Conserved charge fluctuations in the chiral limit

Mugdha Sarkar
(HotQCD Collaboration)

Criticality in QCD and the Hadron Resonance Gas

UNIVERSITÄT BIELEFELD
Fakultät für Physik

CRC–TR 211
Strong-interaction matter under extreme conditions

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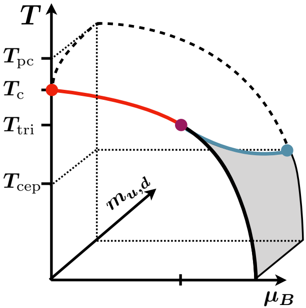
Chiral phase transition in the limit of vanishing up and down quark masses



[F. Karsch, arxiv:1905.03936]

Expected to belong to the **universality class** of $3d\ O(4)$ spin model

[R.D. Pisarski, F. Wilczek, PRD 29 338 (1984)]

- Determination of the order and nature of the chiral phase transition
- Imprint of the criticality on the thermodynamics at physical quark masses

$T_c = 132^{+3}_{-6}$ MeV

Anirban's & Christian's talk

$T_{pc} = 156.5(1.5)$ MeV

[HotQCD, PRL 123, 062002 (2019)]

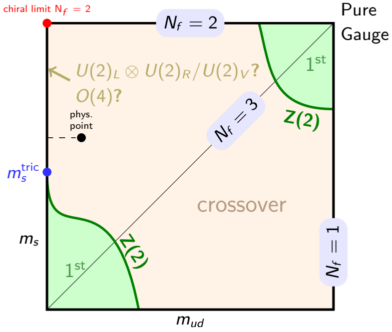
[HotQCD, PLB 795 (2019) 15–21]

Exploring the chiral phase transition

- measurements towards the chiral limit with strange mass fixed at physical value

- investigate behavior of conserved charge fluctuations, which are also measurable in HIC, in the chiral limit

- alternative scenario : $3d$ Ising universality transition at non-zero mass, with a first-order chiral limit transition



[O. Philipsen and C. Pinke, PRD93, 114507, 2016]

Lattice Setup

- Gauge ensembles generated with HISQ fermion discretization and Symanzik-improved gauge action, used in chiral T_c determination [HotQCD, arxiv:1905.11610].
- Ensembles for smaller-than-physical quark (up, down) masses $m_l = ms/27, m_s/40, m_s/80, m_s/160$, keeping strange quark mass m_s *fixed at physical value*.
- Corresp. pion masses : 140 MeV, 110 MeV, 80 MeV, 55 MeV
- Thermodynamic and continuum limit not yet performed. Measurements done at the largest simulated volumes for each mass at fixed time extent $N_\tau = 8$.
- Computing resources : Jülich, Piz Daint, JLAB, Bielefeld and Wuhan supercomputing facilities.

Critical behavior of thermodynamic quantities I

Analysis of QCD thermodynamics in the vicinity of the chiral phase transition, acc. to Wilson's RG approach

infinite volume

$$\frac{p}{T^4} = \frac{1}{VT^3}\ln Z(T,\vec{\mu},m_l) = \boldsymbol{h^{(2-\alpha)/\beta\delta} f_f(z)} + f_r(T,\vec{\mu},m_l), \ \boldsymbol{z \equiv t/h^{1/\beta\delta}}$$

singular **regular** $\longrightarrow$ *no linear m_l dependence*

$$t = \frac{1}{t_0}\left(\frac{T-T_c}{T_c} + \kappa_2^X \left(\frac{\mu_X}{T}\right)^2\right)$$

"energy-like" coupling

$$h = \frac{1}{h_0}\frac{m_l}{m_s}$$

X=B,Q,S

"magnetic-like" coupling

Chiral phase transition at
$\boldsymbol{m_l \equiv m_u = m_d = 0 \ (h = 0)}$
$\boldsymbol{T = T_c \ (t = 0)}$ at $\mu = 0$

critical exponents

	α	β	δ
$O(4)$	-0.21	0.38	4.82
$O(2)$	-0.017	0.349	4.78
$Z(2)$	+0.109	0.325	4.8

Critical behavior of thermodynamic quantities II

Conserved charge fluctuations are energy-like w.r.t to chiral phase transition (also Polyakov loop, see David's talk)	(Anirban's talk) **magnetic-like**	**mixed**	**energy-like**
	$\frac{\partial^2 \ln Z}{\partial h^2}$	$\frac{\partial^2 \ln Z}{\partial h \partial t}$	$\frac{\partial^2 \ln Z}{\partial t^2}$
	$\sim h^{1/\delta - 1}$	$\sim h^{(\beta-1)/\beta\delta}$	$\sim h^{-\alpha/\beta\delta}$
	$\sim h^{-0.79}$	$\sim h^{-0.34}$	$\sim h^{+0.11}$
divergence :	**strong**	**moderate**	**vanishes**

