Higher order cumulants of electric charge and strangeness fluctuations on the crossover line

J. Goswami[1], F. Karsch[1], S. Mukherjee[2], C. Schmidt[1] and D. Bollweg[1]
HotQCD Collaboration

[1]Bielefeld University, [2]Brookhaven National Lab

Criticality in QCD and the Hadron Resonance Gas, Wroclaw | 30.07.2020

UNIVERSITÄT BIELEFELD
Faculty of Physics

CRC-TR 211
Strong-interaction matter under extreme conditions

Outline

QCD Phase Diagram

- Smooth crossover for $\mu_B < 300$ MeV.
- Recent result from lattice QCD [arXiv:1812.08235]:

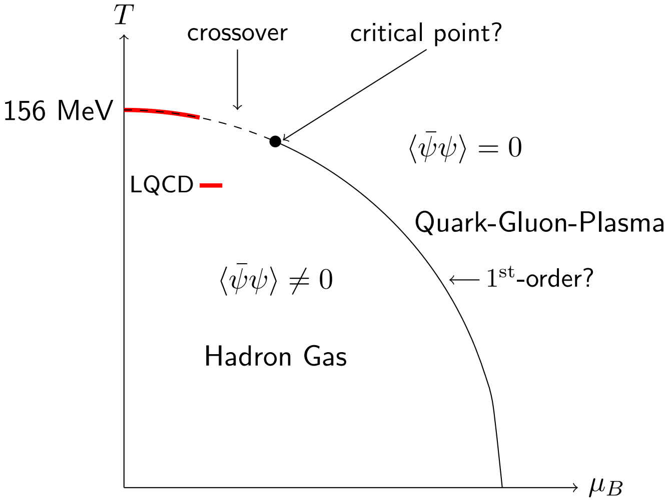
$$T_{pc}(\mu_B) = T_{pc,0}\left(1 - \kappa_2^{B,f}\bar{\mu}_B^2 - \kappa_4^{B,f}\bar{\mu}_B^4\right)$$

$$\text{with } \bar{\mu}_B = \frac{\mu_B}{T_{pc,0}}, T_{pc,0} = 156.5 \pm 1.5\ \text{MeV},$$

$$\kappa_2^{B,f} = 0.012 \pm 0.004,$$

$$\kappa_4^{B,f} = 0.000 \pm 0.004$$

- Rest of the phase diagram largely unknown.



QCD Phase Diagram

- Chiral crossover overlaps with chemical freeze-out in heavy ion collisions: $T_{cf}(\mu_B \sim 0) = 156.5$ MeV [Andronic et al. Nature 2018].
- Transition region is accessible through HIC experiments.
- Cumulants of conserved charge fluctuations are ideal probes to study phase diagram: maxima along crossover line, divergence at CEP.

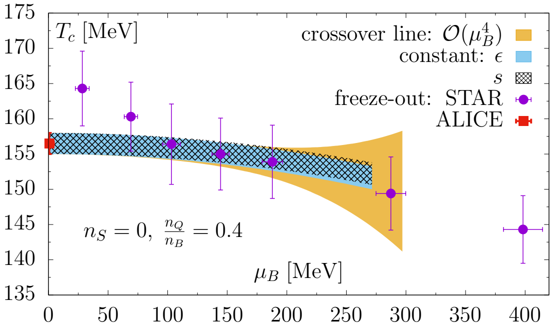


Figure: Freeze-out vs. chiral transition temperature from HotQCD [arXiv:1812.08235].

Goal: first-principle QCD predictions for cumulant ratios $M/\sigma^2, S\sigma, \kappa\sigma^2, \ldots$ etc.

