Unified quark-hadron EoS and CEP in the QCD phase diagram [1]

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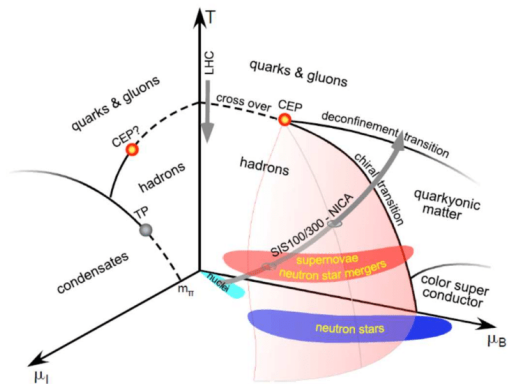Workshop on Criticality in QCD and the Hadron Resonance Gas
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[1]Collab: N.-U. Bastian, **K. Devyatyarov**, A. Dubinin, A. Friesen, G. Röpke, ...

Plan of the talk

1. Motivation: 3D phase diagram and astrophysics of deconfinement in supernovae, mergers and neutron stars

2. How to construct a unified approach ?
   - Cluster decomposition: quarks, gluons, mesons, baryons → Beth-Uhlenbeck approach
   - Two challenges:
     a) confinement of quarks and gluons and
     b) Mott dissociation of hadrons
   - Backreaction via order parameters:
     → Polyakov-loop and chiral condensate (quark masses)

3. A simple model for quark-nuclear matter at low temperatures
   → phase diagram: CEP and crossover

4. Cluster decomposition for quark-gluon matter with Mott-HRG
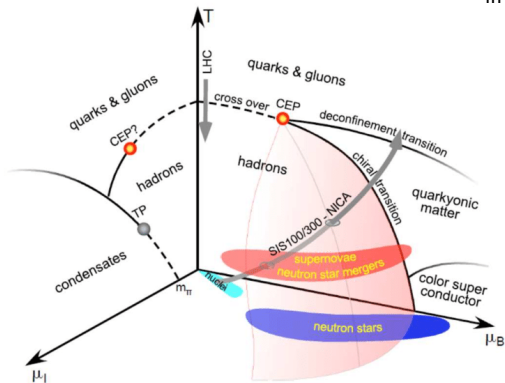   → comparison to lattice QCD

QCD Phase Diagram: Astrophysics vs. Heavy-Ion Coll.

From: NuPECC Long Range Plan 2017

QCD Phase Diagram: Astrophysics vs. Heavy-Ion Coll.

From: NuPECC Long Range Plan 2017

Prominent contributions to deconfinement in modern multimessenger Astrophysics:

- Quark deconfinement transition triggers the supernova explosion of a very massive ($M = 50 M_\odot$) blue supergiant progenitor star
  T. Fischer et al., Nature Astron. 2 (2018) 960

- Unambiguous signal of a strong phase transition in the postmerger GW from a binary NS merger predicted
  A. Bauswein et al., Phys. Rev. Lett. 122 (2019) 061102

- Strong deconfinement phase transition in NS can be detected by observing the mass twin star phenomenon
  D. B. et al., Universe 6 (2020) 81

Φ—Derivable Approach to the Cluster Virial Expansion

$$\Omega = \sum_{l=1}^{A} \Omega_l = \sum_{l=1}^{A} \left\{ c_l \left[\text{Tr} \ln \left(-G_l^{-1} \right) + \text{Tr} \left(\Sigma_l \; G_l \right) \right] + \sum_{\substack{i,j \\ i+j=l}} \Phi[G_i, G_j, G_{i+j}] \right\} ,$$

$$G_A^{-1} = G_A^{(0)^{-1}} - \Sigma_A , \; \Sigma_A(1 \ldots A, 1' \ldots A', z_A) = \frac{\delta \Phi}{\delta G_A(1 \ldots A, 1' \ldots A', z_A)}$$
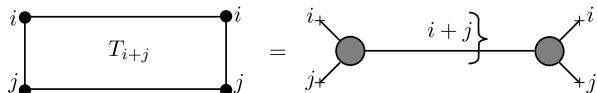
Stationarity of the thermodynamical potential is implied

$$\frac{\delta \Omega}{\delta G_A(1 \ldots A, 1' \ldots A', z_A)} = 0 .$$

Cluster virial expansion follows for this $\Phi-$ functional

Figure: The Φ functional for $A-$particle correlations with bipartitions $A = i + j$.