Conserved charge fluctuations at $\mu = 0$ (Singular part) :

$$\chi_{2n}^{X} = - \frac{\partial^{2n} f/T^4}{\partial(\mu_X/T)^{2n}}\bigg|_{\mu_X=0} \sim \; - (2\kappa_2^X)^n \, h^{(2-\alpha-n)/\beta\delta} f_f^{(n)}(z)$$

Scaling Expectation : $\frac{\partial}{\partial T} \sim \kappa_2^X \frac{\partial^2}{\partial \mu^2}$

$\Rightarrow \chi_2 \sim$ Energy density, $\chi_4 \sim$ Specific heat

Scaling at physical quark masses

➪ Scaling expectation for singular part : $\frac{\partial}{\partial T} \sim \kappa \frac{\partial^2}{\partial \mu^2}$.

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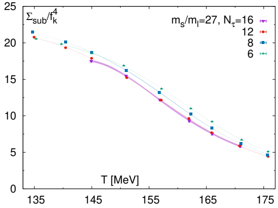

Chiral condensate

[HotQCD, PLB 795 (2019) 15]

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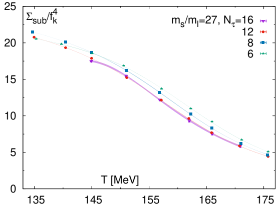
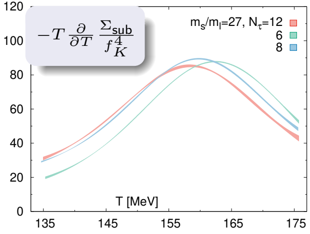


Chiral condensate

$$\frac{\partial}{\partial T}$$



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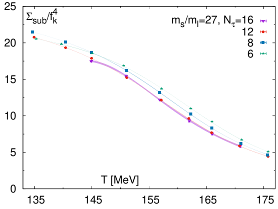

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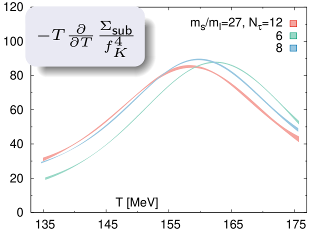
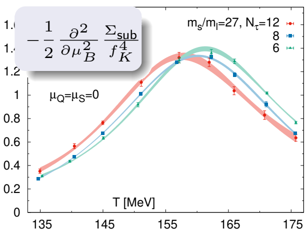
$\frac{\partial}{\partial T}$

$\frac{\partial^2}{\partial \mu^2}$





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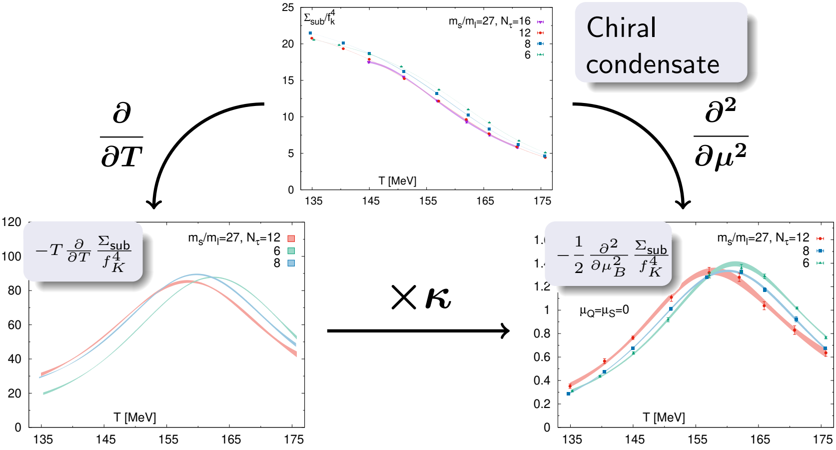

