

The physics of binary neutron star mergers

Lecture II

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Plan of the lectures

* Lecture I: the **math** of neutron-star mergers

* Lecture II: the **physics** of neutron-star mergers

* Lecture III: the **astrophysics** of neutron-star mergers

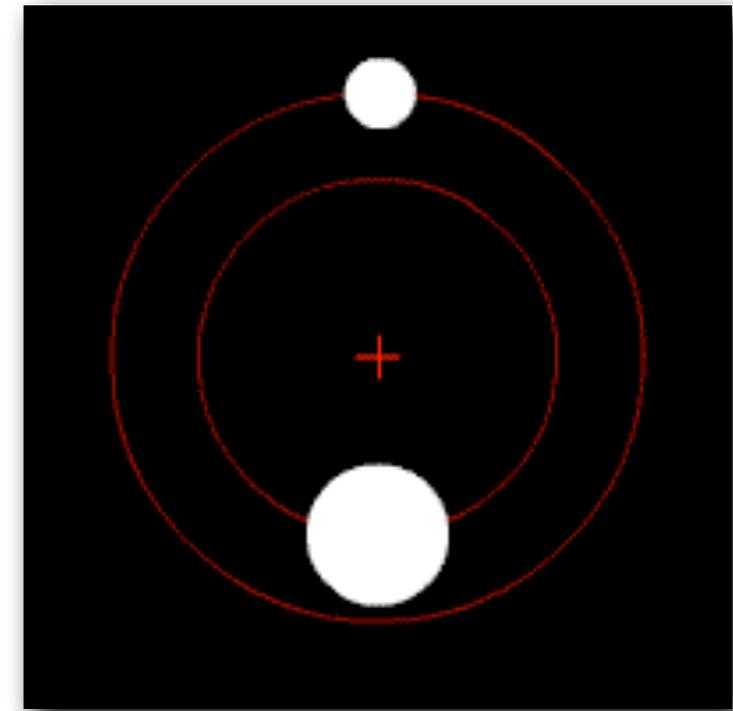
* L. Baiotti and L. Rezzolla, *Rep. Prog. Phys.* **80**, 096901, 2017

* V. Paschalidis, *Classical Quantum Gravity* **34**, 084002 2017

* Rezzolla and Zanotti, *“Relativistic Hydrodynamics”*, Oxford University Press, 2013

The two-body problem: Newton vs Einstein

Take two objects of mass m_1 and m_2 interacting only gravitationally



In **Newtonian gravity** solution is analytic: there exist **closed** orbits (circular/elliptic) with

$$\ddot{\mathbf{r}} = -\frac{GM}{d_{12}^3} \mathbf{r}$$

where $M \equiv m_1 + m_2$, $\mathbf{r} \equiv \mathbf{r}_1 - \mathbf{r}_2$, $d_{12} \equiv |\mathbf{r}_1 - \mathbf{r}_2|$.

In **Einstein's gravity** no analytic solution! **No closed** orbits: the system loses energy/angular momentum via gravitational waves.

The two-body problem in GR

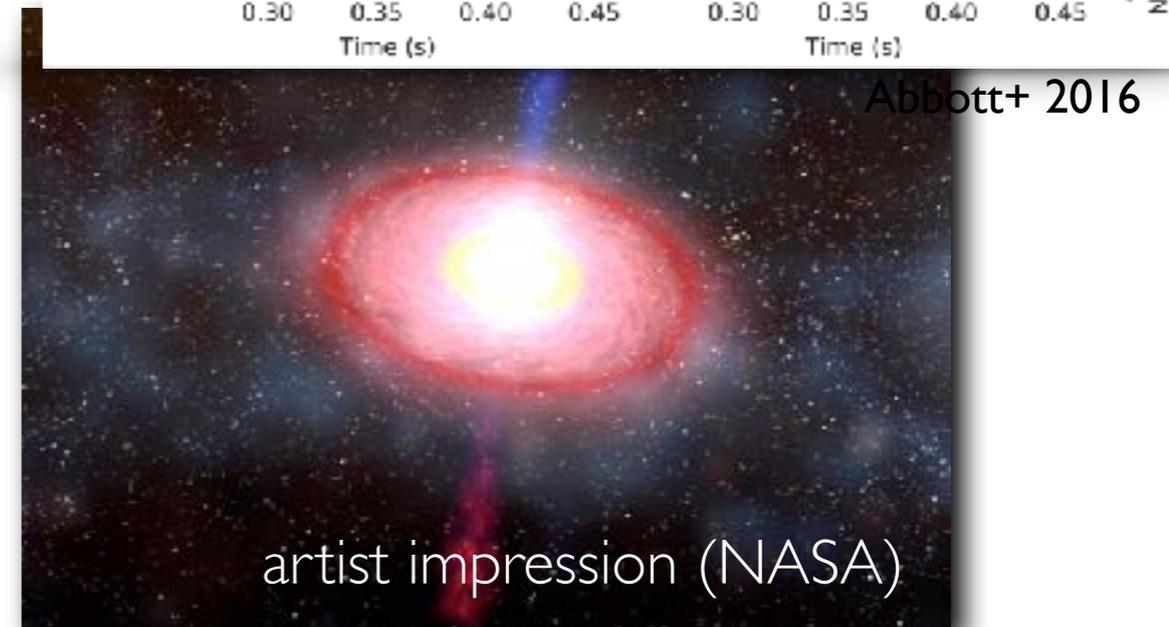
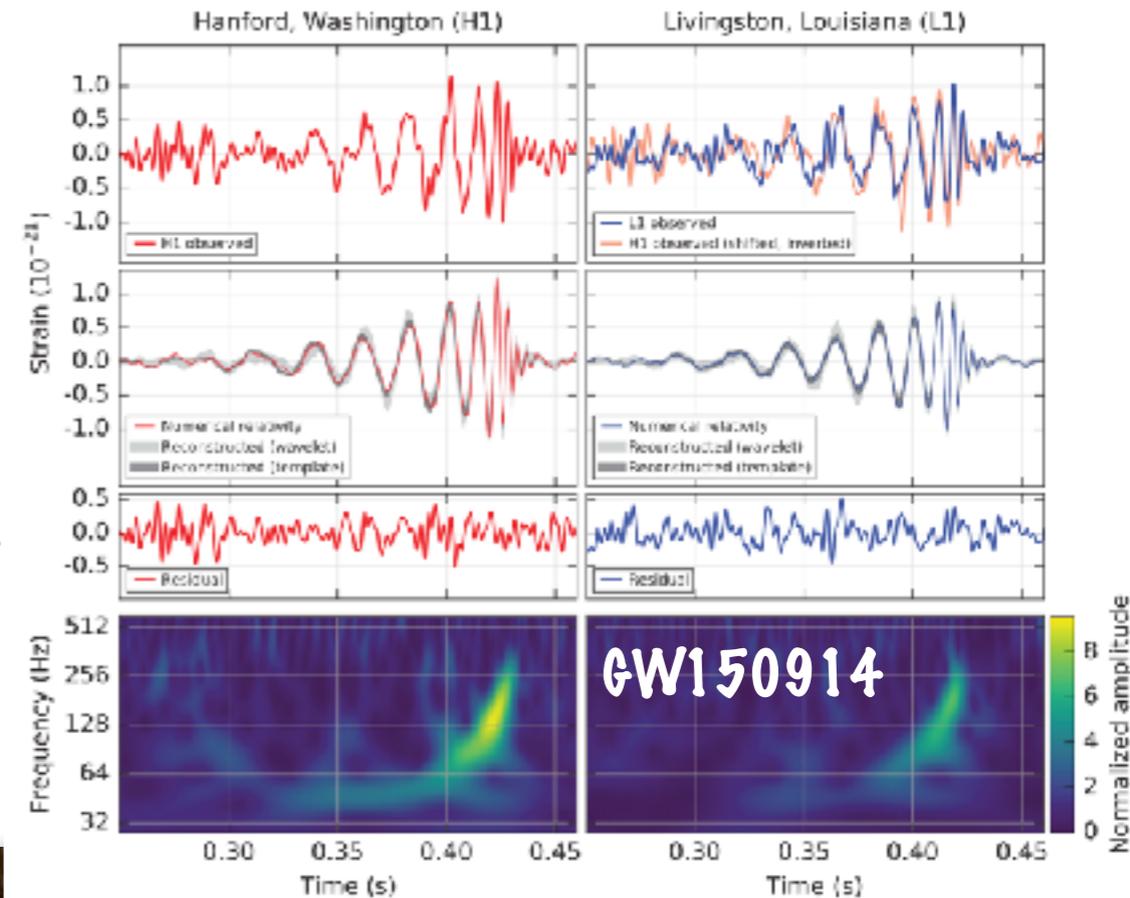
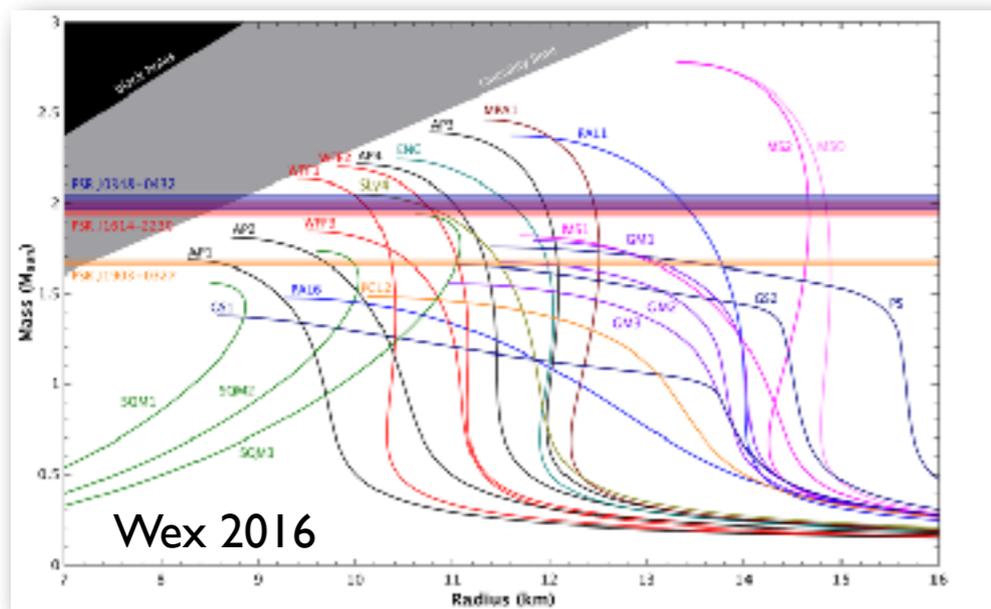
- For BHs we know what to **expect**:

$$\text{BH} + \text{BH} \longrightarrow \text{BH} + \text{GWs}$$

- For NSs the question is more **subtle**: hyper-massive neutron star (HMNS), ie

$$\text{NS} + \text{NS} \longrightarrow \text{HMNS} + \dots ? \longrightarrow \text{BH} + t$$

- **HMNS** phase can provide clear information on **EOS**



- **BH+torus** system may tell us on the central engine of **GRBs**

The two-body problem in GR

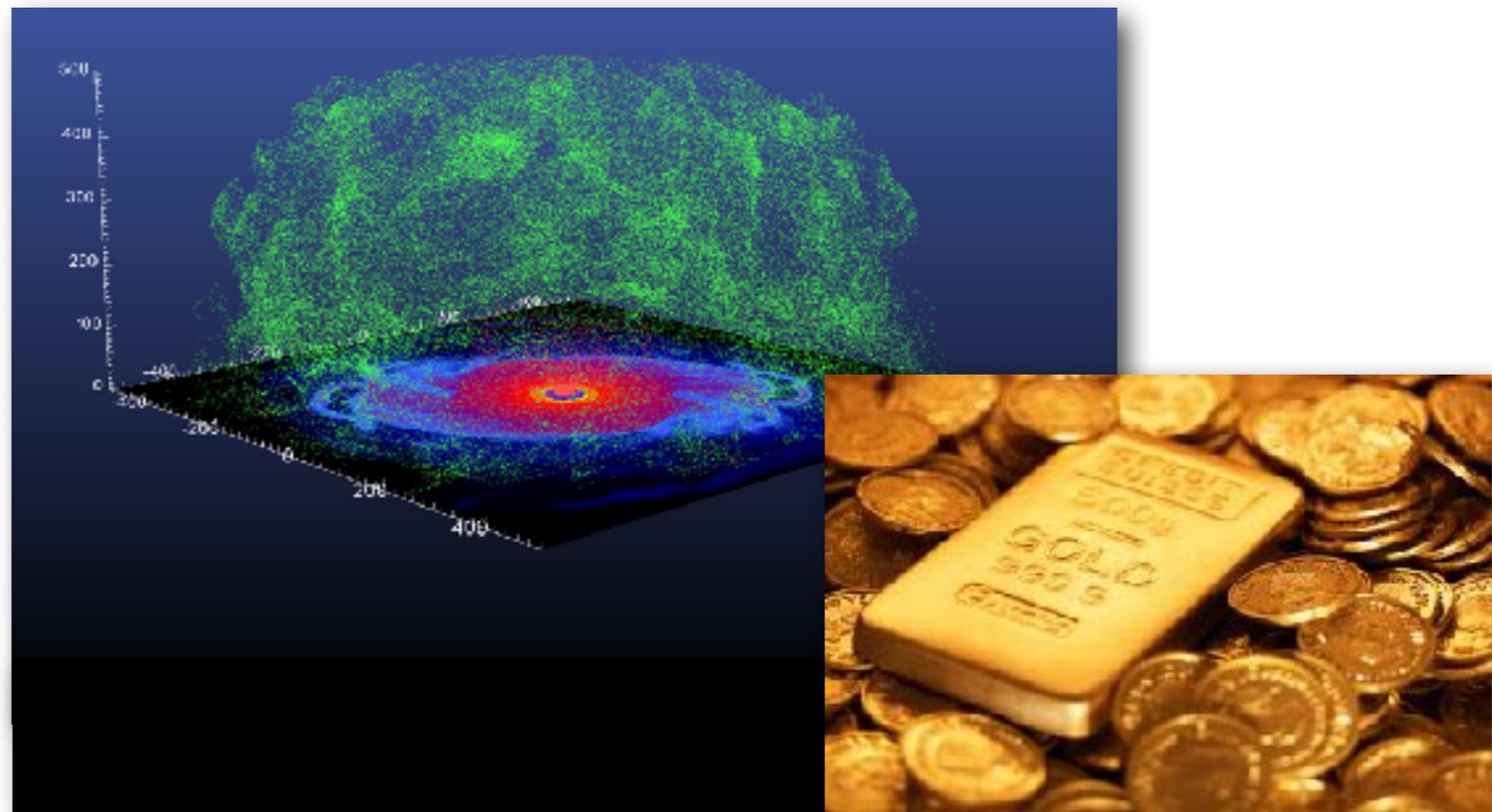
- For BHs we know what to **expect**:



- For NSs the question is more **subtle**: the merger leads to an hyper-massive neutron star (HMNS), ie a metastable equilibrium:



- **ejected matter** undergoes nucleosynthesis of heavy elements



The equations of numerical relativity

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi T_{\mu\nu}, \quad (\text{field equations})$$

$$\nabla_{\mu}T^{\mu\nu} = 0, \quad (\text{cons. energy/momentum})$$

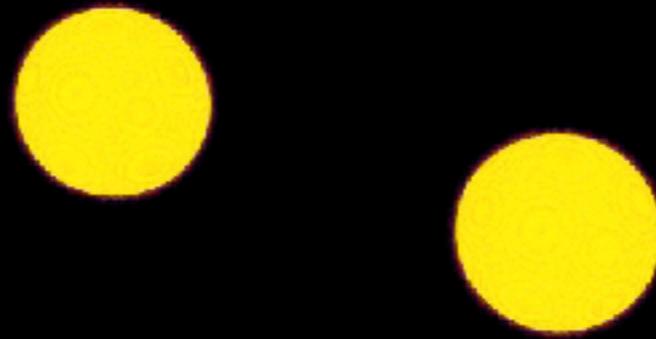
$$\nabla_{\mu}(\rho u^{\mu}) = 0, \quad (\text{cons. rest mass})$$

$$p = p(\rho, \epsilon, Y_e, \dots), \quad (\text{equation of state})$$

$$\nabla_{\nu}F^{\mu\nu} = I^{\mu}, \quad \nabla_{\nu}^*F^{\mu\nu} = 0, \quad (\text{Maxwell equations})$$

$$T_{\mu\nu} = T_{\mu\nu}^{\text{fluid}} + T_{\mu\nu}^{\text{EM}} + \dots \quad (\text{energy - momentum tensor})$$

In GR these equations do not possess an analytic solution in the regimes we are interested in

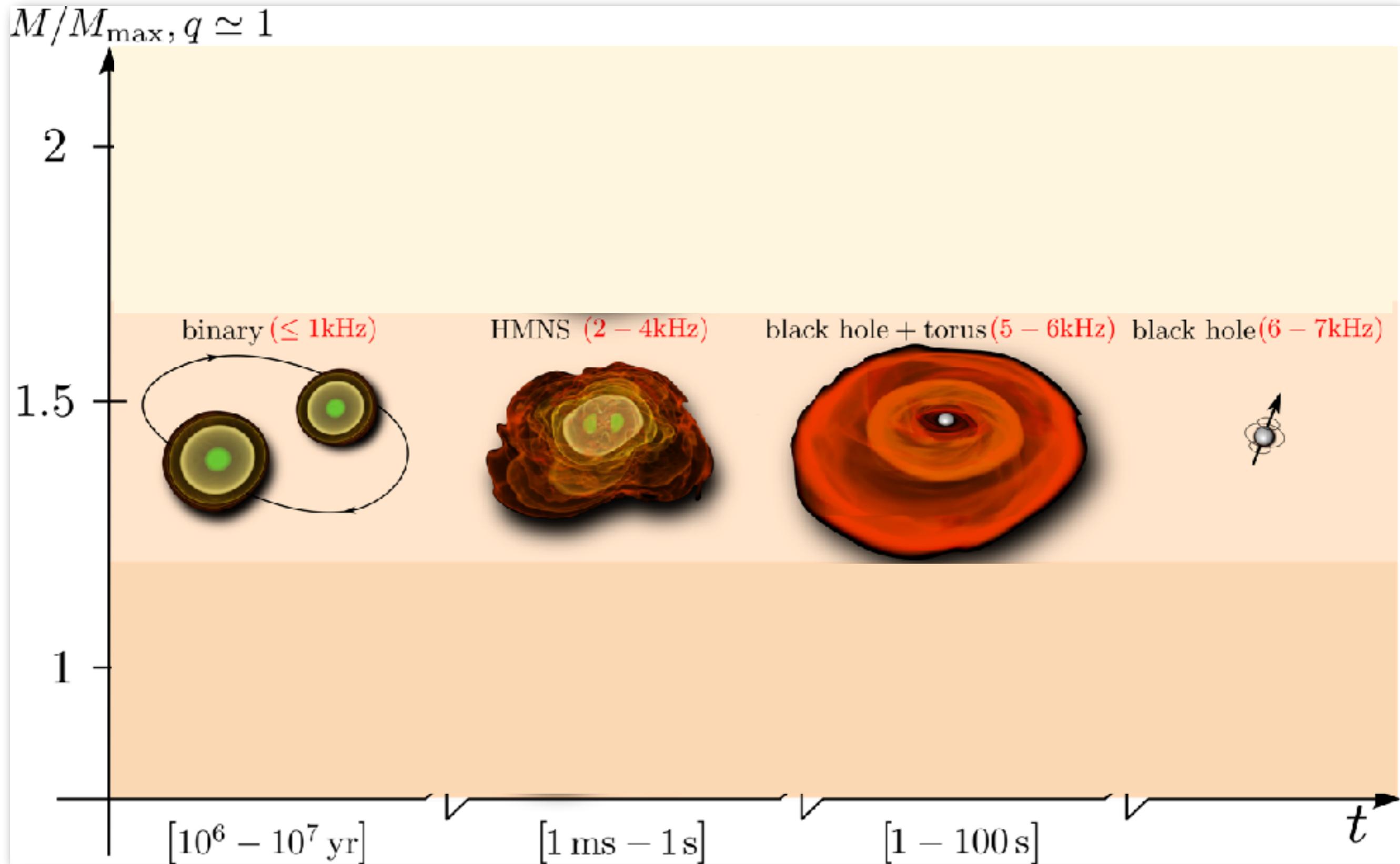


merger → HMNS → BH + torus

Quantitative differences are produced by:

- **total mass** (prompt vs delayed collapse)

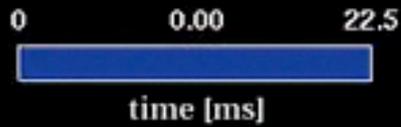
Broadbrush picture



merger → HMNS → BH + torus

Quantitative differences are produced by:

- total mass (prompt vs delayed collapse)
- mass asymmetries (HMNS and torus)



Total mass : $3.37 M_{\odot}$; mass ratio :0.80;



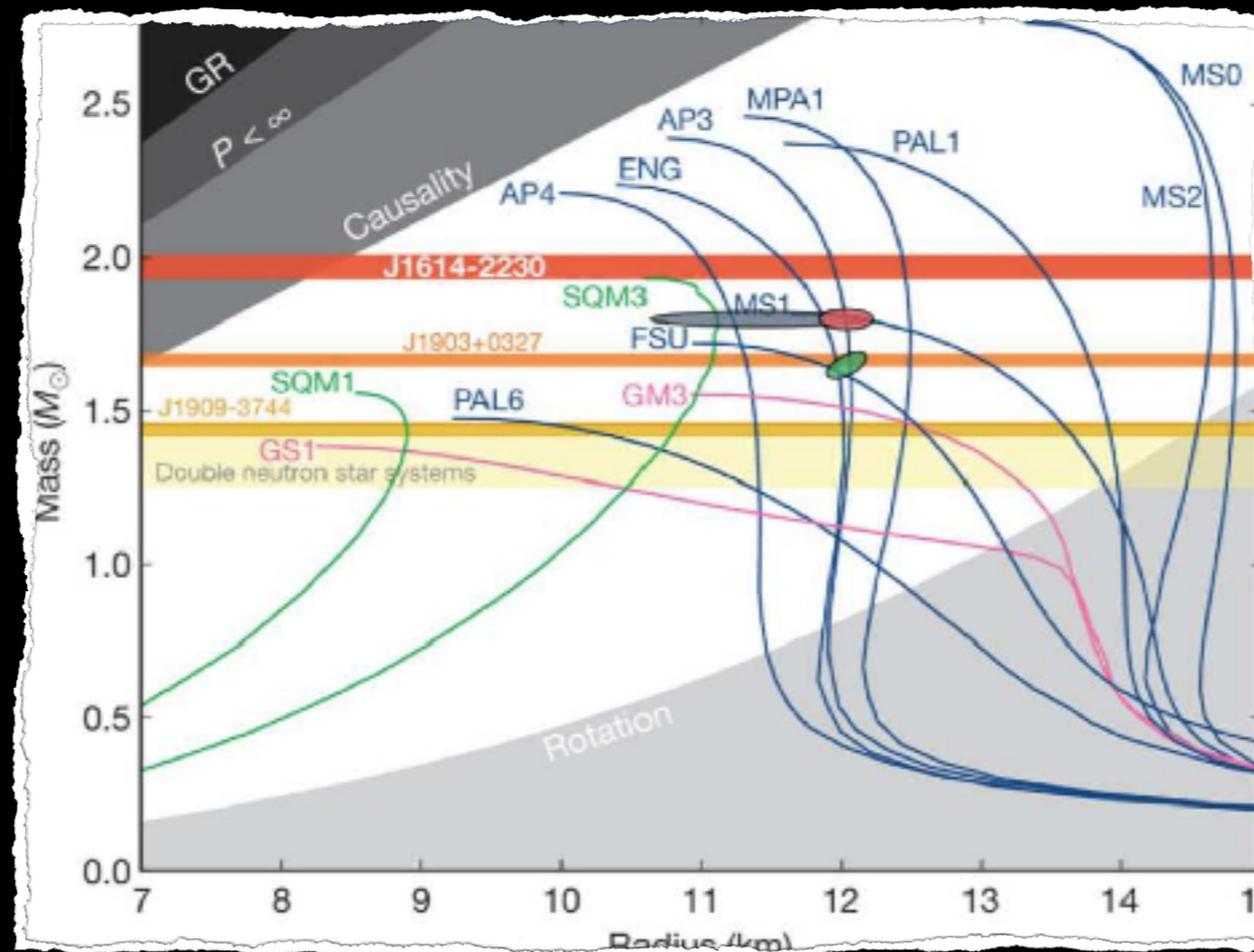
- * the torii are generically **more massive**
- * the torii are generically **more extended**
- * the torii tend to stable **quasi-Keplerian** configurations
- * overall unequal-mass systems have all the ingredients needed to create a GRB

merger → HMNS → BH + torus

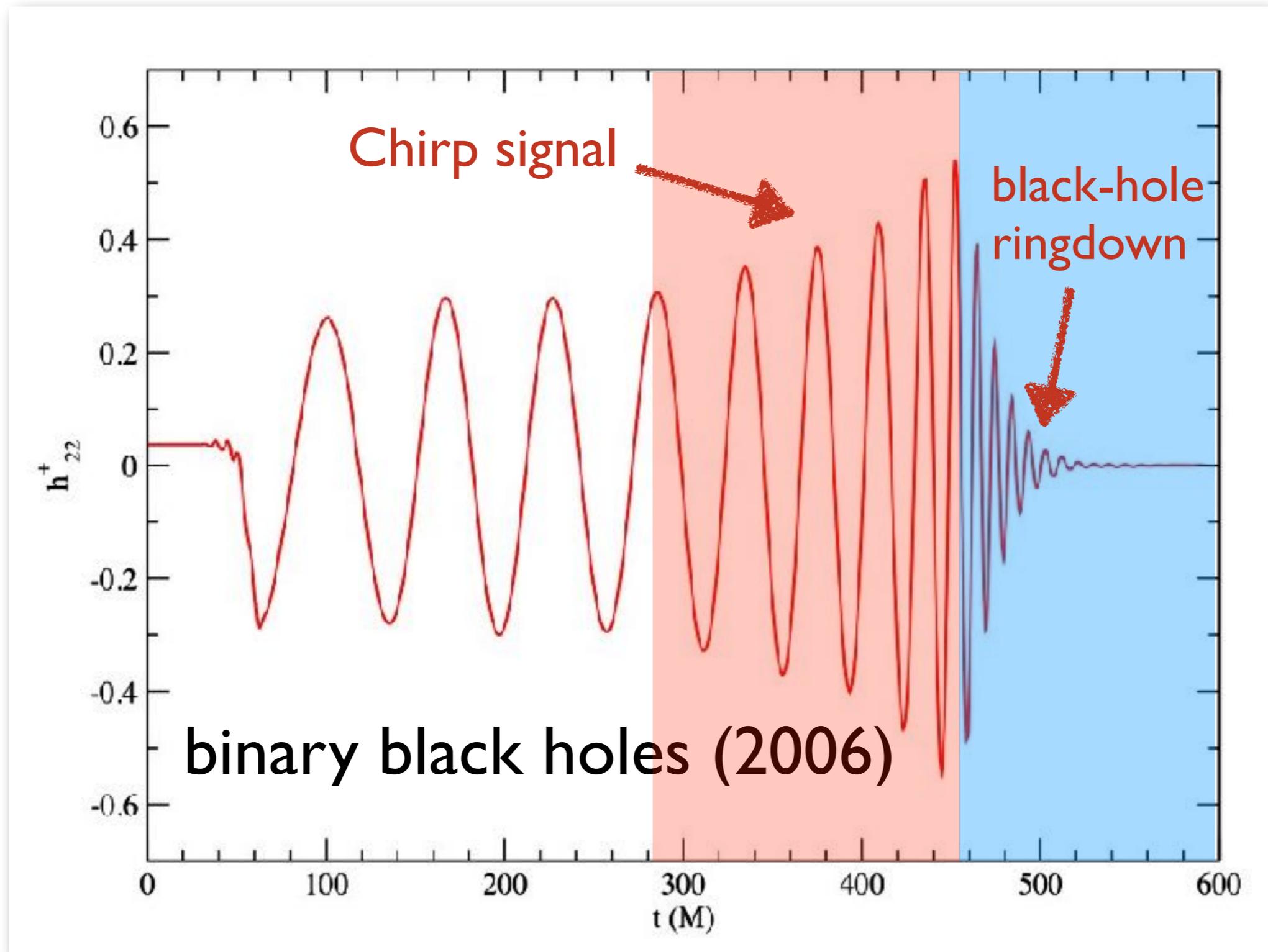
Quantitative differences are produced by:

- total **mass** (prompt vs delayed collapse)
- mass **asymmetries** (HMNS and torus)
- soft/stiff **EOS** (inspiral and post-merger)
- **magnetic fields** (equil. and EM emission)
- **radiative** losses (equil. and nucleosynthesis)