Fluctuations via lattice QCD

$M/\sigma^2, S\sigma, \kappa\sigma^2$ are accessible via *generalized susceptibilities* χ:

$$\chi_{ijk}^{BQS} \equiv \frac{1}{VT^3} \frac{\partial^{i+j+k} \log \mathcal{Z}}{\partial \hat{\mu}_B^i \partial \hat{\mu}_Q^j \partial \hat{\mu}_S^k}, \quad \hat{\mu}_X \equiv \frac{\mu_X}{T}$$

$$M_X/\sigma_X^2 = \frac{\chi_1^X}{\chi_2^X}, \; S_X\sigma_X = \frac{\chi_3^X}{\chi_2^X}, \; \kappa_X\sigma_X^2 = \frac{\chi_4^X}{\chi_2^X} \; \text{with} \; X = B, Q, S$$

- Finite-density sign problem renders direct simulations at $\mu_B > 0$ impossible.
- Use constrained Tayor-Expansion in μ to access cumulants at finite density.
- Impose strangeness neutrality $n_S = 0$ and $\frac{n_Q}{n_B} = 0.4$ order by order - corresponds to thermal conditions in HIC (e.g. Pb + Pb or Au + Au).

Fluctuations via lattice QCD - Setup

- Dynamical Fermions (HISQ) with $N_f = 2+1$: two light Quarks (up + down) and a strange Quark with mass ratio $\frac{m_s}{m_l} = 27$. $\Rightarrow$ physical meson masses in the continuum limit!
- Lattice sizes $32^3 \times 8$, $48^3 \times 12$ and $64^3 \times 16$ at 9 different temperatures each.
- Large simulation campaign on Summit in 2019 & 2020: Compared to our earlier analysis of baryon skewness and kurtosis [arXiv:1708.04897v3] we increased statistics in the vicinity of T_{pc} on $N_t = 8$ lattices by a factor 3-4 and on $N_t = 12$ lattices by a factor 6-8.

	$N_t = 8$	$N_t = 12$	$N_t = 16$
No. of Conf.	$1.2 \cdot 10^6$	$2 - 4 \cdot 10^5$	10^4

- High statistics data enable us to calculate cumulants up to $\mathrm{N^3LO}$ in μ_B (previous studies: NLO).
- **All data on fluctuations in this presentation are HotQCD preliminary.**

- Cumulant ratios are scanned in μ_B

$$R^X_{nm}(T,\mu_B) = \frac{\sum_i \frac{1}{i!}\chi_n^{X,i}\left(\frac{\mu_B}{T}\right)^i}{\sum_j \frac{1}{j!}\chi_m^{X,j}\left(\frac{\mu_B}{T}\right)^j}$$

- Results like Fig. 2 for each N_t are jointly fitted assuming $1/N_t^2$ corrections → continuum extrapolation.
- The fitted surface can then be evaluated along arbitrary lines in (T,μ_B) if desired. In the following: $T_{\rm pc}(\mu_B)$

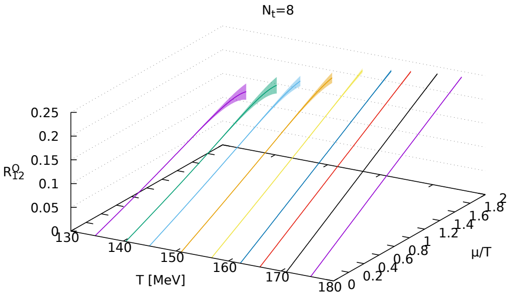
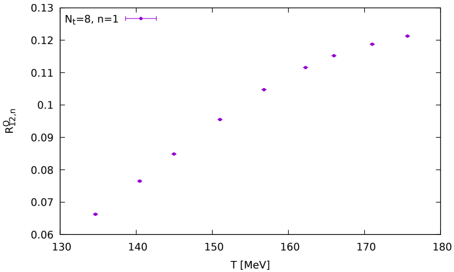
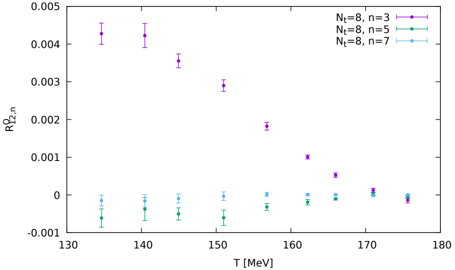


Figure: R_{12}^Q for $N_t = 8$ scanned in μ_B/T.