Green's function and T-matrix: separable approximation

The T_A matrix fulfills the Bethe-Salpeter equation in ladder approximation

$$T_{i+j}(1, 2, \ldots, A; 1', 2', \ldots A'; z) = V_{i+j} + V_{i+j} G_{i+j}^{(0)} T_{i+j} \,,$$

which in the separable approximation for the interaction potential,

$$V_{i+j} = \Gamma_{i+j}(1, 2, \ldots, i; i+1, i+2, \ldots, i+j)\Gamma_{i+j}(1', 2', \ldots, i'; (i+1)', (i+2)', \ldots, (i+j)') \,,$$

leads to the closed expression for the T_A matrix

$$T_{i+j}(1, 2, \ldots, i+j; 1', 2', \ldots (i+j)'; z) = V_{i+j} \left\{ 1 - \Pi_{i+j} \right\}^{-1} \,,$$

with the generalized polarization function

$$\Pi_{i+j} = \mathrm{Tr} \left\{ \Gamma_{i+j} G_i^{(0)} \Gamma_{i+j} G_j^{(0)} \right\}$$

The one-frequency free $i-$particle Green's function is defined by the $(i-1)$-fold Matsubara sum

$$
\begin{aligned}
G_i^{(0)}(1, 2, \ldots, i; \Omega_i) &= \sum_{\omega_1 \ldots \omega_{i-1}} \frac{1}{\omega_1 - E(1)} \frac{1}{\omega_2 - E(2)} \cdots \frac{1}{\Omega_i - (\omega_1 + \ldots \omega_{i-1}) - E(i)} \\
&= \frac{(1-f_1)(1-f_2)\ldots(1-f_i) - (-)^i f_1 f_2 \ldots f_i}{\Omega_i - E(1) - E(2) - \ldots E(i)} \,.
\end{aligned}
$$

Useful relationships for many-particle functions

$$G_{i+j}^{(0)} = G_{i+j}^{(0)}(1, 2, \ldots, i+j; \Omega_{i+j}) \quad = \quad \sum_{\Omega_i} G_i^{(0)}(1, 2, \ldots, i; \Omega_i) G_j^{(0)}(i+1, i+2, \ldots, i+j; \Omega_j) \ .$$

Another set of useful relationships follows from the fact that in the ladder approximation both, the full two-cluster $(i + j$ particle) T matrix and the corresponding Greens' function

$$G_{i+j} = G_{i+j}^{(0)} \left\{ 1 - \Pi_{i+j} \right\}^{-1} \tag{1}$$

have similar analytic properties determined by the $i + j$ cluster polarization loop integral and are related by the identity

$$T_{i+j} G_{i+j}^{(0)} = V_{i+j} G_{i+j} \ . \tag{2}$$

which is straightforwardly proven by multiplying Equation for the $T_{i+j}-$ matrix with $G_{i+j}^{(0)}$ and using Equation (1). Since these two equivalent expressions in Equation (2) are at the same time equivalent to the two-cluster irreducible Φ functional these functional relations follow

$$T_{i+j} = \delta\Phi / \delta G_{i+j}^{(0)} \ ,$$
$$V_{i+j} = \delta\Phi / \delta G_{i+j} \ .$$

Next we prove the relationship to the Generalized Beth-Uhlenbeck approach!

GBU EoS from the Φ−derivable approach

Consider the partial density of the $A-$particle state defined as

$$n_A(T, \mu) = -\frac{\partial \Omega_A}{\partial \mu} = -\frac{\partial}{\partial \mu} d_A \int \frac{d^3 q}{(2\pi)^3} \int \frac{d\omega}{2\pi} \left[\ln\left(-G_A^{-1}\right) + \text{Tr}\left(\Sigma_A \; G_A\right) \right] + \sum_{\substack{i,j \\ i+j=A}} \Phi[G_i, G_j, G_{i+j}] \, . \tag{3}$$

Using spectral representation for $F(\omega)$ and Matsubara summation

$$F(iz_n) = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \frac{\text{Im}F(\omega)}{\omega - iz_n} \, , \quad \sum_{z_n} \frac{c_A}{\omega - iz_n} = f_A(\omega) = \frac{1}{\exp[(\omega - \mu)/T] - (-1)^A}$$

with the relation $\partial f_A(\omega)/\partial \mu = -\partial f_A(\omega)/\partial \omega$ we get for Equation (3) now

$$n_A(T, \mu) = -d_A \int \frac{d^3 q}{(2\pi)^3} \int \frac{d\omega}{2\pi} f_A(\omega) \frac{\partial}{\partial \omega} \left[\text{Im} \ln\left(-G_A^{-1}\right) + \text{Im}\left(\Sigma_A \; G_A\right) \right] + \sum_{\substack{i,j \\ i+j=A}} \frac{\partial \Phi[G_i, G_j, G_A]}{\partial \mu} \, ,$$

where a partial integration over ω has been performed For two-loop diagrams of the sunset type holds a cancellation[2] which we generalize here for cluster states

$$d_A \int \frac{d^3 q}{(2\pi)^3} \int \frac{d\omega}{2\pi} f_A(\omega) \frac{\partial}{\partial \omega} \left(\text{Re}\Sigma_A \; \text{Im}G_A \right) - \sum_{\substack{i,j \\ i+j=A}} \frac{\partial \Phi[G_i, G_j, G_A]}{\partial \mu} = 0 \, .$$