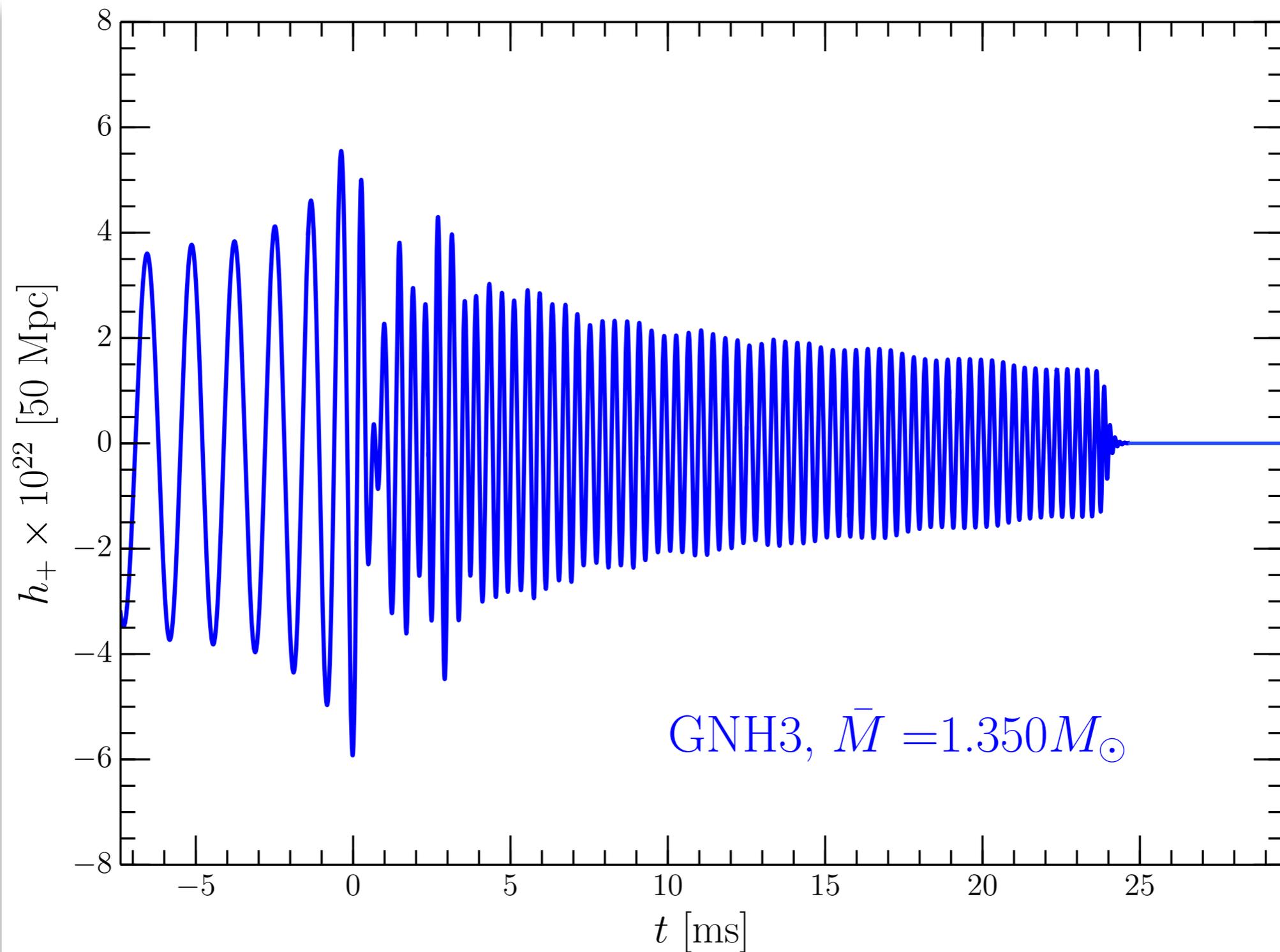
How to constrain the EOS from the GWs



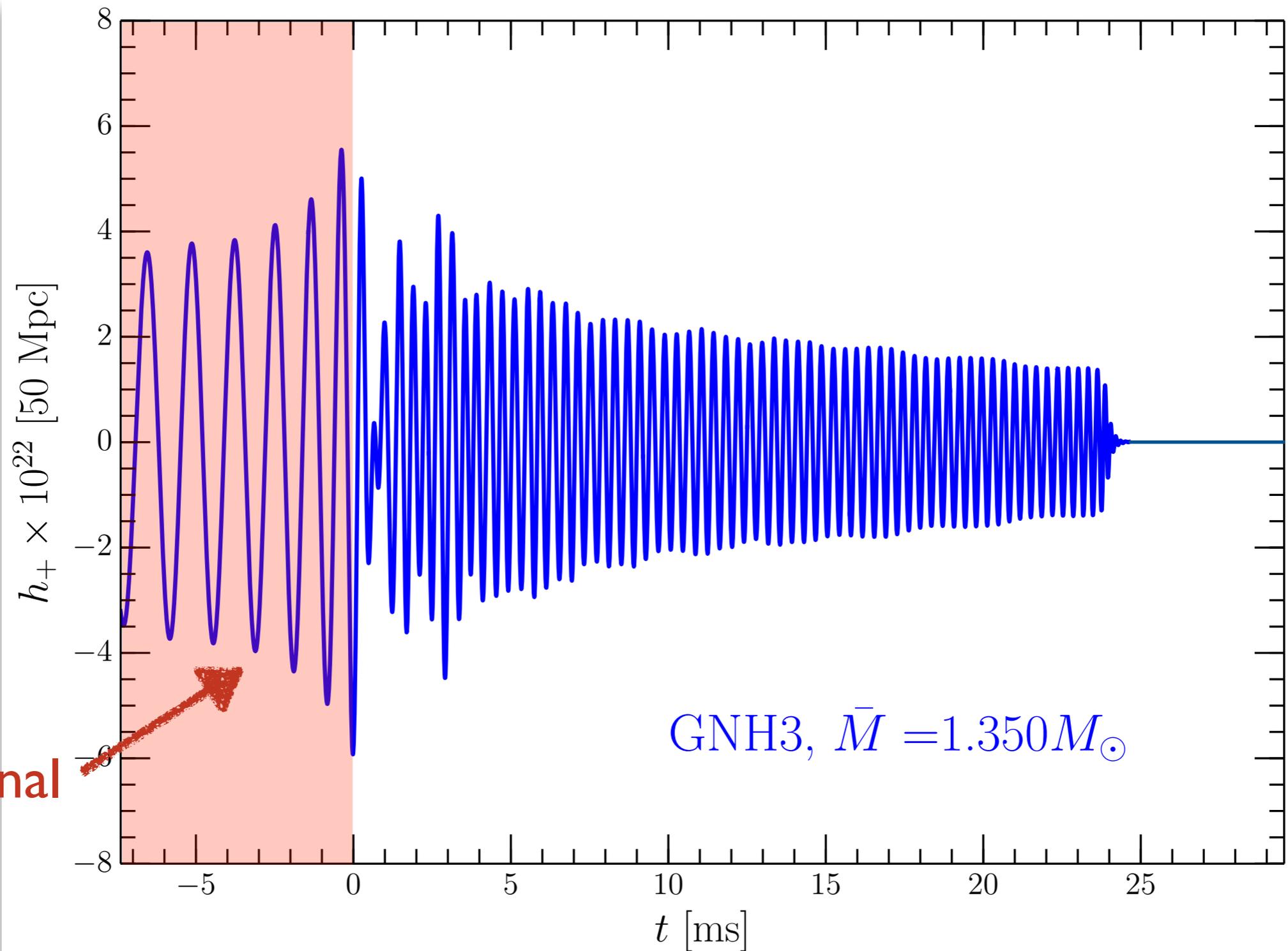
Anatomy of the GW signal



Anatomy of the GW signal



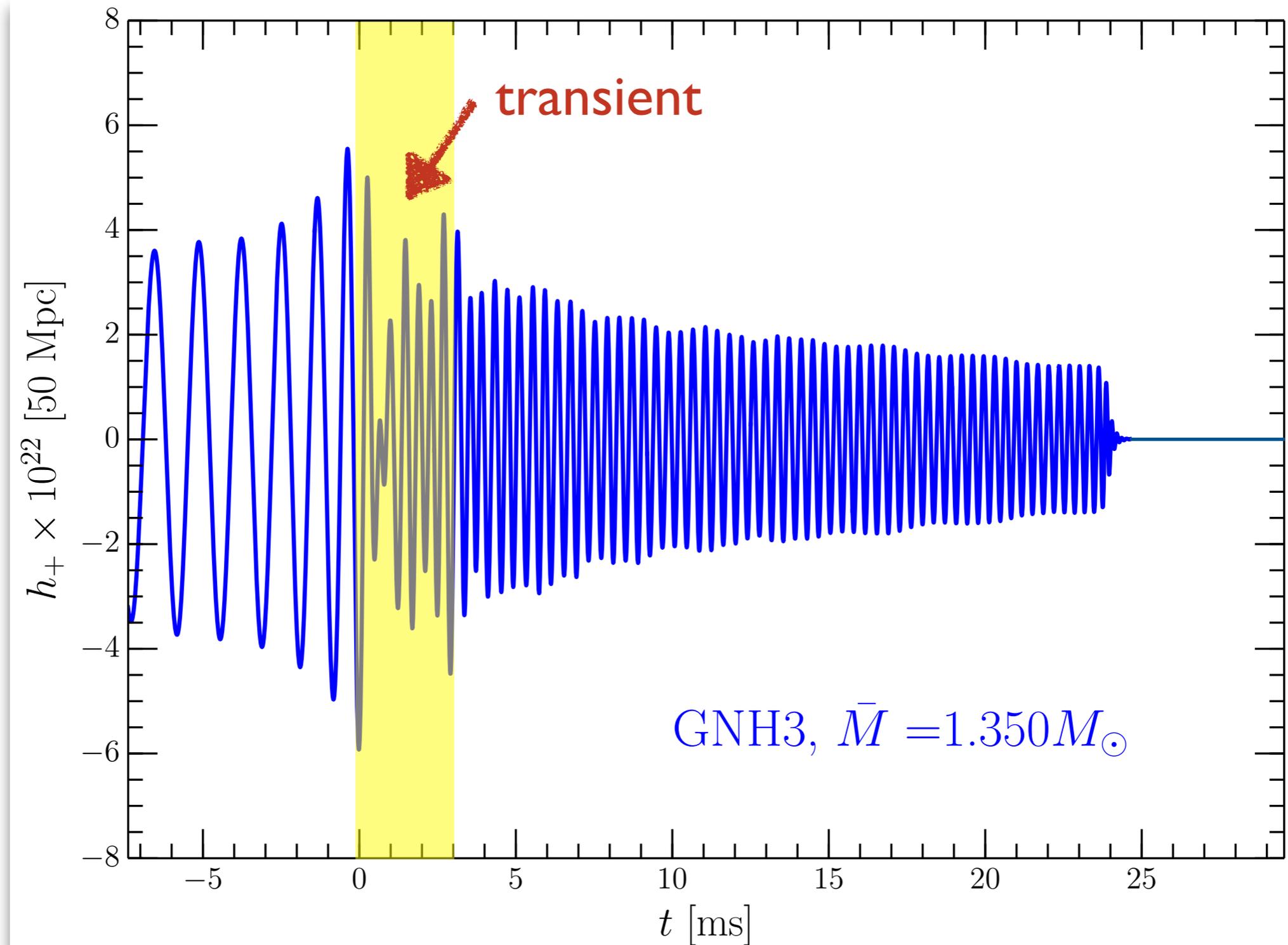
Anatomy of the GW signal



Chirp signal

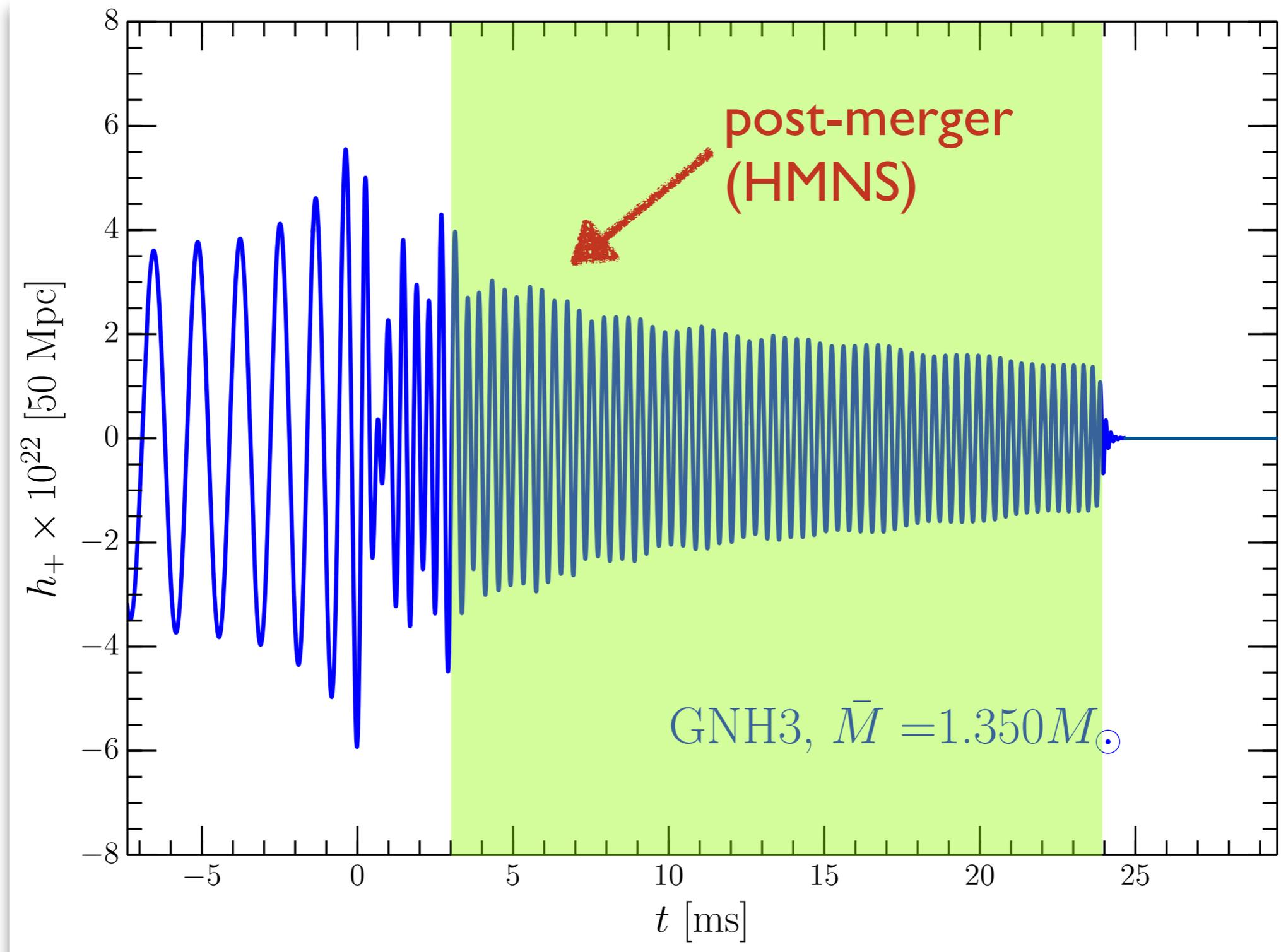
Inspiral: well approximated by PN/EOB; tidal effects important

Anatomy of the GW signal



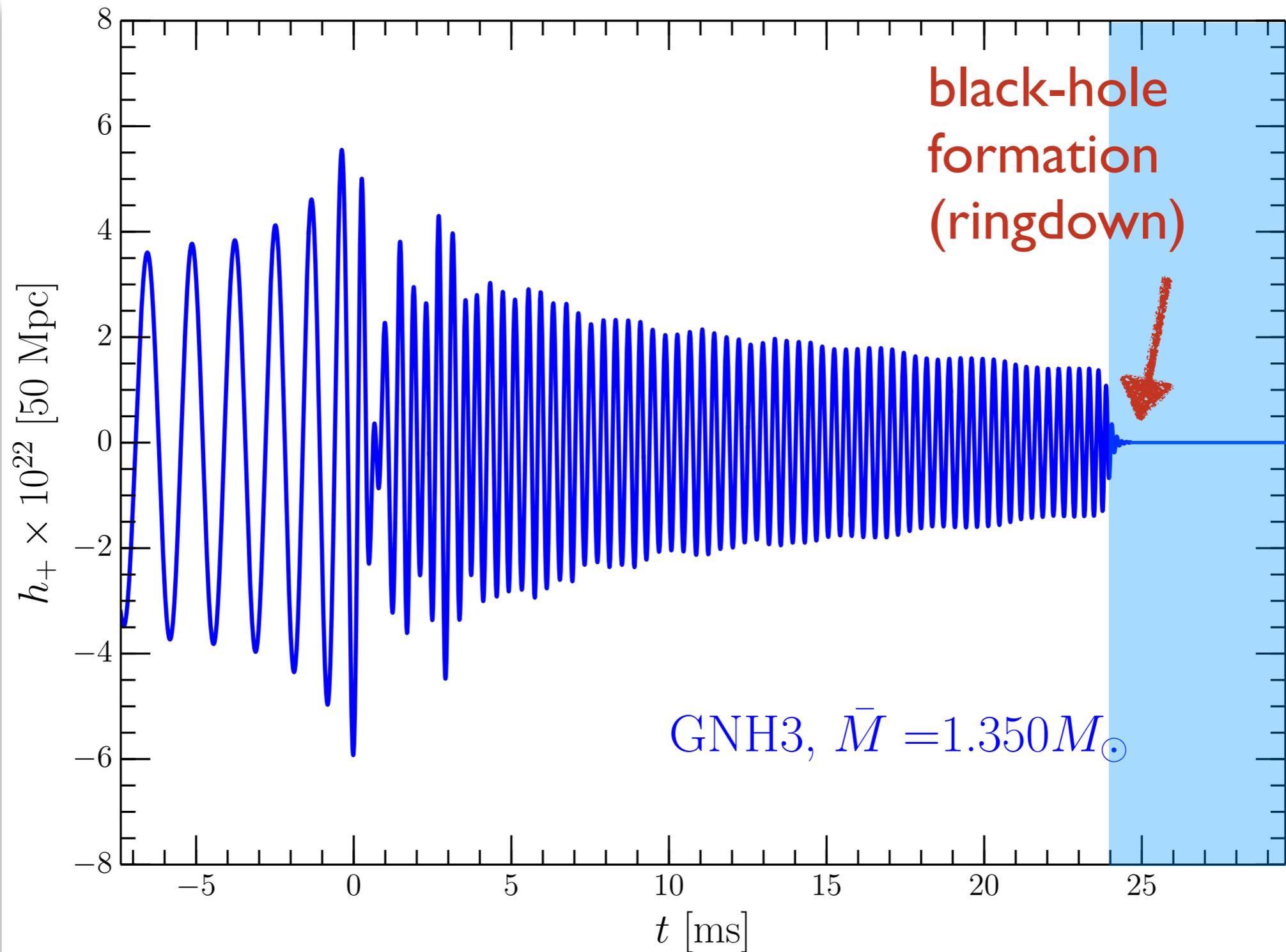
Merger: highly nonlinear but analytic description possible

Anatomy of the GW signal



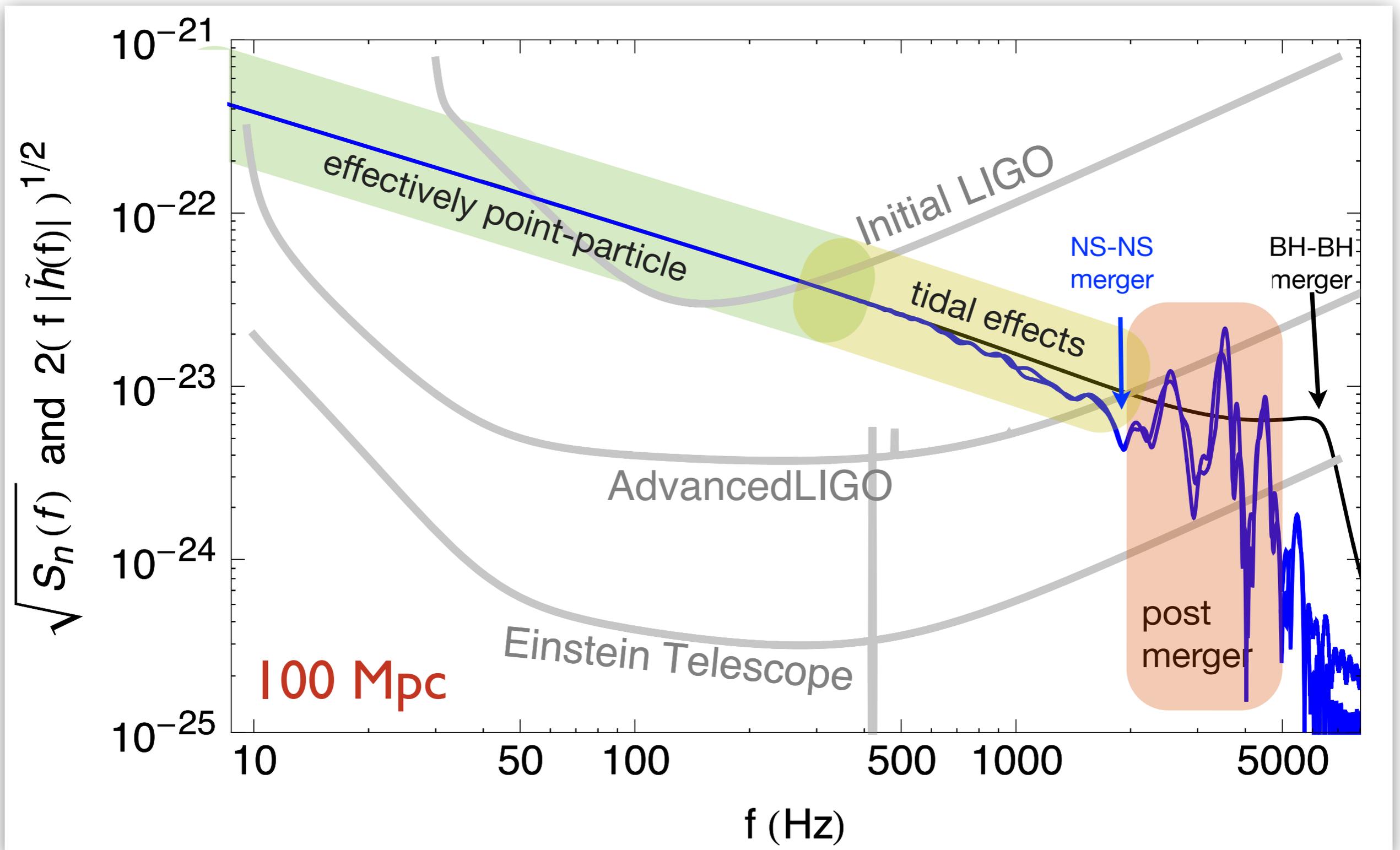
post-merger: quasi-periodic emission of bar-deformed HMNS

Anatomy of the GW signal



Collapse-ringdown: signal essentially shuts off.

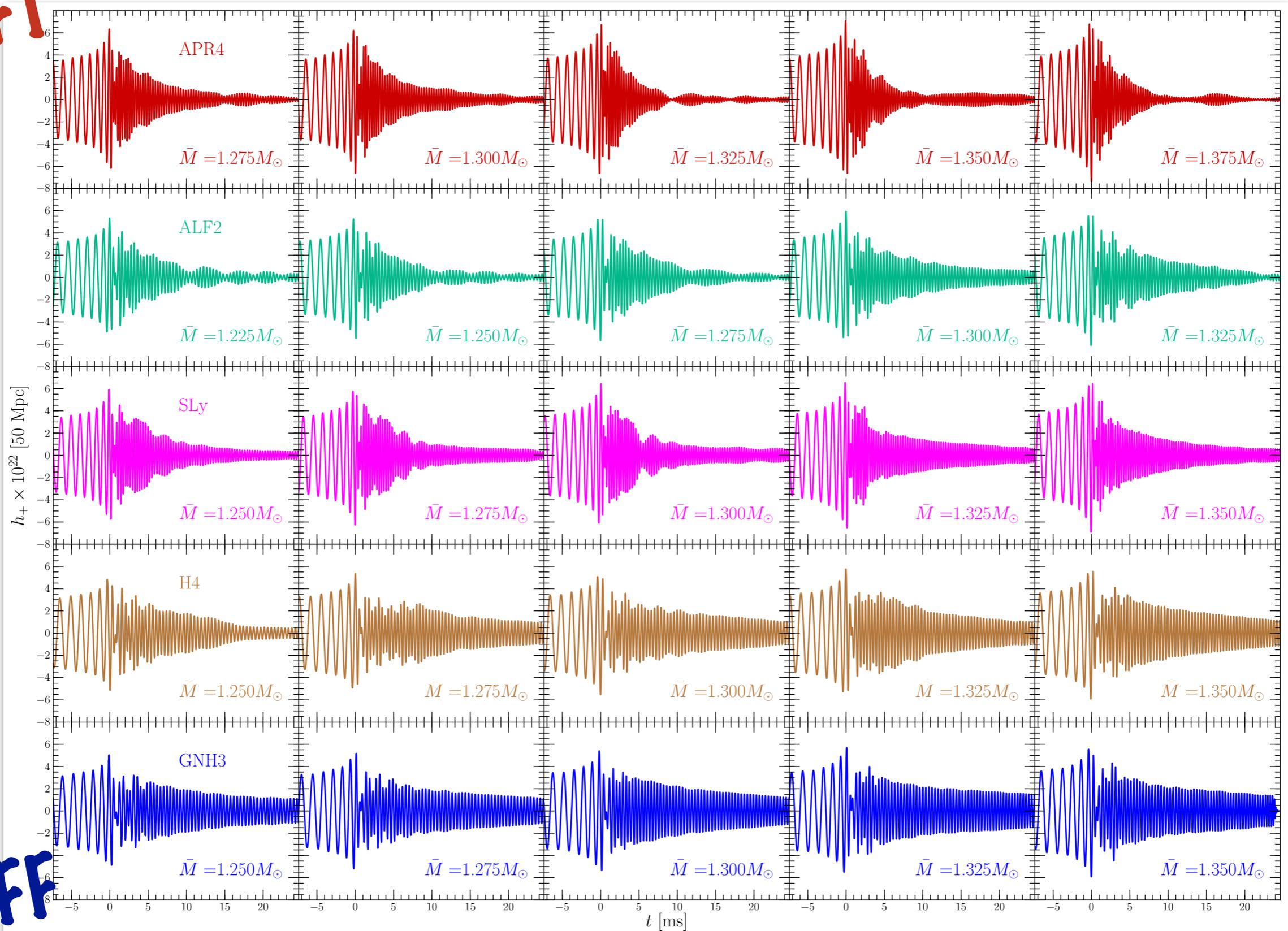
In frequency space



What we can do nowadays

Takami, LR, Baiotti (2014, 2015), LR+ (2016)

SOFT

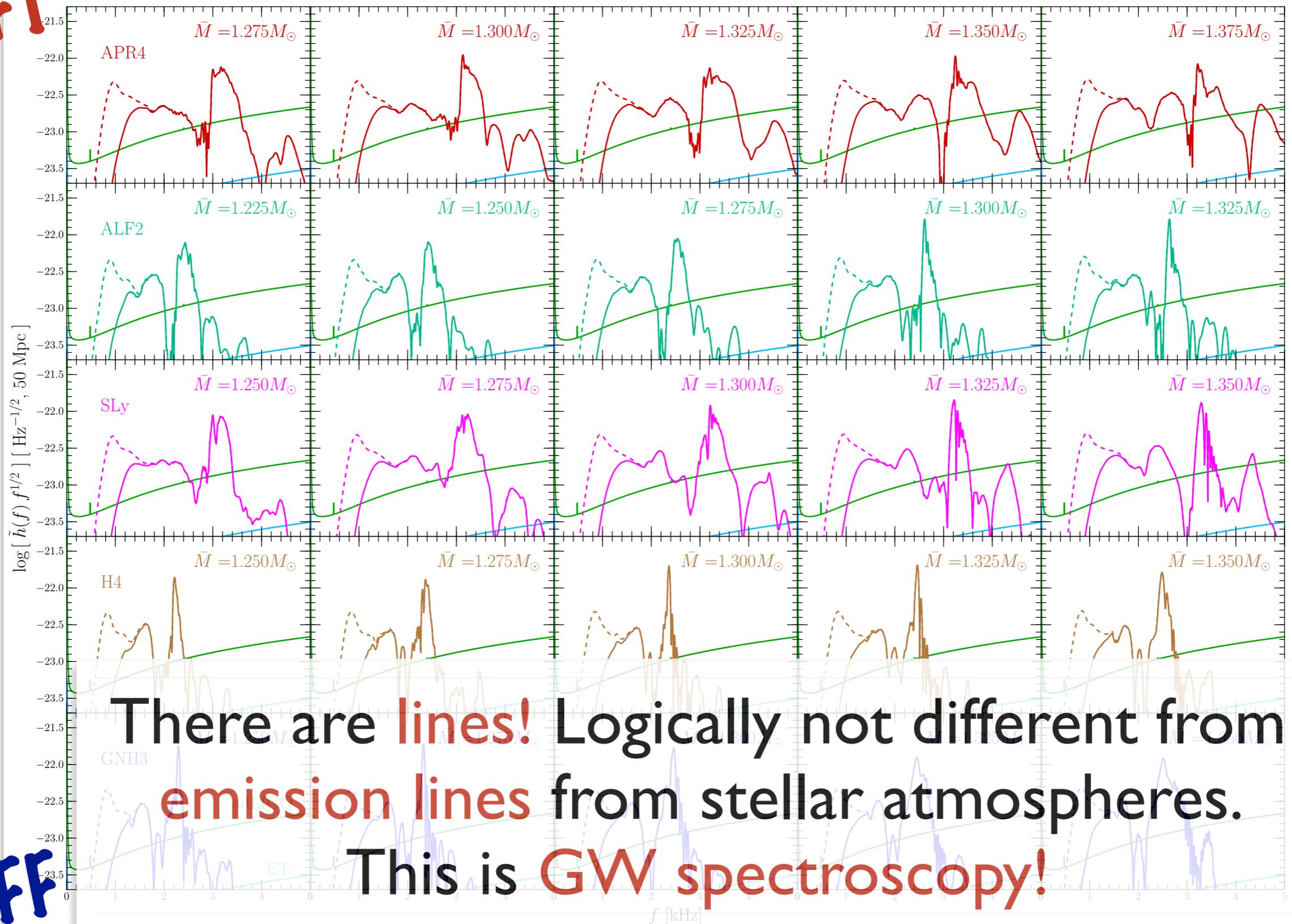


STIFF

Extracting information from the EOS

Takami, LR, Baiotti (2014, 2015), LR+ (2016)

SOFT



There are **lines!** Logically not different from **emission lines** from stellar atmospheres.

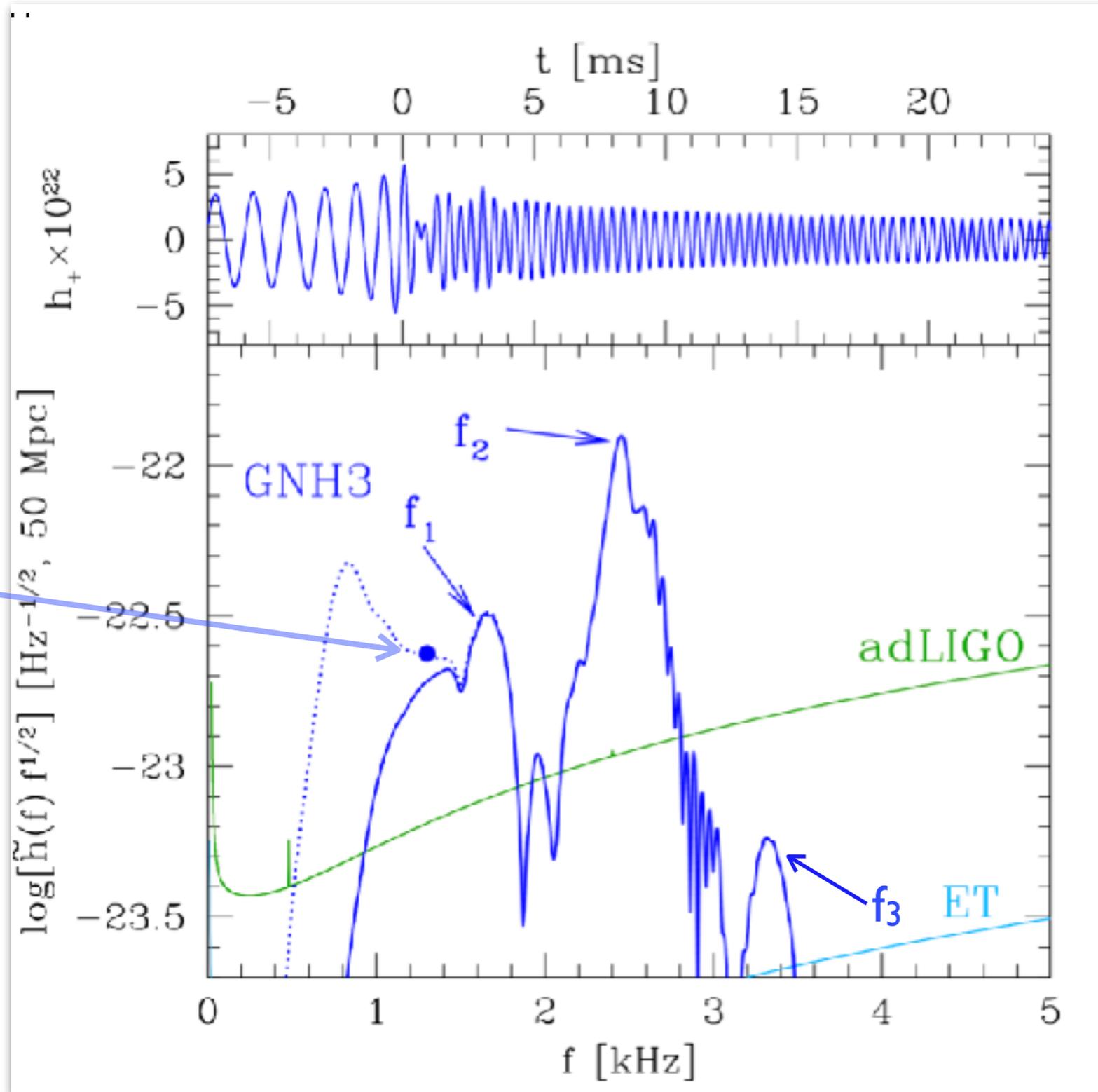
This is **GW spectroscopy!**

STIFF

A new approach to constrain the EOS

Oechslin+2007, Baiotti+2008, Bauswein+ 2011, 2012, Stergioulas+ 2011, Hotokezaka+ 2013, Takami 2014, 2015, Bernuzzi 2014, 2015, Bauswein+ 2015, Clark+ 2016, LR+2016, de Pietri+ 2016, Feo+ 2017, Bose+ 2017 ...

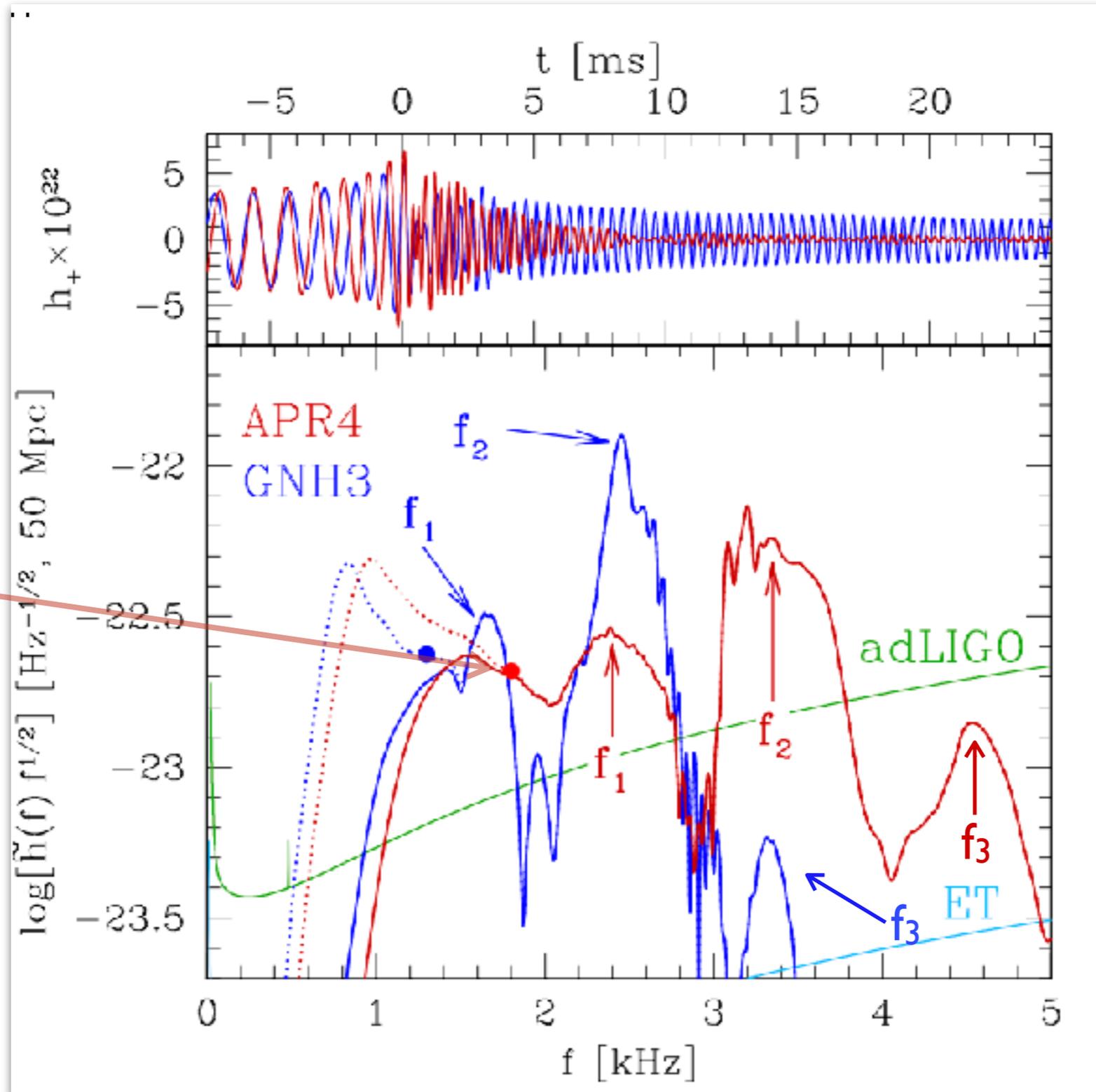
merger
frequency



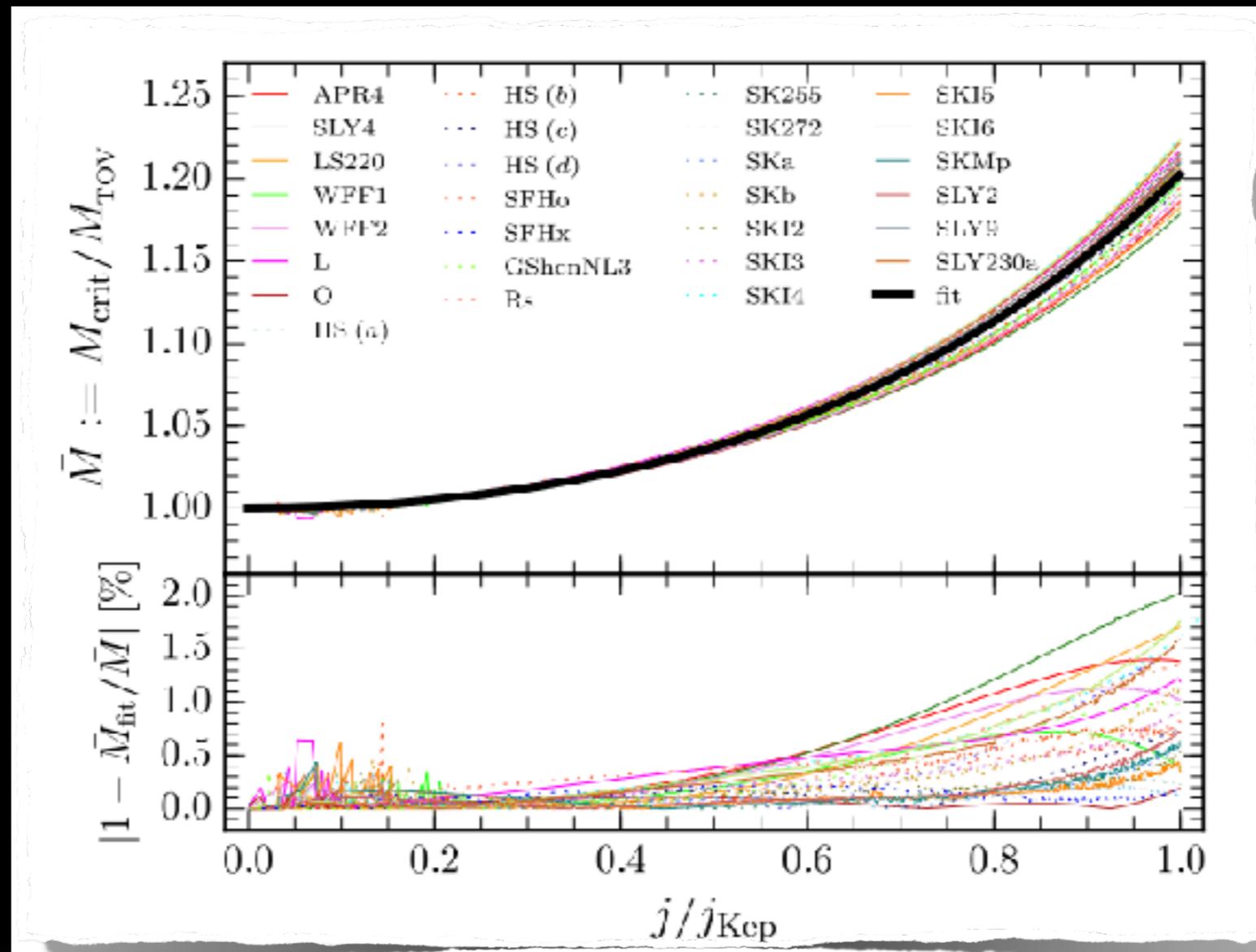
A spectroscopic approach to the EOS

Oechslin+2007, Baiotti+2008, Bauswein+ 2011, 2012, Stergioulas+ 2011, Hotokezaka+ 2013, Takami 2014, 2015, Bernuzzi 2014, 2015, Bauswein+ 2015, Clark+ 2016, LR+2016, de Pietri+ 2016, Feo+ 2017, Bose+ 2017 ...

