Nuclear Science Computing Center at CCNU

QCD phase structure in a background magnetic field

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Work done in collaboration with **Sheng-Tai Li**, Swagato Mukherjee, Qi Shi, Akio Tomiya, **Xiao-Dan Wang**, Yu Zhang

Workshop on criticality in QCD and the Hadron Resonance Gas

29-31 July, 2020

Outline

- Chiral properties of QCD at T=0 and B=/=0

HTD, S.-T. Li, A. Tomiya, X.-D. Wang & Y. Zhang, arXiv:2008.00493

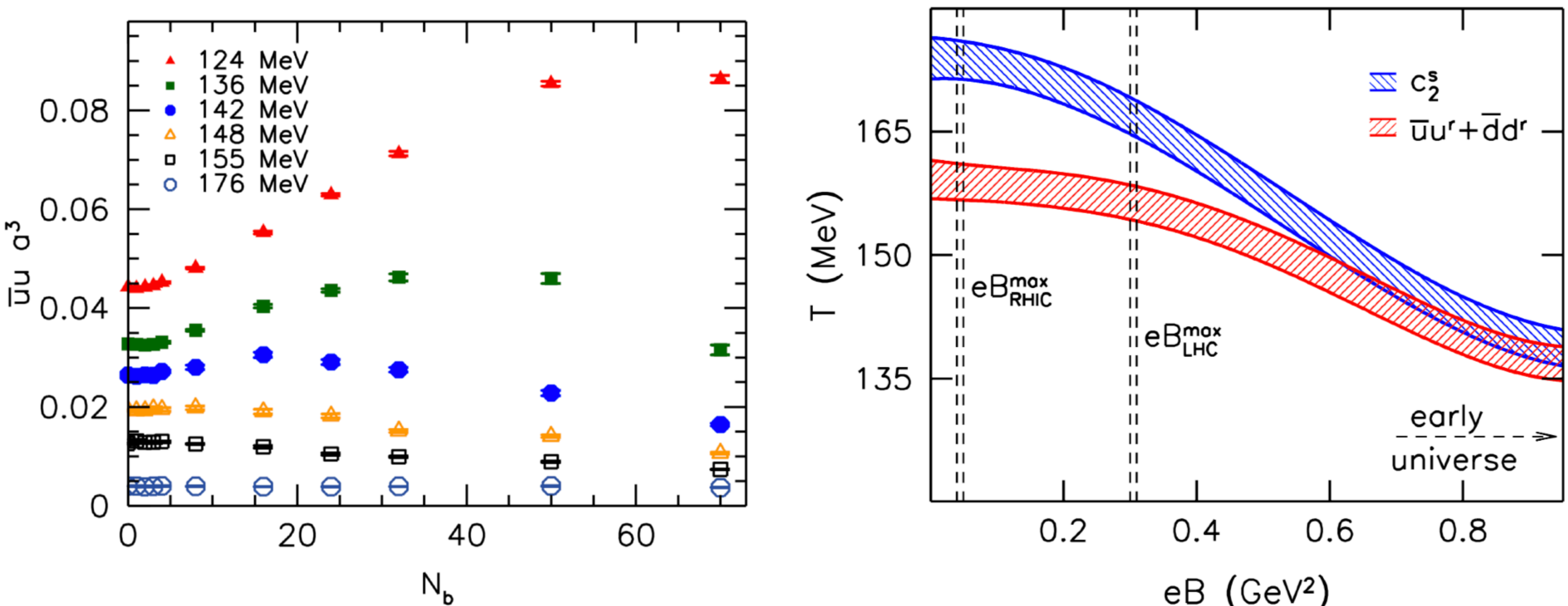
  - Gell-Mann-Oakes-Renner relation
  - qB scaling of chiral condensates, neutral pion mass & decay constants
  - masses, magnetic polarizability decay constants of neutral pion and kaon

- Fluctuations of conserved charges at T=/=0 and B=/=0

HTD, S.-T. Li, Q. Shi, X.-D. Wang, work in progres

  - 2nd order diagonal and off-diagonal fluctuations

(Inverse) magnetic catalyses v.s. reduction of T_{pc} in a background magnetic field



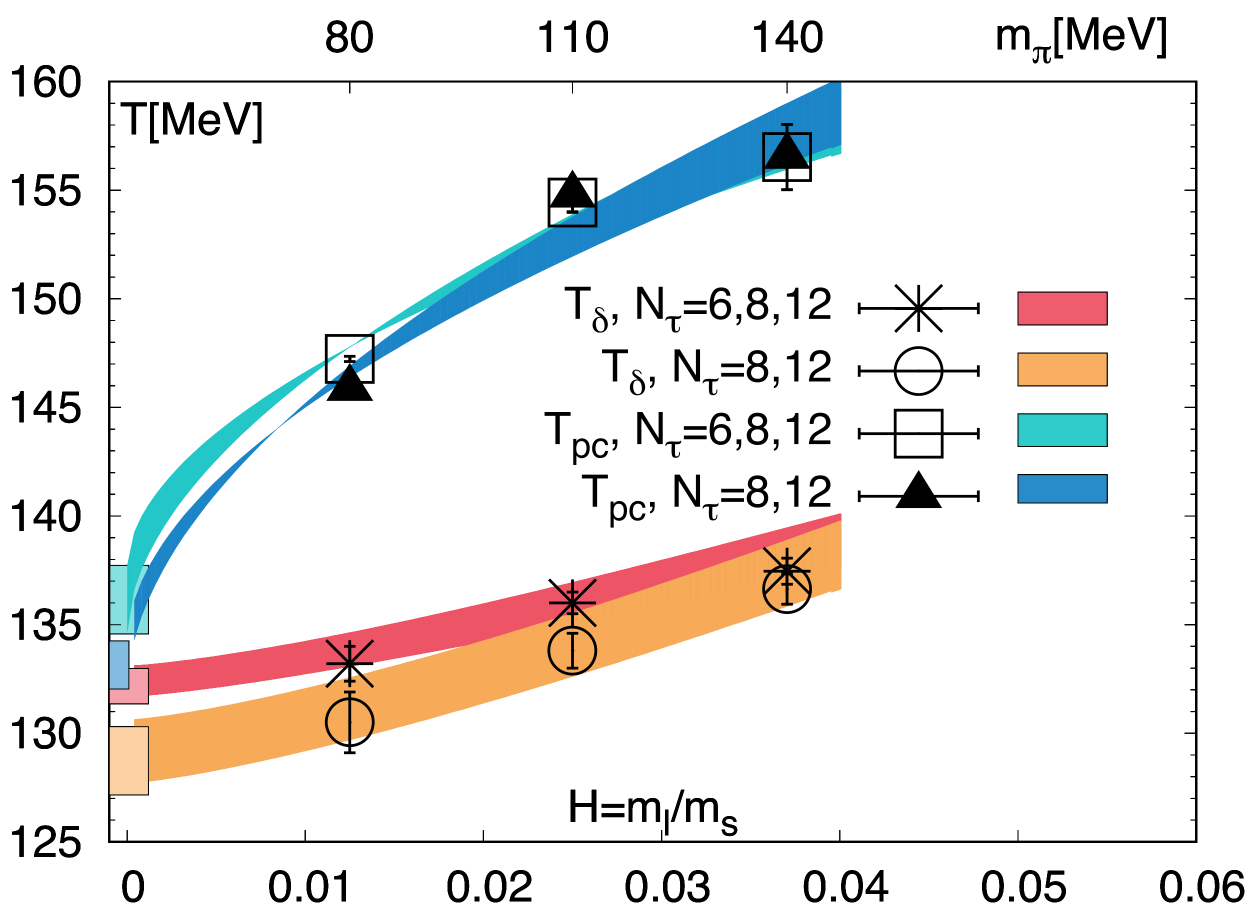
Bali et al., JHEP02(2012)044

Chiral condensate always increases as eB at T=0

Its connection to the reduction of T_{pc} is highly non-trivial

Reduction of T_{pc} v.s. lighter pion

eB=0

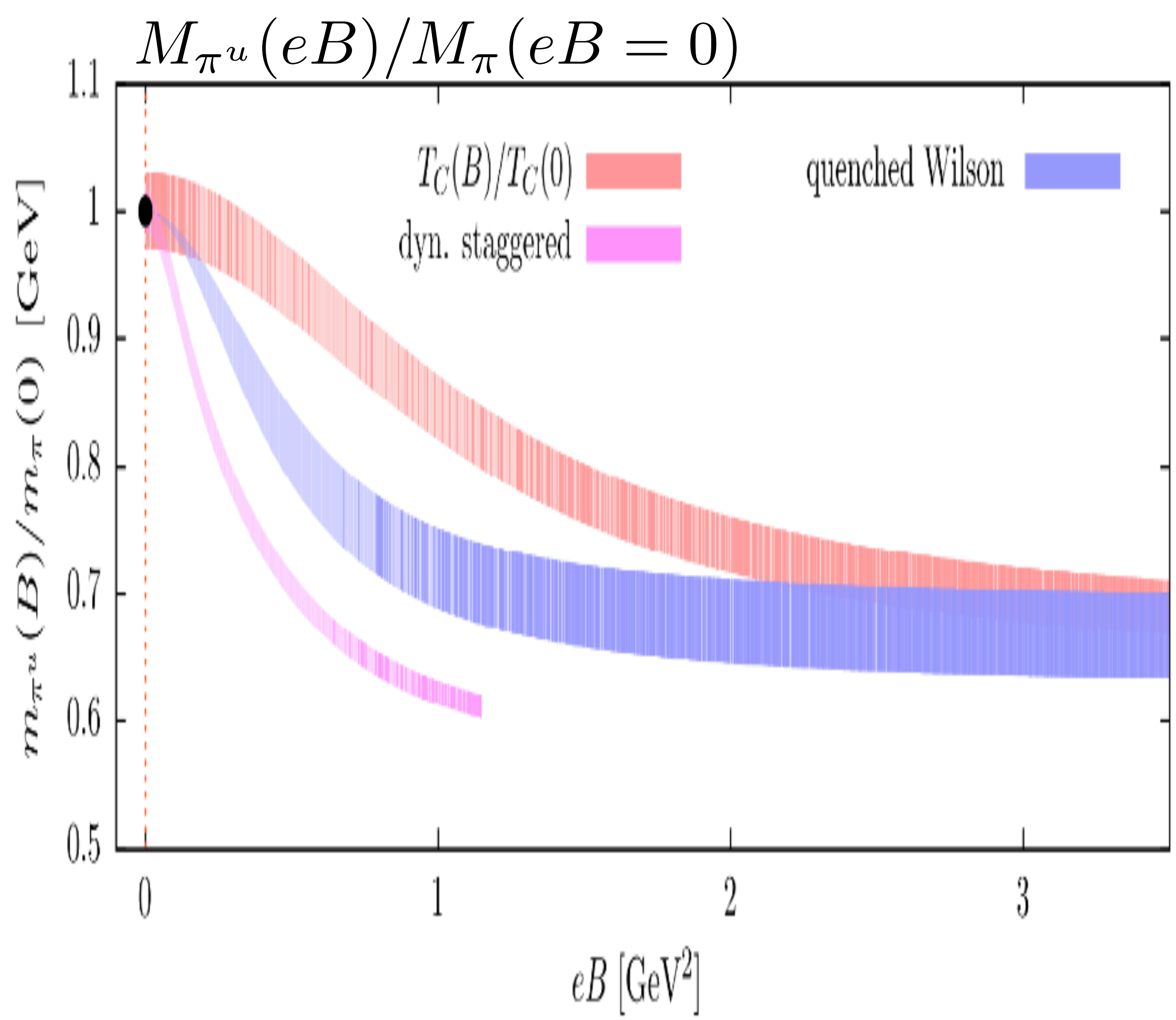


HTD, P. Hegde, O. Kaczmarek et al.[HotQCD], Phys. Rev. Lett. 123 (2019) 062002

HTD, arXiv:2002.11957

See also Anirban's talk on day1

eB=/=0



Bali et al., PRD 97, 034505 (2018)

Is (neutral) pion still a Goldstone boson at eB=/=0 ?

