

THERMODYNAMICS OF COUPLED CHANNEL SYSTEM

POK MAN LO (盧博文)

University of Wrocław

29-31 JULY 2020,
CRITICALITY IN QCD AND
THE HADRON RESONANCE GAS

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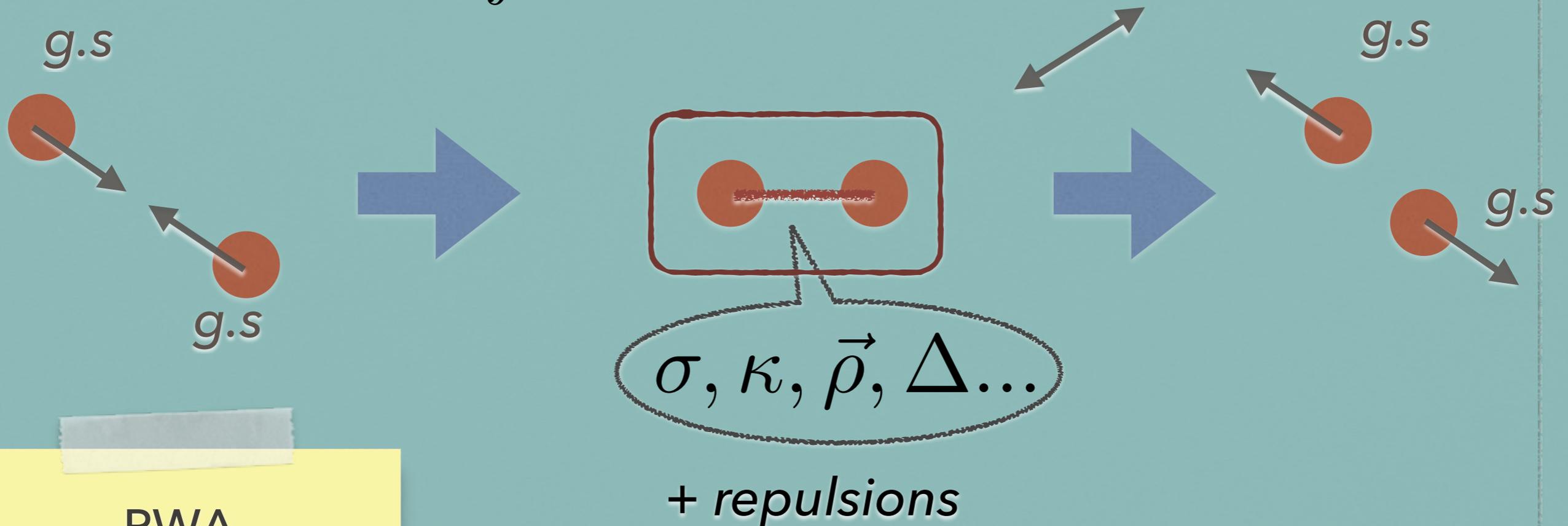
Peter Petreczky

Jean Cleymans

S-MATRIX FORMULATION OF STATISTICAL MECHANICS

S-MATRIX FORMULATION OF STATISTICAL MECHANICS

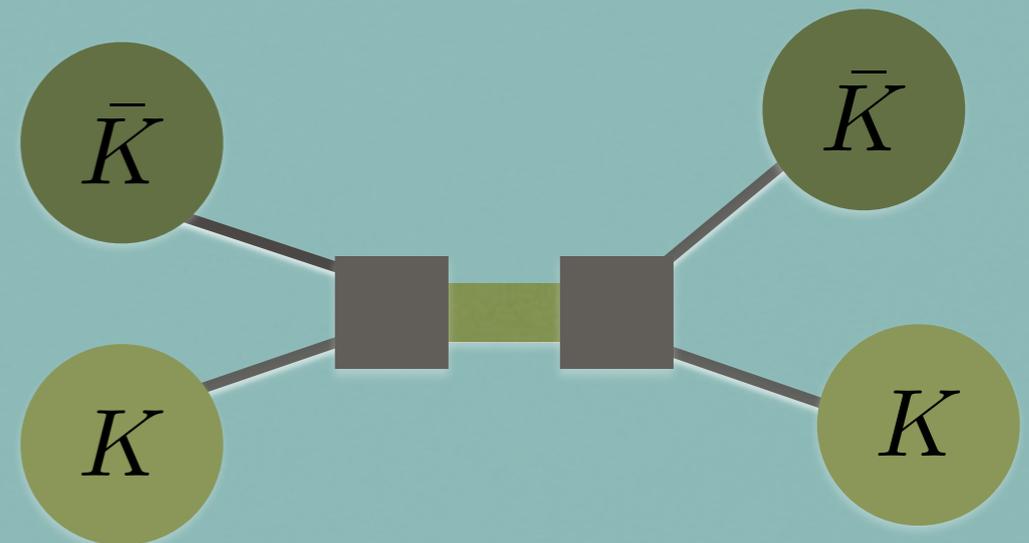
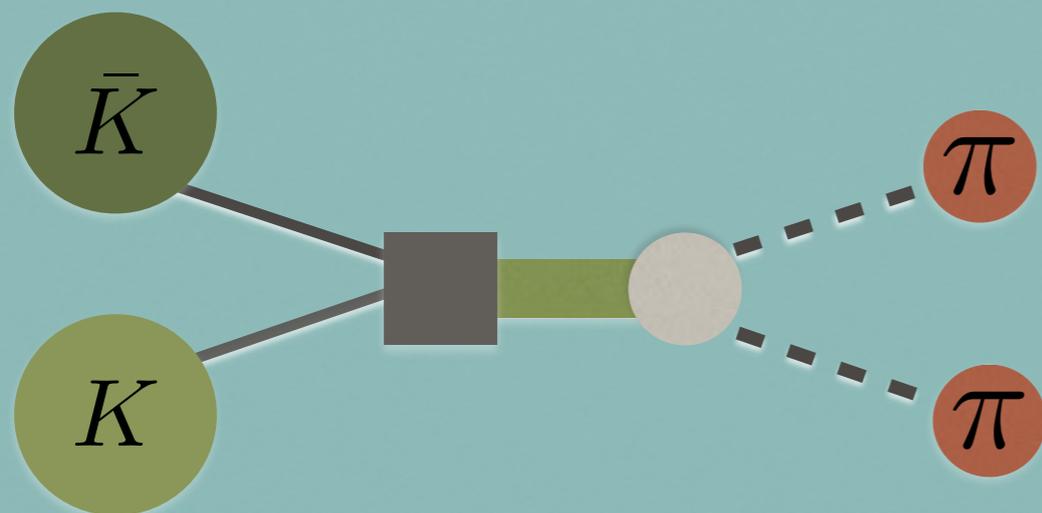
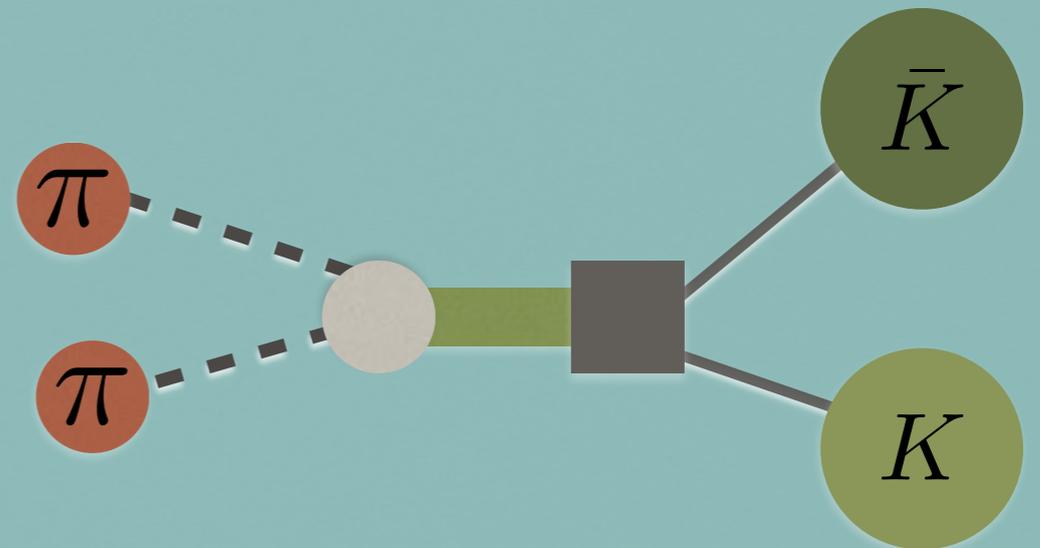
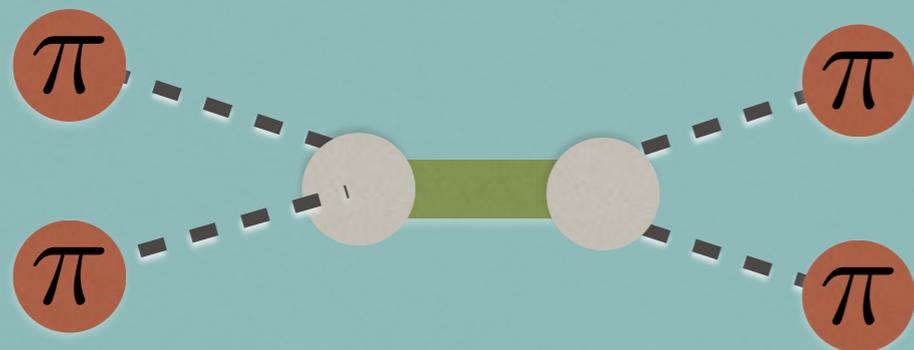
$$\Delta \ln Z = \int dE e^{-\beta E} \times \frac{1}{\pi} \frac{\partial}{\partial E} \text{tr} (\delta_E).$$



PWA
~~X~~
 S-matrix thermo.

$$\delta \longrightarrow Q(M) \equiv \frac{1}{2} \text{Im} (\text{tr} \ln S)$$

1 RES. 2 CHANNELS PROBLEM



COUPLED-CHANNEL PROBLEM

$$\{\gamma_1, \gamma_2, m_{\text{res}}\} \longleftrightarrow \{\delta_1, \delta_2, \eta\}$$

$$S = \begin{pmatrix} \eta e^{2i\delta_I} & i\sqrt{1-\eta^2} e^{i(\delta_I+\delta_{II})} \\ i\sqrt{1-\eta^2} e^{i(\delta_I+\delta_{II})} & \eta e^{2i\delta_{II}} \end{pmatrix}$$

branching ratio

$f_0(980)$ system

$$\pi\pi \rightarrow \begin{pmatrix} \pi\pi \\ K\bar{K} \end{pmatrix} \rightarrow \pi\pi$$

$$K\bar{K} \rightarrow \begin{pmatrix} \pi\pi \\ K\bar{K} \end{pmatrix} \rightarrow K\bar{K}$$

COUPLED-CHANNEL PROBLEM

$$\{\gamma_1, \gamma_2, m_{\text{res}}\} \longleftrightarrow \{\delta_1, \delta_2, \eta\}$$

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f₀(980) system

$$\begin{aligned} Q(M) &\equiv \frac{1}{2} \text{Im} (\text{tr} \ln S) \\ &= \frac{1}{2} \text{Im} (\ln \det [S]) \\ &= \delta_I + \delta_{II}. \end{aligned}$$

$$\begin{aligned} \pi\pi &\rightarrow \begin{pmatrix} \pi\pi \\ K\bar{K} \end{pmatrix} \rightarrow \pi\pi \\ K\bar{K} &\rightarrow \begin{pmatrix} \pi\pi \\ K\bar{K} \end{pmatrix} \rightarrow K\bar{K} \end{aligned}$$

HOW TO COUNT? S-MATRIX POINT OF VIEW

indecently optimistic...

S-Matrix Theory of Strong Interactions without Elementary Particles*†

Geoffrey F. Chew

Department of Physics and Lawrence Radiation Laboratory, University of California, Berkeley, California

1. INTRODUCTION

IN this paper I present an indecently optimistic view of strong interaction theory. My belief is that a major breakthrough has occurred and that within a relatively short period we are going to achieve a depth of understanding of strong interactions that a few years ago I, at least, did not expect to see within my lifetime. I know that few of you will be convinced by the arguments given here, but I would be masking my feelings if I were to employ a conventionally cautious attitude in this talk. I am bursting with excitement, as are a number of other theorists in this game.

tell me that this is a fetish, that field theory is an equally suitable language, but to me the basic strong-interaction concepts, simple and beautiful in a pure S-matrix approach, are weird, if not impossible, for field theory. It must be said, nevertheless, that my own awareness of these concepts was largely achieved through close collaboration with three great experts in field theory, M. L. Goldberger, Francis Low, and Stanley Mandelstam. Each of them has played a major role in the development of the strong interaction theory that I describe,¹ even though the language of my description may be repugnant to them. Murray Gell-Mann, also, although he has not actu-

PARTICLES AS S-MATRIX POLES; HADRON DEMOCRACY *

satisfy unitarity. There is no "reason" for any others. **Similarly, as Feynman and Heisenberg have both emphasized, there is no reason why some particles should be on a different footing from others. The elementary particle concept is unnecessary, at least for baryons and mesons.**

The second assumption may turn out to be closely related to the first, perhaps even a consequence, but

Geoffrey F. Chew



Chew at his California home on July 2014

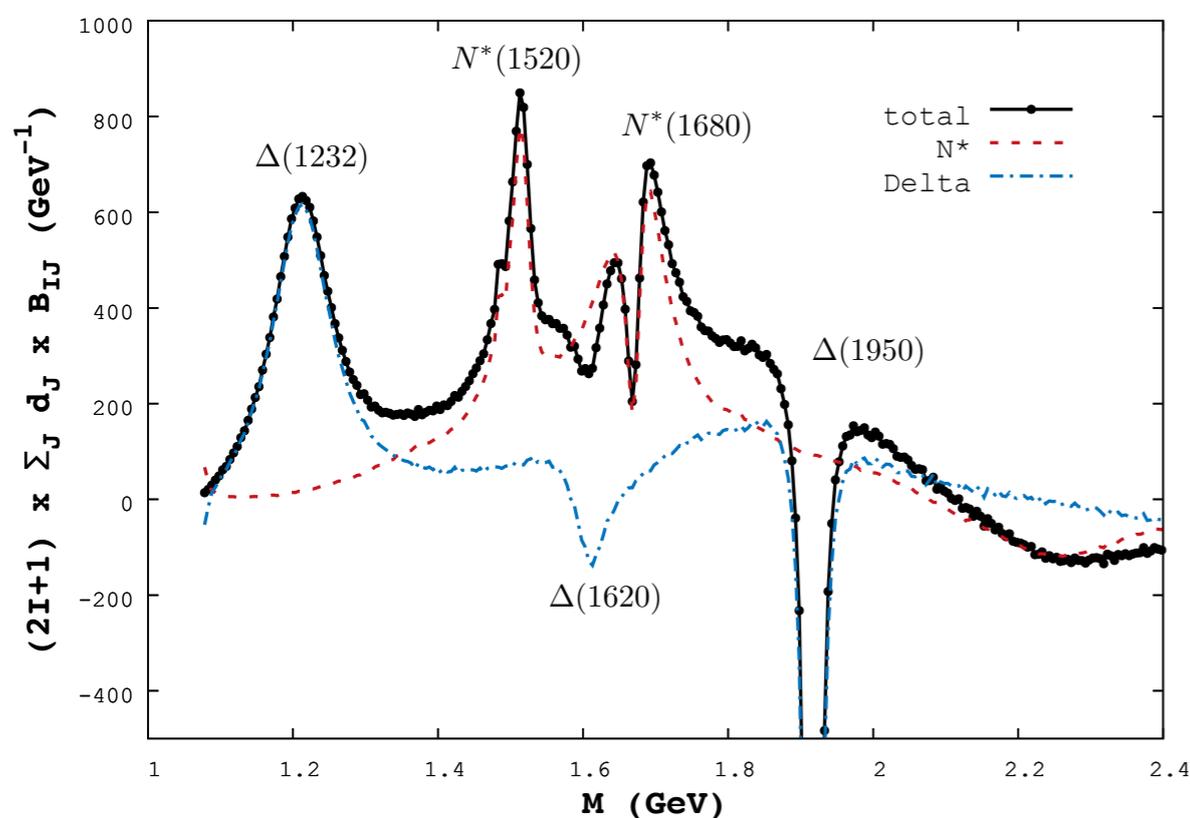
Born	June 5, 1924 Washington, D.C., United States
Died	April 12, 2019 (aged 94) Berkeley, California, United States
Nationality	American
Alma mater	University of Chicago
Known for	S-matrix theory, bootstrap theory, strong interactions, Chew–Frautschi plot
Awards	Hughes Prize (1962) Lawrence Prize (1969) Majorana Prize (2008)
Scientific career	
Fields	Theoretical physics
Institutions	University of Illinois UC Berkeley
Doctoral advisor	Enrico Fermi
Doctoral students	David Gross John H. Schwarz John R. Taylor

but some are more equal than the others?

RESONANCES / EXCITATIONS VIA SCATTERING STATES

- broad /overlapping resonances
- molecular states
- threshold effects /cusps

non-resonant interactions: +/-



The neutral partner of the $Z_c(3900)$

UNIVERSITÄT
DUISBURG ESSEN

- Observation of $Z_c(3900)^0 \rightarrow J/\psi \pi^0$
 - in $e^+e^- \rightarrow J/\psi \pi^0 \pi^0$ GeV (2.8 fb⁻¹, 10.4σ)
 - confirms earlier evidence in CLEO-c data

"When I see a bird that walks like a duck and swims like a duck and quacks like a duck, I call that bird a duck."

— James Whitcomb Riley
Indiana Poet

[PRL 115 (2015) 112603]

Frank Nerling

Spectroscopy of exotic charmonia with BES III/PANDA, 14

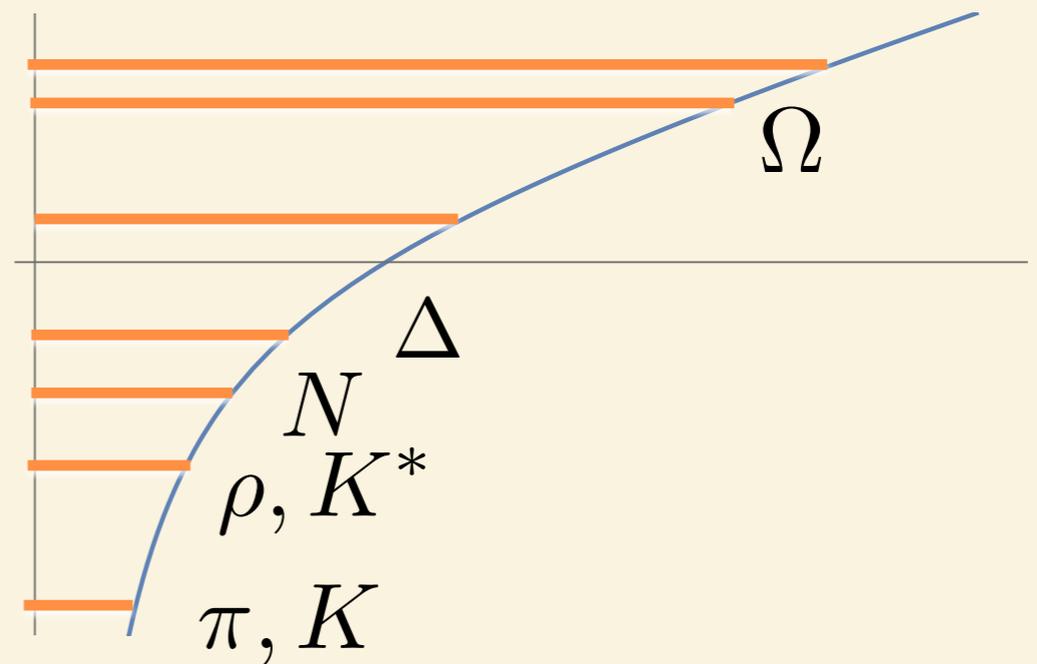
04/12/2019

DENSITY OF STATES

QUENCHED QCD & CONSTITUENT QUARK MODELS

$$Z = \sum_{\alpha=B,M} \langle \alpha | e^{-\beta H} | \alpha \rangle$$

QCD spectrum



COUPLING TO CONTINUUM

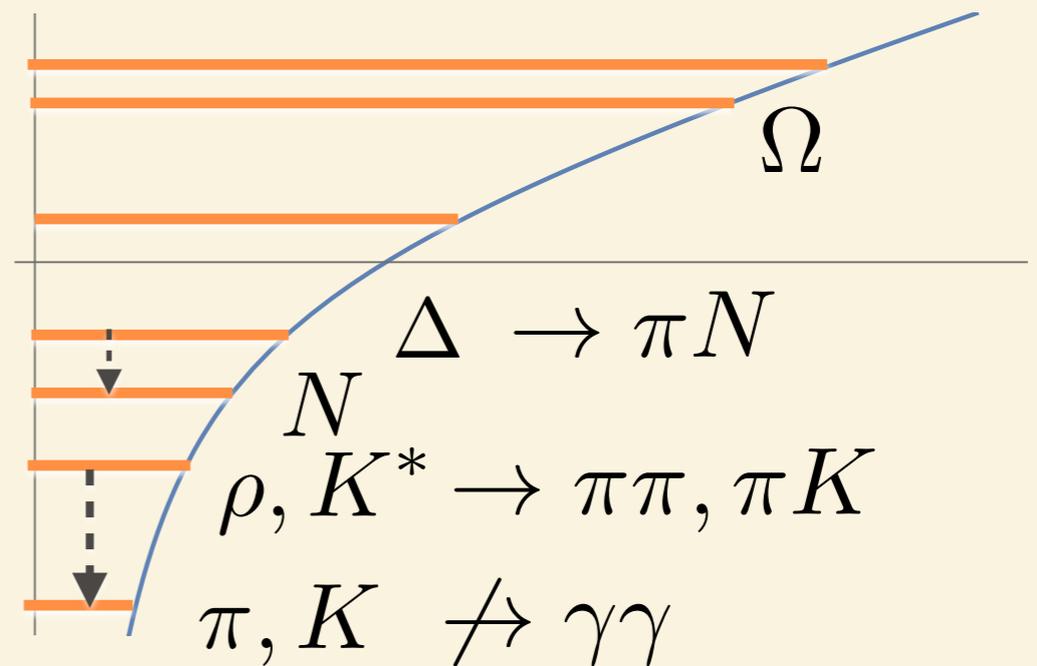
$$Z = \sum_{\alpha=B,M} \langle \alpha | e^{-\beta H} | \alpha \rangle$$

meson loops effects:
shift in hadron masses

S-matrix issues:

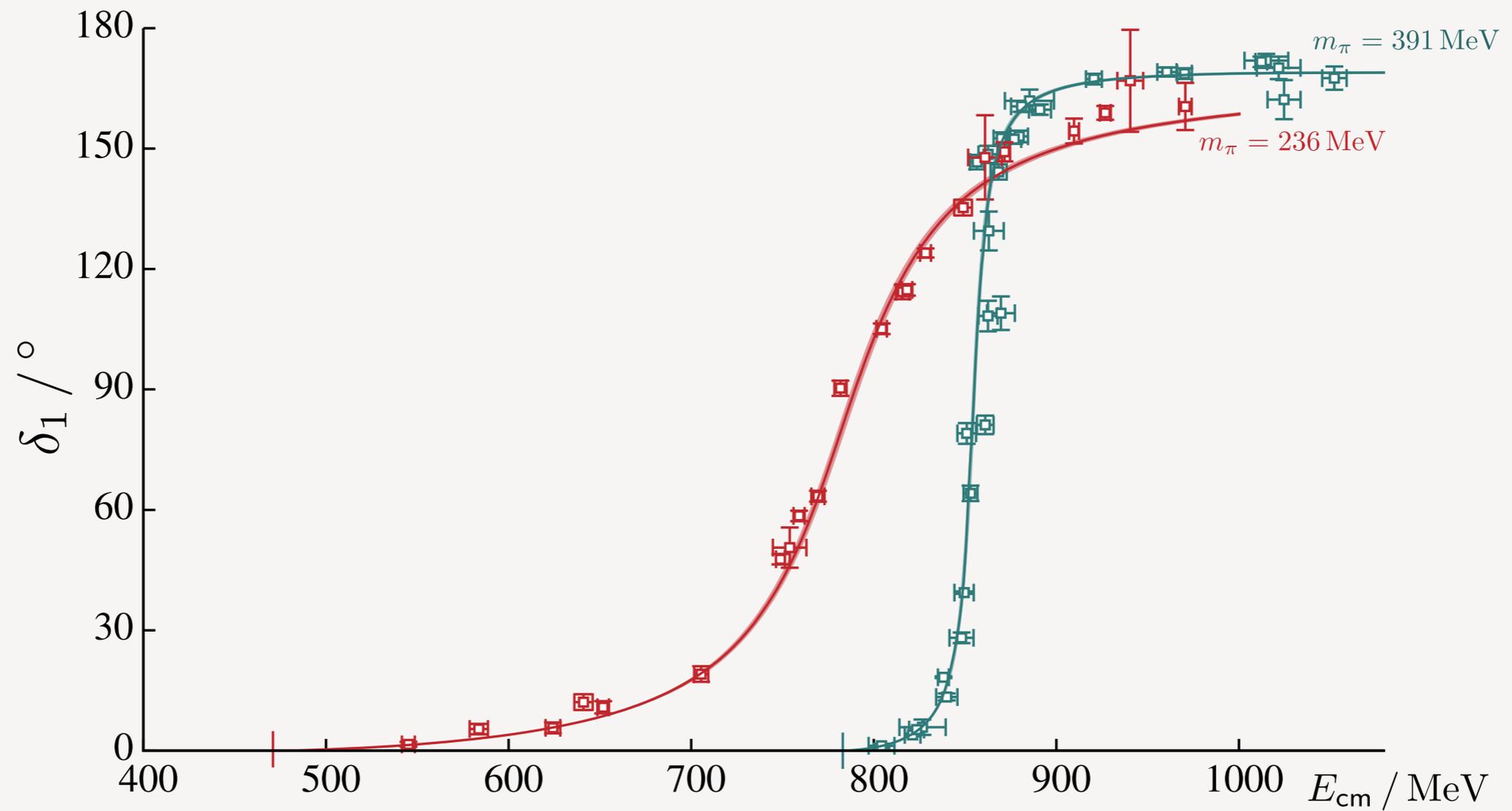
how resonances are expressed
by the scattering states?

QCD spectrum



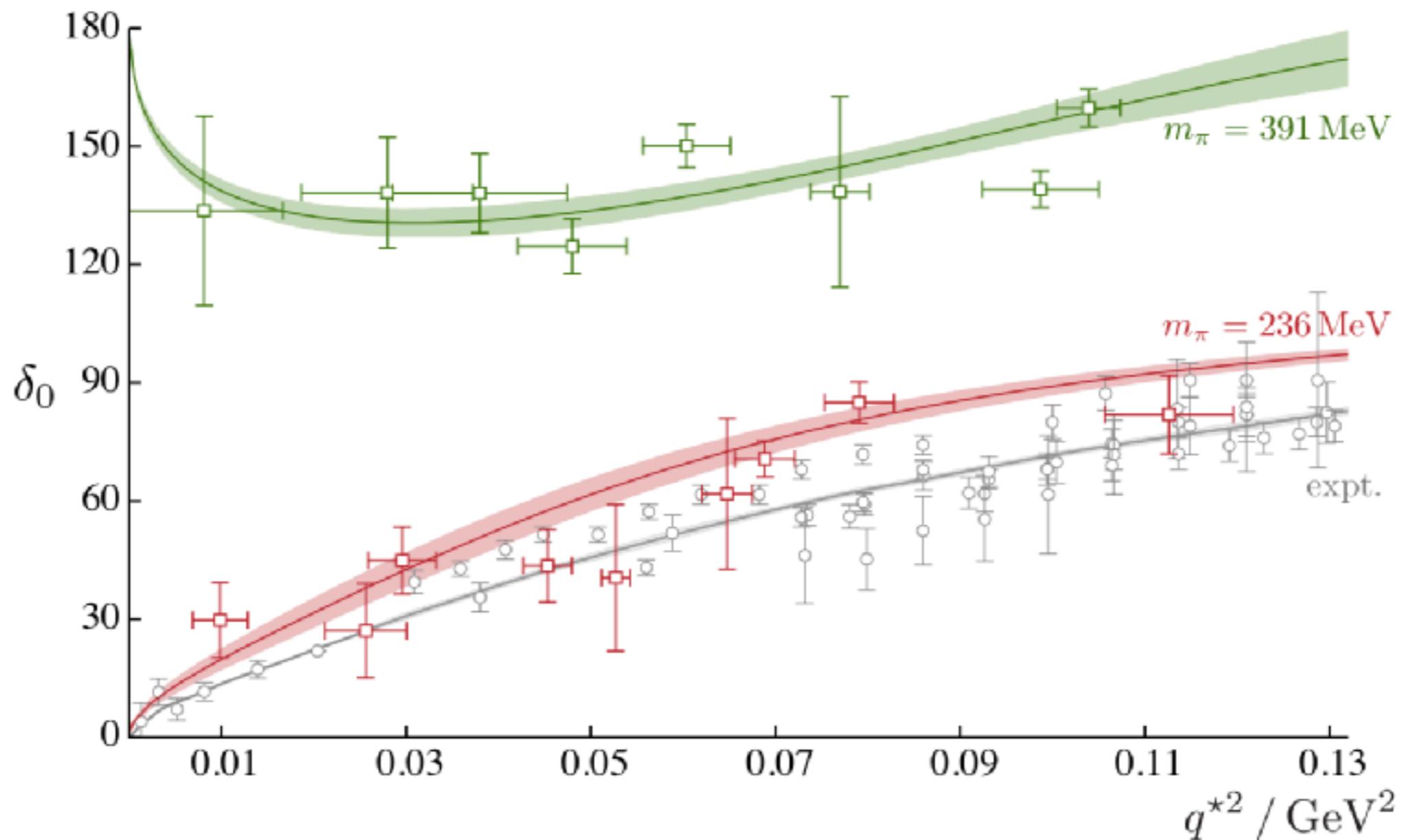
LATTICE COMPUTATIONS ON PHASE SHIFT

WILSON *et al.*



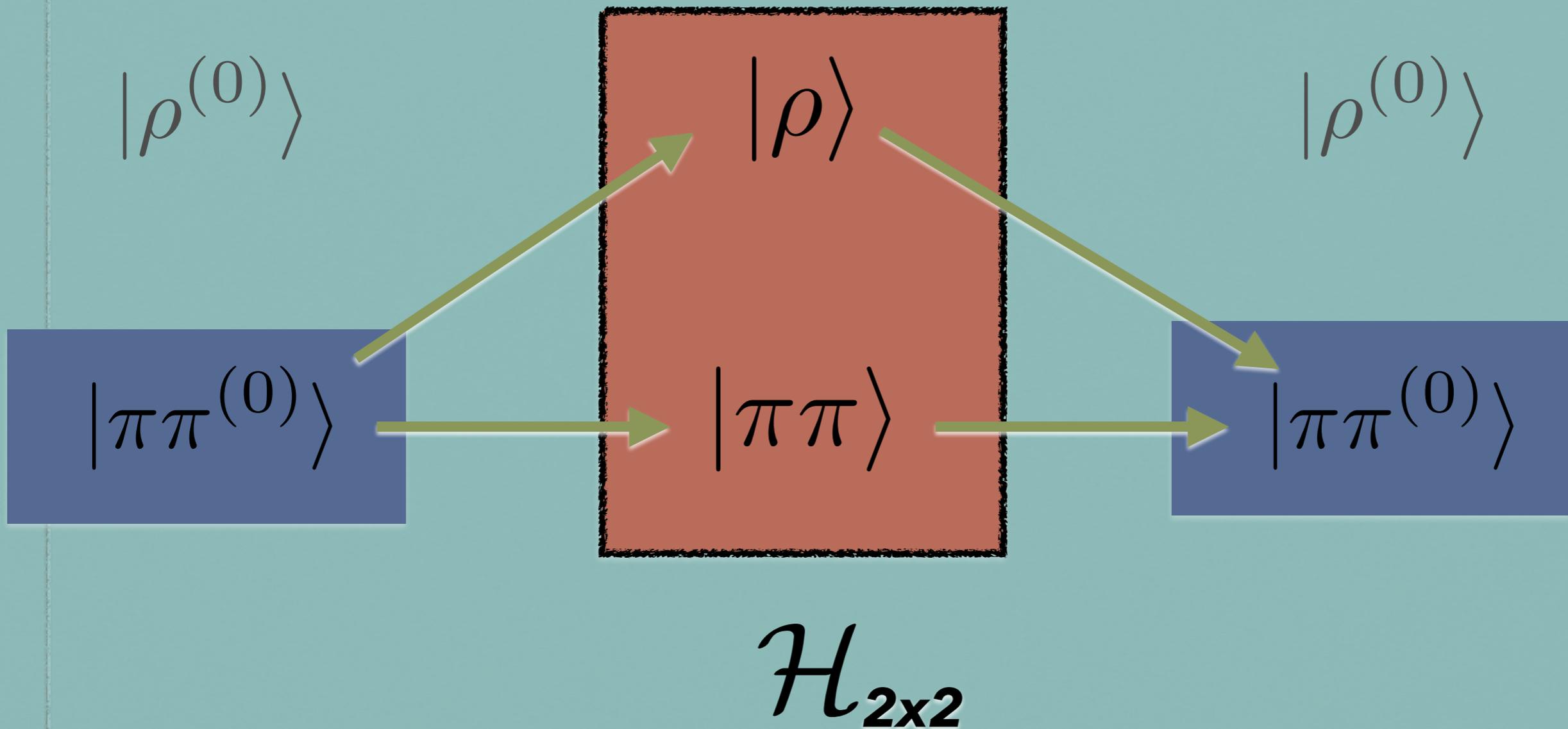
LATTICE COMPUTATIONS ON PHASE SHIFT

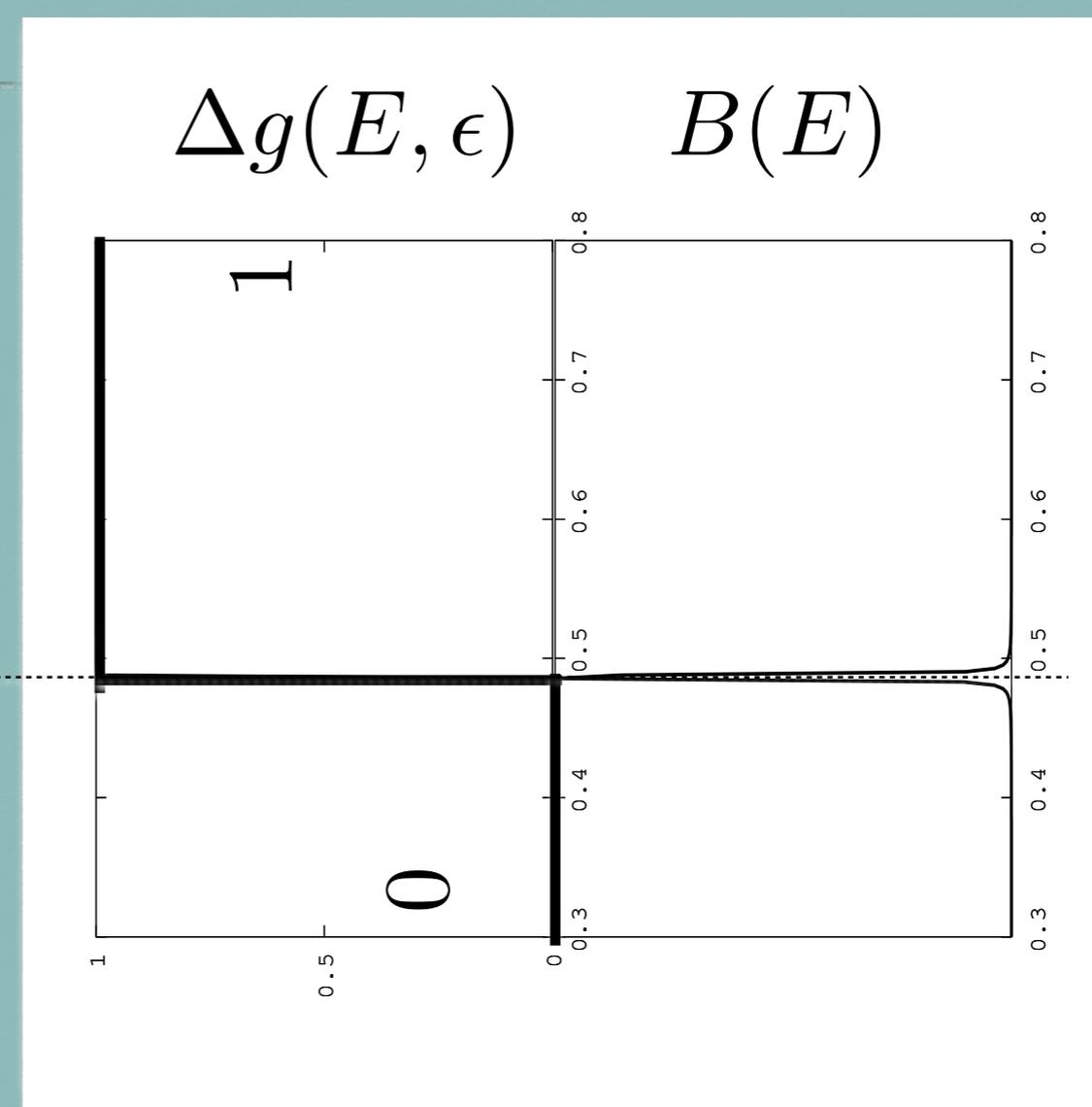
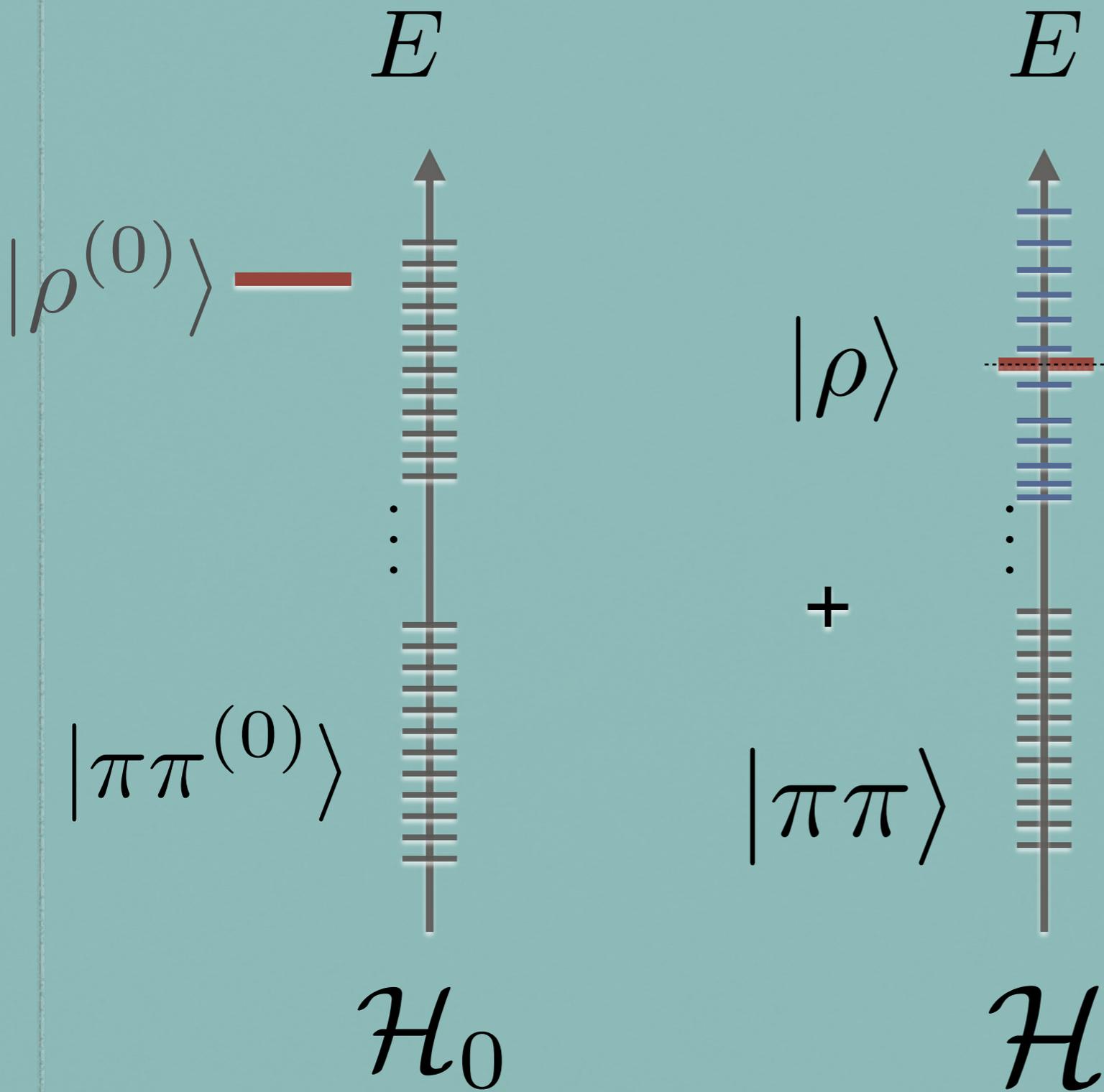
deuteron physics?