Using generalized optical theorems we can show that $(G_A = |G_A| \exp(i\delta_A))$

$$\frac{\partial}{\partial \omega} \left[\text{Im} \ln\left(-G_A^{-1}\right) + \text{Im}\Sigma_A \; \text{Re}G_A \right] = 2\text{Im} \left[G_A \; \text{Im}\Sigma_A \frac{\partial}{\partial \omega} G_A^* \; \text{Im}\Sigma_A \right] = -2\sin^2 \delta_A \frac{\partial \delta_A}{\partial \omega} \, .$$

The density in the form of a generalized Beth-Uhlenbeck EoS follows

$$n(T, \mu) = \sum_{i=1}^{A} n_i(T, \mu) = \sum_{i=1}^{A} d_i \int \frac{d^3 q}{(2\pi)^3} \int \frac{d\omega}{2\pi} f_i(\omega) 2 \sin^2 \delta_i \frac{\partial \delta_i}{\partial \omega} \, .$$

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[2] B. Vanderheyden & G. Baym, J. Stat. Phys. (1998), J.-P. Blaizot et al., PRD (2001)

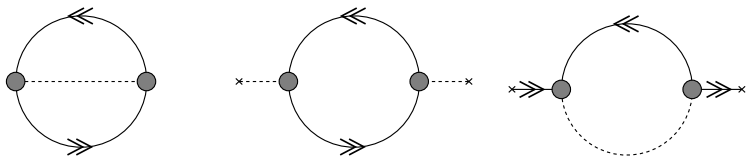# Example: Mott Dissociation of Pions in Quark Matter

Figure: The Φ functional (left panel) for the case of mesons in quark matter, where the bosonic meson propagator is defined by the dashed line and the fermionic quark propagators are shown by the solid lines with arrows. The corresponding meson and quark selfenergies are shown in the middle and right panels, respectively.

Mott Dissociation of Pions in Quark Matter

The meson polarization loop $\Pi_M(q, z)$ enters the definition of the meson T matrix

$$T_M^{-1}(q, \omega + i\eta) = G_S^{-1} - \Pi_M(q, \omega + i\eta) = |T_M(q, \omega)|^{-1} e^{-i\delta_M(q,\omega)} \ ,$$

which in the polar representation introduces a phase shift $\delta_M(q, \omega) = \arctan(\Im T_M / \Re T_M)$, that results in a generalized Beth-Uhlenbeck equation of state for the thermodynamics of the consistently coupled quark-meson system

$$\Omega = \Omega_{\mathrm{MF}} + \Omega_M \ ,$$

where the selfconsistent quark meanfield contribution is

$$\Omega_{\mathrm{MF}} = \frac{\sigma_{\mathrm{MF}}^2}{4 G_S} - 2 N_c N_f \int \frac{d^3 p}{(2\pi)^3} \left[E_p + T \ln \left(1 + e^{-(E_p - \Sigma_+ - \mu)/T} \right) + T \ln \left(1 + e^{-(E_p + \Sigma_- + \mu)/T} \right) \right] \ ,$$

The mesonic contribution to the thermodynamics is

$$\Omega_M = d_M \int \frac{d^3 k}{(2\pi)^3} \int \frac{d\omega}{2\pi} \left\{ \omega + 2T \ln \left[1 - e^{-\omega/T} \right] \right\} 2 \sin^2 \delta_M(k, \omega) \, \frac{\delta_M(k, \omega)}{d\omega} \ ,$$

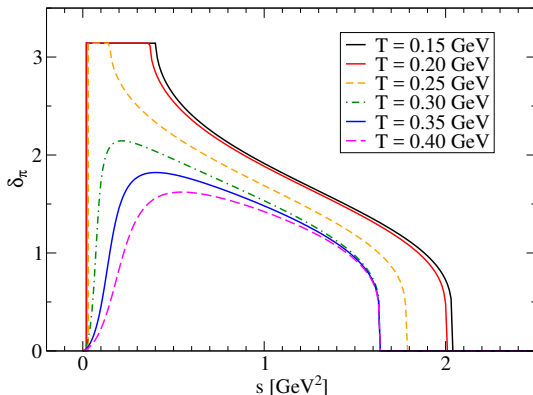# Mott Dissociation of Pions in Quark Matter

Figure: Phase shift of the pion as a quark-antiquark state for different temperatures, below and above the Mott dissociation temperature.
From D.B., M. Buballa, et al., Ann. Phys. 348 (2014) 228.
See also D.B., A. Dubinin, D. Ebert, A.V. Friesen, PPN Lett. 15 (2018) 230.