Gell-Mann-Oakes-Renner relation

M. Gell-Mann, R. J. Oakes, and B. Renner, Phys. Rev. 175, 2195 (1968)

J. Gasser and H. Leutwyler, Nucl. Phys. B 250, 465 (1985)

- At T=0, eB=0, next-to-leading order chiral correction δ_π=(6.2±1.6)%, δ_K=(55±5)% Bordes et al., JHEP05(2010)064, JHEP10(2012)102

2-flavor: $$(m_u + m_d)\ \left(\langle\bar{\psi}\psi\rangle_u + \langle\bar{\psi}\psi\rangle_d\right) = 2f_\pi^2 M_\pi^2\,(1-\delta_\pi)$$

3-flavor: $$(m_s + m_d)\ \left(\langle\bar{\psi}\psi\rangle_s + \langle\bar{\psi}\psi\rangle_d\right) = 2f_K^2 M_K^2\,(1-\delta_K)$$

- At T=0, in the weak magnetic field the 2-flavor GMOR relation holds true for chiral pions from LO ChPT

Shushpanov and Smilga, PLB402(1997)351

- At eB=/=0, additional pion decay constants appear due to a nonzero pion-to-vacuum transition via the vector electroweak current Fayazbakhsh & Sadooghi, PRD 88(2013)065030

Bali et al., 121(2018)072001, Coppola et al., PhysRevD.99 (2019)0540312

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Shushpanov and Smilga, PLB402(1997)351

We focus on the neutral pion & kaon decay constants which have same definitions at eB=0

a nonzero pion-to-vacuum transition via the vector electroweak current Fayazbakhsh & Sadooghi, PRD 88(2013)065030

Bali et al., 121(2018)072001, Coppola et al., PhysRevD.99 (2019)0540312

Lattice setup

- Symanzik-improved gauge action with HISQ fermions
- 32^3x96 lattices, with a=0.117 fm (a^{-1}=0.17 GeV), ml/ms =1/10 (M_π =220 MeV)
- In our setup f_π= 96.93(2) MeV, f_K=112.50(2) MeV, f_K/f_π=1.1606(3)

FLAG 2019: At physical mass point f_π= 92.1(6) MeV, f_K=110.1(5) MeV, f_K/f_π=1.1917(37)

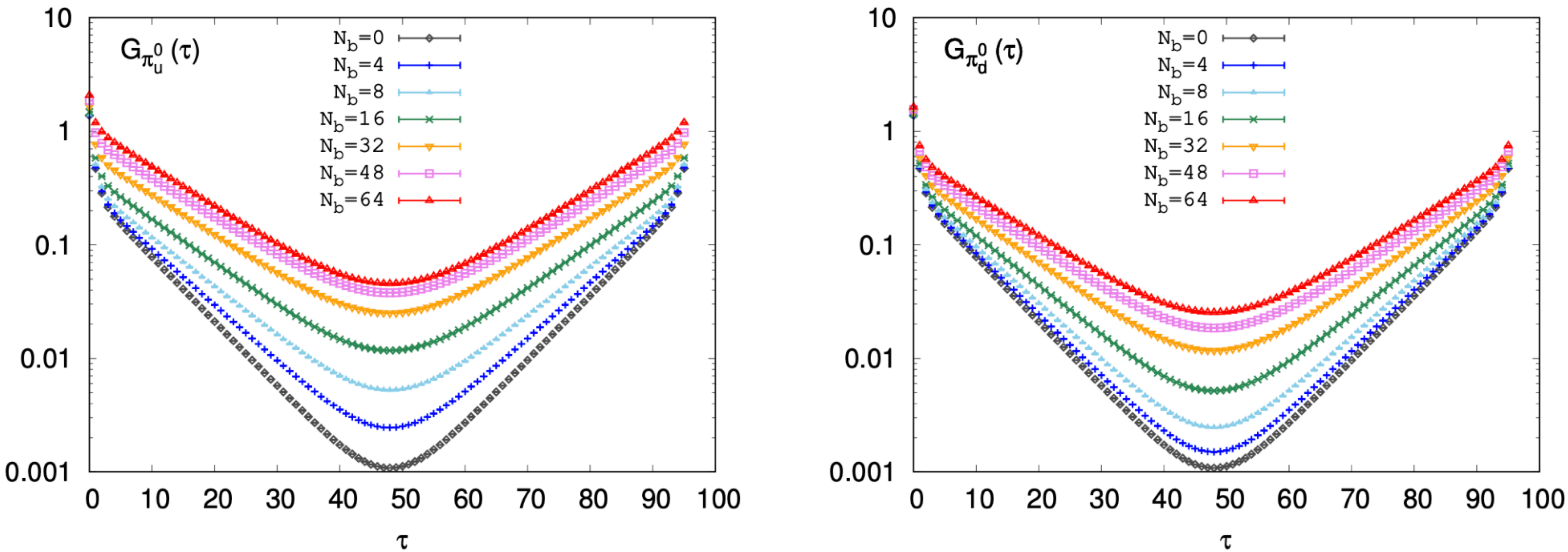
✦ Magnetic field is quantized as $eB = \frac{6\pi N_b}{N_x N_y} a^{-2}$

✦ Magnetic flux: Nb=0,1,2,3,4,6,8,10,12,16,20,24,32,48 & 64

✦ $0 \leq eB \leq 3.35$ GeV^2 (~60 M_π^2)

Meson masses extracted from temporal correlation functions

Neutral pion becomes iso-singlet: u and d flavor components

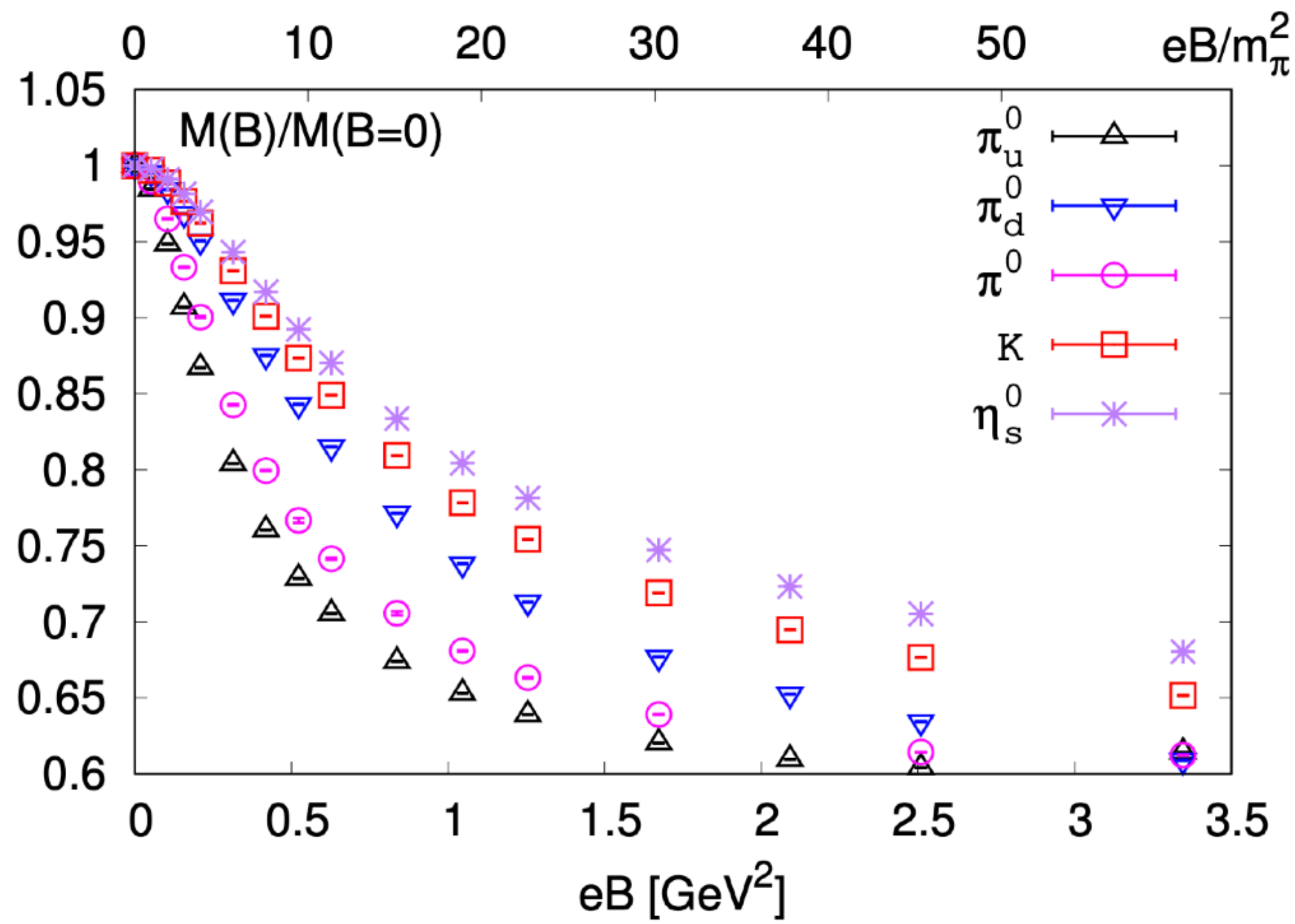


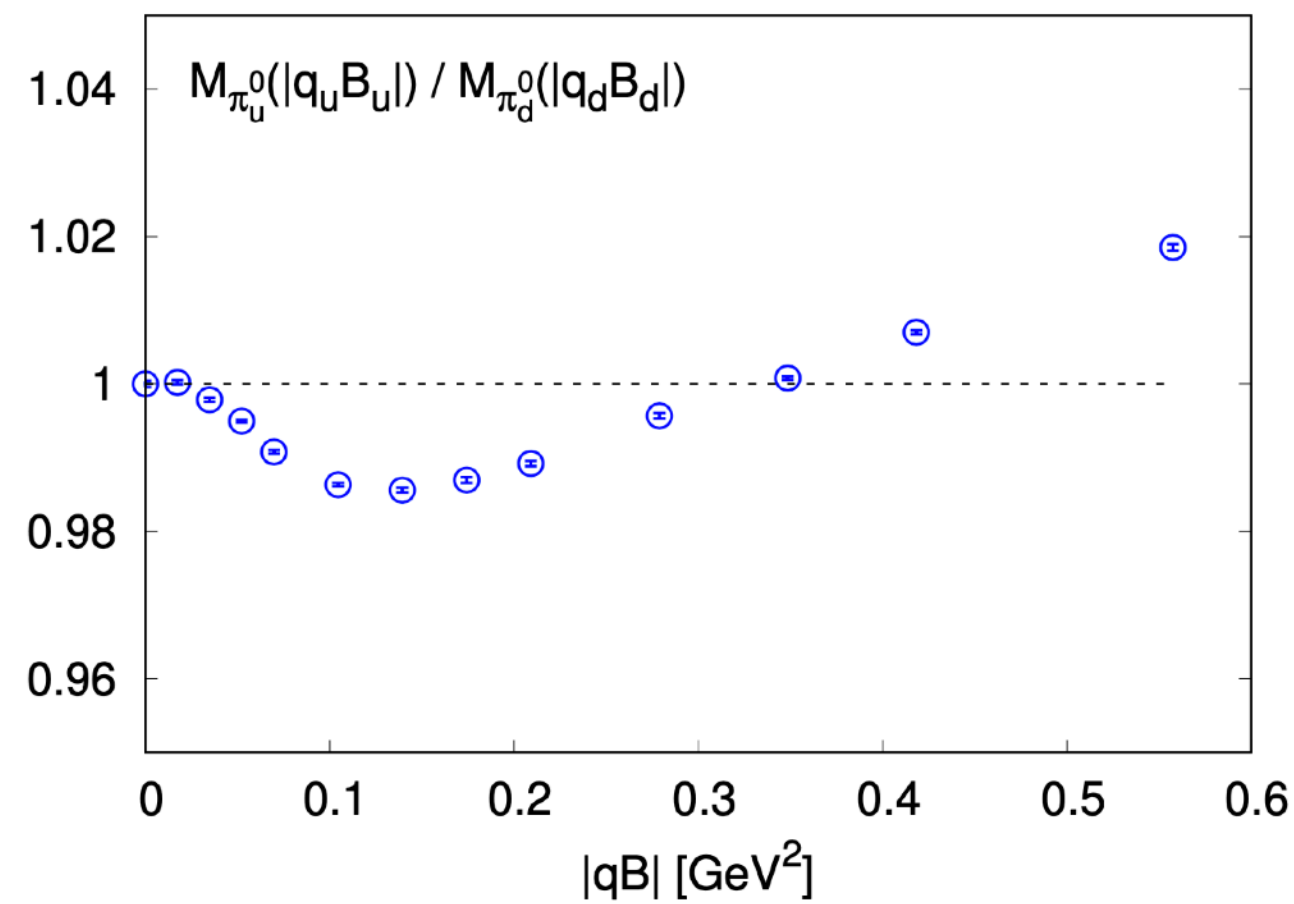
$$G(n_\tau) = \sum_{i=1}^{N_{nosc}} A_{nosc,i} \exp\left(-M_{nosc,i}\, n_\tau\right) - (-1)^{n_\tau} \sum_{i=0}^{N_{osc}} A_{osc,i} \exp\left(-M_{osc,i}\, n_\tau\right).$$