R. A. Briceno, J. J. Dudek and R. D. Young, arXiv:1706.06223 [hep-lat].

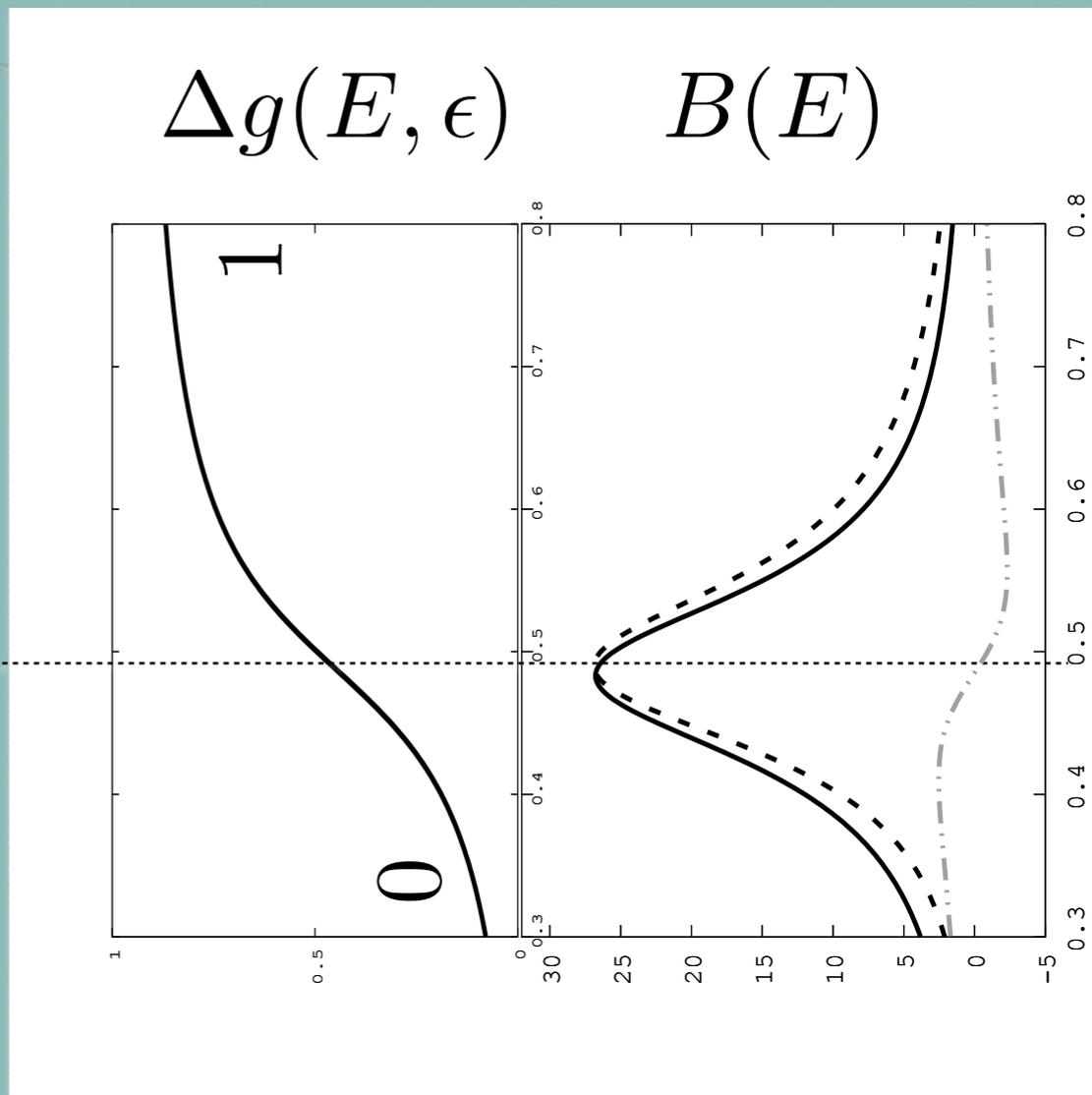
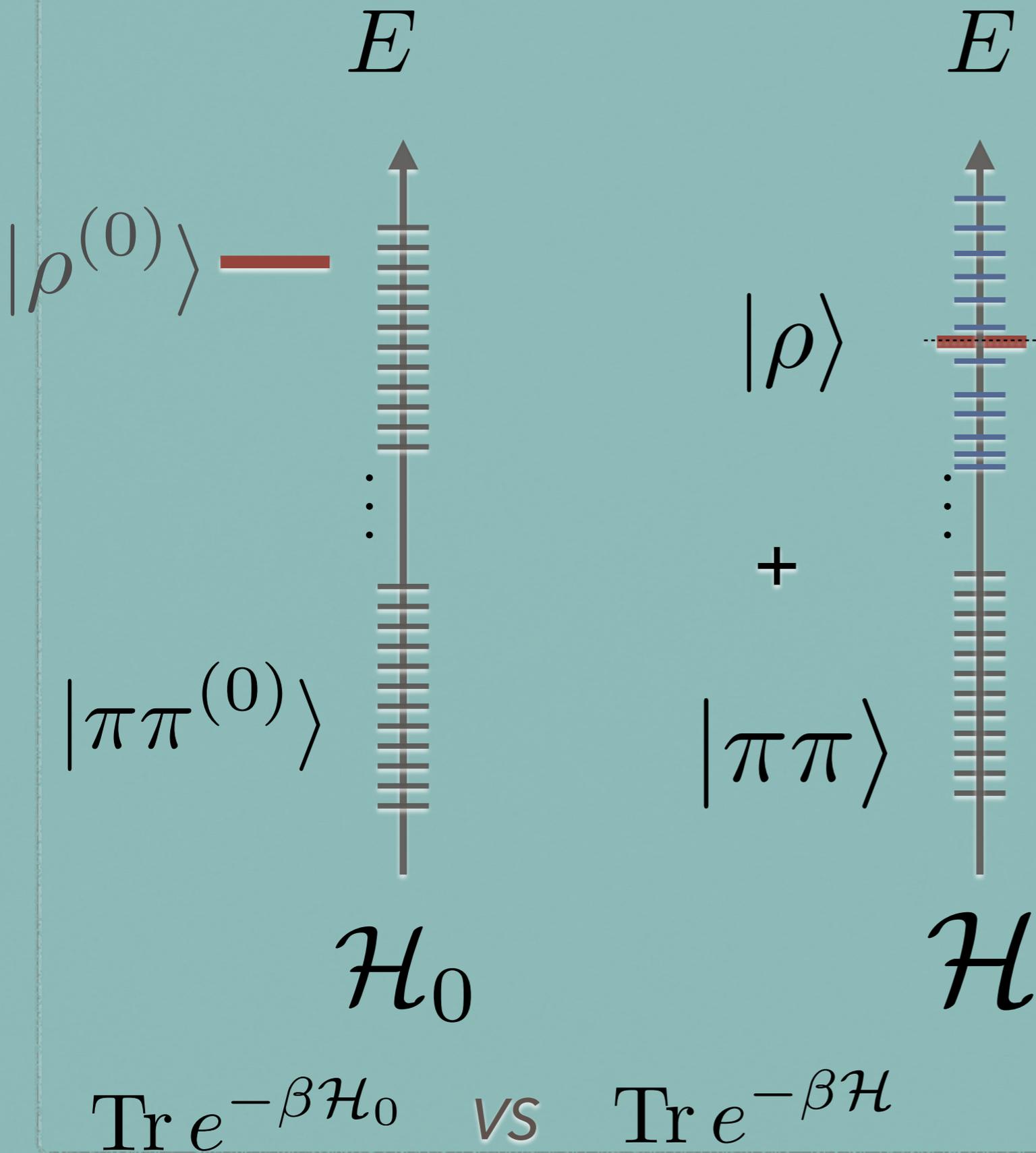
SCATTERING THEORY VS HAMILTONIAN (LEE MODEL)





$$g(E, \epsilon) = \sum_n \theta_\epsilon(E - E_n)$$

$$B(E) = 2\pi \frac{d}{dE} \Delta g(E, \epsilon)$$



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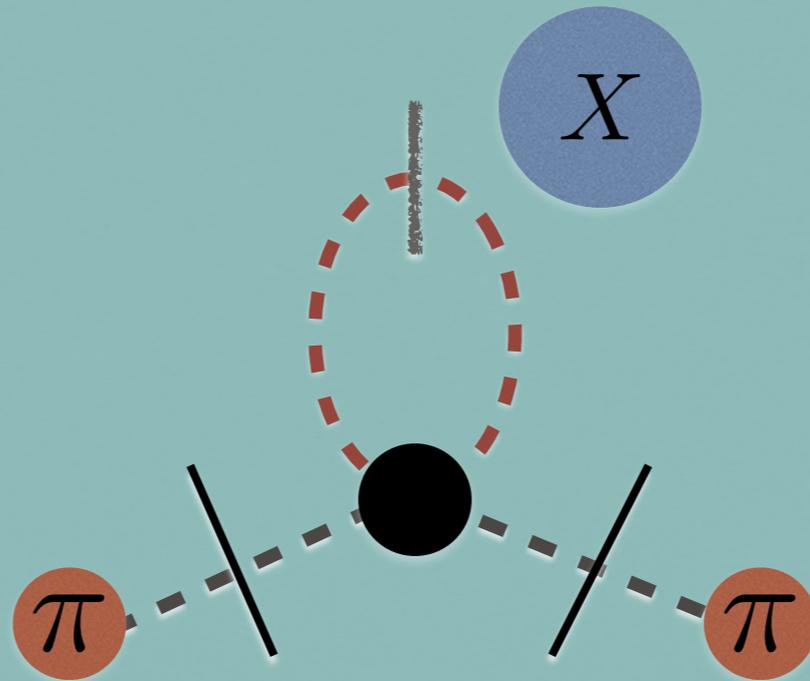
$$B(E) = 2\pi \frac{d}{dE} \Delta g(E, \epsilon)$$

$$= A_\rho + \Delta A_{\pi\pi}$$

medium effects

IN-MEDIUM EFFECTS FROM S-MATRIX

$$\Sigma_\pi =$$



$$\propto \int \frac{d^3q}{\omega_q} n_X T_{\pi X}(s)$$

forward amplitude

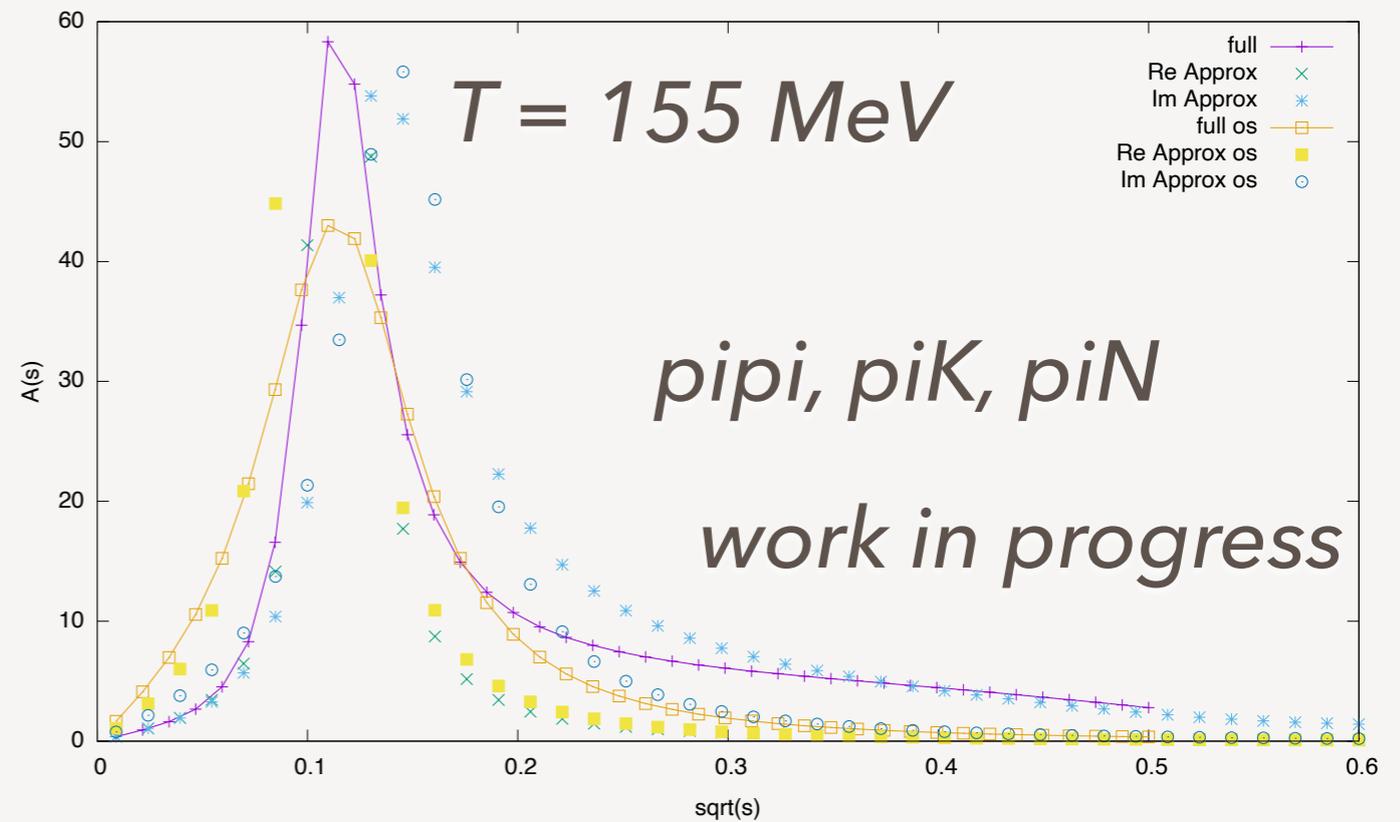
A. Schenk NPB 363 (1991)

S. Jeon and P. J. Ellis PRD 58 045013 (1998)

IN-MEDIUM EFFECTIVE S-MATRIX

$$\Sigma_{\pi} =$$

π



$$\propto \int \frac{d^3 q}{\omega_q} n_X T_{\pi X}(s)$$

forward amplitude

A. Schenk NPB 363 (1991)

S. Jeon and P. J. Ellis PRD 58 045013 (1998)

VACUUM PHYSICS?

Quantum statistical mechanics of gases in terms of dynamical filling fractions and scattering amplitudes

André LeClair

Newman Laboratory, Cornell University, Ithaca, NY, USA

Received 22 November 2006, in final form 3 May 2007

Published 19 July 2007

Online at stacks.iop.org/JPhysA/40/9655

It helps to realize that at least in principle it is possible to decouple the zero temperature dynamics and the quantum statistical sums. The argument is simple: the computation of the partition function $Z = \text{Tr}(e^{-\beta H})$ is in principle possible from the complete knowledge of the zero temperature eigenstates of the Hamiltonian H . In practice this is rather difficult and one resorts to perturbative methods such as the Matsubara method, which unfortunately entangles the zero temperature dynamics from the quantum statistical mechanics. However,

S-MATRIX FORMULATION OF THERMODYNAMICS

thermo-statistical

dynamical

$$\Delta \ln Z = \int dE e^{-\beta E} \frac{1}{4\pi i} \text{tr} \left\{ S_E^{-1} \frac{\partial}{\partial E} S_E \right\}_c$$

R. Dashen, S. K. Ma and H. J. Bernstein,
Phys. Rev. 187 (1969) 345.

A SIMPLE TRICK

$$\frac{1}{4\pi i} \operatorname{tr} \left\{ S_E^{-1} \overleftrightarrow{\partial} S_E \right\}_c$$

$$= \frac{1}{2\pi} \times 2 \frac{\partial}{\partial E} \left[\frac{1}{2} \operatorname{Im} \operatorname{tr} \{ \ln S_E \} \right]$$

$$S_E = e^{2i\delta_E}$$

$$\Delta \ln Z = \int dE e^{-\beta E} \times \frac{1}{\pi} \frac{\partial}{\partial E} \operatorname{tr} (\delta_E).$$

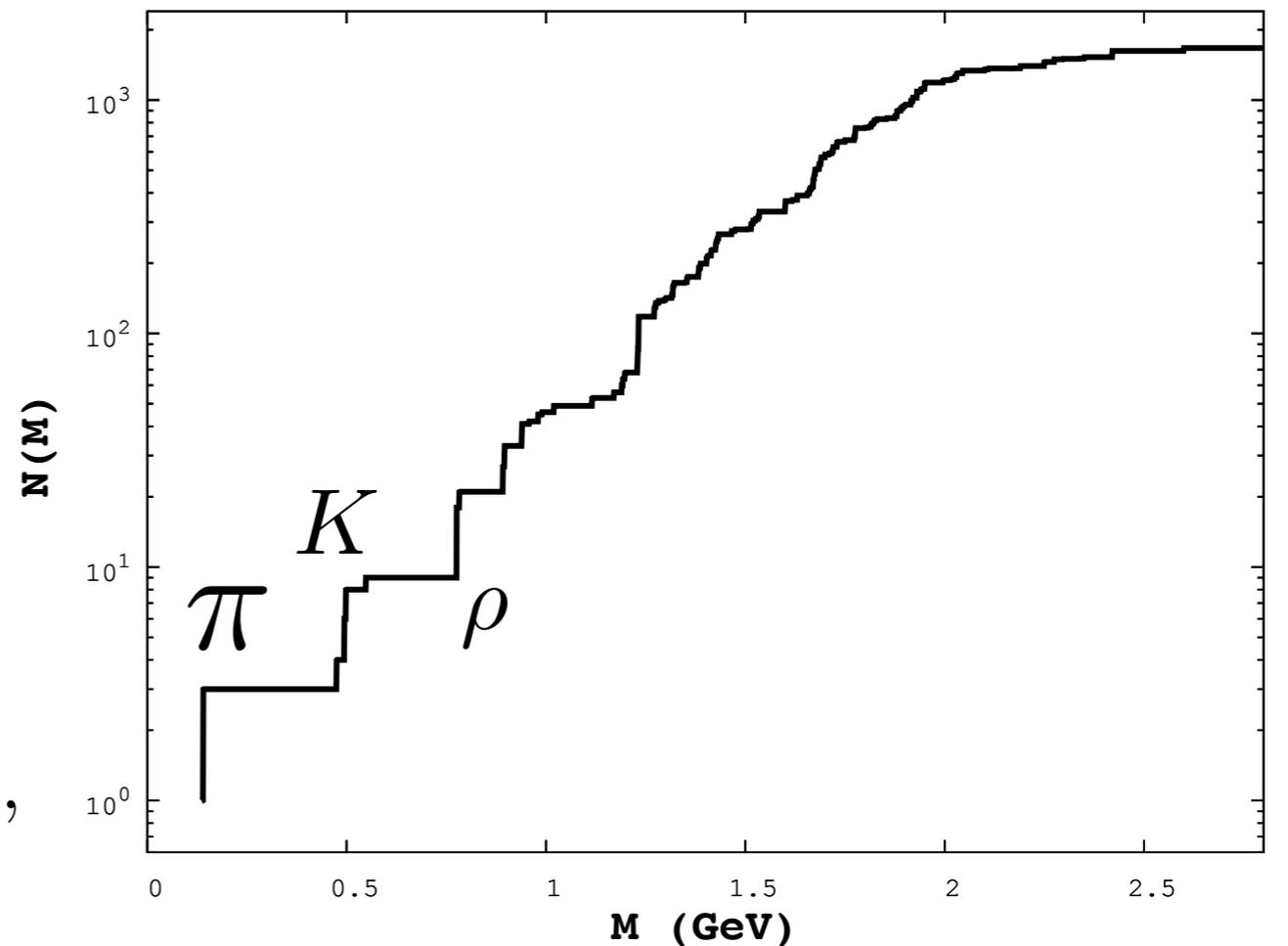
E. Beth and G. Uhlenbeck,
Physica (Amsterdam) 4, 915 (1937).

HRG AS AN S-MATRIX SCHEME

$$\det S(E) = \prod_{\{\text{res}\}} \frac{z_{\text{res}}^* - E}{z_{\text{res}} - E}, \quad z_{\text{res}} \approx m_{\text{res}} - i0^+.$$

$$Q(M) \equiv \frac{1}{2} \text{Im} (\text{tr} \ln S)$$

$$Q_{\text{HRG}}(E) = \sum_{\text{res}} d_{IJ} \times \pi \theta(E - m_{\text{res}}),$$



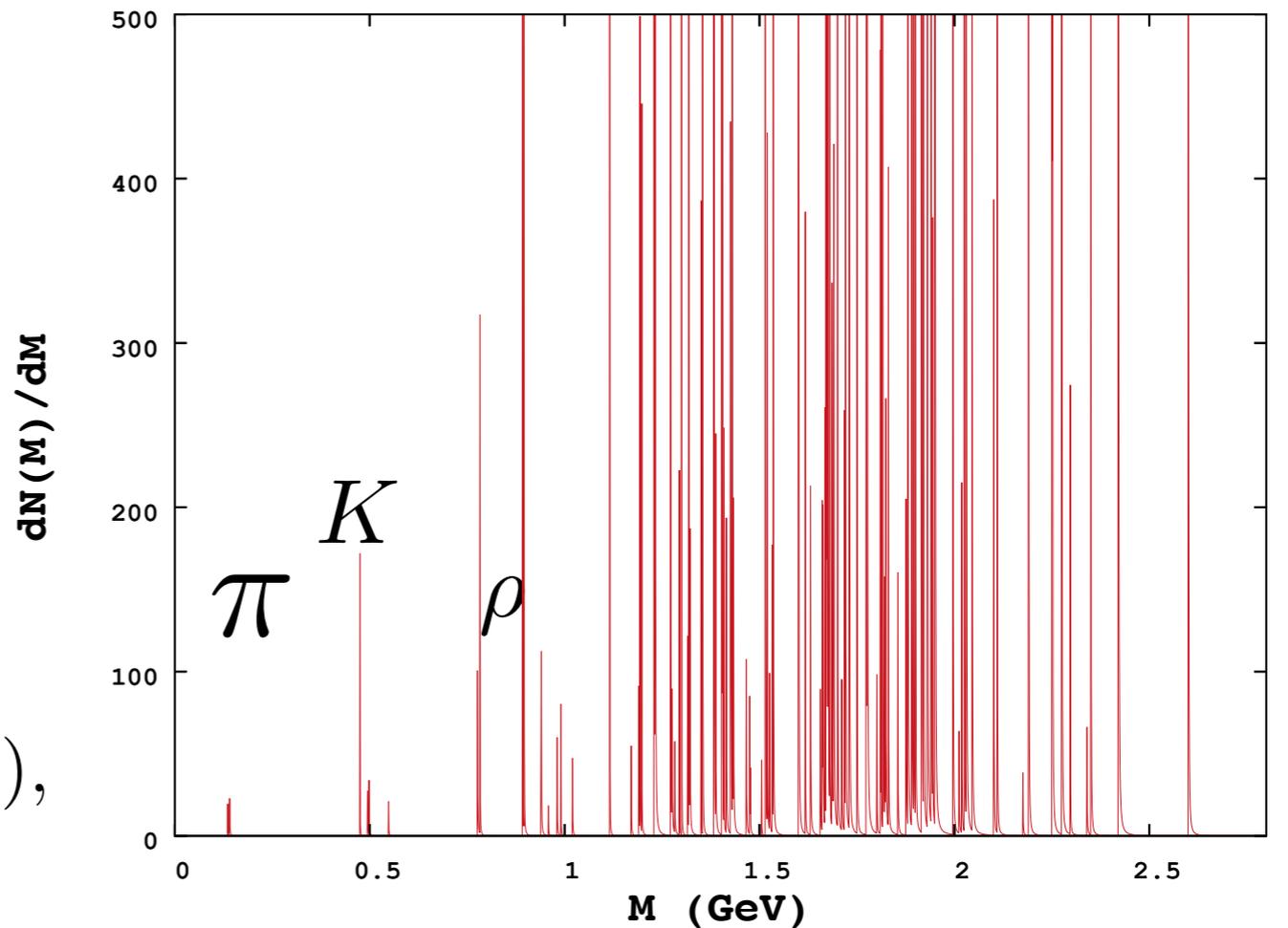
HRG AS AN S-MATRIX SCHEME

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$$Q(M) \equiv \frac{1}{2} \text{Im} (\text{tr} \ln S)$$

$$\frac{\partial}{\partial E}$$

$$Q_{\text{HRG}}(E) = \sum_{\text{res}} d_{IJ} \times \pi \theta(E - m_{\text{res}}),$$

