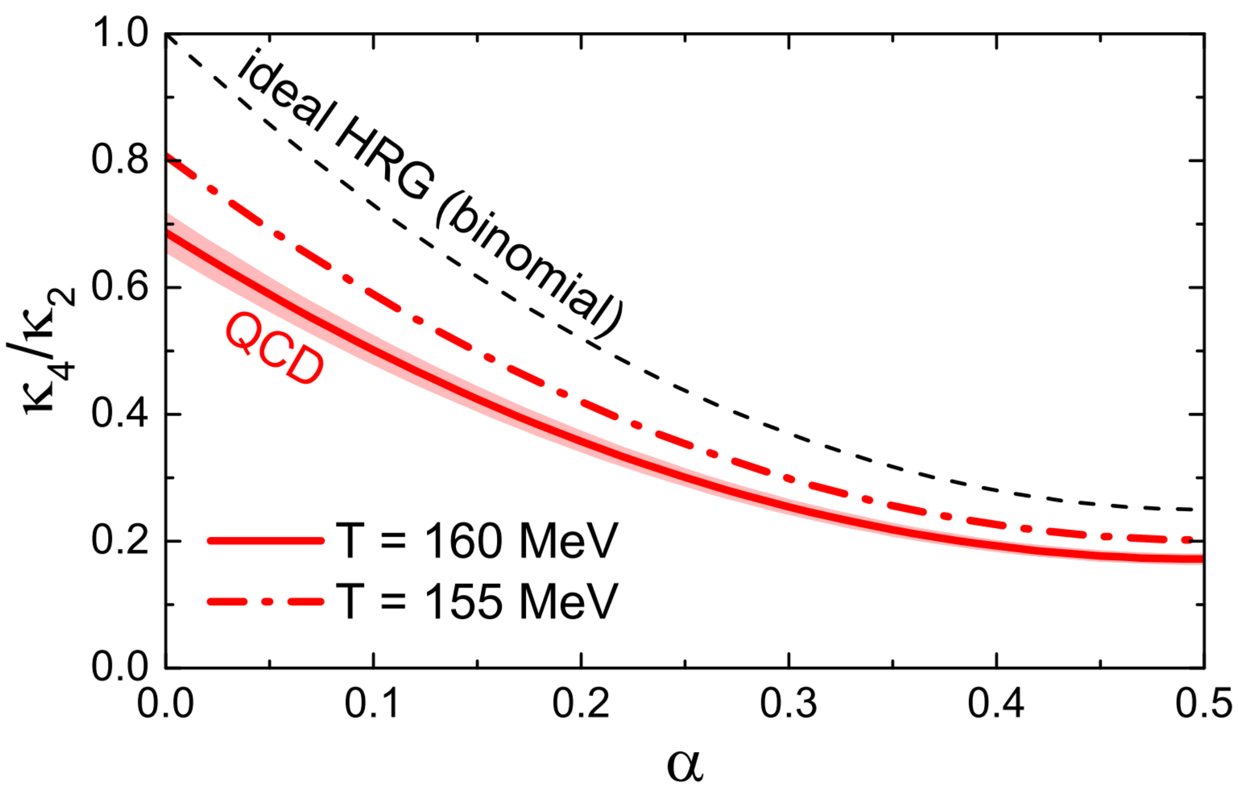
skewness
$$\frac{\kappa_3[B_1]}{\kappa_2[B_1]} = (1-2\alpha)\,\frac{\chi_3^B}{\chi_2^B},$$