Meson mass: exponential decay of temporal correlation functions

AICc fit to obtain meson masses

Masses of neutral pseudo scalar mesons

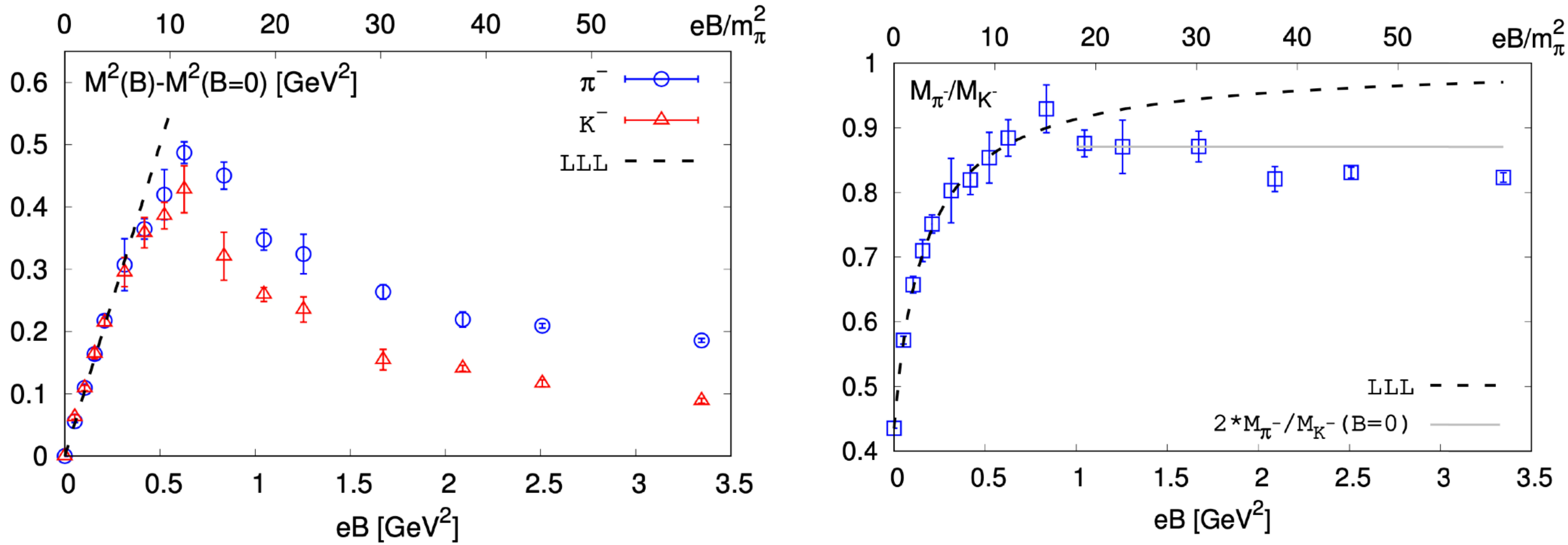




- The mass drops at most to 60% of its value at B=0
- Light meson are more affected by B

⭐ **qB scaling is observed in u and d flavor components of M_π**

Masses of charged pseudo scalar mesons

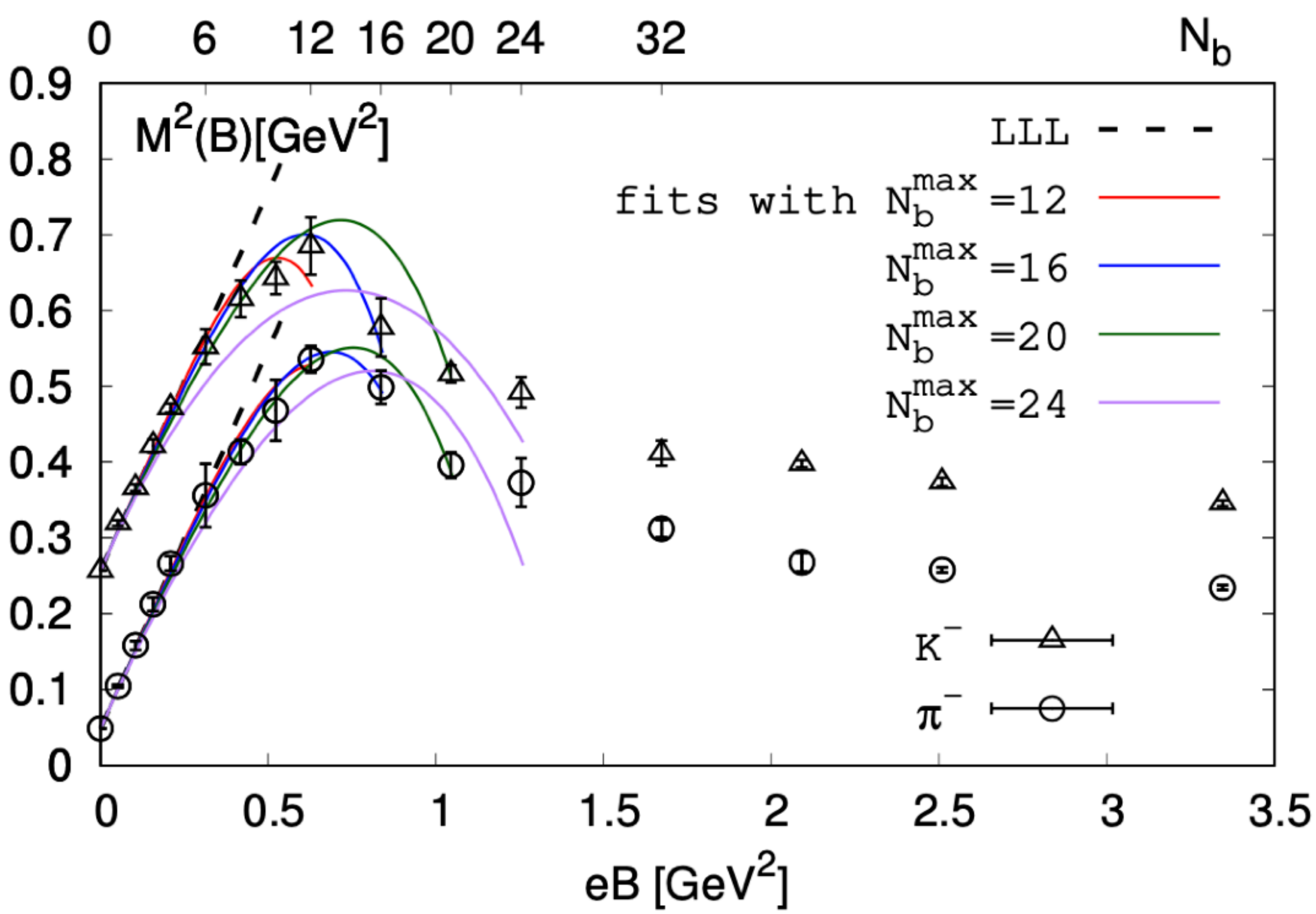


Lowest Landau Level approximation for point charged PS mesons:
$M^2(eB) = M^2(eB=0) + eB$

- $M^2(eB) - M^2(eB=0)$ firstly increases and then decreases with eB. The decreasing behavior is not observed in the previous LQCD simulations
- At eB ≳ 0.3 GeV^2, charged pion and K cannot be considered as point particles any more
- Ratio of charged pion and Kaon increases with eB and then saturate

Magnetic polarizability β_m of charged pion & kaon

$$M^2(B) = M^2(B=0) + |qB| - 4\pi M(B=0)\,\beta_m\,(eB)^2 - 4\pi M(B=0)\,\beta_m^{1h}\,(eB)^4 + \mathcal{O}((eB)^6)$$



β_m from our best fits:

- pion: $(-0.00\pm0.04)\times10^{-4}$ fm^3

  At odd with experiment results

- kaon: $(-2\pm2)\times10^{-4}$ fm^3

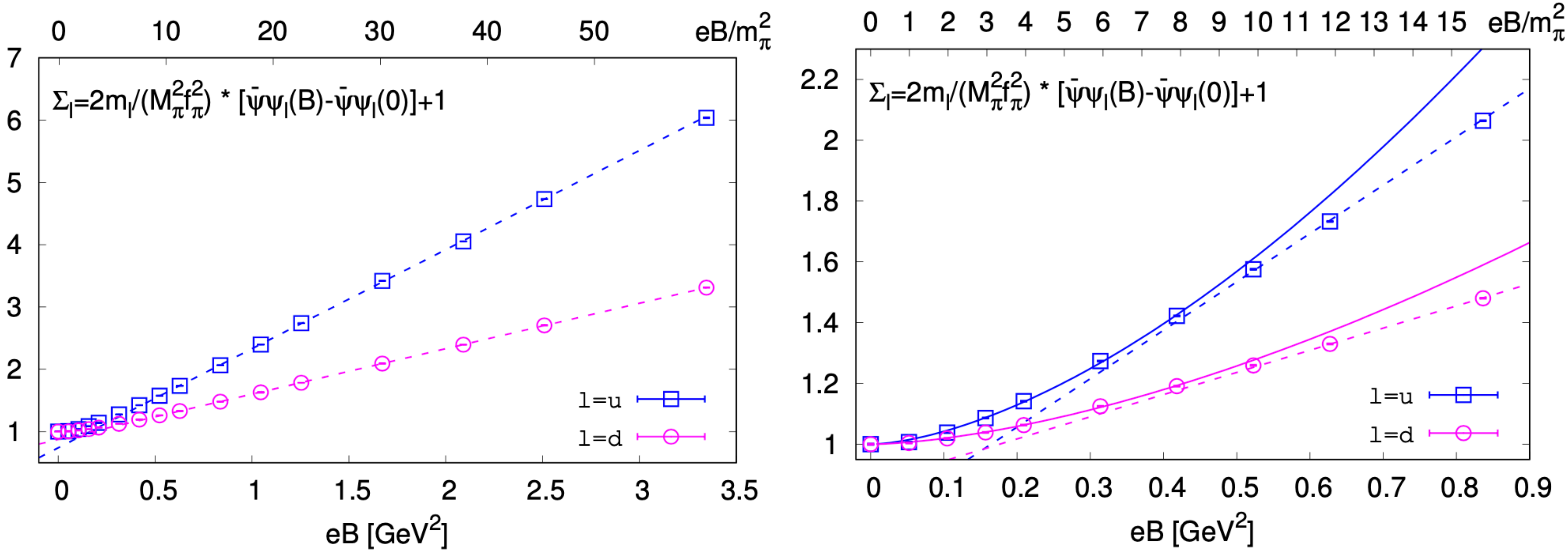
- COMPASS experiment for pion: $\beta_m = (-2\pm0.6_{stats}\pm0.7_{syst})\times10^{-4}$ fm^3 assuming $\beta_m = -\alpha_m$

  [Compass], PRL 114 (2015) 062002

- Quenched lattice QCD for pion: $\beta_m = (-2.06\pm0.76)\times10^{-4}$ fm^3 on 18^4 and $\beta_m = (-1.15\pm0.31)\times10^{-4}$ fm^3 on 20^4 lattices

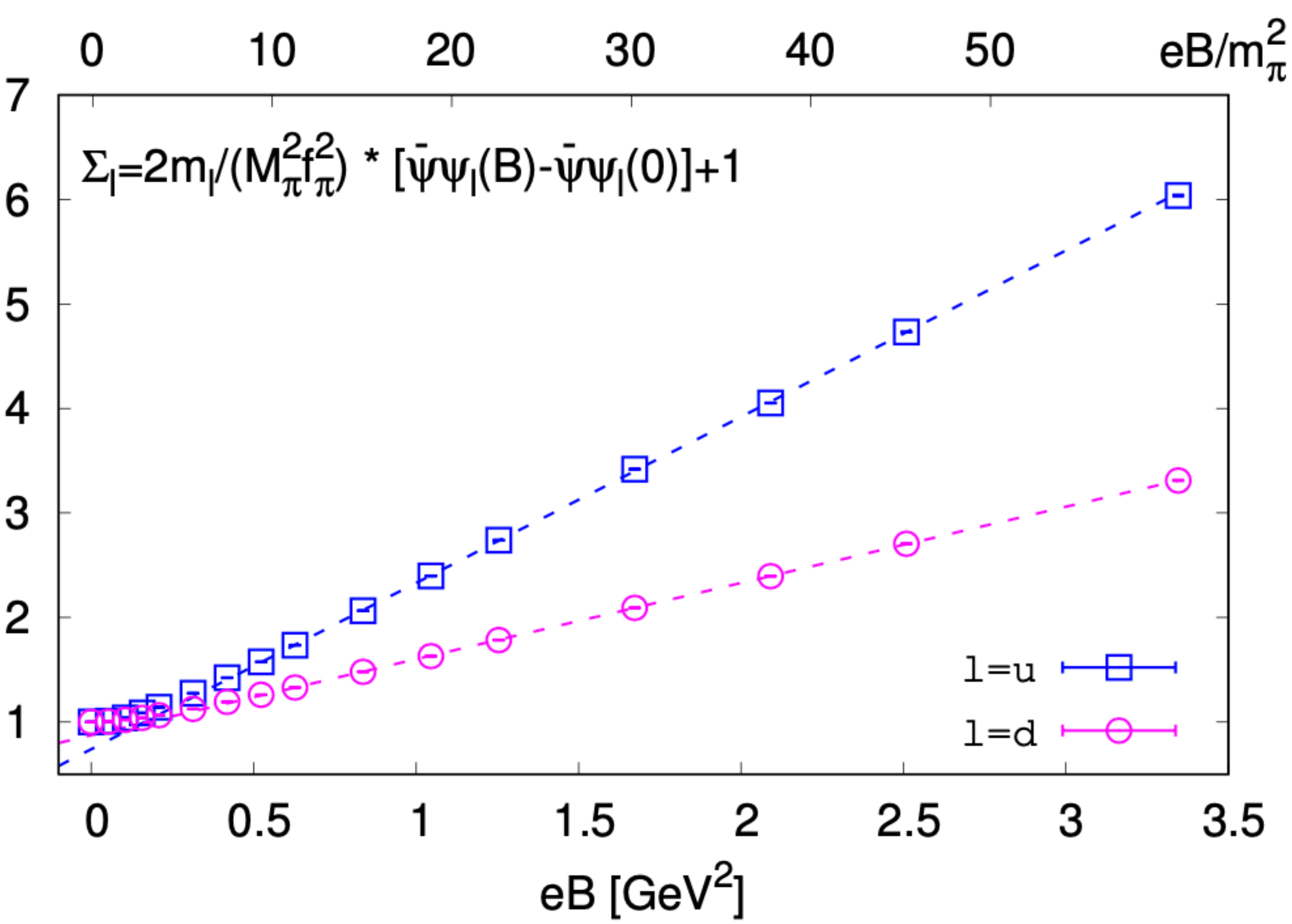
  Luschevskaya, Solovjeva & O.V. Teryaev, PLB 761 (2016) 393