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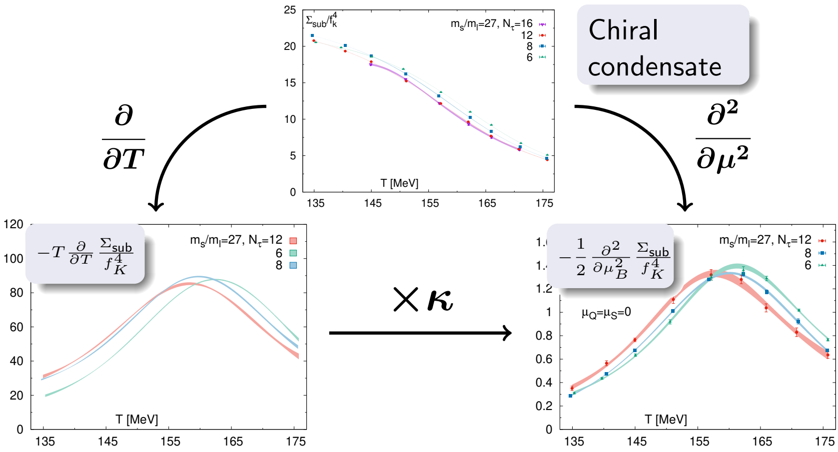

[HotQCD, PLB 795 (2019) 15]

Scaling expectation already holds at physical quark masses

Scaling expectation for χ_2^Q

- Scaling expectation if singular part dominates : $\frac{\partial}{\partial T} \sim \kappa \frac{\partial^2}{\partial \mu^2}$.

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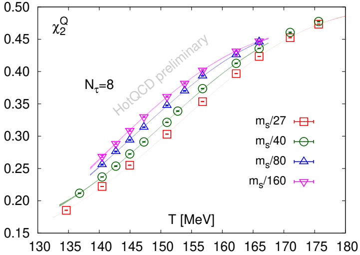

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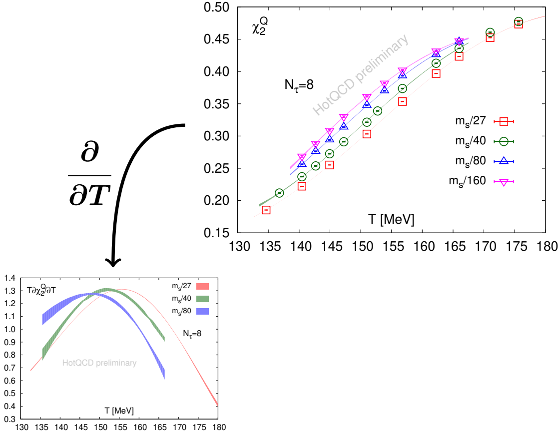

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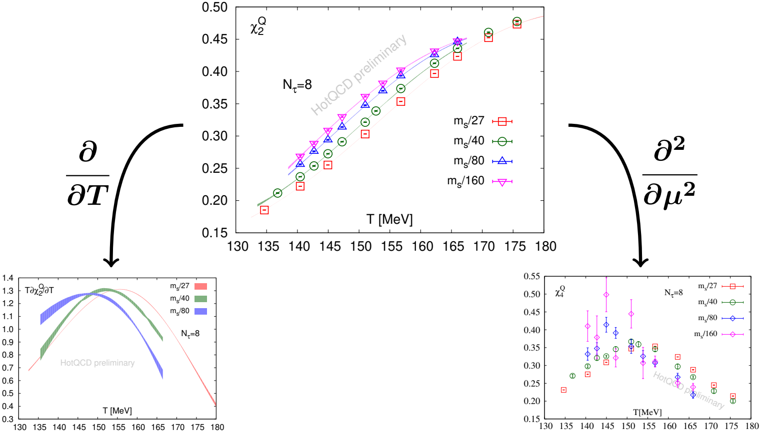

