Equation of state for nuclear matter with correlations and clustering

I Quantum statistical approach to interacting fermion systems

II Nuclear systems

III Applications

nucleons, neutrons, protons

single particle states $|1\rangle = |\vec{p}_1, \sigma_1, \tau_1\rangle = |\hbar\vec{k}_1, \text{spin}, \text{isospin}\rangle$ (q k, p,n)

creation/destruction operators a_1^+, a_1 occupation nr. $n_1 = a_1^+ a_1$

$\{a_1, a_{1'}^+\}_+ = \delta_{11'}, \ldots$

Thermodynamic equilibrium: statistical operator ρ

averages $\langle A \rangle = \mathrm{Tr}\{\rho A\}$

grand canonical ensemble $\rho(\beta,\mu) = \dfrac{e^{-\beta(H-\mu N)}}{\mathrm{Tr}\, e^{-\beta(H-\mu N)}}$

Conserved quantities H, N (different components τ_1: $\sum_{\tau_1} \mu_{\tau_1} N_{\tau_1}$)

maximum of entropy $S = -k_B \mathrm{Tr}\{\rho \ln \rho\}$ at given $\langle H \rangle, \langle N \rangle$

volume V

Lagrange parameters $\beta = \frac{1}{k_B T}$, μ

$\frac{1}{V} \mathrm{Tr}\{\rho H\} = \frac{\langle H \rangle}{V} = u(\beta,\mu)$, $\frac{1}{V} \mathrm{Tr}\{\rho N\} = \frac{\langle N \rangle}{V} = n(\beta,\mu)$

caloric eos: internal energy   density eos $\to \mu(T,n)$

thermodynamic potential

$J(T,V,\mu) = -p(T,\mu)V = -\int_{-\infty}^{\mu} n(T,\mu')\,d\mu' \cdot V$

free energy density

$$f(T,n) = \frac{F(T,N,V)}{V} = f(T,n_0) + \int_{n_0}^{n} \mu(T,n')\,dn'$$

$$p(T,n) = n\,\mu(T,n) - f(T,n)$$

entropy, ...

thermodynamic stability : $\left.\frac{\partial \mu(nT)}{\partial n}\right|_T \geq 0$

Solve $n(\beta,\mu) = \frac{1}{V} \mathrm{Tr}\{\varrho N\} = \frac{1}{V} \sum_\lambda \langle a_\lambda^+ a_\lambda \rangle$

$E_\lambda = \frac{p_\lambda^2}{2m}$ $\quad H^0 = \sum_\lambda E_\lambda a_\lambda^+ a_\lambda \qquad H = H^0 + V$

ideal Fermi gas: $\cancel{V}$ $\qquad n^0(\beta,\mu) = \int \frac{d^3k}{(2\pi)^3} \frac{1}{e^{(E_\lambda-\mu)/T}+1}$

Fermi distribution

interaction:

numerical simulations

path integrals

Green's function method (perturbative, partial summations)

Spectral density:

$$A(\tau) = e^{\tau(H-\mu N)} A e^{-\tau(H-\mu N)}$$

$$(H-\mu N)|n\rangle = \varepsilon_n |n\rangle \qquad \tau \to \frac{i}{\hbar} t$$

$$\langle a_{1'}^{+} a_1(\tau)\rangle = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} I(1\,1',\omega) e^{-\omega\tau}$$

$$I(1\,1',\omega) = \frac{2\pi}{Z} \sum_{nm} e^{-\beta\varepsilon_n} \langle n| a_{1'}^{+} |m\rangle \langle m| a_1 |n\rangle \delta(\varepsilon_n - \varepsilon_m - \omega)$$

$$Z = \sum_n e^{-\beta\varepsilon_n}$$

thermodynamic Green's function

$$G(1\tau_1, 1'\tau_1') = -\mathrm{Tr}\{\rho T[a_1(\tau_1)\, a^\dagger_{1'}(\tau_1')]\}$$

$$T[\ldots] = \begin{cases} a_1(\tau_1)\, a^\dagger_{1'}(\tau_1') & \text{for } \tau_1' < \tau_1 \\ -a^\dagger_{1'}(\tau_1')\, a_1(\tau_1) & \text{for } \tau_1 < \tau_1' \end{cases}$$