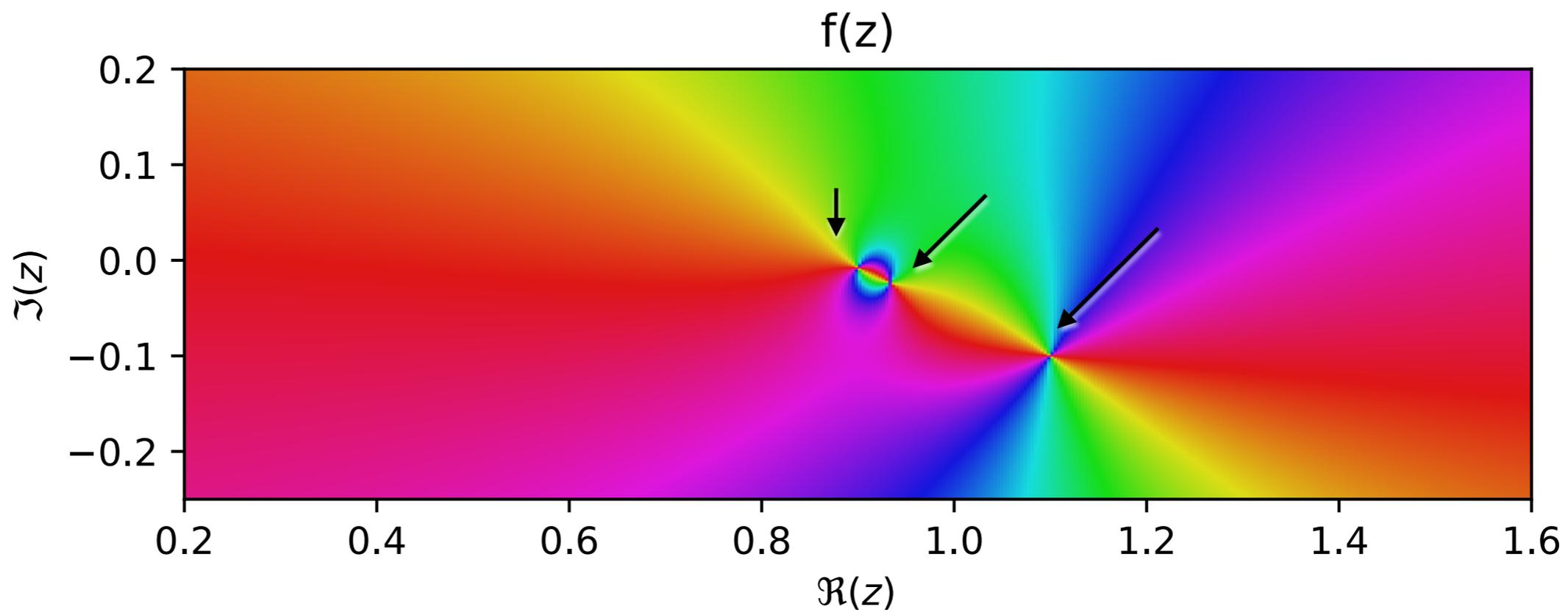
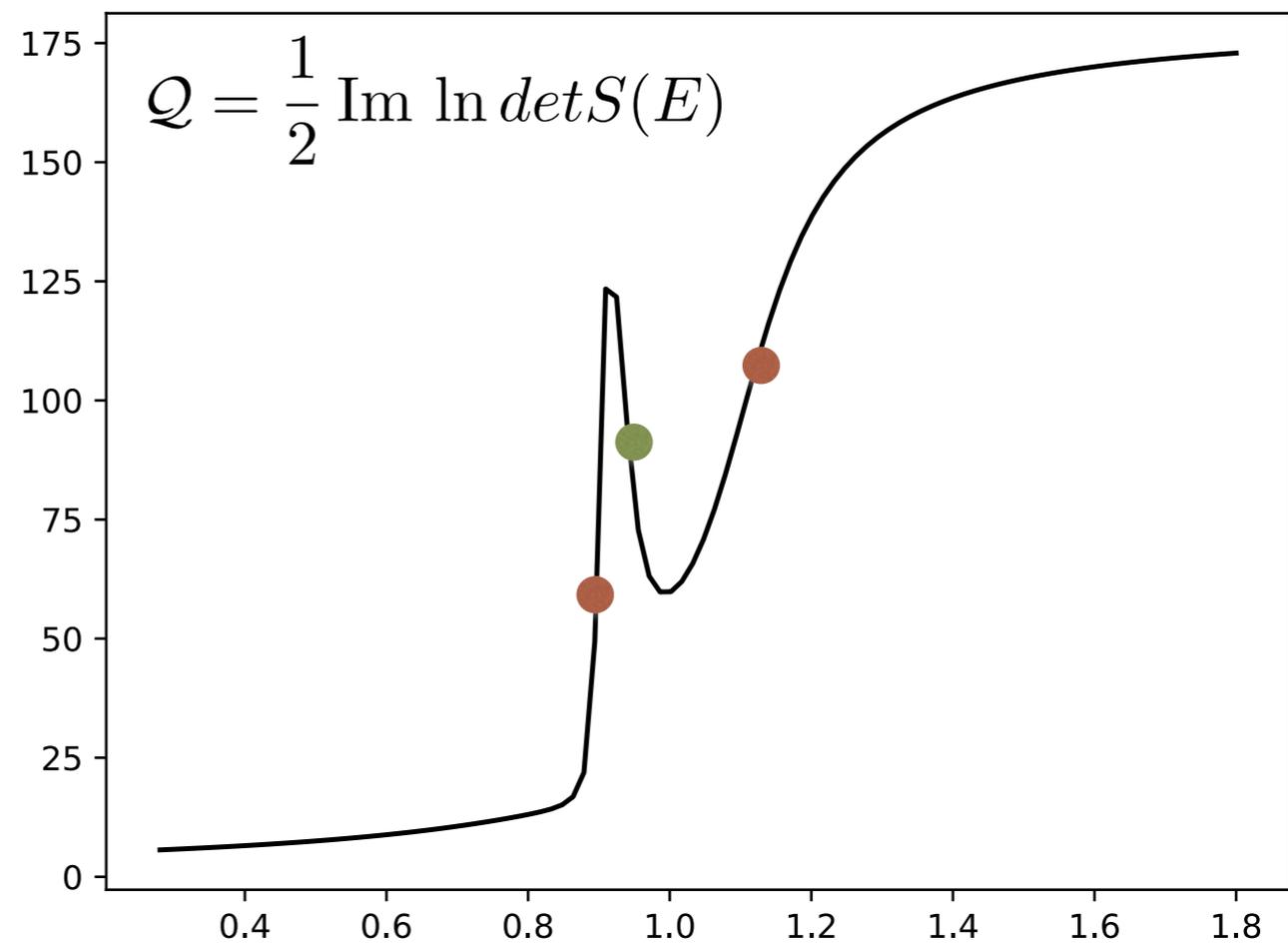


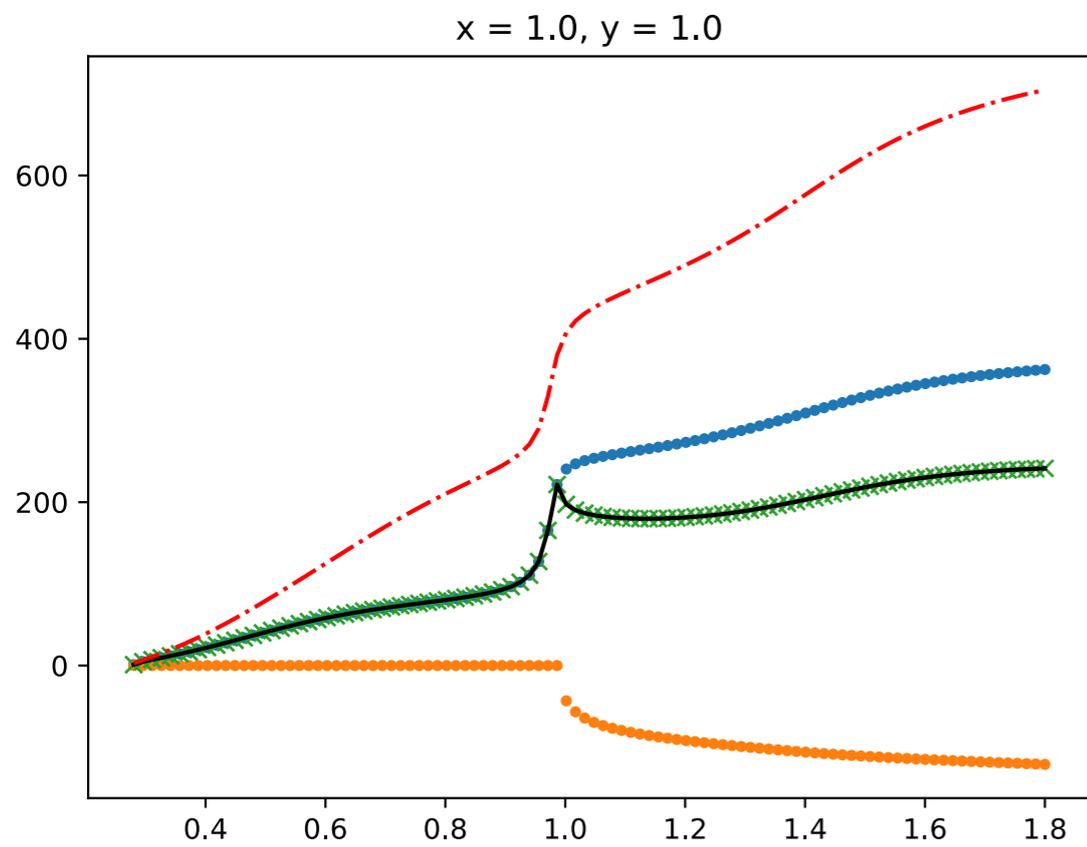
ROOTS IN S-M

2 pole + 1 root

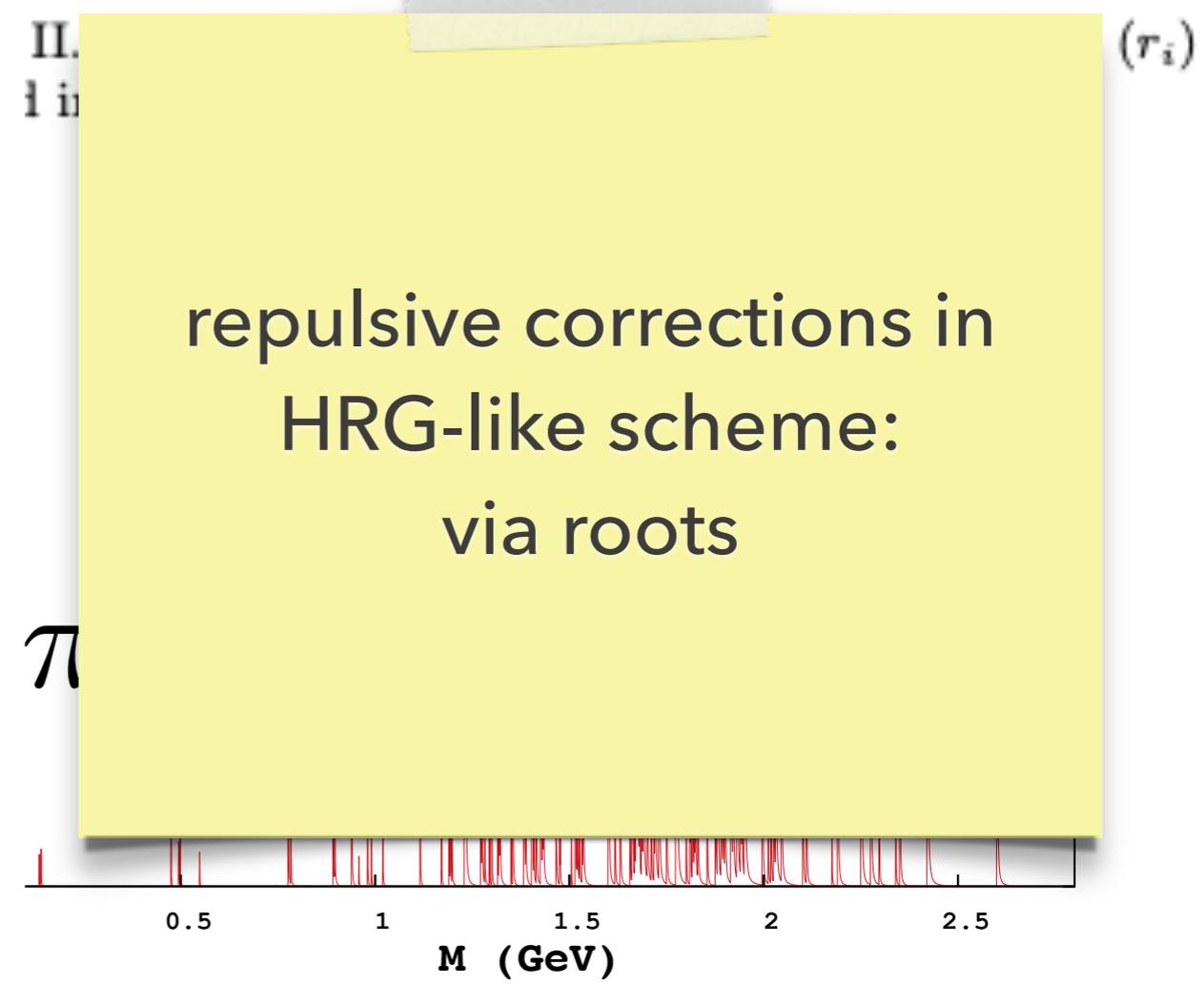
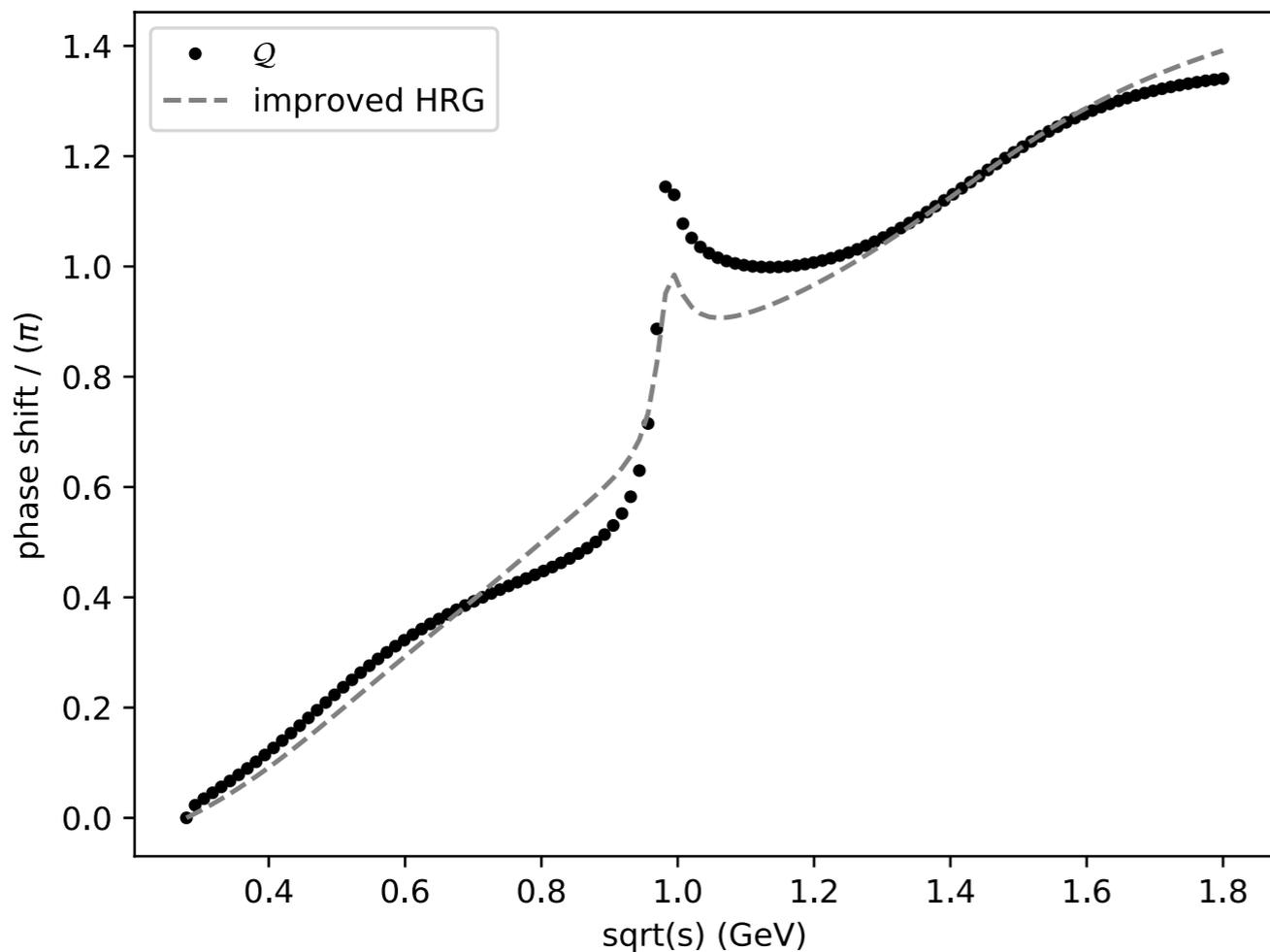
$$\det S \propto \frac{g_1}{E - p_1} + \frac{g_2}{E - p_2}$$

$$\det S \propto \frac{g_1}{E - p_1} \times \frac{g_2}{E - p_2}$$





	$\text{Re } \sqrt{s}$	$\text{Im } \sqrt{s}$	sheet
p1	0.4637	-0.2357	II
p2	0.975	-0.0164	II
p3	1.401	-0.249	II
p4	0.6654	-0.2263	III
p5	1.4176	-0.2640	III
r1	0.787	+0.259	I
r2	1.410	+0.691	I
r3	0.981	-0.032	II
r4	1.393	-0.669	II
r5	0.918	+0.248	IV



DYNAMICAL GENERATION OF BS / RESONANCES

- dynamical generation of bound states / resonances:
 - f(980) close to $K \bar{K}$ threshold
 - f(500) dynamically generated
- coupling of open channels: $\pi\pi$, kk with a $|q\bar{q}\rangle$ state

what you give \neq what you get

1 in 5 out!

$$\frac{1}{E - \mathcal{H}_0} = \begin{matrix} |\pi\pi\rangle \\ |K\bar{K}\rangle \\ |R^0\rangle \quad (|q\bar{q}\rangle) \end{matrix} \left[\begin{array}{ccc} \Pi_{\pi\pi}(E) & & \\ & \Pi_{K\bar{K}}(E) & \\ & & \frac{1}{E - m_{res}^0} \end{array} \right]$$

$$V_{int} = \left[\begin{array}{ccc} g_{\pi\pi} & g_{\pi K} & g_{\pi R} \\ g_{\pi K} & g_{K K} & g_{K R} \\ g_{\pi R} & g_{K R} & \end{array} \right]$$

$$G = G_0 + G_0 V_{int} G$$

From Hamiltonian to Scattering Matrix

$$\frac{1}{E - \mathcal{H}_0 \pm i\delta}$$


$$\begin{aligned}\tilde{S} &= (I - G_-^0 V) (I + G_+^0 T) \\ &= I - G_-^0 V + G_+^0 T - G_-^0 V G_+^0 T \\ &= I - G_-^0 V + G_+^0 V + G_+^0 V G_+^0 T - G_-^0 V G_+^0 T \\ &= I + (G_+^0 - G_-^0) V + (G_+^0 - G_-^0) V G_+^0 T \\ &= I + (G_+^0 - G_-^0) T \\ &\rightarrow I + 2i \operatorname{Im} (G_+^0) \times T. \quad \text{on-shell limit}\end{aligned}$$

TESTING THE ROBUSTNESS

$$Q(E) = \frac{1}{2} \text{Im Tr} \{ \ln S_E \}$$

*Getting
Effective DOS
on
REAL Energy*

effective DOS

$$B = 2 \frac{d}{dE} Q$$

what is being counted?

can it handle dynamically generated states?

TESTING THE ROBUSTNESS

$$Q(E) = \frac{1}{2} \text{Im} \text{Tr} \{ \ln S_E \}$$

effective DOS

$$B = 2 \frac{d}{dE} Q$$

*Getting
Effective DOS
on
REAL Energy*

Need to find all the
poles...

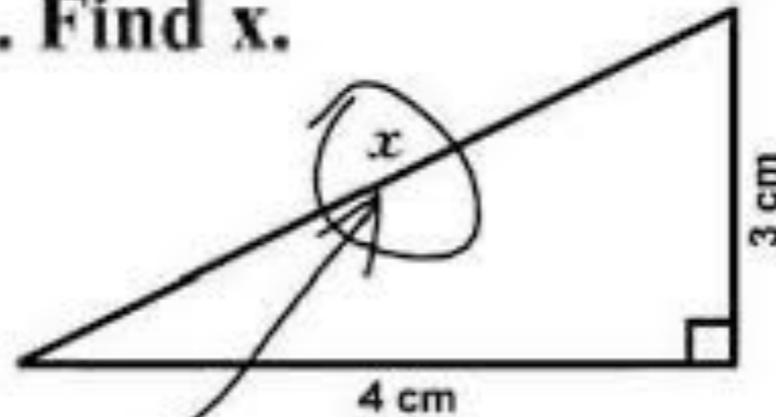
/?

ally generated st



How to find poles (and roots, and sheets, and many other?)