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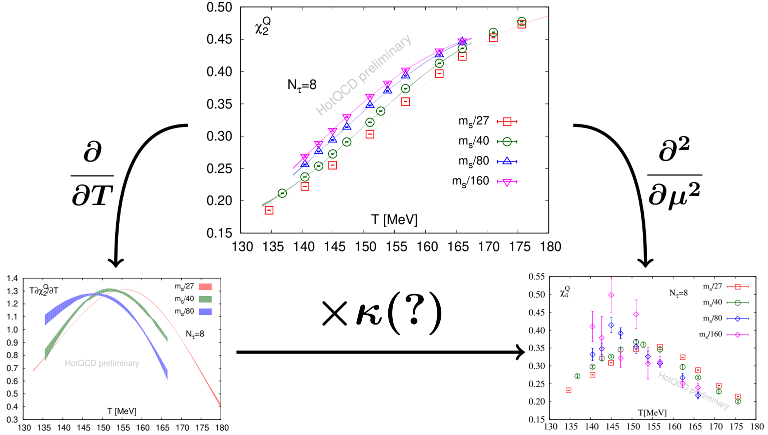

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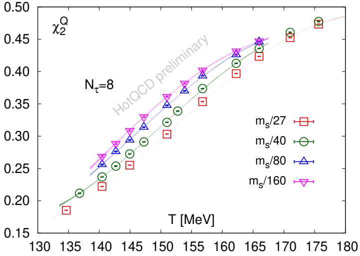

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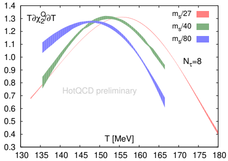
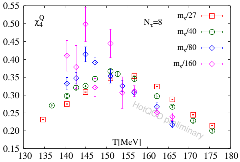


$\frac{\partial^2}{\partial \mu^2}$



$\times \kappa(?)$

Singular term in χ_2 (energy-like) is not dominant unlike in order parameter

Estimation of the singular contribution to χ_2^B

$$\chi_2^X(T_c, m_l) \sim -\kappa_2^X h^{(1-\alpha)/\beta\delta} f_f^{(1)}(0) + \text{const. reg. term} + \mathcal{O}(m_l^2)$$

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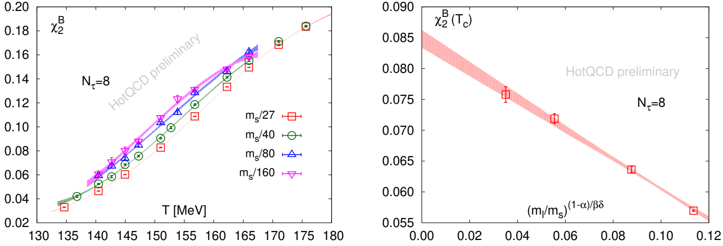

$$\chi_2^X(T_c, m_l = 0) - \chi_2^X(T_c, m_l = m_s/27) = \text{Singular part of } \chi_2^X$$

Linear in H plot

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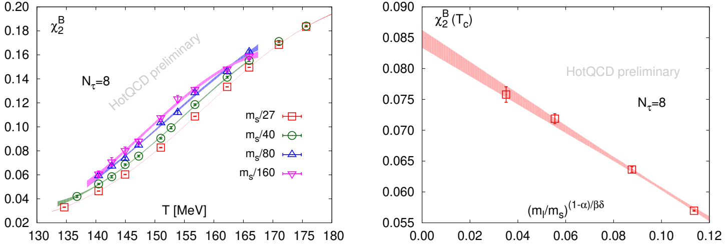

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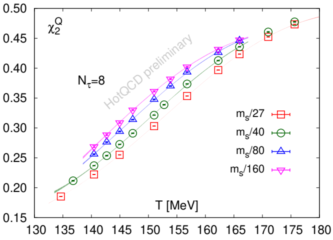
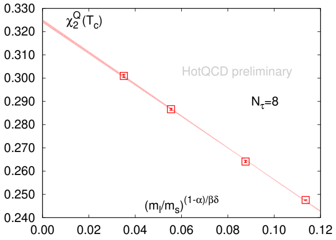
Singular contribution to χ_2^B at physical masses $\sim$ 50%

