Karpacz, June 15, 2021

Equation of state for nuclear matter with correlations and clustering

Gerd Röpke, Rostock

Part II: Nuclear systems. Outline

1. Properties of nuclei – empirical data
2. Finite temperatures – empirical data
3. Quantum statistical approach
4. In-medium effects: self-energy, Pauli blocking
5. Equation of state including correlations

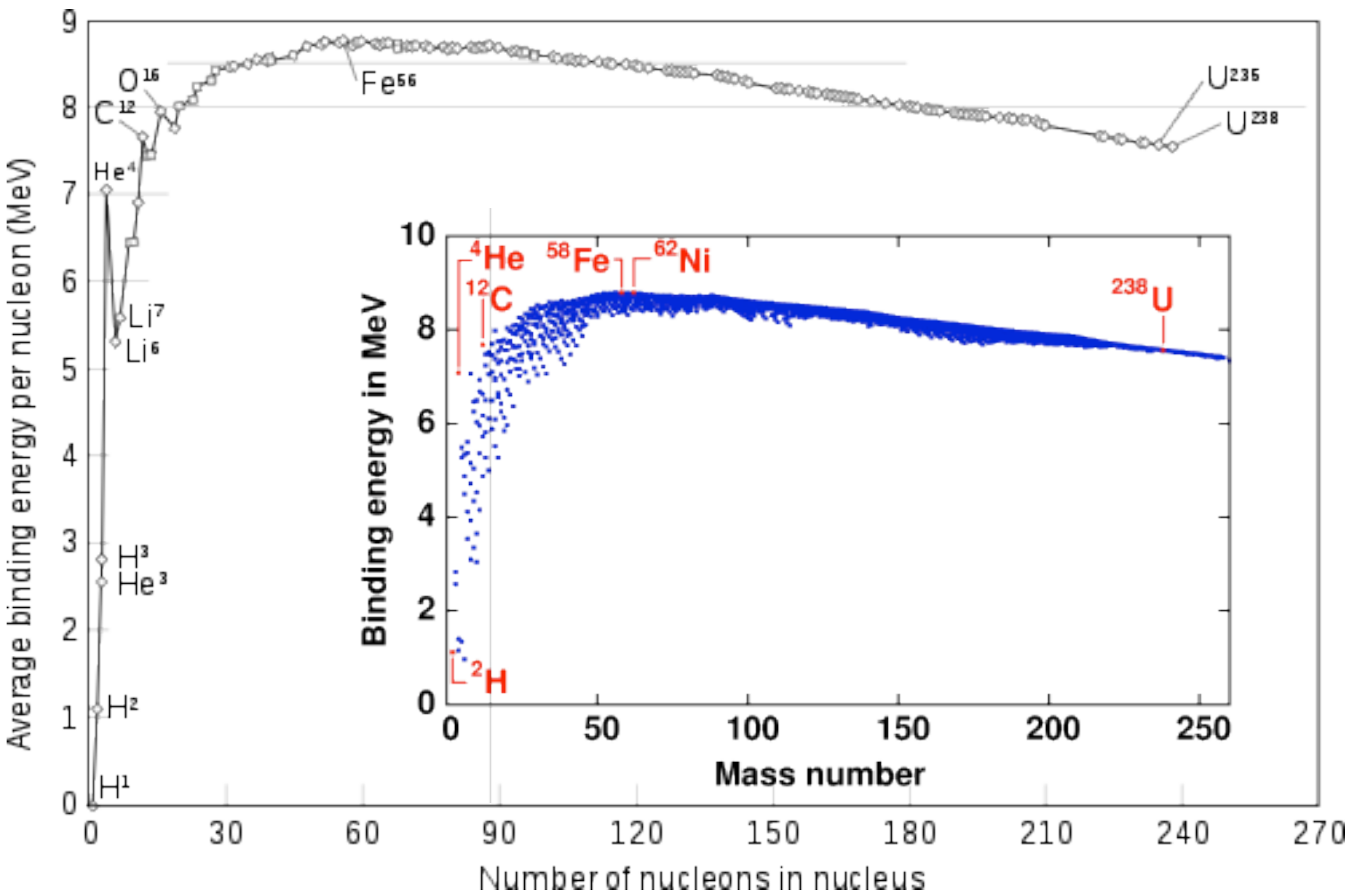
Problem: single (quasi-) particle approach
to describe the properties of nuclear systems
(mean-field approximation).
Are correlations of relevance? How to calculate?

1. Properties of nuclei, correlations

- Binding energy (mass)
- Radii of nuclei
- Shell model and correlations
- Excited states
- Stability and decay modes

G. Audi et al., Nucl. Phys. A **729**, 3 (2003)

Binding energy per nucleon



Semi-empirical mass formula

Liquid drop model: Bethe-Weizsaecker mass formula

$$B(A,Z) = a_V A - a_S A^{2/3} - a_C \frac{Z(Z-1)}{A^{1/3}} - a_A \frac{(A-2Z)^2}{A} + a_P \frac{1}{A^{1/2}}$$

bulk contribution: $a_V = 15.75$ MeV
surface contribution: $a_S = 17.8$ MeV
Coulomb repulsion: $a_C = 0.711$ MeV
asymmetry term: $a_A = 23.7$ MeV
pairing: $a_P = 11.18$ MeV (even-even),
$= 0$ (even-odd), $= -11.18$ MeV (odd-odd)
shell structure and magic numbers

proton fraction $$Y_p = \frac{Z}{A} = \frac{Z}{N+Z}, \quad \frac{N}{Z} = 1 + \frac{a_C}{2a_A} A^{2/3}$$

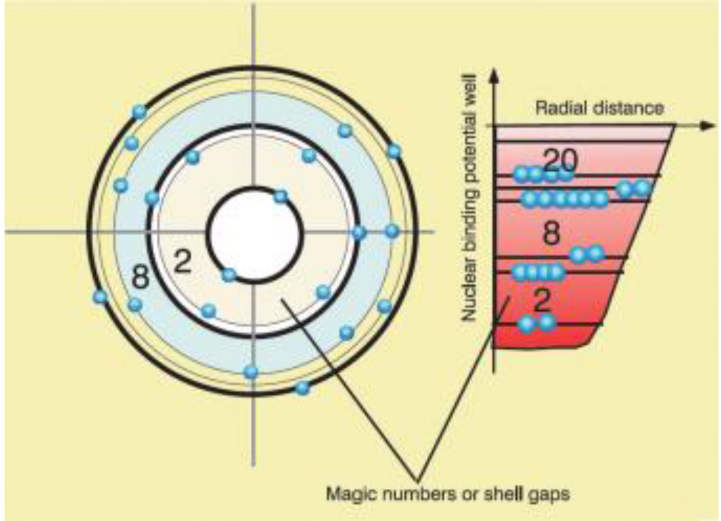
Models of nuclei

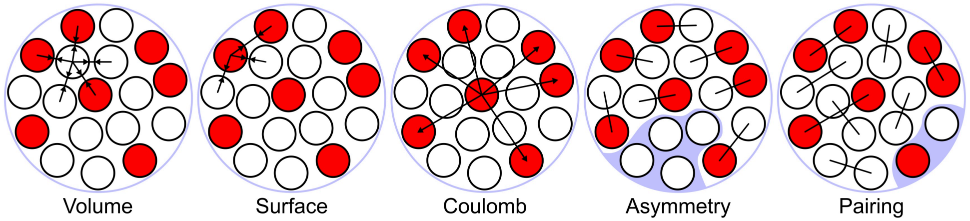
Constituents:
protons, neutrons

Shell model of nucleus:
potential well

Droplet model:
Bethe-Weizsäcker-Formel

C. F. von Weizsäcker:
Zur Theorie der Kernmassen.
In: *Zeitschrift für Physik.* **96** (1935), S. 431–458.





magic numbers:
2; 8; 20; 28; 50; 82; 126

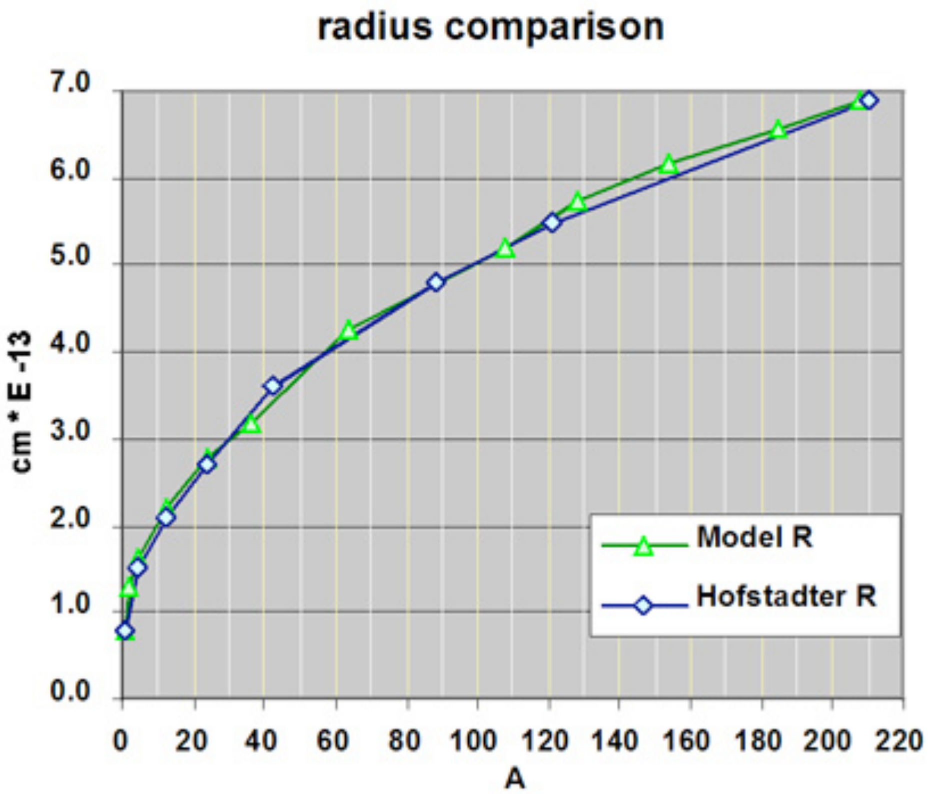
Hans Jensen, Maria Goeppert-Mayer

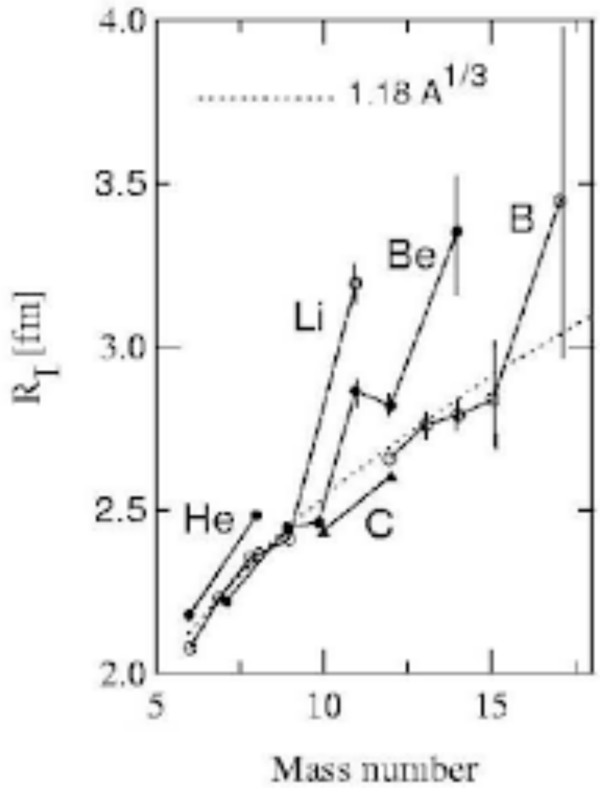
O. Haxel, J.H.D. Jensen, H. E. Suess
Zur Interpretation der ausgezeichneten Nukleonenzahlen im Bau der Atomkerns,
Die Naturwissenschaften, Band **35**, (1949) S.376

Nuclear radii

root mean square radius (charge or point): $$\mathrm{rms}^2 = \frac{\int_0^\infty dr\; r^4\, \rho(r)}{\int_0^\infty dr\; r^2\, \rho(r)}$$

mass – radius relation: R = 1.18 $A^{1/3}$ [fm] → n_B = 0.15 fm^{-3} = ρ_{sat}





I. Angeli, Atomic Data and Nuclear Data Tables **87**, (2004)

nucleon-nucleon interaction potential

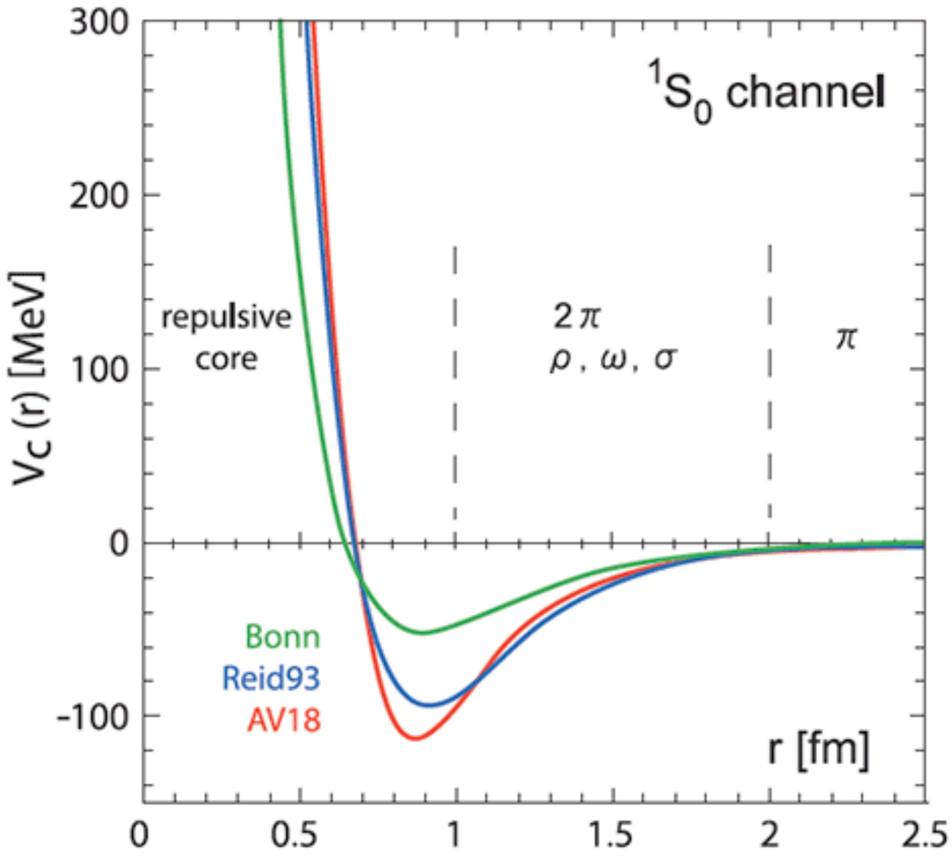
- Effective potentials
  (like atom-atom potential)
  binding energies, scattering

- non-local, energy-dependent?
  QCD?

- microscopic calculations
  (AMD, FMD)

- single-particle descriptions:
  Thomas-Fermi approximation
  shell model
  density functional theory (DFT)

- correlations, clustering
  low-density nα nuclei, Volkov



Separable interaction (Yamaguchi)

$$V^{\text{sep}}(p,p') = -\lambda/\Omega w(p) w(p')$$

Exact solution in closed form, including scattering states.
Theorem of Ernst, Shakin and Thaler: each potential can be represented as a sum of separable potentials.

- **general form:**

$$V_\alpha(p,p') = \sum_{i,j=1}^{N} w_{\alpha i}(p)\lambda_{\alpha ij} w_{\alpha j}(p') \qquad \text{uncoupled}$$

and

$$V_\alpha^{LL'}(p,p') = \sum_{i,j=1}^{N} w_{\alpha i}^{L}(p)\lambda_{\alpha ij} w_{\alpha j}^{L'}(p') \qquad \text{coupled}$$

PEST (Paris),
BEST (Bonn),
...

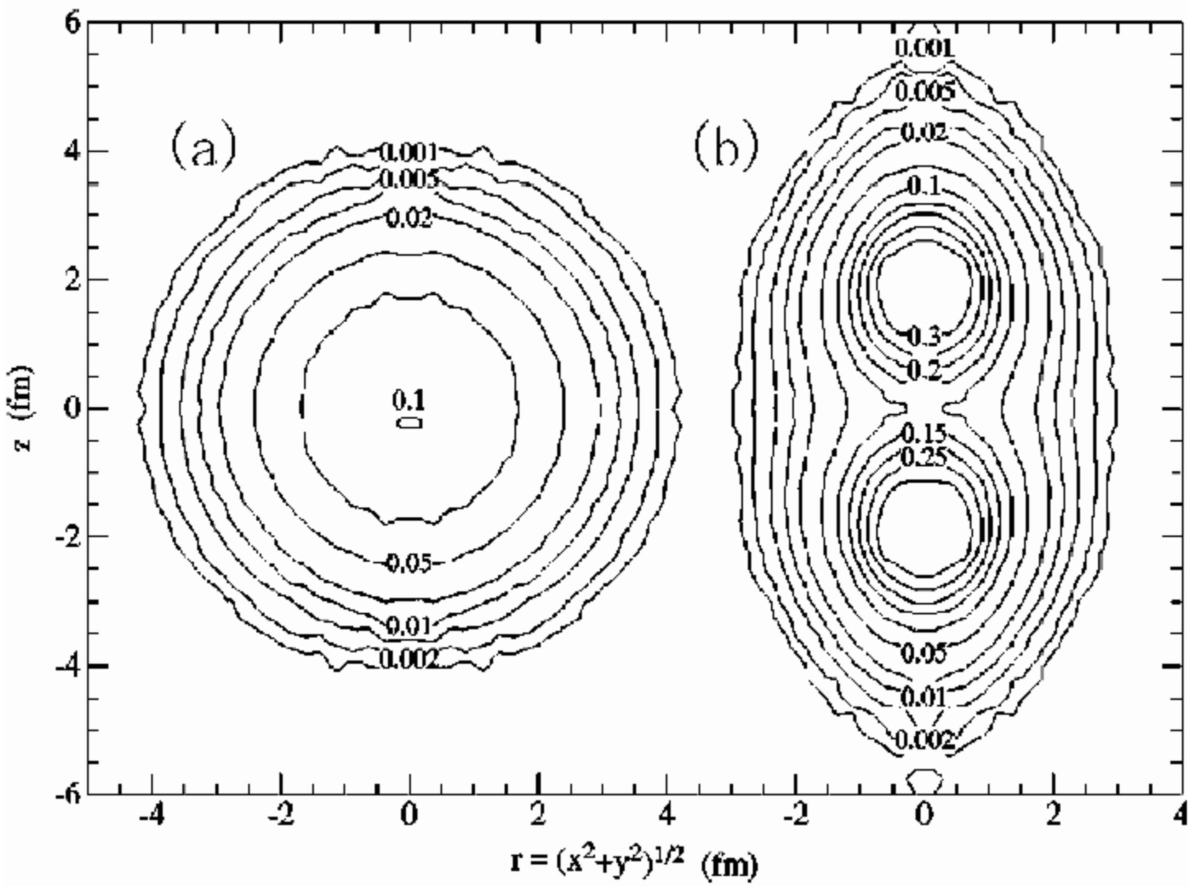
$p,\ p'$		in- and outgoing relative momentum
α	...	channel
N	...	rank
$\lambda_{\alpha ij}$	.	coupling parameter
$L,\ L'$		orbital angular momentum

D. J. Ernst, C. M. Shakin, R. M. Thaler, Phys. Rev. C **8**, 46 (1973).

Correlations in nuclei

- Liquid droplet (Bethe – Weizsaecker)
- Shell model (Jensen)
- Pairing (odd-even staggering) - quartetting
- Hoyle state in ^{12}C
- α – formation and α - decay

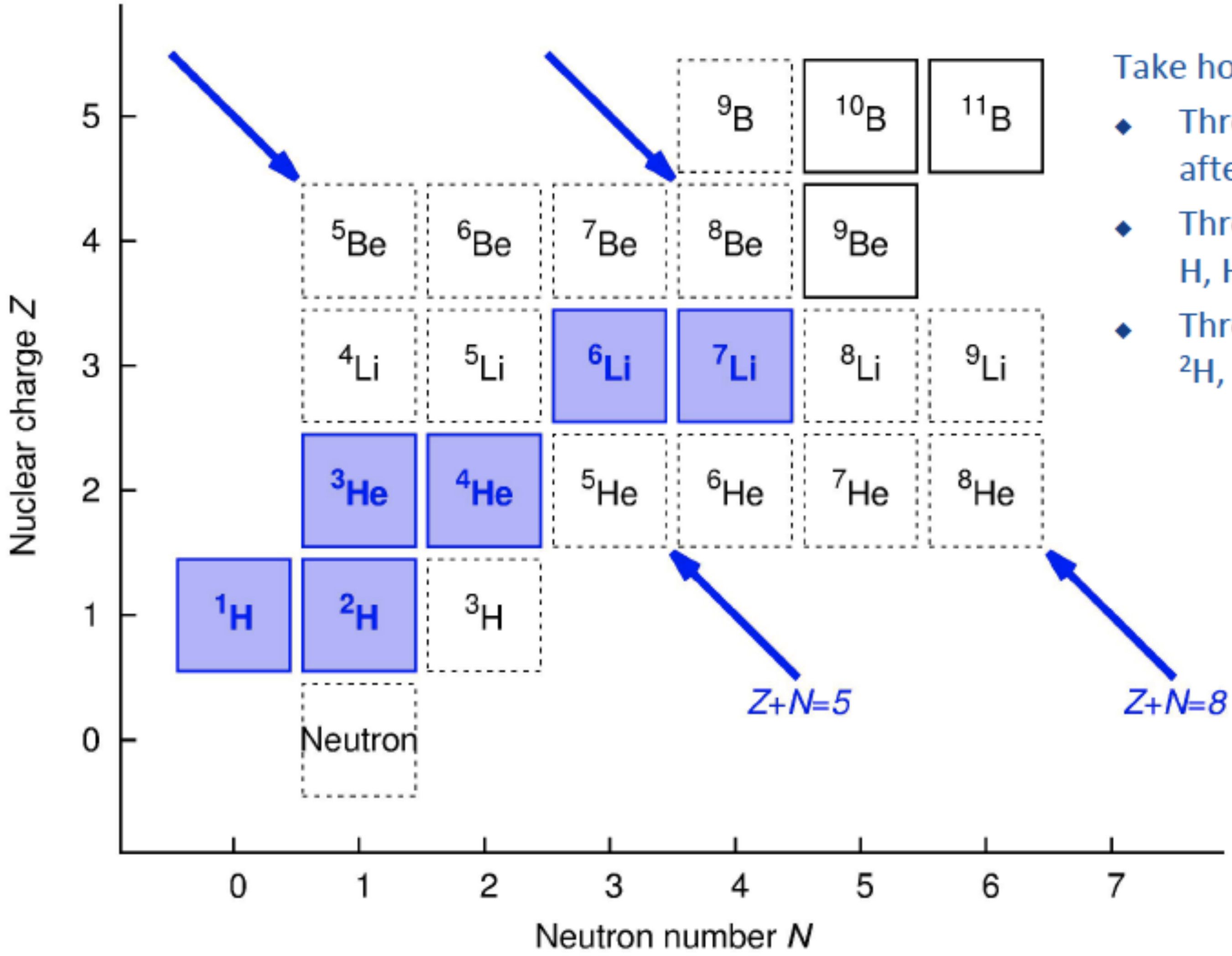
α cluster structure of ^{8}Be



R.B. Wiringa et al.,
PRC **63**, 034605 (01)

Contours of constant density, plotted in cylindrical coordinates, for ^{8}Be(0+) .
The left side is in the laboratory frame while the right side is in the intrinsic frame.