Φ-derivable cluster expansion for quark-nuclear matter

Cluster decomposition of the thermodynamic potential

$$
\begin{aligned}
\Omega &= \sum_i \Omega_i \\
&= \sum_i \Big\{ c_i \left[\text{Tr} \ln \left(-G_i^{-1} \right) + \text{Tr} \left(\Sigma_i \, G_i \right) \right] + \sum_{\substack{j,k \\ j+k=i}} \Phi[G_j, G_k, G_{j+k}] \Big\} \ ,
\end{aligned}
$$

generates the partial densities

$$
\begin{aligned}
n_j(T, \mu) = -\frac{\partial \Omega}{\partial \mu_j} &= \sum_{i=1} A_{ij} n_i(T, \mu) \\
&= \sum_{i=1} A_{ij} g_i \int \frac{\mathrm{d}^3 p}{(2\pi)^3} \int \frac{\mathrm{d}E}{2\pi} f_i(E) 2 \sin^2 \delta_i(E) \frac{\mathrm{d}\delta_i(E)}{\mathrm{d}E} \ ,
\end{aligned}
$$

Based on: N.-U. F. Bastian and D.B., arXiv:1812.11889

Φ-derivable cluster expansion for quark-nuclear matter

Figure:
Upper row: full set of sunset type diagrams.
Lower row: after collapsing the diquark and meson-propagators ($\rightarrow$ mean fields).

Relativistic Density Functional Approach to Nuclear Matter

In case of color confinement all closed loop diagrams with Q- and D-lines vanish. The system reduces to the M-B- system. The Φ−functional becomes a density functional.

$$\Phi = \qquad\qquad\qquad \implies \qquad \mathcal{U} =$$

$$\Omega = T \sum_{i=n,p,\Lambda,\dots} c_i \left[\mathrm{Tr}\ln S_{i,qu}^{-1} + \sum_{j=S,V} n_{i,j}\Sigma_{i,j} \right] + U\left[\{n_{i,S}, n_{i,V}\}\right] \ ,$$

$$\frac{\partial\Omega}{\partial n_{i,S}} = \frac{\partial\Omega}{\partial n_{i,V}} = 0 \ , \quad i = n, p, \Lambda, \dots \ , \qquad\qquad \frac{\partial U}{\partial n_{i,S}} = \Sigma_{i,S} \ , \quad \frac{\partial U}{\partial n_{i,V}} = \Sigma_{i,V} \ .$$

The baryon quasiparticle propagators fulfill the Dyson equations $S_{i,qu}^{-1} = S_{i,0}^{-1} - \Sigma_{i,S} - \Sigma_{i,V}$,

Φ-derivable cluster expansion for quark-nuclear matter

$$n_i^{\text{free}} = g_i \int \frac{\mathrm{d}^3 p}{(2\pi)^3} \int \frac{\mathrm{d}M}{2\pi} f_i \left(\sqrt{p^2 + M^2} + V_i \right) 2 \sin^2 \delta_i(M) \frac{\mathrm{d}\delta_i(M)}{\mathrm{d}M}$$

Phase shift model:

$\delta_{i=\text{p,n}}(M) = \pi \, \Theta(M - M_i) \, \Theta(M_i^{\text{thr}} - M)$ with $M_{\text{p}}^{\text{thr}} = 2M_{\text{u}} + M_{\text{d}}$ etc.

$$n_{i=\text{p,n}} = g_i \int \frac{\mathrm{d}^3 p}{(2\pi)^3} \left[f_i(\sqrt{p^2 + M_i^2} + V_i) - f_i(\sqrt{p^2 + (M_i^{\text{thr}})^2} + V_i) \right] \Theta(M_i^{\text{thr}}$$

$$= \left(n_{\text{N}}^{\text{qu}} - n_{\text{N}}^{\text{thr}} \right) \Theta(M_i^{\text{thr}} - M_i)$$

Selfenergies: Relativistic Density Functionals

Quark Matter: $U_{\mathrm{SFM}} = D(n_{\mathrm{v}}) n_{\mathrm{q,s}}^{2/3} + a n_{\mathrm{q,v}}^2$.

$$S_{\mathrm{SFM}} = \frac{2}{3} D(n_{\mathrm{q,v}}) \, n_{\mathrm{q,s}}^{-1/3} \; , \quad D(n_{\mathrm{v}}) = D_0 \exp(-\alpha n_{\mathrm{q,v}}^2) \tag{5}$$

Nuclear Matter:

$$U_{\mathrm{DD2}} = -\frac{1}{2} \frac{\Gamma_\sigma^2}{m_\sigma^2} n_{\mathrm{s}}^2 + \frac{1}{2} \frac{\Gamma_\omega^2}{m_\omega^2} n_{\mathrm{v}}^2 + \frac{1}{2} \frac{\Gamma_\rho^2}{m_\rho^2} n_{\mathrm{vi}}^2 \; , \tag{6}$$

Scalar self-energy: $S_{\mathrm{DD2}} = -(\Gamma_\sigma^2/m_\sigma^2) n_{\mathrm{s}}$, Vector self-energy:

$$V_{\mathrm{DD2},i} = \frac{\Gamma_\omega^2}{m_\omega^2} n_{\mathrm{v}} + \tau_i \frac{\Gamma_\rho^2}{m_\rho^2} n_{\mathrm{vi}} - \frac{\Gamma_\sigma \Gamma_\sigma'}{m_\sigma^2} n_{\mathrm{s}}^2 + \frac{\Gamma_\omega \Gamma_\omega'}{m_\omega^2} n_{\mathrm{v}}^2 + \frac{\Gamma_\rho \Gamma_\rho'}{m_\rho^2} n_{\mathrm{vi}}^2 \; . \tag{7}$$