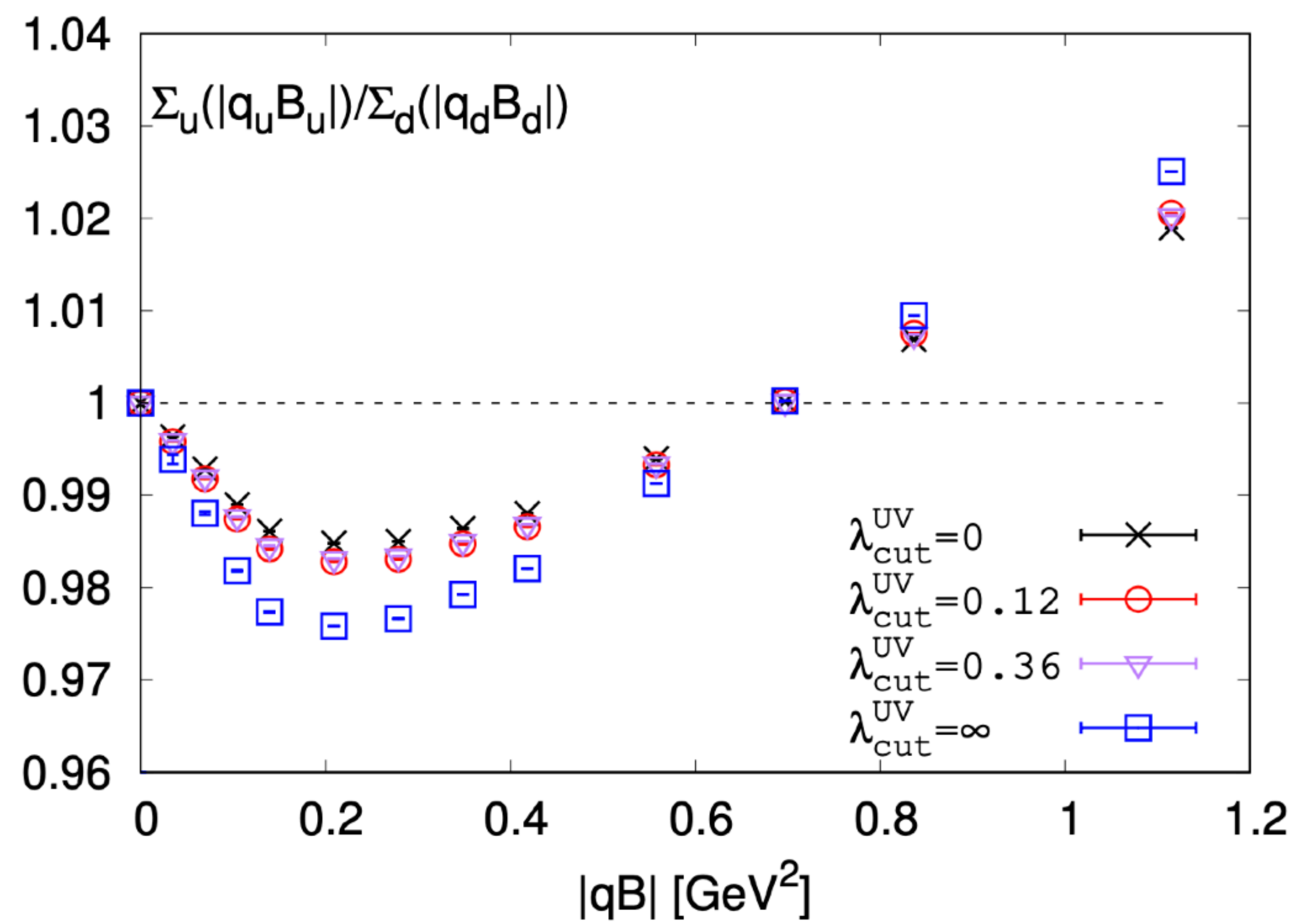
Light quark chiral condensates



- Magnetic catalysis: light quark chiral condensate increase linear with eB at large eB, and increases with a power-law behavior at small eB