Electric charge fluctuations - R_{12}^Q





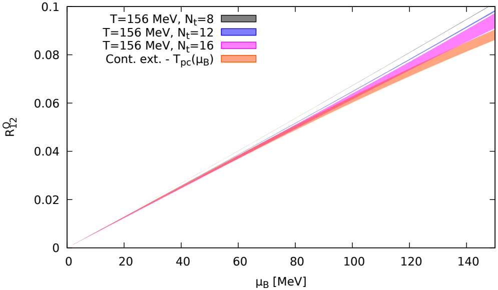


Figure: Left: Contributions to R_{12}^Q sorted by order in μ_B. Right: R_{12}^Q along $T_{\rm pc}(\mu_B)$.

Electric charge fluctuations - R_{12}^{Q}

- R_{12}^{Q} dominated by leading order contribution $\sim \mu_B$.
- NLO contributions smaller by an order of magnitude.
- Mild temperature dependence.
- Ideal for extracting freeze-out chemical potential $\mu_{B,f} \Rightarrow$ "Baryometer"

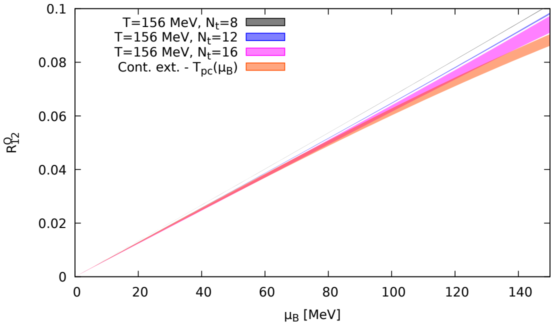


Figure: R_{12}^{Q} for different N_t and continuum extrapolation along $T_{pc}(\mu_B)$.

Electric charge fluctuations - R_{12}^Q

Freeze-out chemical potential extracted by comparing to data from STAR[1]:

$\sqrt{s_{NN}}$ [GeV]	$\mu_{B,f}$ [MeV]
200	19.4(1)
62.4	58(1)
39	92(2)
27	131(3)

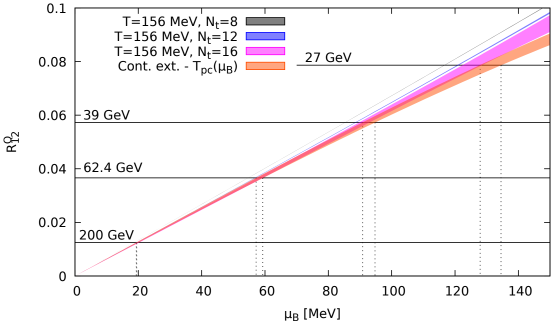


Figure: R_{12}^Q for different N_t and continuum extrapolation along $T_{pc}(\mu_B)$.

[1] [PRL 113, 092301 (2014)]

Electric charge fluctuations - R_{31}^{Q}

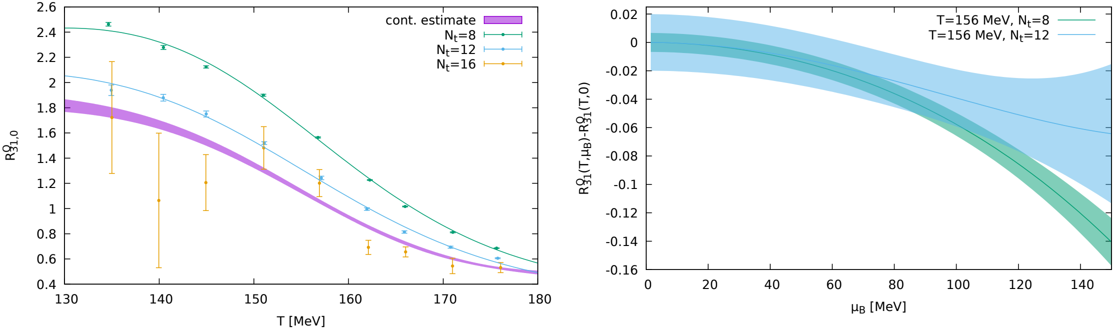


Figure: Left: R_{31}^{Q} at $\mu_B = 0$. Right: μ_B dependence around T_{pc}.