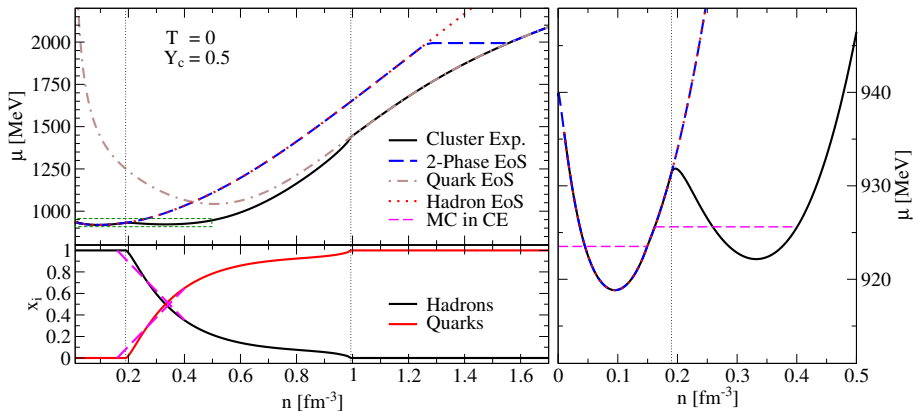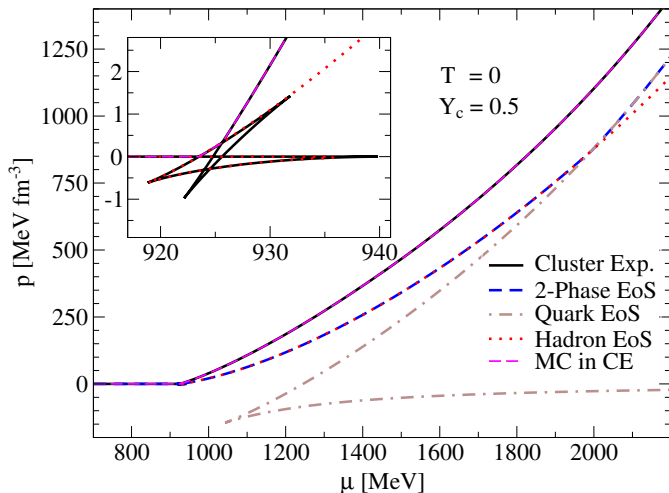# Thermodynamics of quark-nuclear matter

Figure: Van der Waals type thermodynamic behaviour with two first order phase transitions: liquid-gas and deconfinement. Note the wide density range for quark-hadron mixture after the deconfinement transition.

Maxwell construction for quark-nuclear matter EoS
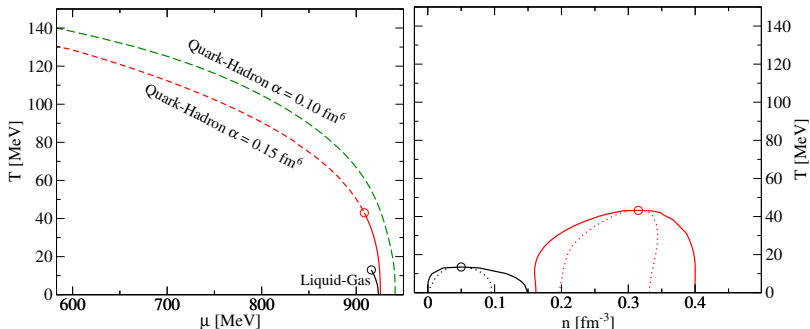
Phase diagram of quark-nuclear matter

Figure: Phase diagram of the cluster expansion, with the liquid-gas phase transition (black) and two parametrizations of quark-hadron transition. One features a first order phase transition (red) with mixed phase and a critical endpoint at $T_C = 43$ MeV and $\mu_C = 909$ MeV. The other represents a crossover all over scenario (green) without critical endpoint. Right panel: the borders of the phase coexistence regions (binodals) for liquid-gas (black) and quark-hadron (red) transitions; dotted lines show the spinodal region of thermodynamic instability ($\partial\mu/\partial n < 0$).