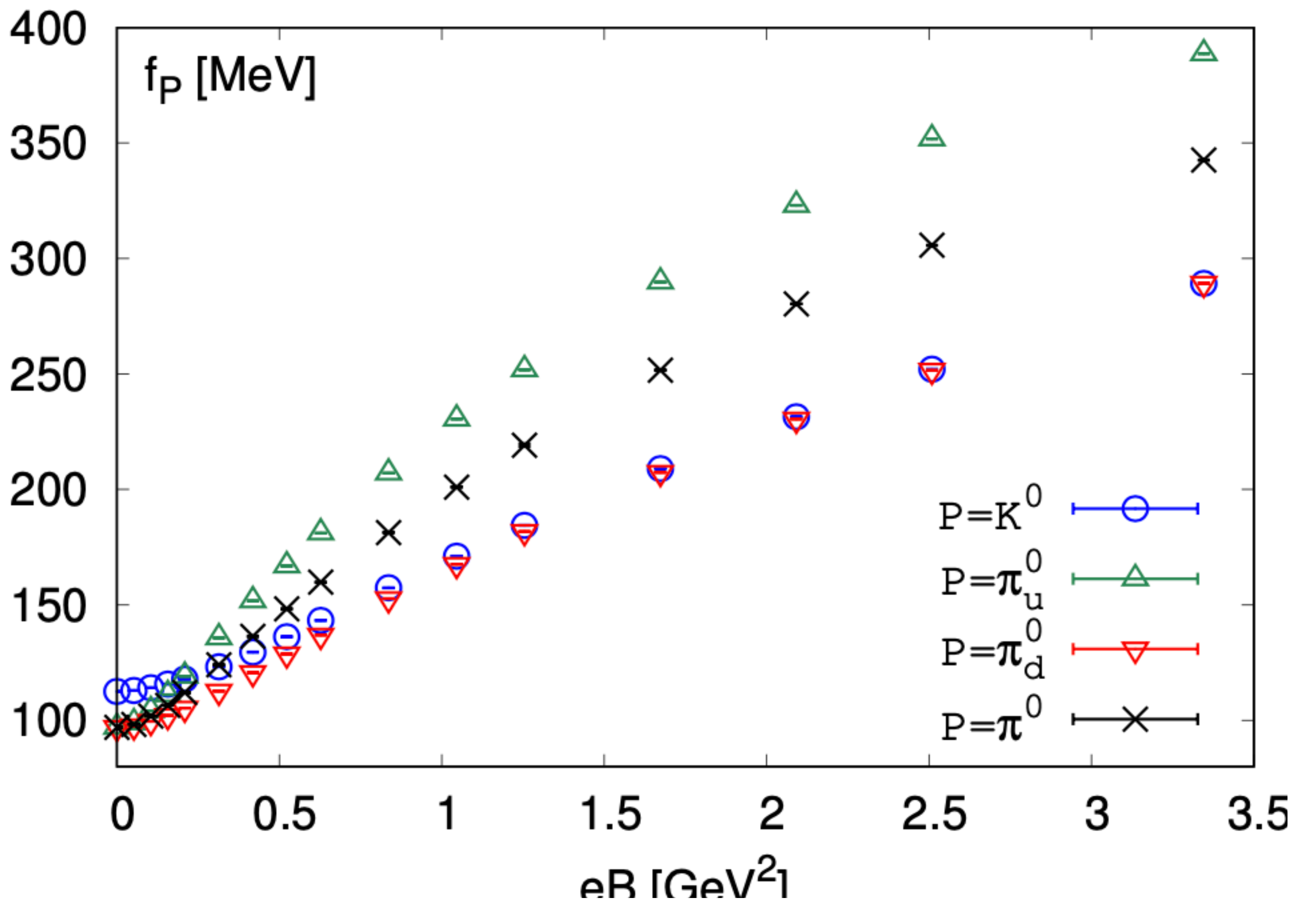
Light quark chiral condensates

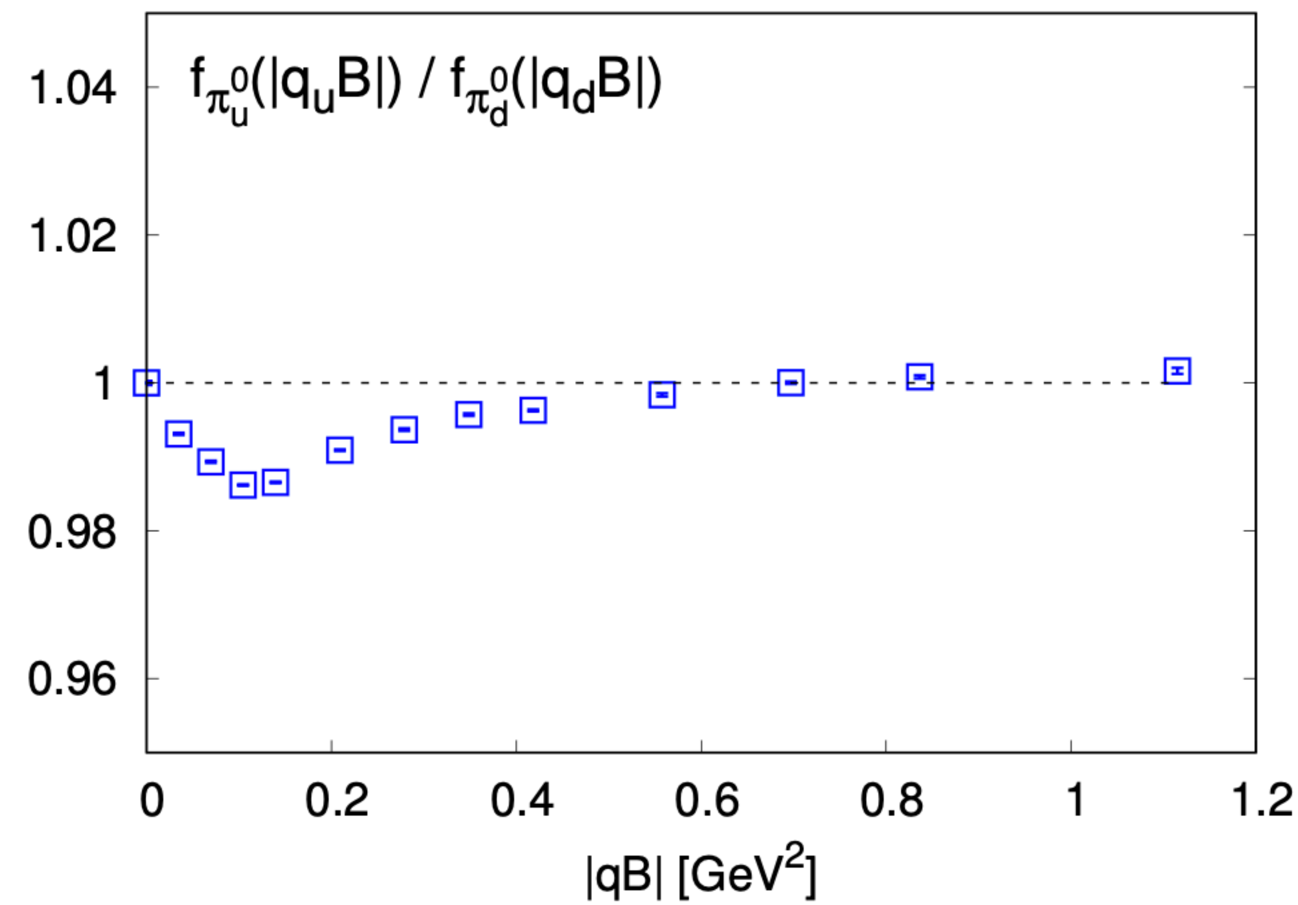


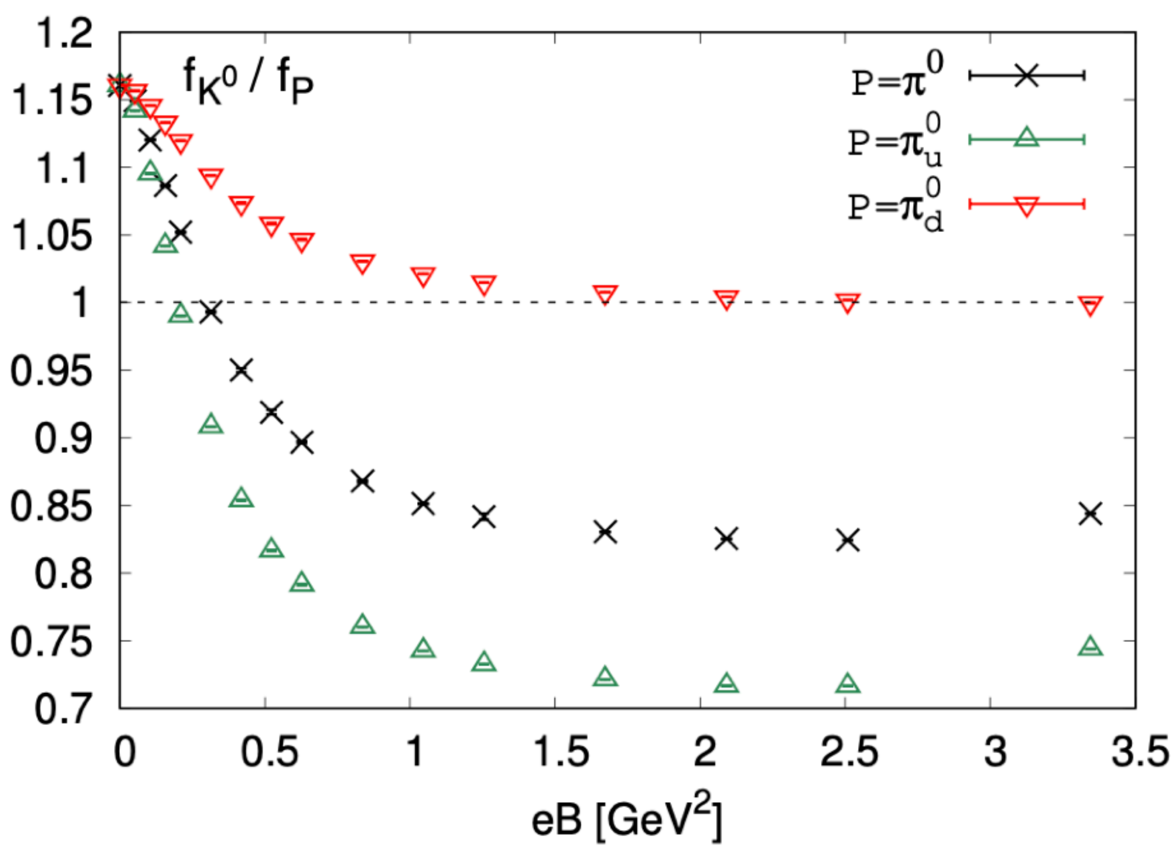


- Magnetic catalysis: light quark chiral condensate increase linear with eB at large eB, and increases with a power-law behavior at small eB
- qB scaling of u and d quark chiral condensates: same at same value of |qB|

Decay constants of neutral pion and kaon





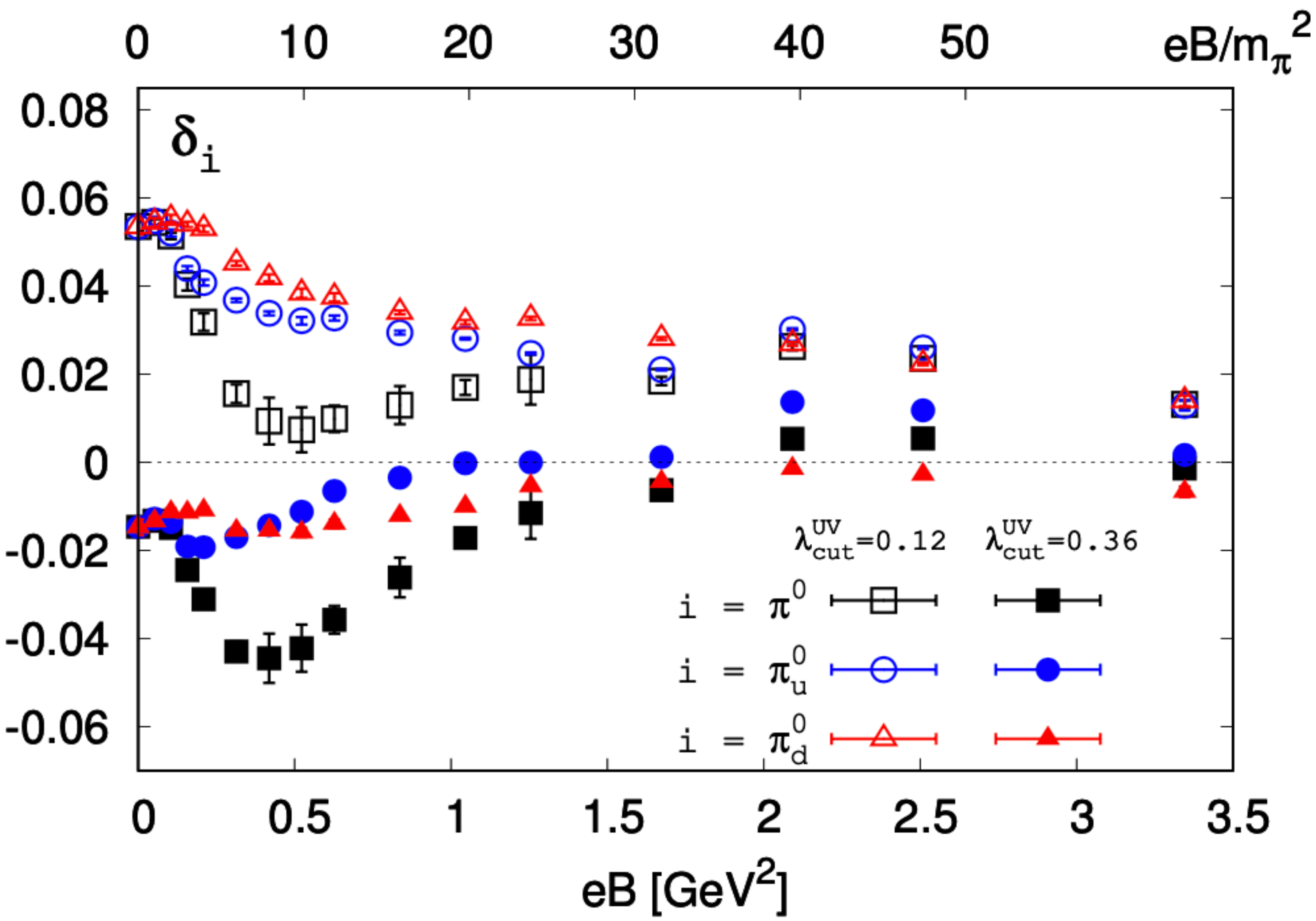


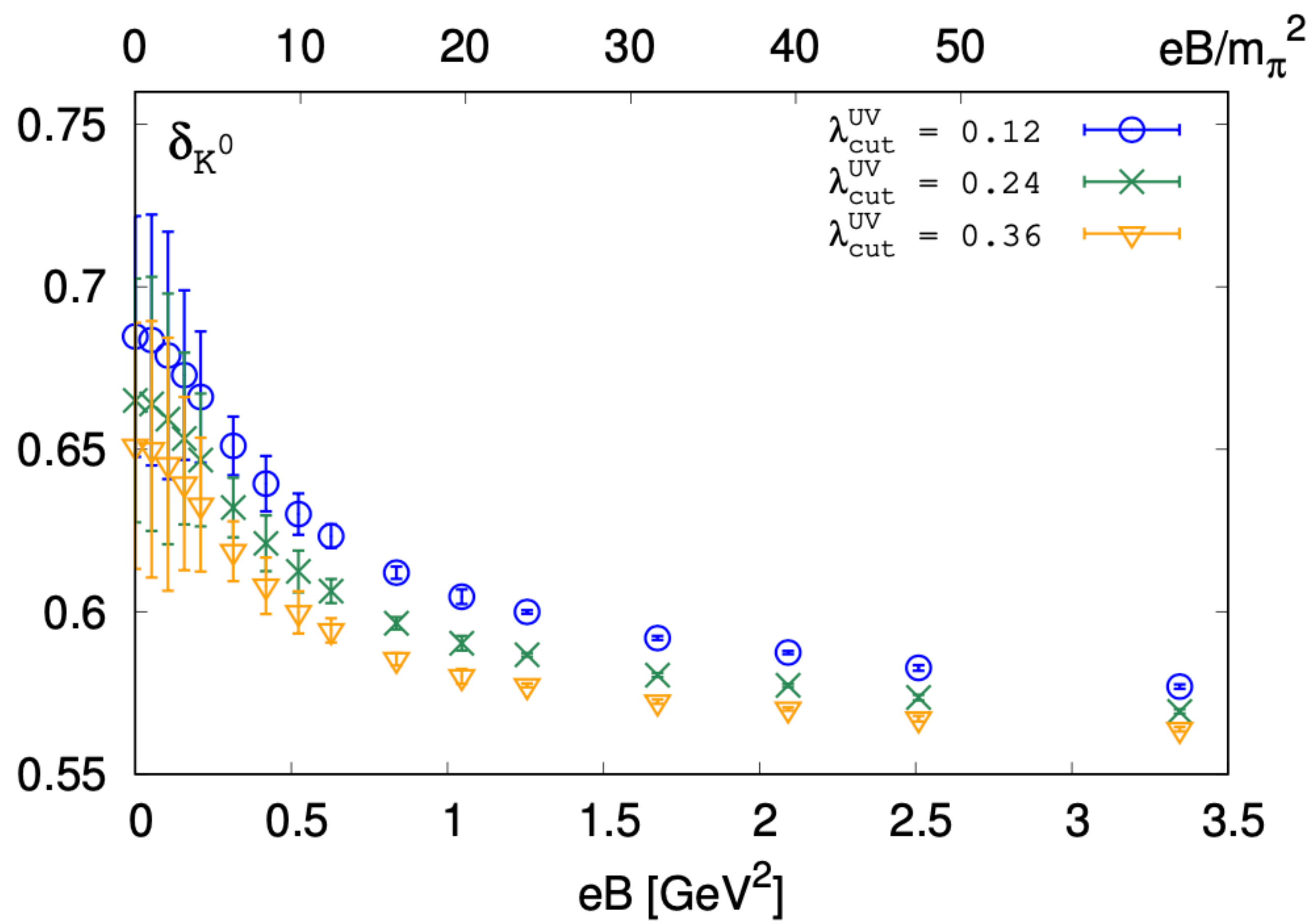
- All the decay constants increase with eB
- qB scaling seen in u and d quark flavor components of fpi
- f_K/f_π decreases with eB and then saturate

Gell-Mann-Oakes-Renner relation

$$4m_u\ \langle\bar{\psi}\psi\rangle_u = 2f^2_{\pi^0_u}M^2_{\pi^0_u}\,(1-\delta_{\pi^0_u})$$
$$4m_d\ \langle\bar{\psi}\psi\rangle_d = 2f^2_{\pi^0_d}M^2_{\pi^0_d}\,(1-\delta_{\pi^0_d})\,.$$
$$(m_u+m_d)\ \left(\langle\bar{\psi}\psi\rangle_u+\langle\bar{\psi}\psi\rangle_d\right) = 2f^2_{\pi}M^2_{\pi}\,(1-\delta_{\pi})$$