Big-Bang nucleosynthesis: H, He, Li, __________



Take home textbook knowledge

- Three minutes after the Big Bang
- Three chemical elements: H, He, Li
- Three observed abundances: ^{2}H, ^{4}He, ^{7}Li

Courtesy: D. Bemmerer, lecture at Karpacz-2019

The Hoyle state in ^{12}C

^{12}C: from astrophysics: excited state predicted near the 3 α threshold energy (F. Hoyle).

a 0^+ state at 0.39 MeV above the 3 α threshold energy has been found.

not described by shell structure calculations,
3 α cluster interact predominantly in relative S waves,
gas-like structure, THSR state

A. Tohsaki et al., PRL **87**, 192501 (2001)

α−particle condensation in low-density nuclear matter,
ρ below $\rho_{sat}/5$

nα nuclei: ^{8}Be, ^{12}C, ^{16}O, ^{20}Ne, ^{24}Mg, …
cluster type structures near the n α breakup threshold energy

Excited light nuclei

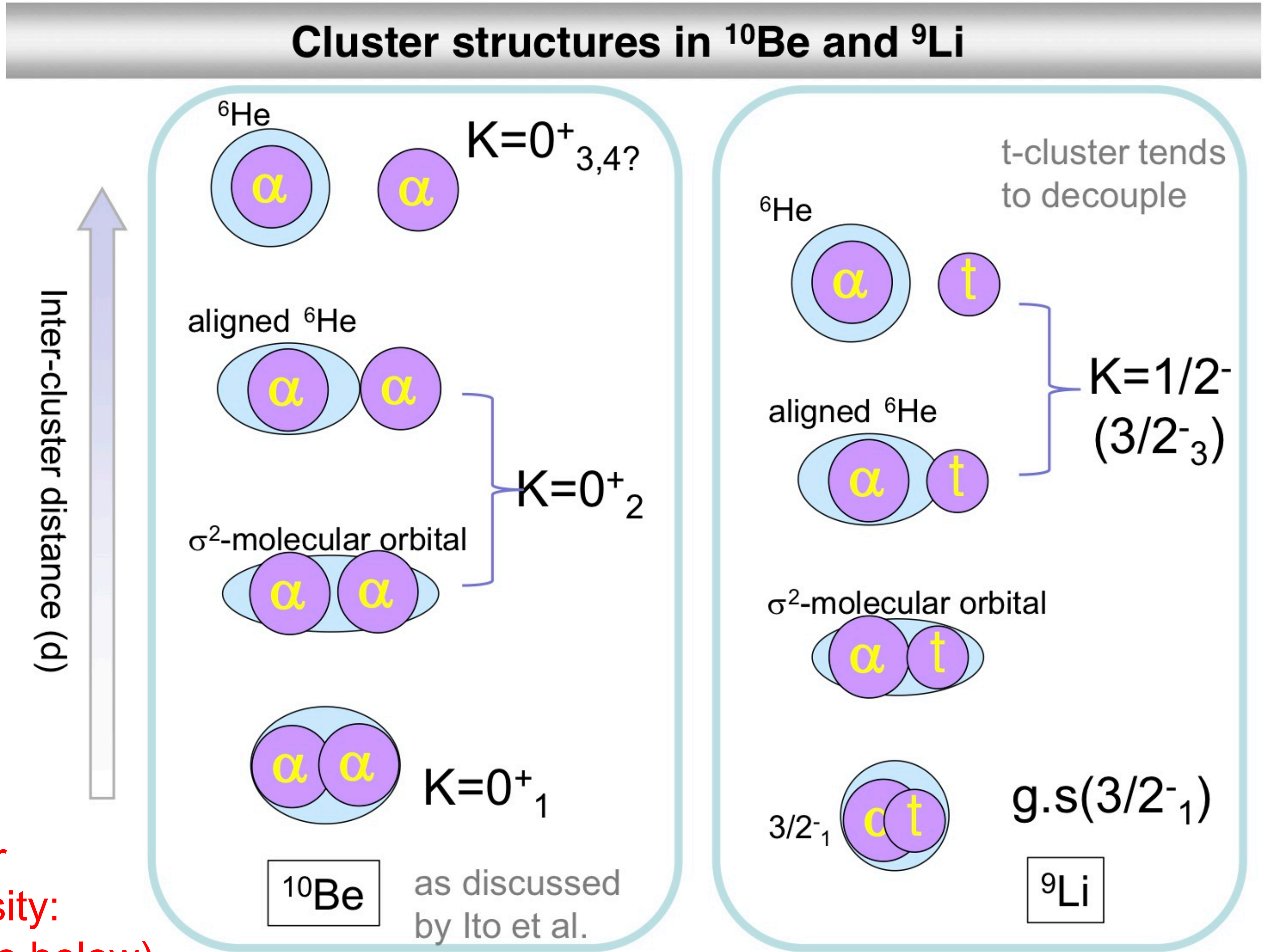
Yoshiko Kanada-En'yo
Cluster2012,Debrecen

decreasing
density

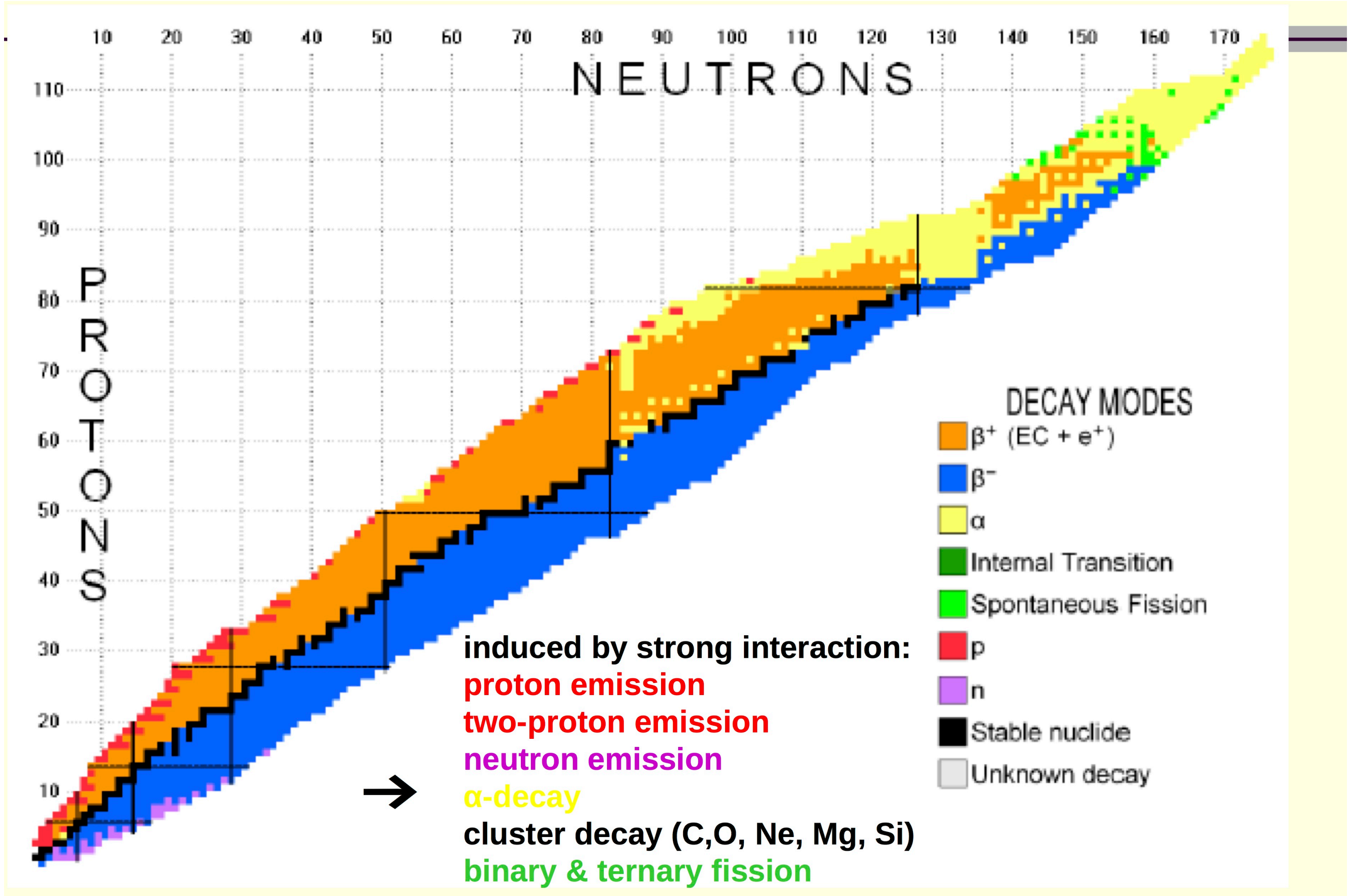
systematics in
weakly bound
light elements

clustering at
low densities

clusters disappear
at increasing density:
Pauli blocking (see below)



Decay modes of nuclei



Half-lives of nuclei

radioactive decay of instable isotopes

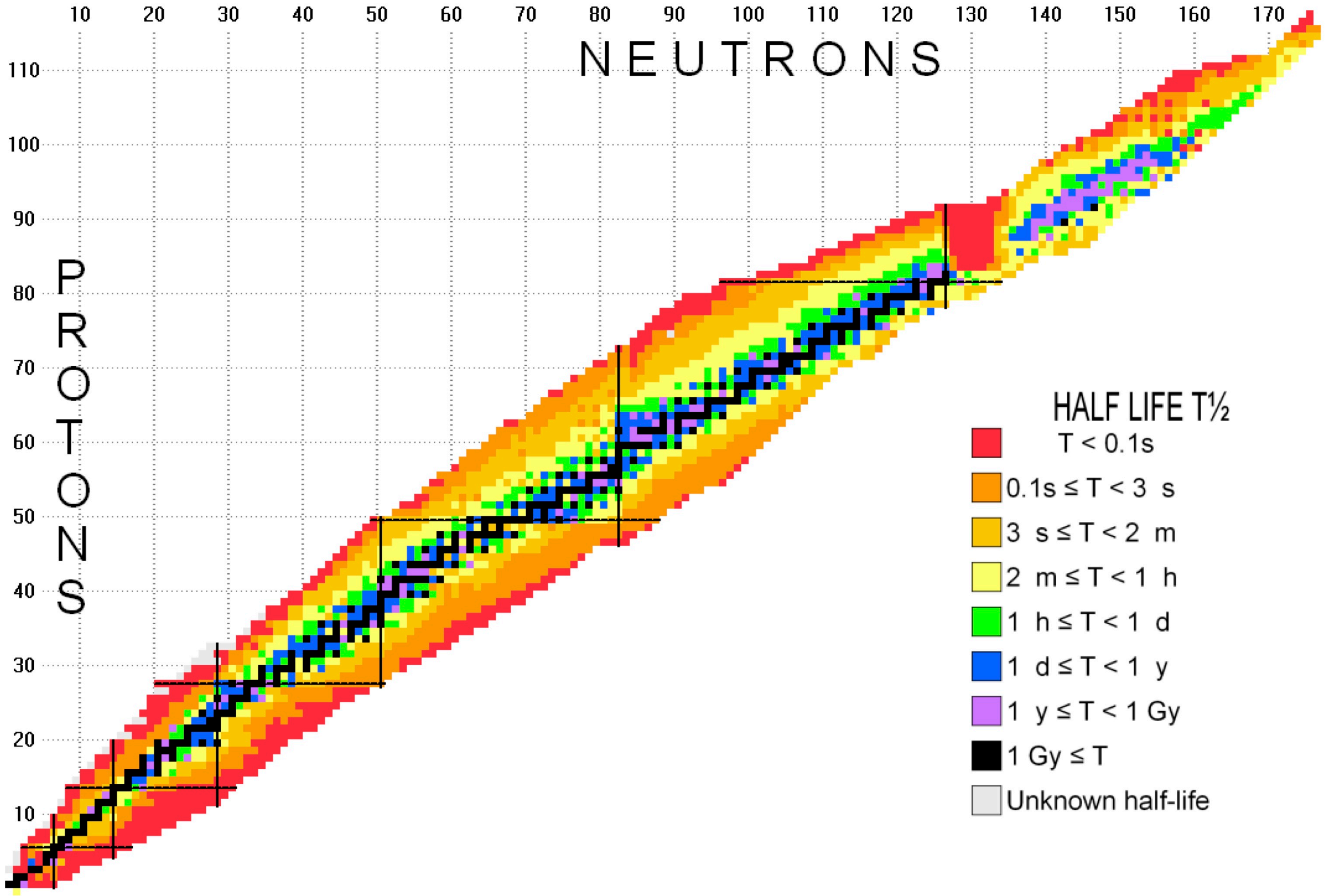
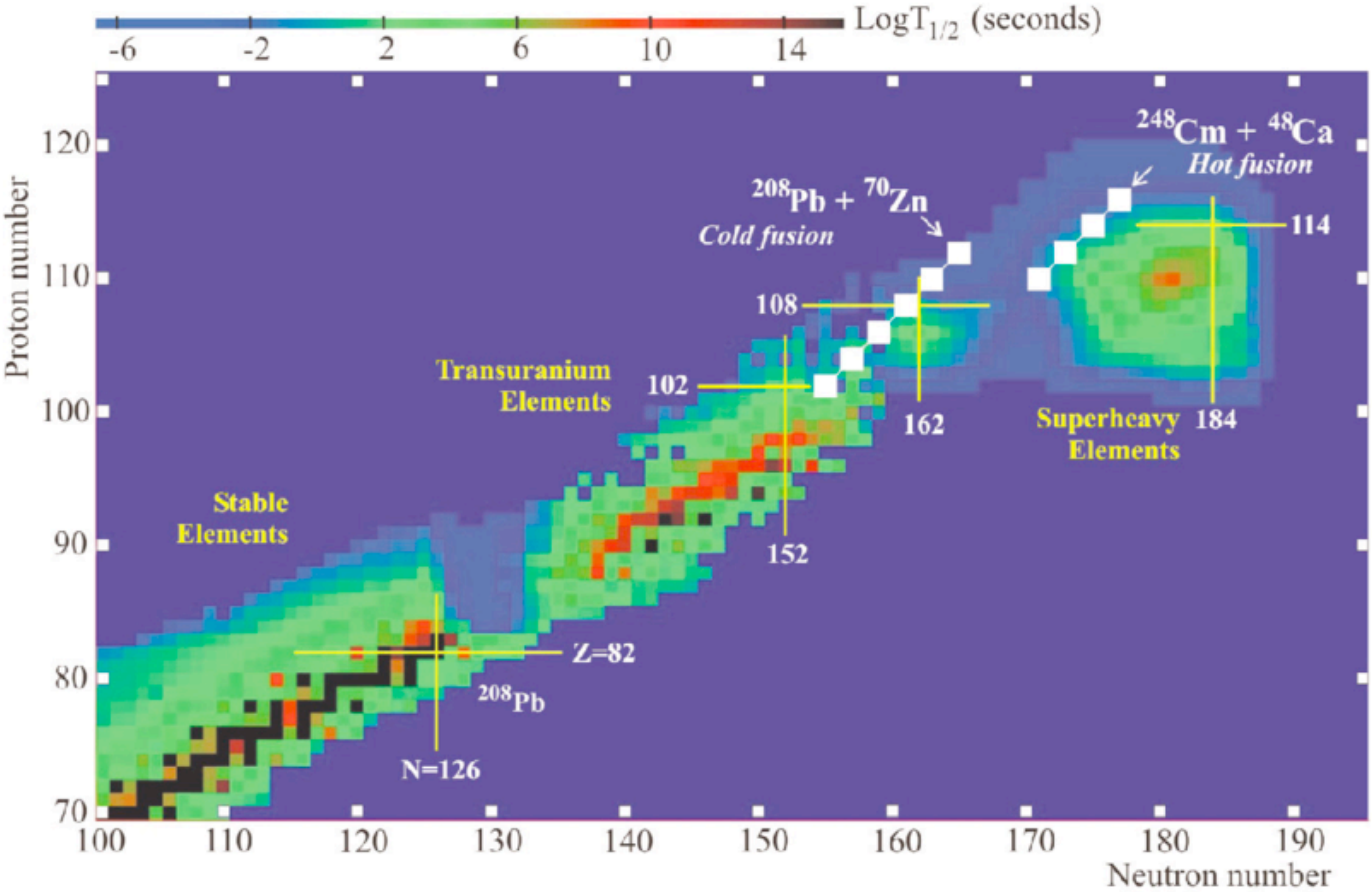


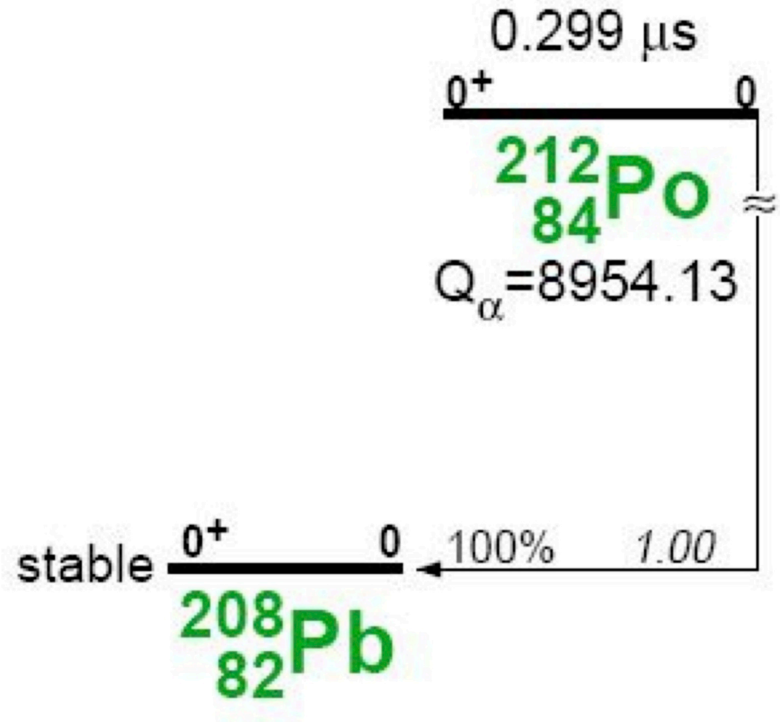
Figure 2: Chart of the nuclides for half-lives (created by NUCLEUS-AMDC).