More details in: N.-U. F. Bastian and D.B., arXiv:1812.11889

Unified q-g-H EoS in the Beth-Uhlenbeck approach

$$P_{\text{total}}(T) = P_{\text{MHRG}}(T) + P_{\text{PNJL}}(T) + P_{\text{pert}}(T) , \qquad (8)$$

Beth-Uhlenbeck model for HRG with Mott dissociation

$$P_{\text{MHRG}}(T) = \sum_{i=M,B} P_i(T) , \qquad (9)$$

$$P_i(T) = \mp d_i \int_0^\infty \frac{dp\ p^2}{2\pi^2} \int_0^\infty \frac{dM}{\pi}\ T \ln\left(1 \mp e^{-\sqrt{p^2+M^2}/T}\right) \frac{d\delta_i(M^2;T)}{dM}$$

Simple phase shift model (see above), fulfilling Levinson's theorem

$$\delta_i(M^2;T) = \pi\left[\Theta(M - M_i) - \Theta(M - M_{\text{thr},i}(T))\right]\Theta(M_{\text{thr},i}(T) - M_i).$$

$$P_i(T) = \mp d_i \int_0^\infty \frac{dp\ p^2}{2\pi^2}\ T \ln\left(\frac{1 \mp e^{-\sqrt{p^2+M_i^2}/T}}{1 \mp e^{-\sqrt{p^2+M_{\text{thr},i}(T)^2}/T}}\right)\Theta(M_{\text{thr},i}(T) - M_i)$$

$$(10)$$

Model for the continuum threshold

$$M_{\mathrm{thr},i}(T) = \sqrt{2}\left[(N_i - N_s)m_l(T) + N_s m_s(T)\right], \ q = l, s$$

$$m_q(T) = \Delta m \, \Delta_{l,s}(T) + m_{q,0}, \ m_{l,0} = 5 \ \mathrm{MeV}, \ m_{s,0} = 135 \ \mathrm{MeV}$$
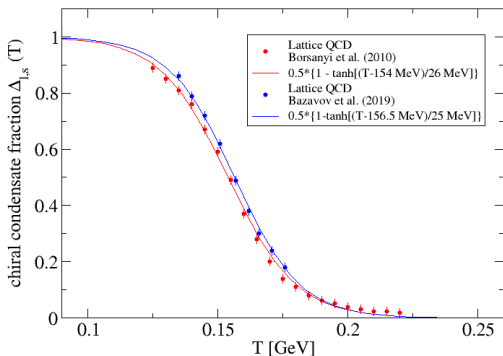

Figure: Chiral condensate fraction and fit function.
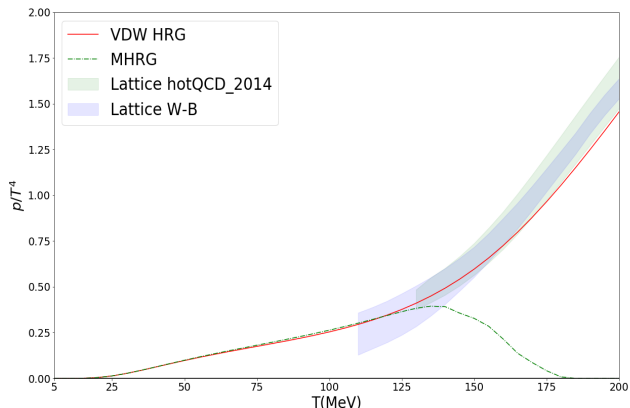
HRG with Mott dissociation

Figure: Hadron resonance gas model without and with Mott dissociation of hadrons, comparison with lattice QCD thermodynamics.
From K. Devyatyarov, Master Thesis, NRNU (MEPhI) Moscow (2020).

Polyakov-loop improved Nambu–Jona-Lasinio model

The underlying quark and gluon thermodynamics is divided into a perturbative contribution $P_{\mathrm{pert}}(T)$ which is treated as virial correction in two-loop order and a nonperturbative part described within a PNJL model

$$P_{\mathrm{PNJL}}(T) = P_Q(T) + U(\phi, T) \tag{11}$$

where the quark quasiparticle pressure contribution is

$$P_Q(T) = 4N_c \sum_{q=u,d,s} \int \frac{dp\, p^2}{2\pi^2} \frac{T}{3} \ln\left[1 + 3\phi(1 + Y)Y + Y^3\right], \ Y = e^{-E(p)/T} \tag{12}$$

and $U(\phi; T)$ is the nonperturbative gluon background in a meanfield approximation using the polynomial fit of Ratti et al. PRD **73** (2006)

$$U(\phi, T) = \frac{b_2(T)}{2}\phi^2 + \frac{b_3}{3}\phi^3 - \frac{b_4}{4}\phi^4 \tag{13}$$

Perturbative contribution: two-loop diagram

The 2-loop contribution to the pressure (quark-gluon sunset diagram) in the massless limit is defined by

$$P_{pert}(\phi, T) = -\frac{8}{\pi}\alpha_s T^4 \left\{ I_\Lambda(\phi, T) + \frac{3}{\pi^2}\left[I_\Lambda(\phi, T)\right]^2 \right\} \qquad (14)$$

with the integral

$$I_\Lambda(\phi, T) = \int_{\Lambda/T}^{\infty} \mathrm{d}x \; x \; f_\phi(x), \qquad (15)$$

where the Polyakov-loop generalized Fermi distribution function is

$$f_\phi(x) = [\phi(1 + 2Y)Y + Y^3]/[1 + 3\phi(1 + Y)Y + Y^3], \; Y = \exp(-x) \;(16)$$

and $\Lambda = m_l(T)$ is the momentum range below which nonperturbative physics dominates, accounted for by a dynamically generated quark mass. We use here a temperature dependent, regularized running coupling

$$\alpha_s = \frac{g^2}{4\pi} = \frac{12\pi}{11N_c - 2N_f}\left(\frac{1}{\ln(r^2/c^2)} - \frac{c^2}{r^2 - c^2}\right), \qquad (17)$$

where $r = 3.2T$, $c = 350$ MeV and $N_c = N_f = 3$.