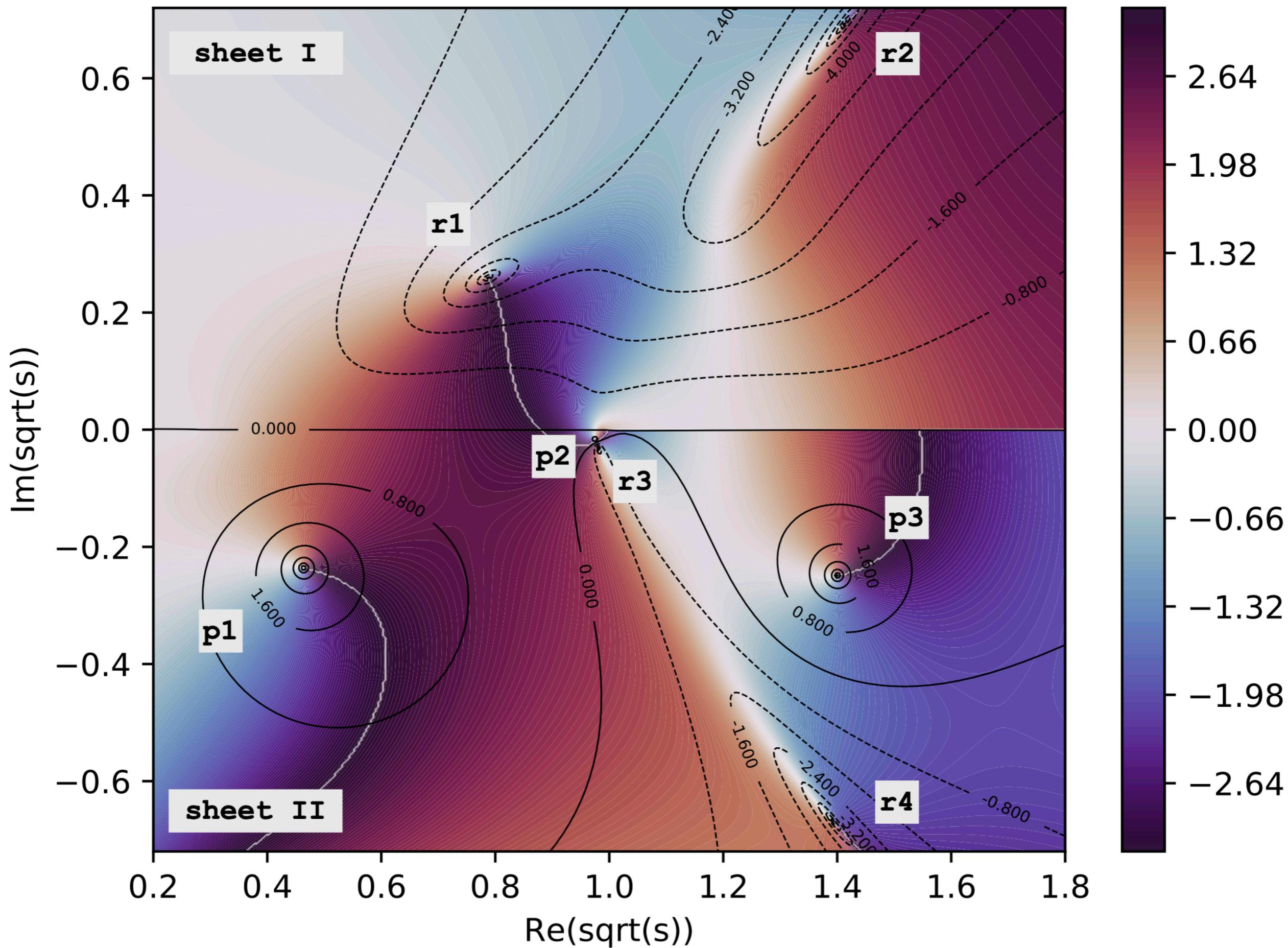
3. Find x .

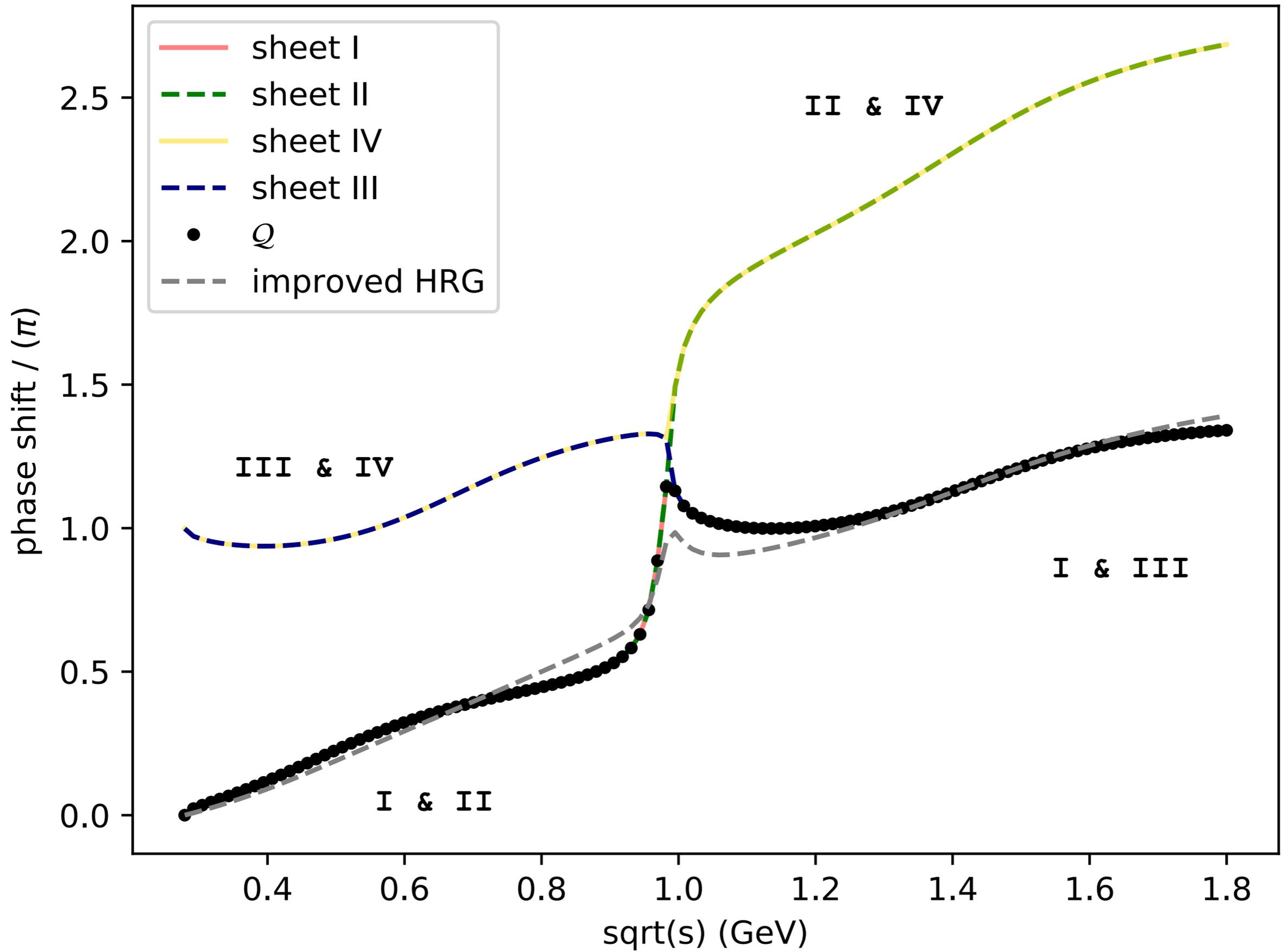


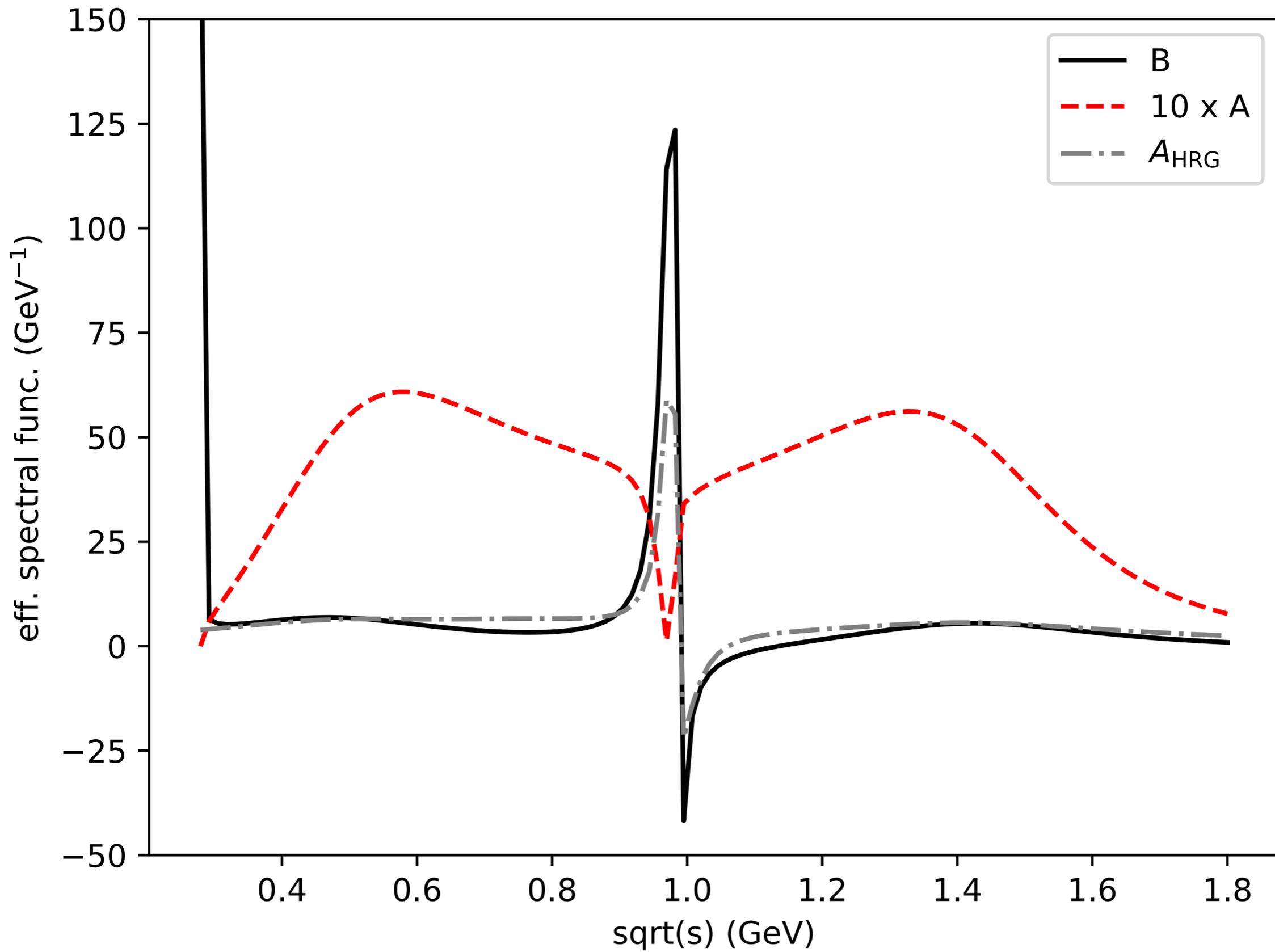
Here it is

The power of graphical solution

detS(sqrt(s))







PROTON PUZZLE

BQ CORRELATION

$$\chi_{BQ}/T^2 = \sum_{I_z; B} d_J \frac{BQ}{I_z} \times \int_{m_{\text{th}}}^{\infty} \frac{d\sqrt{s}}{2\pi} B_{I,J}(s) \\ \times \frac{1}{T^3} \int \frac{d^3p}{(2\pi)^3} \frac{e^{\beta\sqrt{p^2+s}}}{(e^{\beta\sqrt{p^2+s}} + 1)^2}$$

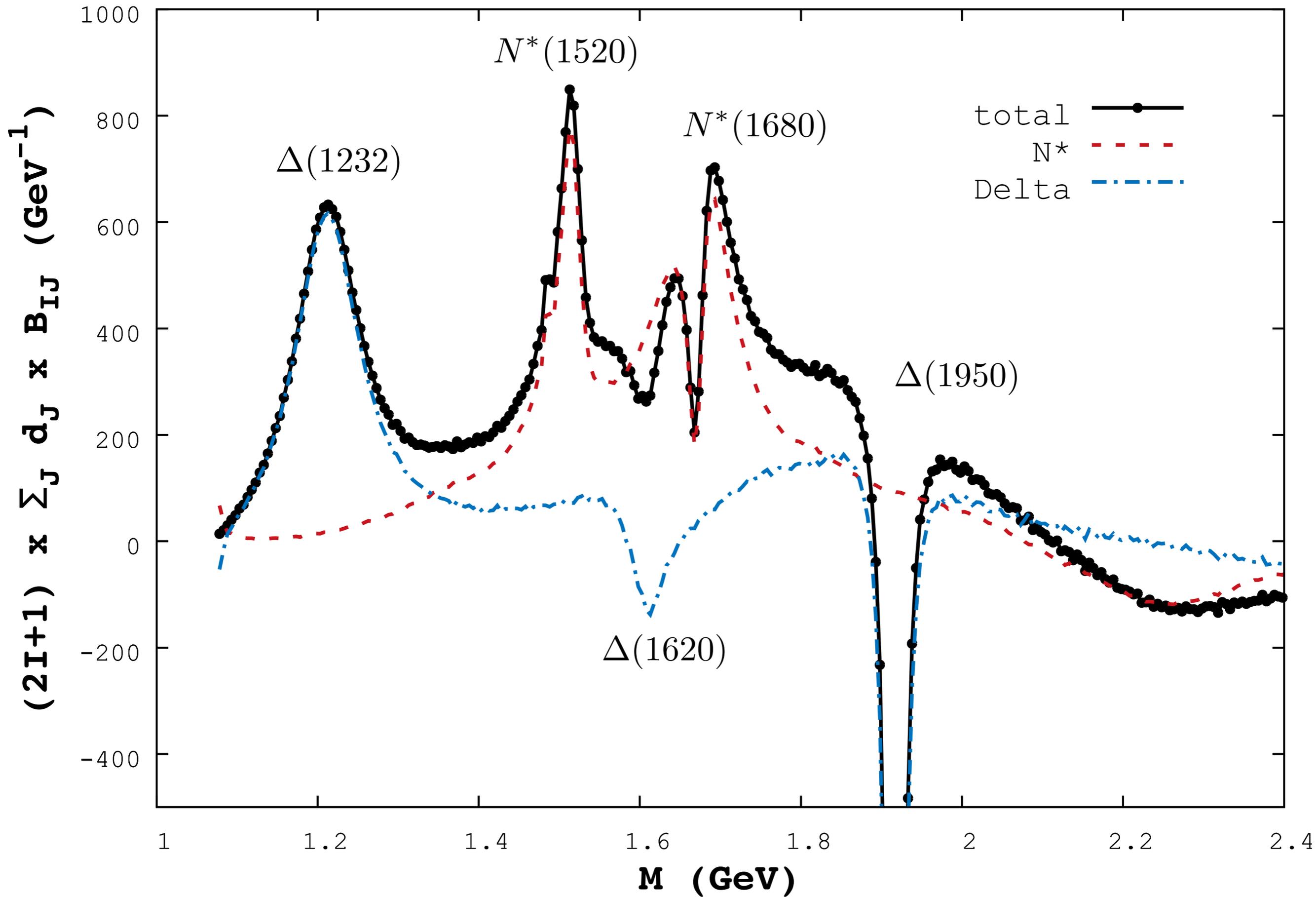
- (in)famous resonances:

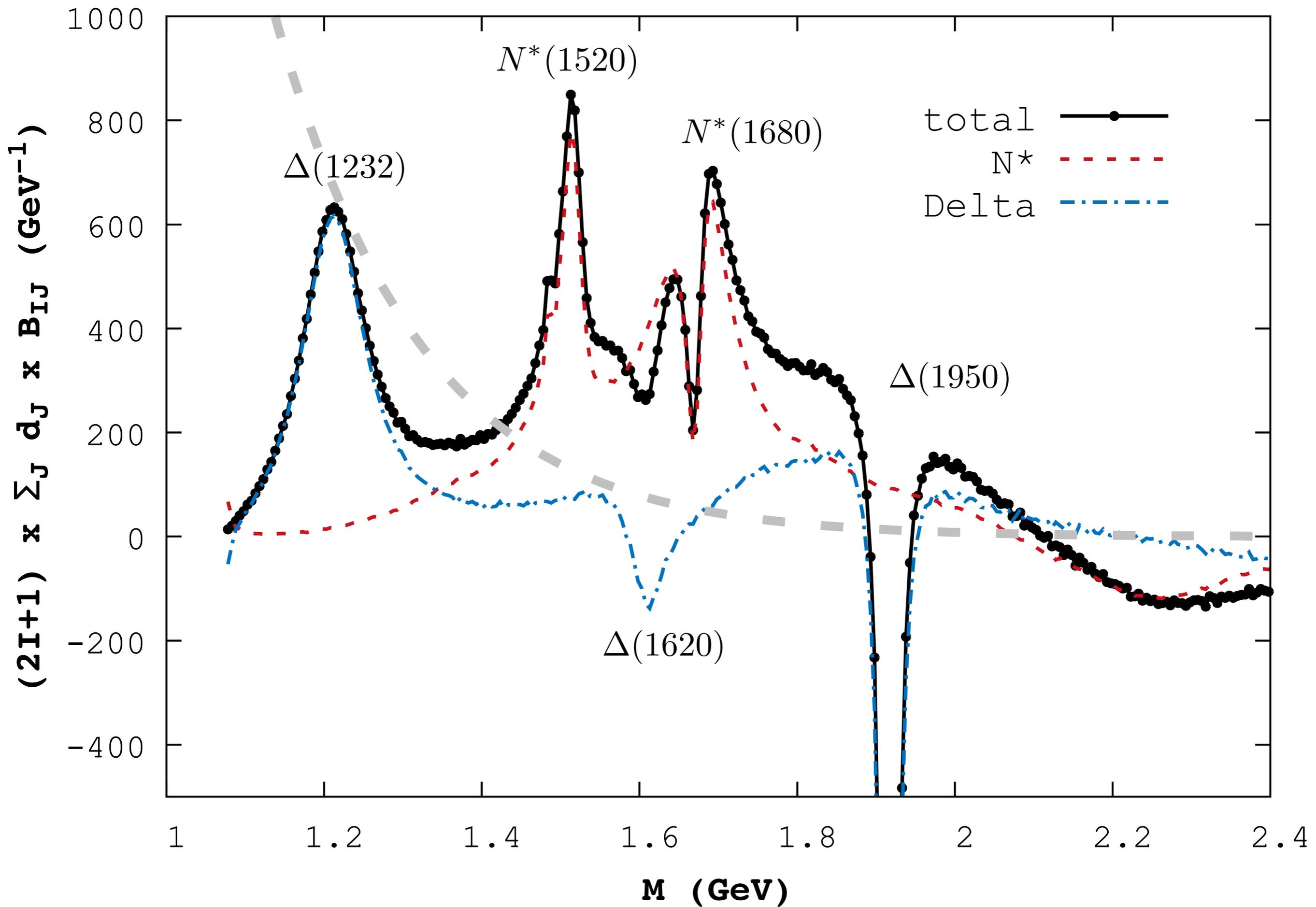
N*: 1535 (S11), 1440 (P11), 1520 (D13) ...