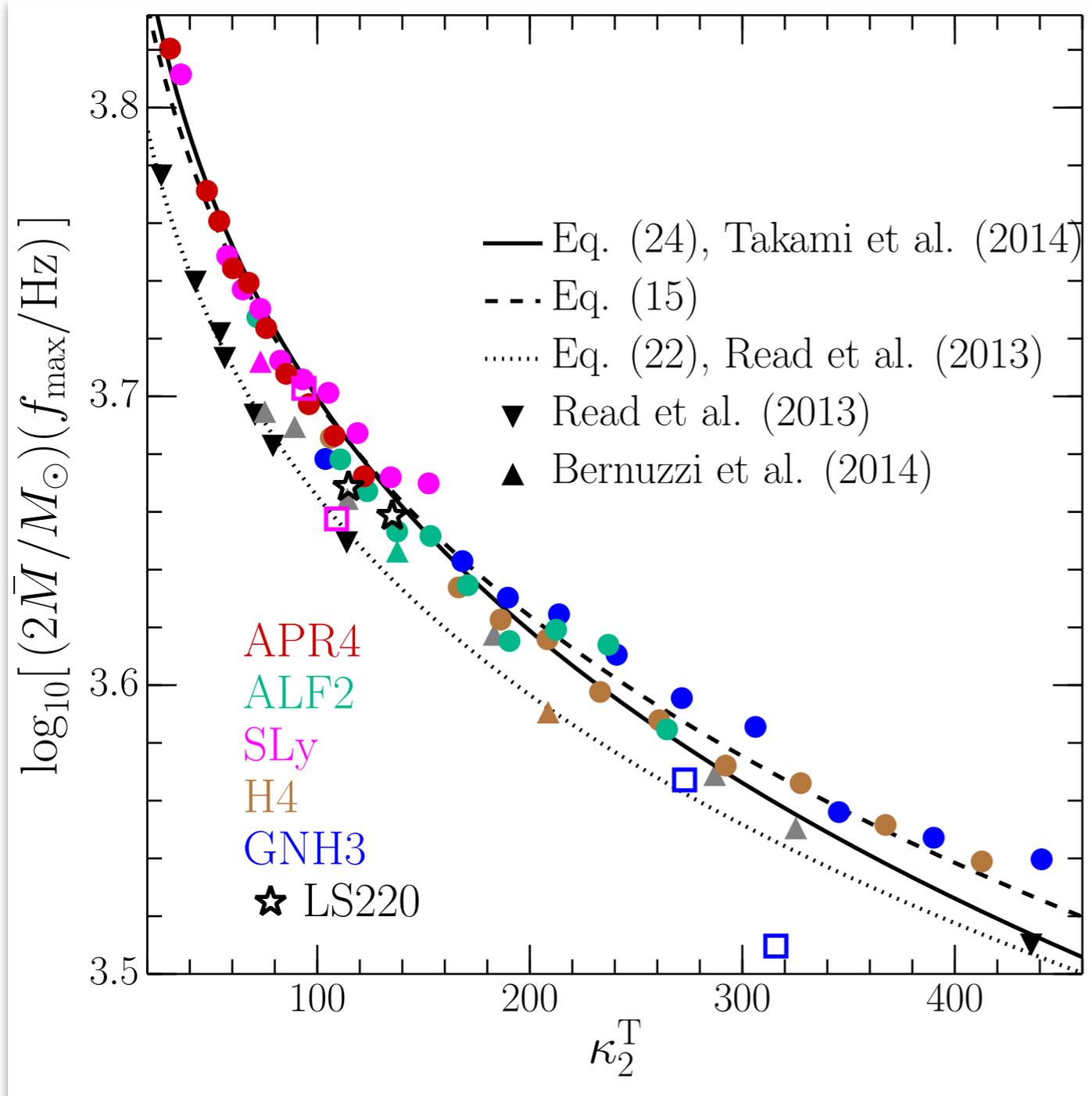
merger
frequency



Quasi-universal behaviour



Quasi-universal behaviour: **inspiral**



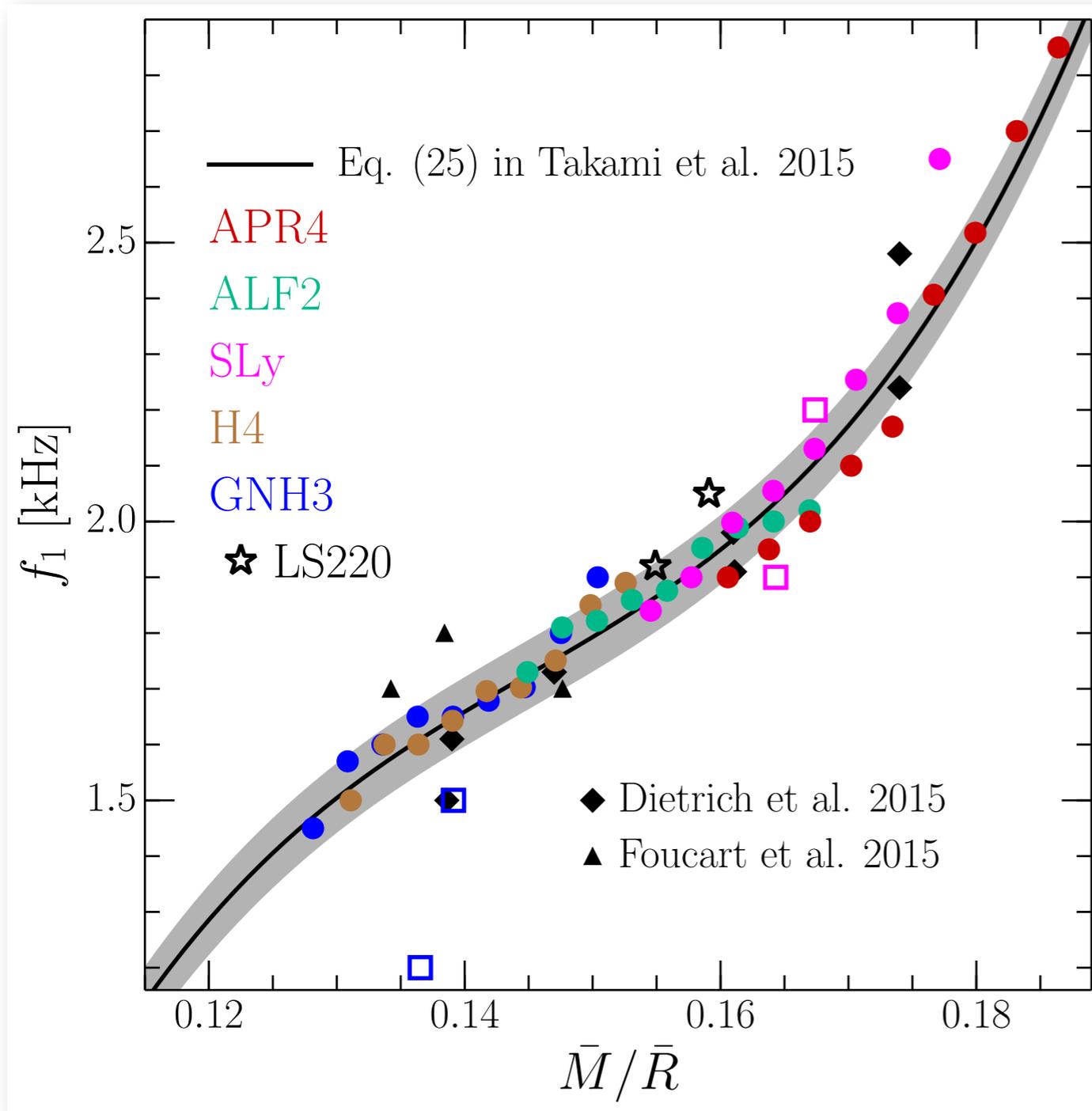
“surprising” result: **quasi-universal** behaviour of GW frequency at amplitude peak (Read+2013)

Many other simulations have confirmed this (Bernuzzi+ 2014, Takami+ 2015, LR+2016).

Quasi-universal behaviour in the **inspiral** implies that once **f_{\max}** is measured, so is tidal deformability, hence $I, Q, M/R$

$$\Lambda = \frac{\lambda}{\bar{M}^5} = \frac{16}{3} \kappa_2^T \quad \text{tidal deformability or Love number}$$

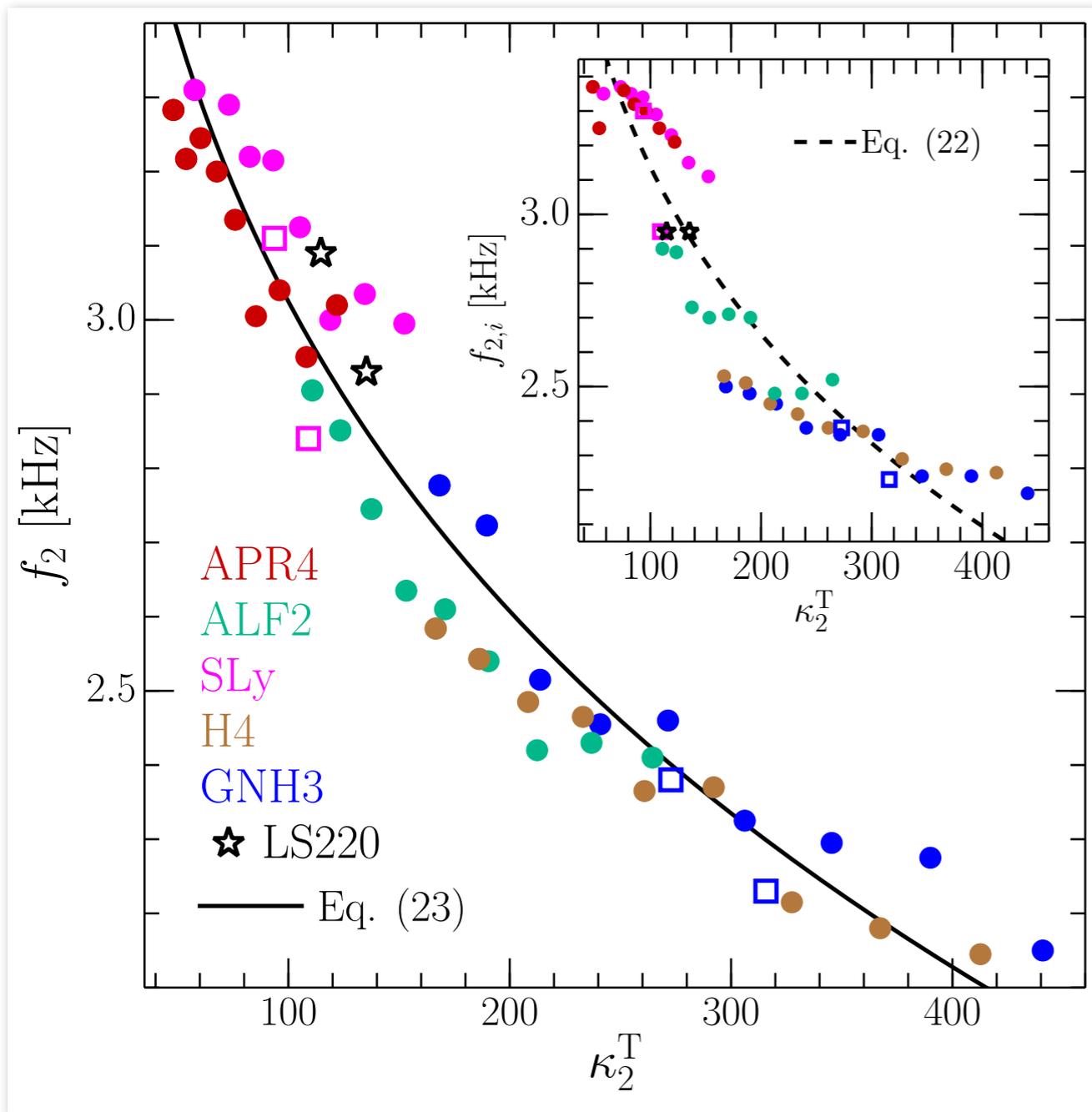
Quasi-universal behaviour: post-merger



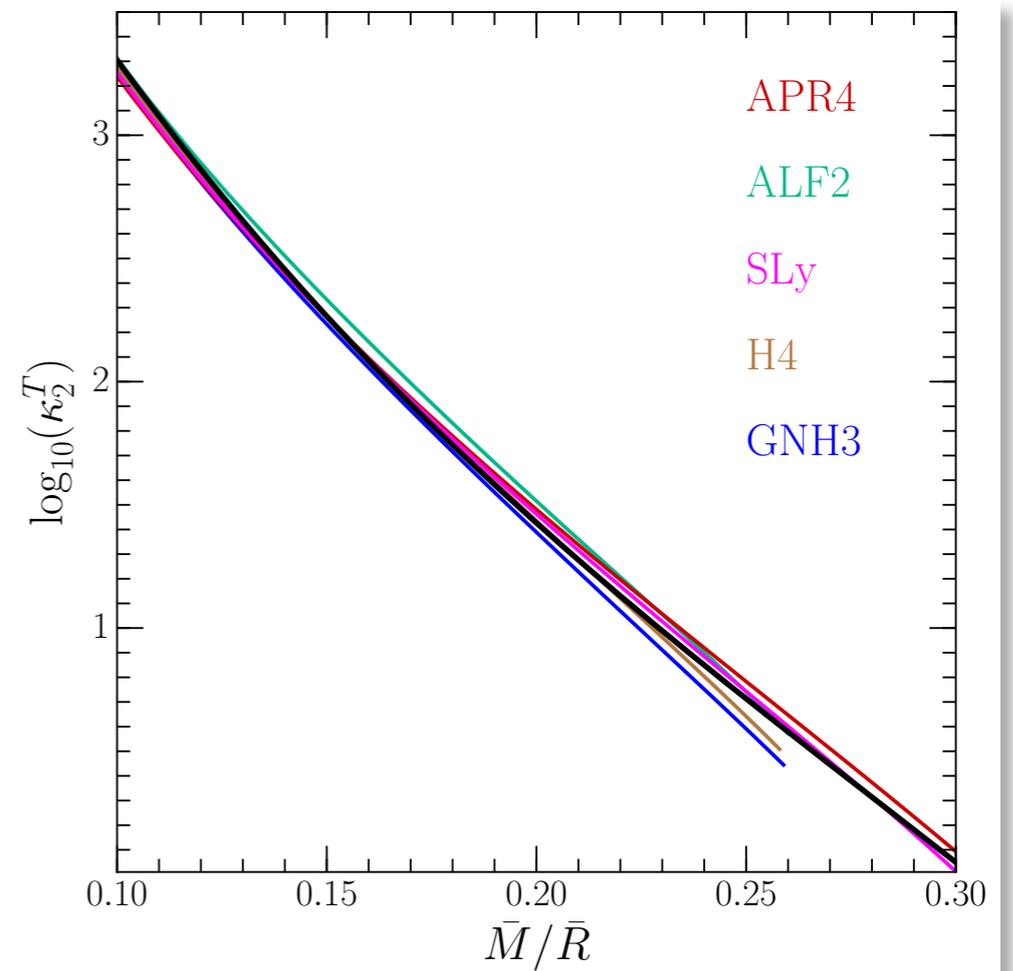
We have found **quasi-universal behaviour**: i.e., the properties of the spectra are only weakly dependent on the EOS.

This has profound implications for the analytical modelling of the GW emission: “what we do for one EOS can be extended to all EOSs.”

Quasi-universal behaviour: post-merger



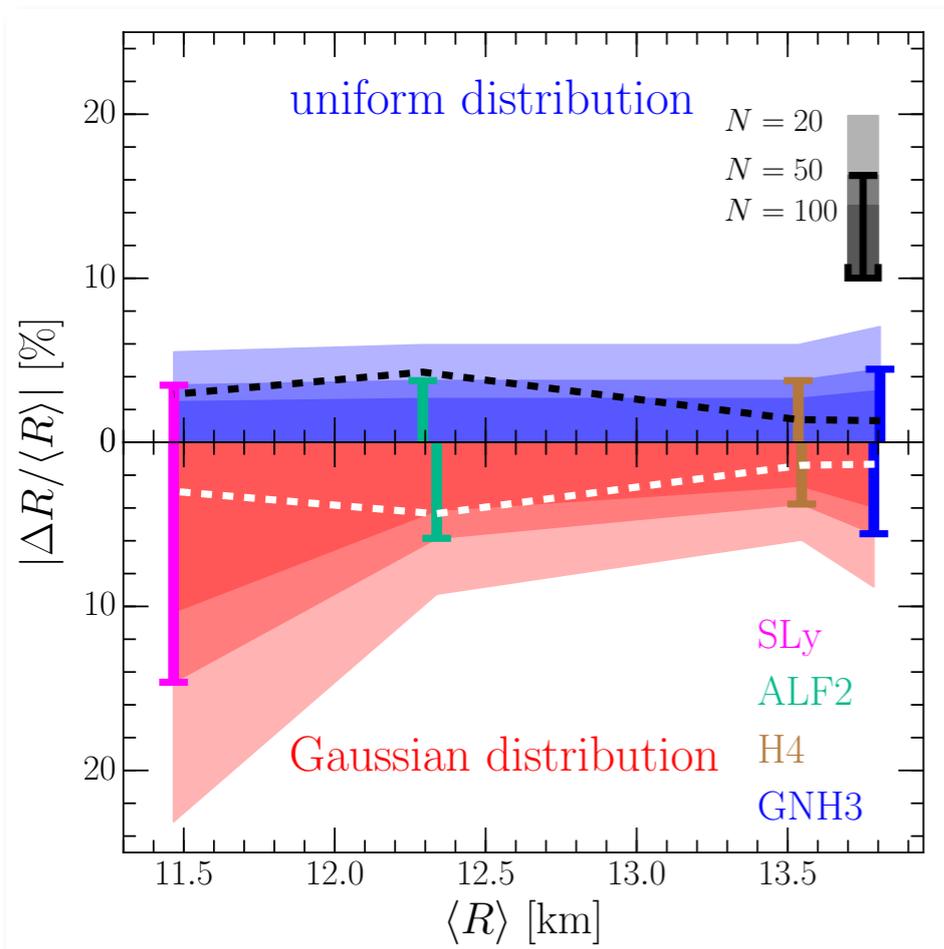
- Correlations with Love number found also for high frequency peak f_2 .
- This and other correlations are **weaker** but equally useful.



- Important correlation also between **compactness** and **deformability**

A spectroscopic approach to the EOS

- **Universal behaviour** and analytic modelling of post-merger relates position of these peaks with the EOS.
- Observation of the post-merger signal would constrain significantly the stellar radius; given **N detections**.



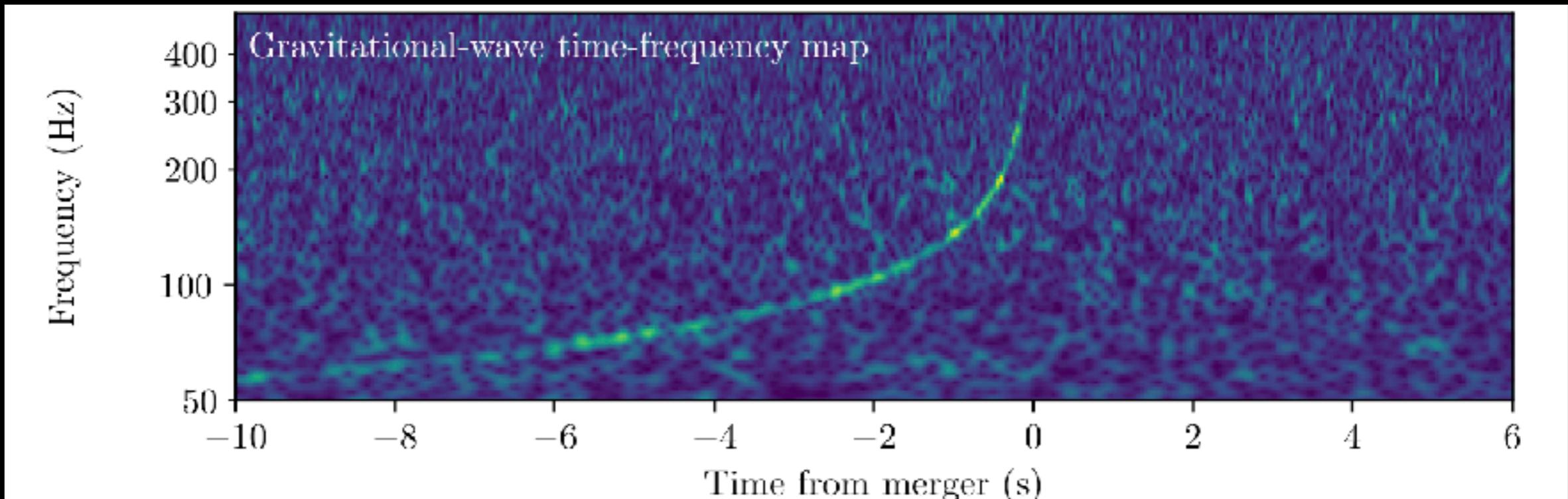
- **stiff EOSs:** $|\Delta R / \langle R \rangle| < 10\%$ **$N \sim 20$** detect.
- **soft EOSs:** $|\Delta R / \langle R \rangle| \sim 10\%$ **$N \sim 50$** detect.
- discriminating stiff/soft EOSs possible even with moderate **N**
- golden binary: **SNR ~ 6 at 30 Mpc**
 $|\Delta R / \langle R \rangle| \lesssim 2\%$ at 90% confidence

GW170817, maximum mass, radii and tidal deformabilities

LR, Most, Weih, ApJL (2018)

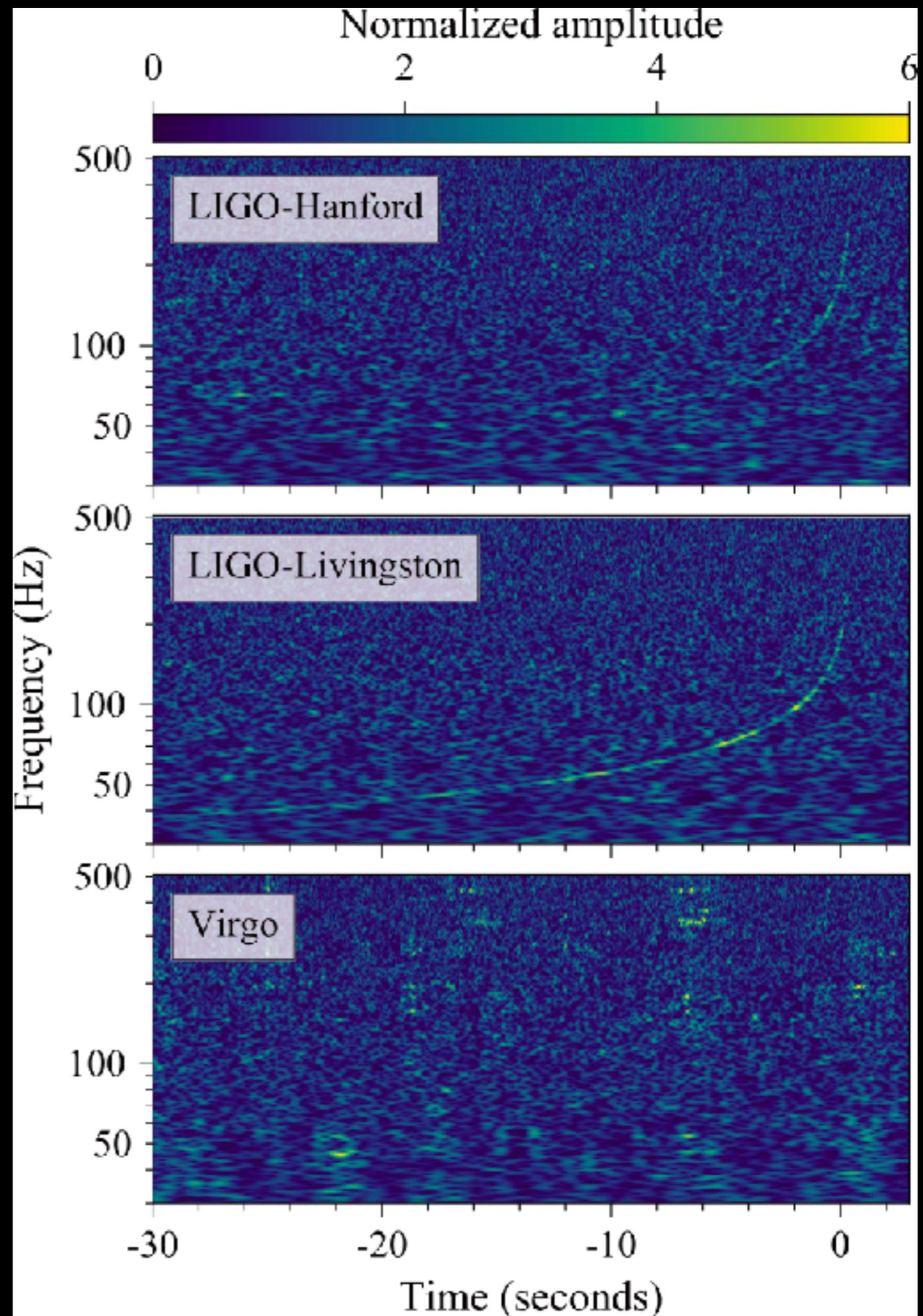
Most, Weih, LR, Schaffner-Bielich, PRL (2018)

Köppel, Bovard, LR, ApJL (2018)



GW170817: the first binary neutron-star system

- * Unfortunately only the **inspiral** signal was detected.
- * Fortunately this was **sufficient** to set a number of constraints on max. mass, tidal deformability, radii, etc.