Linear in H plot

Scaling behavior of χ_2^Q

$$\chi_2^X(T_c, m_l) \sim -\kappa_2^X h^{(1-\alpha)/\beta\delta} f_f^{(1)}(0) + \text{const. reg. term} + \mathcal{O}(m_l^2)$$





$$\frac{\text{Singular part of } \chi_2^Q}{\text{Singular part of } \chi_2^B} = \frac{\kappa_2^Q}{\kappa_2^B} \sim 2.6$$

Close to result for physical mass :

$$1.81 \pm 0.40$$

[HotQCD, Phys. Lett. B 795 (2019) 15]

Singular contribution to χ_2^Q at physical masses $\sim 30\%$

Linear in H plot

Scaling behavior of χ_2^Q

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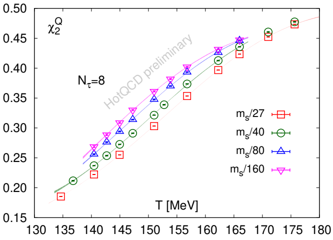

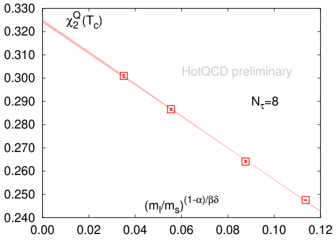

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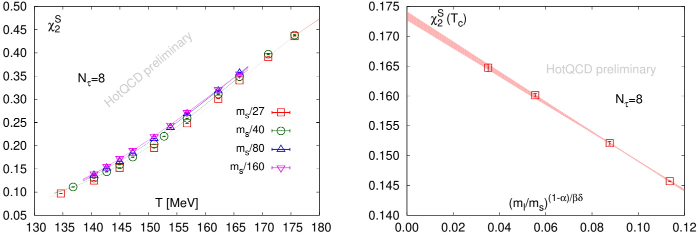


Singular contributions in χ_2^B and χ_2^S are similar.

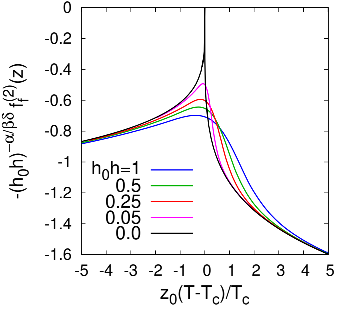
Consistent with previous result for physical mass

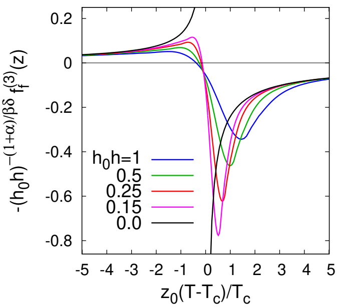
[HotQCD, Phys. Lett. B 795 (2019) 15]

Singular contribution to χ_2^S at physical masses $\sim$ 20%

Derivatives of the singular part for $3d\ O(4)$ universality class

$h^{0.116} f_f^{(2)}$ $h^{-0.429} f_f^{(3)}$





[B. Friman, F. Karsch, K. Redlich,V. Skokov, Eur. Phys. J. C (2011) 71:1694]

$\sim \chi_4$ $\sim \chi_6$

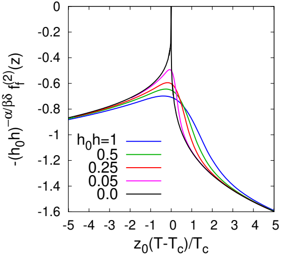
Singular part :

$$\chi_4^Q \sim h^{-\alpha/\beta\delta} f_f^{(2)}(z)$$

$$O(4) : -\alpha/\beta\delta = 0.116$$

Not divergent

– but pronounced spike



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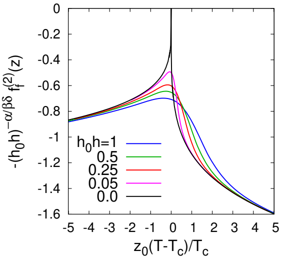
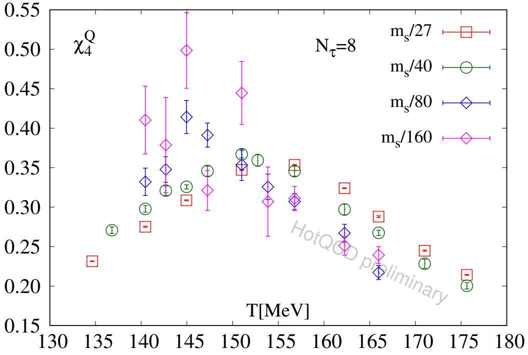