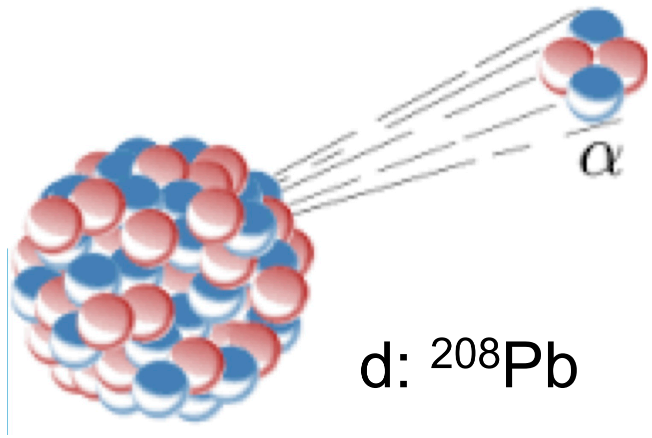
G. Audi et al. / Nuclear Physics A 729 (2003) 3–128

Island of Stability

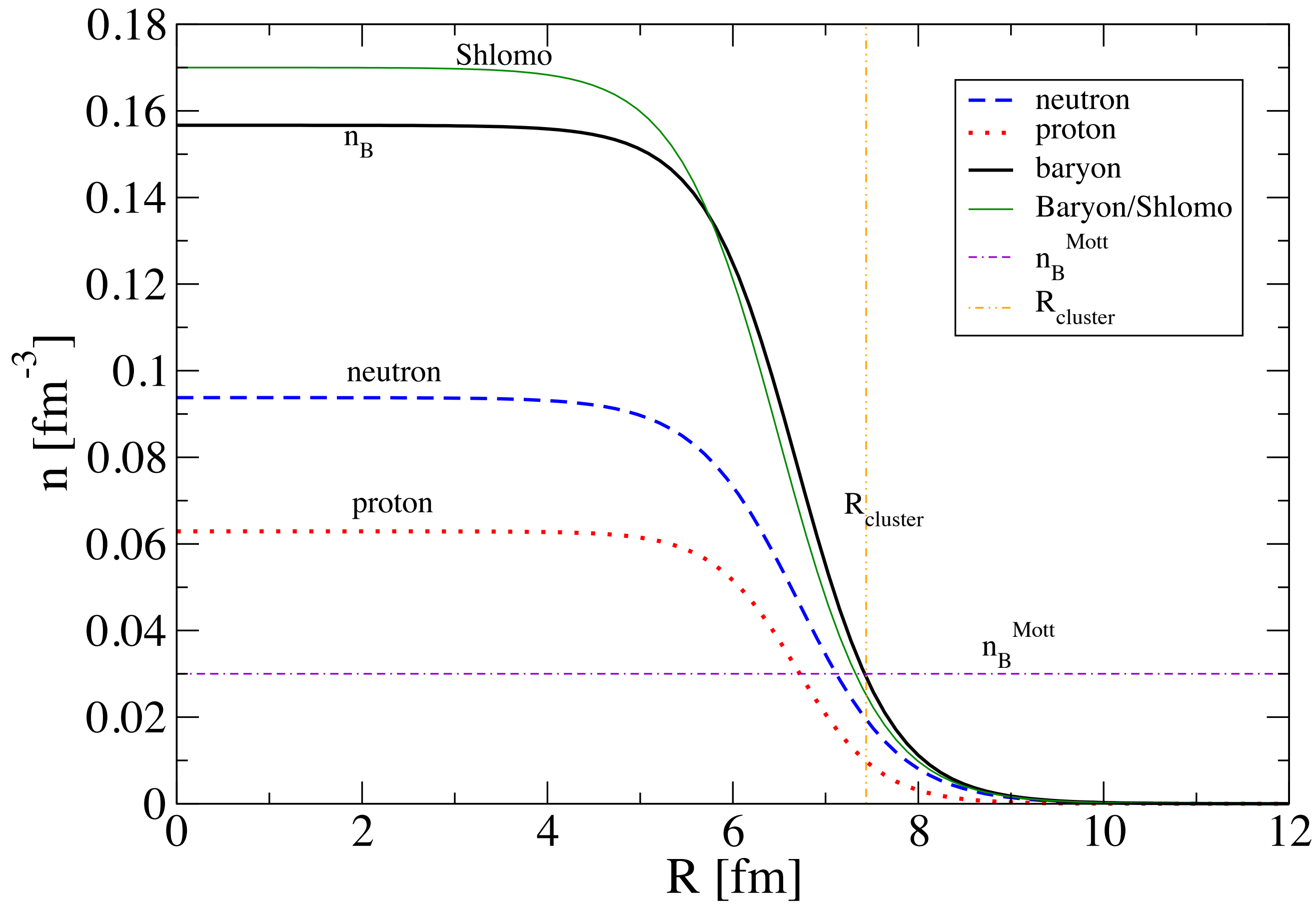


Preformation: α decay of ^{212}Po





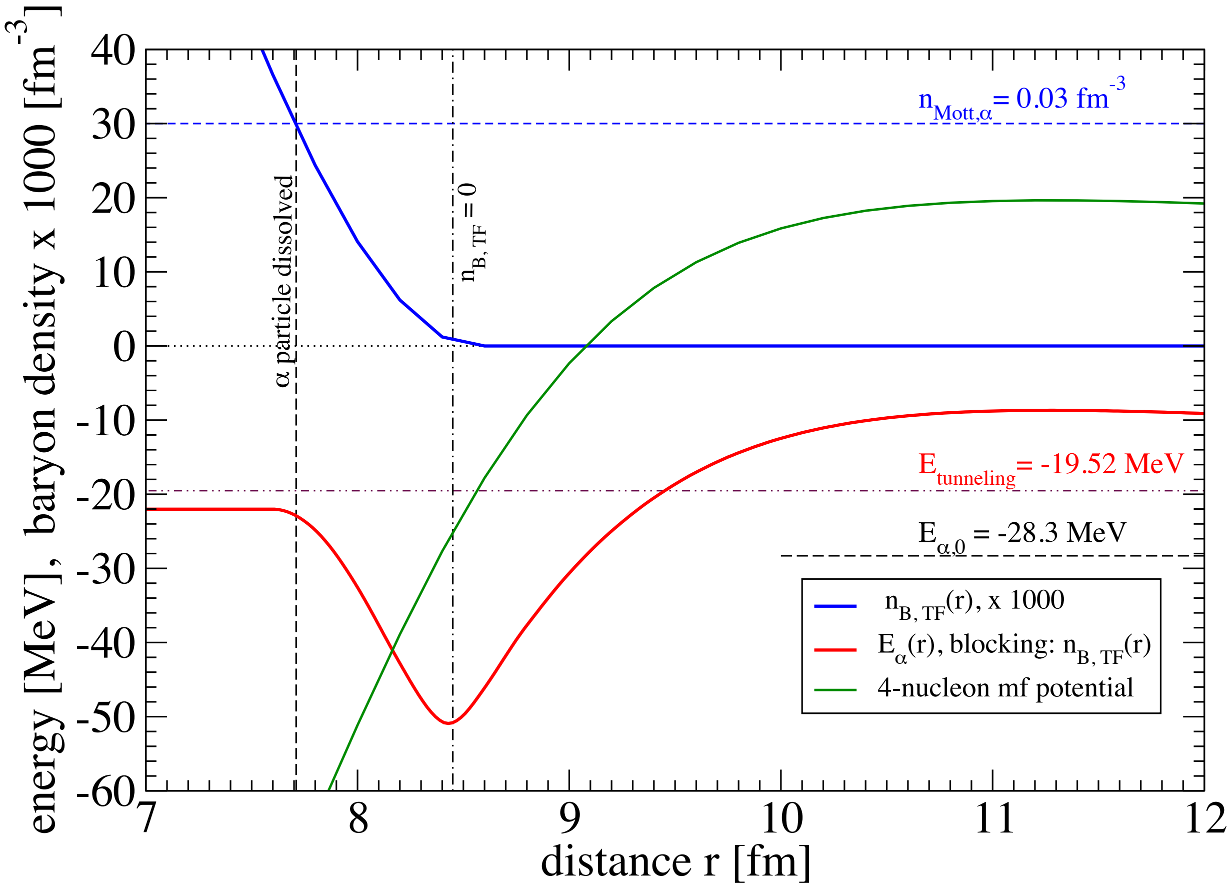
Nucleon density of the ^{208}Pb core



C. M. Tarbert et al., Phys. Rev. Lett. 112, 242502 (2014)

^{212}Po: α(^{4}He) on top of ^{208}Pb

Bound state (quartet) in a dense environment



Coulomb repulsion
+ nuclear attraction
(double folding potential)

density of the lead core

effective potential
of the quartet,
Including intrinsic
(α -like) binding energy

1 fm = 10^{-15} m

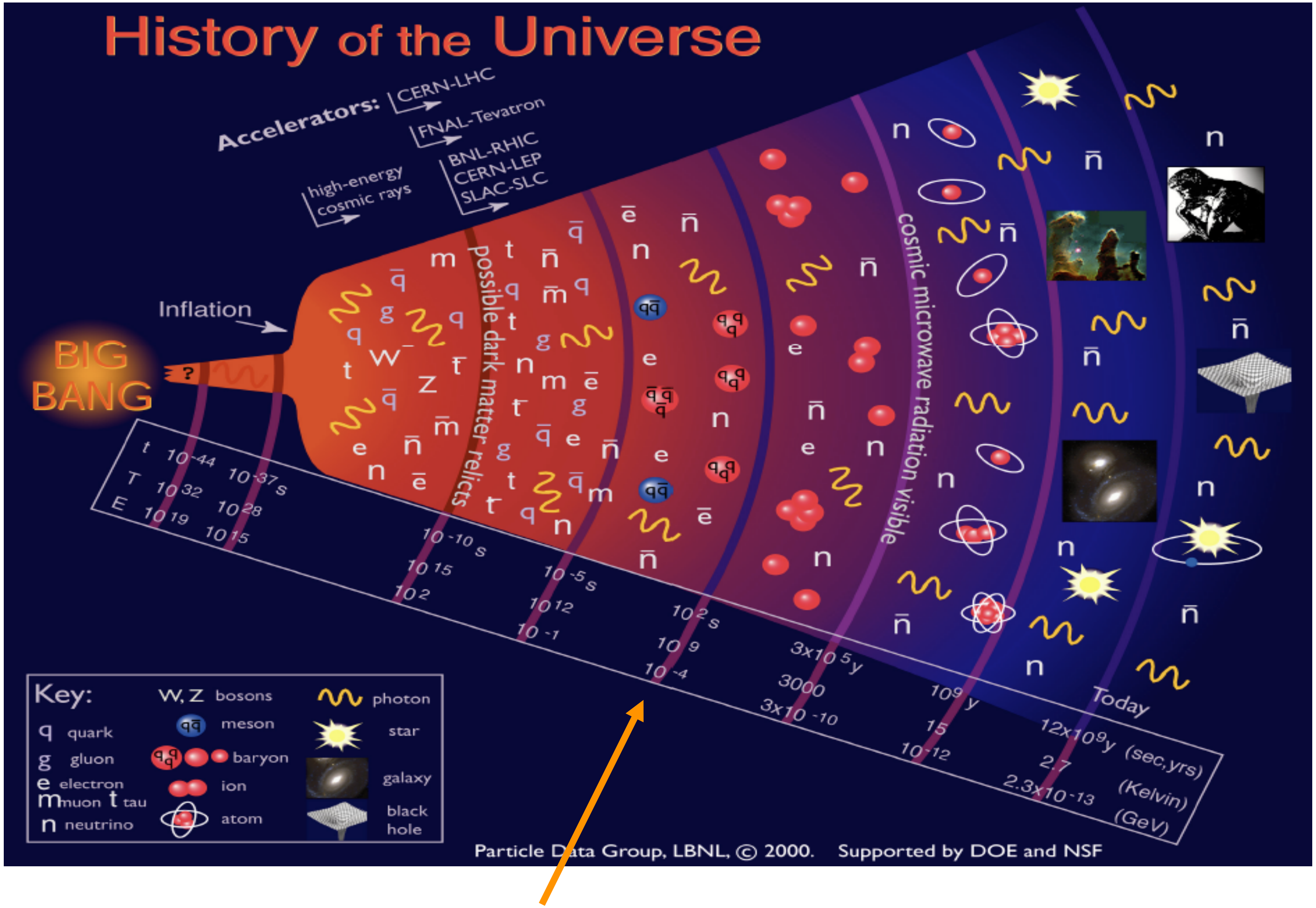
G. R. et al., Physical Review C **90**, 034304 (2014)

α decay to doubly magic core in Quartetting Wave Function Approach arXiv1912.01151: ^{104}Te (submitted)

2. Finite temperatures

- Early universe
- compact objects in astrophysics
- Heavy ion collisions
- Spontaneous fission

Origin of chemical elements



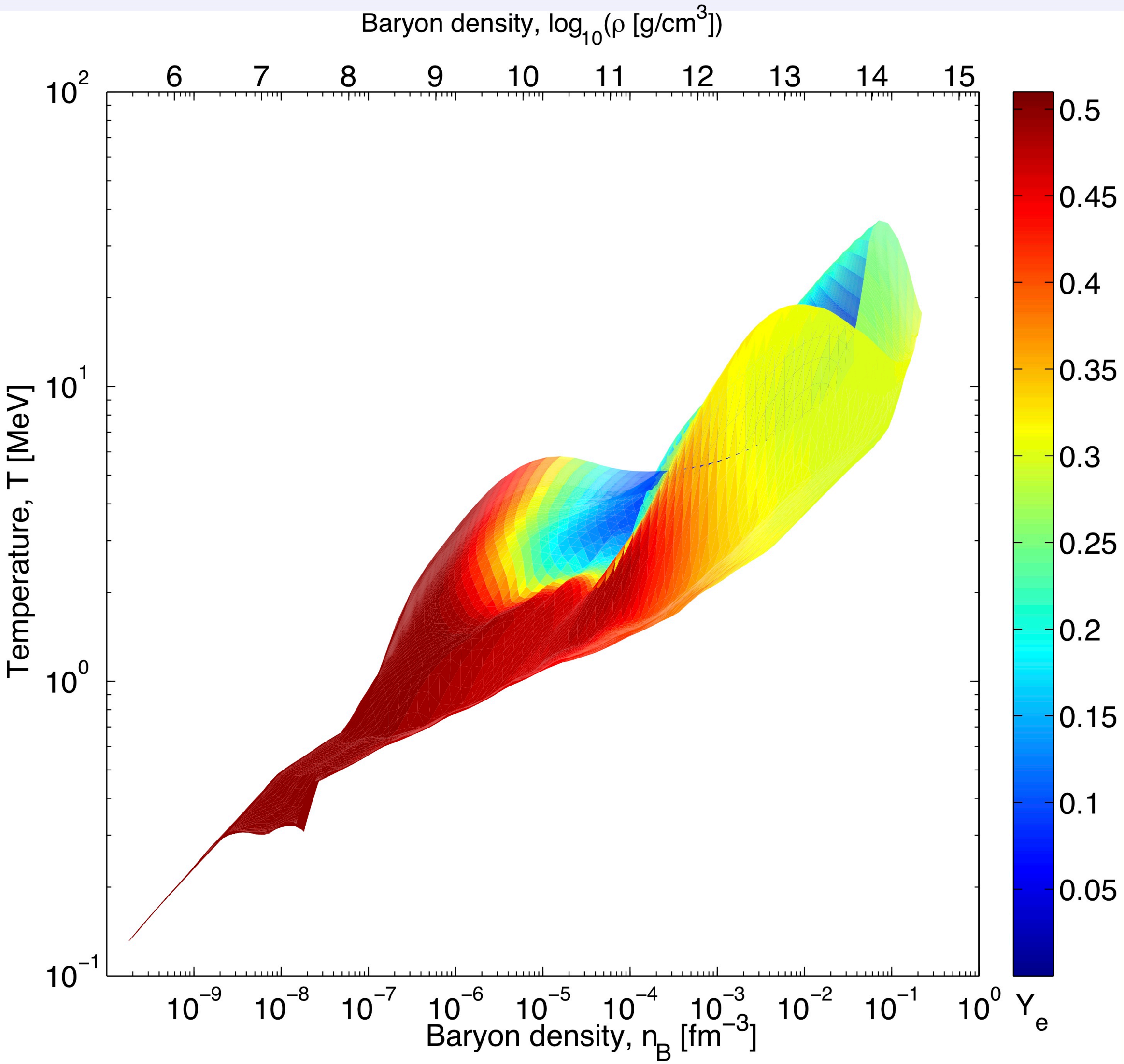
Big-Bang nucleosynthesis, time ~ 100 sec, temperature ~ 10^9 K

Nuclear matter phase diagram

Core collapse supernovae

Relevant Parameters:

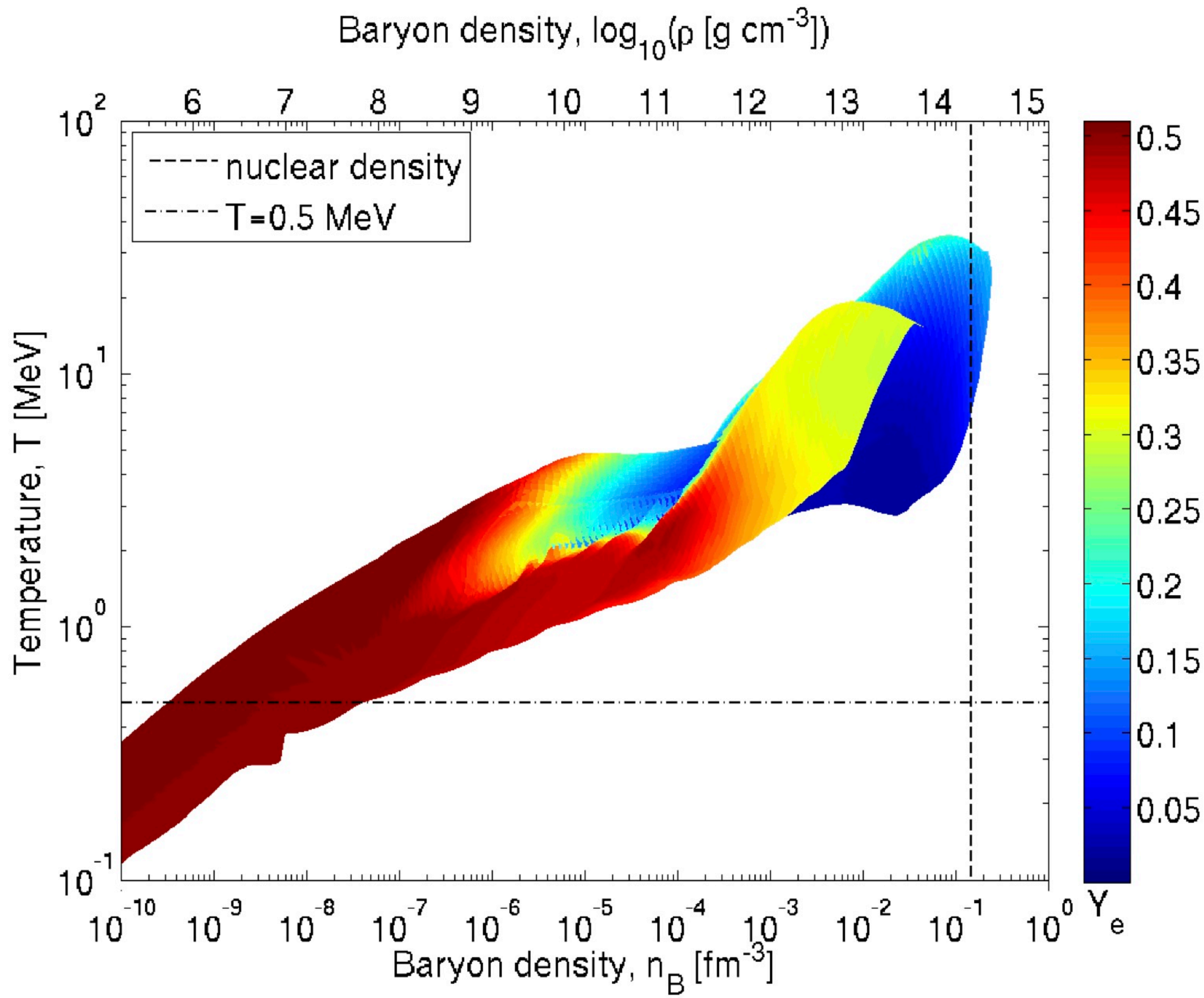
- **density**:
  $10^{-9} \lesssim \varrho/\varrho_{\rm sat} \lesssim 10$
  with nuclear saturation density
  $\varrho_{\rm sat} \approx 2.5 \cdot 10^{14}$ g/cm^3
  ($n_{\rm sat} = \varrho_{\rm sat}/m_n \approx 0.15$ fm^{-3})
- **temperature**:
  0 MeV $\leq k_B T \lesssim 50$ MeV
  ($\hat{=}\ 5.8 \cdot 10^{11}$ K)
- **electron fraction**:
  $0 \leq Y_e \lesssim 0.6$



T. Fischer, arXiv 1307.6190

Nuclear matter phase diagram

Exploding supernova



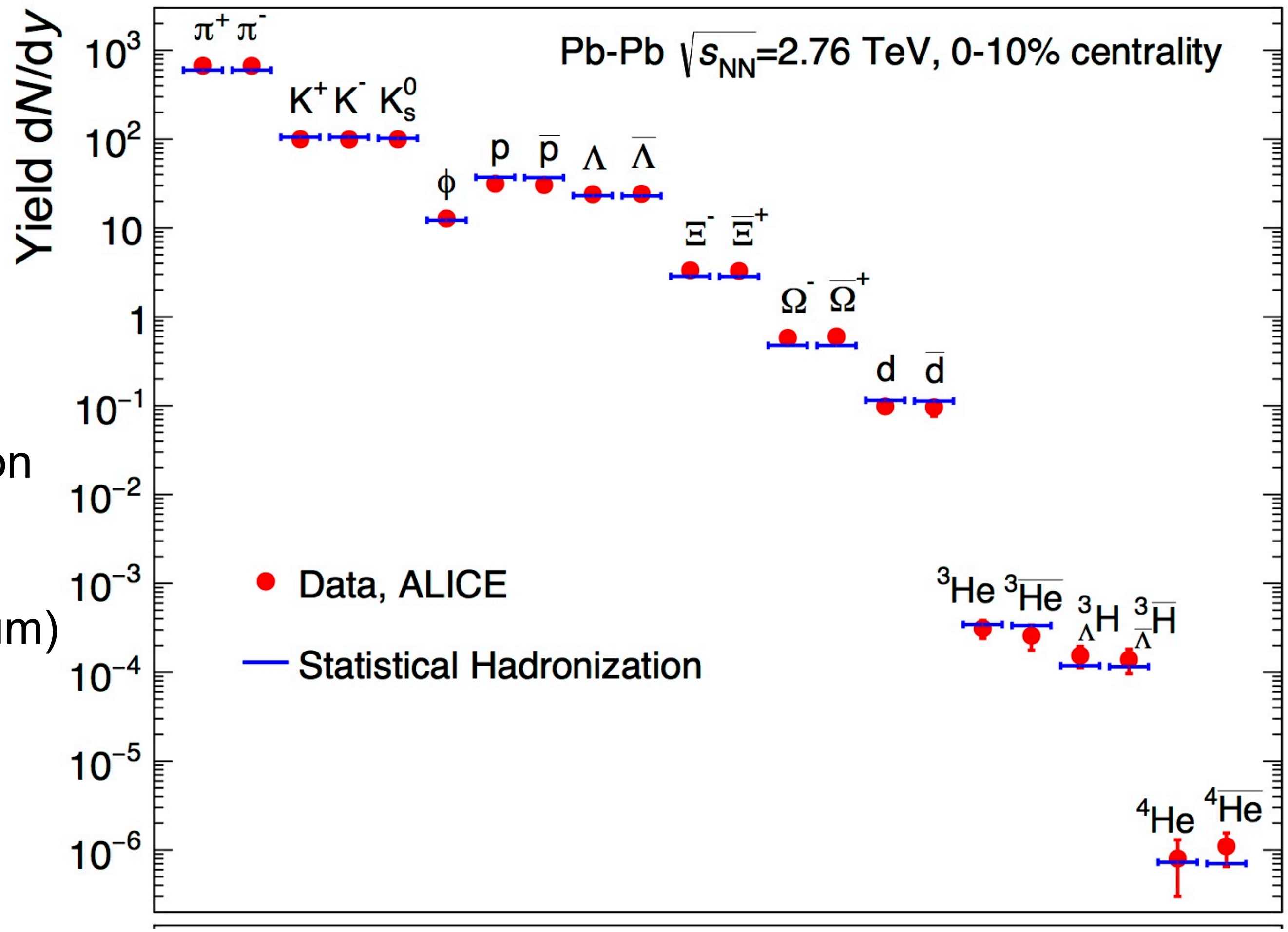
T. Fischer et al., arXiv 1307.6190

Cluster formation at LHC/CERN

ALICE@LHC

Excellent description of data by the Statistical model (chemical equilibrium)

T = 156 MeV



A. Andronic, P. Braun-Munziger, K. Redlich, J. Stachel, Nature 561, 321 (2018)

Freeze-out at heavy ion collisions

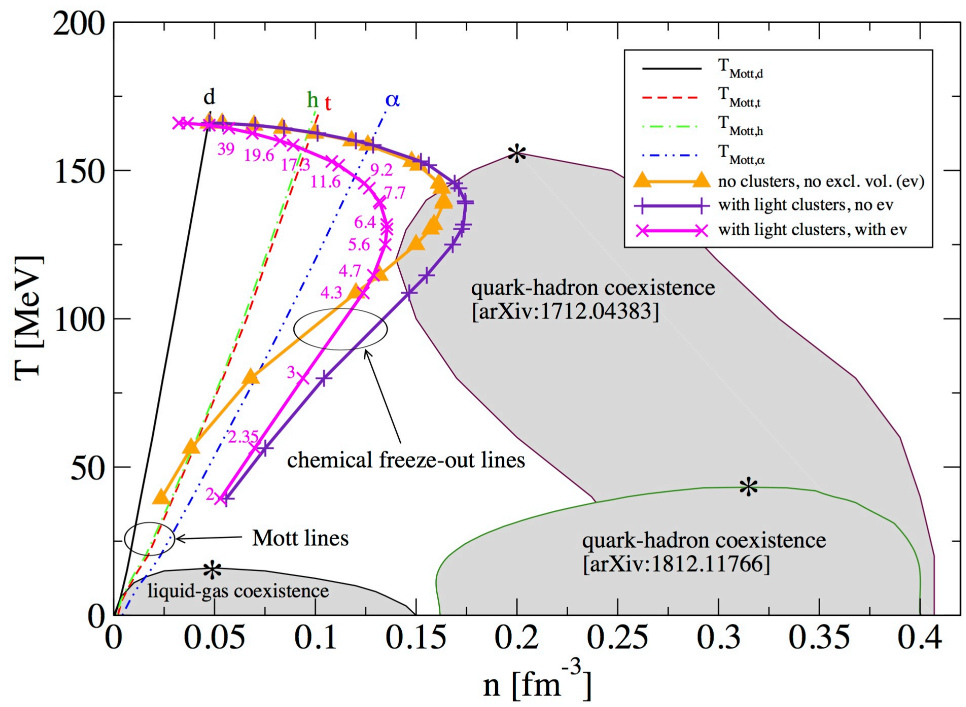


Fig. 1. Chemical freezeout lines in the temperature density plane (phase diagram) together with Mott lines for light clusters. The coexistence regions for the nuclear gas-liquid transition and for two examples of the hadron-quark matter transition are shown as grey shaded regions together with their critical endpoints. For details, see text.