$$= G(11', \tau_1 - \tau_1')$$

Kubo-Martin-Schwinger condition (KMS)

$$G(11', \beta - \tau) = -G(11', -\tau)$$: quasi-periodicity

$$G(11', \tau) = \frac{1}{\beta} \sum_\nu G(11', iz_\nu)\, e^{-iz_\nu \tau}$$

$z_\nu = \frac{\pi \nu}{\beta}$, $\nu = \pm 1, \pm 3, \ldots$

Matsubara frequencies

Fermi

Bose : $\nu = 0, \pm 2, \ldots$

$$G(11', iz_\nu) = \int_{-\infty}^{\infty} \frac{d\omega'}{2\pi} (1+e^{\beta\omega'}) \frac{I(11',\omega')}{iz_\nu - \omega'}$$

Spectral function $A(11',\omega') = (1+e^{\beta\omega'}) I(11',\omega')$

$$\int \frac{d\omega'}{2\pi} \frac{A(11',\omega)}{iz_\nu - \omega'} \qquad G(11',z) = \int \frac{d\omega'}{2\pi} \frac{A(11',\omega')}{z-\omega'}$$

Cauchy typ

$$\frac{1}{\omega \pm i\epsilon} = \mp i\pi\delta(\omega) + \frac{\mathcal{P}}{\omega}$$

$$G(11',\omega - i\epsilon) - G(11',\omega + i\epsilon) = iA(11',\omega)$$

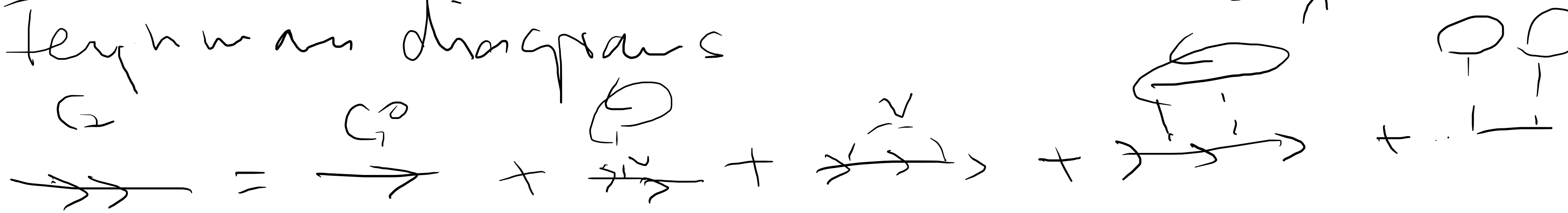
$$n(\beta,\mu) = \sum_1 \int \frac{d\omega}{2\pi} f(\omega) A(11,\omega) \qquad f(\omega) = \frac{1}{e^{\beta\omega}+1}$$

calculate $G(11', iz_\nu)$

ideal case $G^0(11', iz_\nu) = \frac{\delta_{11'}}{iz_\nu - \varepsilon_1}$

$A^0 = 2\pi\delta(\omega - \varepsilon_1)$ $\quad \varepsilon_1 = \frac{p_1^2}{2m_1} - \mu_1$

Feynman diagrams



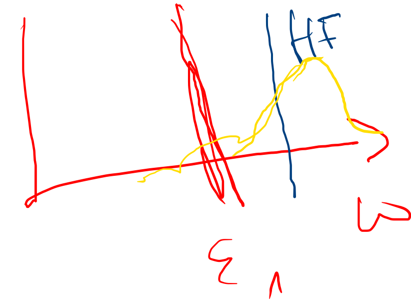
$$G(11', iz_\nu) = \frac{\delta_{11'}}{iz_\nu - \varepsilon_1 - \Sigma(1, iz_\nu)}$$ Dyson

Σ : self-energy :