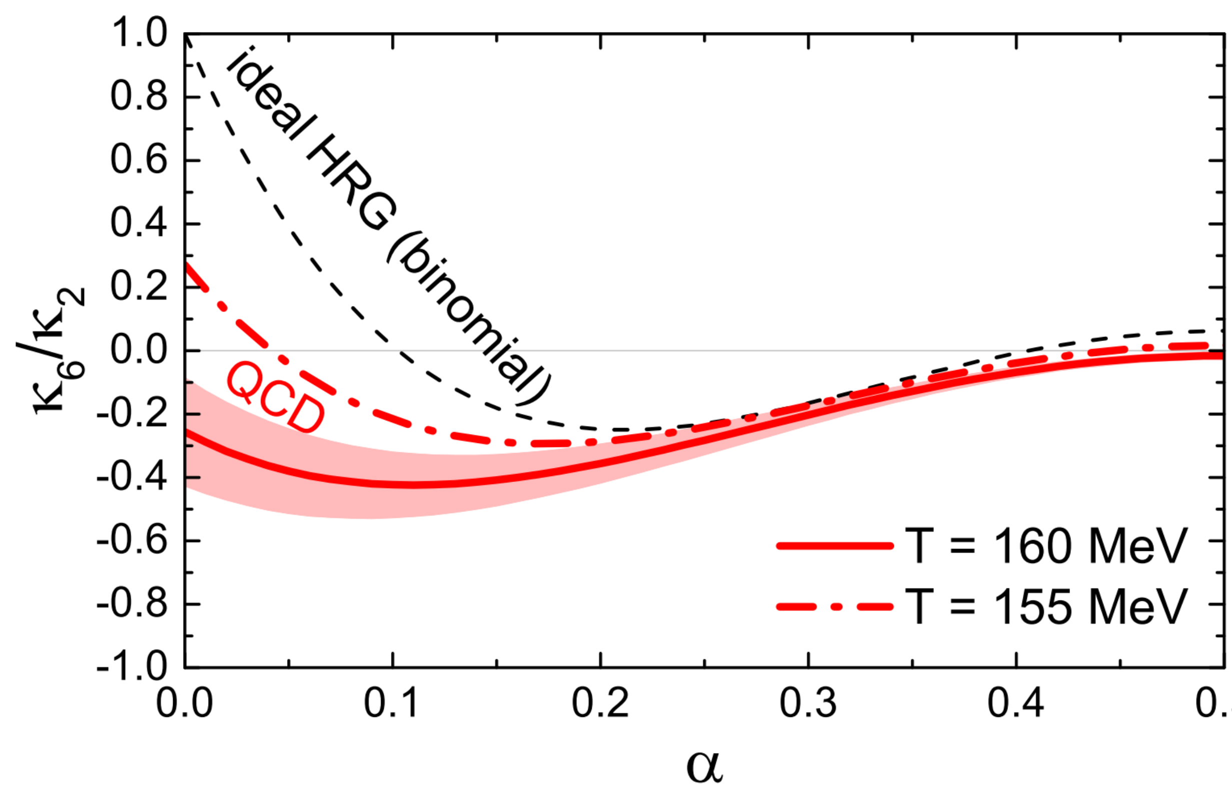
kurtosis
$$\frac{\kappa_4[B_1]}{\kappa_2[B_1]} = (1-3\alpha\beta)\,\frac{\chi_4^B}{\chi_2^B} - 3\alpha\beta\left(\frac{\chi_3^B}{\chi_2^B}\right)^2 .$$

- Global conservation (α) and equation of state (χ_n^B) effects factorize in cumulants up to the 3rd order, starting from κ_4 not anymore
- $\alpha \rightarrow 0$: Grand canonical limit
- $\alpha \rightarrow 1$: canonical limit
- $\chi_{2n} = <N> + <\bar{N}>$; $\chi_{2n+1} = <N> - <\bar{N}>$: recover known results for ideal gas

Net baryon fluctuations at LHC and top RHIC (μ_B=0)

$$\left(\frac{\kappa_4}{\kappa_2}\right)_{LHC} = (1-3\alpha\beta)\,\frac{\chi_4^B}{\chi_2^B} \qquad \left(\frac{\kappa_6}{\kappa_2}\right)_{LHC} = [1-5\alpha\beta(1-\alpha\beta)]\,\frac{\chi_6^B}{\chi_2^B} - 10\alpha(1-2\alpha)^2\beta\left(\frac{\chi_4^B}{\chi_2^B}\right)^2$$