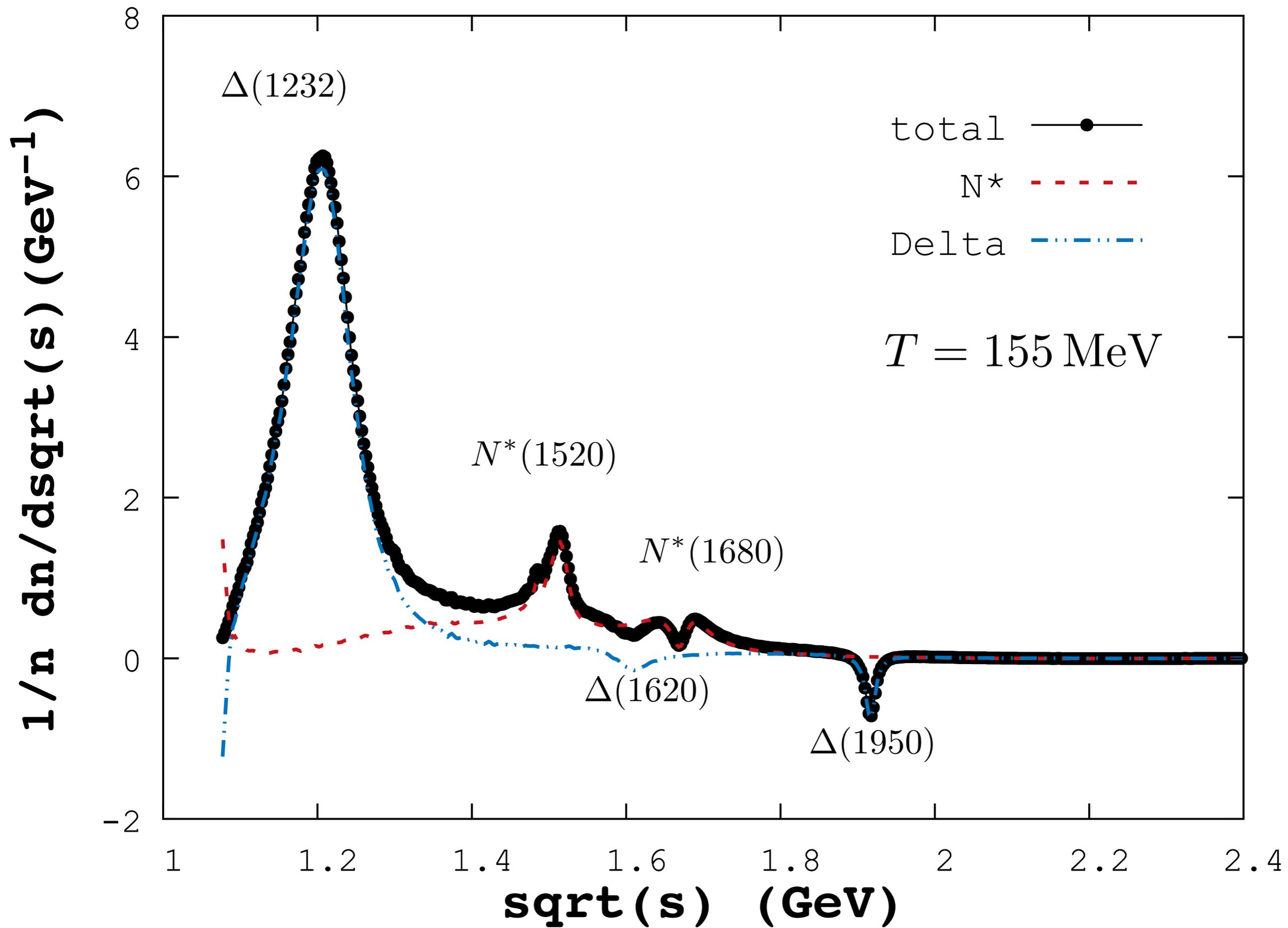
Δ : 1232 (P33), 1620 (S31) ...

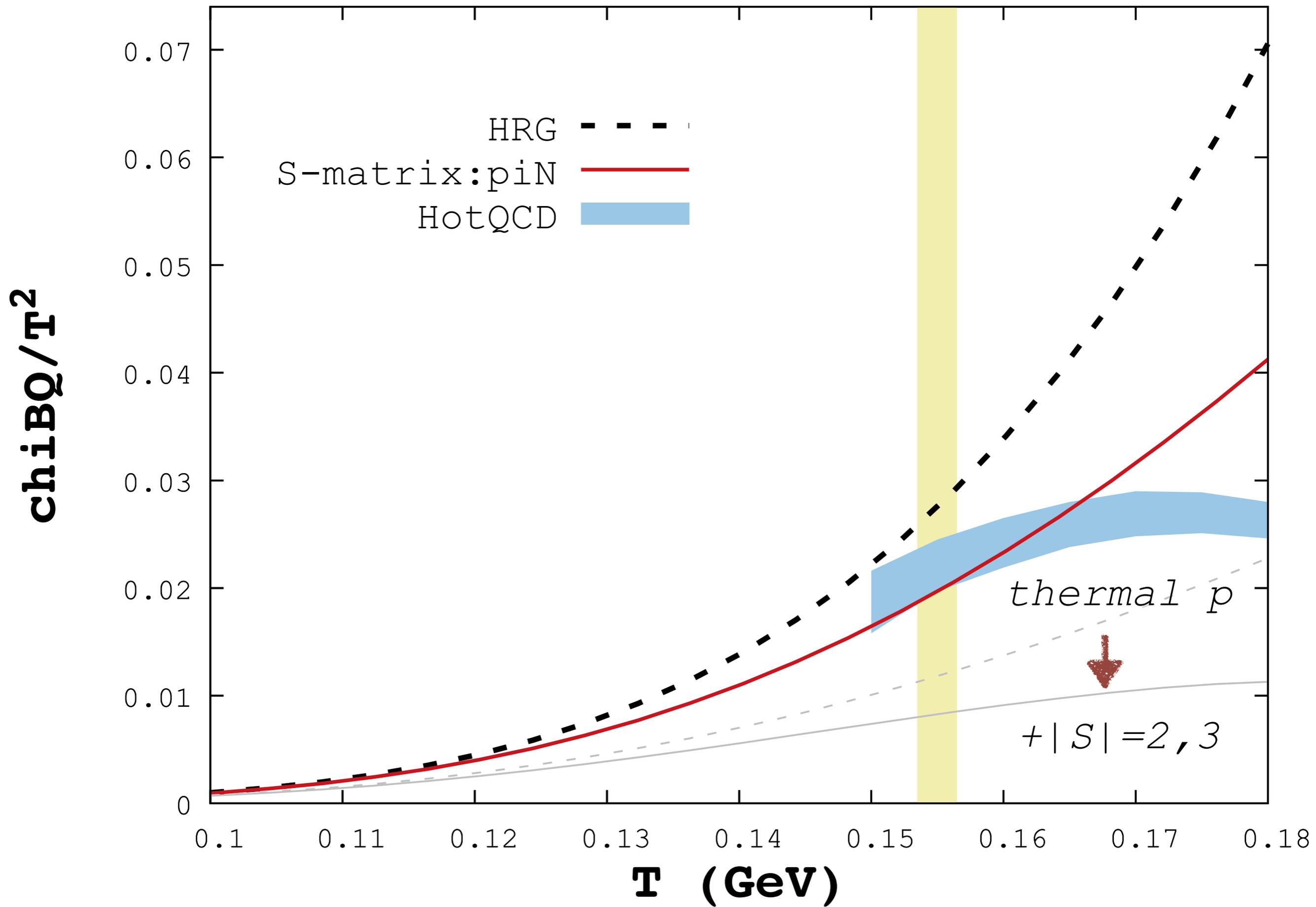
- S = -1 hyperons are excluded:

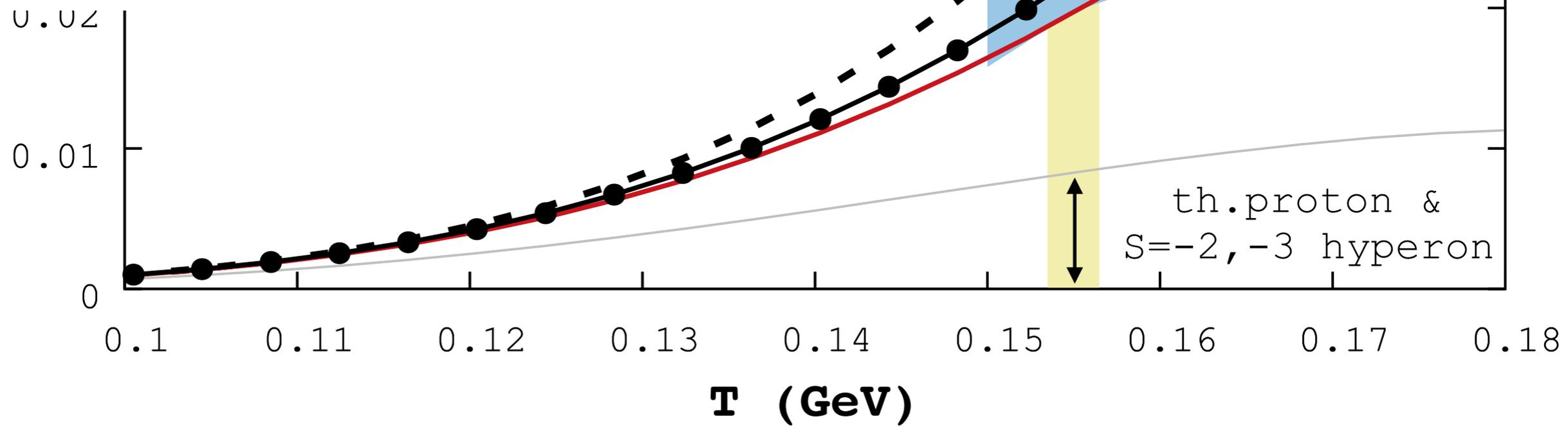
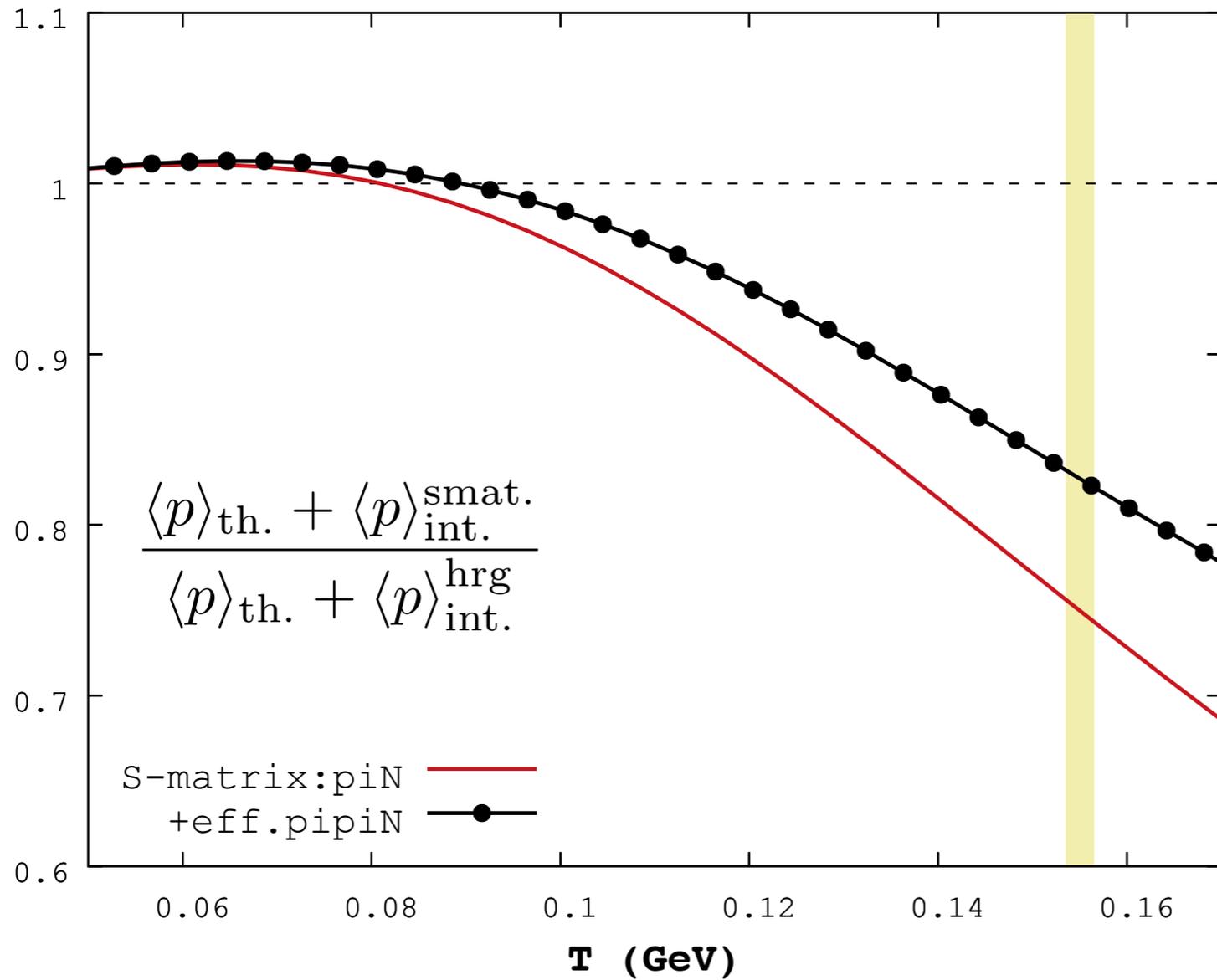
$$Q = I_z + \frac{1}{2} (B + S)$$

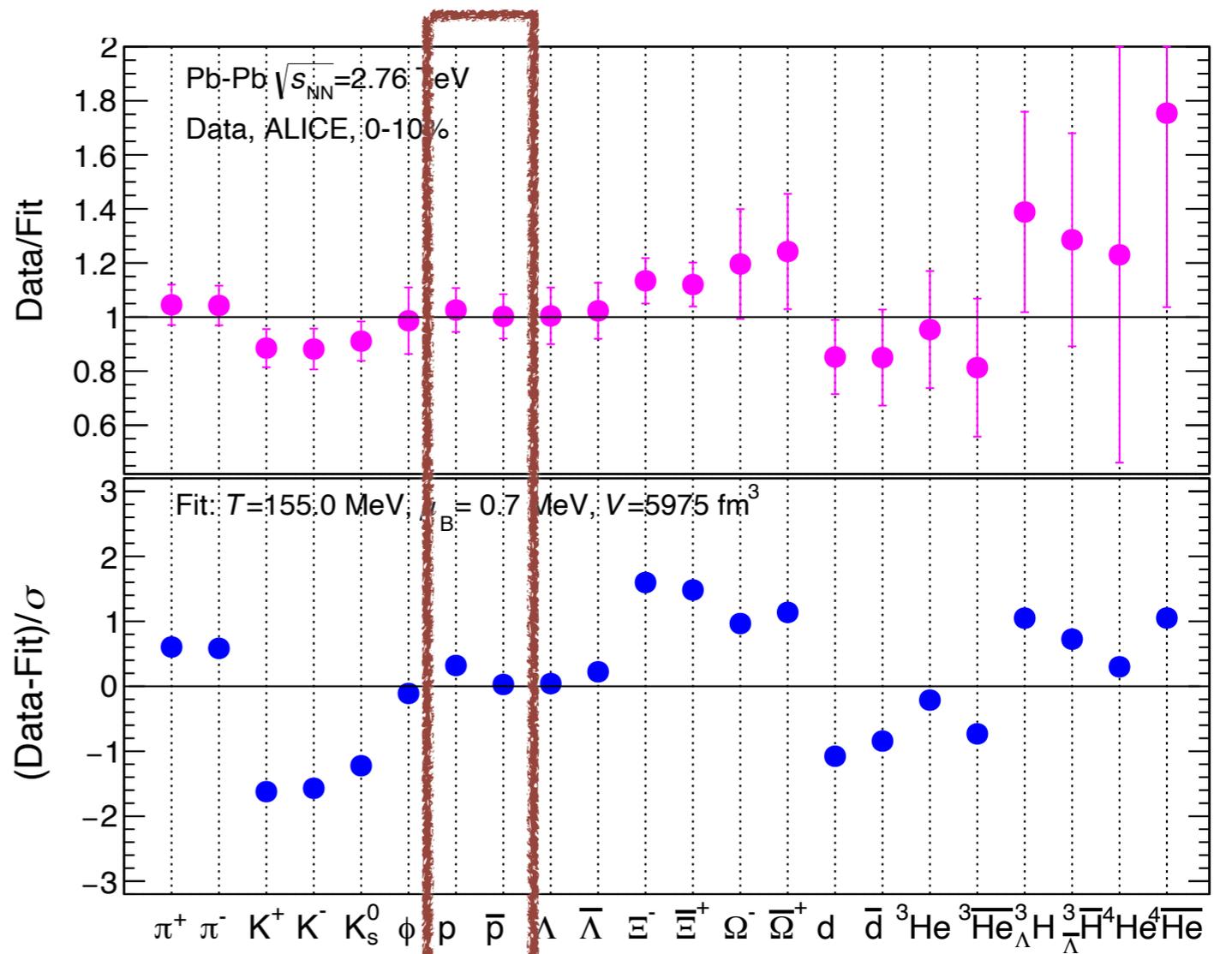
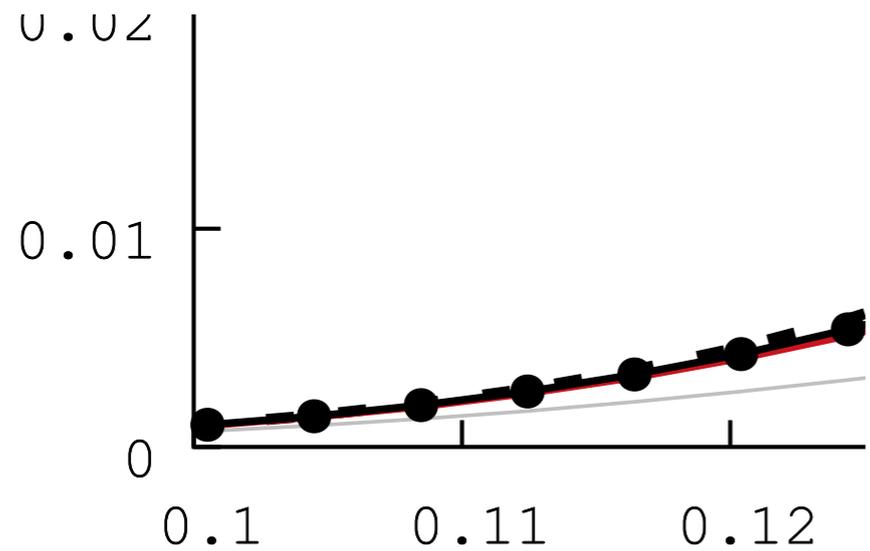
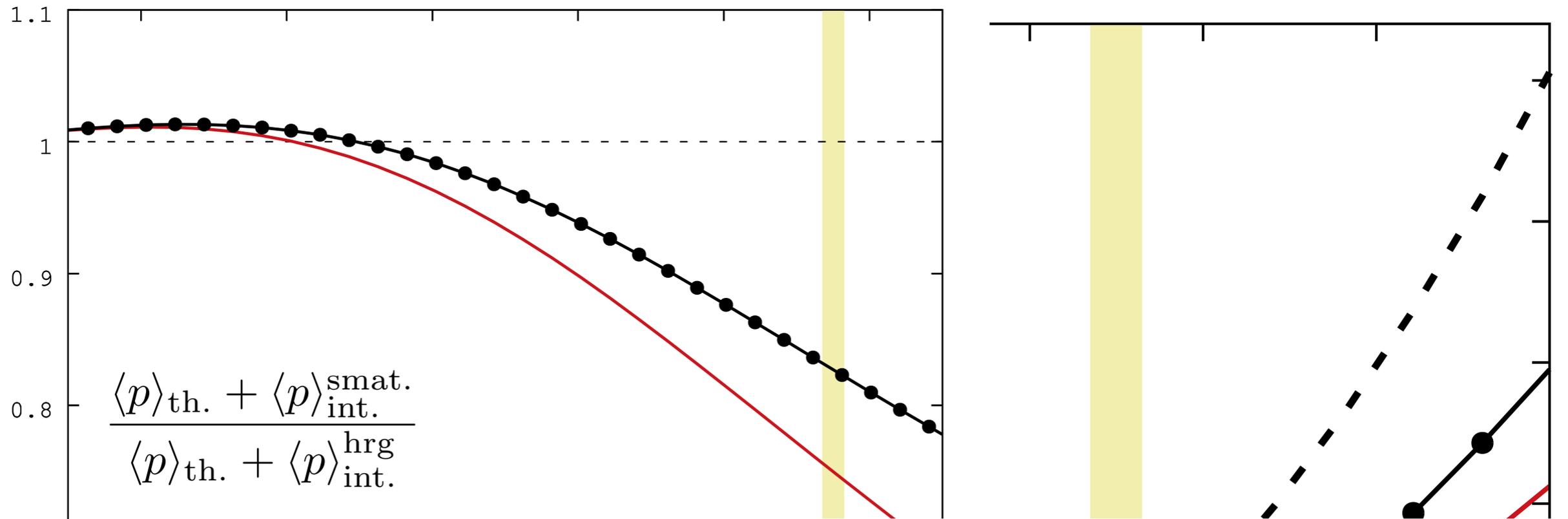