D. Blaschke, G. Ropke, Yu. Ivanov, M. Kozhevnikova, and S. Liebing,
The XVIII International Conference on Strangeness in Quark Matter (SQM 2019)

Nonequilibrium statistical operator (NSO)

principle of weakening of initial correlations (Bogoliubov, Zubarev)

$$\rho_\epsilon(t) = \epsilon \int_{-\infty}^{t} e^{\epsilon(t_1-t)} U(t,t_1)\rho_{\rm rel}(t_1)U^\dagger(t,t_1)\, dt_1$$

time evolution operator $U(t,t_0)$ relevant statistical operator $\rho_{\rm rel}(t)$

selection of the set of relevant observables $\{B_n\}$

self-consistency relations $\mathrm{Tr}\{\rho_{\rm rel}(t)B_n\} \equiv \langle B_n\rangle^t_{\rm rel} = \langle B_n\rangle^t$

maximum of information entropy $S_{\rm rel}(t) = -k_{\rm B}\,\mathrm{Tr}\{\rho_{\rm rel}(t)\log\rho_{\rm rel}(t)\}$

generalized Gibbs distribution $\rho_{\rm rel}(t) = \exp\left\{-\Phi(t) - \sum_n \lambda_n(t)B_n\right\}$

extended von Neumann equation

$$\frac{\partial}{\partial t}\varrho_\varepsilon(t) + \frac{i}{\hbar}[H,\varrho_\varepsilon(t)] = -\varepsilon\left(\varrho_\varepsilon(t) - \varrho_{\rm rel}(t)\right)$$

$\varrho(t) = \lim_{\varepsilon\to 0}\varrho_\varepsilon(t)$ after thermodynamic limit

Relevant statistical operator

State of the system in the past $\mathrm{Tr}\{\rho(t) B_n\} = \langle B_n \rangle^t$

Construction of the relevant statistical operator at time t

$$S_{\rm rel}(t) = -k_{\rm B}\,\mathrm{Tr}\{\rho_{\rm rel}(t) \log \rho_{\rm rel}(t)\}$$ -> maximum

$$\delta[\mathrm{Tr}\{\rho_{\rm rel}(t) \log \rho_{\rm rel}(t)\}] = 0$$

$$\mathrm{Tr}\{\rho_{\rm rel}(t) B_n\} \equiv \langle B_n \rangle^t_{\rm rel} = \langle B_n \rangle^t$$

Generalized Gibbs distribution

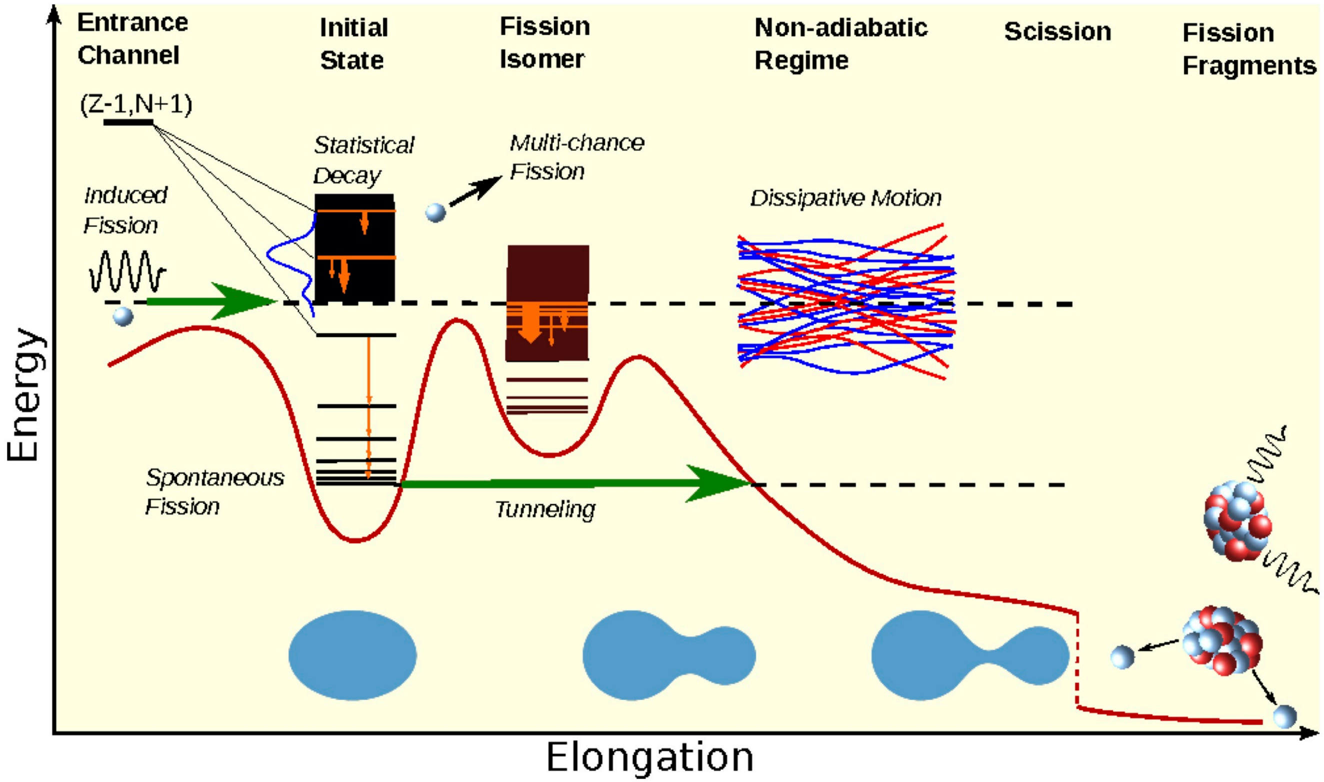
$$\rho_{\rm rel}(t) = \exp\left\{-\Phi(t) - \sum_n \lambda_n(t) B_n\right\}$$

$$\Phi(t) = \log \mathrm{Tr} \exp\left\{-\sum_n \lambda_n(t) B_n\right\}$$

$$\frac{\partial S_{\rm rel}(t)}{\partial t} = \sum_n \lambda_n(t) \langle \dot{B}_n \rangle^t$$

But: von Neumann equation?
Entropy?

Nuclear Fission



quadrupol fluctuations (GDR)
tunneling – deformed droplets, neck formation

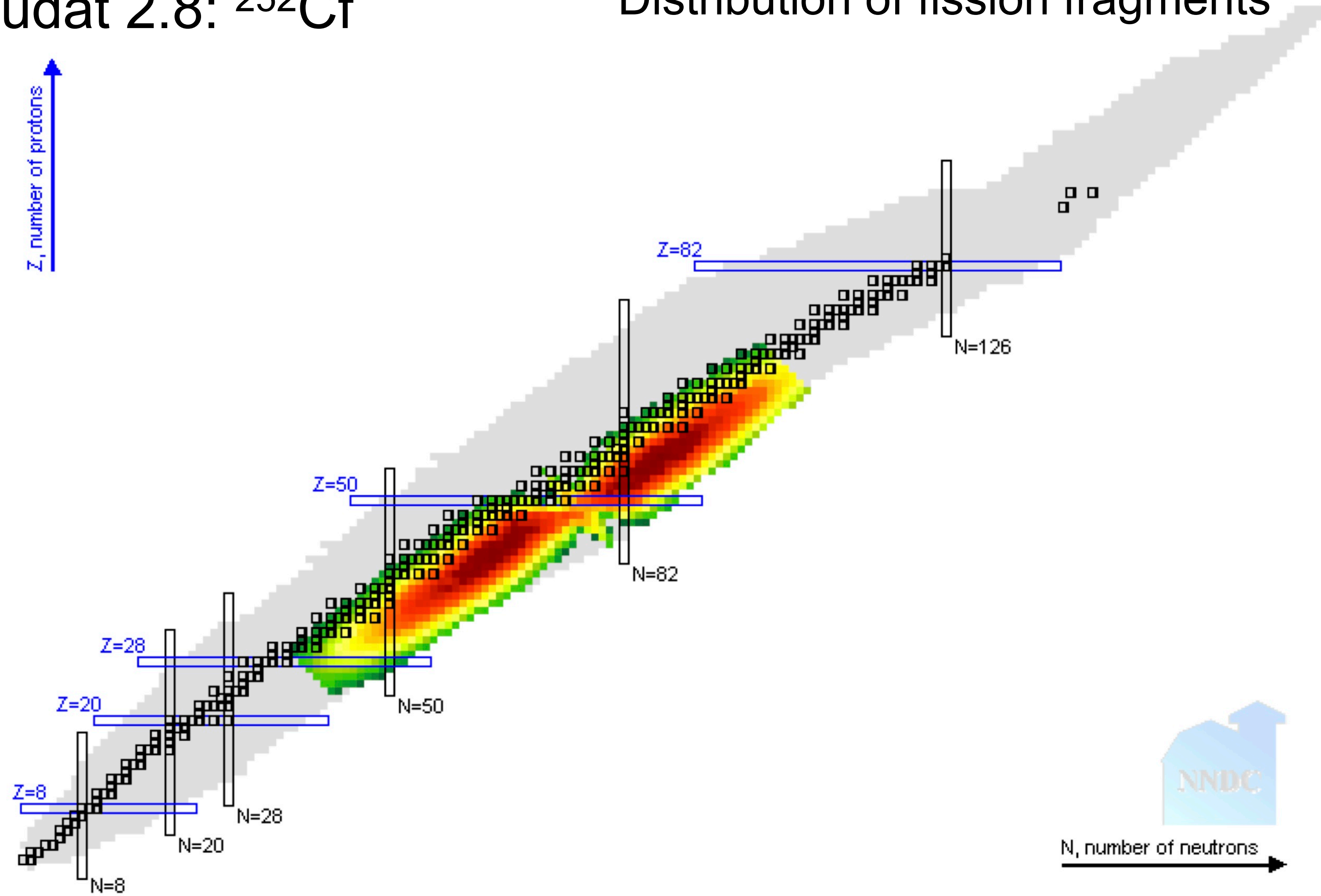
Future of Nuclear Fission Theory

M. Bender et al., J. phys. G: Nucl. Part. Phys. **47**, 113002 (2020)

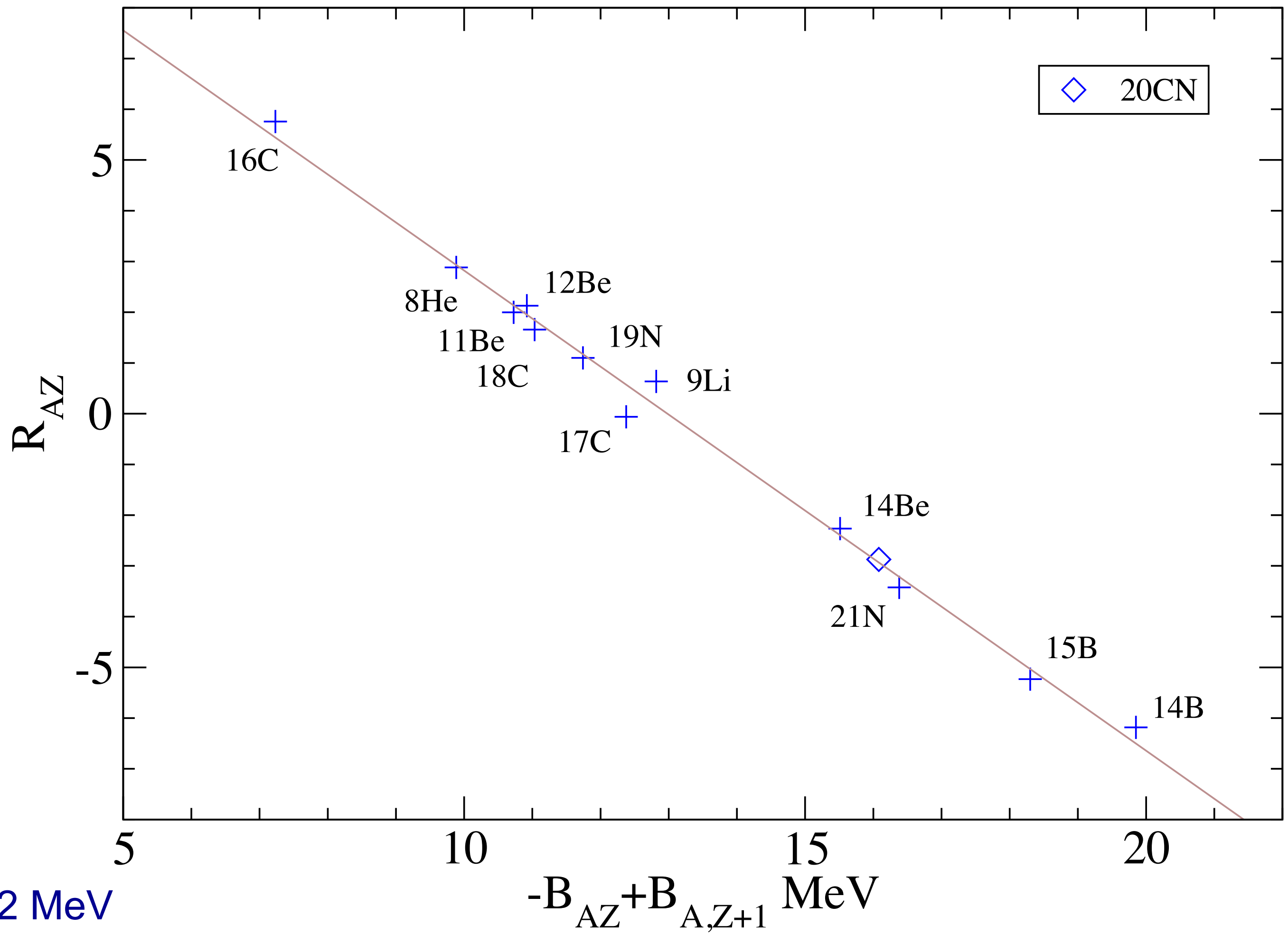
Spontaneous fission of actinides

Nudat 2.8: ^{252}Cf

Distribution of fission fragments



Ternary fission: light cluster yields



Ratio R_{AZ}= $\ln[Y_{AZ}/Y_{A,Z+1}]$

is proportional
to the difference
of binding energies

$(B_{A,Z+1}-B_{AZ})/T$, T=1.2 MeV

Thermodynamics

equation of state $n_B = n_B(T, \mu)$

equation of state $\mu = \mu(T, n_B)$

thermodynamic potential to T, n_B: free energy density

$$f(T, n_B) = \frac{F(T,V,N_B)}{V} = f(T, n_0) + \int_{n_0}^{n_B} \mu(T, n')\, dn'$$

thermodynamic relations:

$$F + pV = G = \mu N$$

equation of state: pressure

$$p(T, n_B) = n_B \mu(T, n_B) - f(T, n_B)$$

consistency

Ideal Fermi gas (neutrons)

equation of state (EoS): energy density is related to pressure (T = 0)
$\rho_{rel}(p)$

nonrelativistic [units MeV, fm] $E_k = \frac{\hbar^2}{2m_n}k^2$

$$N_n = (2s+1)\sum_k f_n(E_k); \quad n_n = \frac{2}{(2\pi)^3}\int_0^{k_F} 4\pi k^2 dk = \frac{1}{3\pi^2}k_F^3$$

$$k_F = (3\pi^2 n_n)^{1/3}$$

chemical potential $\mu(n_n) = E_{k_F} = \frac{\hbar^2}{2m_n}(3\pi^2)^{2/3} n_n^{2/3}$

free energy density $f(n_n) = \frac{\hbar^2}{2m_n}(3\pi^2)^{2/3}\frac{3}{5}n_n^{5/3}$

$$p = K(n_B)^\gamma, \quad \gamma = 5/3, \quad K = \frac{(3^2\pi^4)^{1/3}}{5}\frac{\hbar^2}{m_n} = 79.3609$$

relativistic relation (EoS) from the dispersion relation

$$E_\tau^{(0)}(p) = \sqrt{m_\tau^2 c^4 + \hbar^2 c^2 p^2}, \; \tau = \{n, p\}$$

3. Many-particle theory

$$n_\tau^{\mathrm{tot}}(T,\mu_n,\mu_p) = \frac{1}{\Omega}\sum_{p_1,\sigma_1}\int\frac{d\omega}{2\pi}\frac{1}{e^{(\omega-\mu_\tau)/T}+1}S_\tau(1,\omega)$$

Spectral function S (or A)

- Dyson equation and self energy (homogeneous system)

$$G(1,iz_\nu) = \frac{1}{iz_\nu - E(1) - \Sigma(1,iz_\nu)}$$

- Evaluation of $\Sigma(1,iz_\nu)$:
  perturbation expansion, diagram representation

$$A(1,\omega) = \frac{2\mathrm{Im}\,\Sigma(1,\omega+i0)}{[\omega - E(1) - \mathrm{Re}\,\Sigma(1,\omega)]^2 + [\mathrm{Im}\,\Sigma(1,\omega+i0)]^2}$$

approximation for self energy $\longrightarrow$ approximation for equilibrium correlation functions

alternatively: simulations, path integral methods

Different approximations

Ideal Fermi gas:

protons, neutrons,
(electrons, neutrinos,…)

Different approximations

Ideal Fermi gas:
protons, neutrons,
(electrons, neutrinos,…)

medium effects

Quasiparticle quantum liquid:
mean-field approximation
Skyrme, Gogny, RMF

Medium effects:
Quasiparticle approximation

- Skyrme / Gogny

- relativistic mean field (RMF)

  Lagrangian: non-linear sigma, TM1 parameters,
  single particle modifications, energy shift, effective mass