Hartree Fock

$$\Sigma^{HF}(1, iz_\nu) = \sum_2 V(12, 12)|_{ex} f(\varepsilon_2)$$

$$A(1,\omega) = 2\,\mathrm{Im}\,\frac{1}{\omega - \varepsilon_1 - \Sigma(1,\omega - i0)}$$



$$= \frac{2\,\mathrm{Im}\,\Sigma(1,\omega - i0)}{[\omega - \varepsilon_1 - \mathrm{Re}\,\Sigma(1,\omega)]^2 + [\mathrm{Im}\,\Sigma(1,\omega - i0)]^2}$$

$\mathrm{Im}\,\Sigma$ is assumed to be small

$$= \frac{2\pi\,\delta(\omega - E_1^{quasi})}{1 - \frac{d}{dz}\mathrm{Re}\,\Sigma(1,z)\big|_{z = E_1^{quasi} - \mu}} - 2\,\mathrm{Im}\,\Sigma(1,\omega - i0)\,\frac{d}{d\omega}\,\mathcal{P}\frac{1}{\omega - E_1^{q} + \mu}$$

correlations
bound states

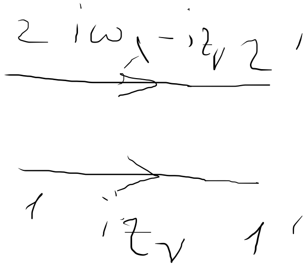
$$E_1^{quasi} = \frac{p_1^2}{2m_1} + \mathrm{Re}\,\Sigma(1,z)\big|_{z = E_1^{quasi} - \mu}$$

Two-particle correlations

different 1, 2

free two-particle propagator

$$\varepsilon_1 = \frac{p_1^2}{2m_1} - \mu_1$$

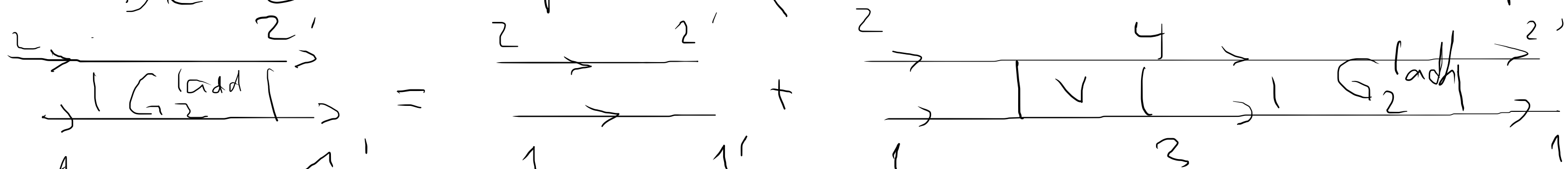


$i\omega_\lambda$ bosonic

$\sum_\nu$

$$G_2^0(12,1'2',i\omega_\lambda) = \frac{1-f(\varepsilon_1)-f(\varepsilon_2)}{i\omega_\lambda-\varepsilon_1-\varepsilon_2}\,\delta_{11'}\delta_{22'}$$

Bethe-Salpeter equation: ladder diagrams



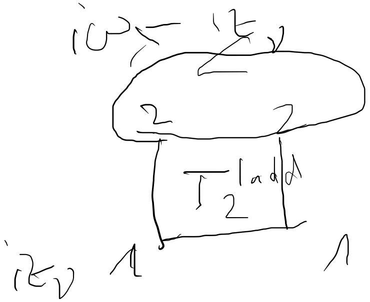
$$G_2^{ladder}(12,1'2',i\omega_\lambda) = \frac{1-f(\varepsilon_1)-f(\varepsilon_2)}{i\omega_\lambda-\varepsilon_1-\varepsilon_2}\left[\delta_{11'}\delta_{22'} + \sum_{34} V(12,34)\,G_2^{ladd}(34,1'2',i\omega_\lambda)\right]$$

solution

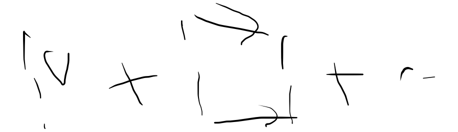
$$G_2^{ladder}(12,1'2',i\omega_\lambda) = \sum_{nP}\psi_{nP}(12)\,\frac{1}{i\omega_\lambda - E_{nP}+\mu_1+\mu_2}\,\psi^*_{nP}(1'2')$$

$$(E_1+E_2-E_{nP})\,\psi_{nP}(12) + (1-f(\varepsilon_1)-f(\varepsilon_2))\sum_{1'2'}V(12,1'2')\,\psi_{nP}(1'2') = 0$$