- Strong temperature dependence/ weak μ_B dependence $\Rightarrow$ “Thermometer”

Electric charge fluctuations - R_{31}^Q

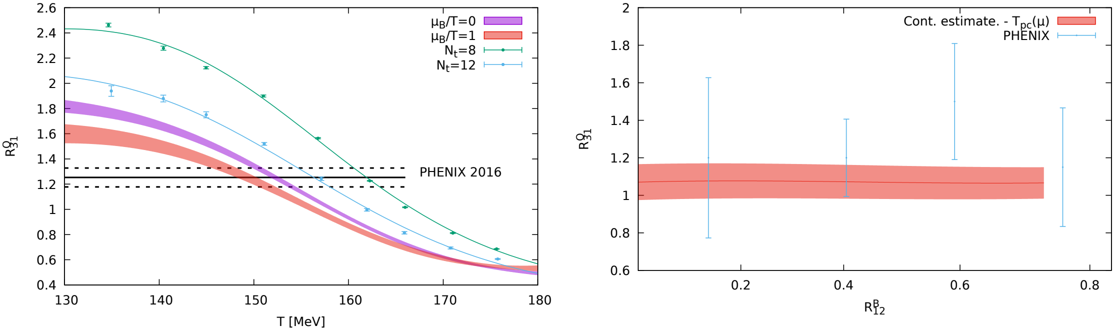


Figure: Cont. estimate of $R_{31}^Q(T_{\rm pc}(\mu_B))$.

- Lattice QCD estimate: $R_{31}^Q(T_{\rm pc}(\mu_B)) = 1.07(9)$.
- PHENIX[1] Measurements of R_{31}^Q consistent with freeze-out at T_{pc}.
- Note: published PHENIX data use $N_T = 8$ lattice results $\rightarrow$ too high T_f.

[1] [Phys. Rev. C 93, 011901(R) (2016)]

Electric charge fluctuations - R_{31}^{Q}

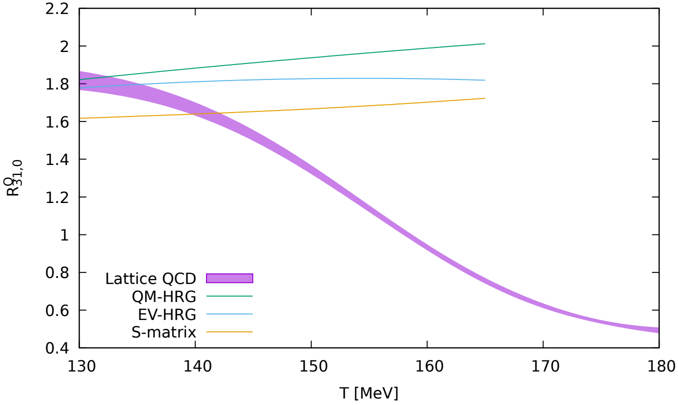
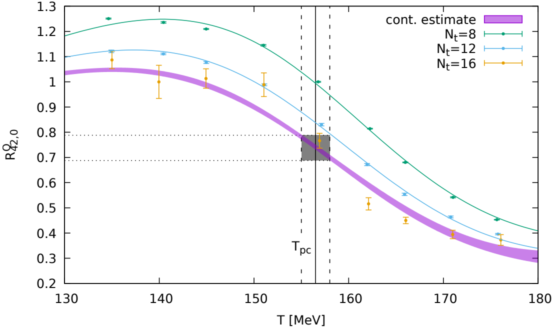


Figure: R_{31}^{Q} via lattice QCD vs. different HRG calculations.