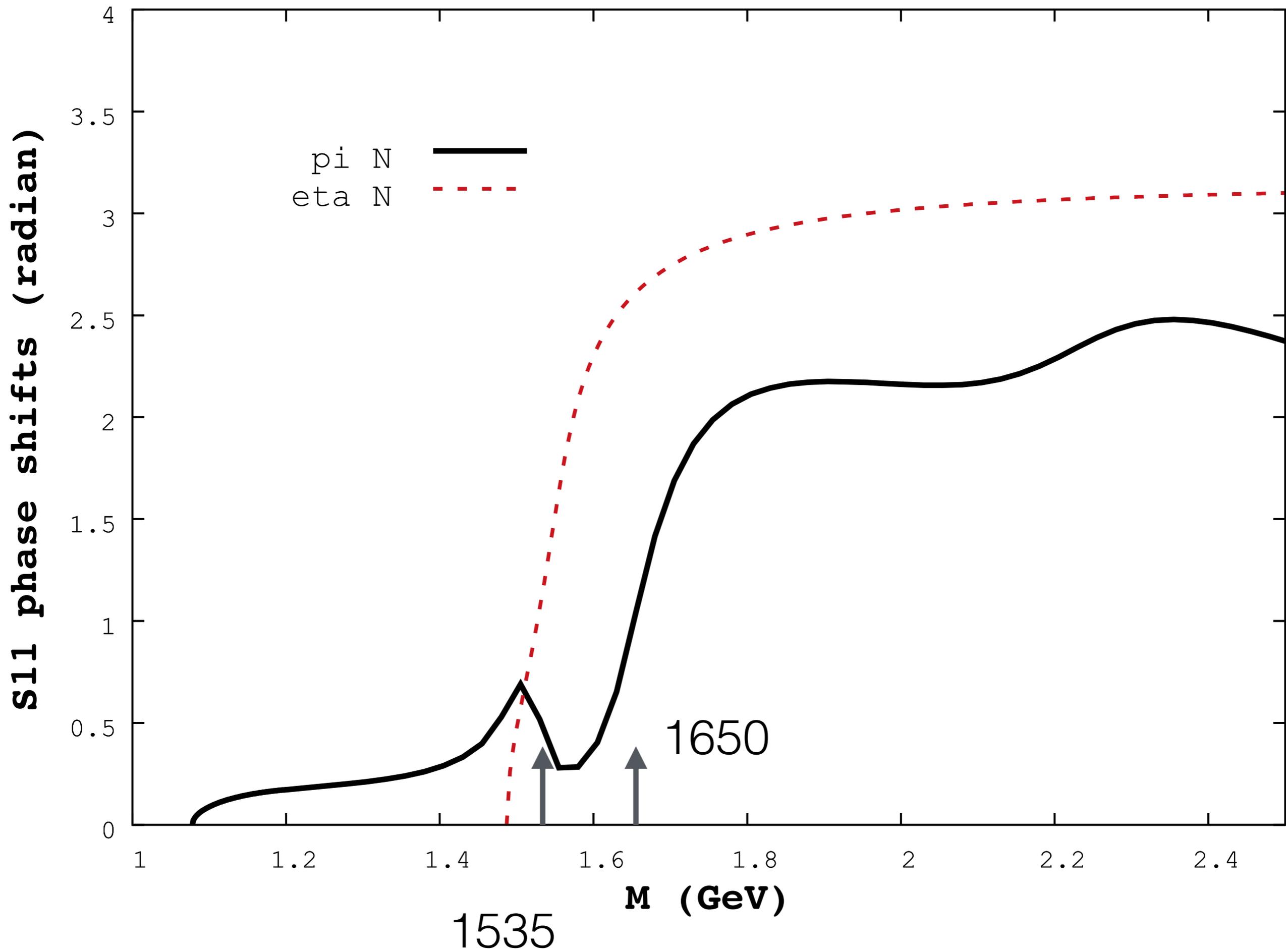


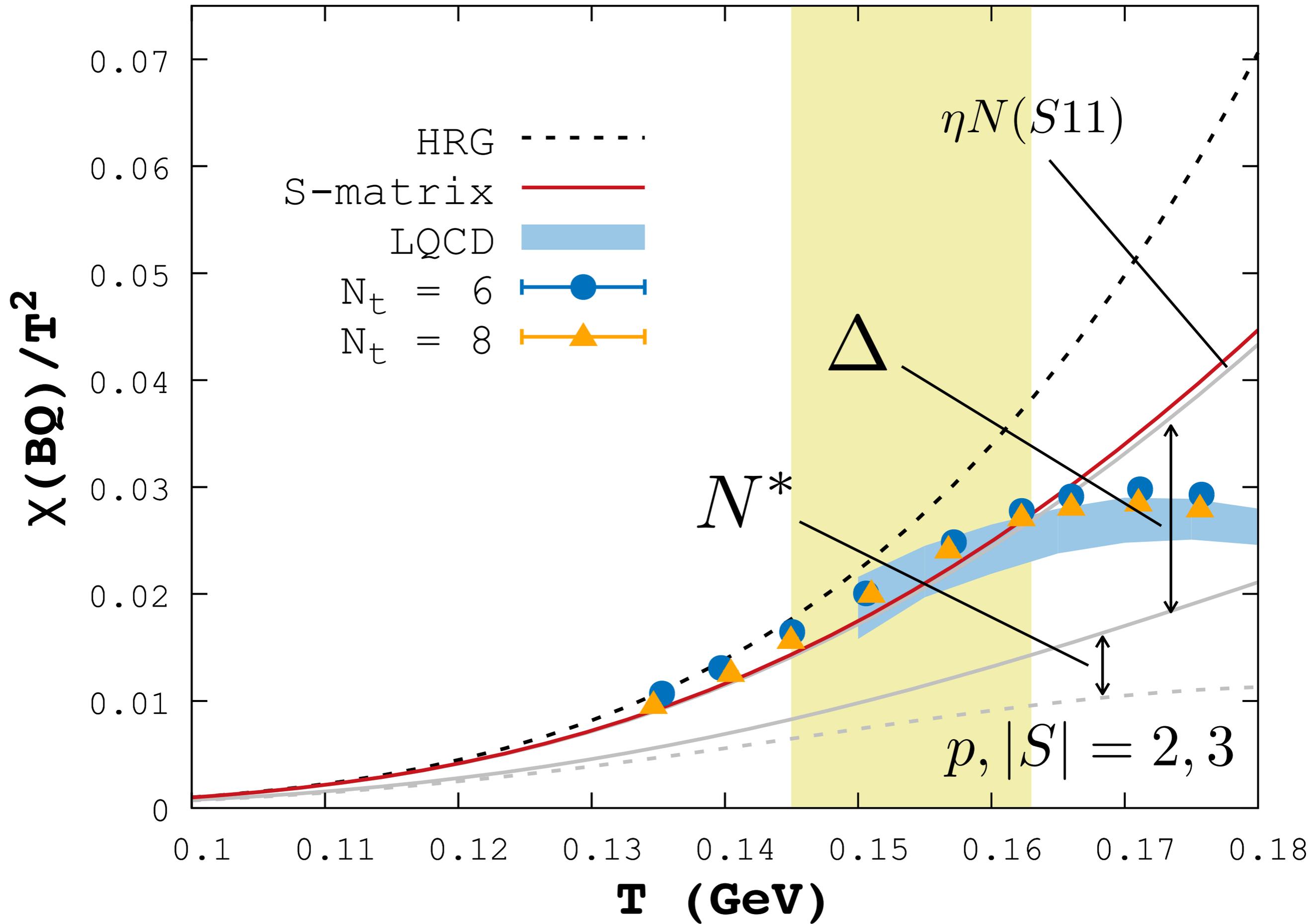












$S = -1$ HYPERONS COUPLED CHANNEL SYSTEM

C. Fernandez-Ramirez, PML, and P. Petreczky,
PRC **98**, 044910 (2018)

PHASE SHIFT FROM PWA

Coupled Channels partial wave calculator for KN scattering

by the Joint Physics Analysis Center (JPAC)

Version: September 1, 2015

Authors:

Cesar Fernandez-Ramirez (Jefferson Lab)

Igor V. Danilkin (Jefferson Lab)

Vincent Mathieu (Indiana University)

Adam P. Szczepaniak (Indiana University and Jefferson Lab)

Citation: Fernandez-Ramirez et al., arxiv:1510.07065 [hep-ph]

First version: Cesar Fernandez-Ramirez (Jefferson Lab)

This version: Cesar Fernandez-Ramirez (Jefferson Lab)

Contact: cefera@gmail.com (Cesar Fernandez-Ramirez)

Disclaimers:

1 - This code follows the 'garbage in, garbage out' philosophy. If your parameters do not make sense, the output will not make sense either.

2 - You can use, share and modify this code under your own responsibility.

3 - This code is distributed in the hope that it will be useful, but WITHOUT ANY WARRANTY; without even the implied warranty of

MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE.

4 - No PhD students or postdocs were severely damaged during the development of this project.

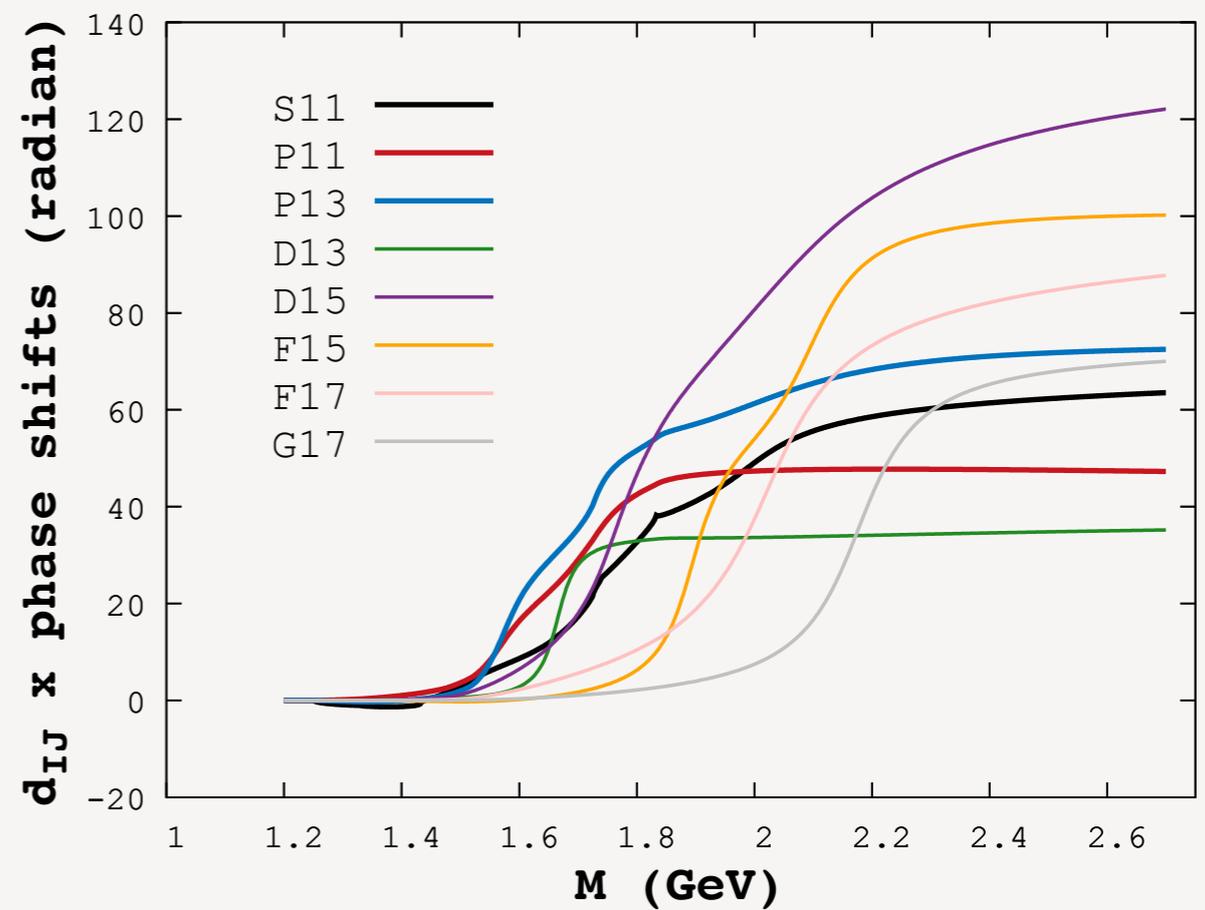
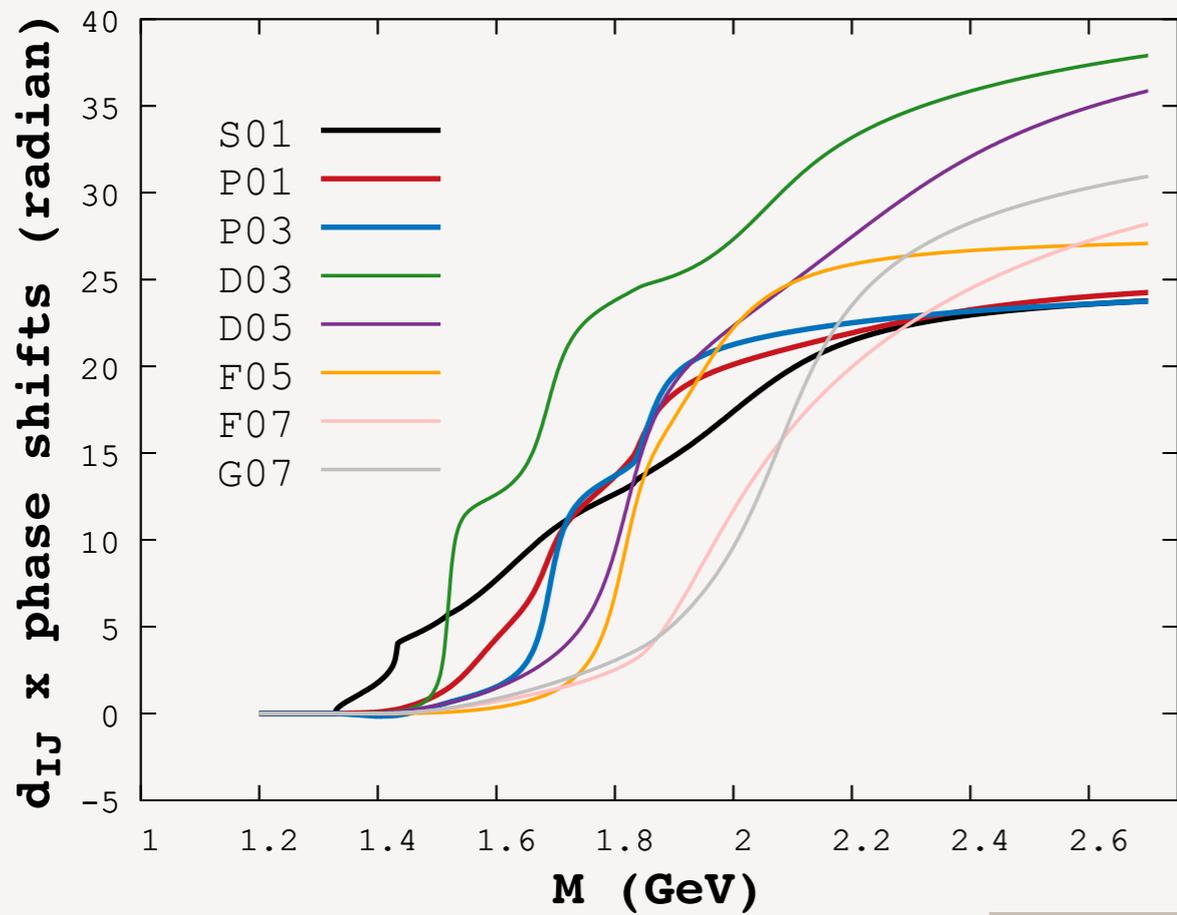
STRANGENESS CONTENT IN A HADRON GAS

- K-N system requires

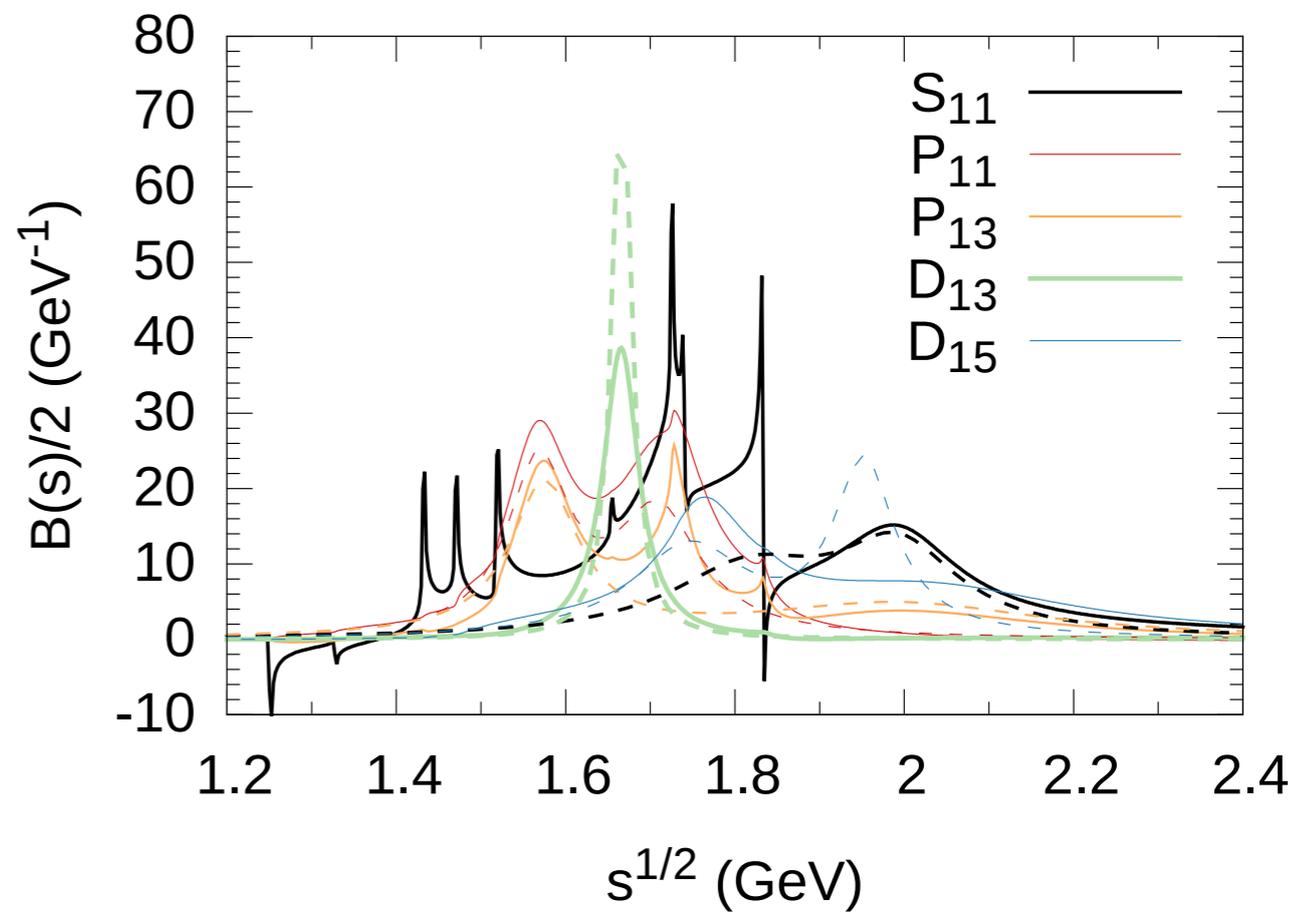
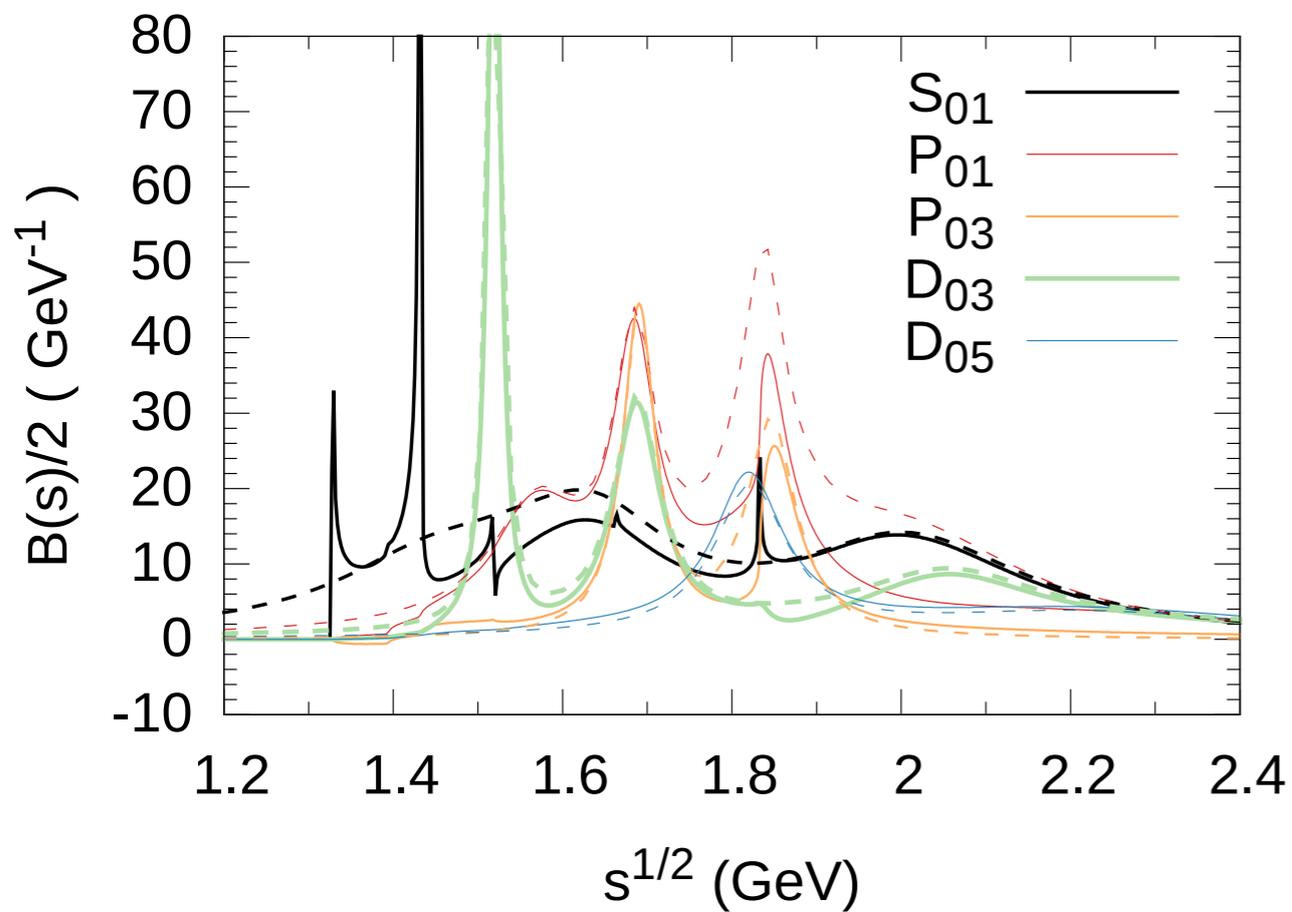
$$|\bar{K}N\rangle, |\pi\Sigma\rangle, |\pi\Lambda\rangle, |\eta\Lambda\rangle$$