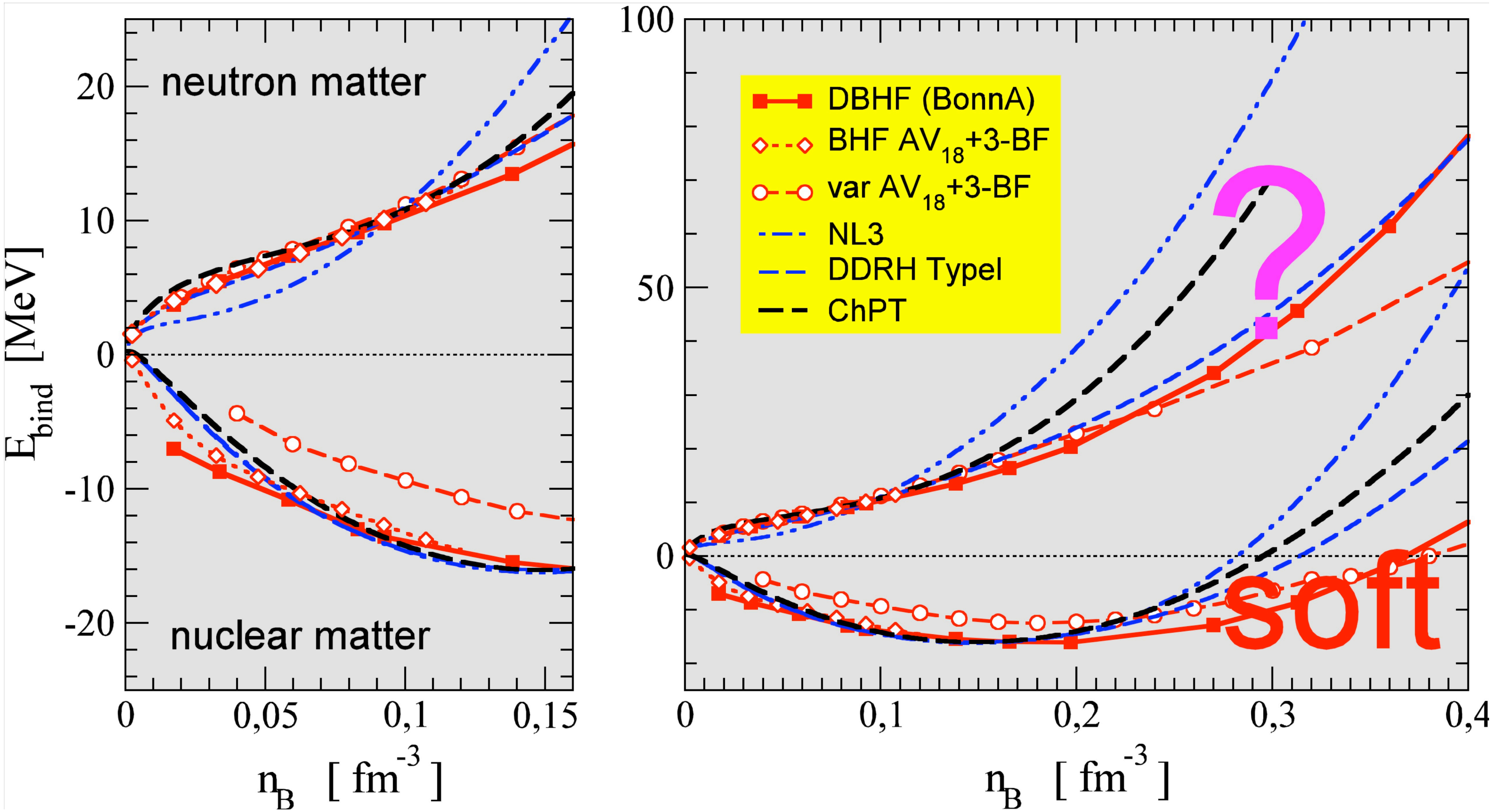
- DD-RMF [S.Typel, Phys. Rev. C **71**, 064301 (2007)]:

  expansion of the scalar field and the vector fields
  in powers of proton/neutron densities

- Dirac-Brueckner Hartree Fock (DBHF)

- Density functional theory

Quasiparticle picture: RMF and DBHF



But: cluster formation

Incorrect low-density limit

C. Fuchs et al.;
J.Margueron et al., Phys.Rev.C **76**,034309 (2007)

Interacting nucleon matter

Ideal, noninteracting Fermi gas: soft EoS, masses too small

microscopic: nucleon-nucleon interaction,
Brueckner Hartree-Fock (BHF)
Hartree-Fock-Bogoliubov (HFB)

Effective interaction models (phenomenologic)
Skyrme: density dependent potential energy, fitted to data

Walecka: effective Lagrangian,
nucleons are coupled to vector and scalar meson fields
masses and coupling constants are fitted to reproduce known properties

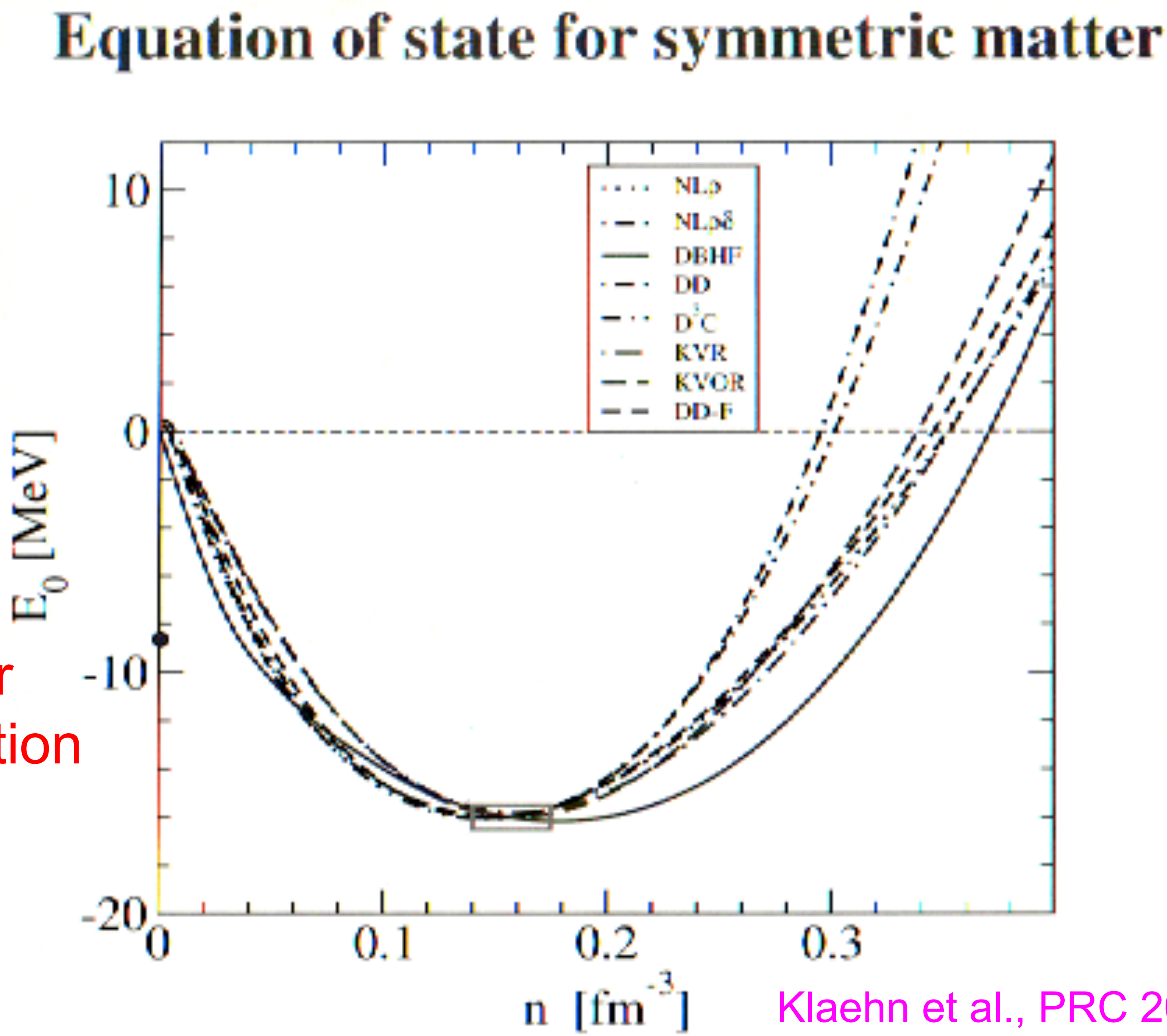
relativistic mean field (RMF) approximation

RMF – DD2: density dependent coupling S. Typel et al., Nucl. Phys. A **656**, 331 (1999)

$$E_\tau(\mathbf{p};T,n_B,Y_p) = \sqrt{[m_\tau c^2 - S(T,n_B,Y_p)]^2 + \hbar^2 c^2 p^2} + V_\tau(T,n_B,Y_p)$$

parametrization of the functions $S(T,n_B,Y_p)$, $V(T,n_B,Y_p)$: G. R., PRC 92,054001 (2015)

Quasiparticle approximation for nuclear matter



But: cluster formation

Incorrect low-density limit

Klaehn et al., PRC 2006

Quasiparticle concept

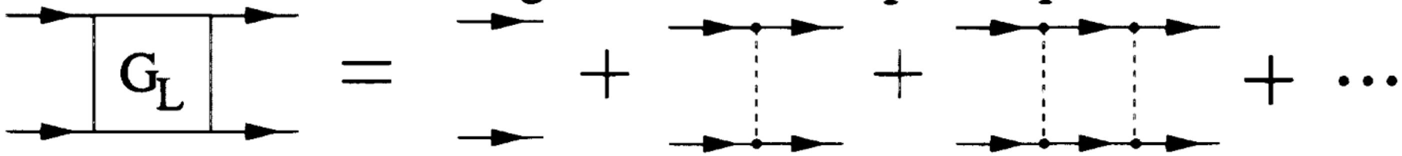
- Expansion for small Im $\Sigma(1, \omega + i\eta)$

$$A(1,\omega) \approx \frac{2\pi\delta(\omega - E^{\rm quasi}(1))}{1 - \frac{\rm d}{{\rm d}z}{\rm Re}\ \Sigma(1,z)|_{z=E^{\rm quasi}-\mu_1}}$$
$$-2{\rm Im}\ \Sigma(1,\omega + i\eta)\frac{\rm d}{{\rm d}\omega}\frac{P}{\omega + \mu_1 - E^{\rm quasi}(1)}$$

quasiparticle energy $E^{\rm quasi}(1) = E(1) + {\rm Re}\ \Sigma(1,\omega)|_{\omega=E^{\rm quasi}}$

- chemical picture: bound states $\hat{=}$ new species

summation of ladder diagrams, Bethe-Salpeter equation

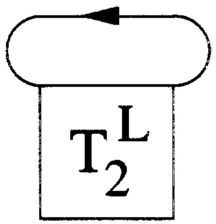


Inclusion of bound states

low density limit:

$$G_2^L(12, 1'2', i\lambda) = \sum_{n\mathbf{P}} \Psi_{n\mathbf{P}}(12) \frac{1}{i\omega_\lambda - E_{n\mathbf{P}}} \Psi^*_{n\mathbf{P}}(12)$$

$\Sigma =$



$$n(\beta, \mu) = \sum_1 f_1(E^{\text{quasi}}(1)) + \sum_{2,n\mathbf{P}}^{\text{bound}} g_{12}(E_{n\mathbf{P}})$$

$$+ \sum_{2,n\mathbf{P}} \int_0^\infty \mathrm{d}k \, \delta_{\mathbf{k},\mathbf{p}_1-\mathbf{p}_2} g_{12}(E^{\text{quasi}}(1) + E^{\text{quasi}}(2)) 2 \sin^2\delta_n(k) \frac{1}{\pi} \frac{\mathrm{d}}{\mathrm{d}k} \delta_n(k)$$

- generalized Beth-Uhlenbeck formula
  correct low density/low temperature limit:
  mixture of free particles and bound clusters

Beth-Uhlenbeck formula

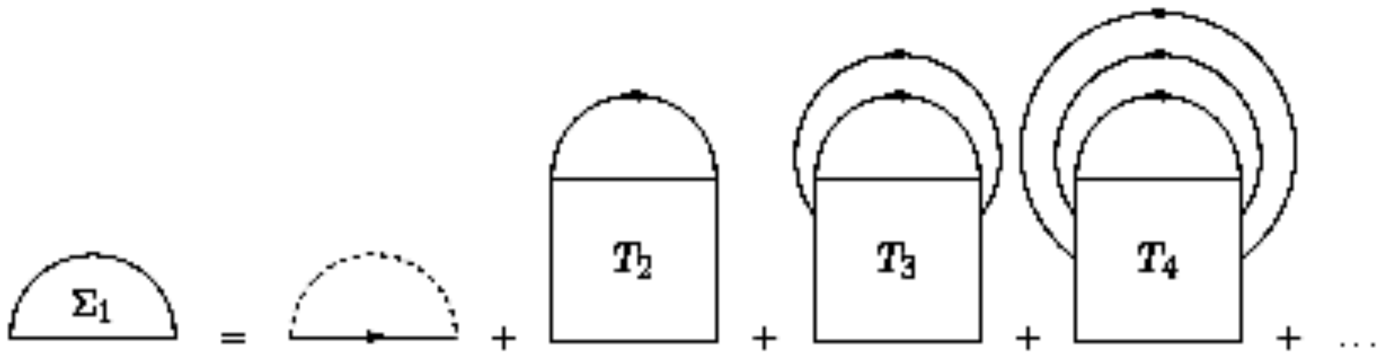
rigorous results at low density: virial expansion

Beth-Uhlenbeck formula

$$\begin{aligned} n(T,\mu) &= \frac{1}{V}\sum_p e^{-(E(p)-\mu)/k_BT} \\ &+ \frac{1}{V}\sum_{nP} e^{-(E_{nP}-2\mu)/k_BT} \\ &+ \frac{1}{V}\sum_{\alpha P}\int_0^\infty \frac{dE}{2\pi} e^{-(E+P^2/4m-2\mu)/k_BT}\frac{d}{dE}\delta_\alpha(E) \\ &+ \ldots \end{aligned}$$

$\delta_\alpha(E)$: scattering phase shifts, channel α

Cluster decomposition of the self-energy



T-matrices: bound states, scattering states
Including clusters like new components
chemical picture,
mass action law, nuclear statistical equilibrium (NSE)

Ideal mixture of reacting nuclides

$$n_p(T,\mu_p,\mu_n) \;=\; \frac{1}{V}\sum_{A,\nu,K} Z_A f_A\{E_{A,\nu K} - Z_A\mu_p - (A-Z_A)\mu_n\}$$

$$n_n(T,\mu_p,\mu_n) \;=\; \frac{1}{V}\sum_{A,\nu,K} (A-Z_A) f_A\{E_{A,\nu K} - Z_A\mu_p - (A-Z_A)\mu_n\}$$

mass number A,
charge Z_A,
energy $E_{A,\nu,K}$,
ν internal quantum number,
~K center of mass momentum

$$f_{A(z)} = \frac{1}{\exp(z/T) - (-1)^A}$$

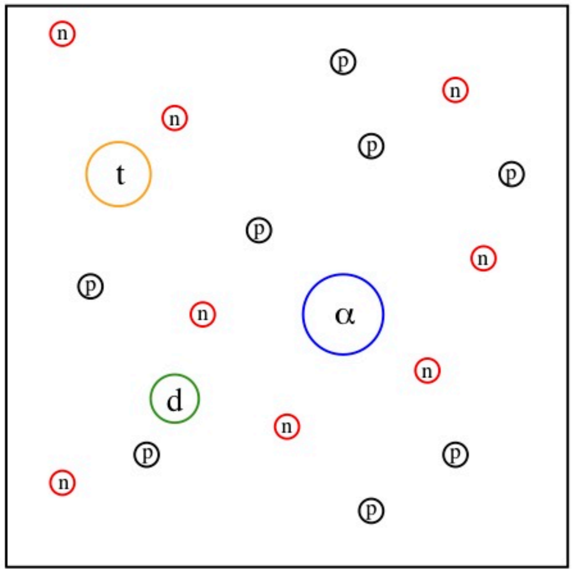
Chemical equilibrium, mass action law,
Nuclear Statistical Equilibrium (NSE)

Nuclear statistical equilibrium (NSE)

Chemical picture:

Ideal mixture of reacting components

Mass action law



Different approximations

medium effects

Ideal Fermi gas:

protons, neutrons,
(electrons, neutrinos,…)

Quasiparticle quantum liquid:

mean-field approximation
Skyrme, Gogny, RMF

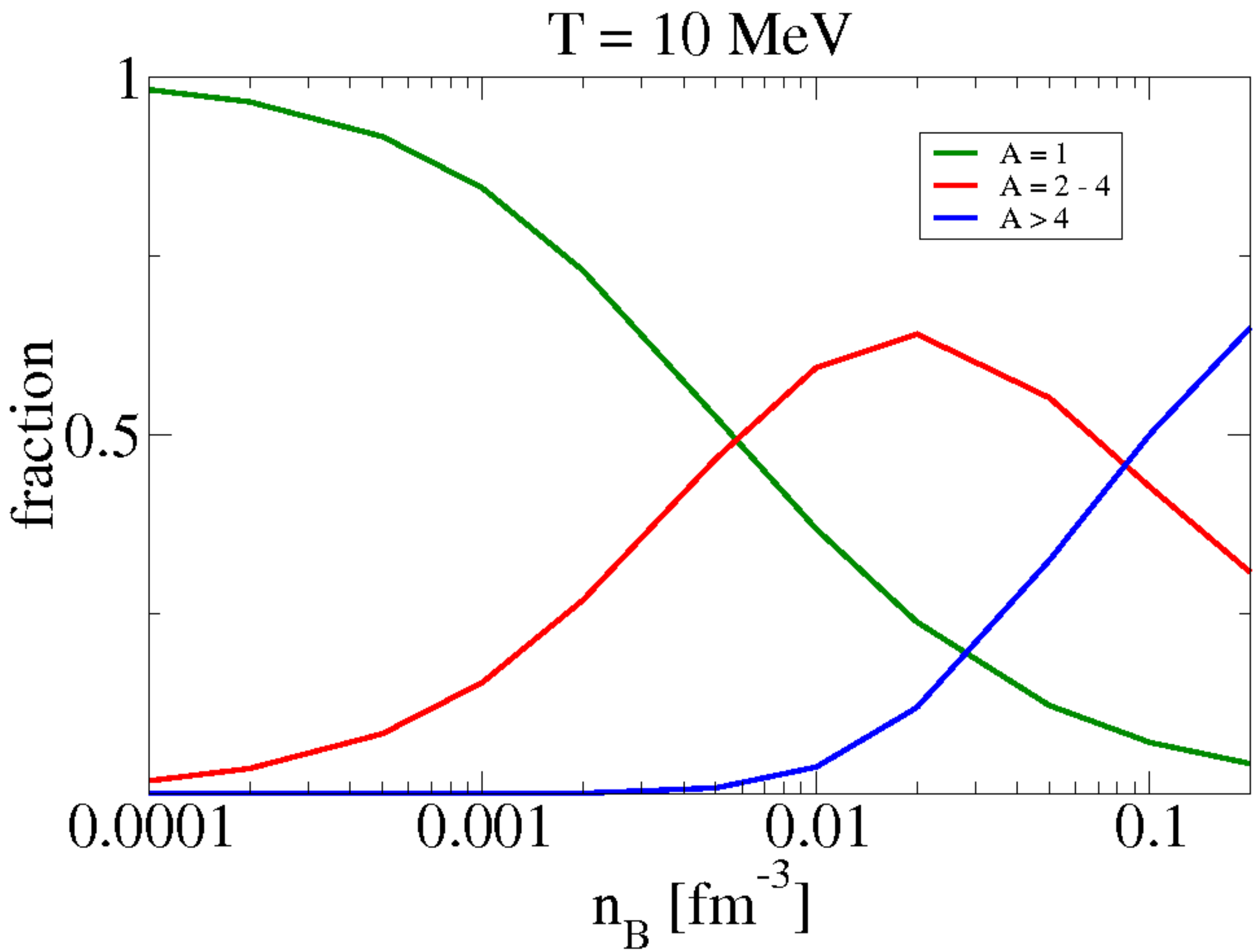
bound state formation

Nuclear statistical equilibrium:

ideal mixture of all bound states
(clusters:) chemical equilibrium

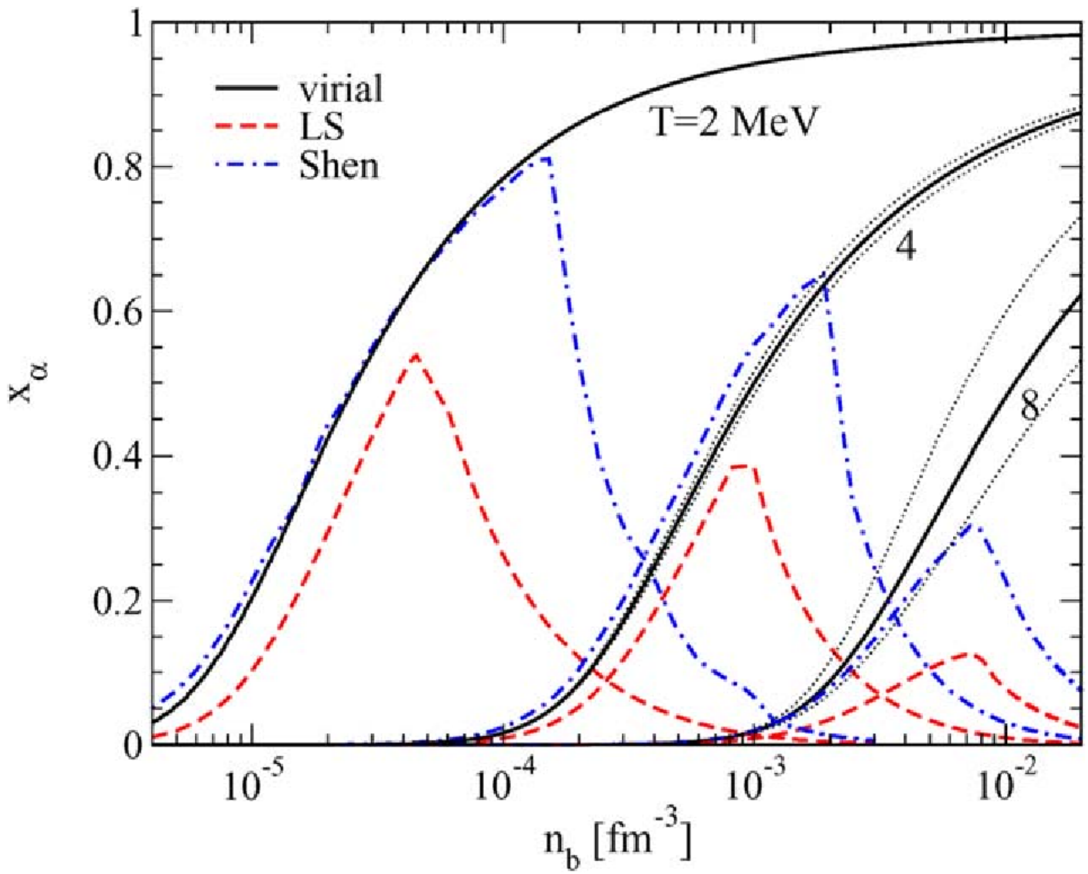
Inclusion of the light clusters (d,t,^{3}He,^{4}He)