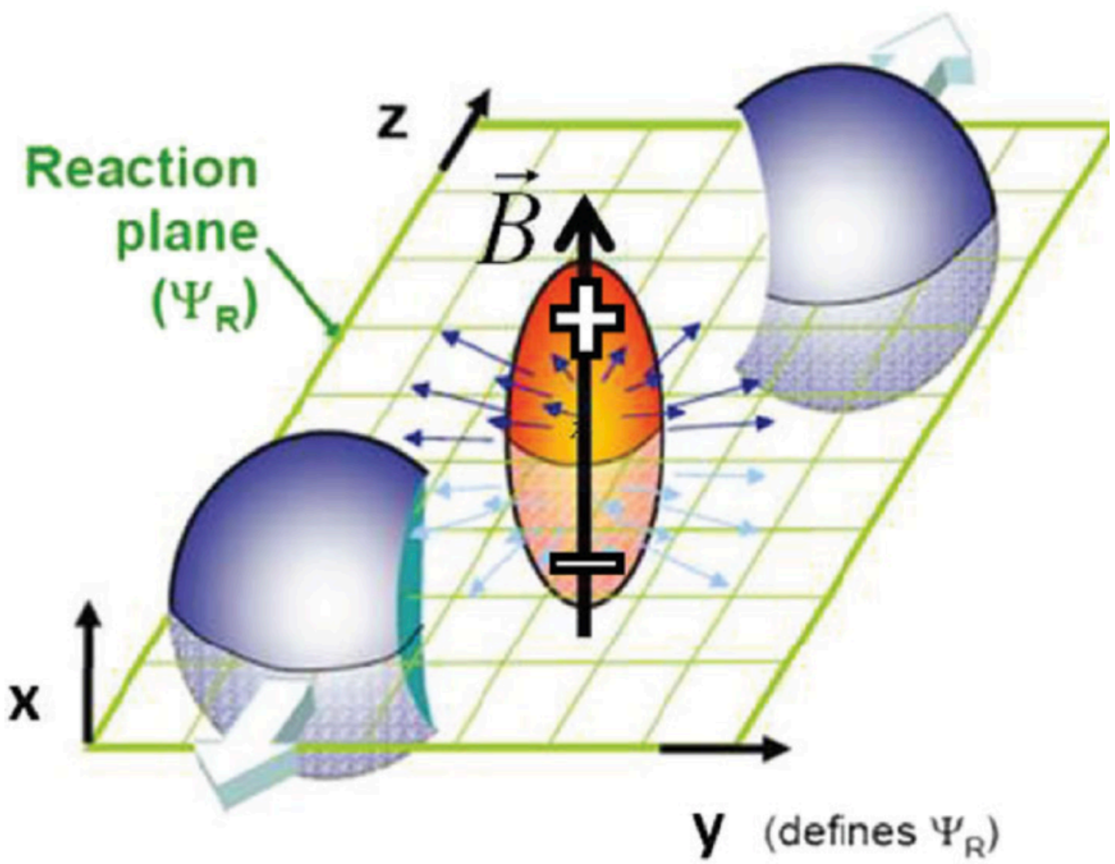
$$(m_s+m_d)\ \left(\langle\bar{\psi}\psi\rangle_s+\langle\bar{\psi}\psi\rangle_d\right) = 2f^2_{K}M^2_{K}\,(1-\delta_{K})$$

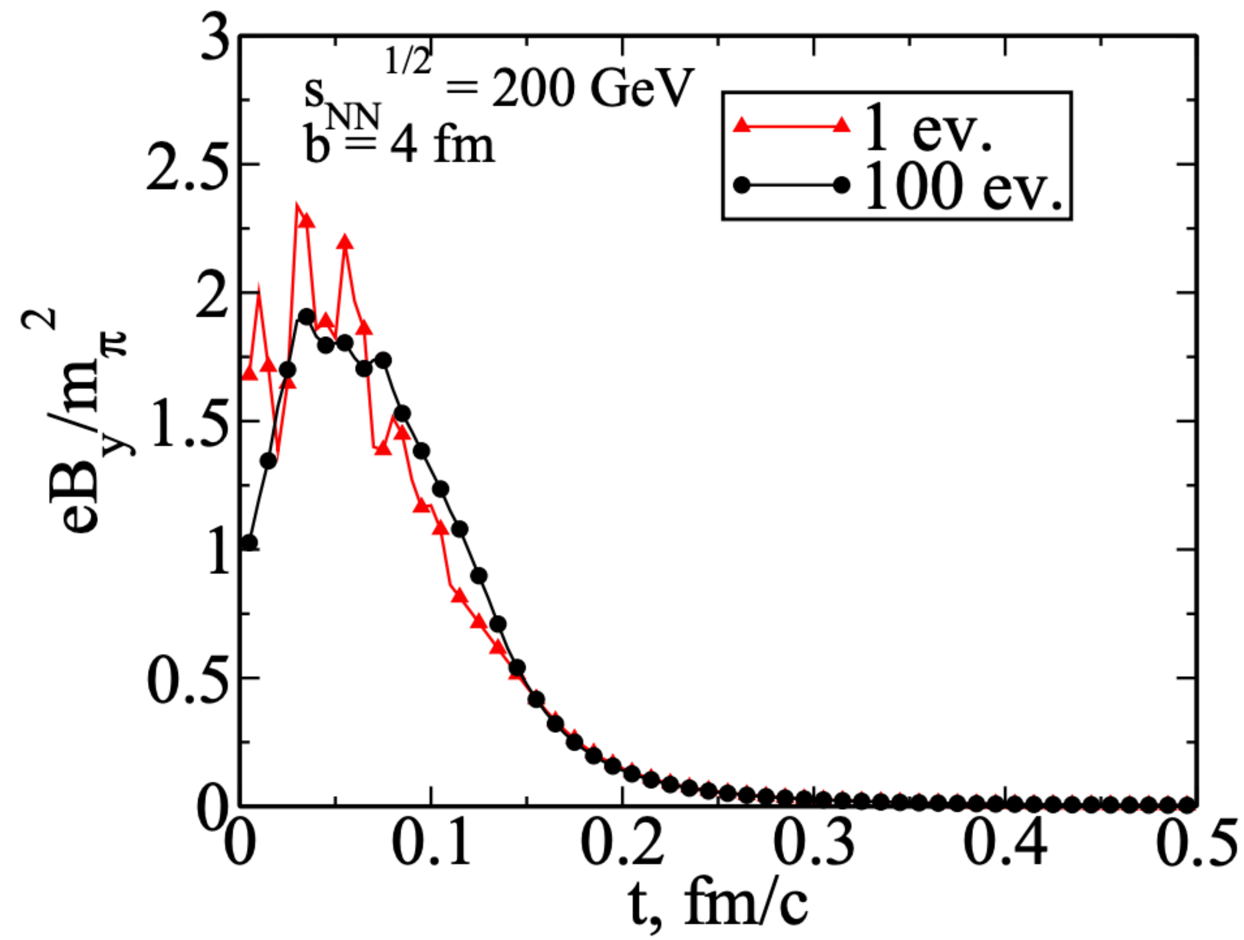




neutral pion remains as a Goldstone boson with eB up to ~3.5 GeV^2

Magnetic fields created in HIC



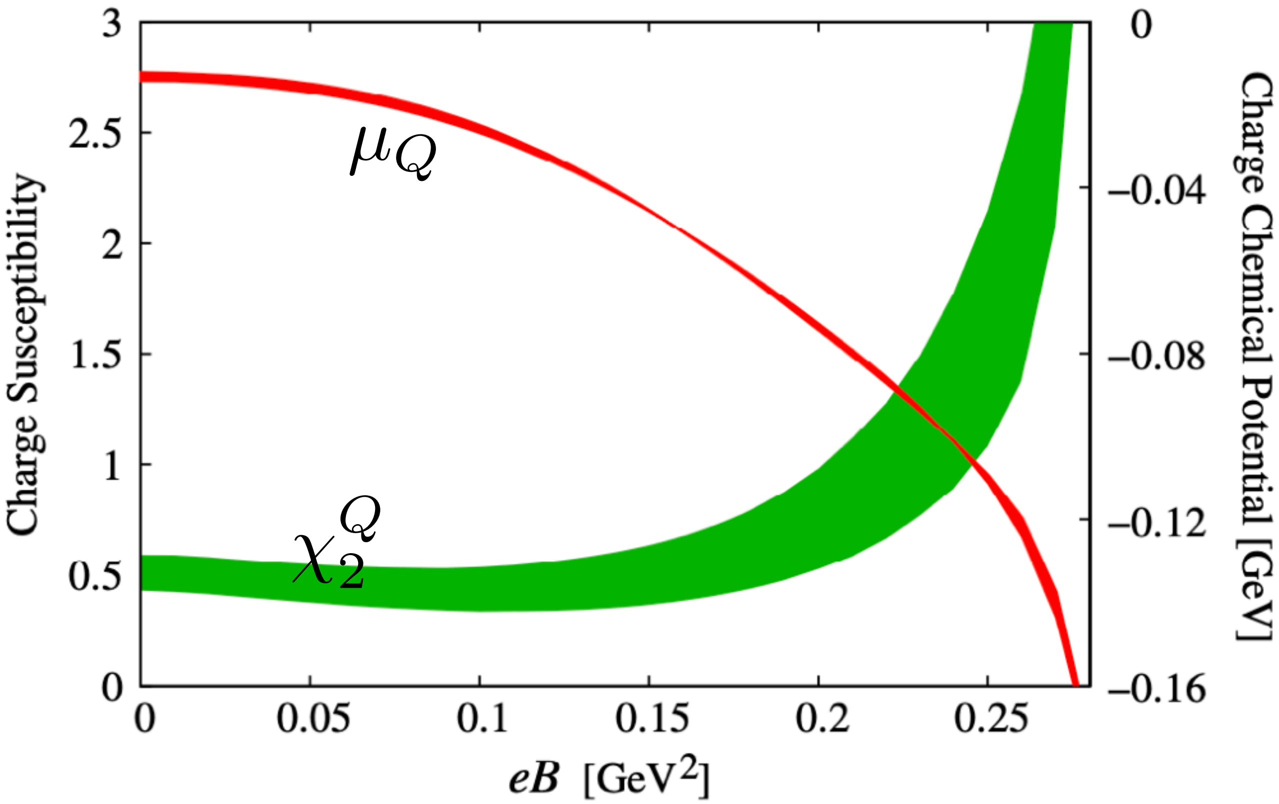


Skokov, Illarionov and V.Toneev, IJMPA 24 (2009) 5925

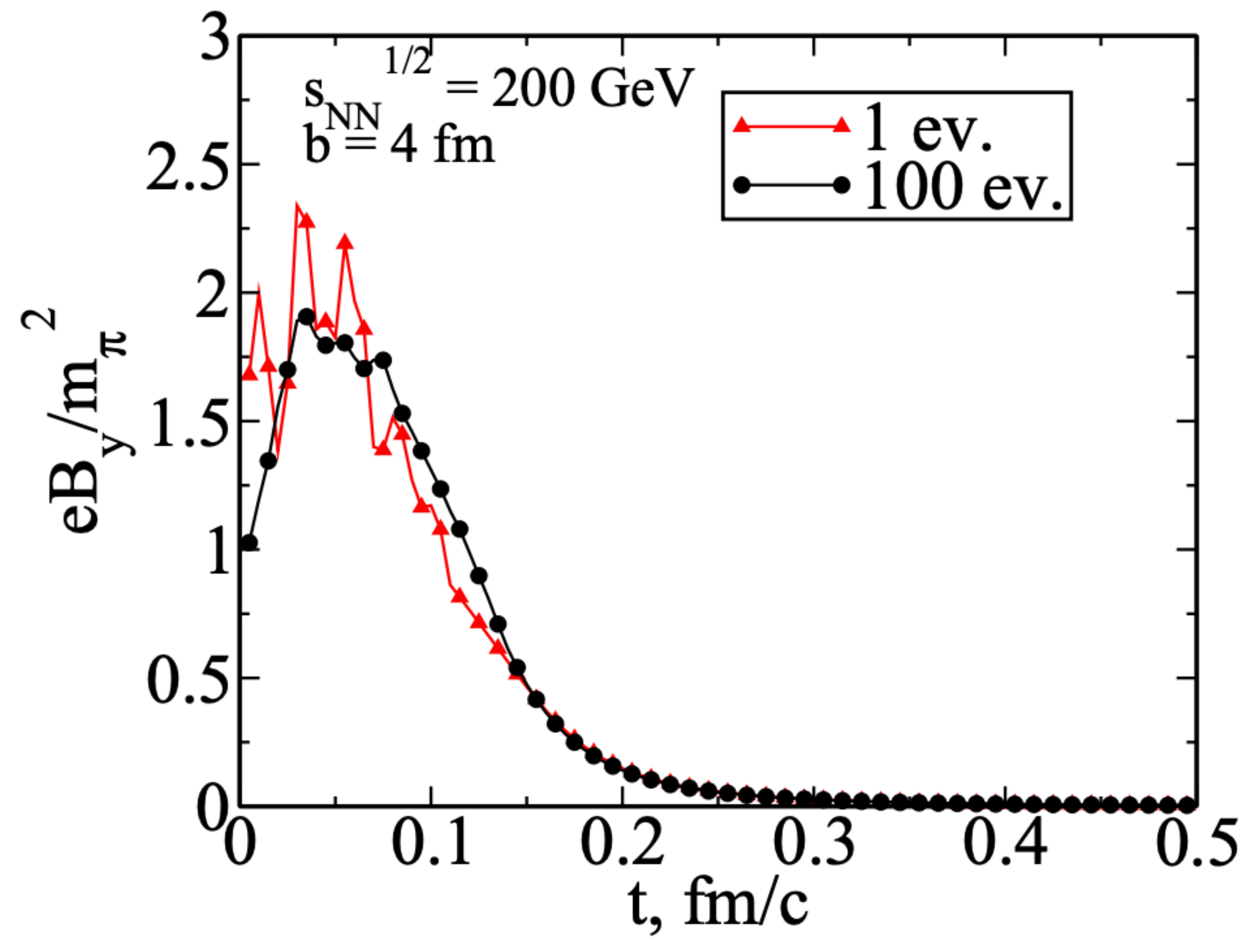
t=0: RHIC: $eB \sim m_\pi^2$

LHC: $eB \sim 15 m_\pi^2$

Magnetic fields created in HIC



Fukushima & Hidaka, PRL 117, 102301 (2016)