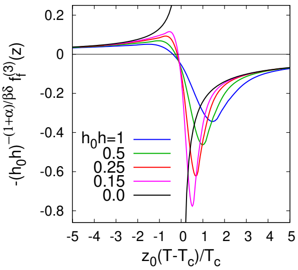


➪ Expected features are apparent for $m_l = m_s/27, m_s/40$

Singular part :

$$\chi_6^Q \sim h^{-(1+\alpha)/\beta\delta} f_f^{(3)}(z)$$
$$\sim h^{-0.429}$$

Moderate divergence



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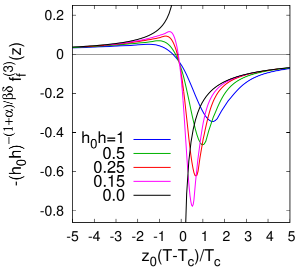

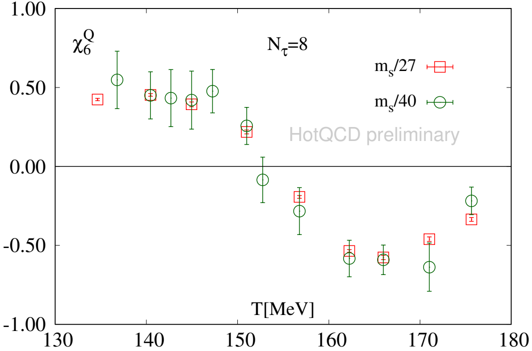

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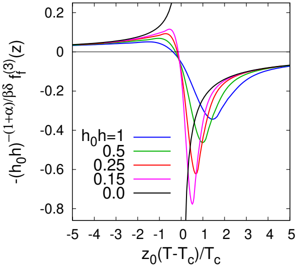
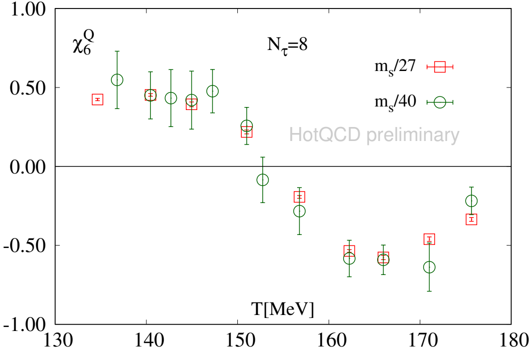




Ratio of peak heights expected from scaling : $(\chi_6^Q)^{max}_{1/40}/(\chi_6^Q)^{max}_{1/27} \sim 1.18$

H-derivatives of energy-like observables

$$\chi_2^S \sim -\kappa_2^S H^{(1-\alpha)/\beta\delta} f_f'(z) + f_{\text{reg}}$$

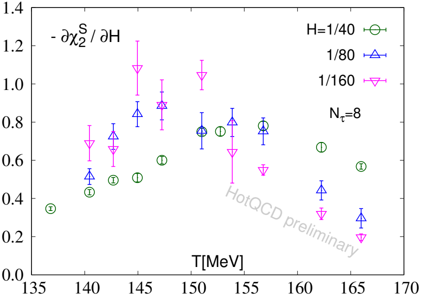
$$H = \frac{m_l}{m_s}$$

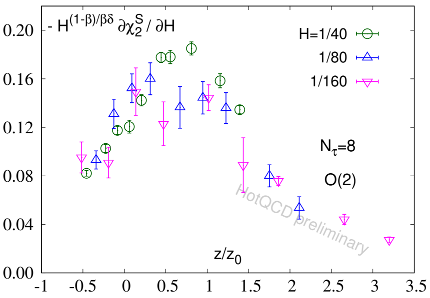
$$z \equiv z_0 \frac{t}{H^{1/\beta\delta}}$$

$$\frac{\partial \chi_2^S}{\partial H} \sim \kappa_2^S H^{(\beta-1)/\beta\delta} f_G'(z) + \frac{\partial f_{\text{reg}}}{\partial H}$$

$$\frac{\beta-1}{\beta\delta} = -0.39, O(2)$$

Divergent already for 2nd order





Dominant singular part

Strange chiral condensate $\Rightarrow$ *energy-like* observable

$$\langle\bar{\psi}\psi\rangle_s = AH^{(1-\alpha)/\beta\delta} f_f'(z) + f_{\rm reg}$$

$$H = \frac{m_l}{m_s}$$

$$\chi_{ls} \equiv \frac{\partial\langle\bar{\psi}\psi\rangle_s}{\partial H} = -AH^{(\beta-1)/\beta\delta} f_G'(z) + \frac{\partial f_{\rm reg}}{\partial H}$$