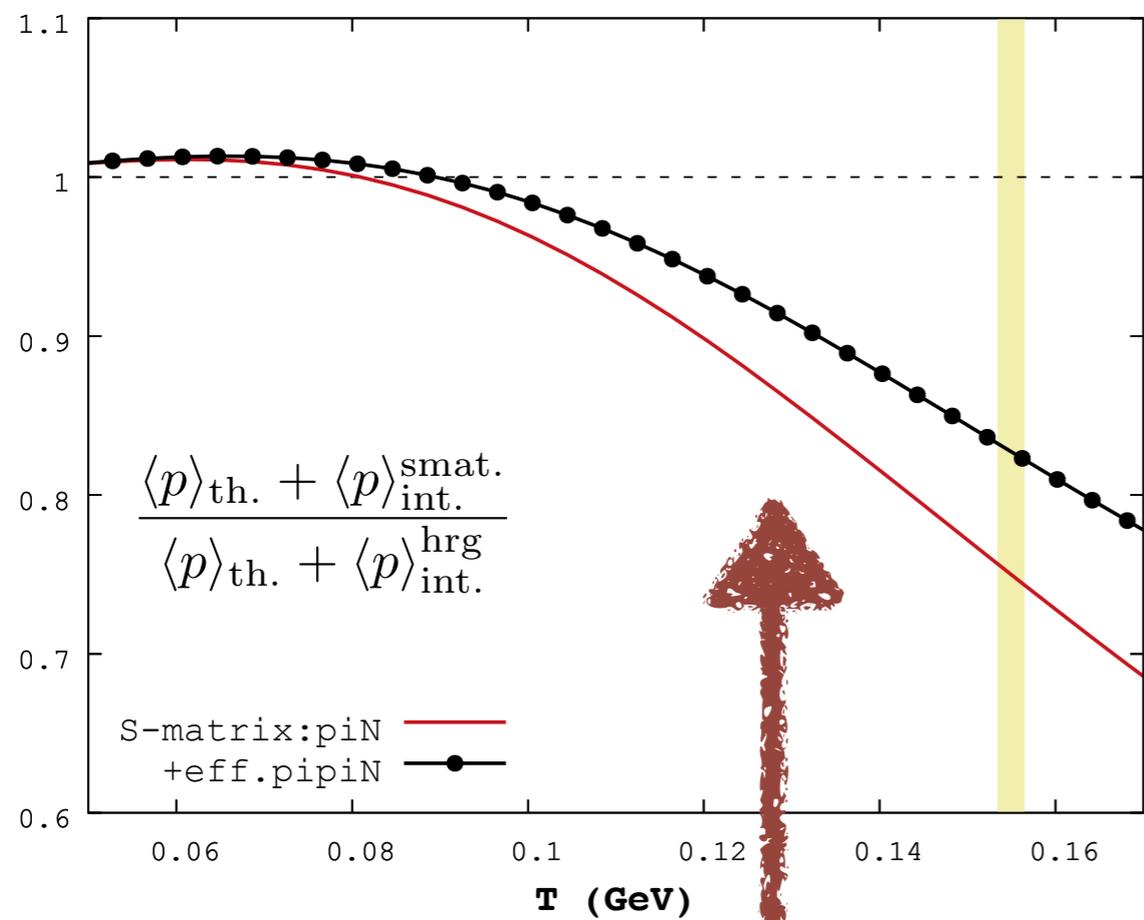
$$\begin{aligned} Q(M) &\equiv \frac{1}{2} \text{Im} (\text{tr} \\ &= \frac{1}{2} \text{Im} (\ln \\ &= \delta_{\bar{K}N} + \delta \end{aligned}$$

- 1 $\rightarrow \bar{K}N$,
- 2 $\rightarrow \pi\Sigma$,
- 3 $\rightarrow \pi\Lambda$,
- 4 $\rightarrow \eta\Lambda$,
- 5 $\rightarrow \eta\Sigma$,
- 6 $\rightarrow \bar{K}_1N$,
- 7 $\rightarrow [\bar{K}_3N]_-$,
- 8 $\rightarrow [\bar{K}_3N]_+$,
- 9 $\rightarrow [\pi\Sigma^*]_-$,
- 10 $\rightarrow [\pi\Sigma^*]_+$,
- 11 $\rightarrow [\bar{K}\Delta]_-$,
- 12 $\rightarrow [\bar{K}\Delta]_+$,
- 13 $\rightarrow [\pi\Lambda(1520)]_-$,
- 14 $\rightarrow [\pi\Lambda(1520)]_+$,
- 15 $\rightarrow \pi\pi\Lambda$,
- 16 $\rightarrow \pi\pi\Sigma$.

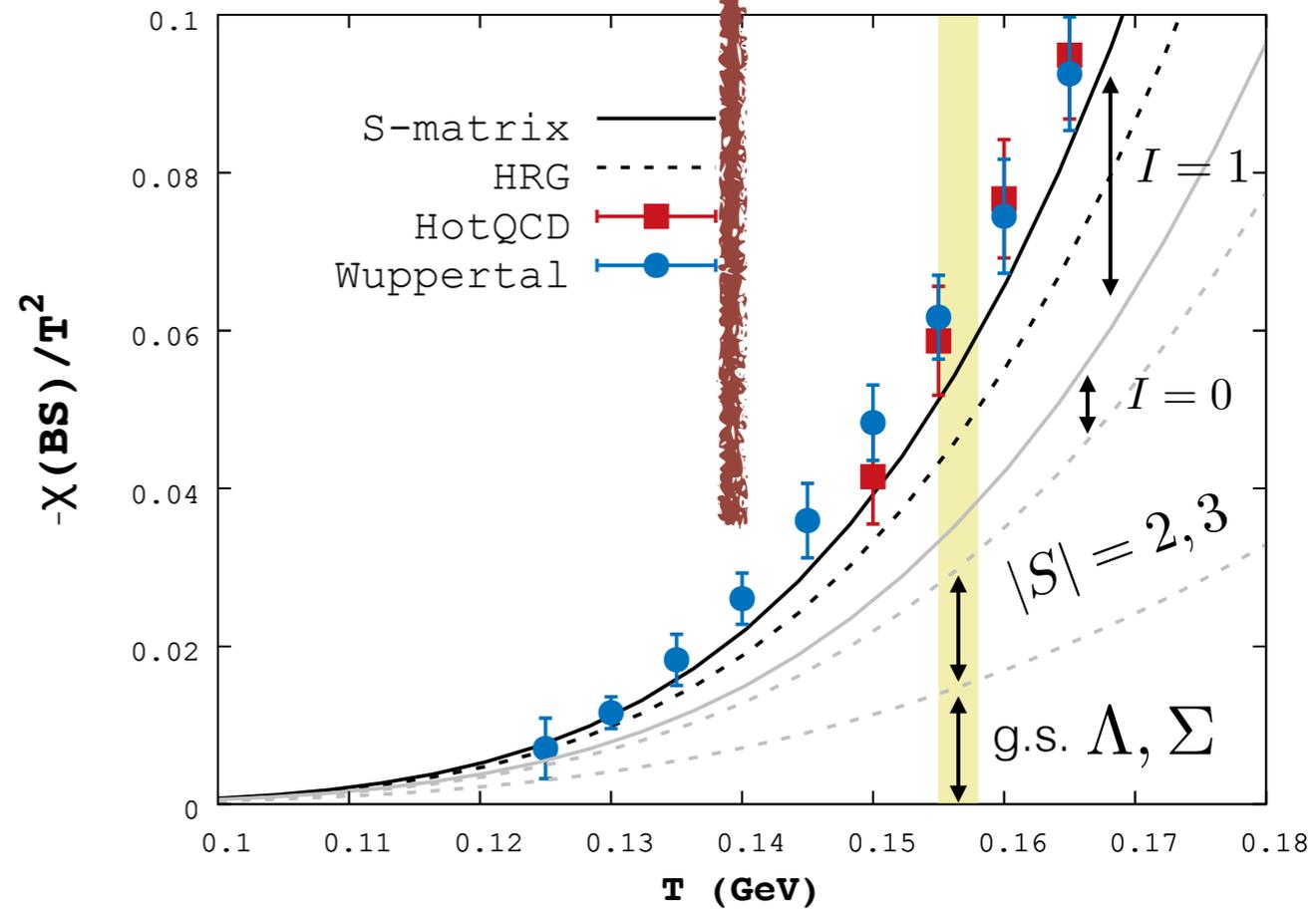
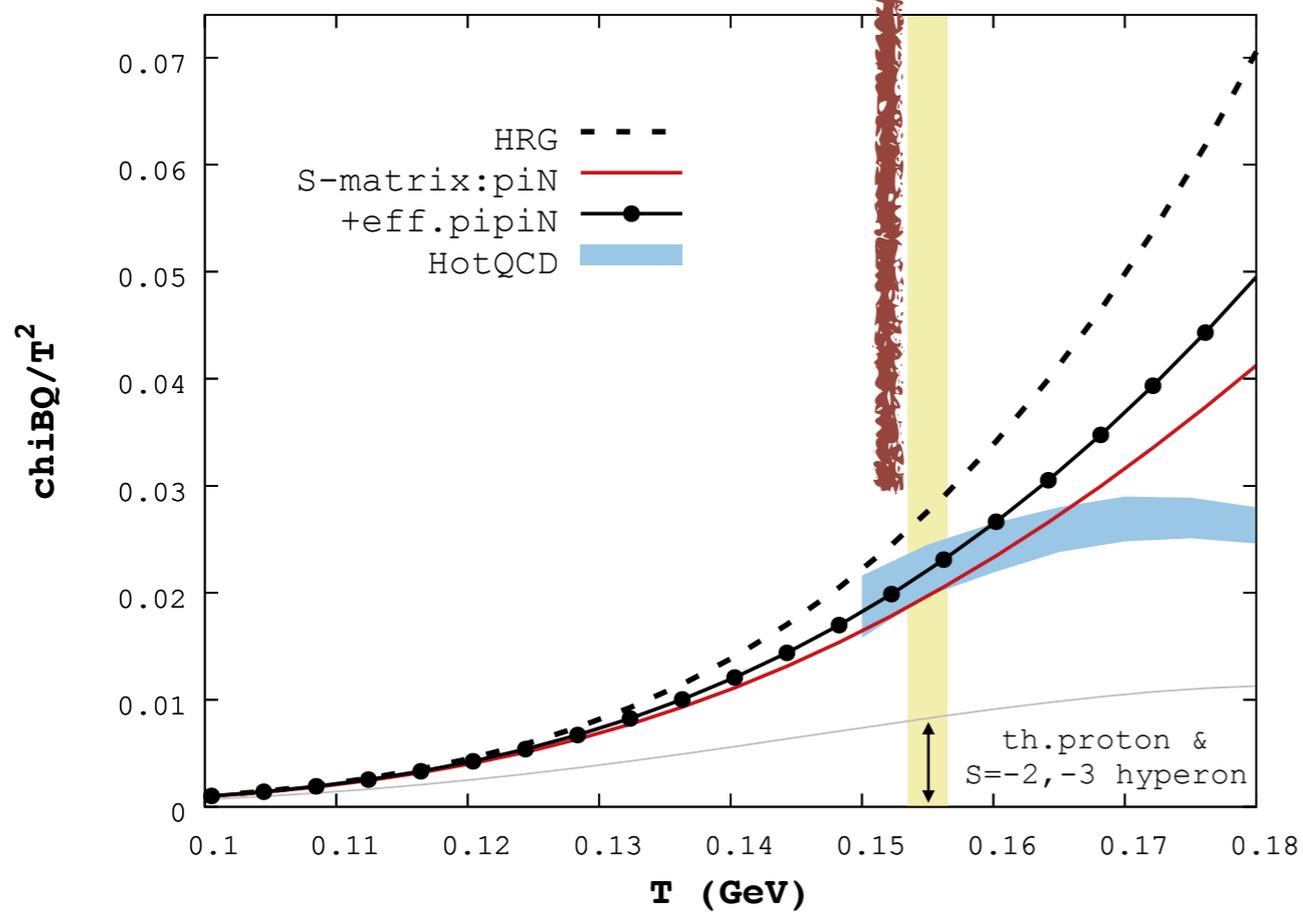


$S=-1$ Hyperon





???



UNQUENCHING THE QUARK MODEL

NO SERIOUS MESON SPECTROSCOPY
WITHOUT SCATTERING*

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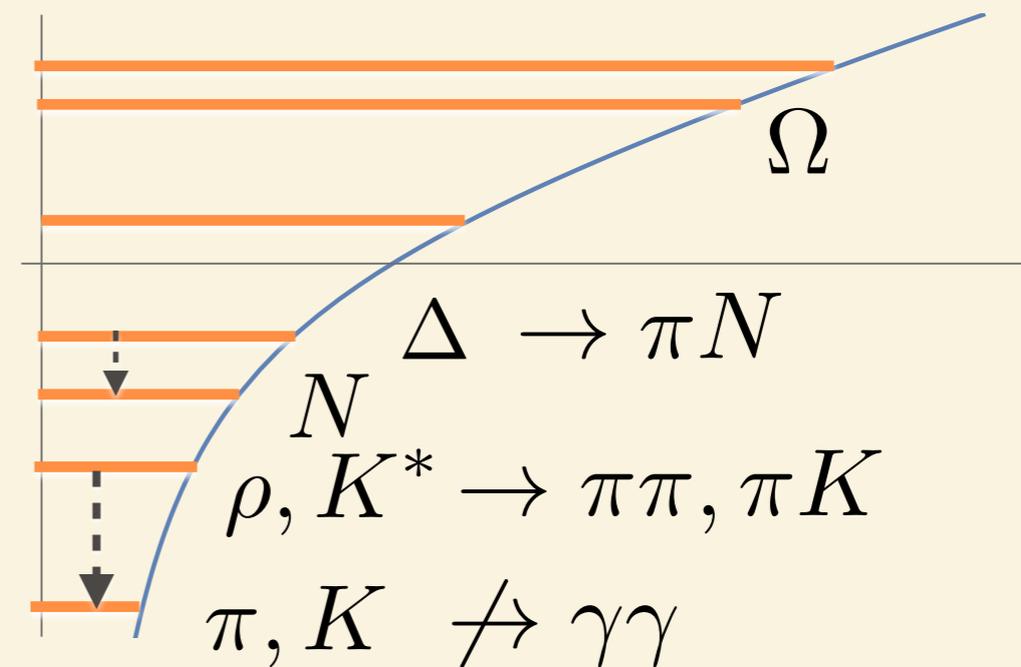
(Received January 26, 2015)

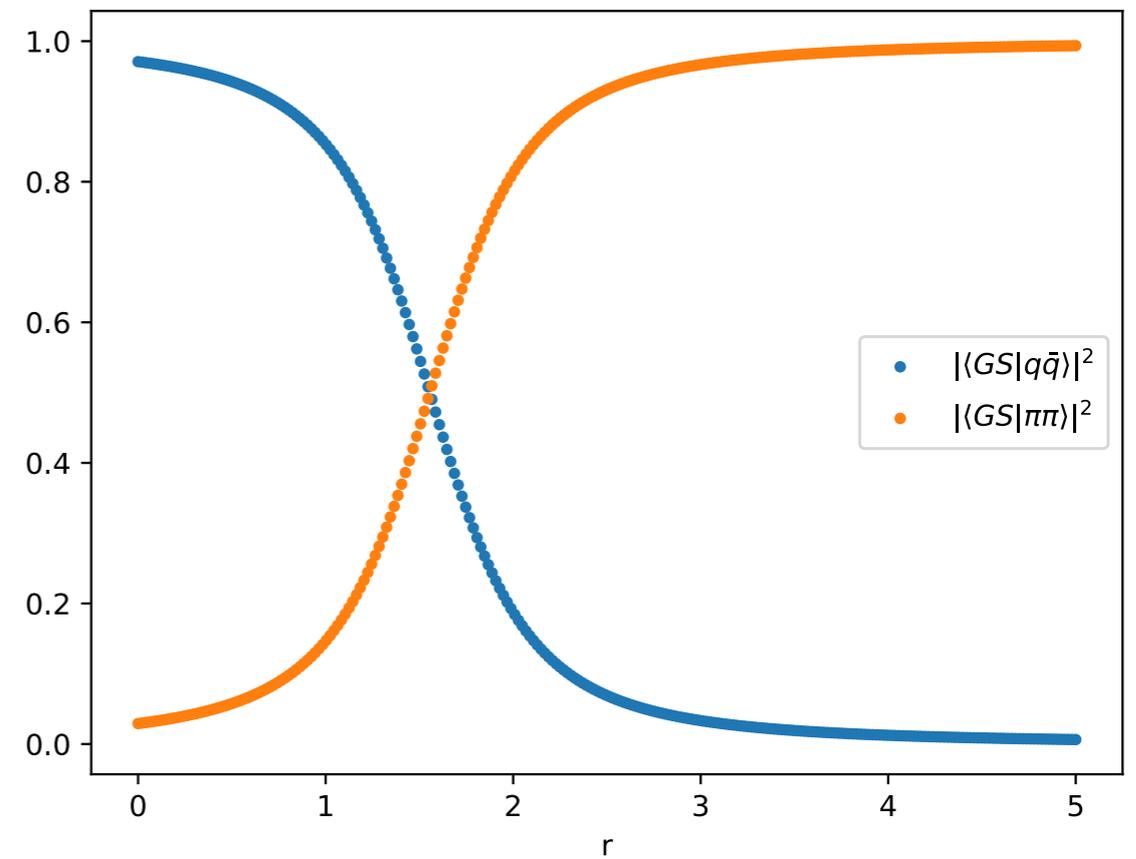
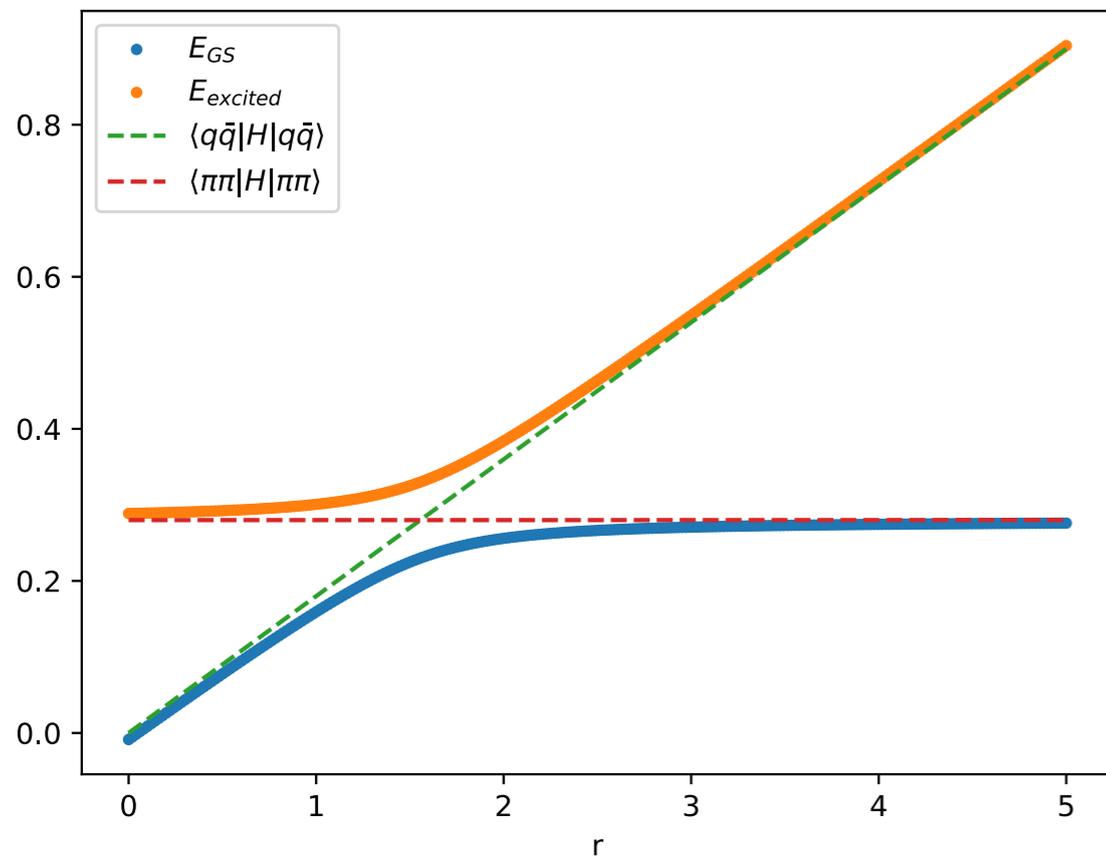
HADRON RESONANCE MODEL

- Confinement

$$Z = \sum_{\alpha=B,M} \langle \alpha | e^{-\beta H} | \alpha \rangle$$

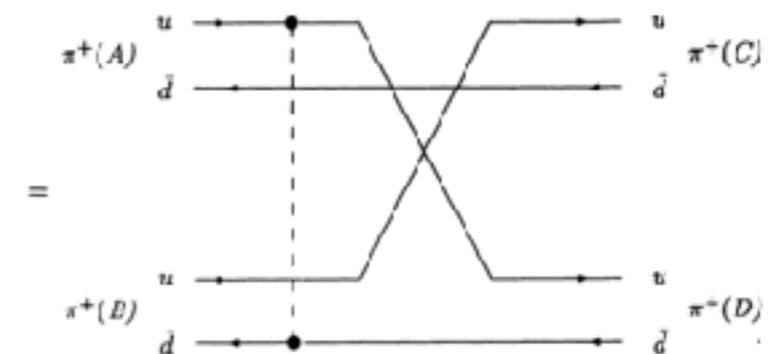
QCD spectrum





need to study the adiabatic surfaces...

DIAGRAMMATIC APPROACH TO MESON-MESON SCATTERING ...



NOW WE KNOW HOW TO COUNT...

- impact of pions on thermodynamics
- lifetimes of unstable states / exotics in a thermal system
- thermal aspects in unquenching the quark model
- S-matrix interpretation of in-medium effect
- N-body forces

THANK YOU