Skokov, Illarionov and V.Toneev, IJMPA 24 (2009) 5925

Based on HRG,
detect eB by comparing 2nd order electrical charge fluctuation in peripheral to central collisions

t=0: RHIC: $eB \sim m_\pi^2$
LHC: $eB \sim 15 m_\pi^2$

Fluctuations of conserved charges

Taylor expansion of the QCD pressure:

Allton et al., Phys.Rev. D66 (2002) 074507
Gavai & Gupta et al., Phys.Rev. D68 (2003) 034506

$$\frac{p}{T^4} = \frac{1}{VT^3} \ln \mathcal{Z}(T, V, \hat{\mu}_u, \hat{\mu}_d, \hat{\mu}_s) = \sum_{i,j,k=0}^{\infty} \frac{\chi_{ijk}^{BQS}}{i!j!k!} \left(\frac{\mu_B}{T}\right)^i \left(\frac{\mu_Q}{T}\right)^j \left(\frac{\mu_S}{T}\right)^k$$

- Taylor expansion coefficients at μ=0 are computable in LQCD, i.e. fluctuations of conserved charges:

$$\chi_{ijk}^{BQS} \equiv \chi_{ijk}^{BQS}(T) = \frac{1}{VT^3} \frac{\partial^{i+j+k} P(T, \mu)/T^4}{\partial \hat{\mu}_B^i \partial \hat{\mu}_Q^j \partial \hat{\mu}_S^k}\Big|_{\hat{\mu}_{B,Q,S}=0}$$

- Magnetic field dependences of these fluctuations

Lattice setup

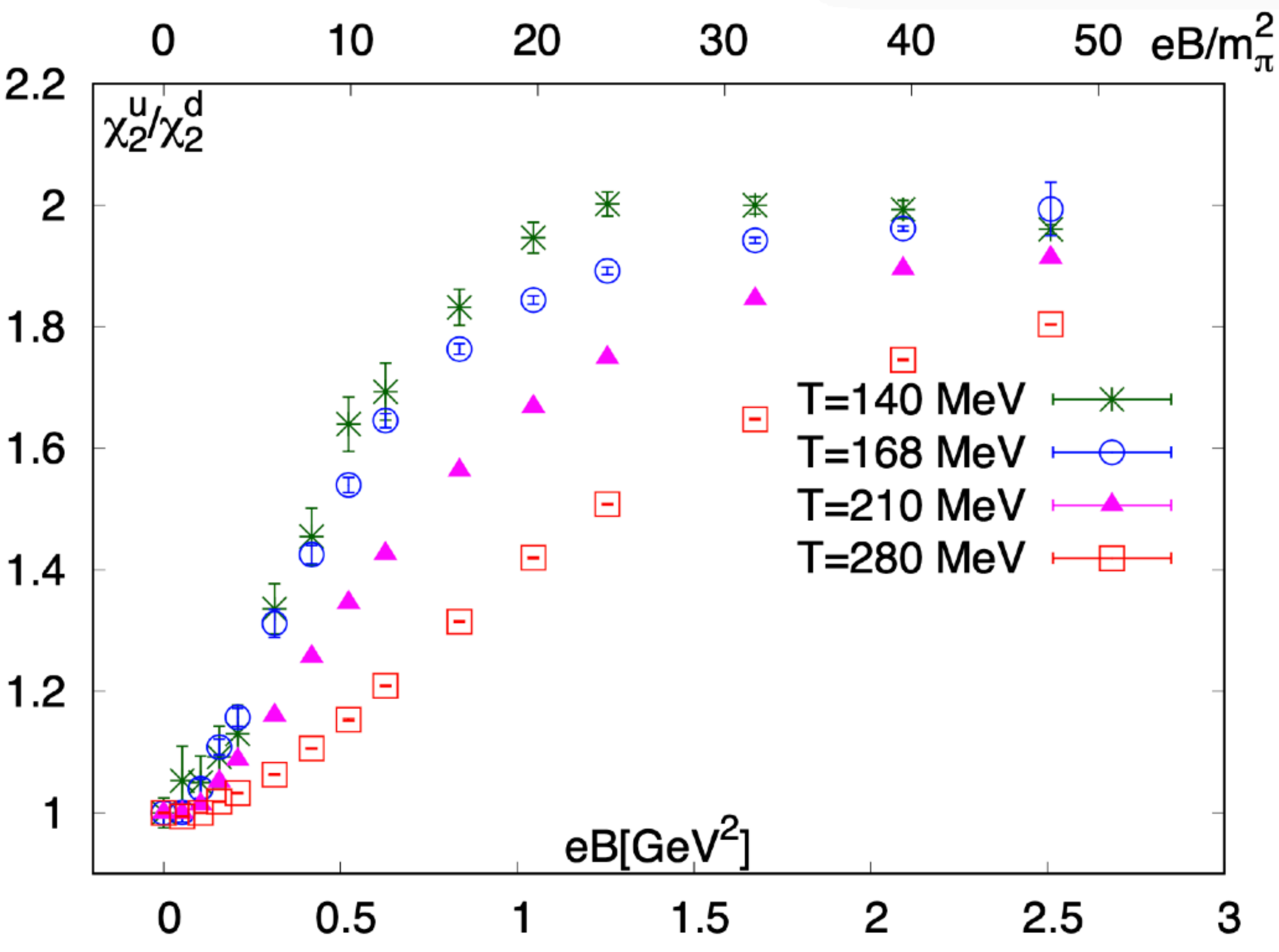
- Same lattice discretization as that at T=0
- Simulations at nonzero T=1/(aNt) with fixed scale approach

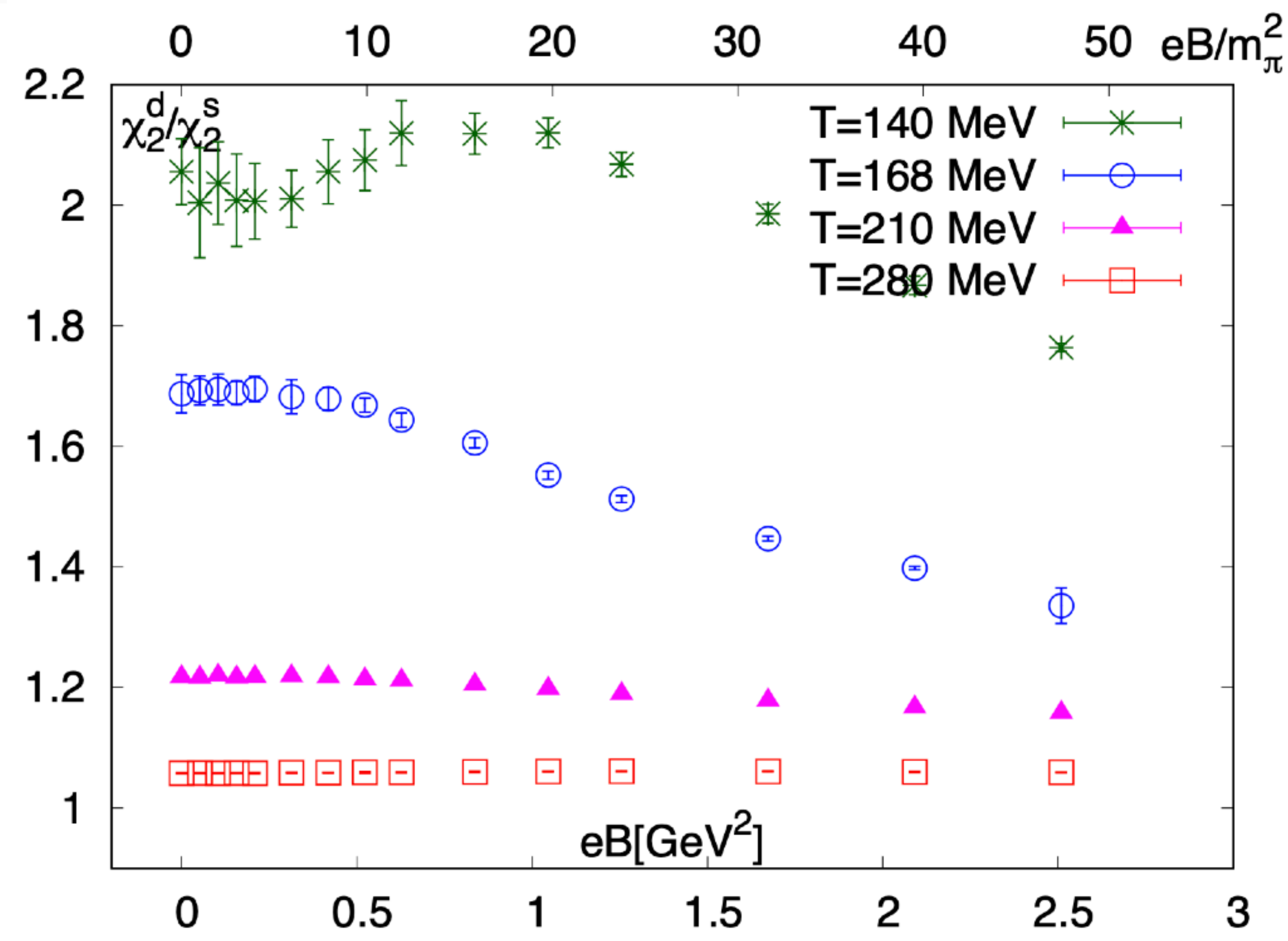
T= 280, 210, 168, 140 MeV with Nt=6,8,10,12

well above T_{pc} ~T_{pc} well below T_{pc}

- $0 \leq eB \leq 2.5$ GeV^2 (~50 M_π^2) with $eB = \frac{6\pi N_b}{N_x N_y} a^{-2}$