- R_{31}^{Q} shows large deviations from HRG in the transition region.
- T-dependence of $R_{31}^{Q}(T, \mu_B = 0)$ is not captured by any of the non-interacting HRG models

Electric charge fluctuations - R_{42}^Q



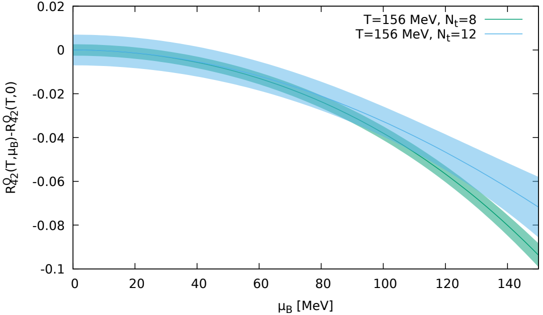


Figure: Left: R_{42}^Q at $\mu_B = 0$. Right: μ_B dependence around T_{pc}.

- Significantly smaller errors compared to R_{31}^Q since noisy baryon correlations do not contribute to LO. LQCD Estimate: $R_{42}^Q(T_{pc}, 0) = 0.73(5)$.
- Avg. over PHENIX data: $R_{42}^Q = 1.29(6)$ inconsistent with lattice results.

Strangeness fluctuations - $\mu_{S,f}$

- Constraint $n_S(\mu_B, \mu_S) \overset{!}{=} 0$ determines μ_S:

$$\frac{\mu_S}{\mu_B} = s_1(T) + s_3(T)\left(\frac{\mu_B}{T}\right)^2 + \mathcal{O}\left(\left(\frac{\mu_B}{T}\right)^4\right).$$

- $s_n(T)$: Consists of combinations of χ_{ijk}^{BQS} directly accessible in LQCD.
- Main contribution to $s_1(T)$ comes from $\frac{\chi_{11}^{BS}}{\chi_2^S}$.
- s_n with $n \geq 3$ almost negligible.
- QCD result on μ_S/μ_B sensitive to strangeness content (in a HRG model).
- Excellent match with QM-HRG.

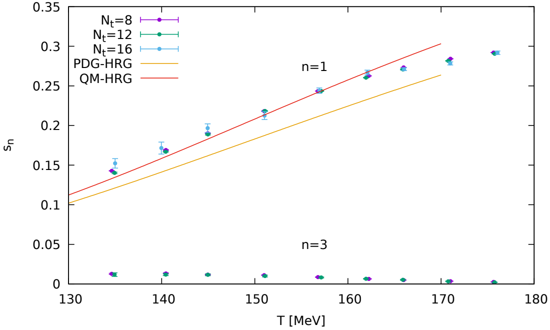


Figure: $s_1(T)$ and $s_3(T)$ from lattice QCD and HRG.

CRC-TR 211

Lattice QCD:

- Constraint $n_S(\mu_B, \mu_S) \stackrel{!}{=} 0$ determines μ_S:

$$\frac{\mu_S}{\mu_B} = s_1(T) + s_3(T)\left(\frac{\mu_B}{T}\right)^2 + \mathcal{O}\left(\left(\frac{\mu_B}{T}\right)^4\right).$$

- $s_n(T)$: Consists of combinations of χ_{ijk}^{BQS} directly accessible in LQCD.
- Main contribution to $s_1(T)$ comes from $\frac{\chi_{11}^{BS}}{\chi_2^S}$.
- s_n with $n \geq 3$ almost negligible.
- QCD result on μ_S/μ_B sensitive to strangeness content (in a HRG model).
- Excellent match with QM-HRG.

Heavy Ion Collisions:

- HRG relation for $\bar{B}$ to B yields can be used:

$$\frac{\bar{B}}{B}(\sqrt{s}) = \exp\left(-\frac{\mu_B}{T}(2 - 2|S|\frac{\mu_S}{\mu_B})\right)$$

- $\frac{\mu_S}{\mu_B}$ obtainable by fitting yields for different particle species in $|S|$.