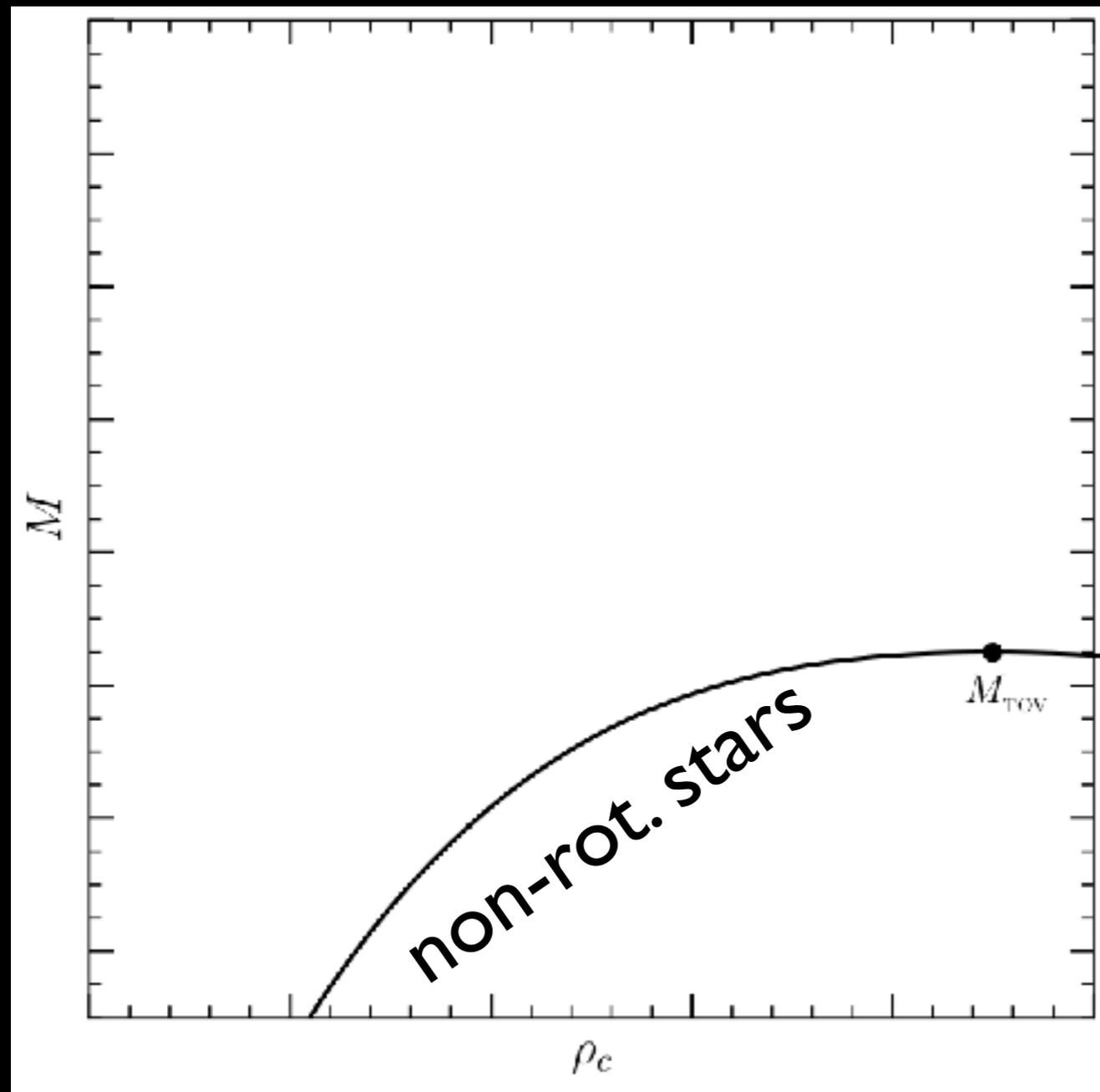


Limits on the maximum mass

- The remnant of GW170817 was a hypermassive star, i.e. a differentially rotating object with initial **gravitational** mass:

$$M_1 + M_2 = 2.74^{+0.04}_{-0.01} M_{\odot}$$

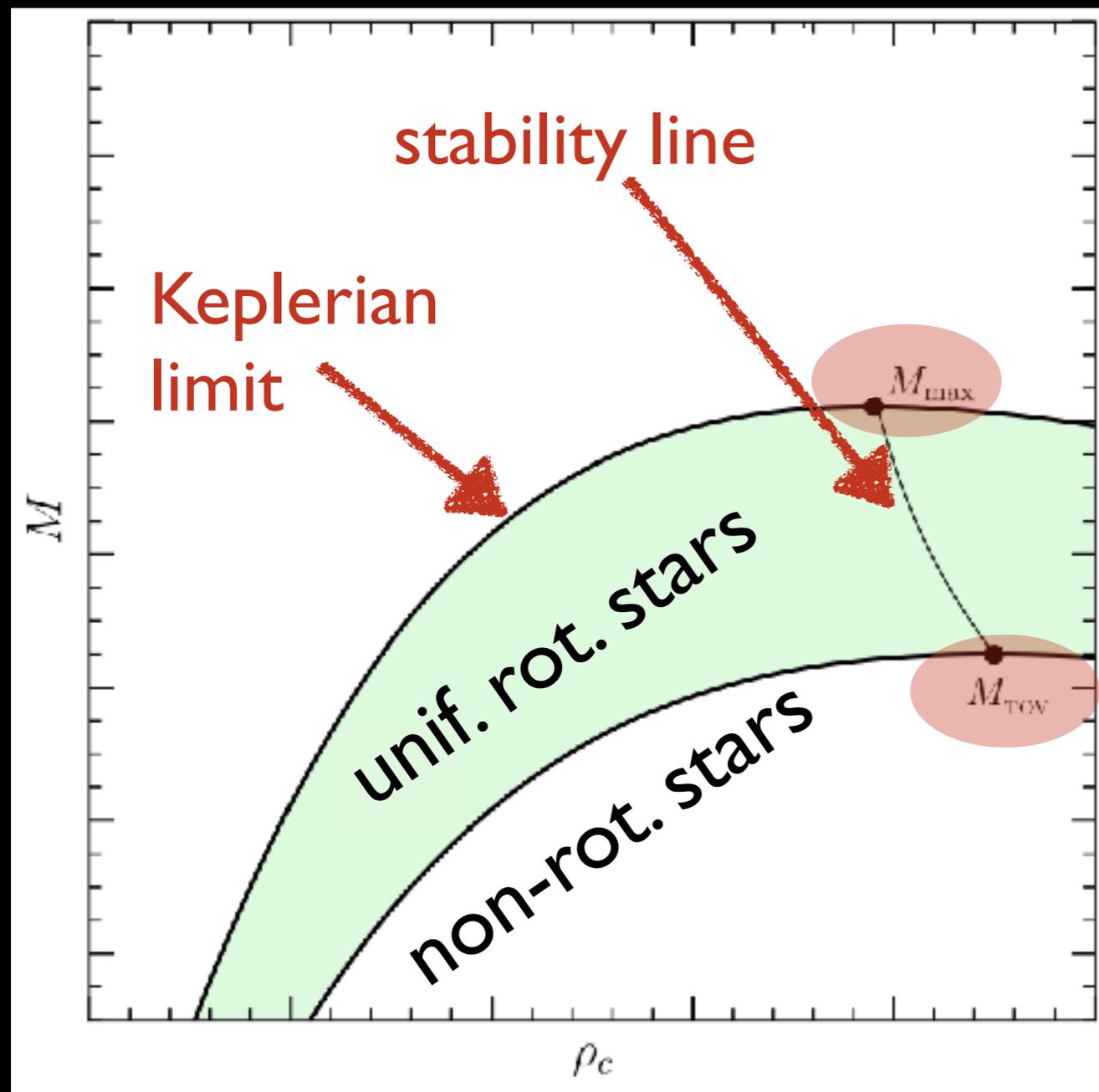
- Sequences of equilibrium models of **nonrotating** stars will have a maximum mass: M_{TOV}



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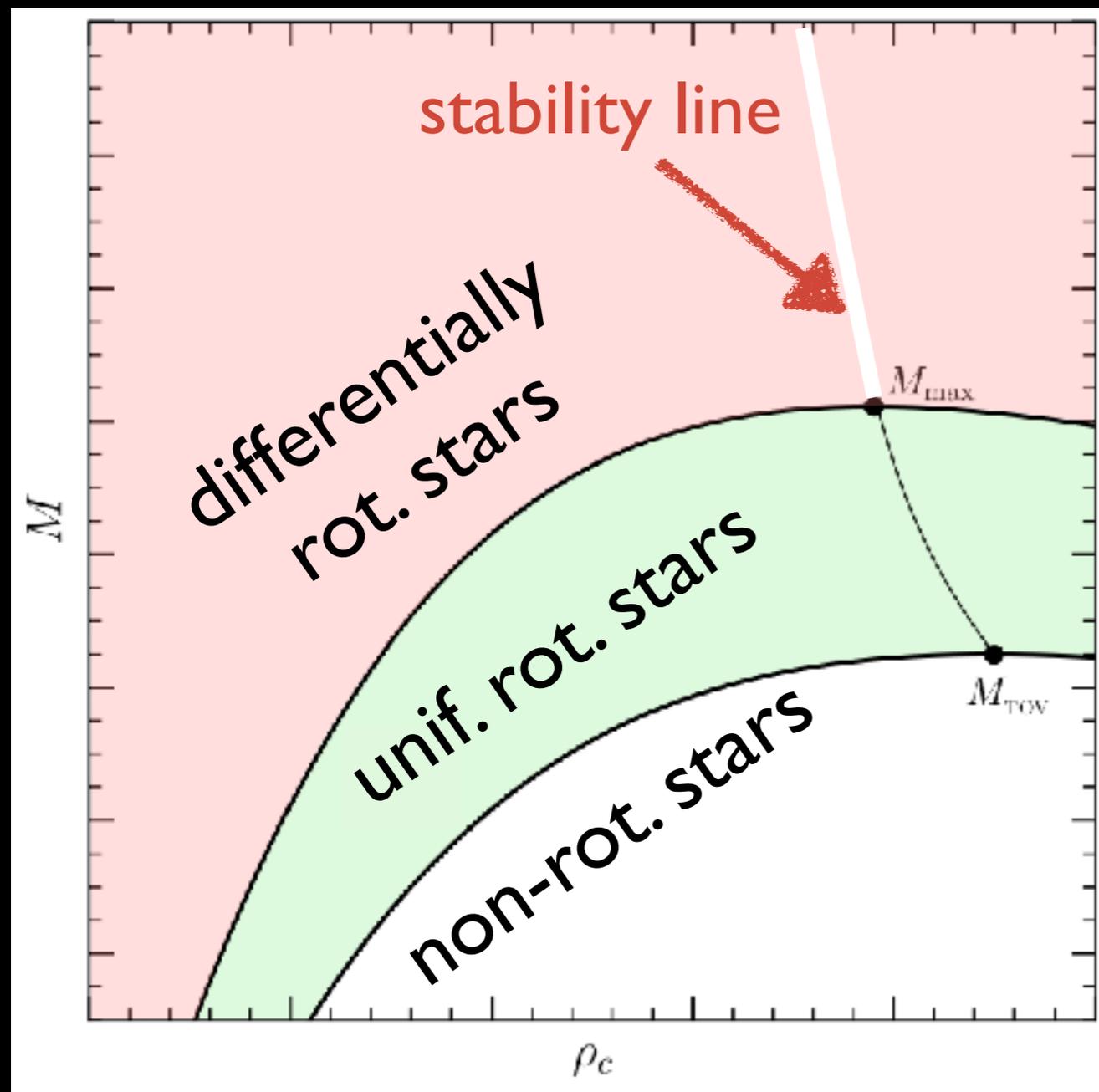
- Sequences of equilibrium models of **nonrotating** stars will have a maximum mass: M_{TOV}
- This is true also for **uniformly** rotating stars at mass shedding limit: M_{max}
- M_{max} simple and **quasi-universal** function of M_{TOV} (Breu & LR 2016)

$$M_{\text{max}} = 1.20_{-0.05}^{+0.02} M_{\odot}$$

Limits on the maximum mass

- The remnant of GW170817 was a hypermassive star, i.e. a differentially rotating object with initial **gravitational** mass:

$$M_1 + M_2 = 2.74_{-0.01}^{+0.04} M_{\odot}$$

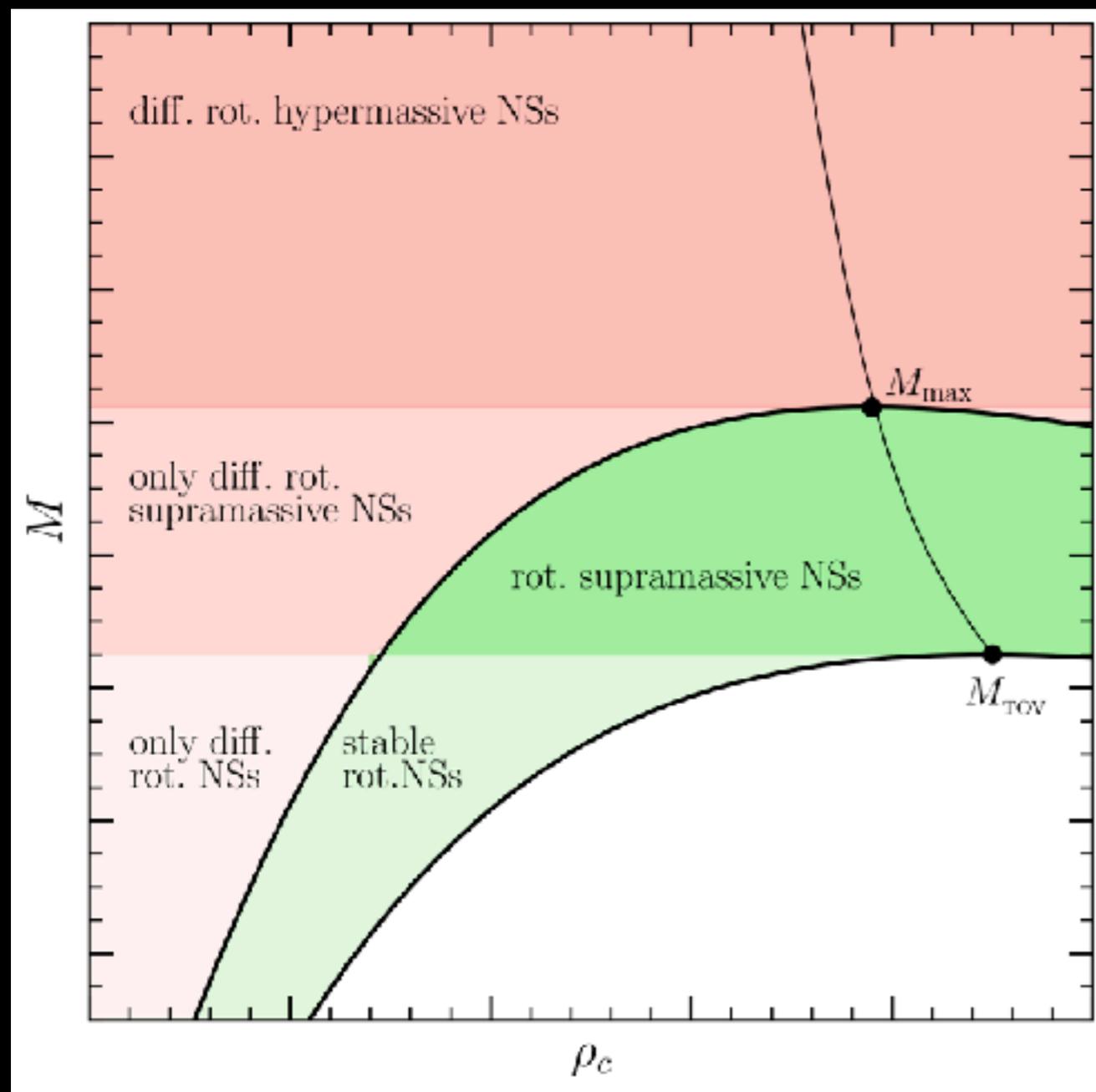


- Green** region is for **uniformly** rotating equilibrium models.
- Salmon** region is for **differentially** rotating equilibrium models.
- Stability line** is simply extended in larger space (Weih+18)

Limits on the maximum mass

- The remnant of GW170817 was a hypermassive star, i.e. a differentially rotating object with initial **gravitational** mass:

$$M_1 + M_2 = 2.74^{+0.04}_{-0.01} M_{\odot}$$



- Green** region is for **uniformly** rotating equilibrium models.
- Salmon** region is for **differentially** rotating equilibrium models.

- Supramassive** stars have:

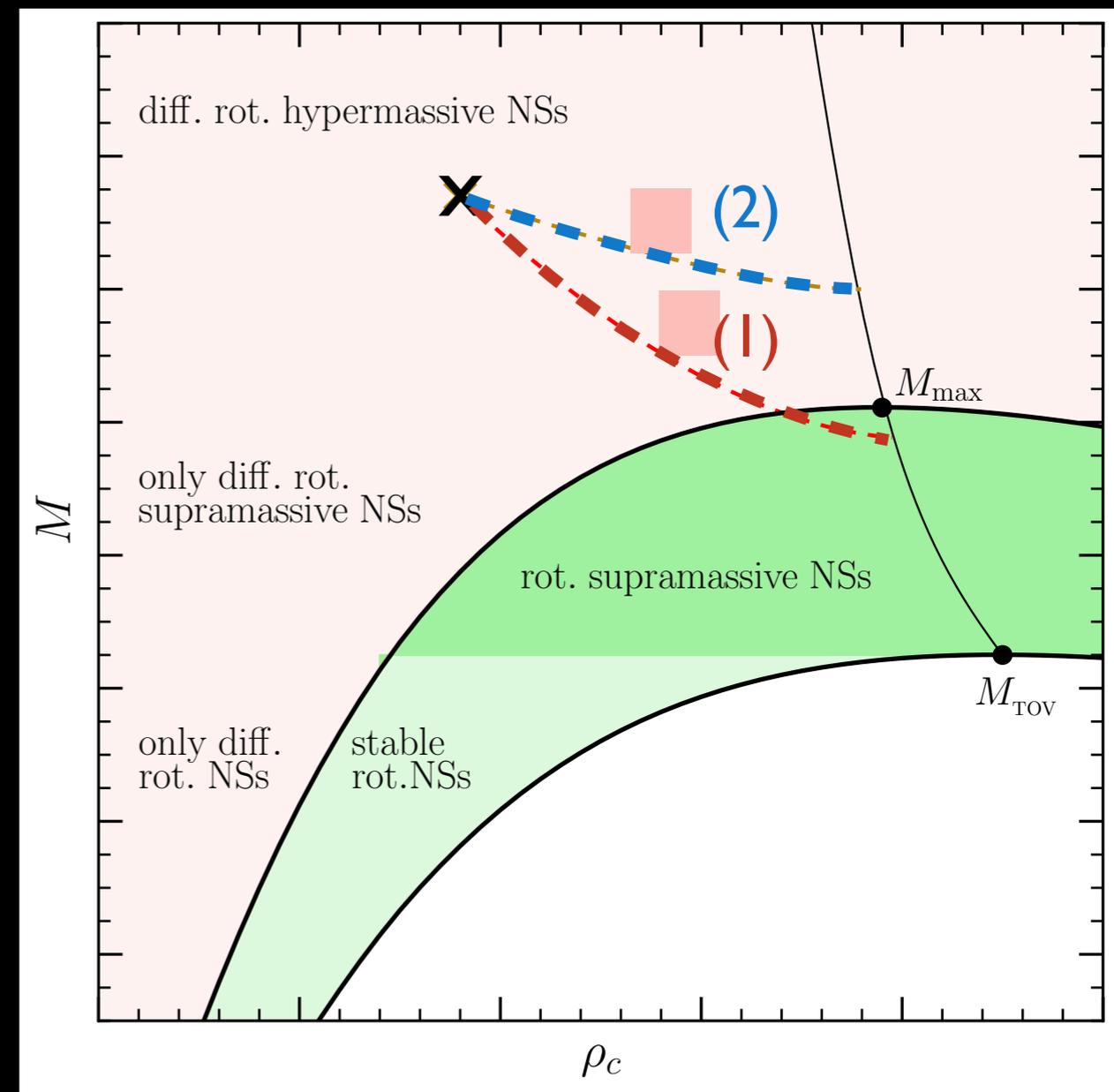
$$M > M_{\text{TOV}}$$

- Hypermassive** stars have:

$$M > M_{\text{max}}$$

Limits on the maximum mass

- GW170817 produced object "X"; GRB implies a BH has been formed: "X" followed two possible tracks: **fast (2)** and **slow (1)**
- It rapidly produced a BH when still **differentially** rotating **(2)**
- It lost differential rotation leading to a **uniformly** rotating core **(1)**.
- **(1)** is much more likely because of large ejected mass (long lived).
- Final mass is near M_{\max} and we know this is universal!



let's recap...

- Consider **evolution track (2)**
- Use measured **gravitational mass** of GW170817
- Remove **rest-mass** deduced from kilonova emission (need conversion)
- Use **universal relations** and account for errors to obtain

pulsar
timing

$$2.01^{+0.04}_{-0.04} \leq M_{\text{TOV}} / M_{\odot} \leq 2.16^{+0.17}_{-0.15}$$

universal relations
and GW170817;
similar estimates
by other groups

Tension on the maximum mass

Nathanail, Most, LR (2021)

- The recent detection of GW190814 has created a significant tension on the maximum mass

$$M_1 = 22.2 - 24.3 M_{\odot}$$

$$M_2 = 2.50 - 2.67 M_{\odot} \quad \text{smallest BH or heaviest NS!}$$

- If secondary in GW190814 was a NS, all previous results on the maximum mass are incorrect.
- No EM counterpart was observed with GW190814 and no estimates possible for ejected matter or timescale for survival.
- **How do we solve this tension?**

Tension on the maximum mass

- We can nevertheless explore impact of larger maximum mass, ie what changes in the previous picture if

$$M_{\text{TOV}}/M_{\odot} \gtrsim 2.5 ?$$

- In essence, this is a multi-dimensional parametric problem satisfying **conservation** of **rest-mass** and **gravitational mass**.
- Observations provide limits on **gravitational** and **ejected mass**.
- Numerical relativity simulations provide limits on **emitted GWs**
- All the rest is contained in **10 parameters** that need to be varied within suitable ranges.

Genetic algorithm

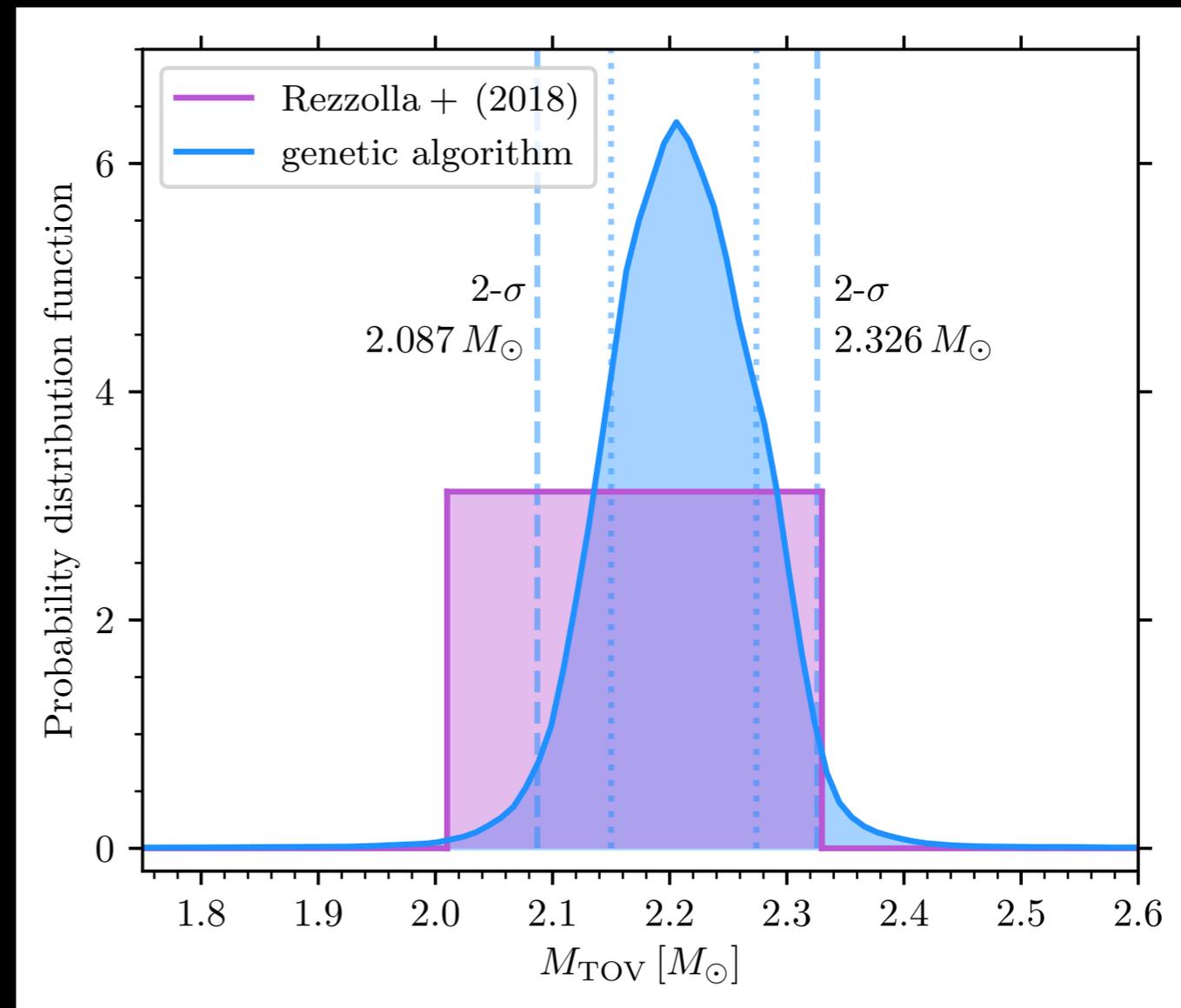
- A **genetic algorithm** is used to sample through the parameter space of the 10 free parameters.

- The algorithm reflects genetic adaptation: given a mutation (i.e. change of parameters) it will be adopted if it provides a better fit to data.

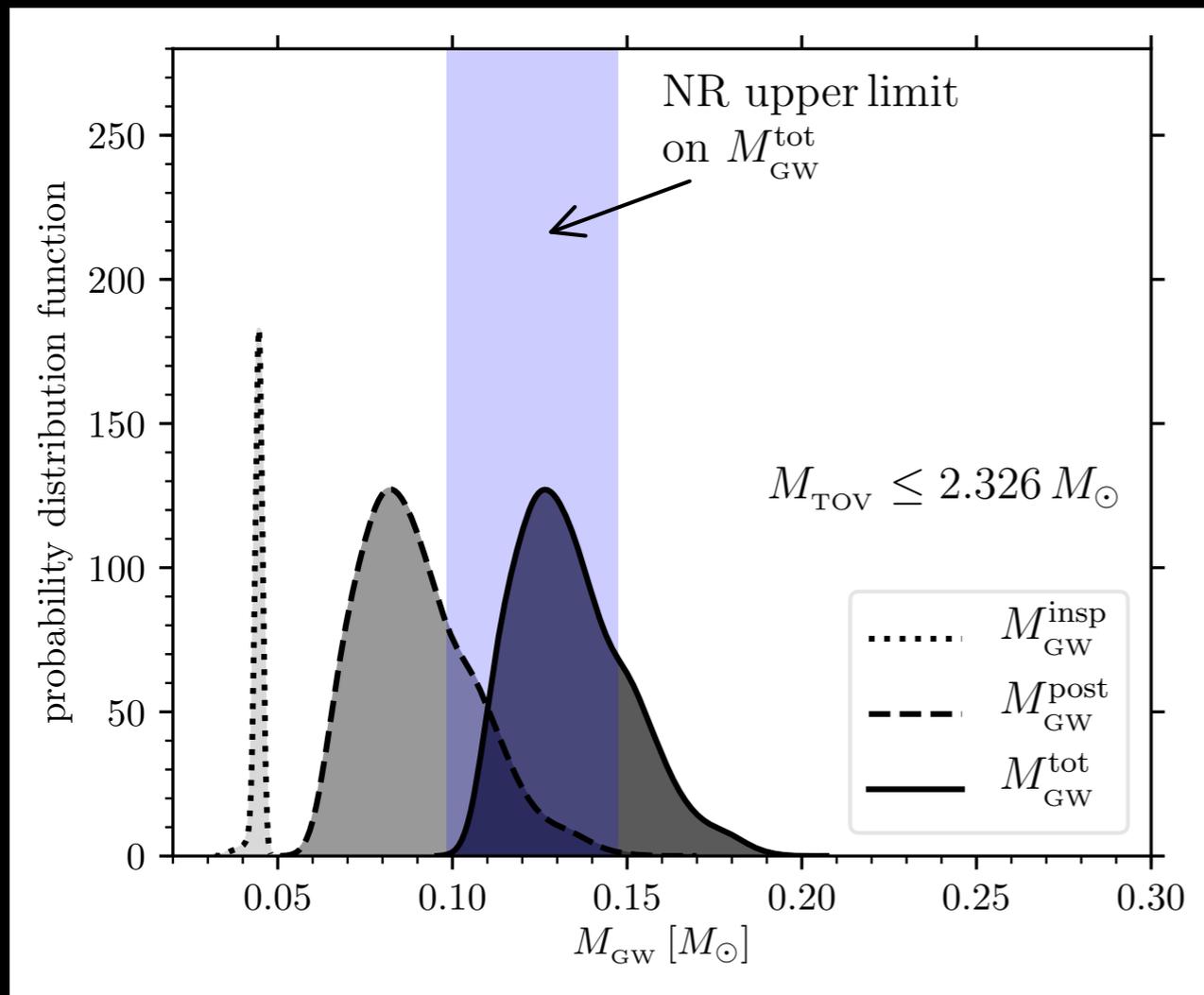
- Consider first previous estimate:

$$M_{\text{TOV}}/M_{\odot} \lesssim 2.3$$

$$M_{\text{TOV}}/M_{\odot} \leq 2.16^{+0.17}_{-0.15}$$

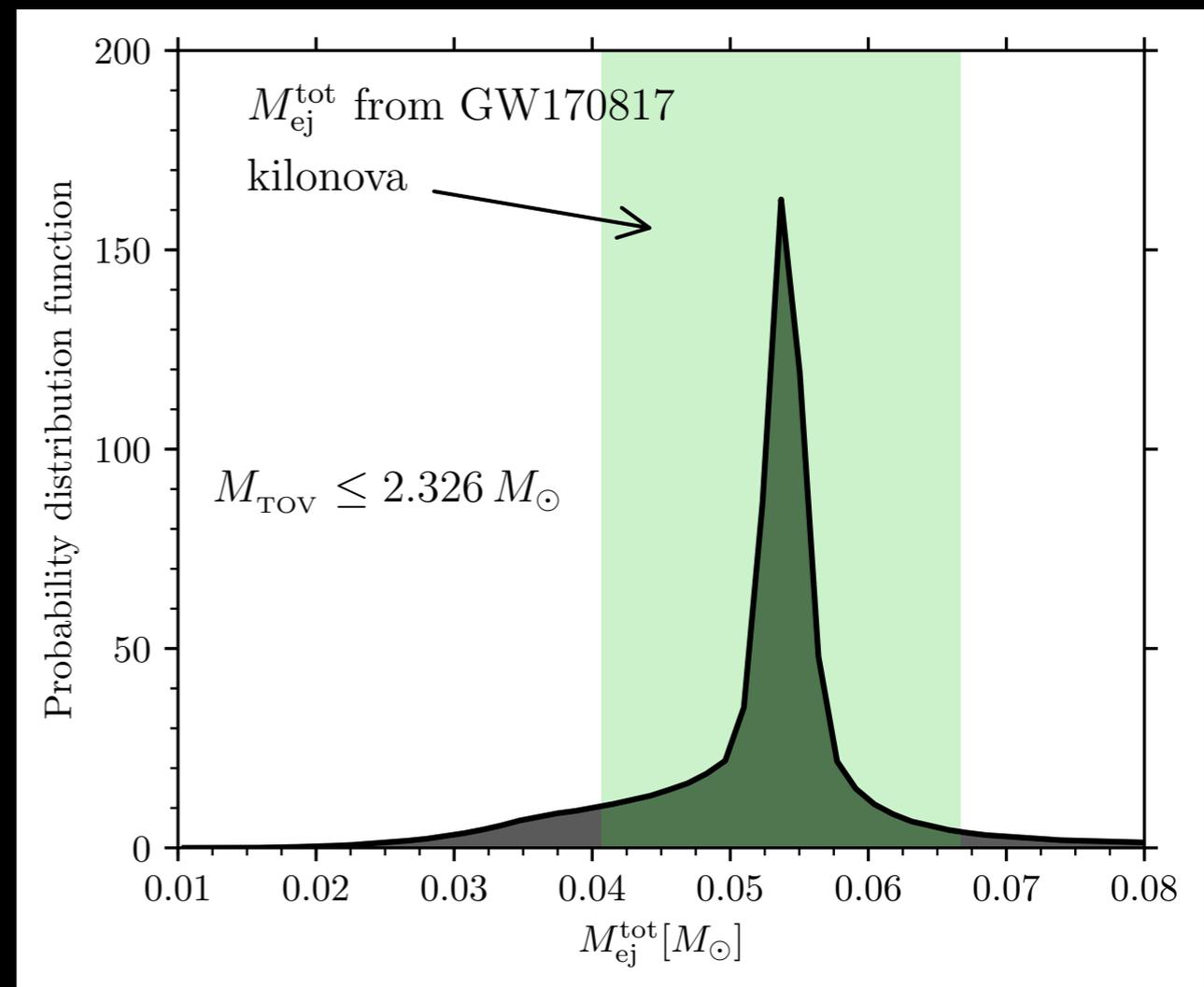


First hypothesis: $M_{\text{TOV}}/M_{\odot} \lesssim 2.3$

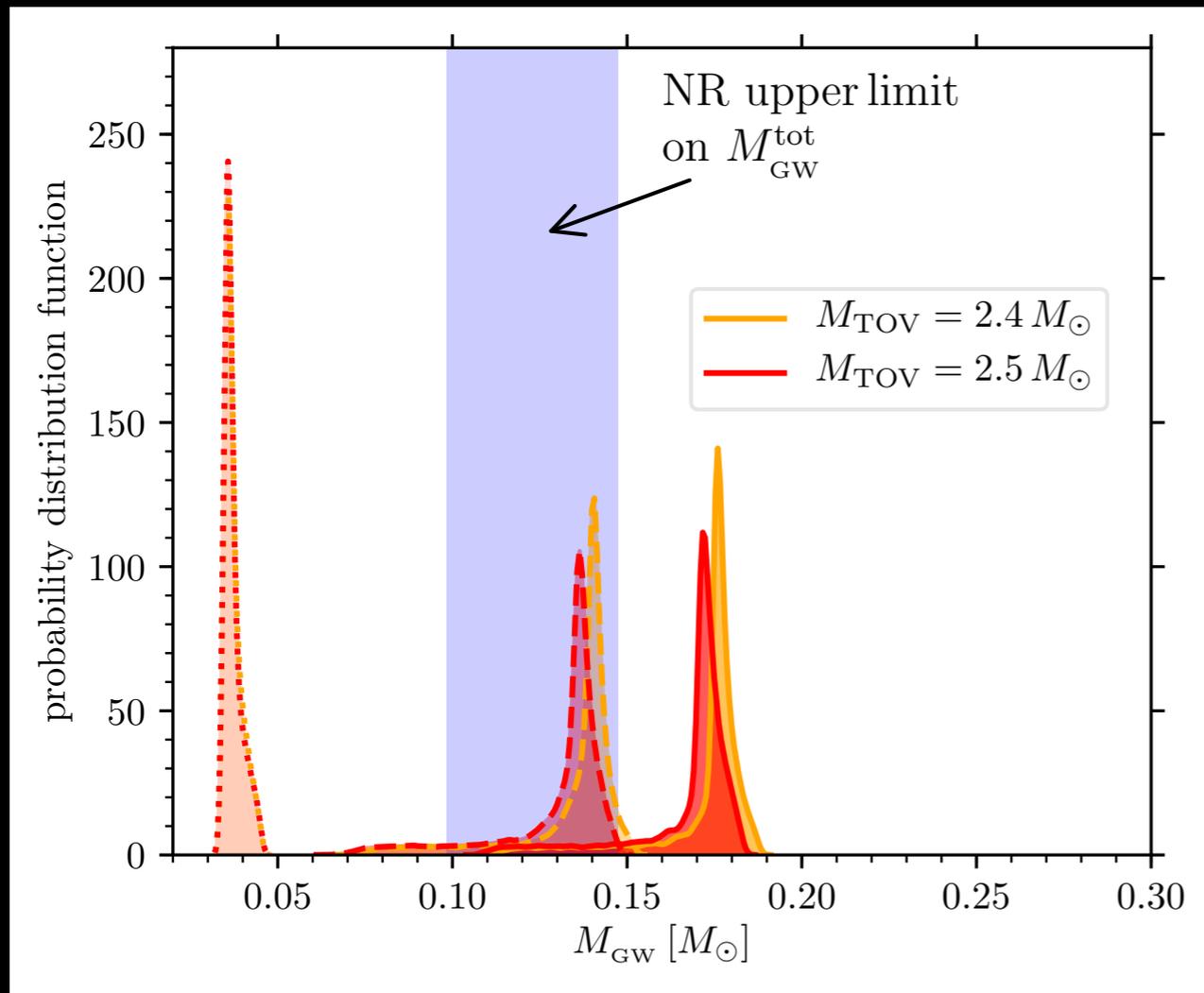


- Total mass ejected is in perfect **agreement** with predictions from kilonova signal

- Total mass emitted in GWs is in perfect **agreement** with predictions from numerical relativity

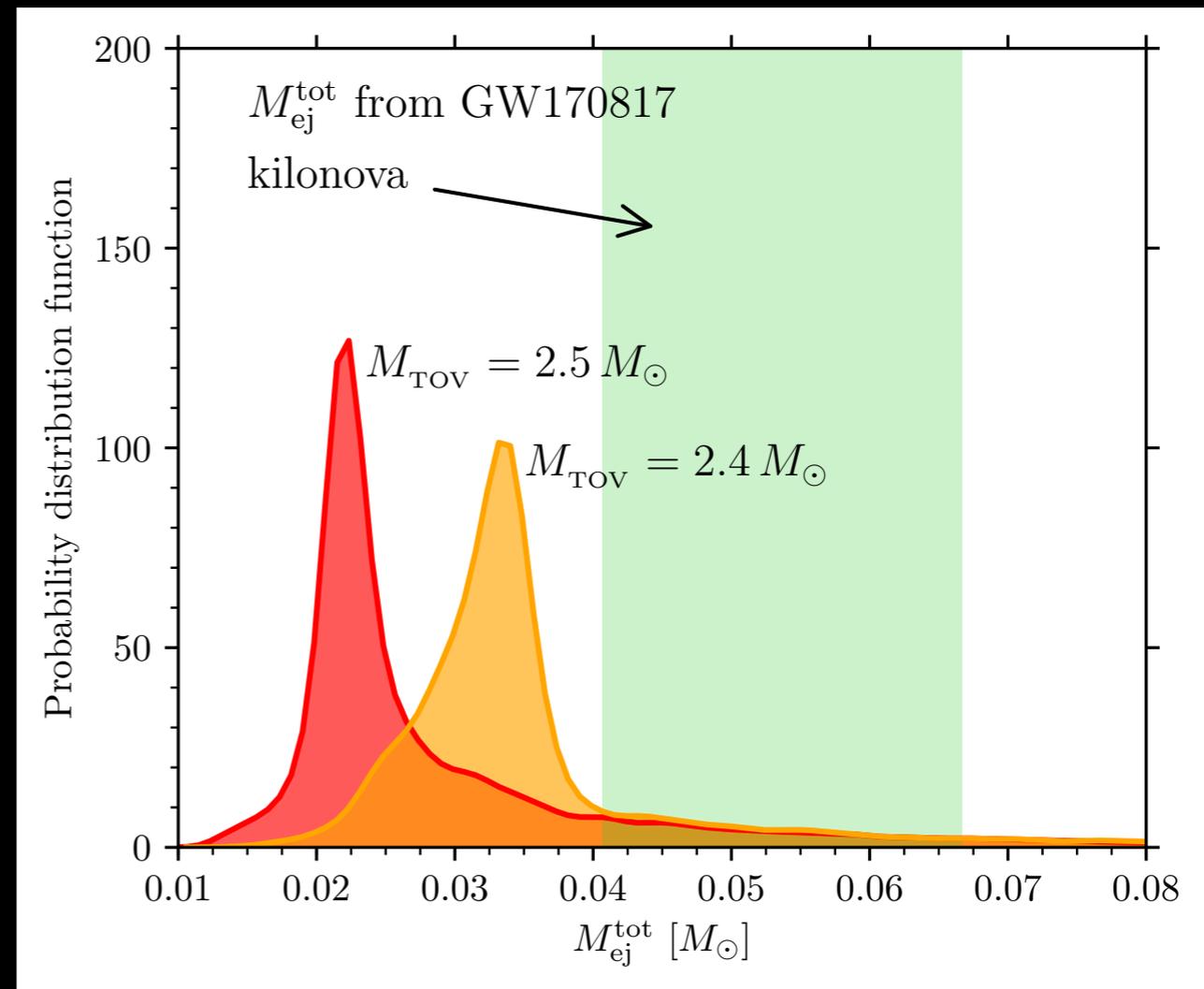


Second hypothesis: $M_{\text{TOV}}/M_{\odot} \gtrsim 2.5$



- Total mass ejected is in perfect **much smaller** than observed from kilonova signal.