Composition of symmetric matter
Ideal mixture of nuclides



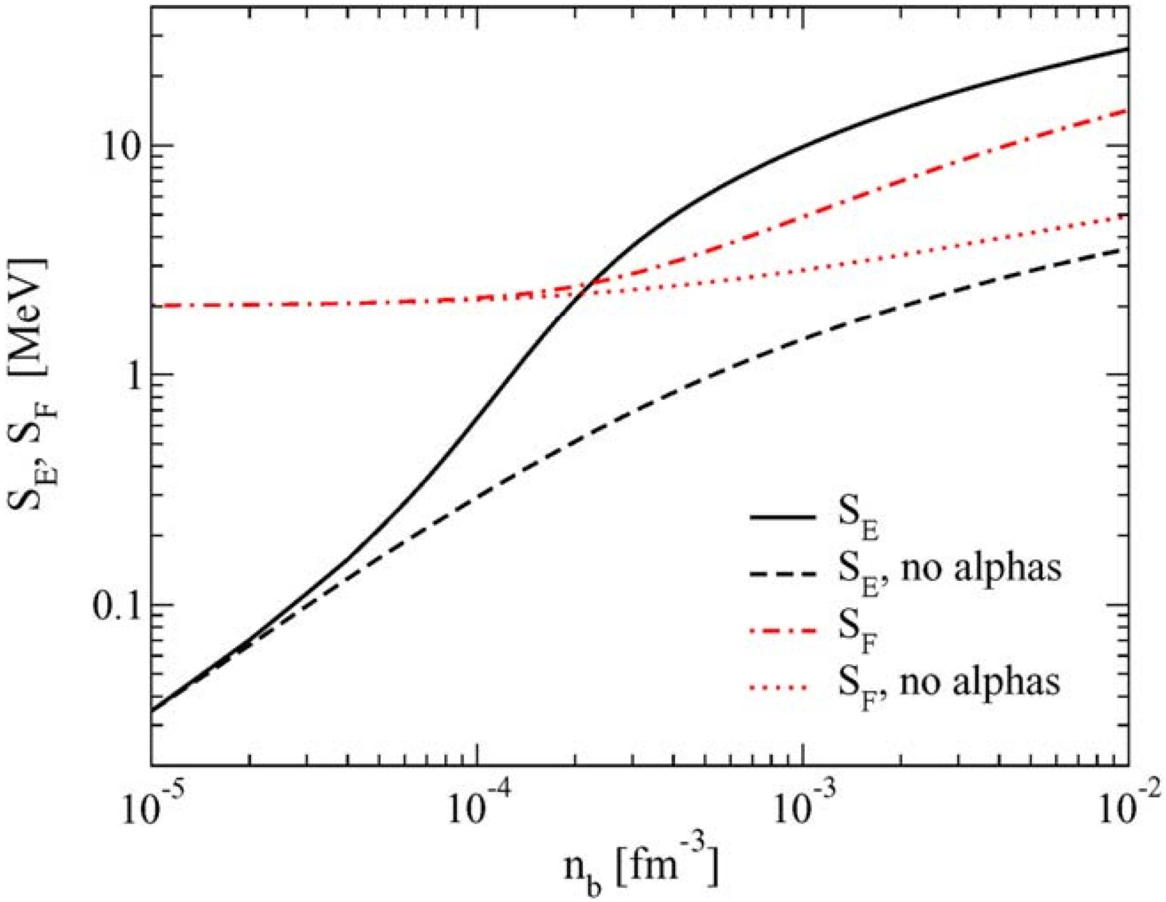
Alpha-particle fraction in the low-density limit

symmetric matter, T=2, 4, 8 MeV



C.J.Horowitz, A.Schwenk, Nucl. Phys. A **776**, 55 (2006)

Symmetry energy and symmetry free energy



T=4MeV

Horowitz & Schwenk, NPA (2006)

4. In-medium effects

- self energy, mean-field approximation
- quasiparticle picture of elementary particles
- full antisymmetrization: Pauli blocking
- bound states as new quasiparticles
- correlated medium

- quantum statistical approach
- excluded volume (Hempel, Schaffner-Bielich,…)
- generalized relativistic mean field:
  clusters as quasiparticles (Typel, Pais,…)

Different approximations

medium effects

Ideal Fermi gas:

protons, neutrons,
(electrons, neutrinos,…)

Quasiparticle quantum liquid:

mean-field approximation
Skyrme, Gogny, RMF

bound state formation

Nuclear statistical equilibrium:

ideal mixture of all bound states
(clusters:) chemical equilibrium

low density limit

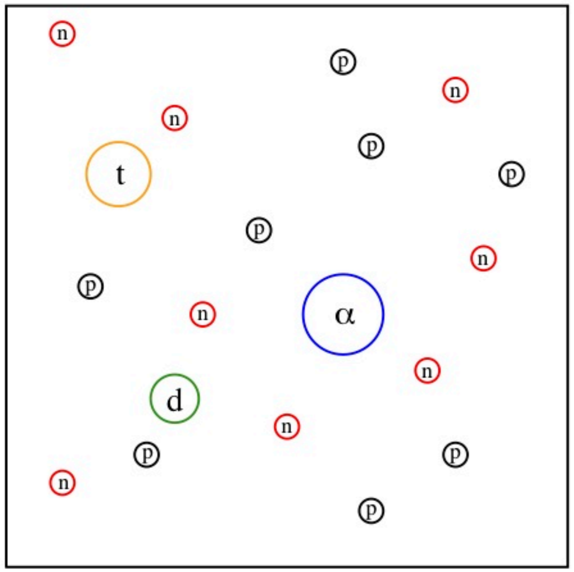
saturation density

Nuclear statistical equilibrium (NSE)

Chemical picture:

Ideal mixture of reacting components

Mass action law

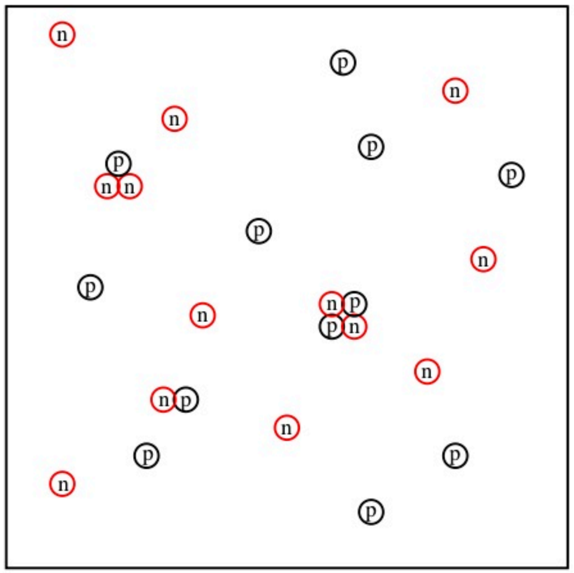


Interaction between the components
internal structure: Pauli principle

Physical picture:

"elementary" constituents

and their interaction



Quantum statistical (QS) approach,
quasiparticle concept, virial expansion

Different approximations

Ideal Fermi gas:
protons, neutrons,
(electrons, neutrinos,…)

medium effects

Quasiparticle quantum liquid:
mean-field approximation
BHF, Skyrme, Gogny, RMF

bound state formation

Nuclear statistical equilibrium:
ideal mixture of all bound states
(clusters:) chemical equilibrium

Chemical equilibrium
with quasiparticle clusters:
self-energy and Pauli blocking

Effective wave equation for the deuteron in matter

In-medium two-particle wave equation in mean-field approximation

$$\left(\frac{p_1^2}{2m_1}+\Delta_1+\frac{p_2^2}{2m_2}+\Delta_2\right)\Psi_{d,P}(p_1,p_2)+\sum_{p_1',p_2'}(1-f_{p_1}-f_{p_2})V(p_1,p_2;p_1',p_2')\Psi_{d,P}(p_1',p_2')$$

$$=E_{d,P}\Psi_{d,P}(p_1,p_2)$$

Add self-energy

Pauli-blocking

Thouless criterion

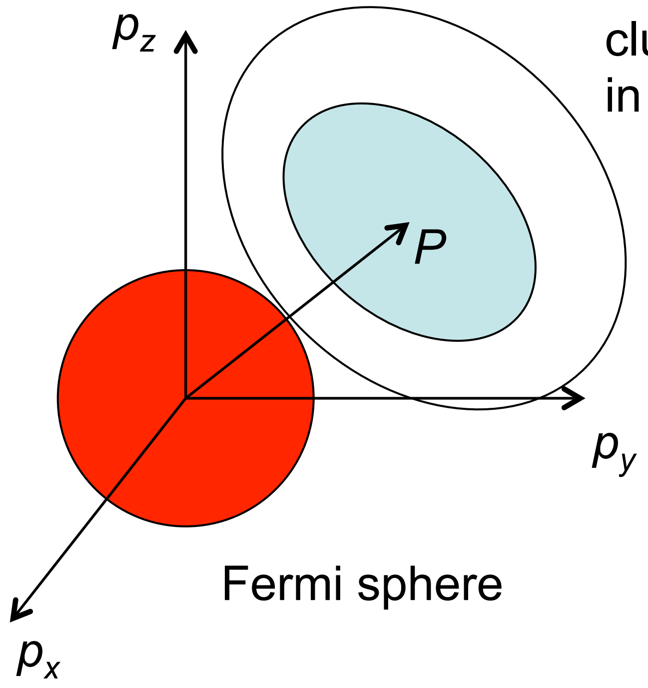
$$E_d(T,\mu)=2\mu$$

Fermi distribution function

$$f_p=\left[e^{(p^2/2m-\mu)/k_BT}+1\right]^{-1}$$

BEC-BCS crossover:
Alm et al.,1993

Pauli blocking – phase space occupation



cluster wave function (deuteron, alpha,…)
in momentum space

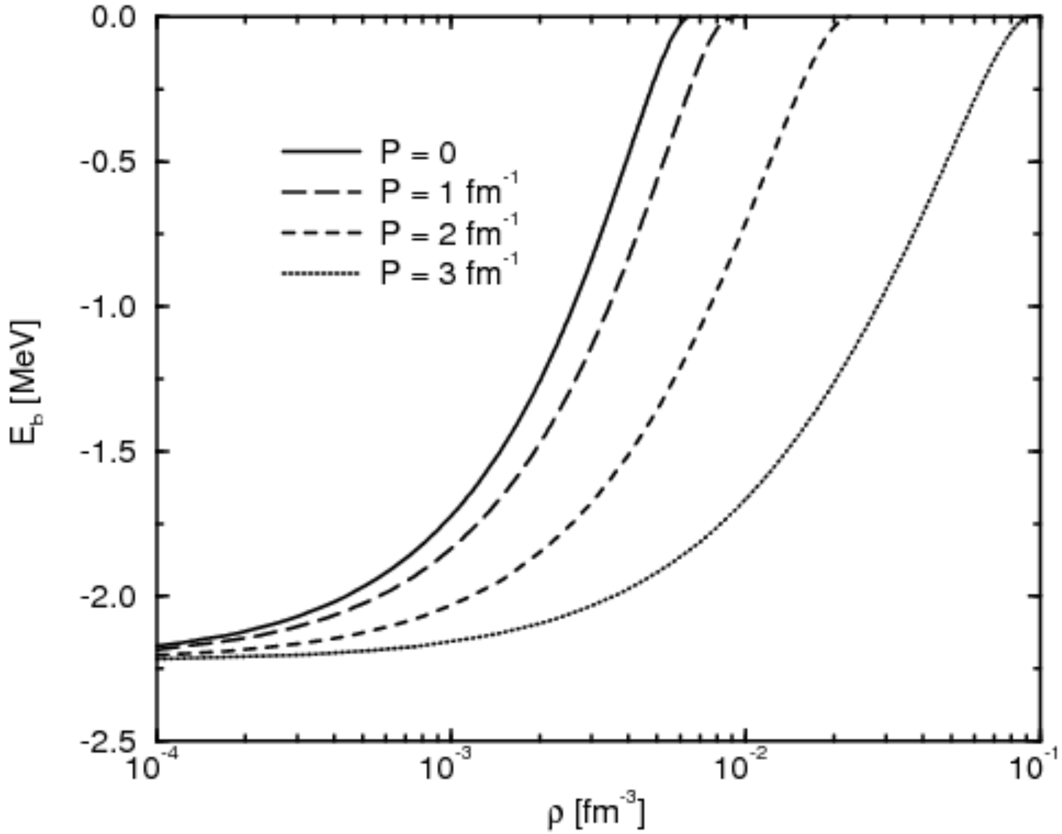
P - center of mass momentum

The Fermi sphere is forbidden,
deformation of the cluster wave function
in dependence on the c.o.m. momentum P

momentum space

The deformation is maximal at $P = 0$.
It leads to the weakening of the interaction
(disintegration of the bound state).

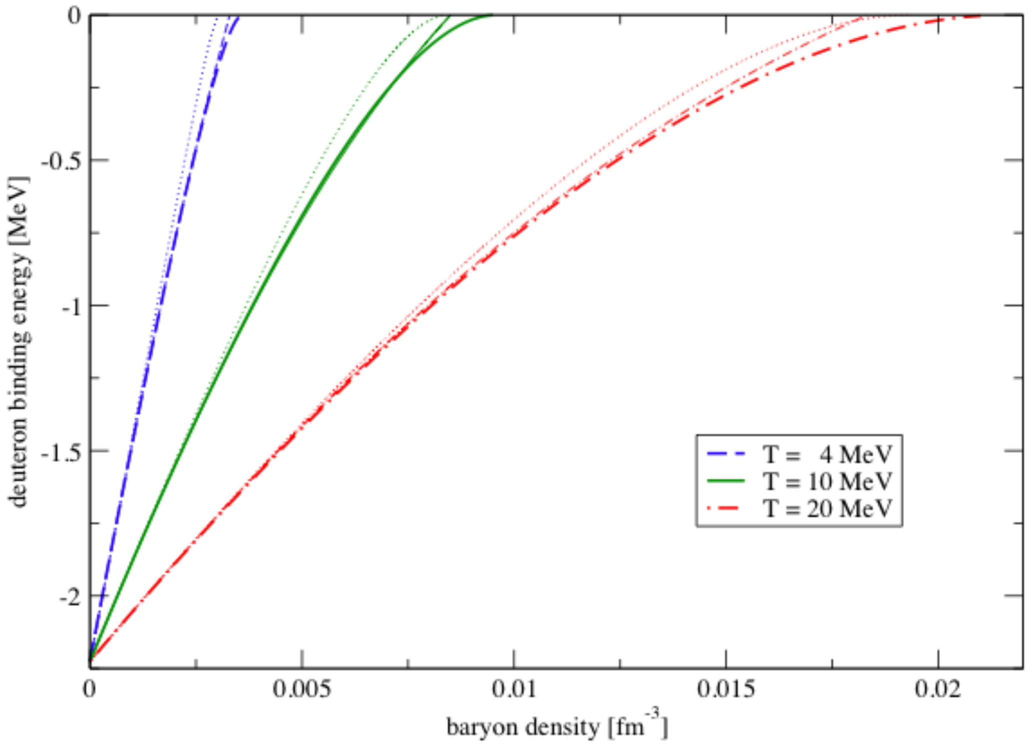
Deuterons in nuclear matter



T=10 MeV, P: center of mass momentum

Shift of the deuteron bound state energy

Dependence on nucleon density, various temperatures,
zero center of mass momentum

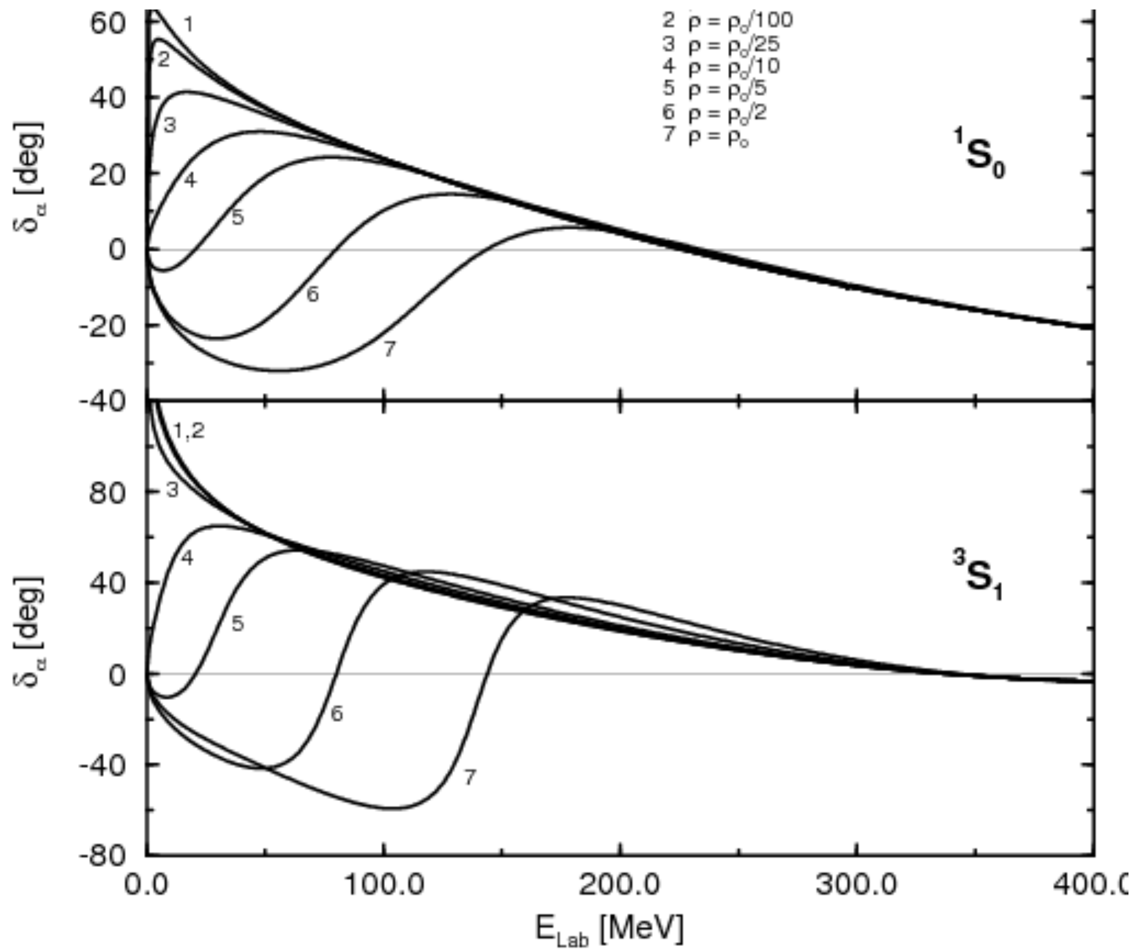


thin lines:

fit formula

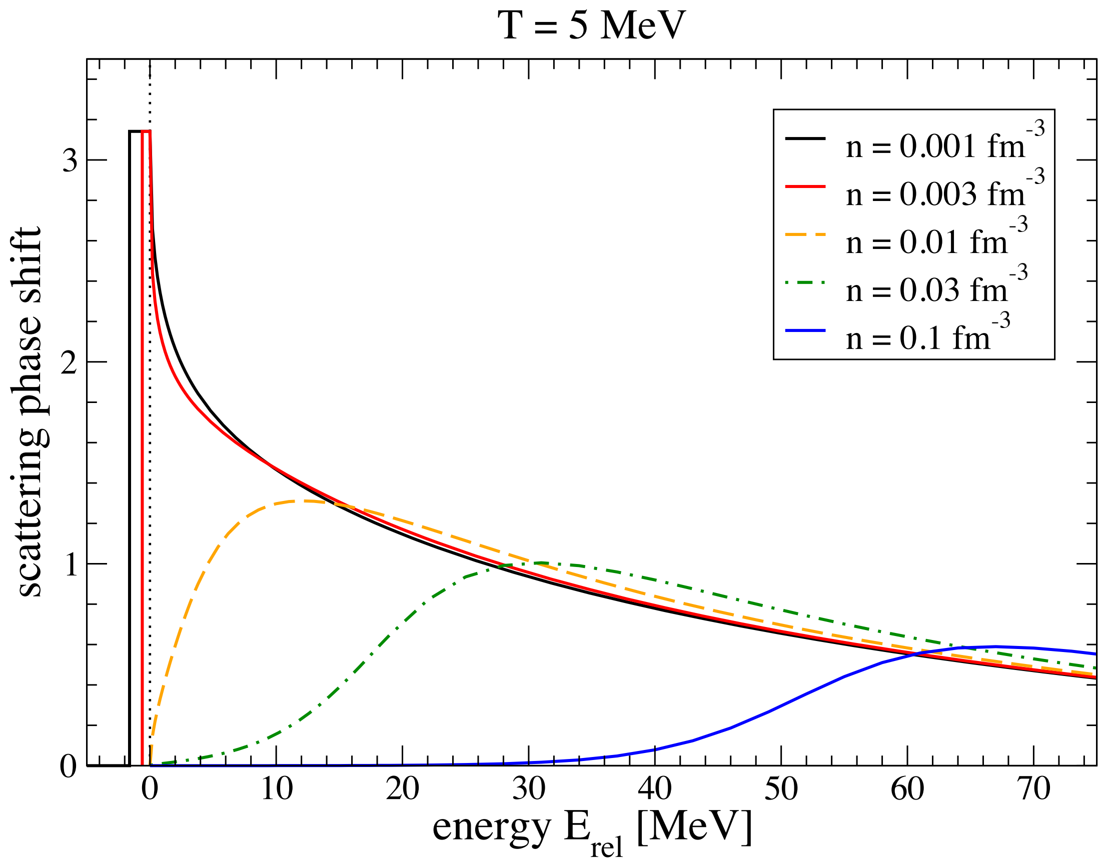
G.R., Nucl. Phys. A **867**, 66 (2011)

Scattering phase shifts in matter



Deuteron-like scattering phase shifts

Virial coeff. ∝ $e^{-E_d^0/T} - 1 + \frac{1}{\pi T}\int_0^\infty dE\, e^{-E/T}\left\{\delta_c(E) - \frac{1}{2}\sin[2\delta_c(E)]\right\}$