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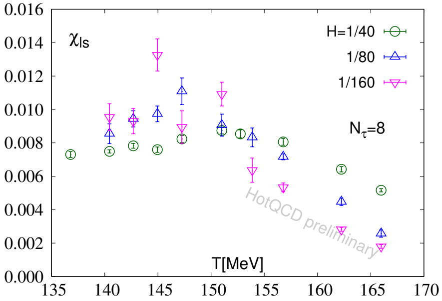
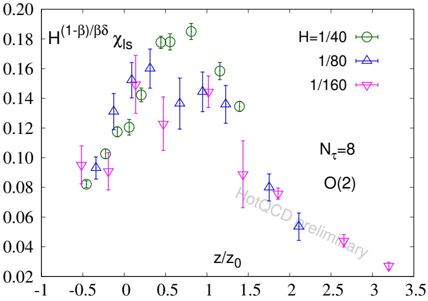
Divergent at 2nd order





Similar divergence, different regular contribution

Conclusions and Outlook

- Fluctuations of conserved charges seem consistent with chiral phase transition belonging to $O(4)$ or $O(2)$ universality class. Expected energy-like behavior of fluctuations w.r.t. chiral phase transition.
- Singular part can be extracted from χ_2^X and may be used to determine the curvature coefficients of the chiral critical line. Singular contribution to 2nd order fluctuations roughly $\leq 50\%$
- Strange fluctuations and strange quark condensate behaves as energy-like quantities in the 2-flavour chiral limit.
- Future comparison with HRG with smaller masses obtained from ChPT.
- Requires more statistics at lower masses and proper continuum and thermodynamic limits.

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Thank you for your attention

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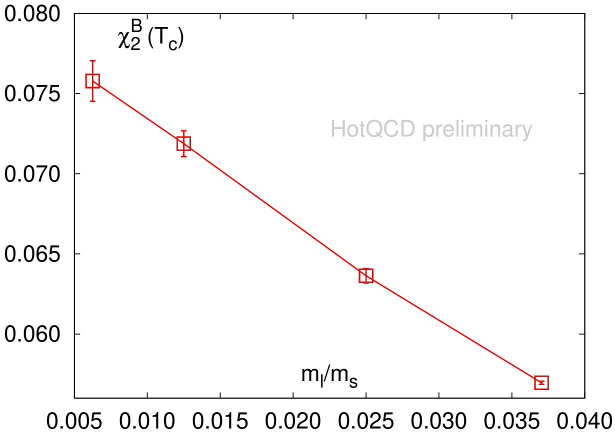
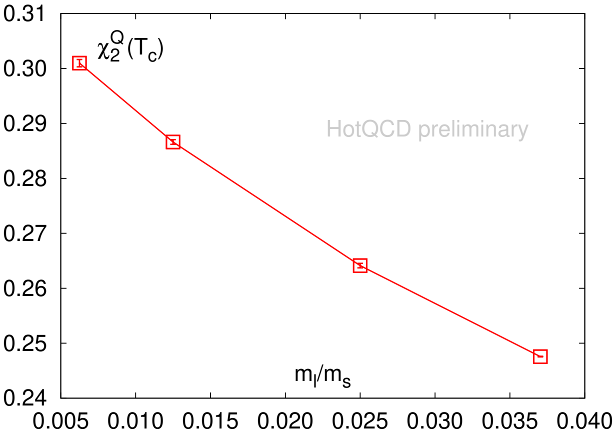


