EXERCISE: PARTIAL SUMMATIONS AND BOUND STATES IN QUANTUM STATISTICS

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15. 06. 2021

(1) Solve the two-particle Green function in ladder approximation and show that in the low-density limit this leads to the the solution of the two-particle Schrödinger equation. Derive the in-medium Schrödinger equation.

(2) Derive the mass action law (MAL) from the equation of state which relates the density to the single-particle spectral function. Consider the low-density limit and only bound states.

(3) Solve the Bethe-Salpeter equation for the two-nucleon problem $p-n$ (proton-neutron: isospin singlet, spin triplet, deuteron) and $n-n$ (isospin triplet, spin singlet, Cooper-pairing). Use $\hbar^2/m = 41.46$ in units MeV, fm, and the Yamaguchi separable interaction (4.10.1) [see Y. Yamaguchi, Phys. Rev. 95, 1628 (1954)]

$$V(p_1p_2, p_1'p_2') = -\frac{\lambda}{\Omega_0}\frac{1}{(p_2-p_1)^2/(2\beta)^2+1}\frac{1}{(p_2'-p_1')^2/(2\beta)^2+1}$$

with the normalization volume Ω_0, the range parameter $\beta = 1.4488$ fm^{-1}. The interaction strengths are adopted as $\lambda_s = 678.6$ MeV fm^3 for the $n-n$, spin singlet, channel (neutron matter), and $\lambda_t = 967.82$ MeV fm^3 for the $n-p$, spin triplet, channel, where the deuteron is found. Neglect the self-energy shifts.
Show that these parameter values give the correct value -2.225 MeV for the deuteron binding energy as well as good approximations for the triplet scattering length $a_t = 5.378$ fm and the singlet scattering length $a_s = -23.69$ fm. Show that the scattering length follows as $a = 2/\beta[1 - 8\pi\hbar^2/(\lambda m\beta)]^{-1}$, the effective range is $r_0 = [1 + 16\pi\hbar^2/(\lambda m\beta)]/\beta$. Compare with values from the literature.

(4) Solve the Gor'kov equation for nuclear matter to determine the critical temperature $T_c(n_B)$ as function of the baryon density n_B. Neglect the self-energy shifts.
Calculate the critical temperature for the formation of a quantum condensate (pairing) for neutron matter, density $n_B = n_n$ and for symmetric matter, $n_B = n_n + n_p = 2n_n$, where n_B is the density of all baryons and n_n, n_p- the neutron/proton densities. Example: $n_B = 0.15$ fm^{-3}.