$\Sigma(1, i\varepsilon_\nu)$:



$\boxed{T_2}^{\text{ladder}} =$



$+ \dots$

$$= \left(G_2^0\right)^{-1} G_2^{\text{ladder}} \left(G_2^0\right)^{-1}$$

$$\Sigma(1, i\varepsilon_\nu) = \sum_{2,\omega_\lambda, n\Omega} \psi_{n\Omega}(12) \frac{1}{i\omega_\lambda - E_{n\Omega} + \mu_1 + \mu_2} \psi^*_{n\Omega}(12) \left[i\omega_\lambda - \varepsilon_1 - \varepsilon_2\right]^2 \frac{1}{i\omega_\lambda - i\varepsilon_\nu - \varepsilon_2}$$

Bose, 2μ small $\quad f(\varepsilon_2)$

$i\omega_\lambda \to i\varepsilon_\nu + \varepsilon_2$

$i\varepsilon_\nu - E_{n\Omega} + \varepsilon_2 - \mu_1$ (Dirac)

$$f(\omega) \frac{\mathrm{Im}\, \Sigma(1, \omega - i0)}{(\omega - \varepsilon_1)^2}$$

δ-fkt $\quad \omega = E_{n\Omega} - \varepsilon_2 + \mu_1$

$$n(\beta, \mu) = \frac{1}{V} \sum_1 f(\varepsilon_1) + \frac{1}{V} \sum_{n\Omega} e^{-(E_{n\Omega} - \mu_1 - \mu_2)}$$

$$\underline{\vec{P}} = \vec{p}_1 + \vec{p}_2$$

$$\vec{p} = \frac{1}{2}(\vec{p}_2 - \vec{p}_1)$$

Separable interaction

$$V(12, 1'2') = -\frac{\lambda}{V} w(p) w(p') \delta_{\underline{P}\,\underline{P}'}$$

Yamaguchi: $$w(p) = \frac{1}{p^2/\gamma^2 + 1}$$

$$\frac{p^2}{m}\psi_n(p) + \sum_{p'} V(p,p')\psi_n(p') = E_n \psi_n(p)$$

$$-\lambda \int \frac{d^3p'}{(2\pi)^3} w(p)\, \overbrace{w(p')\, \psi_n(p')}^{C_n}$$

$$\psi_n(p) = \frac{\lambda w(p) C_n}{p^2/m - E_n}$$

$$\overset{1}{C_n} = \int \frac{d^3p'}{(2\pi)^3} \frac{w(p')\, \lambda\, w(p')\, \overset{1}{C_n}}{p'^2/m - E_n}$$

$$E = -\frac{\gamma^2}{m}\left(\sqrt{\frac{\lambda m \gamma}{2\pi}} - 1\right)^2$$

Scattering states

$$T = V + V G_2^0 T = w(p)\, w(p')\, t(P, z)$$

$$t(P, z) = -\frac{\lambda}{V}\left[1 + \frac{\lambda}{V}\sum_p w^2(p)\, \frac{1 - f(\vec{p}+\vec{P}/2) - f(\vec{p}-\vec{P}/2)}{z - \varepsilon_{p+P/2} - \varepsilon_{p-P/2}}\right]^{-1}$$

$\delta_0(\tilde{E})$:

$$\tan\delta_0(\tilde{E}) = \frac{\operatorname{Im} t(P, \tilde{E})}{\operatorname{Re} t(P, \tilde{E})} = \frac{2\sqrt{\tilde{E}}}{\frac{8\pi\gamma}{\lambda m_\gamma}(1+\tilde{E})^2 - (1-\tilde{E})}$$

$$\tilde{E} = \frac{m}{\gamma^2} E$$