Strangeness fluctuations

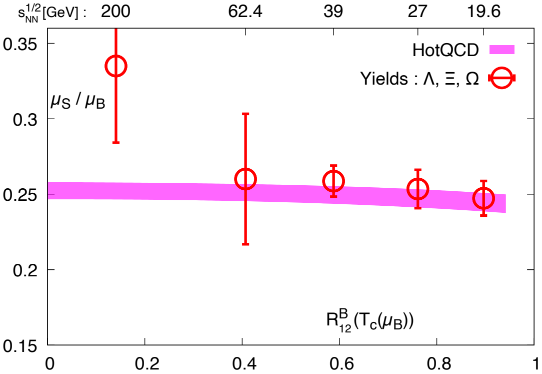


Figure: μ_S/μ_B along $T_{pc}(\mu_B)$ vs. μ_S/μ_B extracted from STAR[1]

- $\mu_{S,f}/\mu_{B,f}$ from strange baryon yields is consistent with lattice QCD results at T_{pc}!

[1] arXiv:1010.0142 & arXiv:1906.03732

Strangeness fluctuations - R_{32}^S and R_{42}^S

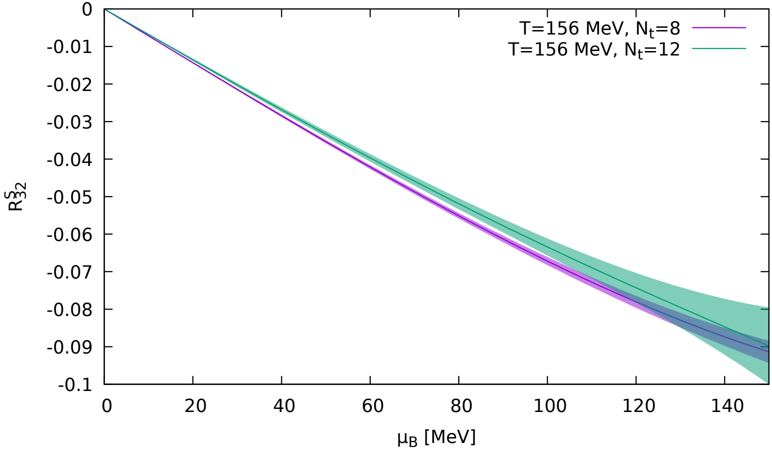


Figure: $R_{32}^S(T_{\mathrm{pc}}, \mu_B)$ for $N_t = 8, 12$ lattices.

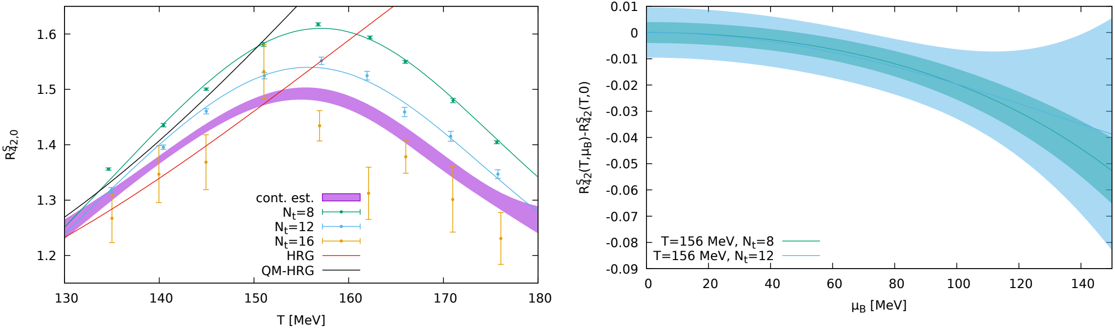


Figure: Left: R^S_{42} at $\mu_B = 0$. Right: μ_B-dependence around $T_{\rm pc}$.

Summary

- Precise calculations of $R_{12}^Q(T_{pc}(\mu_B))$ to NNNLO in μ_B enabled determination of $\mu_{B,f}$ from Event-by-Event fluctuation data by STAR.
- PHENIX data on R_{31}^Q are consistent with $T_f \sim T_{pc}$ when comparing to NNLO lattice QCD continuum estimates.
- Non-interacting HRG models are not suitable to describe thermodynamics of higher order electric charge fluctuations.
- μ_S/μ_B lattice QCD results are well described by QM-HRG which justifies extraction of $\mu_{S,f}$ from experimentally measured strange baryon yields.
- Apart from R_{42}^Q, electric charge and strangeness results are consistent with freeze-out at $T_{pc}(\mu_B)$.