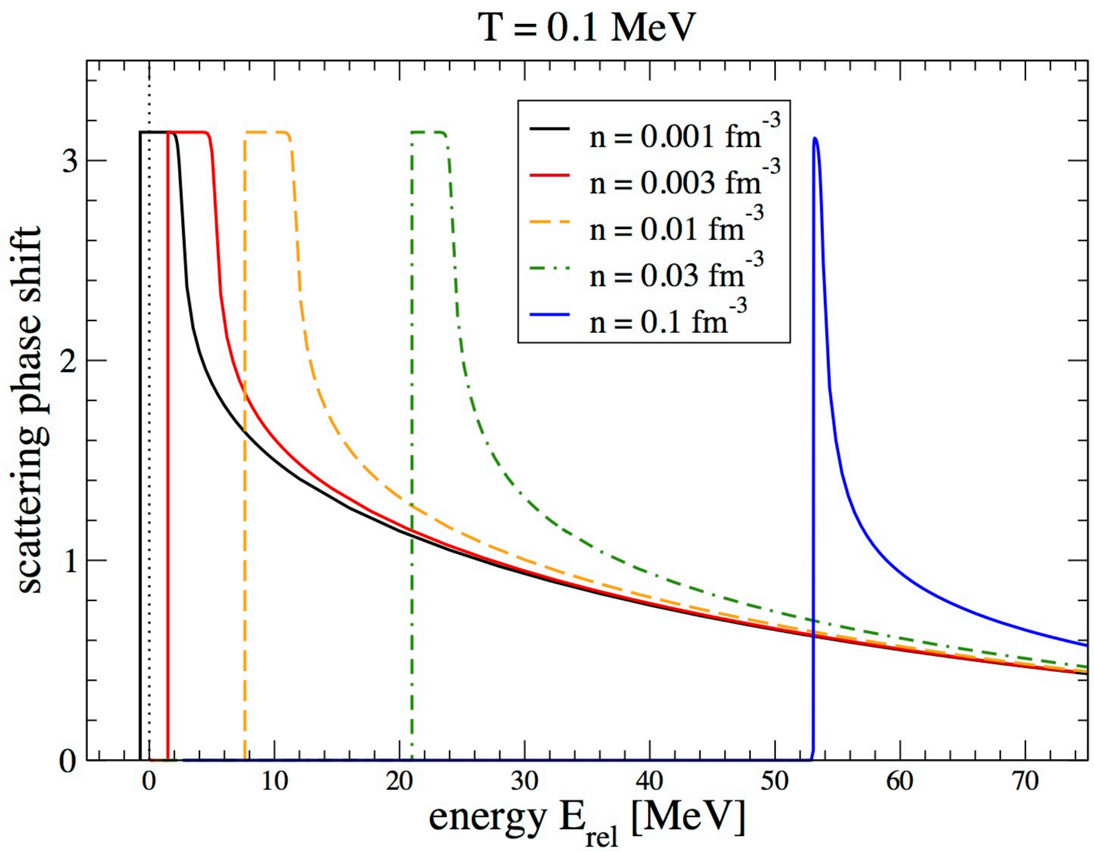
Tamm-Dancoff

deuteron bound state -2.2 MeV

G. Roepke, J. Phys.: Conf. Series 569, 012031 (2014)
Phys. Part. Nucl. 46, 772 (2015) [arXiv:1408.2654]

Deuteron-like scattering phase shifts

Virial coeff. $\propto e^{-E_d^0/T} - 1 + \frac{1}{\pi T}\int_0^\infty dE\, e^{-E/T}\left\{\delta_c(E) - \frac{1}{2}\sin[2\delta_c(E)]\right\}$



Tamm-Dancoff

deuteron bound state -2.2 MeV

G. Roepke, J. Phys.: Conf. Series 569, 012031 (2014)
Phys. Part. Nucl. 46, 772 (2015) [arXiv:1408.2654]

Two-particle correlations

Generalized
Beth-Uhlenbeck Approach
for Hot Nuclear Matter

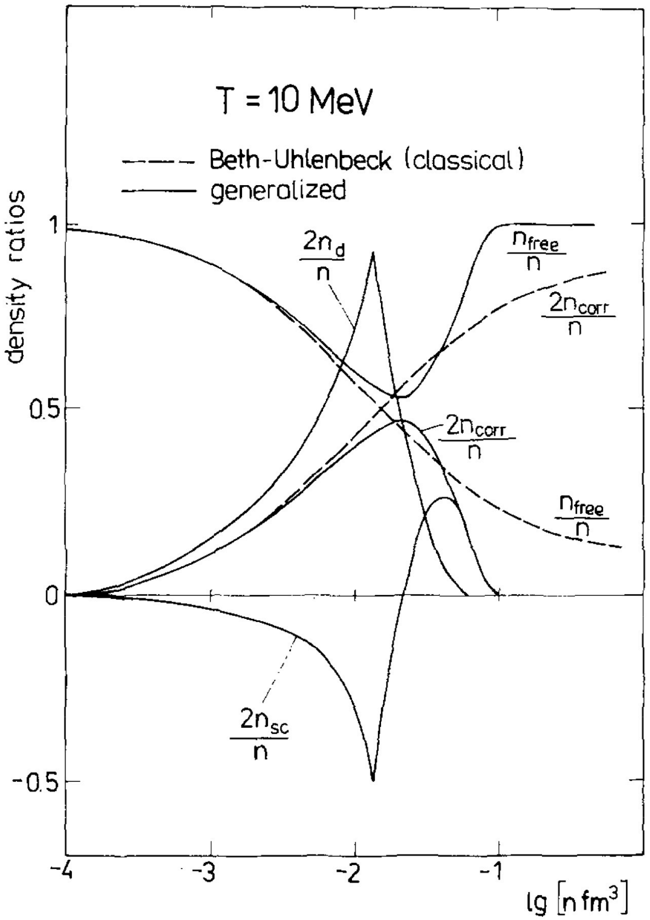


FIG. 7. The composition of nuclear matter as a function of the density n for given temperature $T = 10$ MeV. The solid and dashed lines show the results of the generalized and classical Beth–Uhlenbeck approach, respectively. Note the distinct behavior of n_{free} and n_{corr} predicted by the two approaches in the low and high density limit!

M. Schmidt, G.R., H. Schulz
Ann. Phys. **202**, 57 (1990)

Few-particle Schrödinger equation in a dense medium

4-particle Schrödinger equation with medium effects

$$\left(\left[E^{HF}(p_1)+E^{HF}(p_2)+E^{HF}(p_3)+E^{HF}(p_4)\right]\right)\Psi_{n,P}(p_1,p_2,p_3,p_4)$$

$$+\sum_{p_1',p_2'}(1-f_{p_1}-f_{p_2})V(p_1,p_2;p_1',p_2')\Psi_{n,P}(p_1',p_2',p_3,p_4)$$

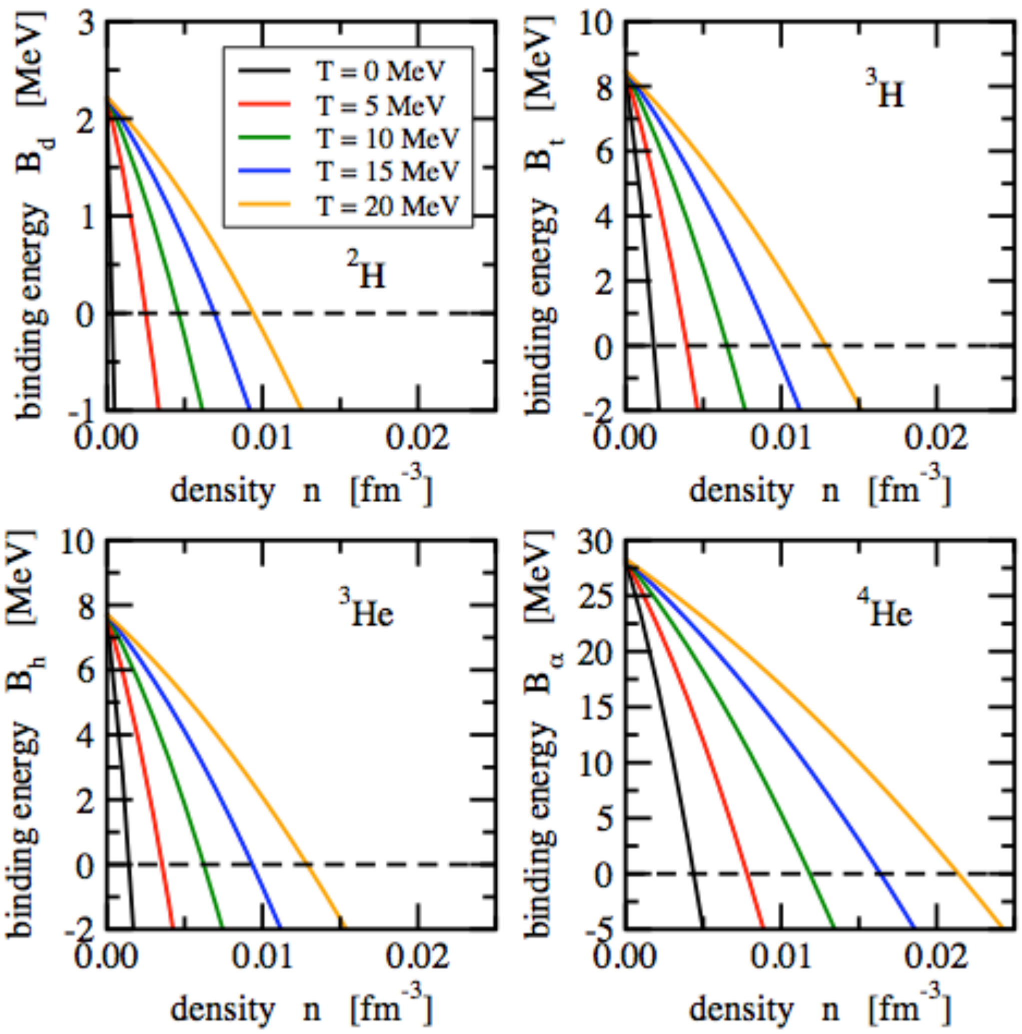
$$+\{permutations\}$$

$$=E_{n,P}\Psi_{n,P}(p_1,p_2,p_3,p_4)$$

Thouless criterion
for quantum condensate:

$E_{n,P=0}(T,\mu) = 4\mu$

Shift of Binding Energies of Light Clusters



Symmetric matter

G.R., PRC **79**, 014002 (2009)
S. Typel et al.,
PRC **81**, 015803 (2010)

Composition of dense nuclear matter

$$n_p(T,\mu_p,\mu_n) = \frac{1}{V}\sum_{A,\nu,K} Z_A f_A\{E_{A,\nu K} - Z_A\mu_p - (A-Z_A)\mu_n\}$$

$$n_n(T,\mu_p,\mu_n) = \frac{1}{V}\sum_{A,\nu,K} (A-Z_A) f_A\{E_{A,\nu K} - Z_A\mu_p - (A-Z_A)\mu_n\}$$

mass number A
charge Z_A
energy $E_{A,\nu,K}$

$$f_{A(z)} = \frac{1}{\exp(z/T) - (-1)^A}$$

ν: internal quantum number
excited states, continuum correlations

- Medium effects: correct behavior near saturation
  self-energy and Pauli blocking shifts of binding energies,
  Coulomb corrections due to screening (Wigner-Seitz, Debye)

Medium modification of light clusters

- Single-particle, two-particle, etc. spectral function
  quasiparticle concept: Peak structures in the few-body spectral function

- Dispersion relation: quasiparticle energy is a function of total few-body momentum K, but also T, n_B, Y_e:
  $E_{A,nu,K}(T, n_B, Y_e)$

- Solution of a few-body equation. For practical use parametrization (like Skyrme or RMF, DFT): G.R., Phys. Rev. C 92,054001 (2015)

- Alternative simple approaches to describe the medium effects:
  excluded volume,
  overlap of the Fermi sphere with the bound state in momentum space,
  momentum dependent cutoff.

EOS: continuum contributions

Partial density of channel A,c at P (for instance, $^3S_1$= d):

$$z_{A,c}^{\rm part}(\mathbf{P};T,\mu_n,\mu_p) = e^{(N\mu_n+Z\mu_p)/T}\left\{\sum_{\nu_c}^{\rm bound} g_{A,\nu_c}\, e^{-E_{A,\nu_c}(\mathbf{P})/T}\,\Theta\left[-E_{A,\nu_c}(\mathbf{P})+E_{A,c}^{\rm cont}(\mathbf{P})\right]+z_{A,c}^{\rm cont}(\mathbf{P})\right\}$$

separation: bound state part – continuum part ?

$$z_c^{\rm part}(\mathbf{P};T,n_B,Y_p) = e^{[N\mu_n+Z\mu_p-NE_n(\mathbf{P}/A;T,n_B,Y_p)-ZE_p(\mathbf{P}/A;T,n_B,Y_p)]/T}$$
$$\times g_c\left\{\left[e^{-E_c^{\rm intr}(\mathbf{P};T,n_B,Y_p)/T}-1\right]\Theta\left[-E_c^{\rm intr}(\mathbf{P};T,n_B,Y_p)\right]+v_c(\mathbf{P};T,n_B,Y_p)\right\}$$

parametrization (d – like):

$$v_c(\mathbf{P}=0;T,n_B,Y_p)\approx\left[1.24+\left(\frac{1}{v_{T_I=0}(T)}-1.24\right)e^{\gamma_c n_B/T}\right]^{-1}.$$

$$v_d^0(T)=v_{T_I=0}^0(T)\approx 0.30857+0.65327\; e^{-0.102424\;T/\mathrm{MeV}}$$

G. Roepke, PRC 92,054001 (2015)

Different approximations

Ideal Fermi gas:
protons, neutrons,
(electrons, neutrinos,…)

bound state formation

Nuclear statistical equilibrium:
ideal mixture of all bound states
(clusters:) chemical equilibrium

continuum contribution

Second virial coefficient:
account of continuum contribution,
scattering phase shifts, Beth-Uhl.E.

medium effects

Quasiparticle quantum liquid:
mean-field approximation
Skyrme, Gogny, RMF

Chemical equilibrium
with quasiparticle clusters:
self-energy and Pauli blocking

Different approximations

medium effects

Ideal Fermi gas:
protons, neutrons,
(electrons, neutrinos,…)

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mean-field approximation
BHF, Skyrme, Gogny, RMF

bound state formation

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ideal mixture of all bound states
(clusters:) chemical equilibrium

Chemical equilibrium of quasiparticle clusters:
self-energy and Pauli blocking

continuum contribution

Second virial coefficient:
account of continuum contribution,
scattering phase shifts, Beth-Uhl.Eq.

Generalized Beth-Uhlenbeck formula:
medium modified binding energies,
medium modified scattering phase shifts

Different approximations

medium effects

Ideal Fermi gas:
protons, neutrons,
(electrons, neutrinos,…)

Quasiparticle quantum liquid:
mean-field approximation
BHF, Skyrme, Gogny, RMF

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Second virial coefficient:
account of continuum contribution,
scattering phase shifts, Beth-Uhl.Eq.

Generalized Beth-Uhlenbeck formula:
medium modified binding energies,
medium modified scattering phase shifts

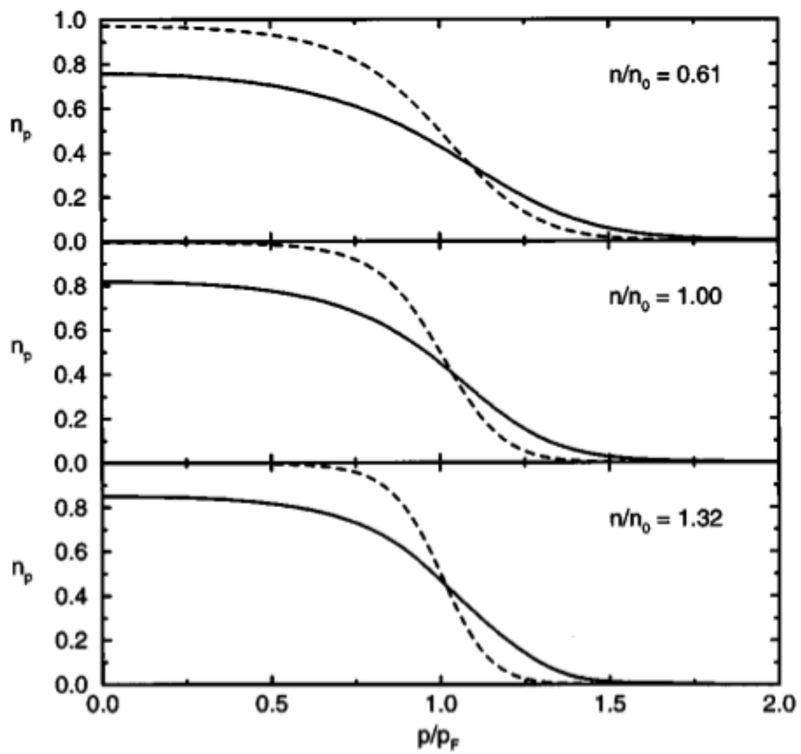
chemical & physical picture

Cluster virial approach:
all bound states (clusters)
scattering phase shifts of all pairs

Correlated medium:
phase space occupation by all bound states
in-medium correlations, quantum condensates

Single nucleon distribution function

Dependence on density

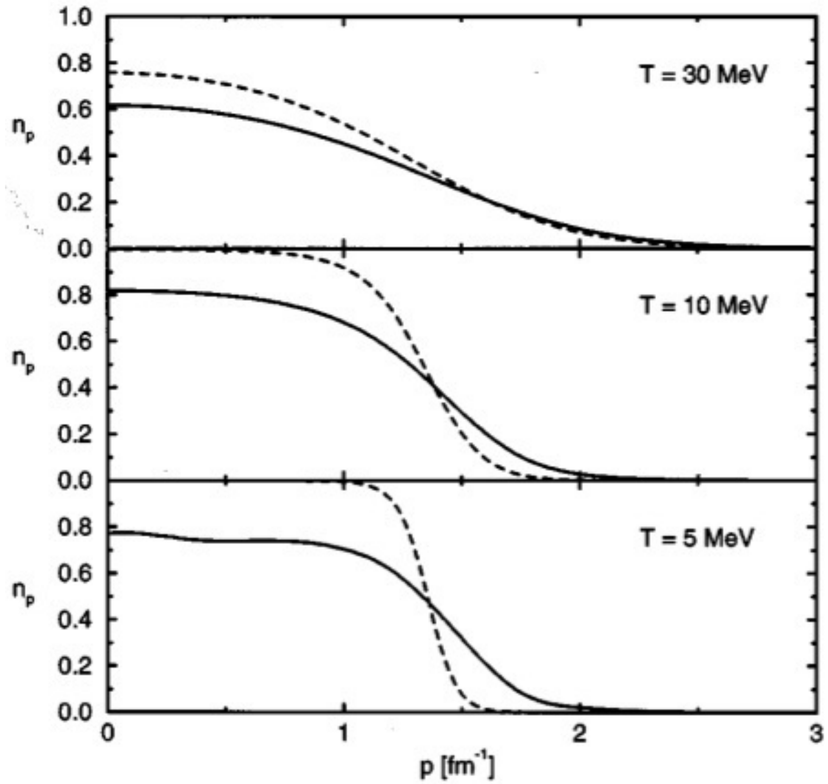


T = 10 MeV

Alm et al., PRC 53, 2181 (1996)

Single nucleon distribution function

Dependence on temperature



saturation density

Alm et al., PRC 53, 2181 (1996)

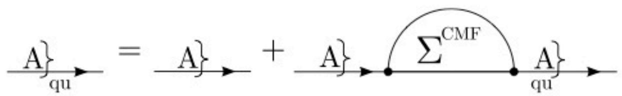
Cluster virial expansion for nuclear matter within a quasiparticle statistical approach

Generalized Beth-Uhlenbeck approach

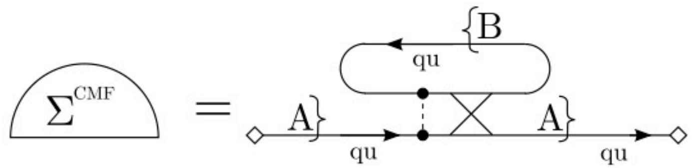
$$n_1^{\mathrm{qu}}(T,\mu_p,\mu_n) = \sum_{A,Z,\nu} \frac{A}{\Omega} \sum_{\substack{\vec{P} \\ P>P_{\mathrm{Mott}}}} f_A\big(E_{A,Z,\nu}(\vec{P};T,\mu_p,\mu_n),\mu_{A,Z,\nu}\big)$$

$$n_2^{\mathrm{qu}}(T,\mu_p,\mu_n) = \sum_{A,Z,\nu} \sum_{A',Z',\nu'} \frac{A+A'}{\Omega} \sum_{\vec{P}} \sum_{c} g_c \frac{1+\delta_{A,Z,\nu;A',Z',\nu'}}{2\pi} \times \int_0^\infty dE\, f_{A+A'}\left(E_c(\vec{P};T,\mu_p,\mu_n)+E,\mu_{A,Z}+\mu_{A',Z'}\right) 2\sin^2(\delta_c)\frac{d\delta_c}{dE}$$

Avoid double counting



Generating functional



G.R., N. Bastian, D. Blaschke, T. Klaehn, S. Typel, H. Wolter, NPA **897**, 70 (2013)