- Total mass emitted in GWs is **much larger** than predicted from simulations;
- Mismatch becomes worse with larger masses



Tension on the maximum mass

Nathanail, Most, LR (2020)

- The recent detection of GW190814 has created a significant tension on the maximum mass

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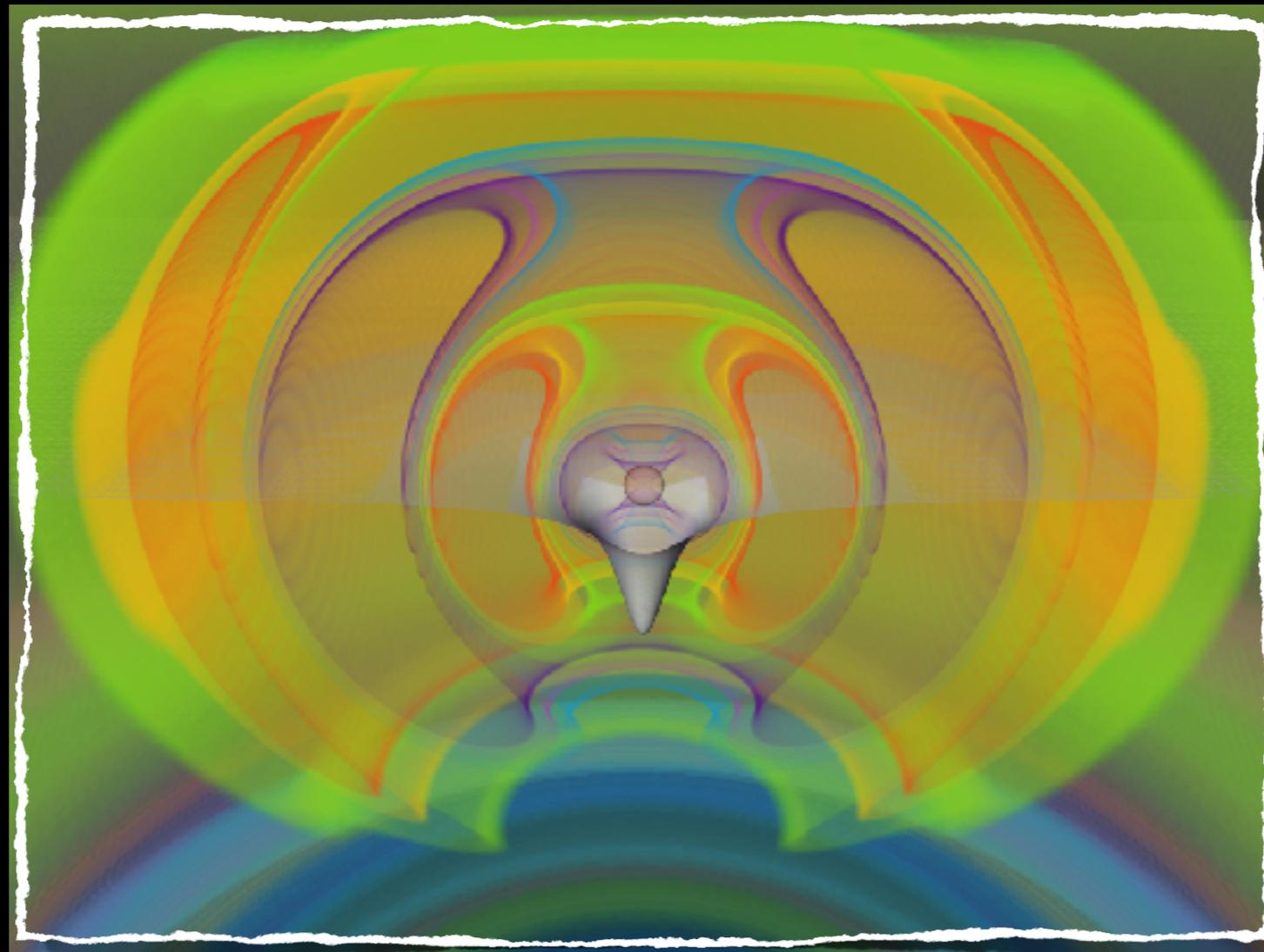
$$M_2 = 2.50 - 2.67 M_{\odot} \quad \text{smallest BH or heaviest NS!}$$

- If secondary in GW190814 was a NS, all previous considerations are incorrect.
- No EM counterpart was observed with GW190814 and no estimates possible on ejected matter or timescale for survival.
- **How do we solve this tension?**
- Solution: secondary in GW190814 was a **BH** at merger but could have been a NS before

GW170817, maximum mass, radii and tidal deformabilities

LR, Most, Weih (2018)

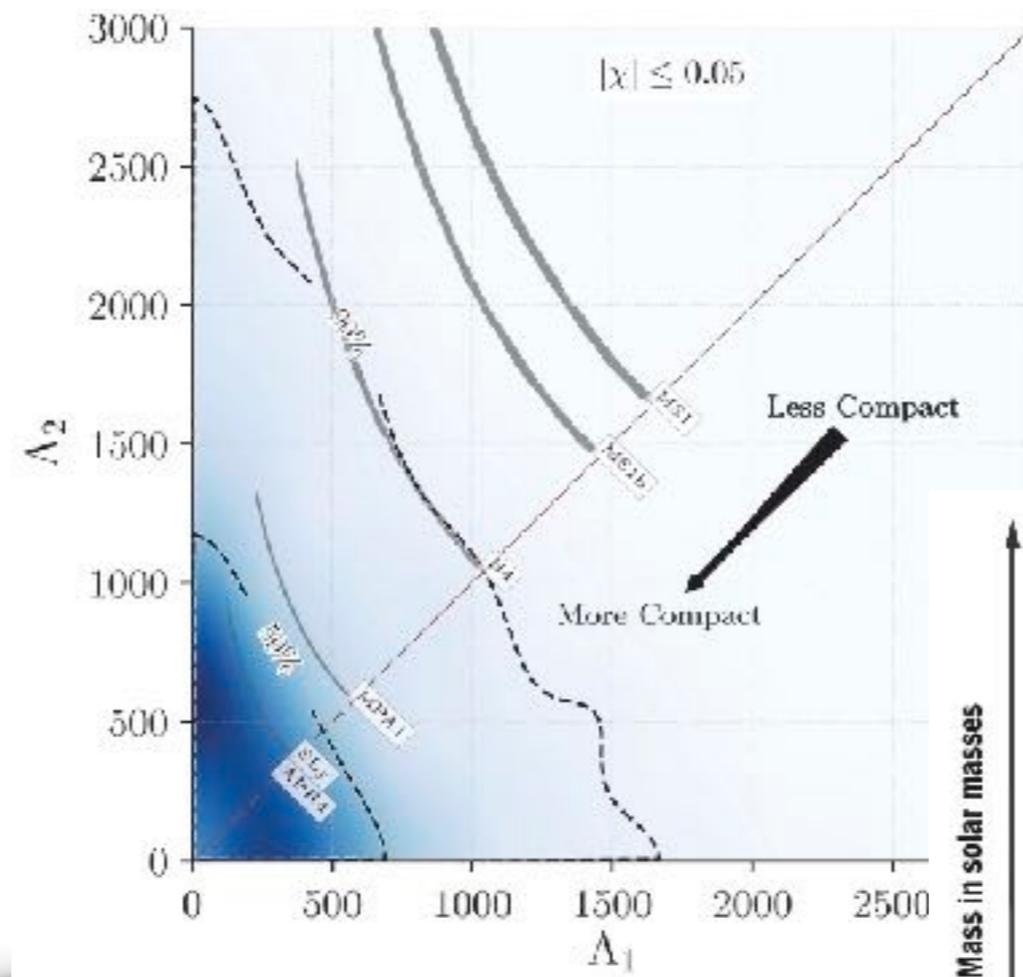
Most, Weih, LR, Schaffner-Bielich (2018)



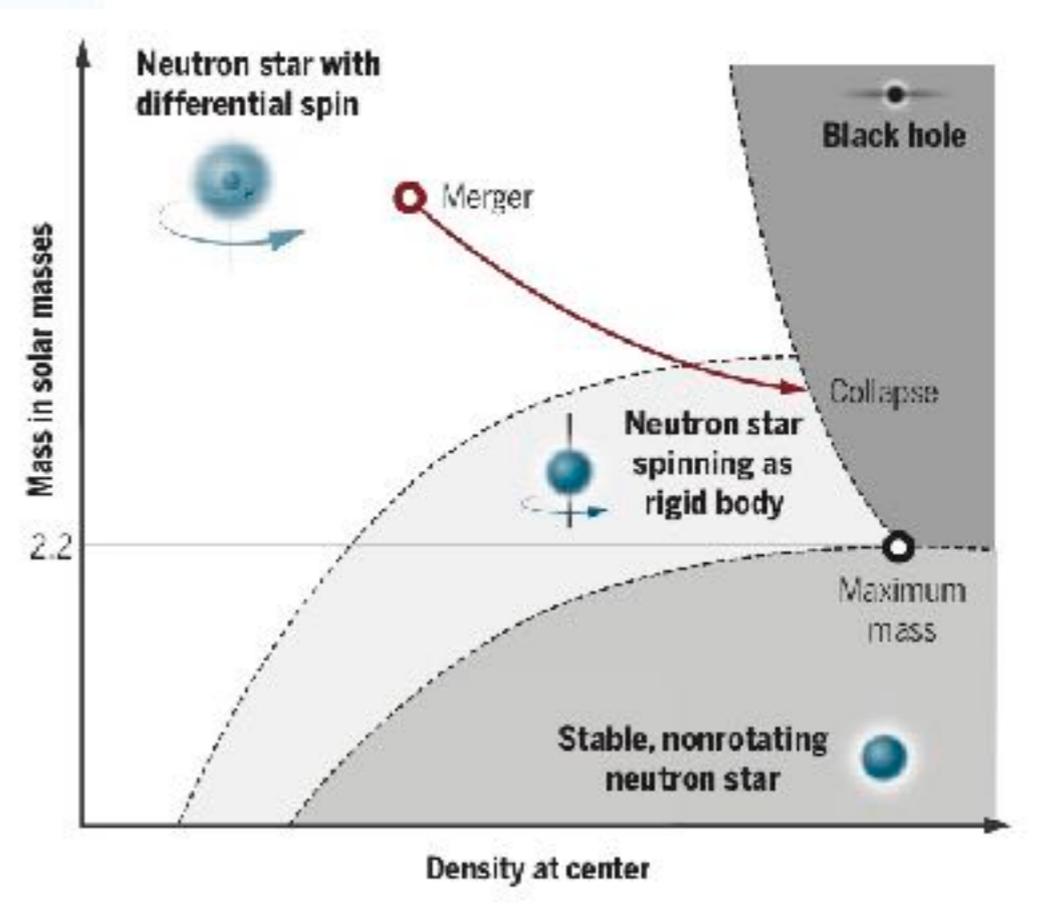
Limits on radii and deformabilities

- We have produced 10^6 EOSs with about 10^9 stellar models.

- Can impose differential constraints from the **maximum mass** and from the **tidal deformability** from **GW170817?**



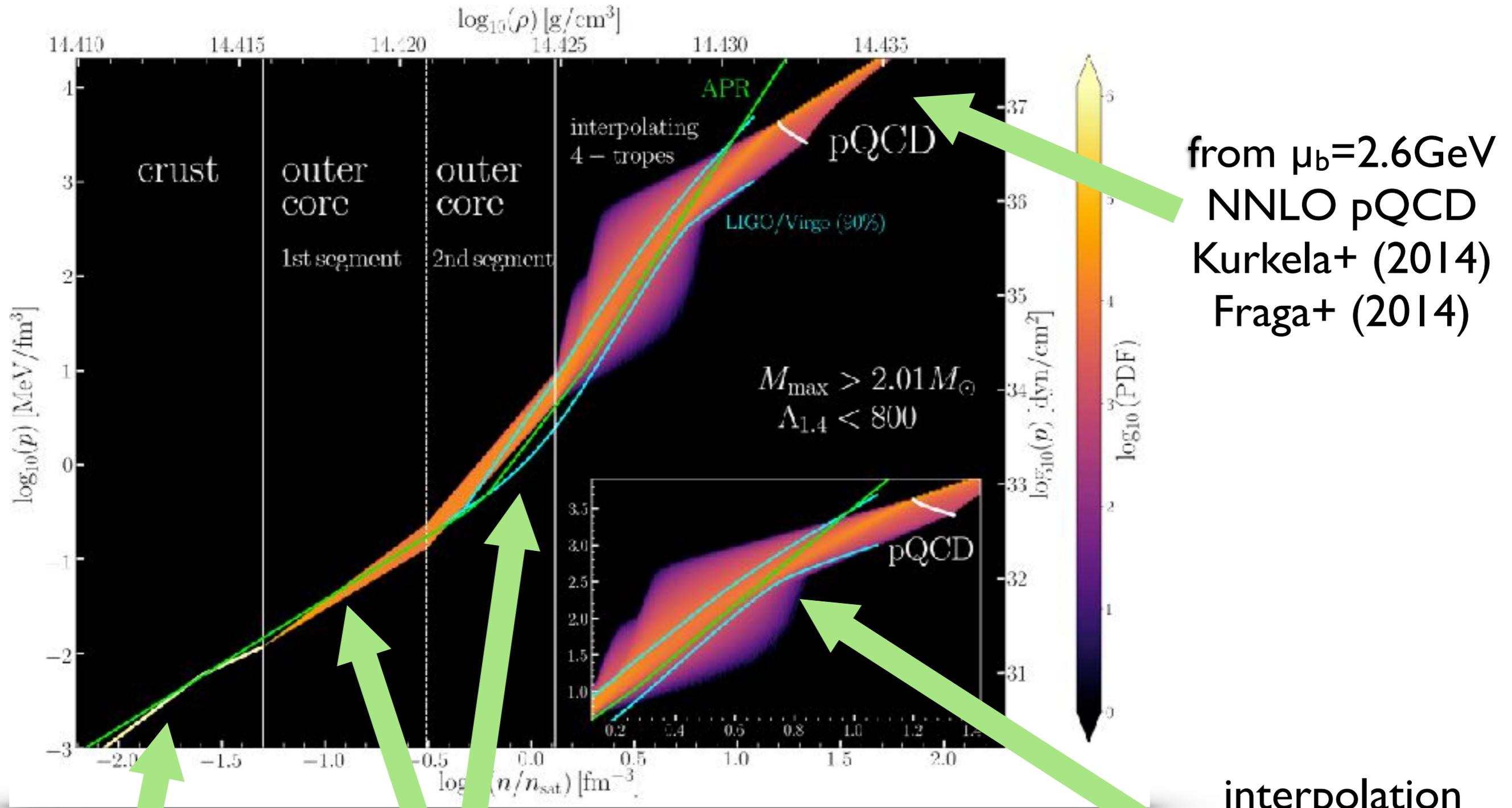
Abbott+ 2017



LR+ 2017

parametrising our ignorance

- Construct most generic family of NS-matter EOs



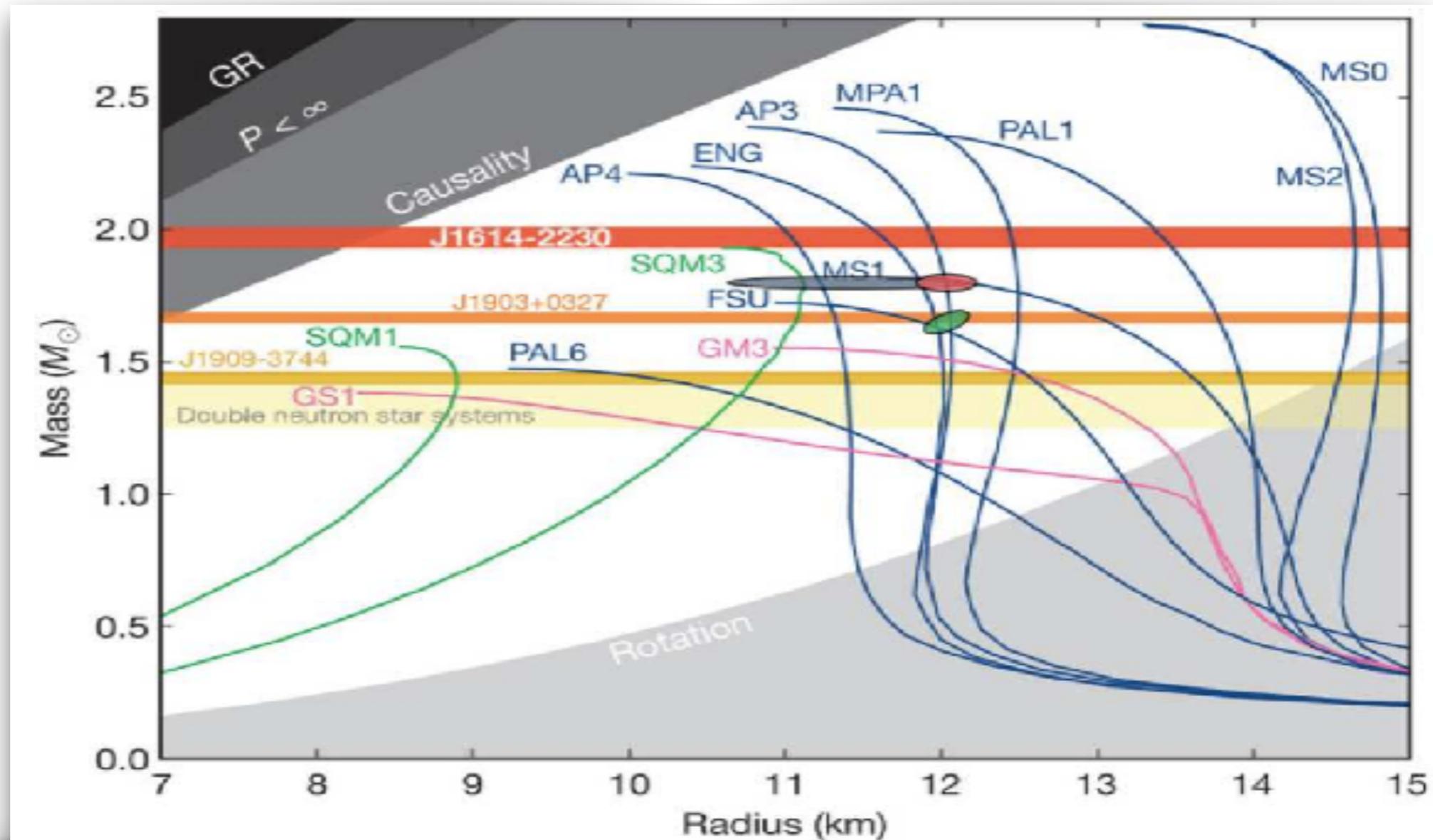
BPS

polytropic fit of Drischler+ (2016)
(large impact on results)

Mass-radius relations

- We have produced 10^6 EOSs with about 10^9 stellar models.

- Can impose differential constraints from the **maximum mass** and from the **tidal deformability** from **GW170817**

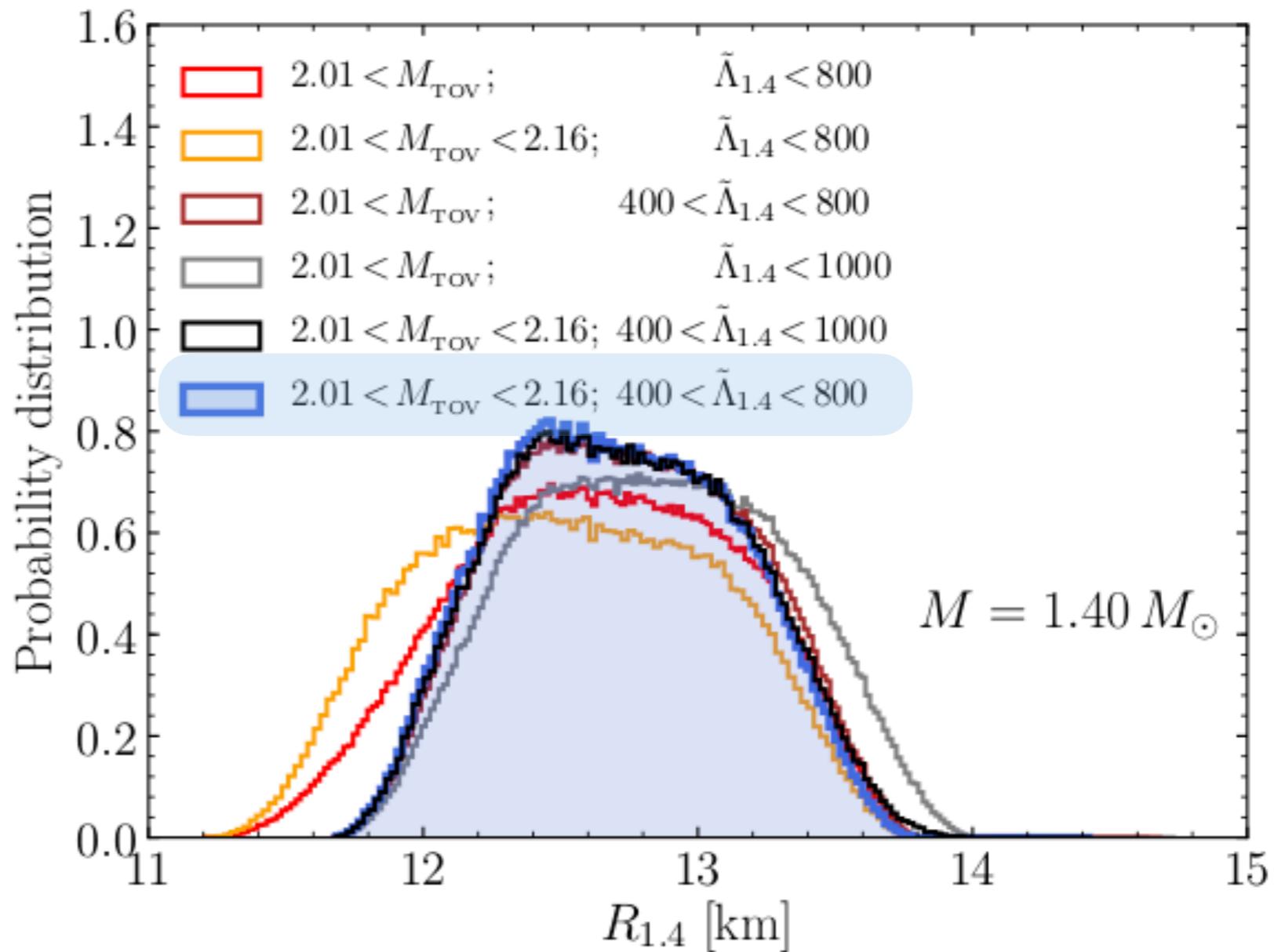


one-dimensional cuts

- Closer look at a mass of $M = 1.40 M_{\odot}$
- Can play with different constraints on maximum mass and tidal deformability.
- Overall distribution is very robust

$$12.00 < R_{1.4}/\text{km} < 13.45$$

$$\bar{R}_{1.4} = 12.45 \text{ km}$$



Constraining tidal deformability

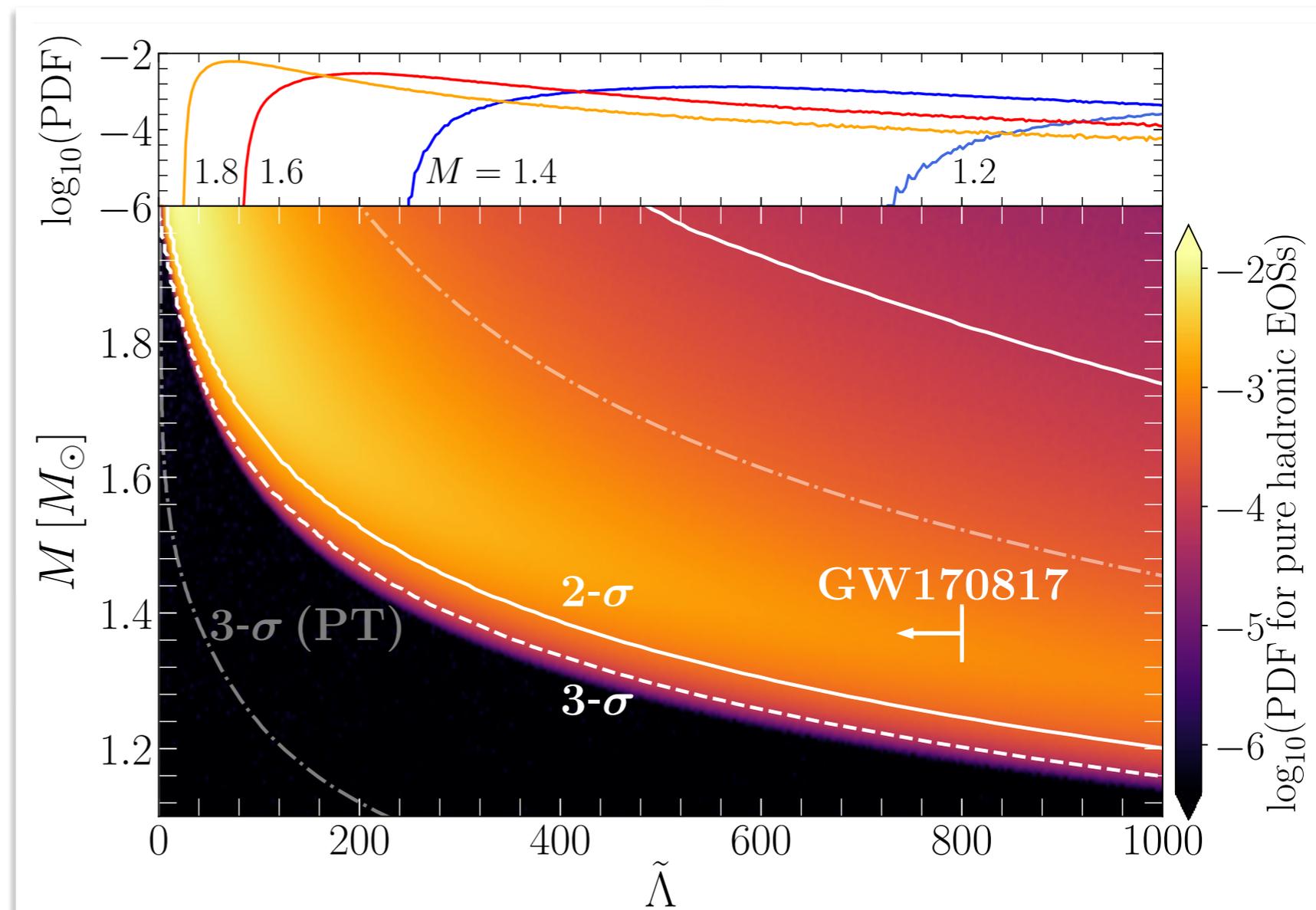
- Can explore statistics of all properties of our 10^9 models.
- In particular can study PDF of tidal deformability: $\tilde{\Lambda}$

- LIGO has already set upper limit:

$$\tilde{\Lambda}_{1.4} \lesssim 800$$

- Our sample naturally sets a lower limit:

$$\tilde{\Lambda}_{1.4} > 375$$

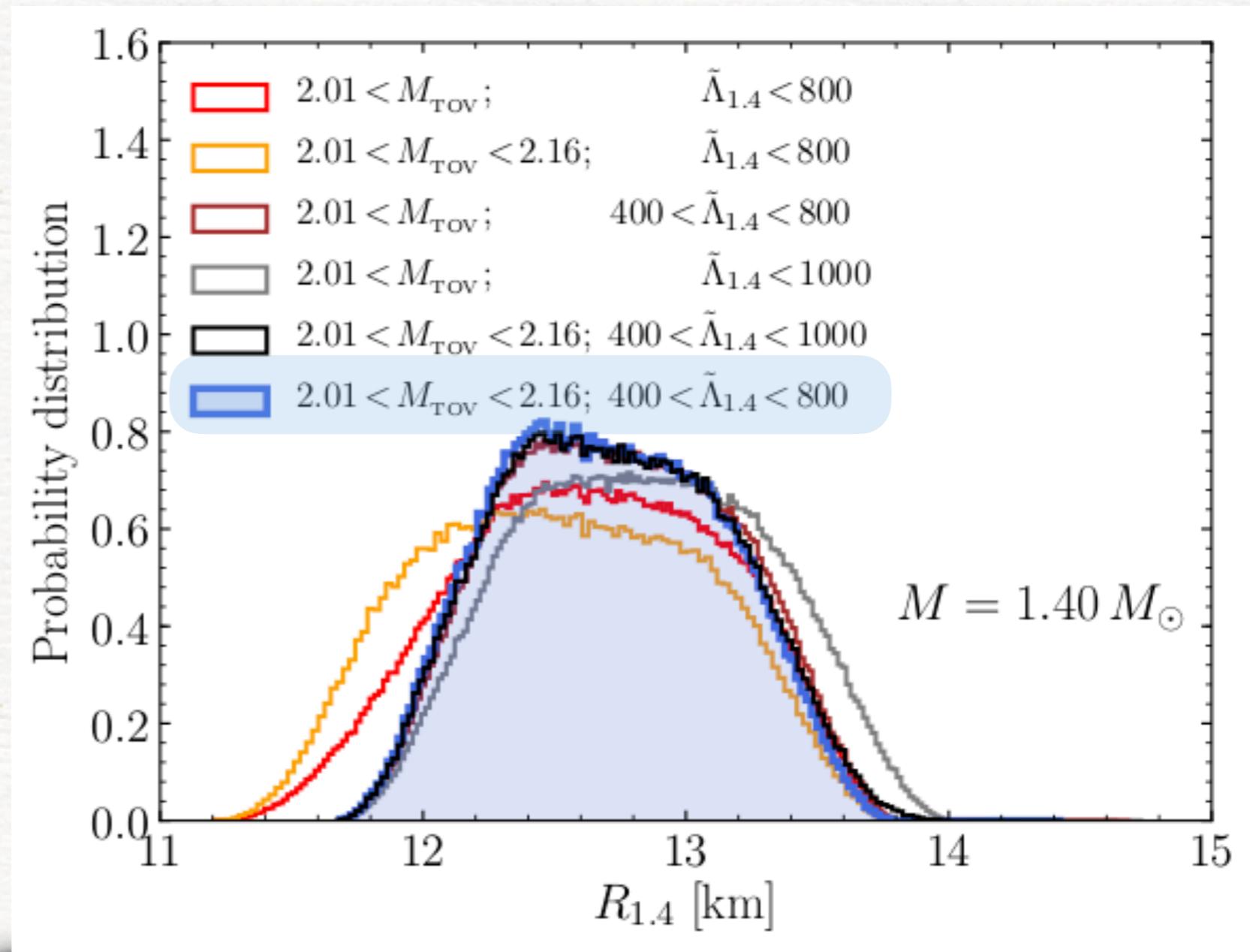


On the importance of a lower limit for $\tilde{\Lambda}$

- Closer look at a mass of $M = 1.40 M_{\odot}$
- Can play with different constraints on maximum mass and tidal deformability.
- Overall distribution is very robust

$$12.00 < R_{1.4}/\text{km} < 13.45$$

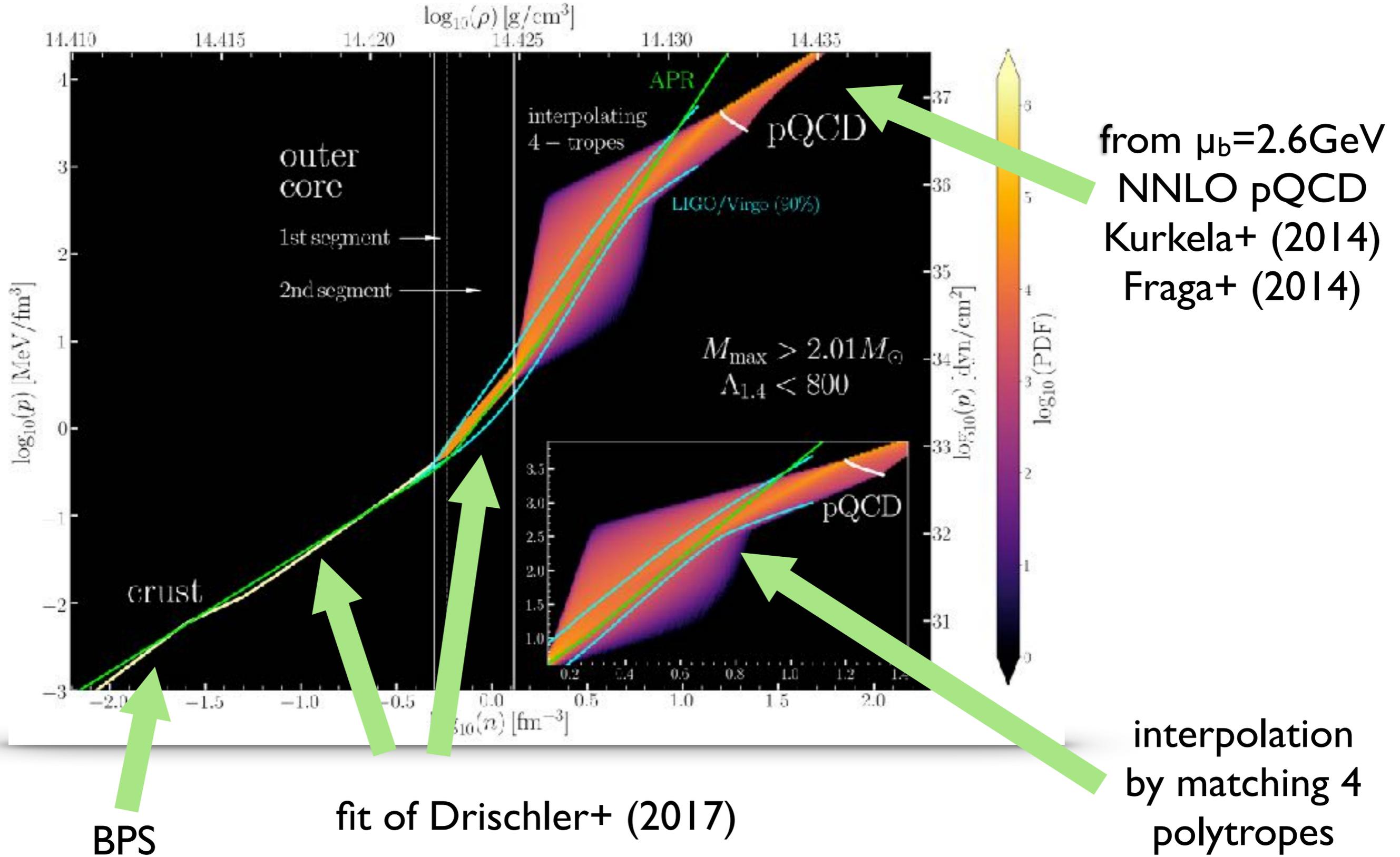
$$\bar{R}_{1.4} = 12.45 \text{ km}$$



In other words, stringent lower limits on $\tilde{\Lambda}$ have huge impact: exclude softest EOSs