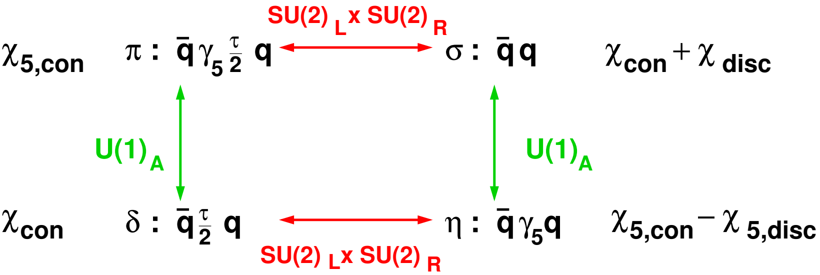
Points are connected to guide the eye. Lines are NOT fits. Go Back

Backup



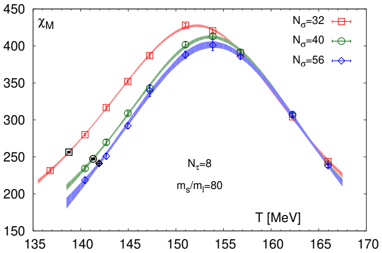
Points are connected to guide the eye. Lines are NOT fits. Go Back

Symmetry transformations



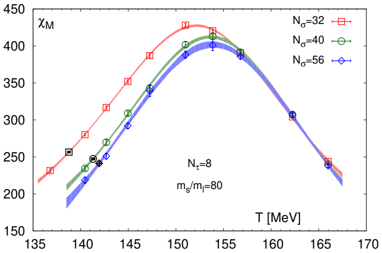
$O(4)$ criticality : Bounds on first order transition

[HotQCD, arxiv:1905.11610]



- No indications of first order volume scaling at $m_\pi = 80$ MeV
- Consistent with $O(4)$ volume scaling at non-zero H

[HotQCD, arxiv:1905.11610]



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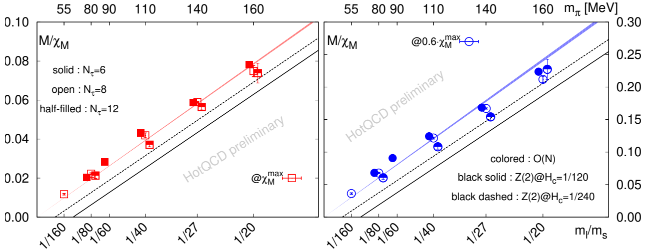


$$\frac{M(T_X)}{\chi_M(T_X)} = (H - H_c)\frac{f_G(z_X)}{f_\chi(z_X)}$$

Seems no indication of a 1st order transition till $m_\pi \sim 46$ MeV

[H.T. Ding et al, Nucl. Phys. A 982 (2019) 211]

$O(4)$ criticality : Chiral T_c estimation

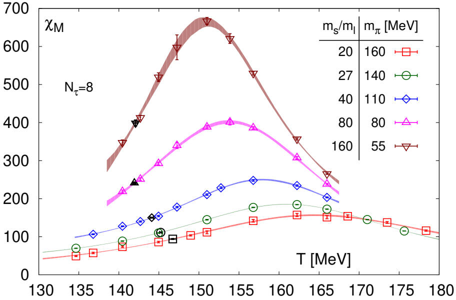
Chiral phase transition at the chiral limit extracted by $O(4)$ scaling arguments.

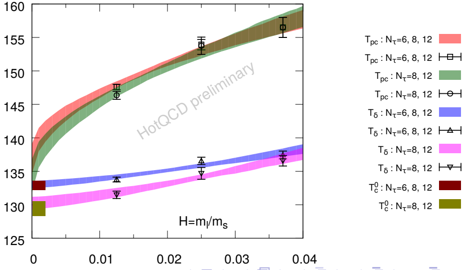
$$T_X(H) = T_c \left(1 + \frac{z_X}{z_0} H^{1/\beta\delta}\right)$$

$$X = \delta, 60, pc$$

$$\frac{H \chi_M(T_\delta, H)}{M(T_\delta, H)} = \frac{1}{\delta}$$

$$\chi_M(T_{60}, H) = 0.6 \chi_M^{max}$$





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$$\chi_M(T_{60}, H) = 0.6\chi_M^{max}$$

$$\mathbf{T_c = 132^{+3}_{-6}}\ \text{MeV}$$

[HotQCD, arxiv:1905.11610]