Cluster - mean field approximation

Cluster (A) interacting with a distribution of clusters (B) in the medium,

fully antisymmetrized

$$\sum_{1'\dots A'} \{H_A^0(1\dots A, 1'\dots A') + \sum_i \Delta_i^{A,mf}\delta_{k,k'} + \frac{1}{2}\sum_{i,j}\Delta V_{ij}^{A,mf}\delta_{l,l'} - E_{A\nu P}\delta_{k,k'}\}\psi_{A\nu P}(1'\dots A') = 0$$

self-energy

$$\Delta_1^{A,mf}(1) = \sum_2 V(12,12)_{ex} f^*(2) + \sum_{B\nu P}\sum_{2\dots B'} f_B(E_{B\nu P})\sum_i V_{1i}(1i,1'i')\psi^*_{B\nu P}(1\dots B)\psi_{B\nu P}(1'\dots B')$$

effective interaction

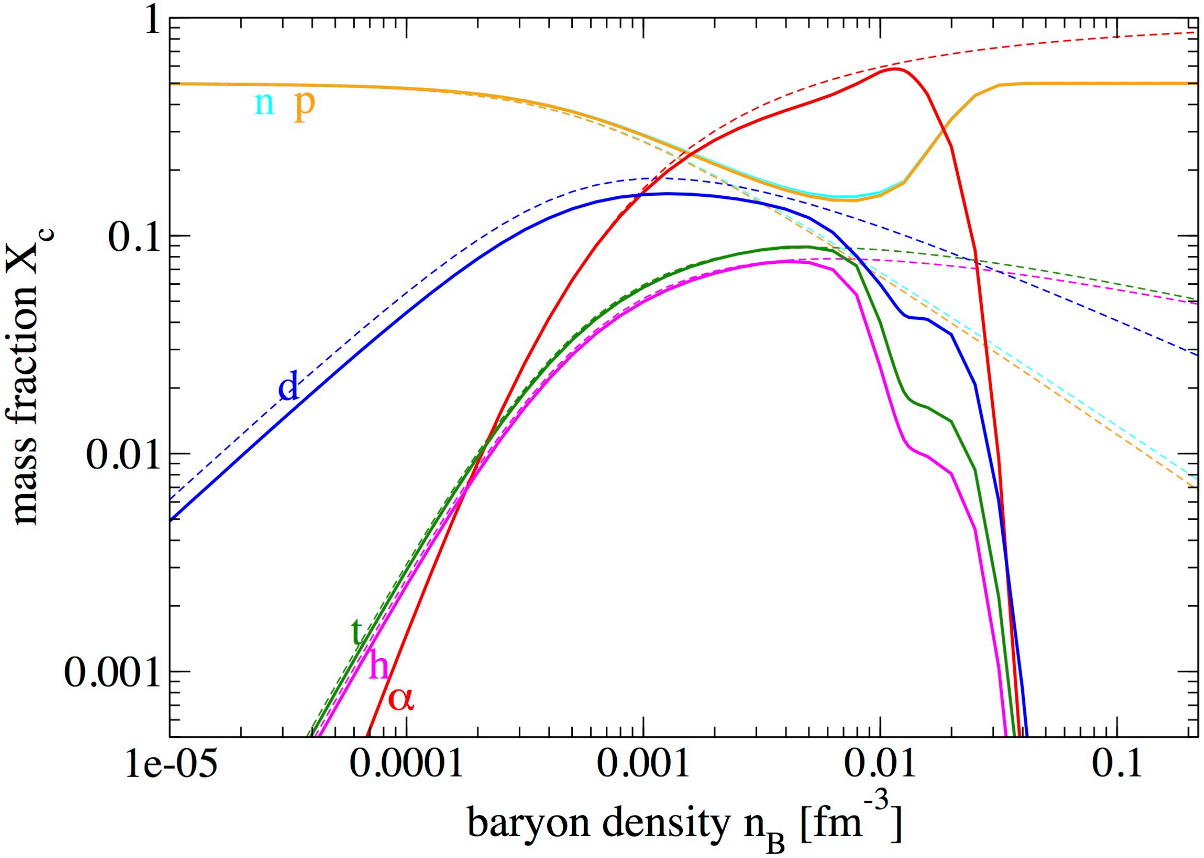
$$\Delta V_{12}^{A,mf} = -\frac{1}{2}[f^*(1) + f^*(2)]V(12,1'2') - \sum_{B\nu P}\sum_{2^*\dots B''} f_B(E_{B\nu P})\sum_i V_{1i}\psi^*_{B\nu P}(22^*\dots B^*)\psi_{B\nu P}(2'2''\dots B'')$$

phase space occupation $\quad f^*(1) = f_1(1) + \sum_{B\nu P}\sum_{2\dots B} f_B(E_{B\nu P})\,|\psi_{B\nu P}(1\dots B)|^2$

5. EoS including correlations

- Composition
- Chemical potential
- Free energy
- Phase transition
- Quantum condensates

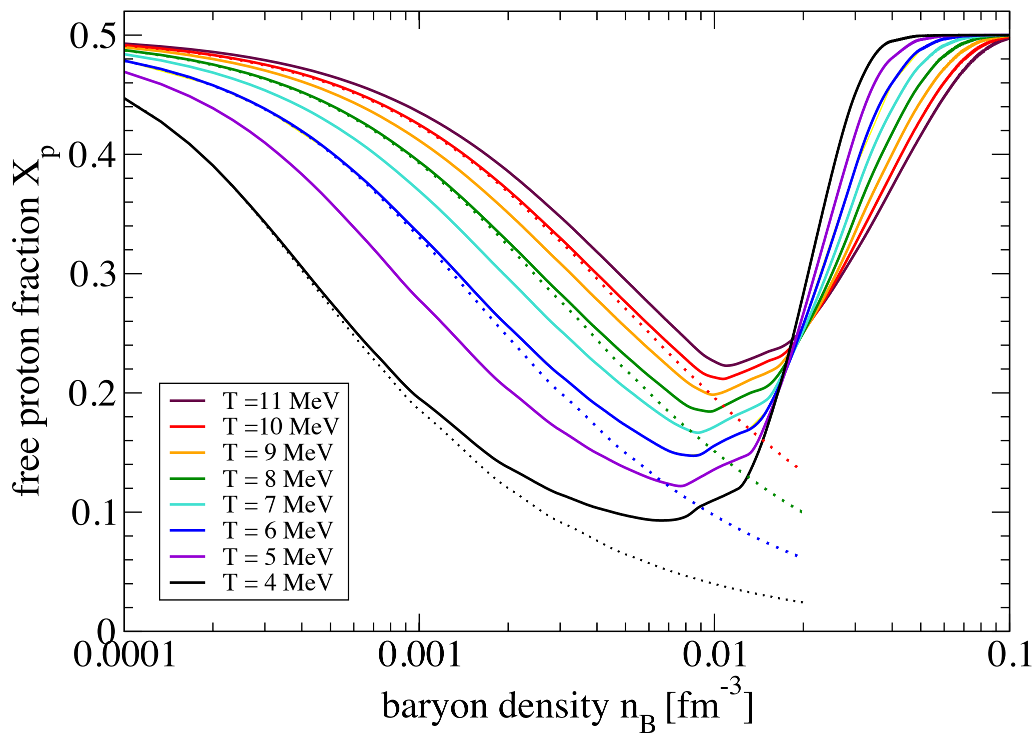
Light Cluster Abundances



Composition of symmetric matter in dependence on the baryon density n_B, T = 5 MeV.
Quantum statistical calculation (full) compared with NSE (dotted).

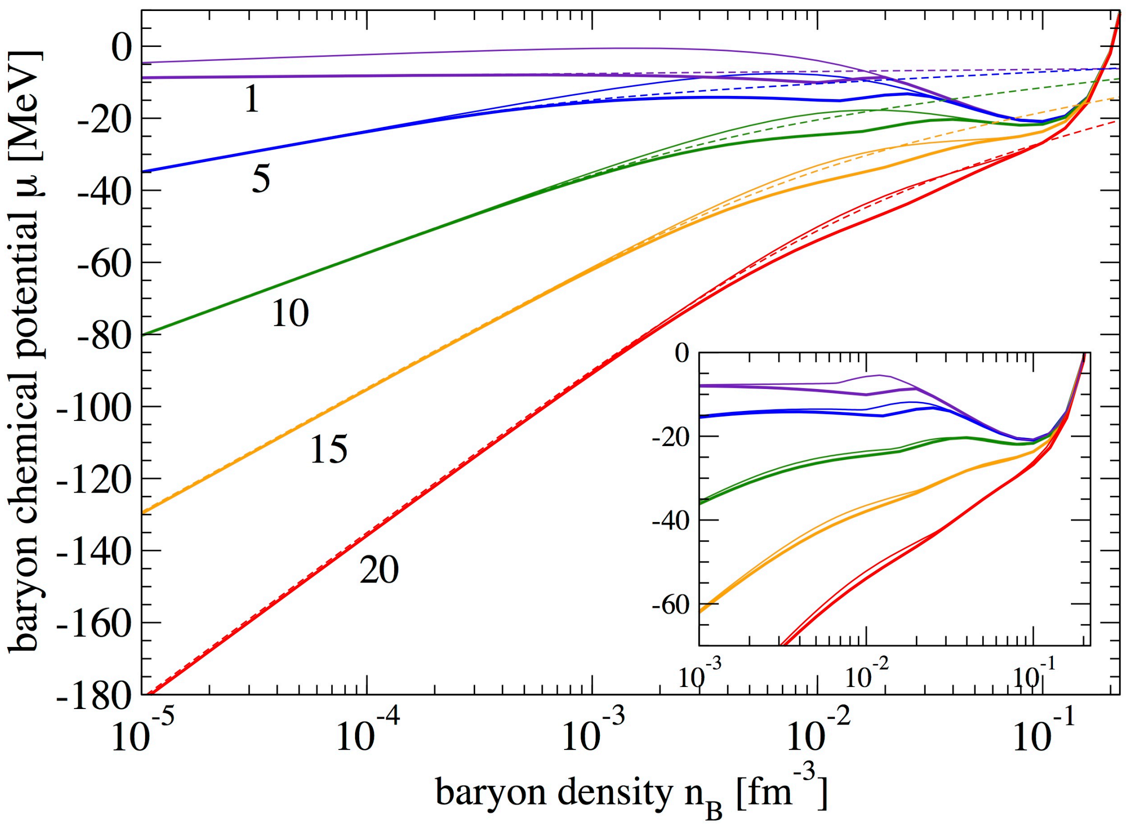
G. R., PRC **92**, 054001 (2015)

Pauli blocking in symmetric matter



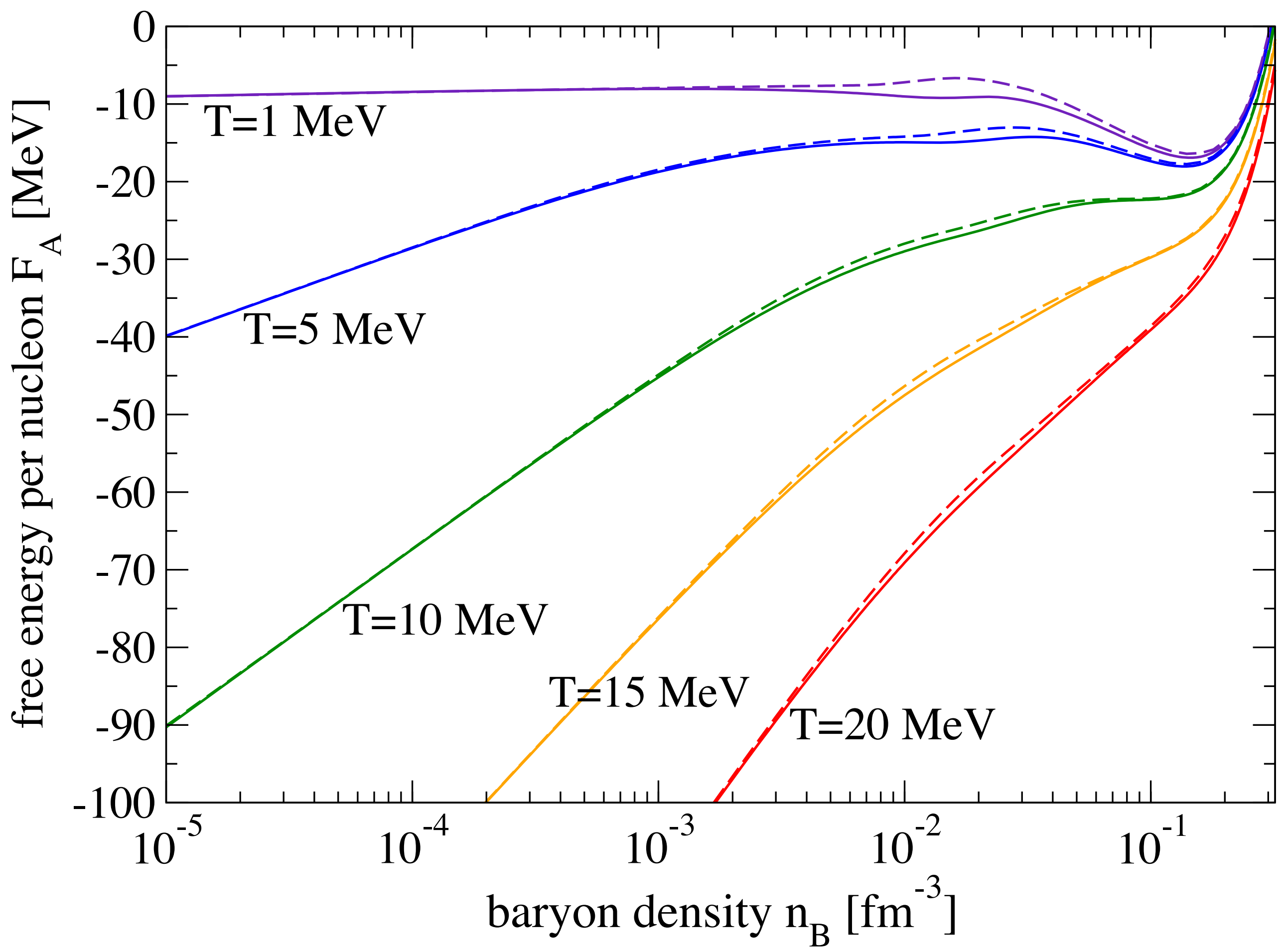
Free proton fraction as function of density and temperature in symmetric matter.
QS calculations (solid lines) are compared with the NSE results (dotted lines).
Mott effect in the region $n_{saturation}/5$.

Equation of state: chemical potential



Chemical potential for symmetric matter. T=1, 5, 10, 15, 20 MeV.
QS calculation compared with RMF (thin) and NSE (dashed).
Insert: QS calculation without continuum correlations (thin lines).

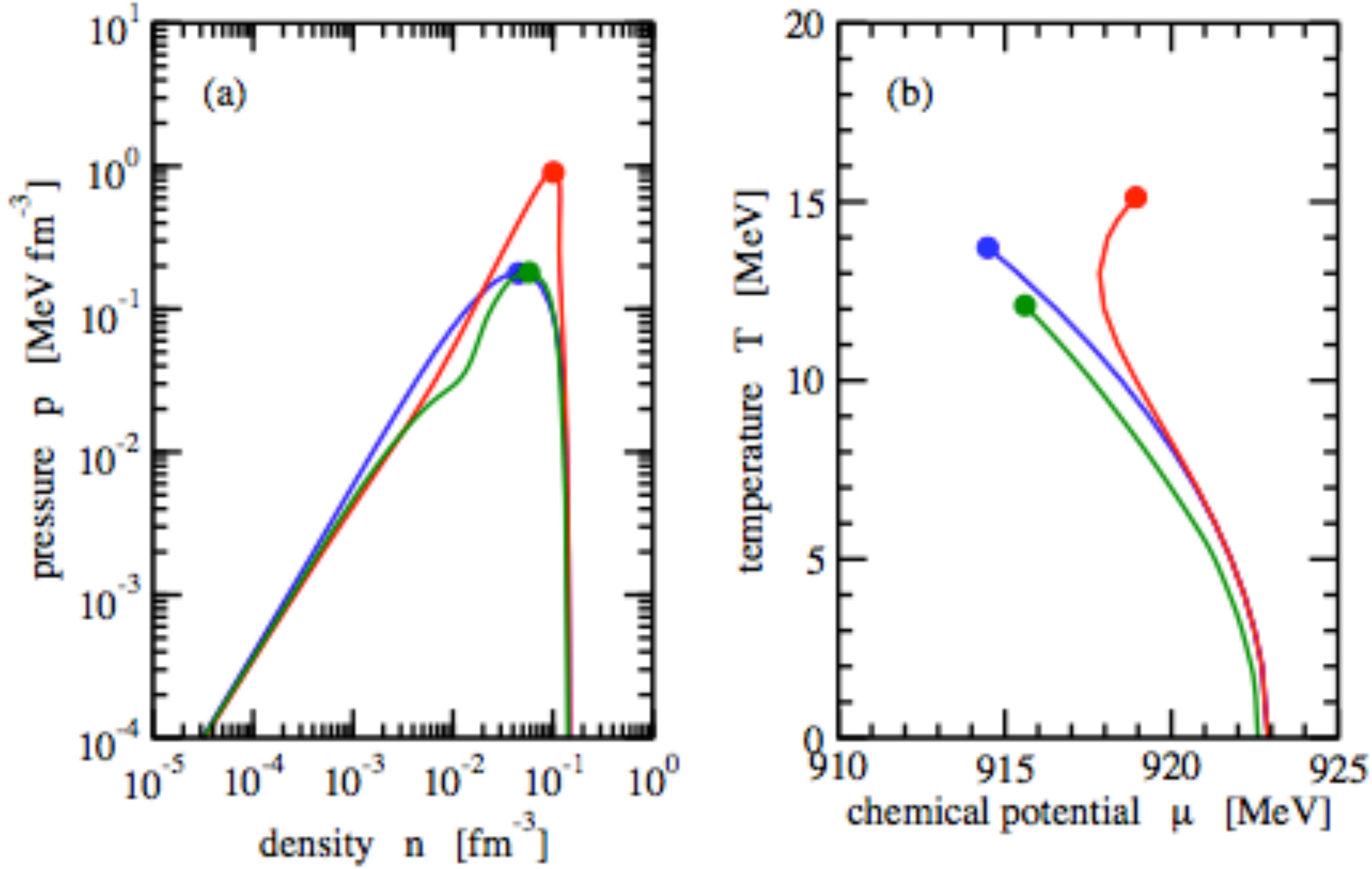
Symmetric matter: free energy per nucleon



Dashed lines: no continuum correlations

G. R., PRC **92**, 054001 (2015)

Liquid-vapor phase transition



blue: no light cluster, green: with light clusters, QS, red: cluster-RMF

S. Typel et al., PRC **81**, 015803 (2010)

Effective wave equation for the deuteron in matter

In-medium two-particle wave equation in mean-field approximation

$$\left(\frac{p_1^2}{2m_1}+\Delta_1+\frac{p_2^2}{2m_2}+\Delta_2\right)\Psi_{d,P}(p_1,p_2)+\sum_{p_1',p_2'}(1-f_{p_1}-f_{p_2})V(p_1,p_2;p_1',p_2')\Psi_{d,P}(p_1',p_2')$$

$$=E_{d,P}\Psi_{d,P}(p_1,p_2)$$

Add self-energy

Pauli-blocking

Fermi distribution function

$$f_p=\left[e^{(p^2/2m-\mu)/k_BT}+1\right]^{-1}$$

Correlated medium?

Thouless criterion

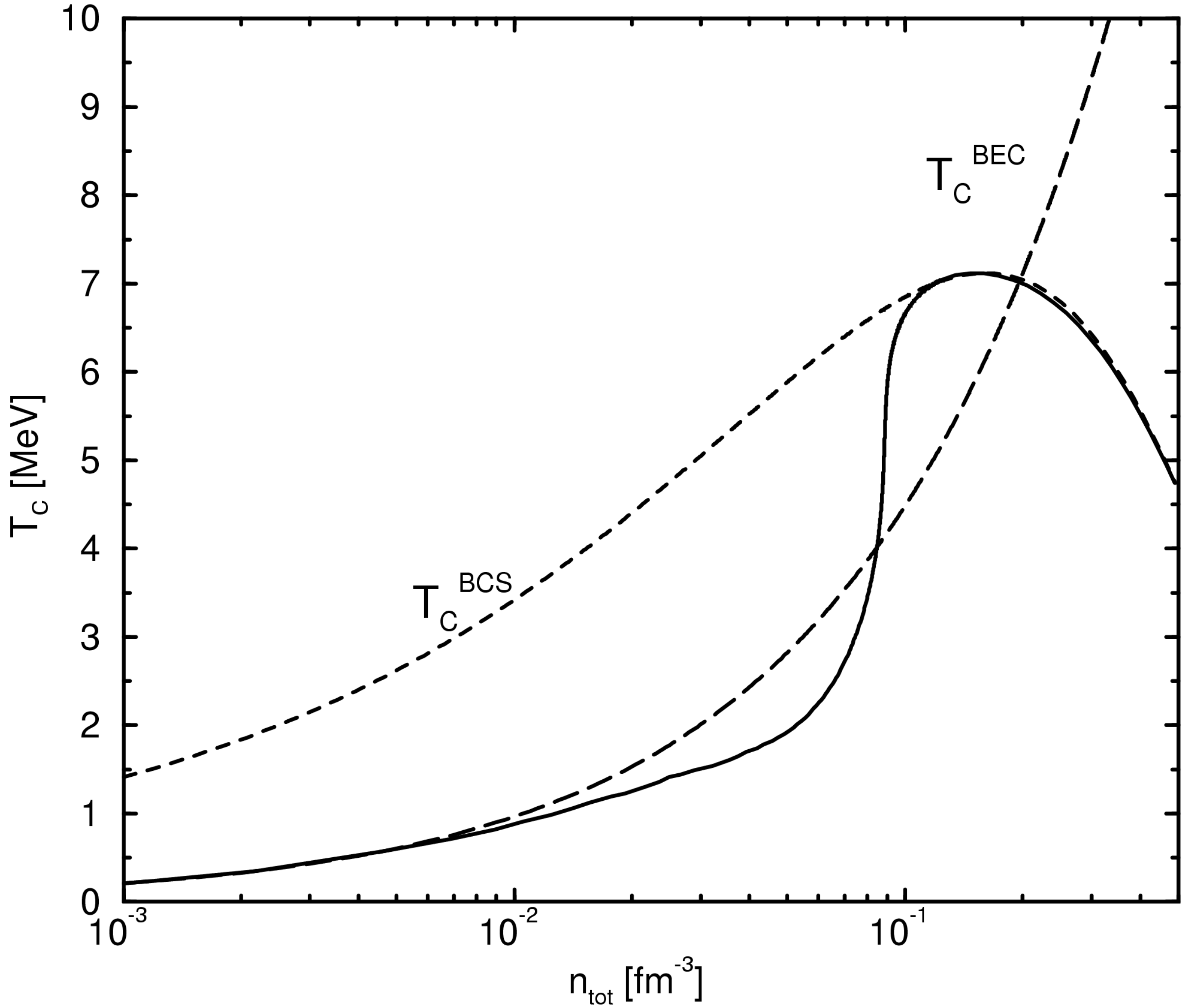
$$E_d(T,\mu)=2\mu$$

BEC-BCS crossover:
Alm et al.,1993

Quantum condensate

Bose-Einstein-
Condensation
of deuterons
(BEC)

Bardeen-Cooper
Schrieffer
pairing
(BCS)



Composition of symmetric nuclear matter

Fraction of correlated matter (virial expansion, Generalized Beth-Uhlenbeck approach, contribution of bound states, of scattering states, phase shifts)

H. Stein et al.,
Z. Phys. **A351**, 259 (1995)