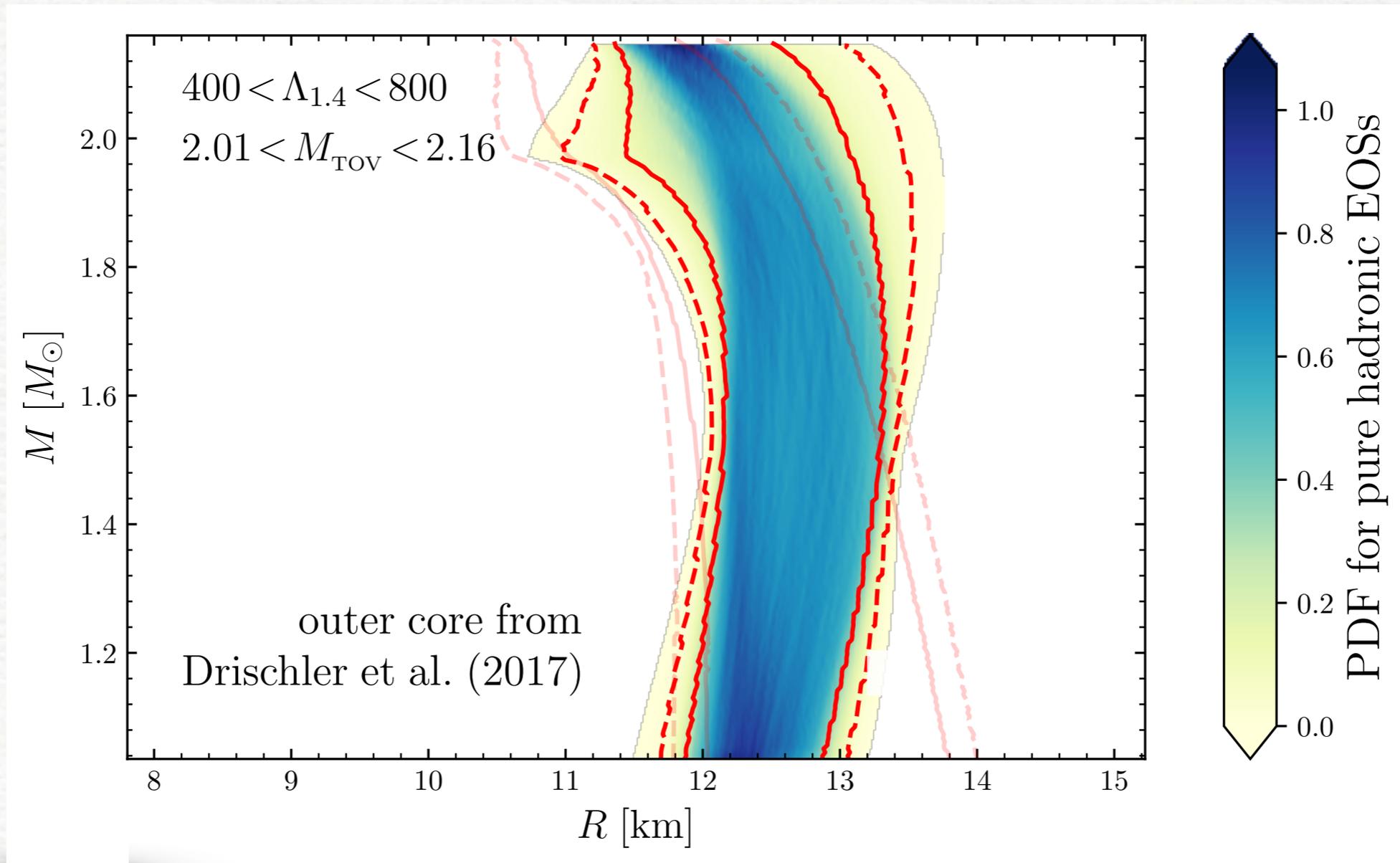
On the importance of the **outer-core**

- Improved prescriptions known for **outer core**, e.g., Drischler+ 2017



On the importance of the **outer-core**

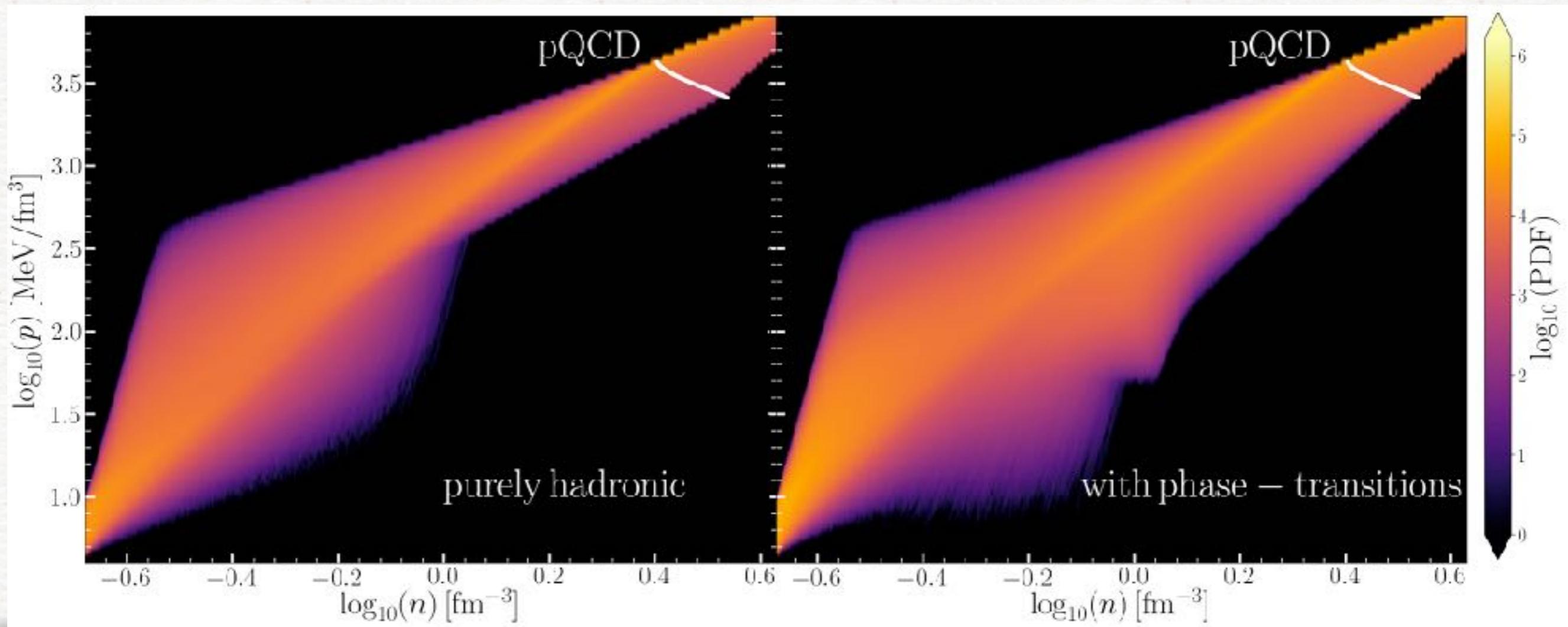
- Improved prescriptions known for **outer core**, e.g., Drischler+ 2017



Lesson: radius constraints depends strongly on stiffness (uncertainty) of EOS for $0.5 \lesssim n/n_s \lesssim 1.3$

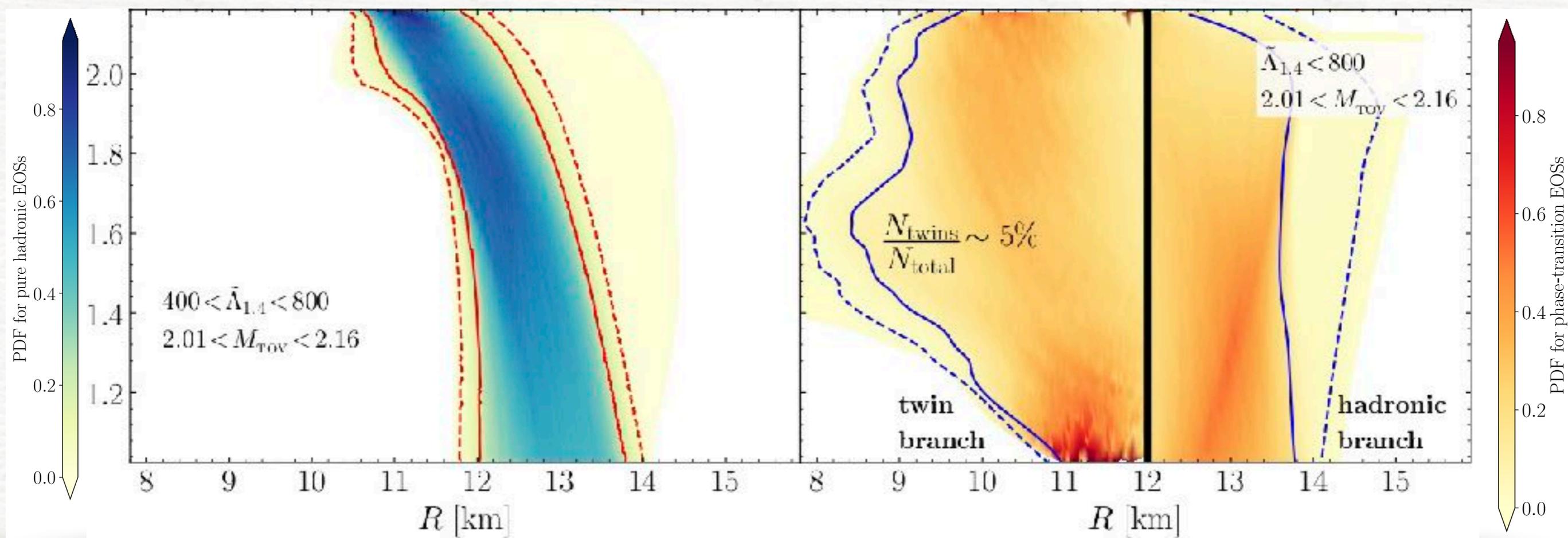
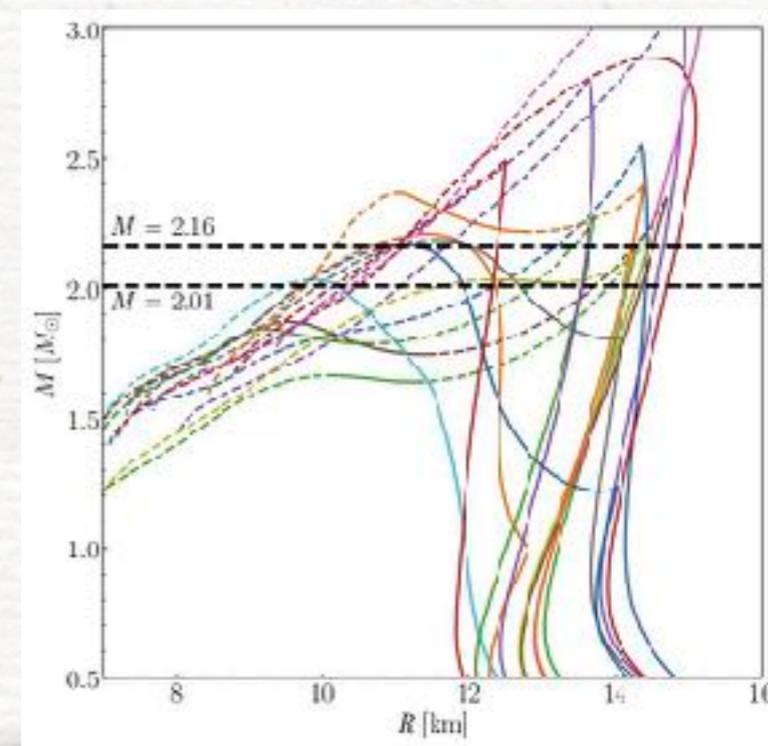
What about **phase transitions**?

- All EOSs so far are purely hadronic; a conservative but probably **reasonable** assumption.
- What about the possibility of **phase transitions**?
- These are not trivial but not too difficult to model.

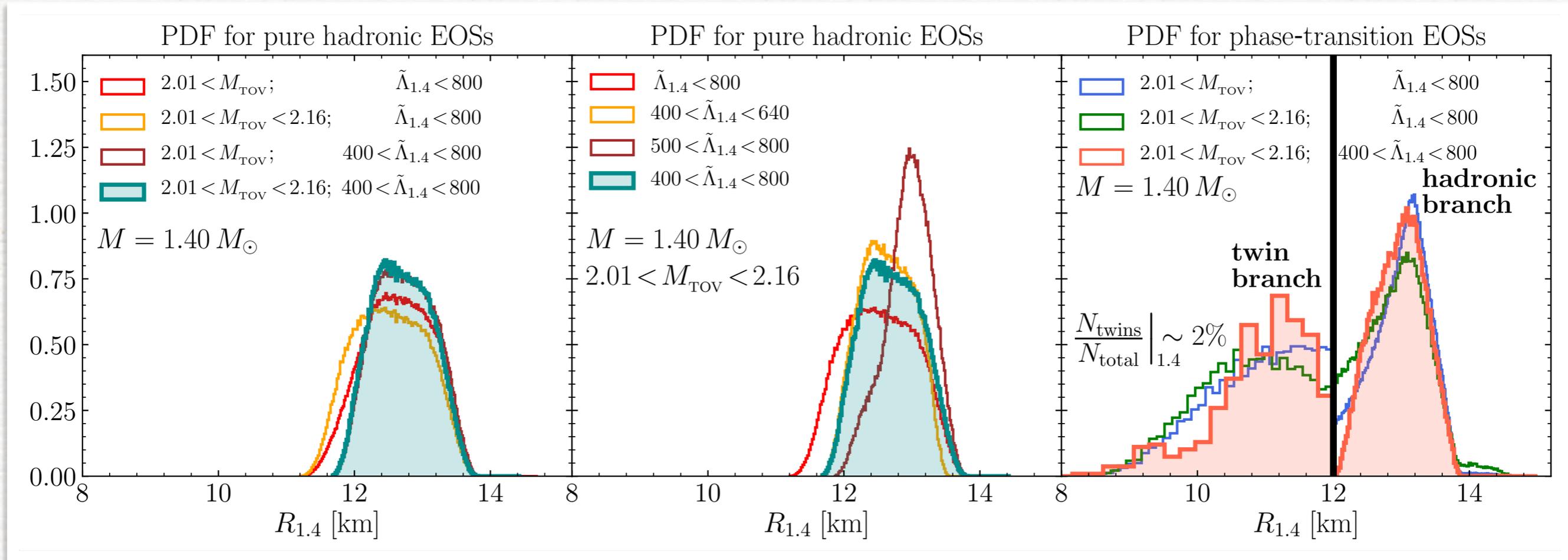


Mass-radius relations

- Presence of a phase transition leads to second stable branch and “**twin-star**” models.



One-dimensional cuts: PTs



Applying all constraints from GW170817:

$$12.00 < R_{1.4}/\text{km} < 13.45$$

$$\bar{R}_{1.4} = 12.45 \text{ km}$$

hadronic EOS

$$8.53 < R_{1.4}/\text{km} < 13.74$$

$$\bar{R}_{1.4} = 13.06 \text{ km}$$

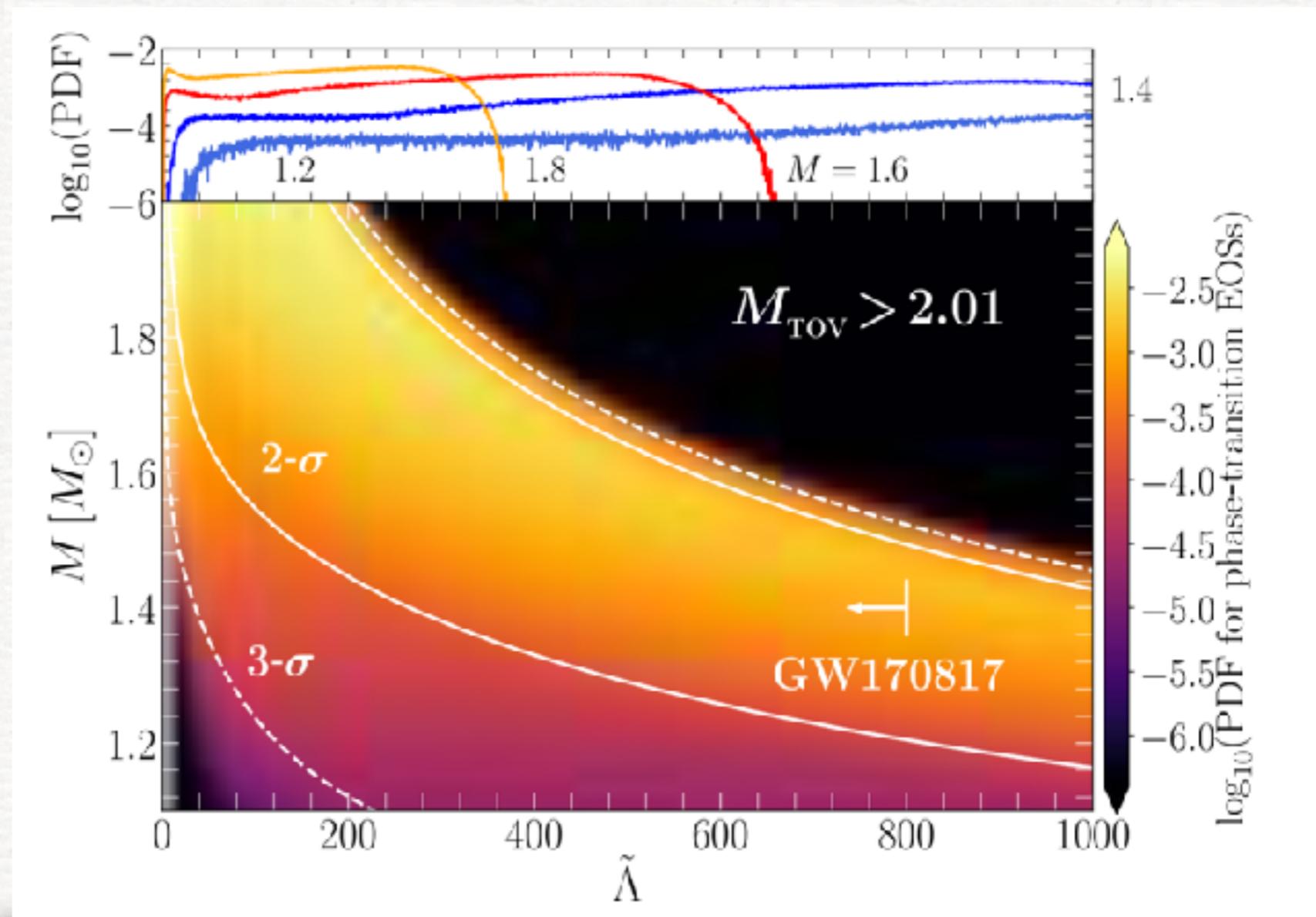
phase
transitions

Constraining tidal deformability: **PTs**

- Can repeat considerations with EOSs having PTs
- Lower limit much weaker: $\tilde{\Lambda}_{1.4} \gtrsim 35$
- Large masses have sharp cut-off on upper limit:

$$\tilde{\Lambda}_{1.7} \lesssim 460$$

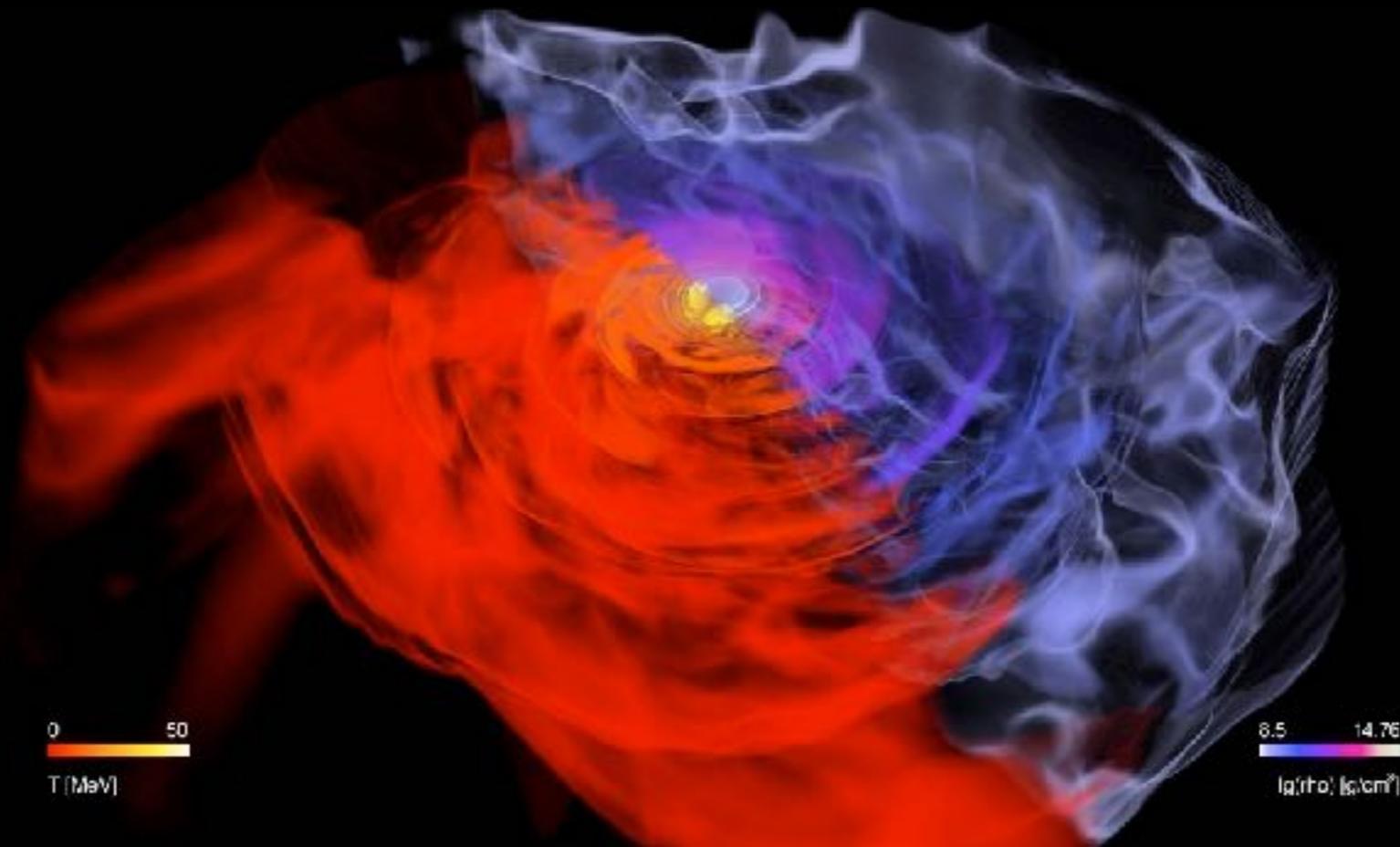
Hence, detection with $\tilde{\Lambda}_{1.7} \sim 600$ would **rule out** twin stars!



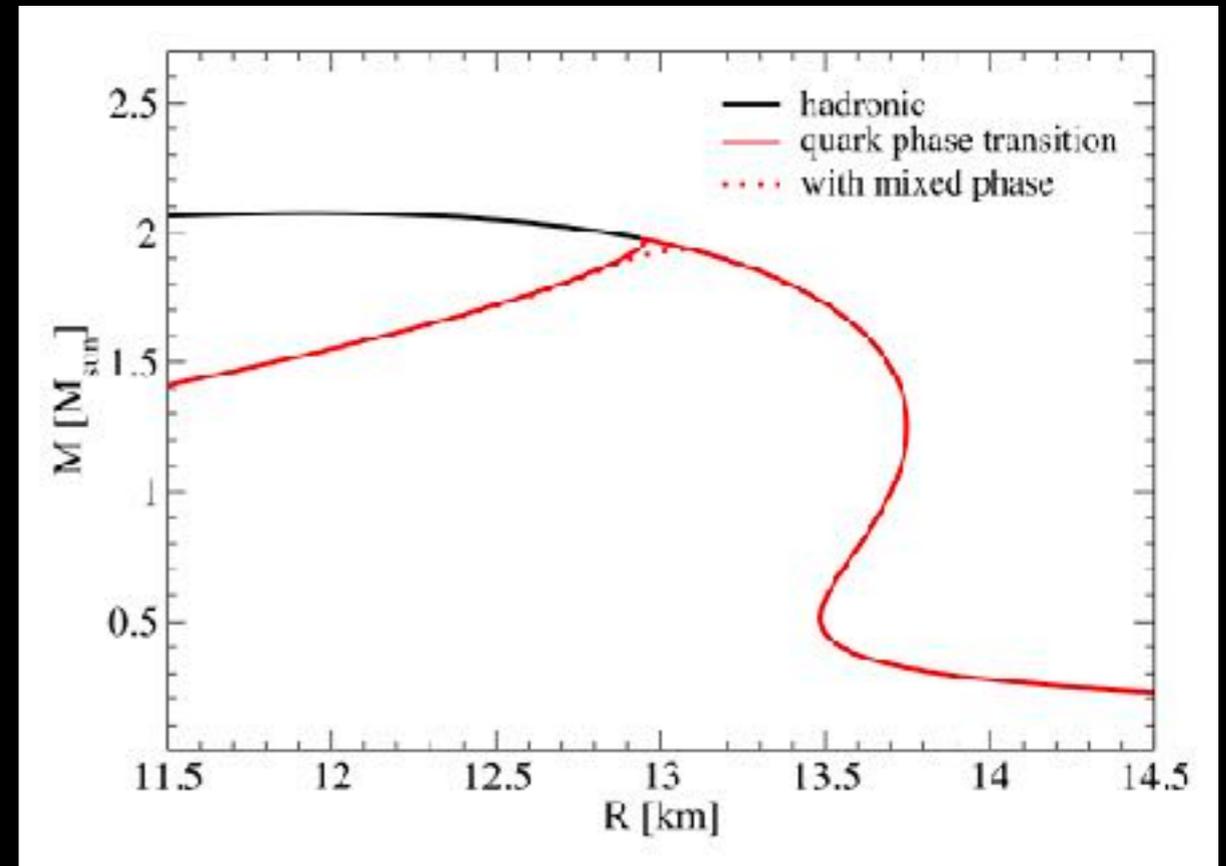
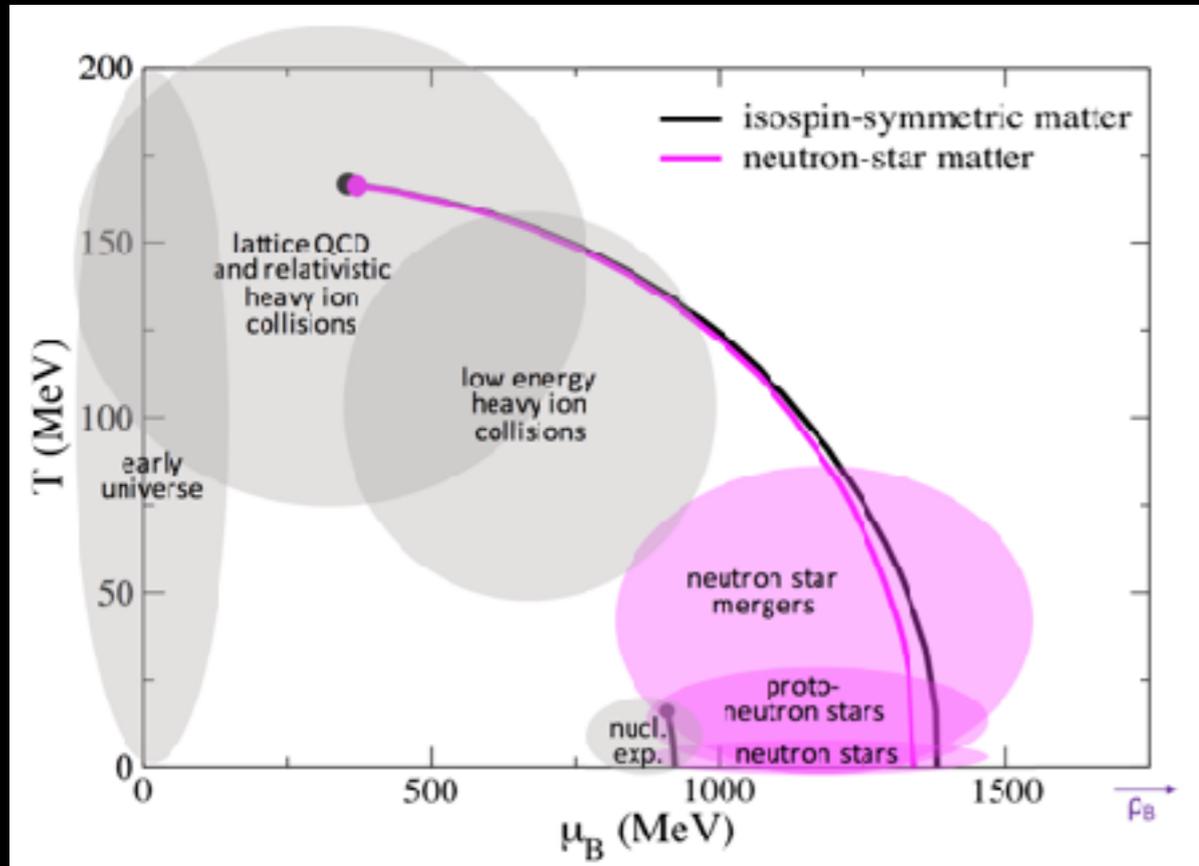
Phase transitions and their signatures

Most, Papenfort, Dexheimer, Hanauske, Schramm, Stoecker, LR (2019)

Weih, Hanauske, LR (2020)

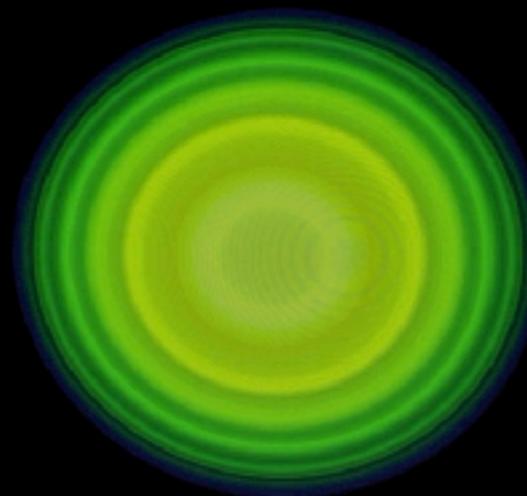
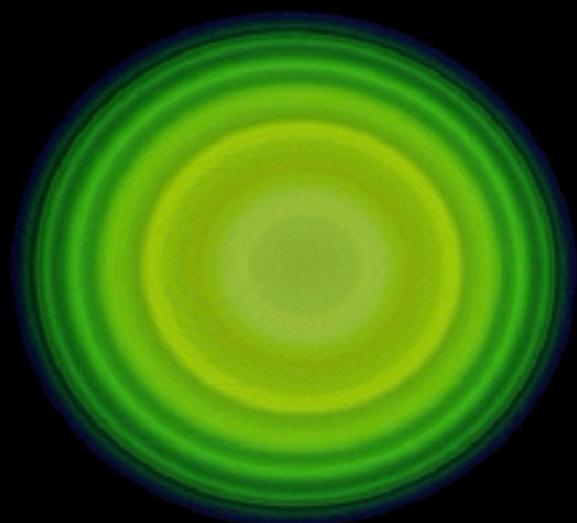


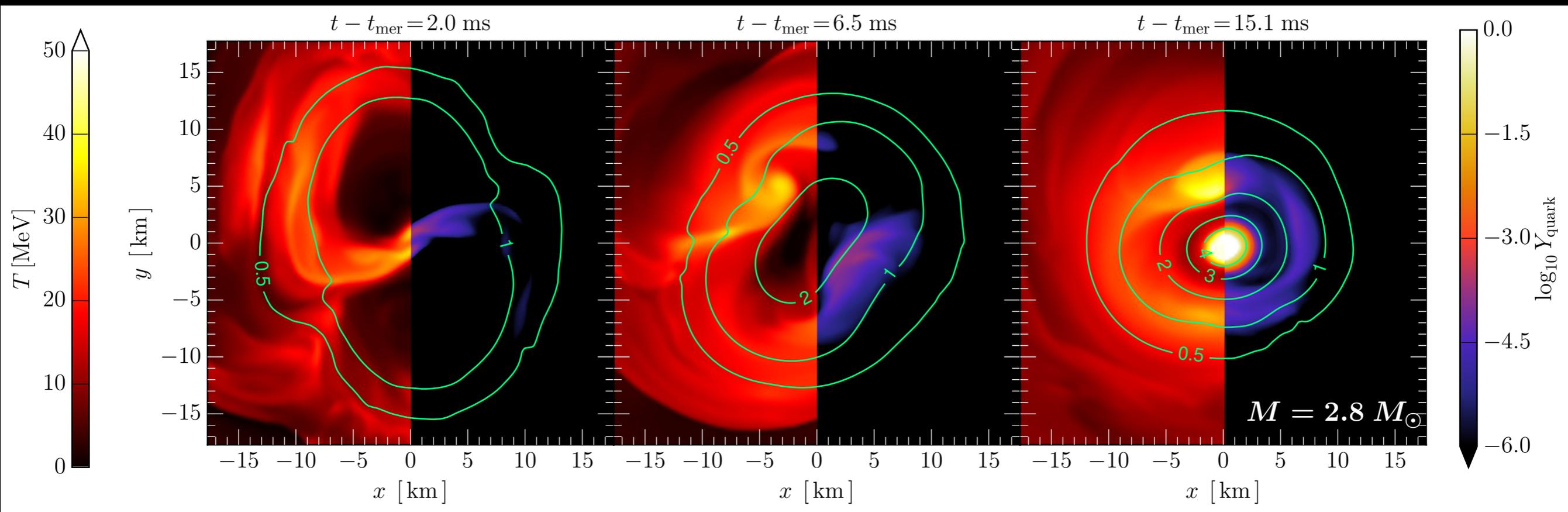
- **Isolated** neutron stars probe a small fraction of phase diagram.
- Neutron-star **binary** mergers reach temperatures up to **80 MeV** and probe regions complementary to experiments.