Lattice data for χ_4^B/χ_2^B and χ_6^B/χ_2^B from Borsanyi et al., 1805.04445

For $\alpha > 0.2$ difficult to distinguish effects of the EoS and baryon conservation in χ_6^B/χ_2^B, $\alpha \leq 0.1$ is a sweet spot where measurements are mainly sensitive to the EoS

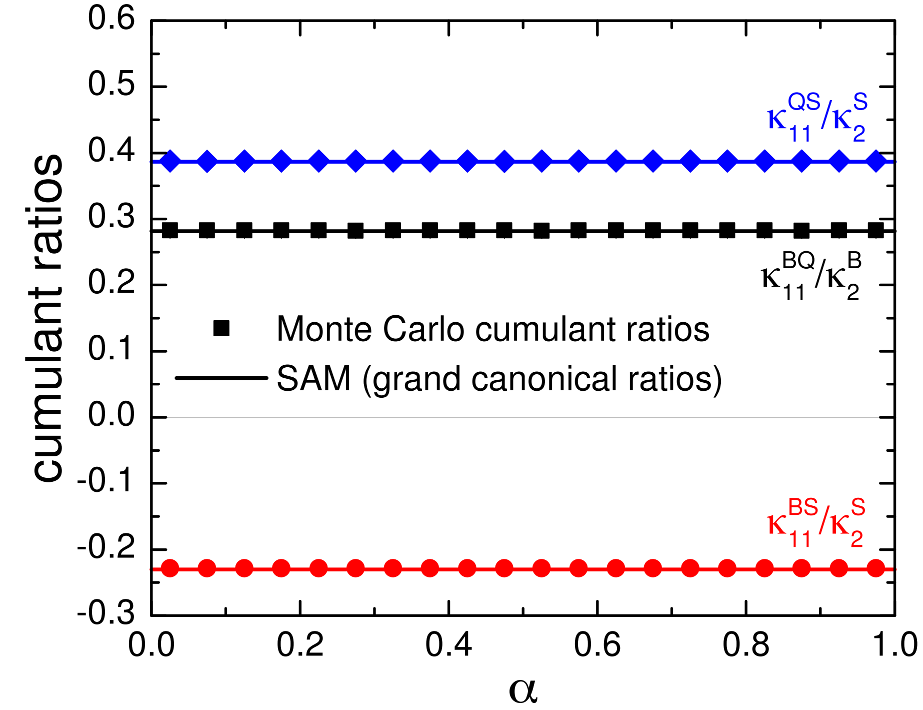
Estimates: $\alpha \approx 0.1$ corresponds to $\Delta Y_{acc} \approx 2(1)$ at LHC (RHIC)

Multiple conserved charges

(Vovchenko, R.Poberezhnyuk, V.K, arXiv:2007.03850)

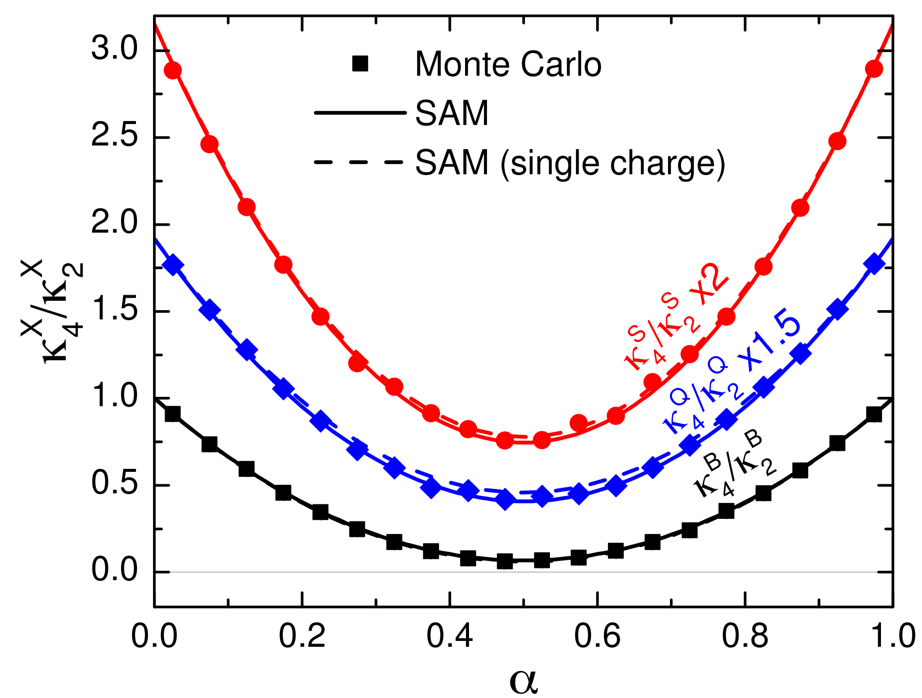
Key findings:

- Ratios of second and third order cumulants are NOT sensitive to charge conservation
  - This is also true for so called “strongly intensive quantities”
  - Requires that acceptance fraction α is the same for both particles (or Q and S)



- For order n>3 charge cumulants “mix”. Effect in HRG is tiny

$$\kappa_4[B^1] = \alpha V T^3 \beta \left[(1-3\alpha\beta)\,\chi_4^B - 3\,\alpha\,\beta\,\frac{(\chi_3^B)^2\chi_2^Q - 2\chi_{21}^{BQ}\chi_{11}^{BQ}\chi_3^B + (\chi_{21}^{BQ})^2\chi_2^B}{\chi_2^B\chi_2^Q - (\chi_{11}^{BQ})^2} \right]$$

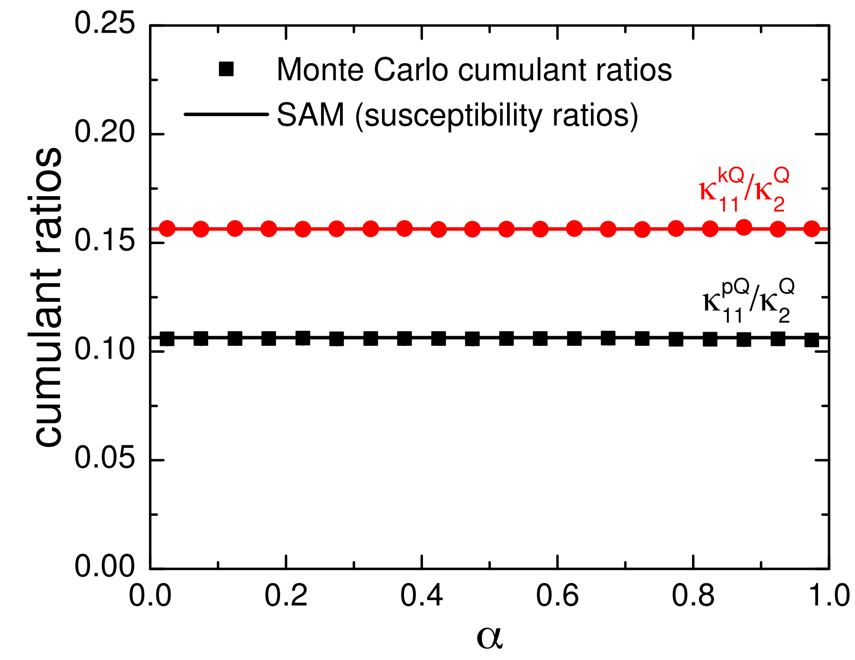


For explicit results up to order n=6, see arXiv:2007.03850

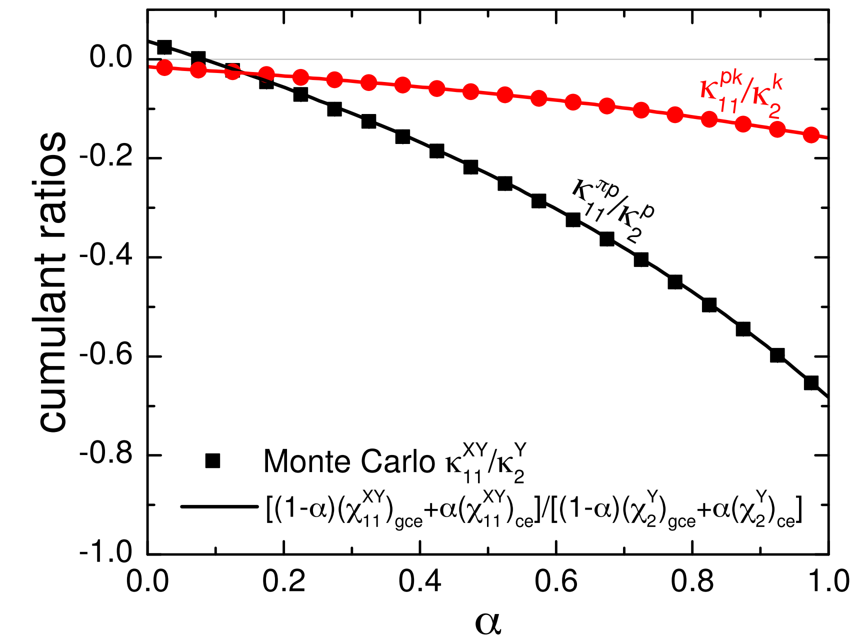
Multiple conserved charges

(Vovchenko, R.Poberezhnyuk, V.K, arXiv:2007.03850)

Also works for non-conserved quantities such as protons, K and Λ



- Mixed cumulants involving one conserved charge such as $\sigma_{1,1}^{p,Q}$ scale like second order charge cumulants
  - Again, same acceptance fraction α for both p and Q, or k and Q