Gap equation for the Polyakov loop

$$\frac{dP_{\text{total}}}{d\phi} = \frac{dU(\phi, T)}{d\phi} + \frac{dP_Q(T)}{d\phi} + \frac{dP_{pert}(T)}{d\phi} = 0$$

$$\frac{dU(\phi, T)}{d\phi} = b_2(T)\phi + b_3\phi^2 - b_4\phi^3$$

$$\frac{dP_Q(T)}{d\phi} = 4N_c \sum_{q=u,d,s} \int \frac{dp\, p^2}{2\pi^2} \frac{(1+Y)Y}{1 + 3\phi(1+Y)Y + Y^3}, \quad Y = e^{-E(p)/T}$$

$$\frac{dP_{\text{pert}}(T)}{d\phi} = -\frac{8}{\pi}\alpha_s T^4 \left[\frac{dI(\phi, T)}{d\phi} + \frac{6}{\pi^2} I(\phi, T)\frac{dI(\phi, T)}{d\phi} \right]$$

$$\frac{dI(\phi, T)}{d\phi} = \int_{\Lambda/T}^{\infty} \mathrm{d}x\, x \frac{df_\phi(\omega)}{d\phi},$$

$$\frac{df_\phi(\omega)}{d\phi} = \frac{Y + 2Y^2 - 2Y^4 - Y^5}{(1 + 3\phi(1+Y)Y + Y^3)^2} = \frac{(1+2Y)Y - (2+Y)Y^4}{(1 + 3\phi(1+Y)Y + Y^3)^2}$$
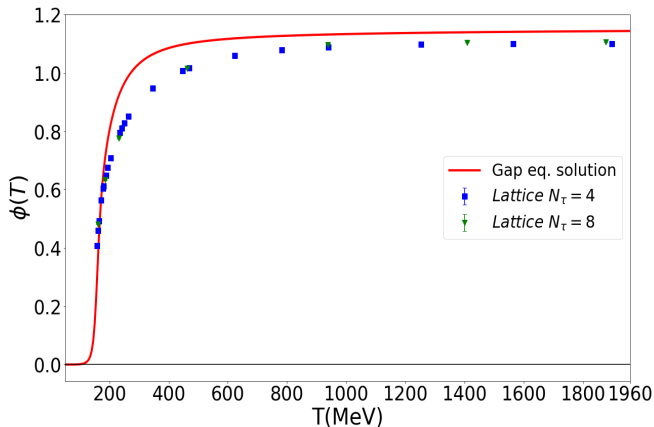
Gap equation for the Polyakov loop

Figure: Temperature dependence of the Polyakov loop (PL) within the unified q-g-H model, comparison with renormalized PL of S. Gupta, K. Huebner and O. Kaczmarek, PRD 77 (2008).
From D.B., K. Devyatyarov and O. Kaczmarek, in preparation (2020).

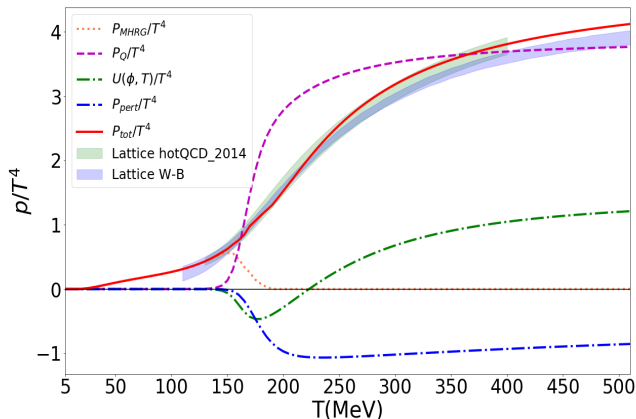# Thermodynamics of the unified q-g-H model