- Considered EOS based on Chiral Mean Field (CMF) model, based on a nonlinear SU(3) sigma model.
- Appearance of quarks can be introduced naturally.

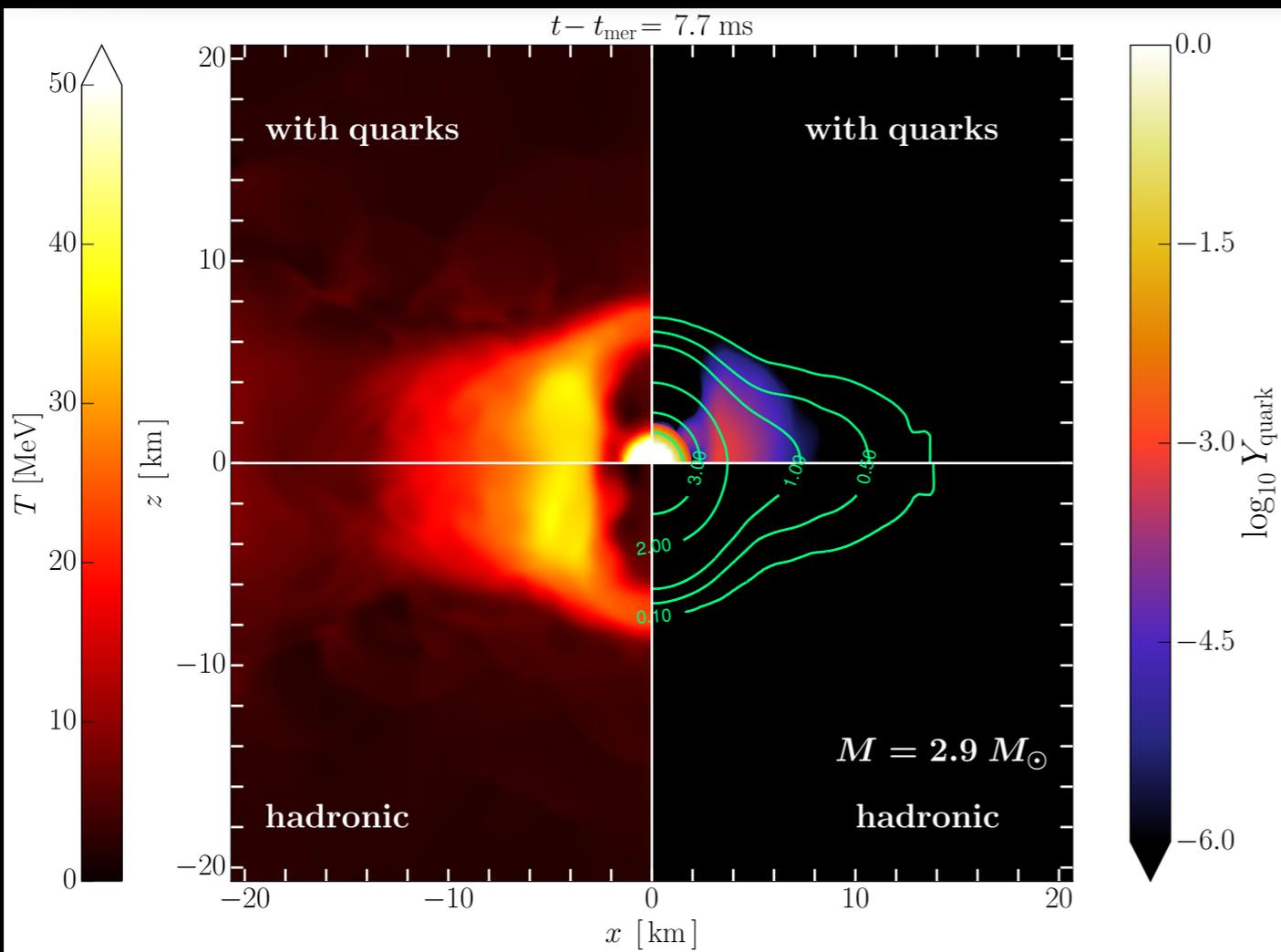
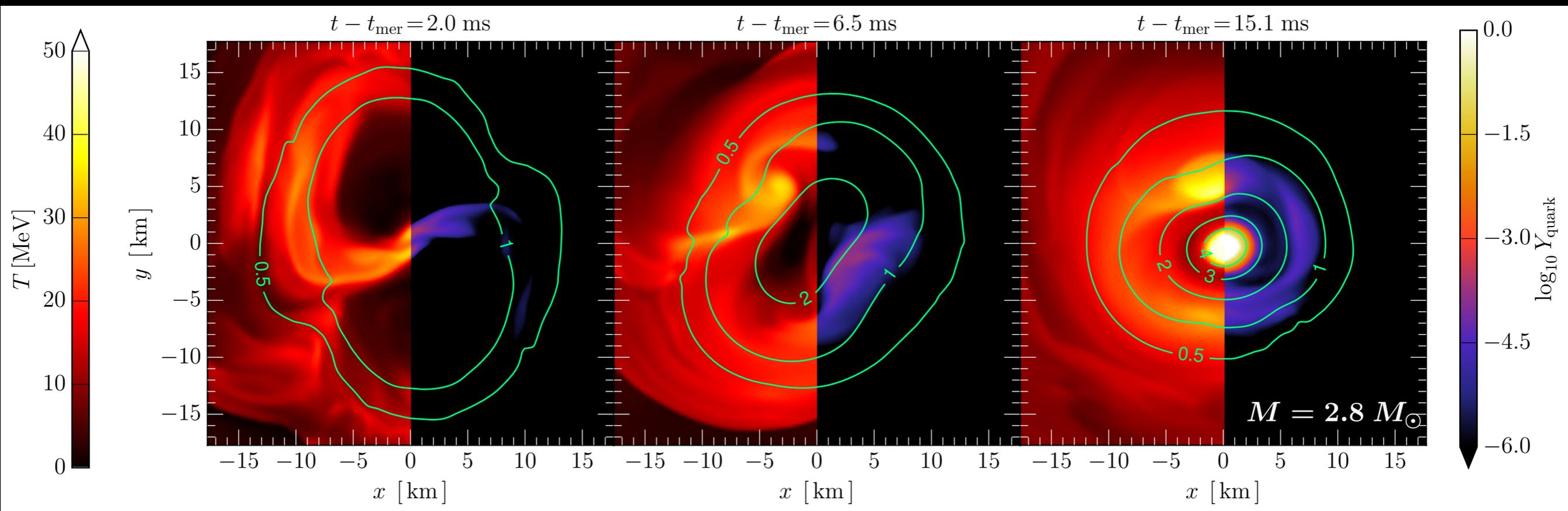
Animations: Weih, Most, LR





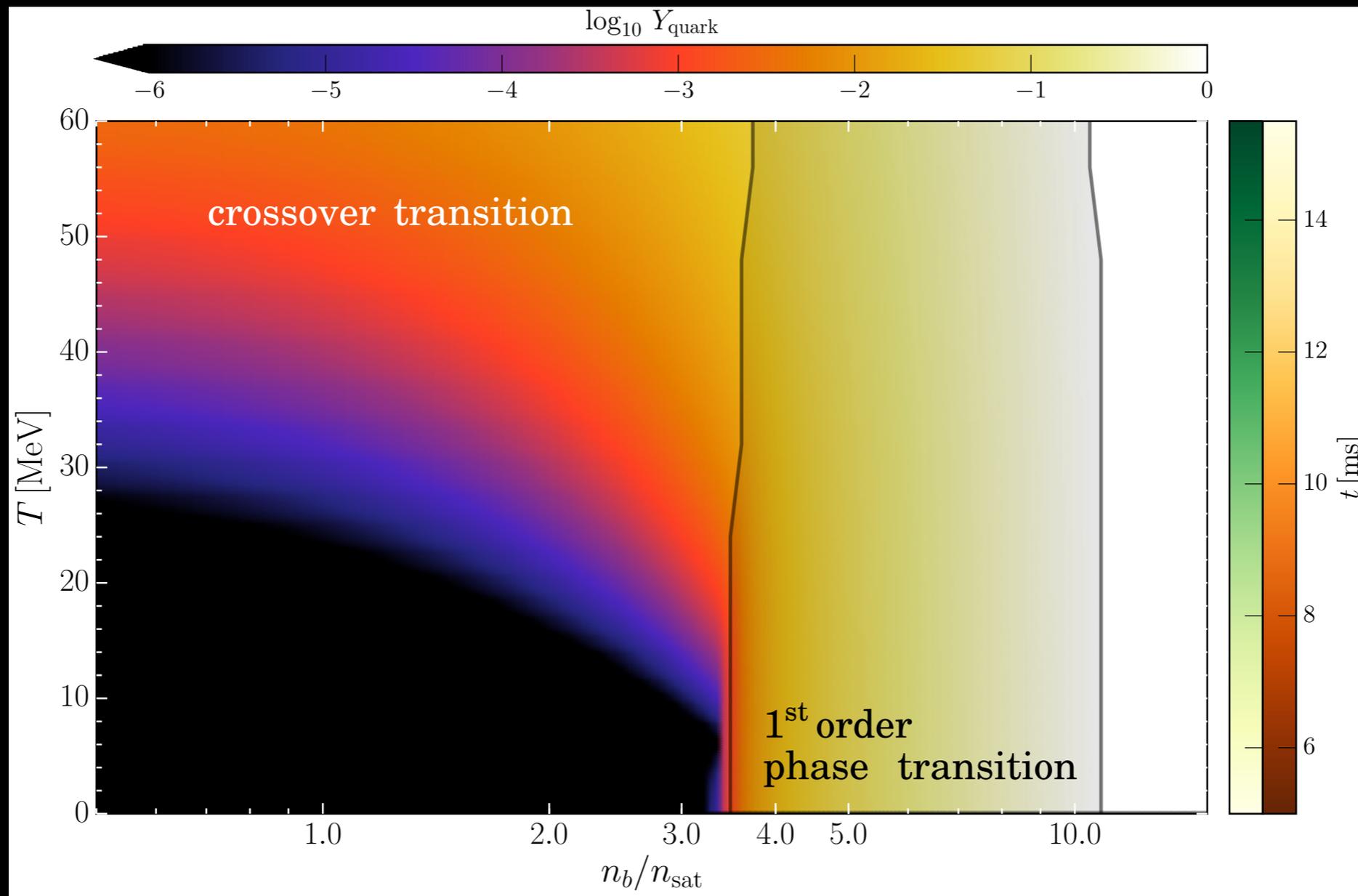
Quarks appear at sufficiently large
temperatures and **densities.**

When this happens the **EOS** is
 considerably **softened.**



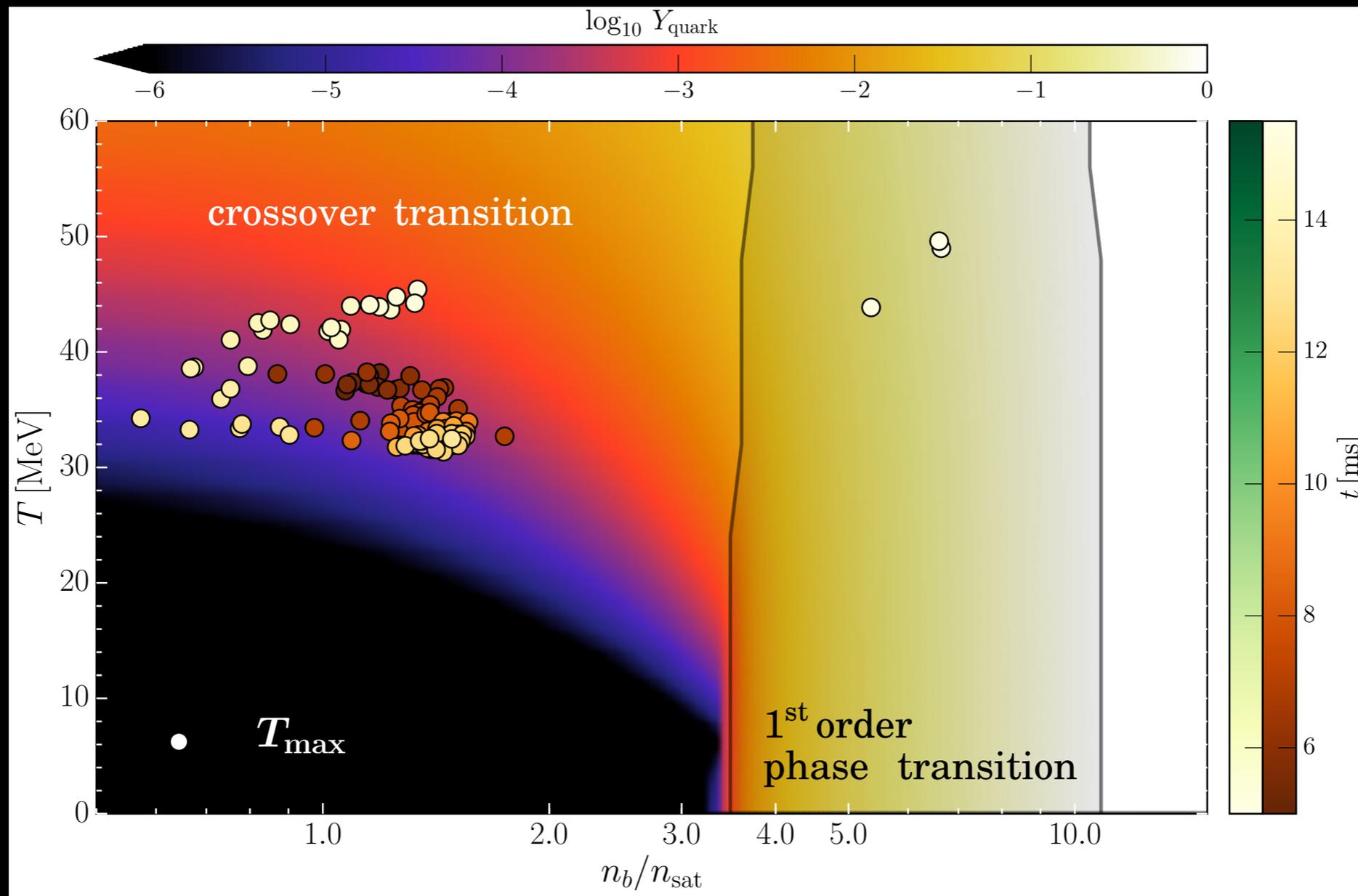
- Quarks appear at sufficiently large temperatures and densities.
- For EOS without quarks, the dynamics (temperature distrib.) is very similar, but no PT.

Comparing with the phase diagram



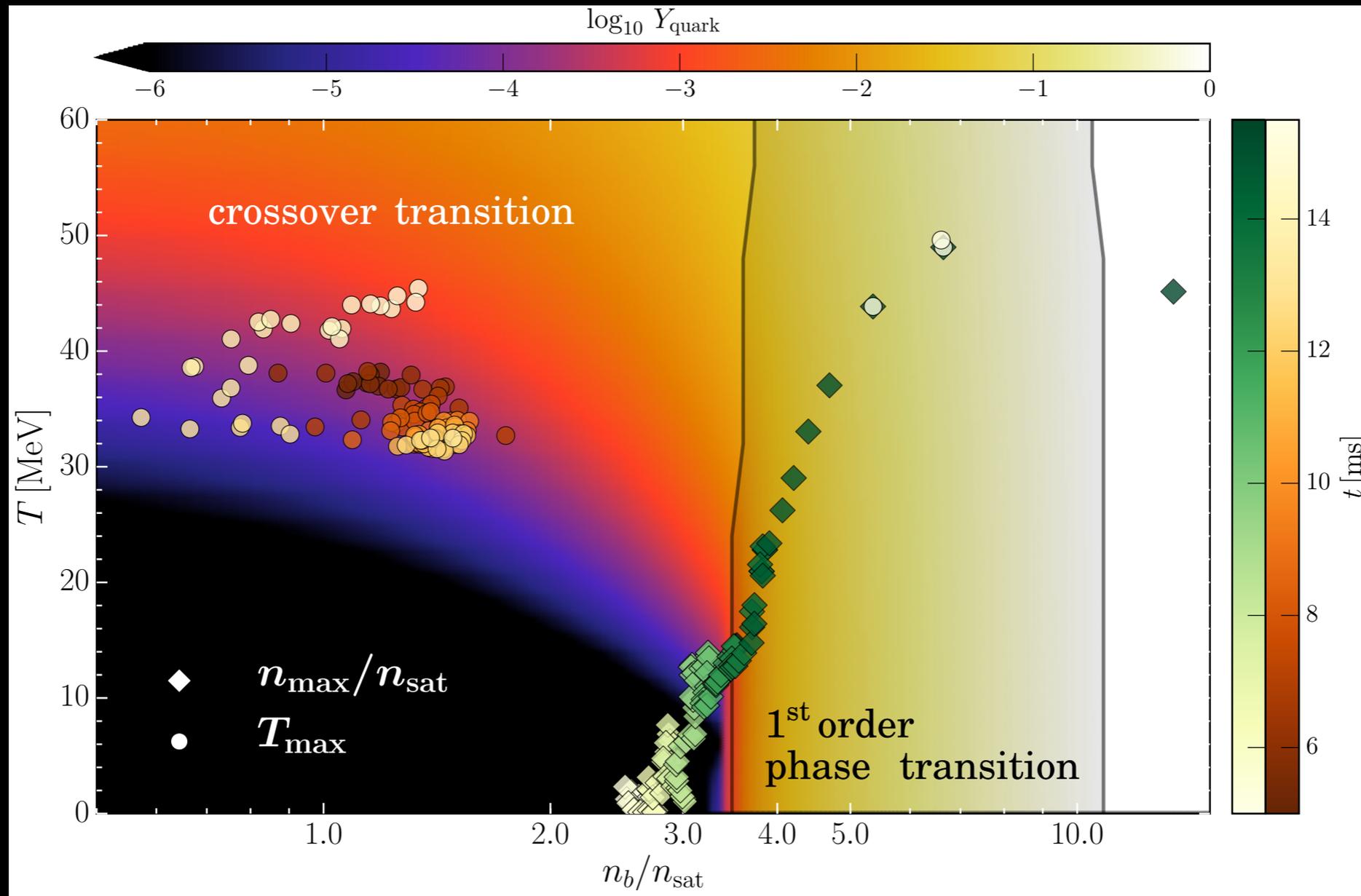
- Phase diagram with quark fraction

Comparing with the phase diagram



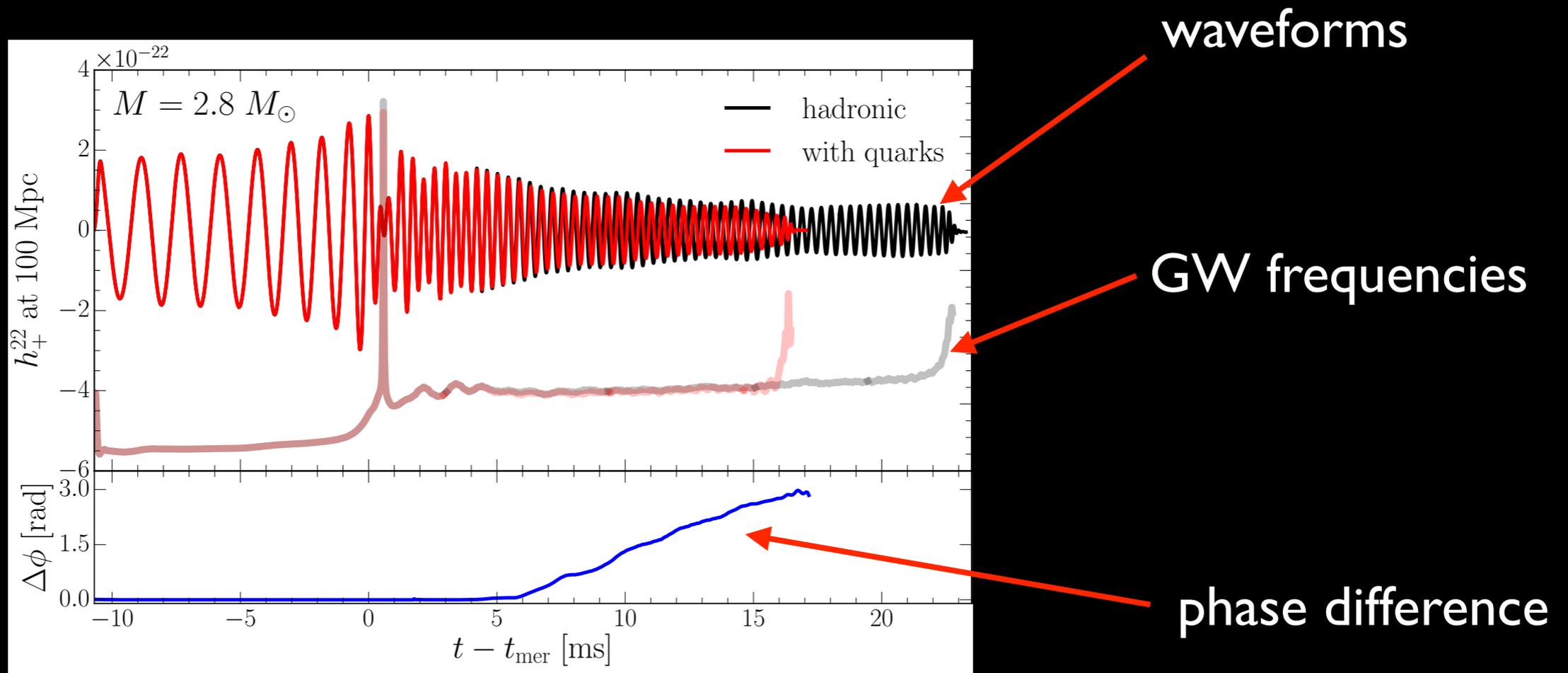
- Phase diagram with quark fraction
- Circles show the position in the diagram of the maximum temperature as a function of time

Comparing with the phase diagram



- Reported are the evolution of the max. temperature and density.
- Quarks appear already early on, but only in small fractions.
- Once sufficient density is reached, a full phase transition takes place.

Gravitational-wave emission



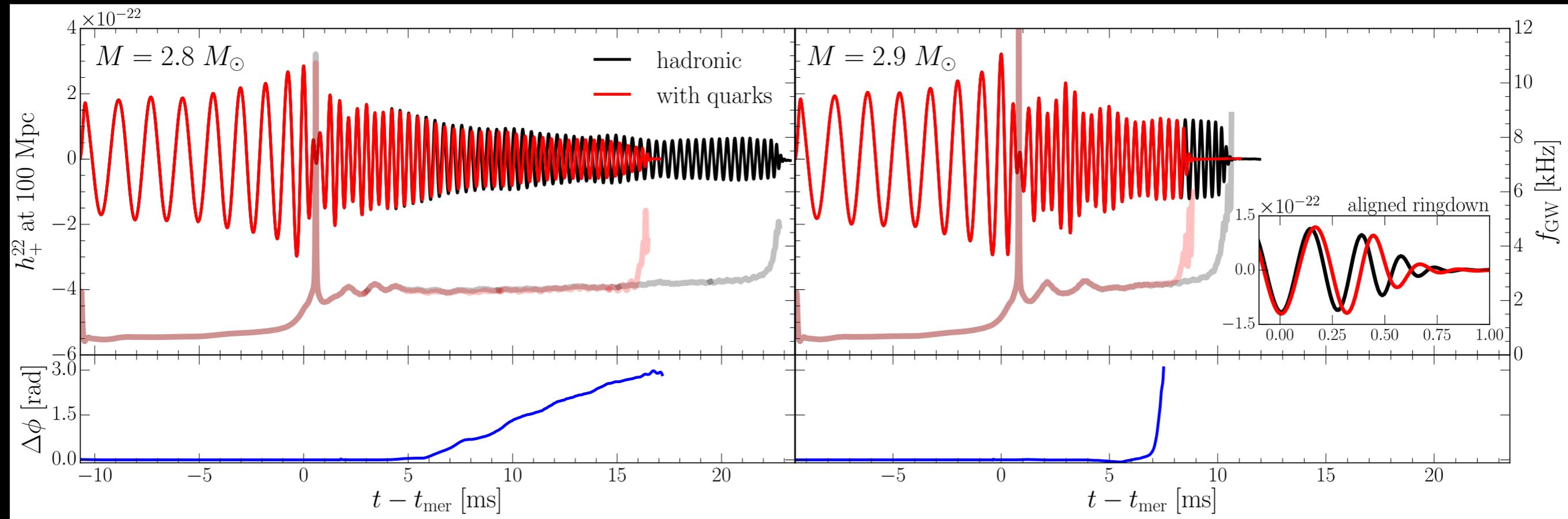
- After ~ 5 ms, quark fraction is large enough to change quadrupole moment and yield differences in the waveforms.
- Sudden softening of the phase transition leads to collapse and **large difference** in phase evolution.

Observing mismatch between **inspiral** (fully hadronic) and **post-merger** (phase transition): clear **signature** of a **PT**

Gravitational-wave emission

“low-mass” binary

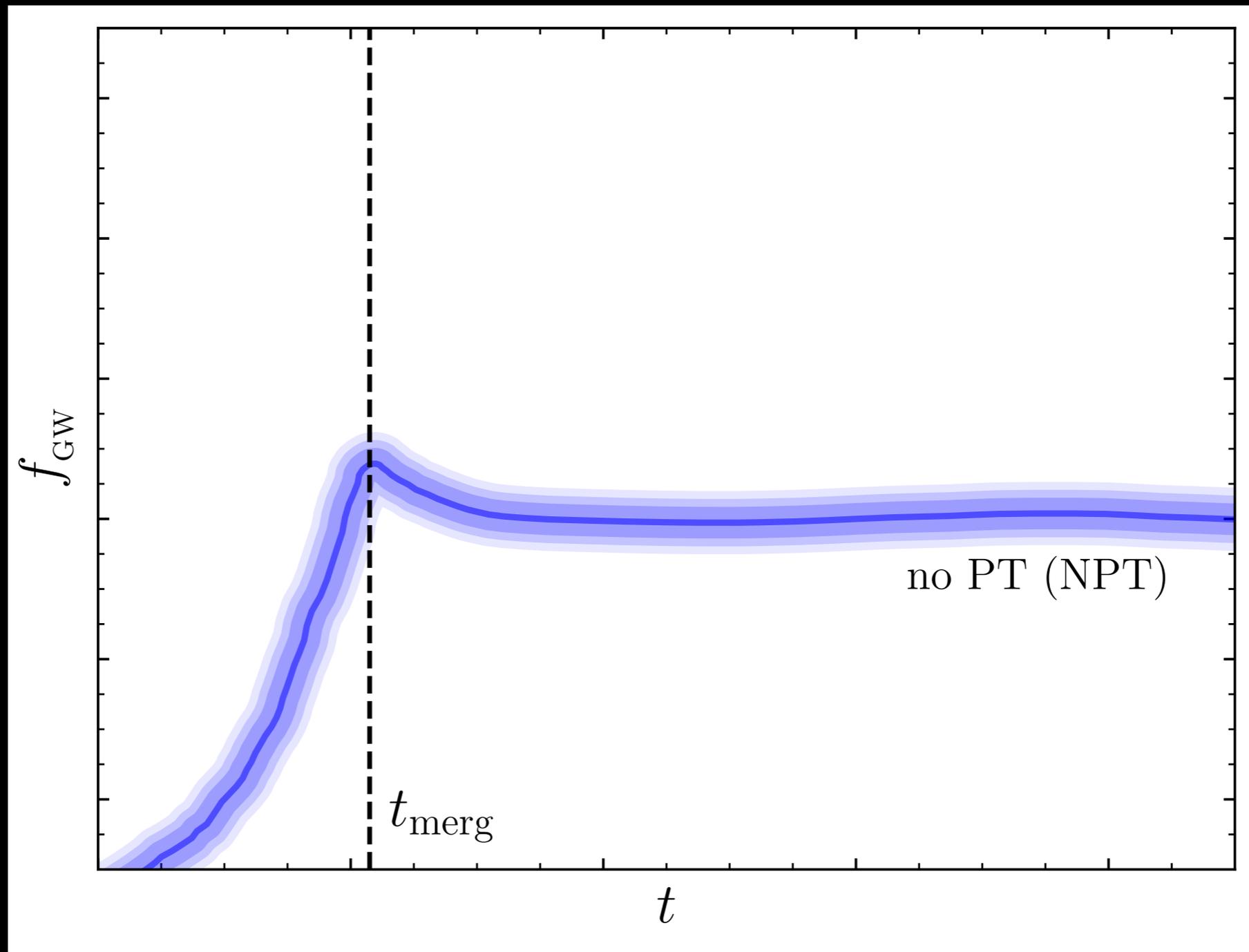
“high-mass” binary



- In **low-mass binary**, after ~ 5 ms, quark fraction is large enough to change quadrupole moment and yield differences in the waveforms.
- In **high-mass binary**, phase transition takes place rapidly after ~ 5 ms. Waveforms are similar but **ringdown** is **different** (free fall for PT). Observing mismatch between **inspiral** (fully hadronic) and **post-merger** (phase transition): clear **signature** of a **PT**

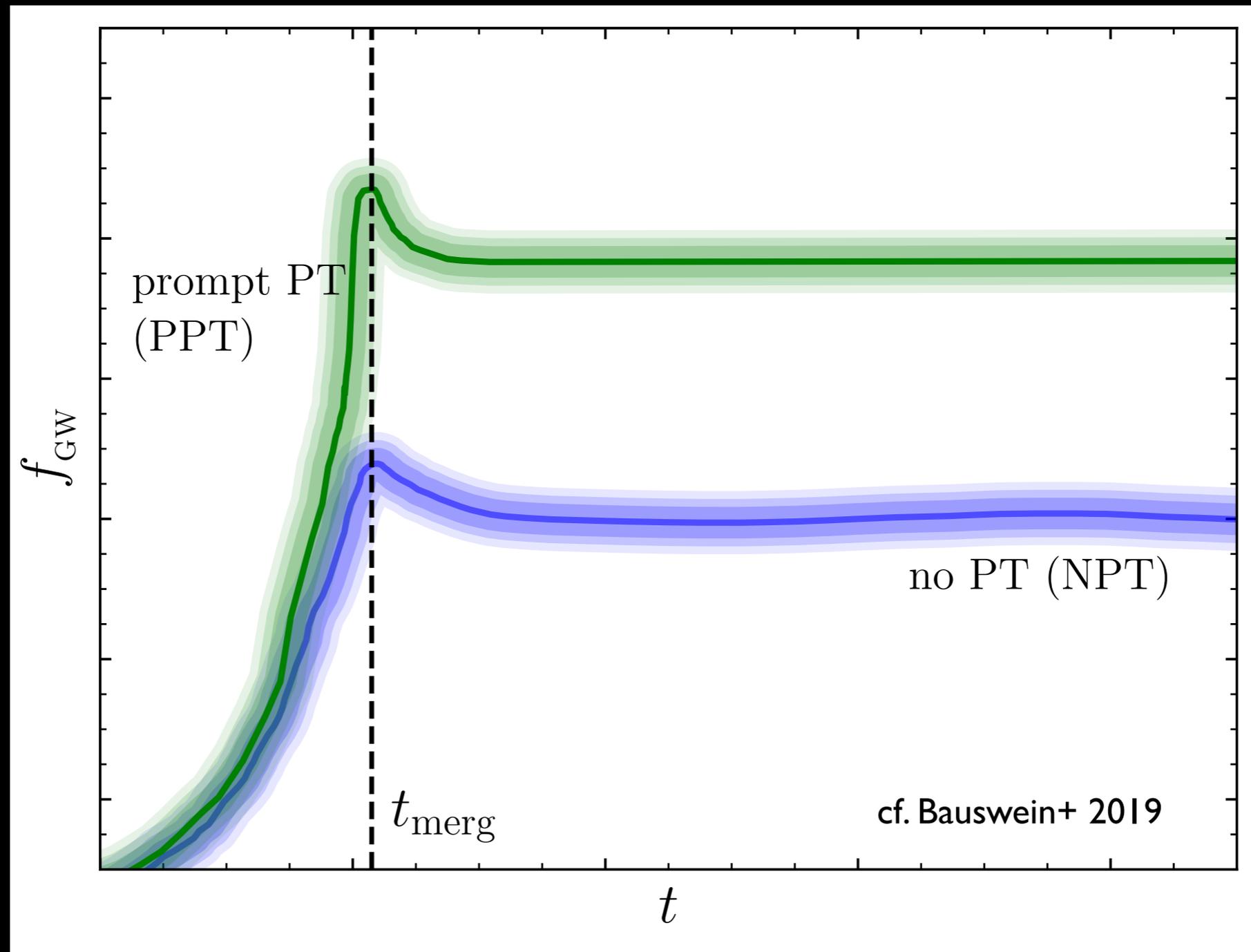
A more comprehensive picture

We have recently added another possible scenario for a post-merger **PT**, which completes the picture of possible scenarios (Weih, Hanauske, LR 2020).