Quark number susceptibilities

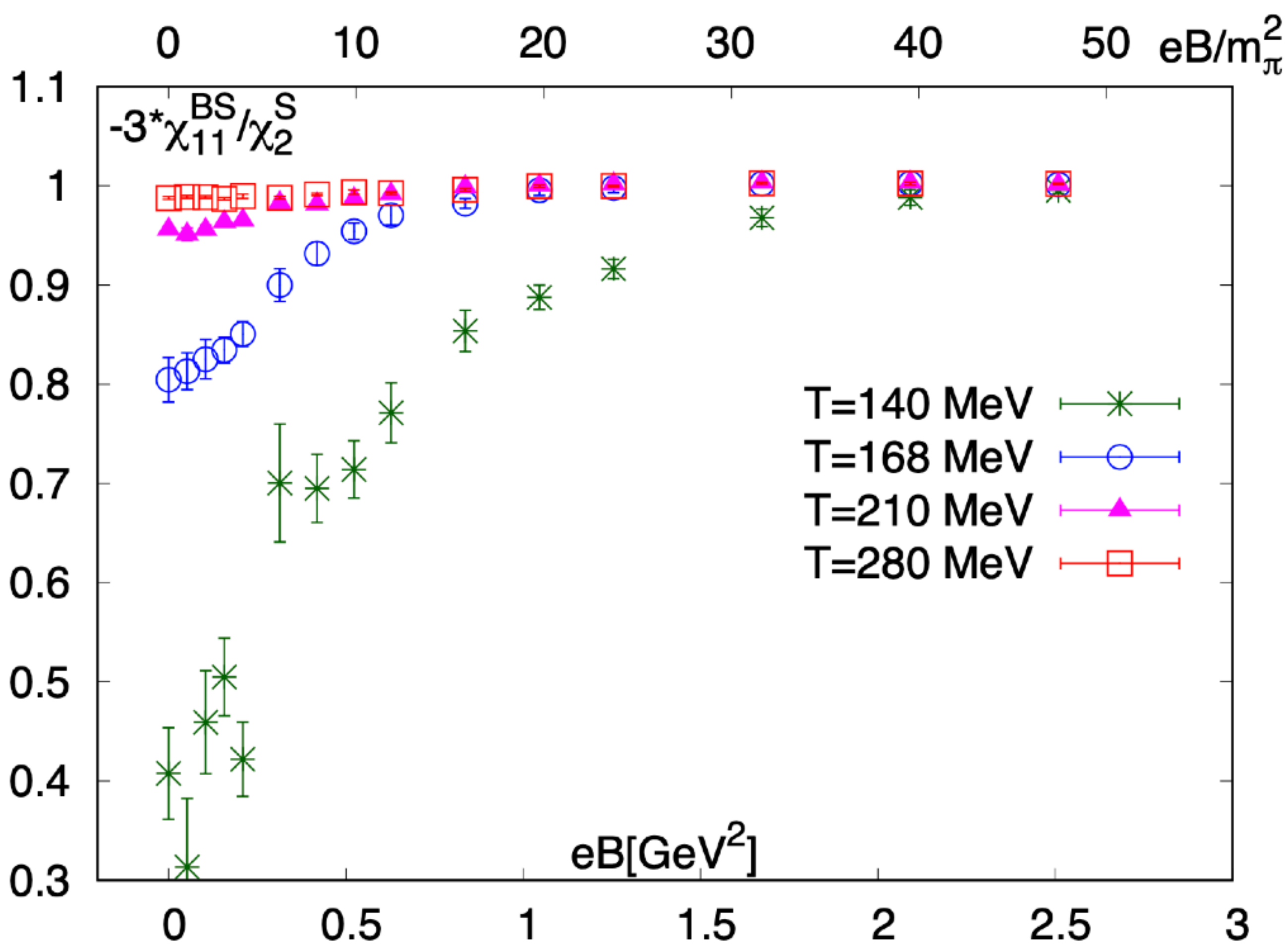


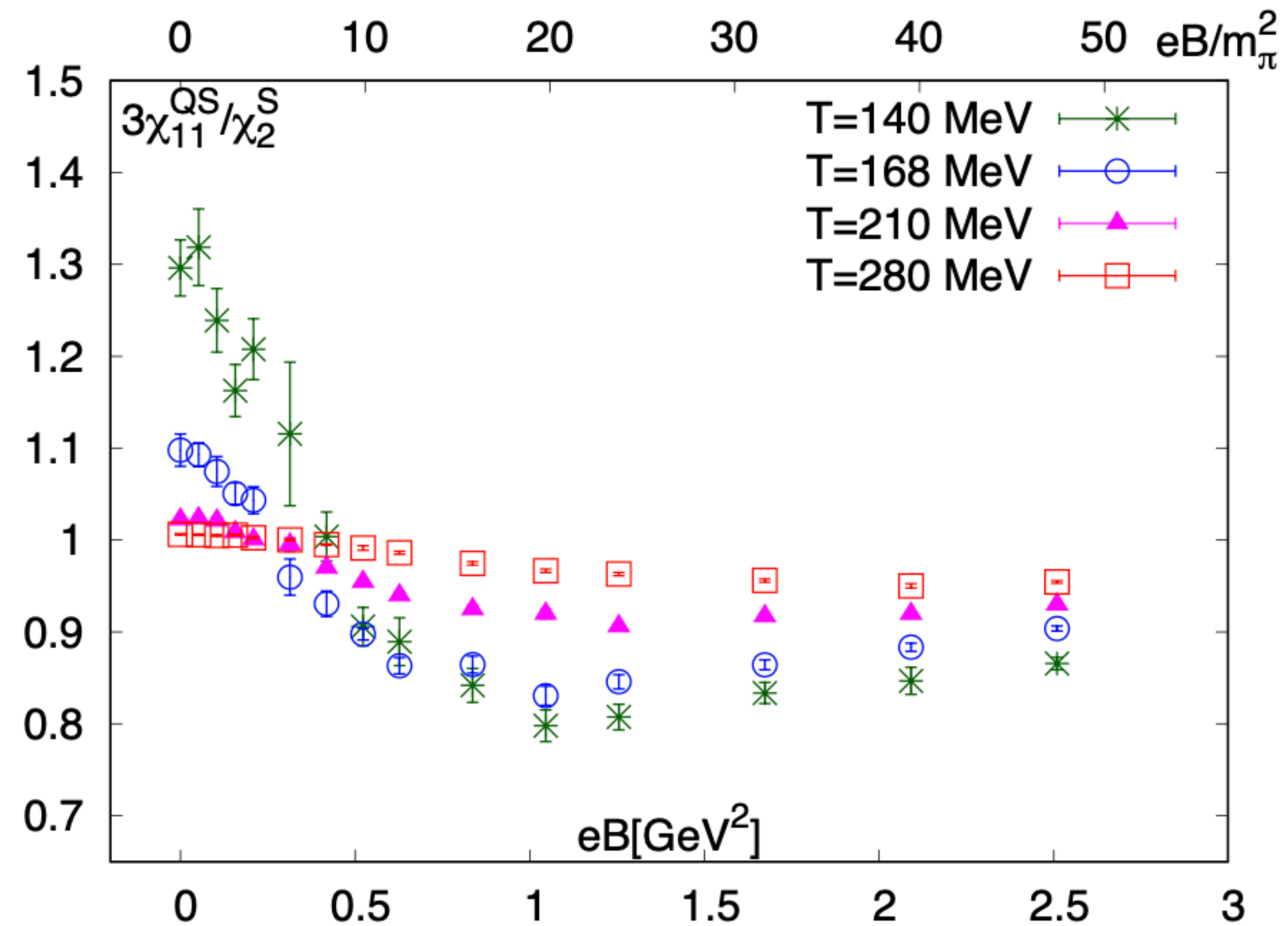


- Up and down quark sus. are denigrate at eB=0 and start to deviate at eB=/=0
- Ratio for down to strange quark sus. is independent on eB at high T, while decreases at two low temperatures

Fluctuations of conserved charges

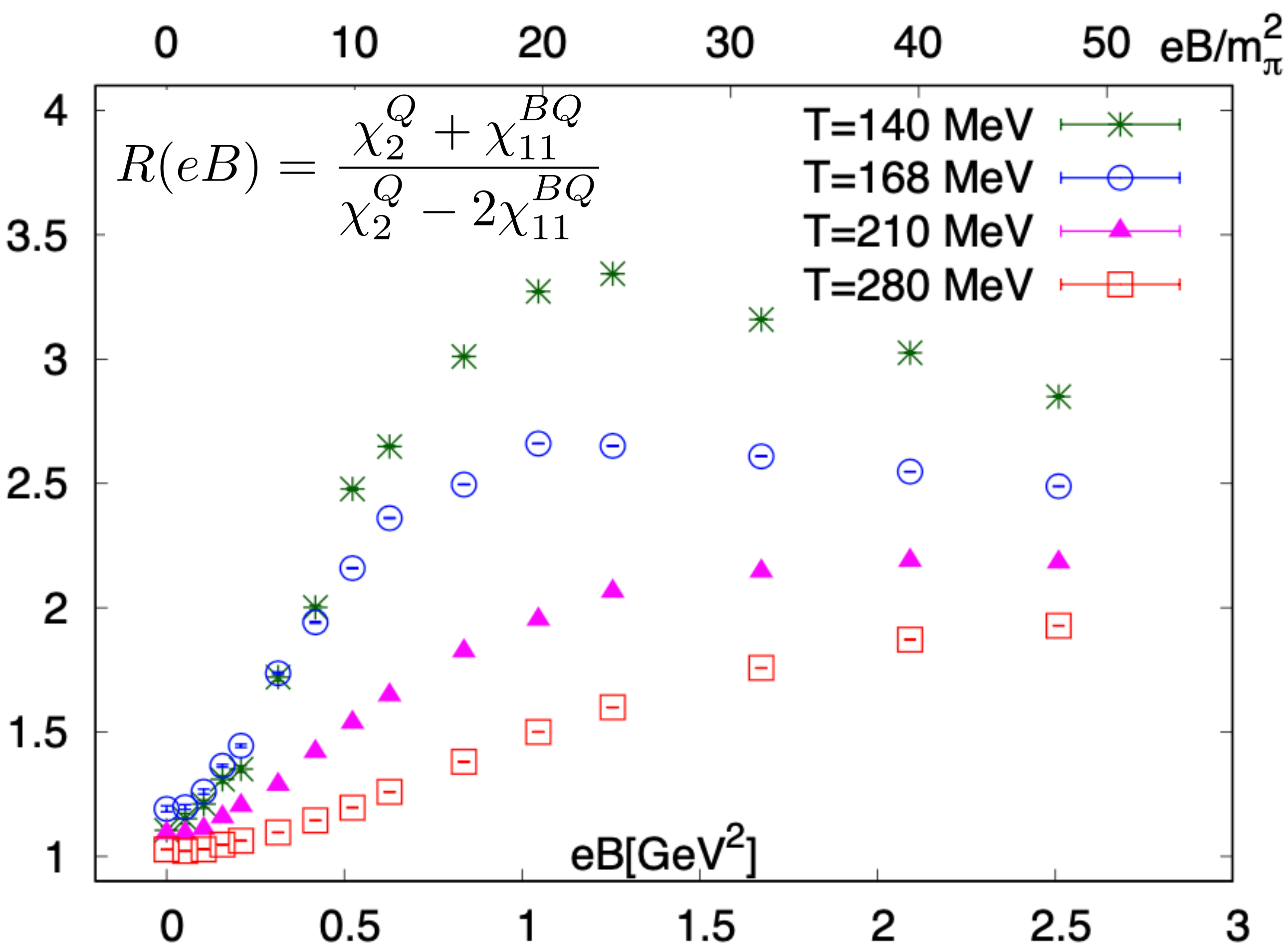
Quantities that reach unity at high T at eB=0

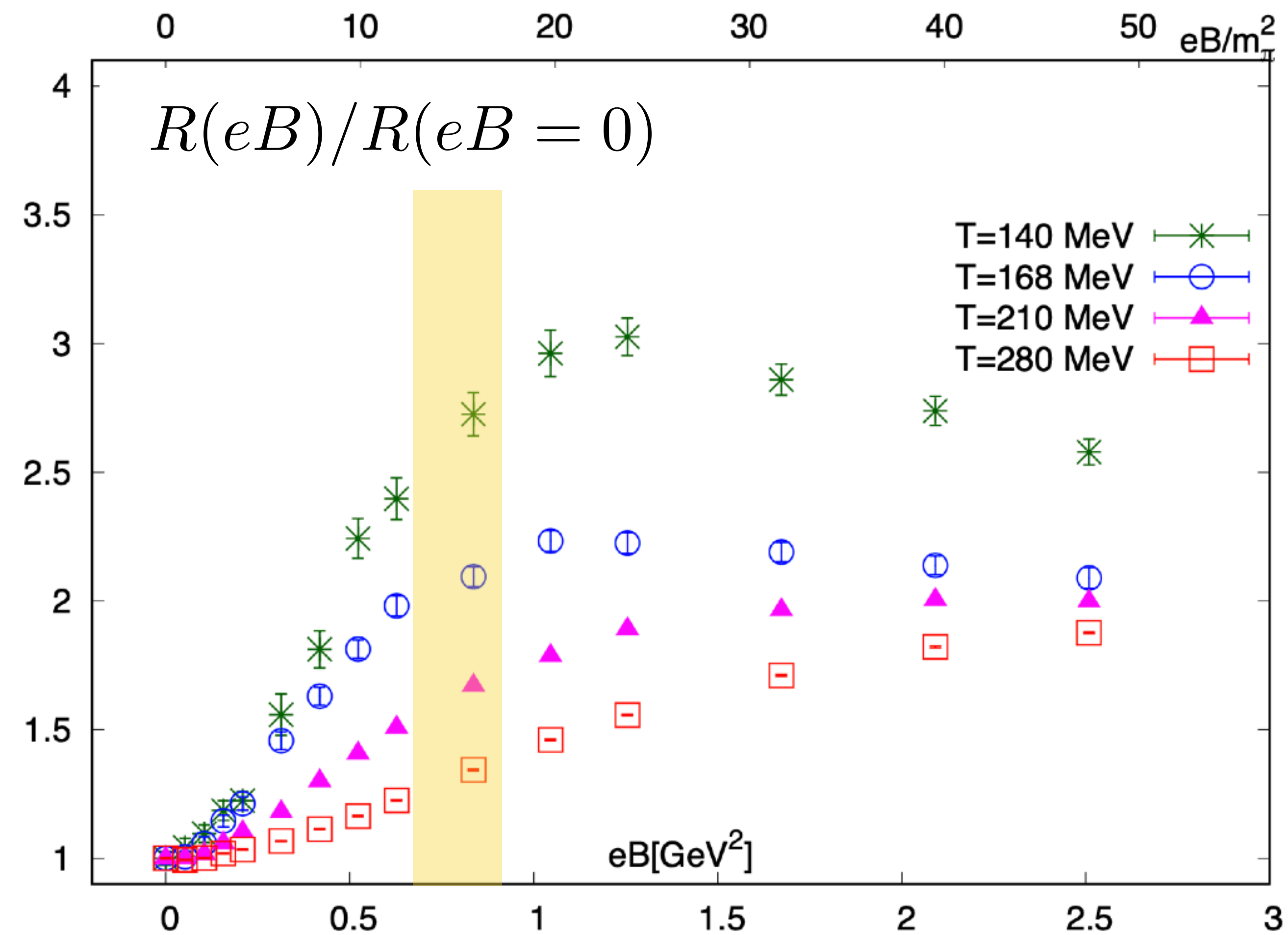




Fluctuations of conserved charges

Quantities that probes χ_2^u/χ_2^d at high T at eB=/=0





- RHIC: $eB \sim m_\pi^2$, LHC: $eB \sim 15m_\pi^2$

Skokov, Illarionov and V.Toneev, IJMPA 24 (2009) 5925

- Comparison between fluctuations in peripheral and central collisions to check the existence of the magnetic field ?

Summary & Conclusion

- ☑ Gell-Mann-Oakes-Renner relation holds true for neutral pion for eB up to ~3.5 GeV^2

- ☑ qB scaling is observed in up and down quark flavor components of neutral pion masses, neutral pion decay constants and quark chiral condensates

- ○ Puzzle in the pion magnetic polarizability: results in full QCD deviates more from the experiment results than that of quenched QCD

- ○ Large effects of magnetic field on the fluctuations of conserved charges are found, which may could be used to detect the existence of magnetic field in experiments

Thanks for your attention!