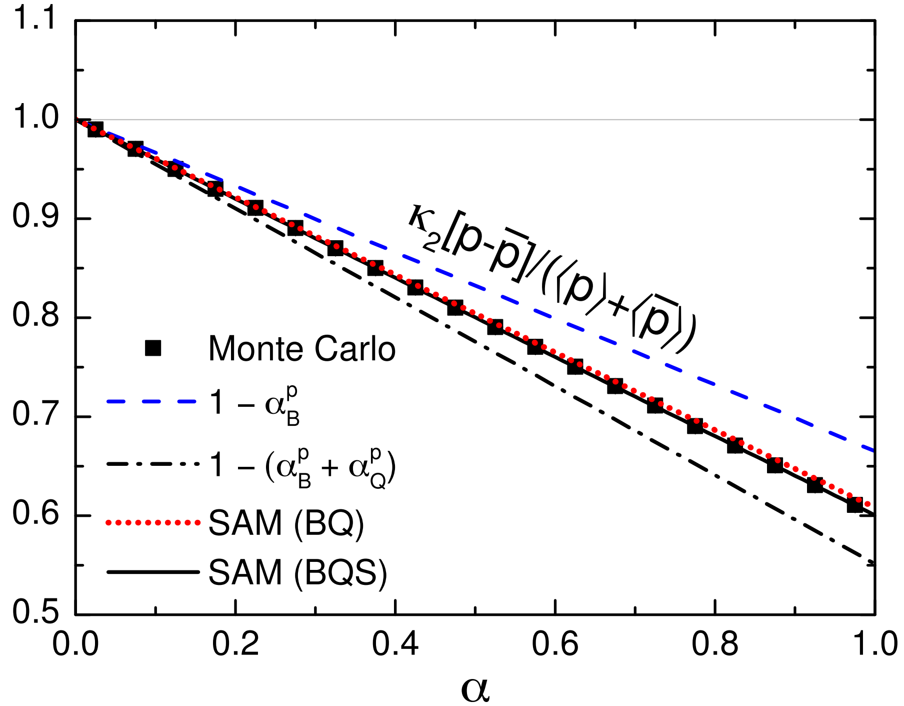


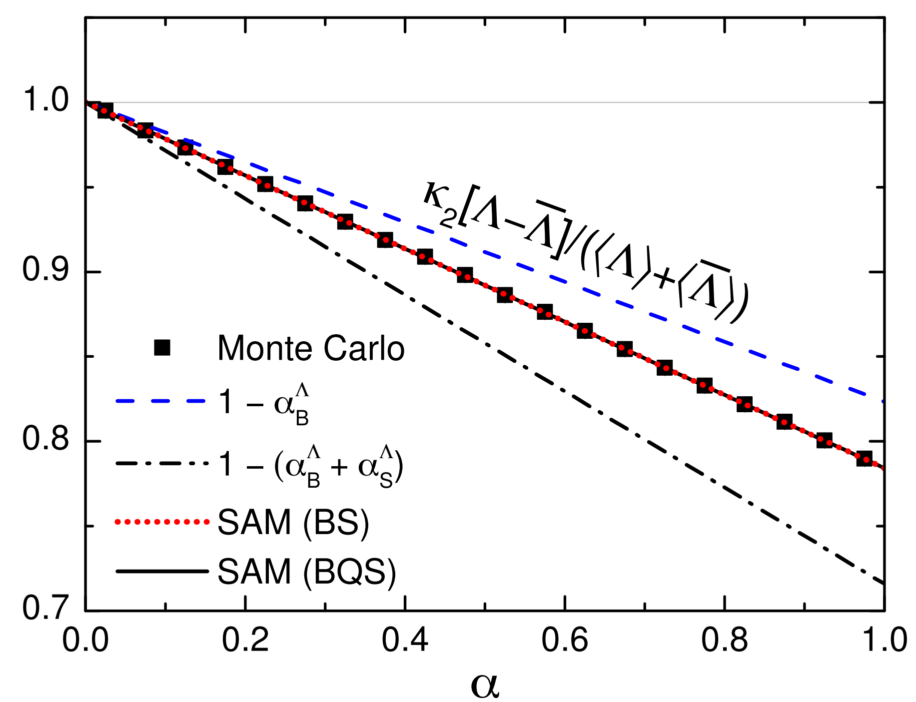
- Does NOT work for two non-conserved charges, such as $\sigma_{1,1}^{p,K}$

Multiple conserved charges

(Vovchenko, R.Poberezhnyuk, V.K, arXiv:2007.03850)

- Allows for corrections due to electric charge (protons) or strangeness (Λ) in addition to baryon number conservation.





Truth lies in between the “naive” corrections
Likely bigger effect for higher orders.

Applicability and limitations

- Argument is based on partition in ***coordinate*** space; experiments partition in ***momentum*** space
  - OK for high energies where we have Bjorken flow
    - Still corrections due to thermal smearing. Under investigation.
  - Limited applicability for lower energies
- Thermodynamic limit i.e. $V_1, V_2 \gg \xi^3$:
  - Lattice calculations work with $V_{lattice} \simeq (5\,\mathrm{fm})^3 = 125\,\mathrm{fm}^3$.
    Chemical freeze out Volume at LHC $\sim 4500\,\mathrm{fm}^3$
- Not addressed: local charge conservation

Summary

- Preliminary STAR data:
  - consistent with “Bi-Modal” distribution at 7.7 GeV
  - Can be tested RIGHT NOW by STAR via higher order factorial cumulants
  - “Final”, efficiency uncorrected multiplicity distribution does support idea of “Bi-Modal” distribution.
    - What about the revised data?

- Corrections for global (multiple) charge conservation in terms of grand canonical susceptibilities for ANY equation of state not just ideal gas
  - connection to lattice results
  - Applicable at top RHIC and LHC
  - Ratios of second and third order cumulants insensitive to conservation effects as long as acceptance fraction is the same

Thank You