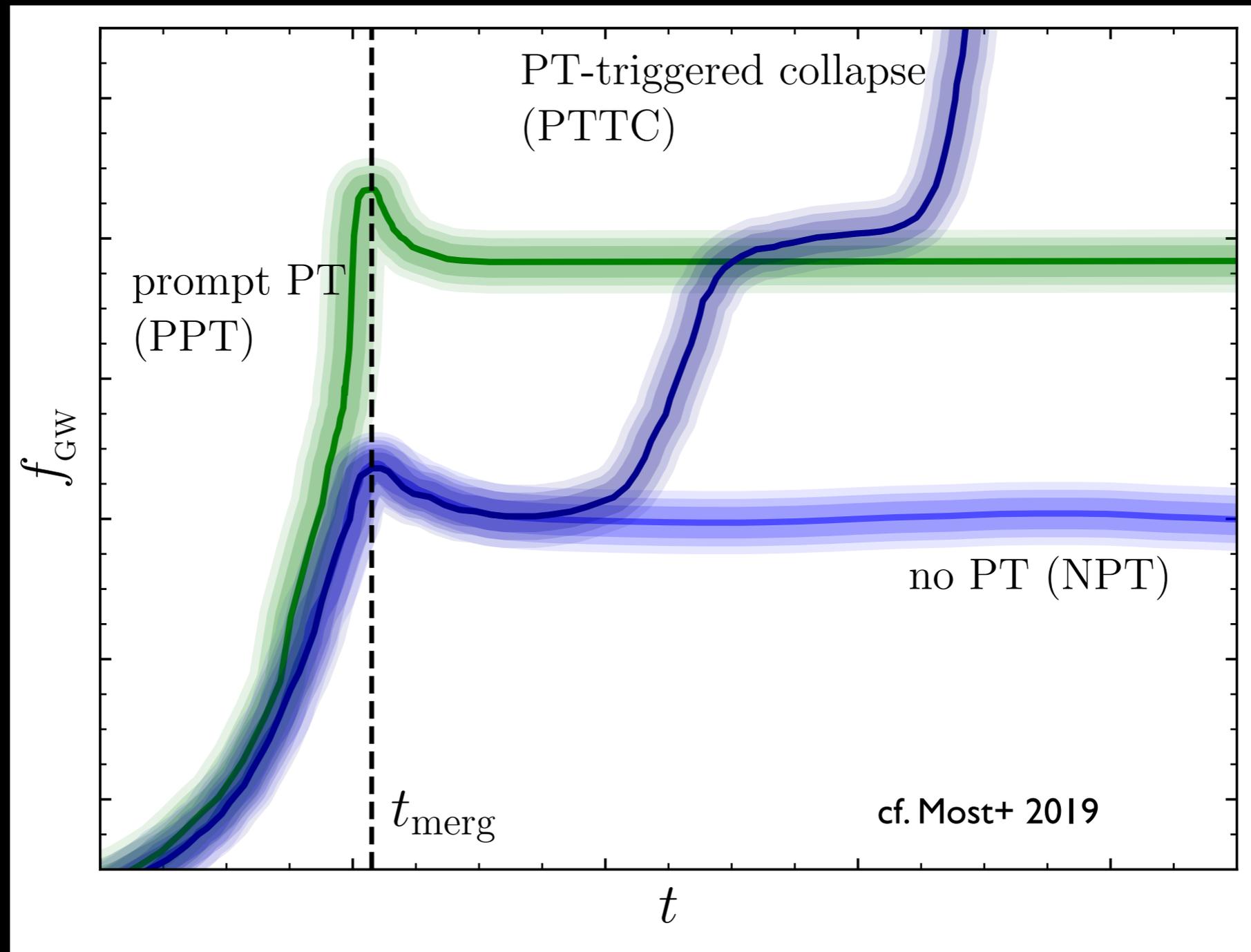
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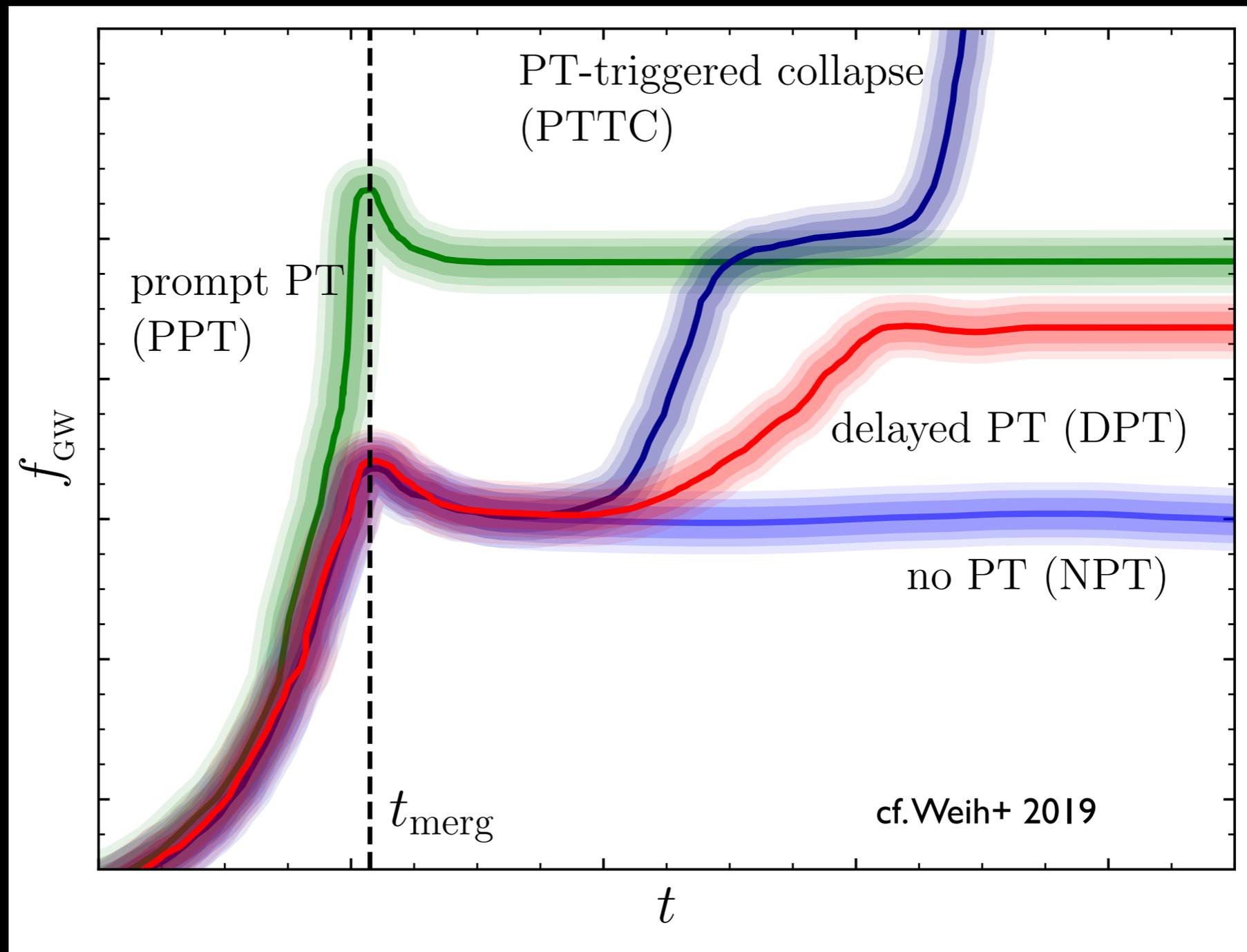
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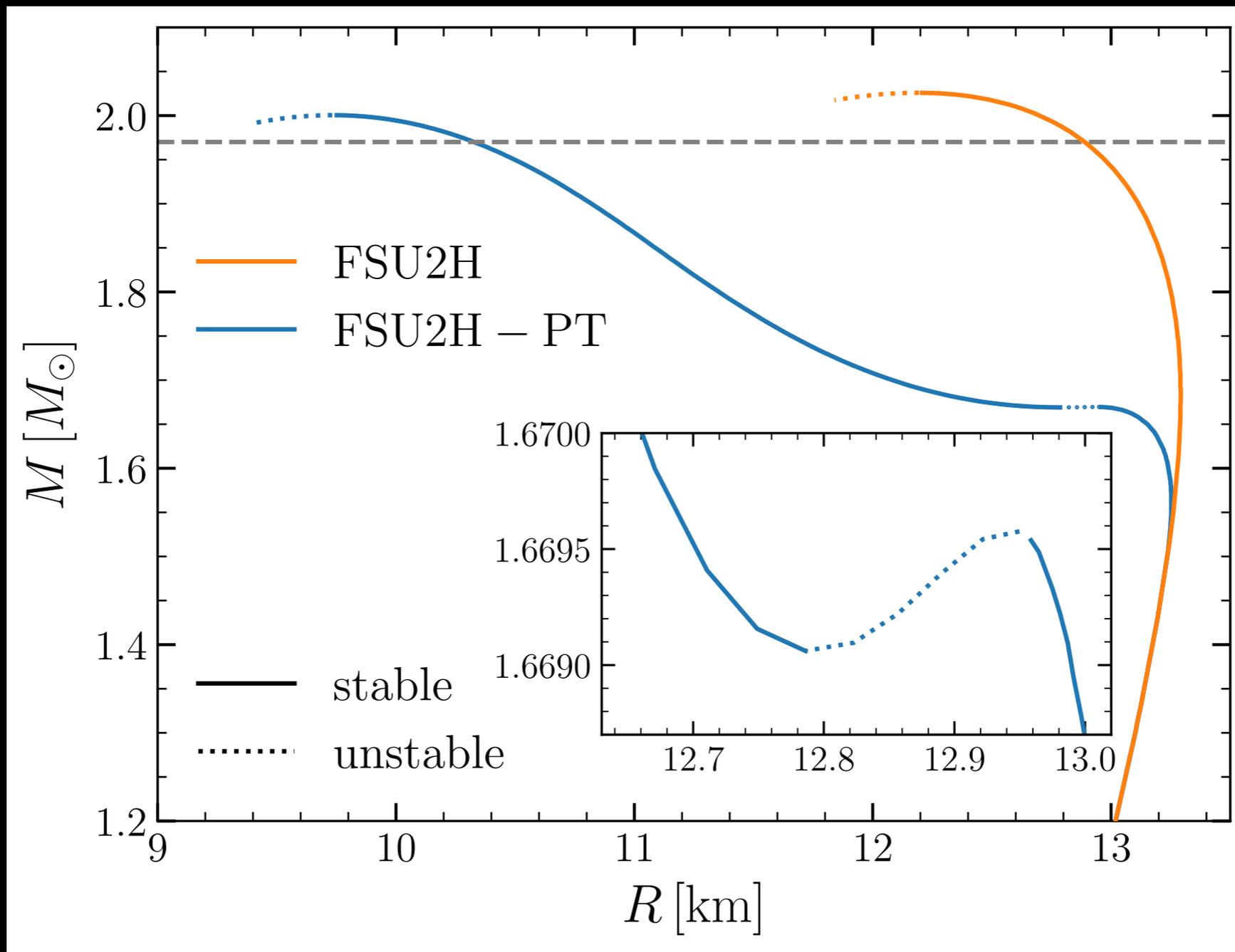
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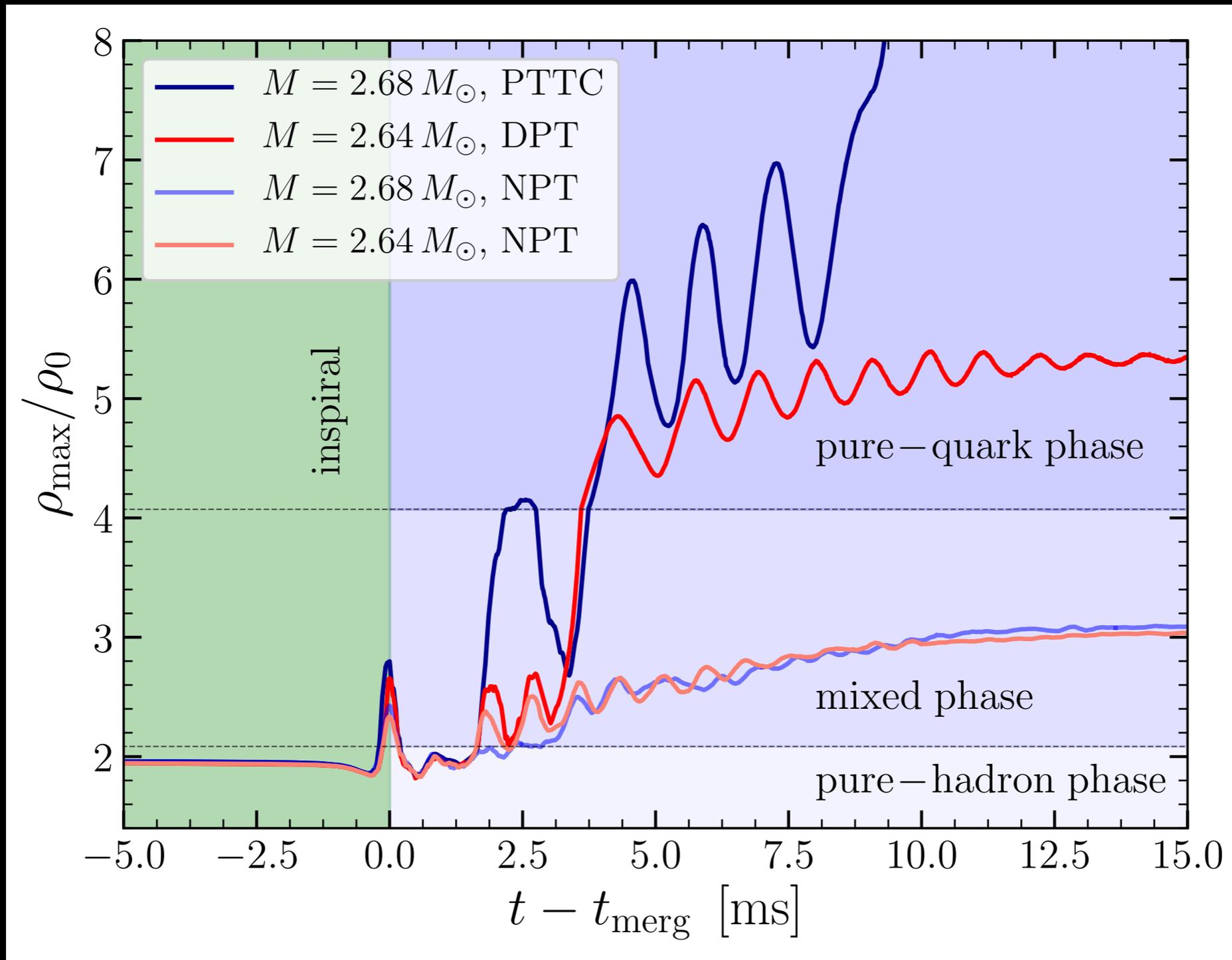
A more comprehensive picture

Characteristic properties of twin-stars: note the presence of a second stable branch of equilibrium configurations



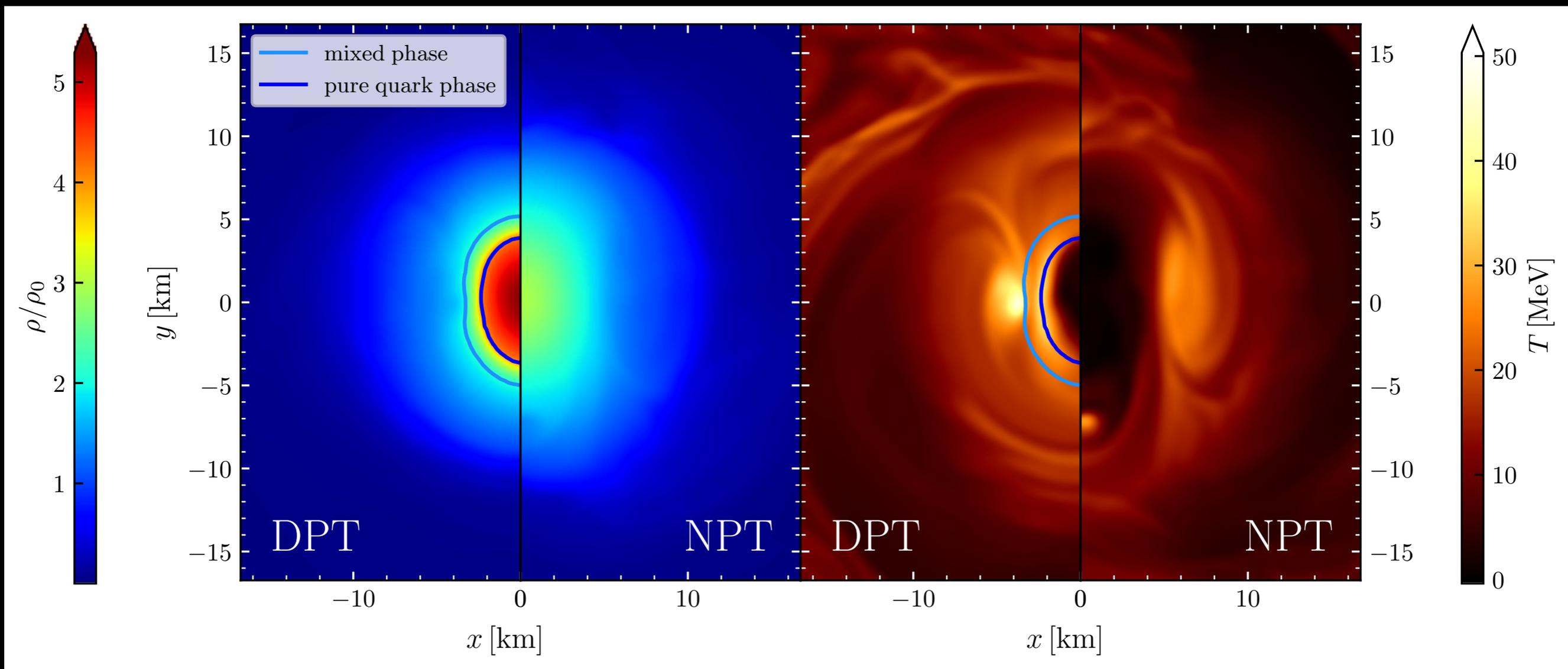
A more comprehensive picture

Best understood in terms of the evolution of the normalise maximum rest-mass density: ρ_{\max}/ρ_0



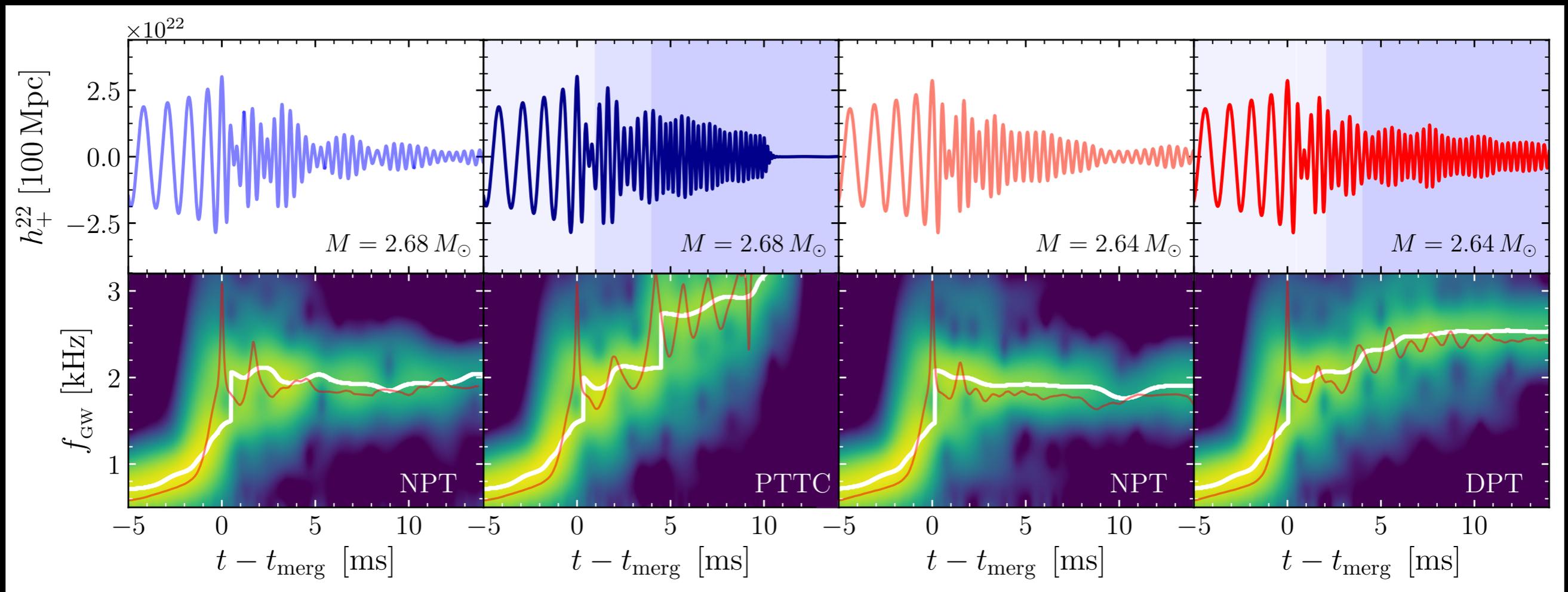
A more comprehensive picture

Comparison of density and temperature distributions on the equatorial plane for binaries with and without a DPT. Note the hot ring in the mixed phase present in the case of DPT



A more comprehensive picture

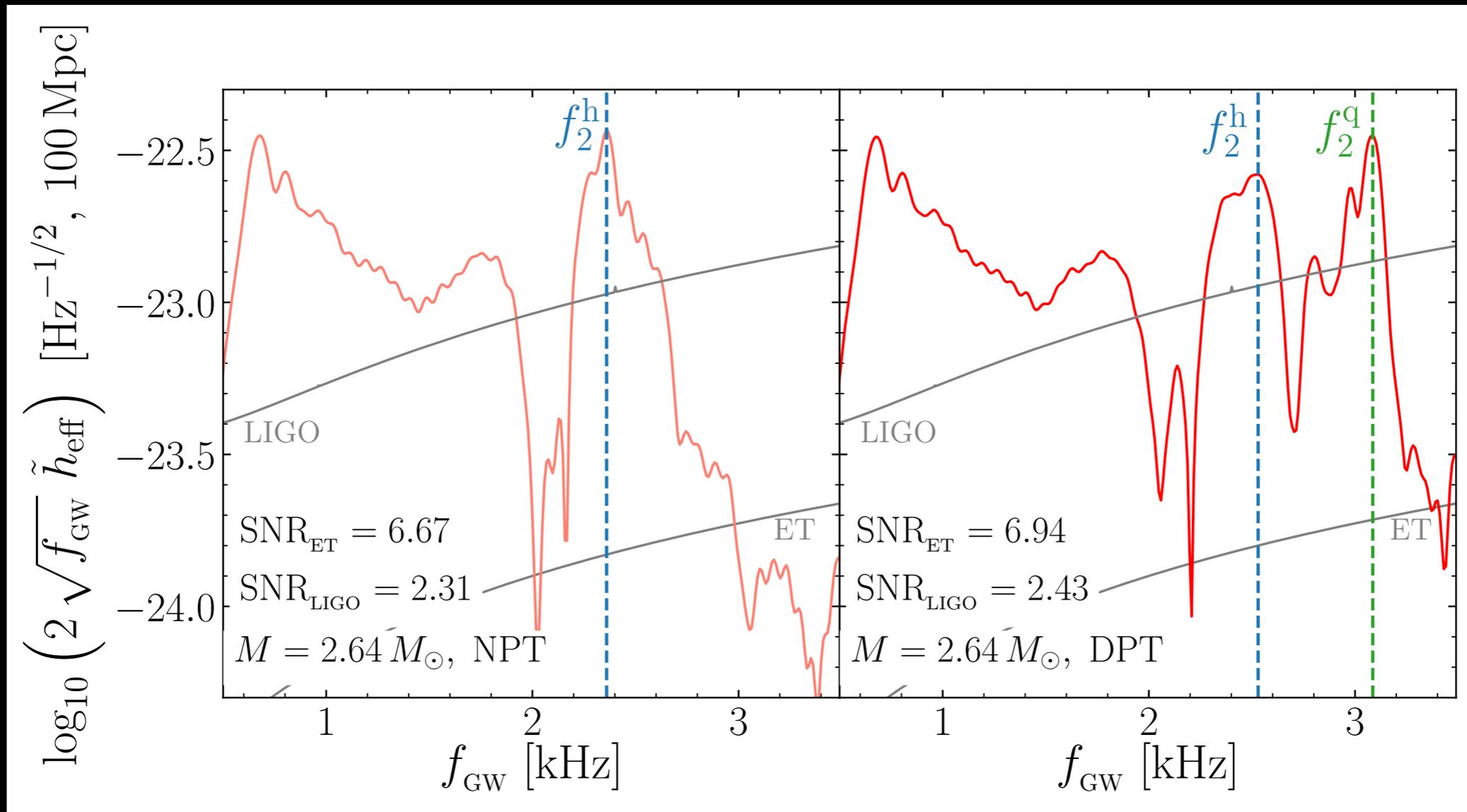
Different signatures are also quite transparent when shown in terms of the gravitational waves and their spectrograms.



Importance of **DPT** is that it leads to **two** different “stable” f_2 **frequencies** that are easily distinguishable in the PSD

A more comprehensive picture

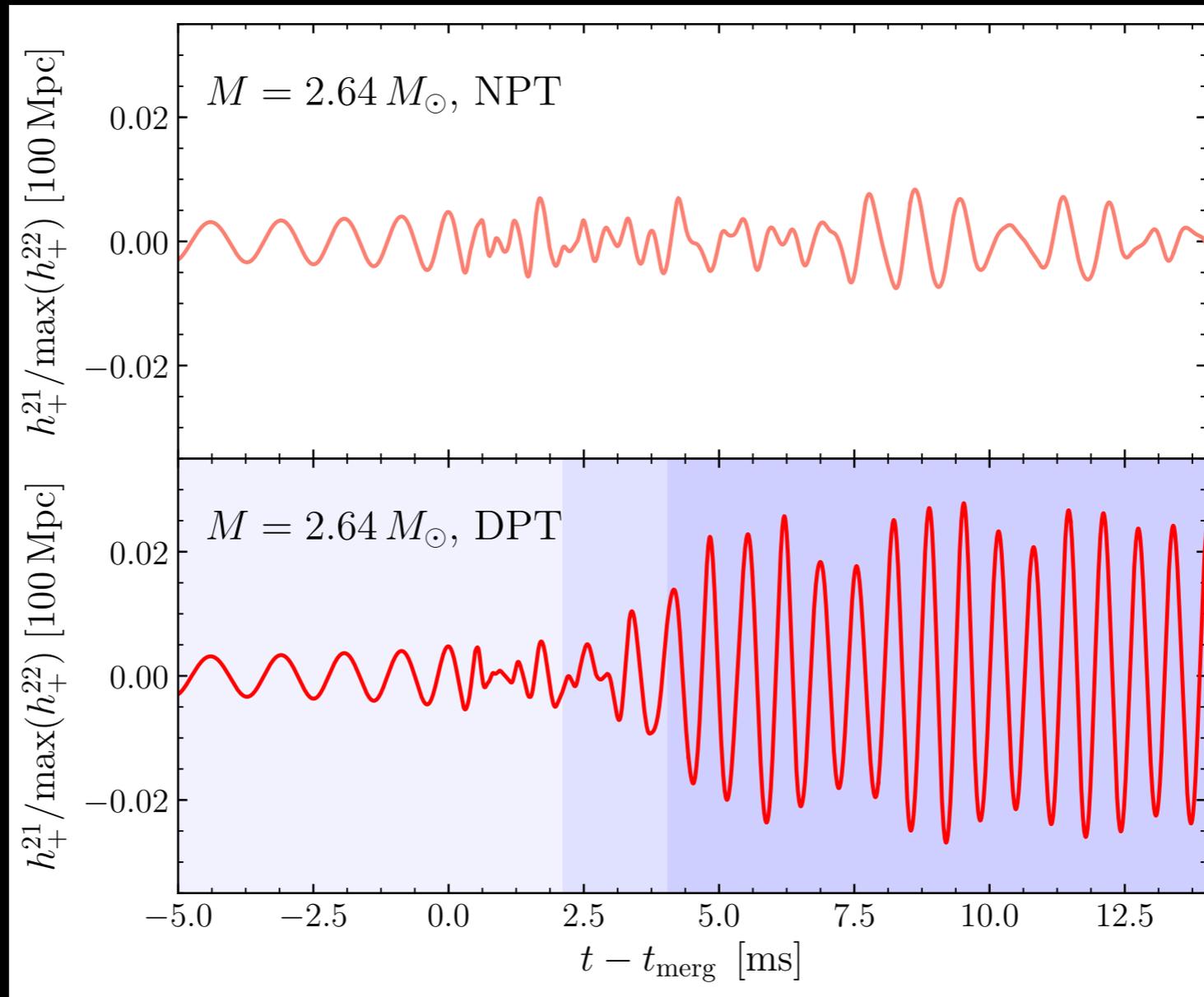
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Importance of **DPT** is that it leads to **two** different “stable” f_2 **frequencies** that are easily distinguishable in the PSD

A more comprehensive picture

Another signature is appearance of an $\ell = 2, m = 1$ mode



The mode is triggered by the PT and the non-axisymmetric deformations it produces.

Conclusions

- * Spectra of post-merger shows peaks, some **"quasi-universal"**.
- * When used together with tens of observations, they will set tight constraints on EOS: radius known with **~1 km** precision.
- * Merging binaries with magnetic fields can lead to the formation of **jet structures** and match phenomenology of SGRBs.
- * **GW170817** has already provided new limits on
 - $2.01^{+0.04}_{-0.04} \leq M_{\text{TOV}}/M_{\odot} \leq 2.16^{+0.17}_{-0.15}$ **maximum mass**
 - $12.00 < R_{1.4}/\text{km} < 13.45$ $\tilde{\Lambda}_{1.4} > 375$ **radius, tidal deformability**
 - $M_{\text{th}}/M_{\text{TOV}} \approx 1.41$ $R_{\text{TOV}} \geq 9.74^{+0.14}_{-0.04} \text{ km}$ **threshold mass**
- * A phase transition after a BNS merger leaves GW **signatures** and opens a gate to access quark matter beyond accelerators

Recap

- ✓ Spectra of post-merger shows clear “**quasi-universal**” peaks
- ✓ GW spectroscopy possible with post-merger signal
- ✓ Unless binary very close, peaks have **SNR ~ 1** . Multiple signals can be stacked and **SNR will increase coherently**.
- ✓ Only inspiral detected in GW170817 but new limits set on:

Maximum mass

$$2.01_{-0.04}^{+0.04} \leq M_{\text{TOV}}/M_{\odot} \lesssim 2.16_{-0.15}^{+0.17}$$

Typical radii and tidal deformabilities

$$12.00 < R_{1.4}/\text{km} < 13.45 \quad \tilde{\Lambda}_{1.4} > 375 \quad \text{hadronic EOSs}$$

$$8.53 < R_{1.4}/\text{km} < 13.74 \quad \tilde{\Lambda}_{1.4} \gtrsim 35 \quad \tilde{\Lambda}_{1.7} \lesssim 460 \quad \text{phase transitions}$$

- ✓ Phase transition can take place after merger leading to clear signatures: mismatch between inspiral and postmerger.