Figure: Temperature dependence of the total pressure and its components within the unified q-g-H model, comparison with lattice QCD data.
From D.B., K. Devyatyarov and O. Kaczmarek, in preparation (2020).
Compare with earlier results: D.B., A. Dubinin, L. Turko, APPB 10, 473 (2017).
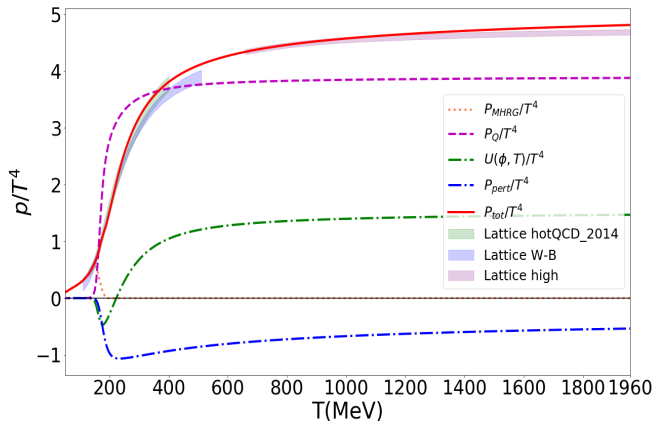
Thermodynamics of the unified q-g-H model

Figure: Temperature dependence of the total pressure and its components within the unified q-g-H model, comparison with lattice QCD data up to $T = 1960$ MeV. From D.B., K. Devyatyarov and O. Kaczmarek, in preparation (2020).

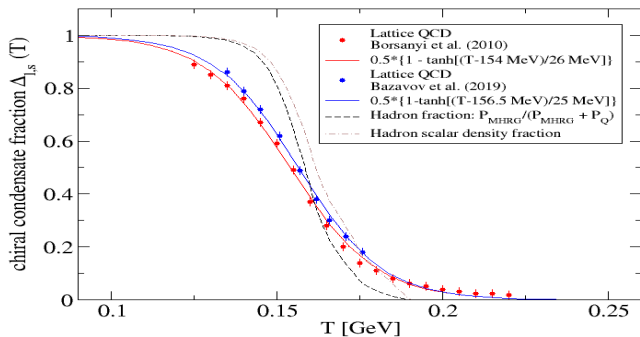# Consistency check: Chiral condensate and hadron fraction

Figure: MHRG pressure fraction and scalar density fraction compared to lattice QCD data for the chiral condensate ratio.
From D.B., K. Devyatyarov and O. Kaczmarek, in preparation (2020).

"In-hadron condensate": Brodsky, Roberts, et al., Phys.Rev.C 85 (2012))

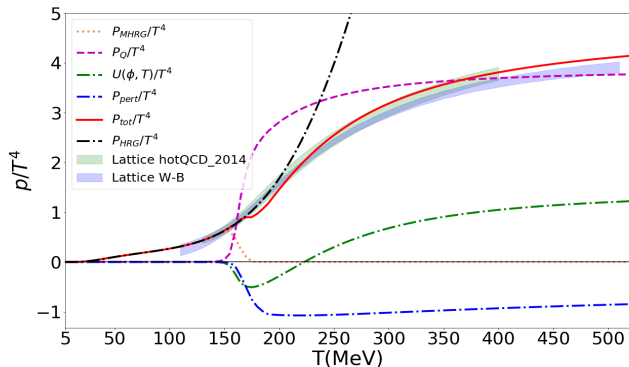# Consistency check: Chiral condensate and hadron fraction

Figure: Temperature dependence of the total pressure and its components within the unified q-g-H model, comparison with lattice QCD data..
From D.B., K. Devyatyarov and O. Kaczmarek, in preparation (2020).
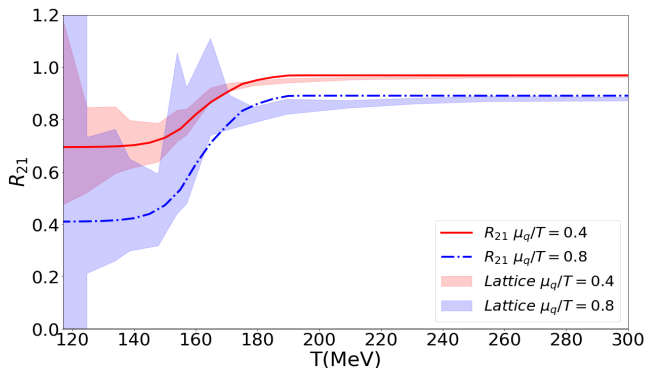
Ratios of susceptibilities - an example

Figure: From D.B., K. Devyatyarov and O. Kaczmarek, in preparation (2020).

Summary

- equivalence of $\Phi-$derivable and generalized Beth-Uhlenbeck approaches shown for Φ functionals that are two-loop diagrams

- cluster virial expansion developed for sunset-type Φ functionals made of cluster Green's functions and a cluster T-matrix

- selfconsistent density-functional approach to quark matter with confinement and chiral symmetry breaking obtained as limiting case

- low-T phase diagram obtained, Existence of CEP depends on details of the density functional

- finite-T thermodynamics of unified q-g-H model in agreement with lattice QCD data

Outlook

- improvement of phase shifts: continuum correlations and hadron broadening

- finite-μ extension straightforward, is there a CEP?

- RDF as confining mechanism at low temperatures - combination with Polyakov-loop?