Equation of state for binary neutron star mergers and core-collapse supernovae: Lecture 1: Hadronic Phases

A Sedrakian

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Armen Sedrakian

Karpacz Winter School, 2021

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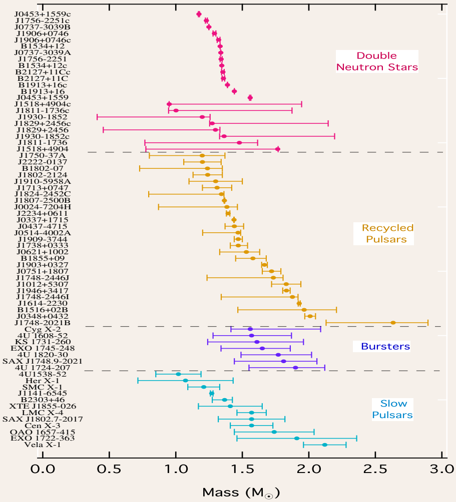
Measured masses and radii



The radii are less well determined, but NICER experiment predicts $\sim$ 13 km for $1.4M_\odot$.

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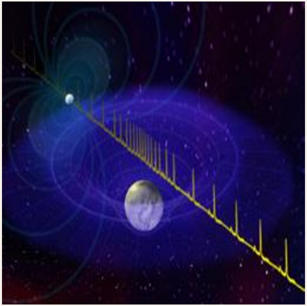
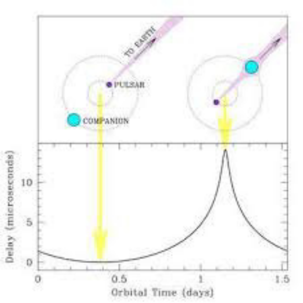
Shapiro delay in neutron star white-dwarf binaries





- The millisecond pulsar J1614-2230 in a binary with a white dwarf, $M = 1.97 \pm 0.04 M_\odot$ (Demorest et al. 2010), Relativistic Shapiro delay.
- The millisecond pulsar J0348+0432 in a binary with a white dwarf $M = 2.01 \pm 0.04 M_\odot$ (Antoniadis et al. 2013) [theor. assumptions about WD cooling.]
- The millisecond pulsar J0740+6620 $M = 2.14^{+0.10}_{-0.09} M_\odot$ (NANOGrav, Cromartie et al. 2019) Relativistic Shapiro delay.

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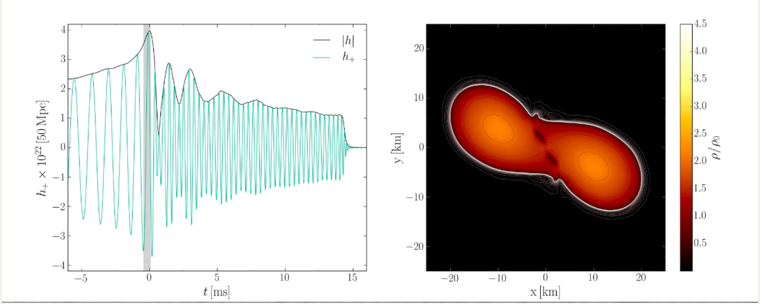
GW170817: First gravitational waves from a neutron star merger (Ligo-Virgo-Collaboration)



The associated EM events observed by over 70 observatories :

- + 2 sec gamma ray burst is detected
- +10 h 52 min bright source in optical
- +11 h 36 min infrared emission; +15 h ultraviolet
- +9 days X-rays; +16 days radio

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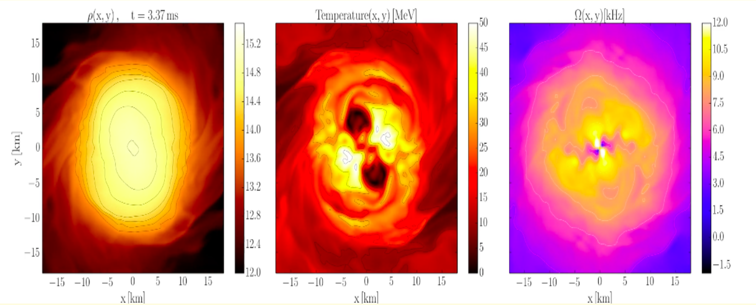
New nuclear physics laboratories



pictures courtesy: J. Pappenfort

- extreme high temperatures $\sim$ 100 MeV
- supra-nuclear densities $\sim 5 \times n_s$
- high and differential rotation rates

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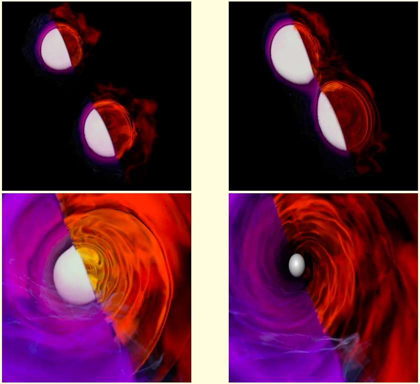
Binary neutron star inspirals



The gravitational wave signal allows for extraction of the tidal deformability of the two neutron stars Λ_1 and Λ_2.

$$Q_{ij} = -\lambda \mathcal{E}_{ij}, \quad \Lambda = \frac{\lambda}{M^5},$$

where Q_{ij} is the induced quadrupole moment, $\mathcal{E}_{ij}$ is the tidal field of the partner.

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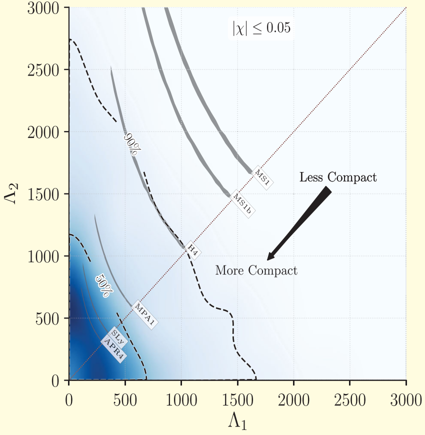
Tidal deformabilities extracted from GW170817 event



Soft EoS are favored by this analysis. There is a strong correlation between the radius and the tidal deformability, i.e., small deformabilities favor more compact stars are favored.

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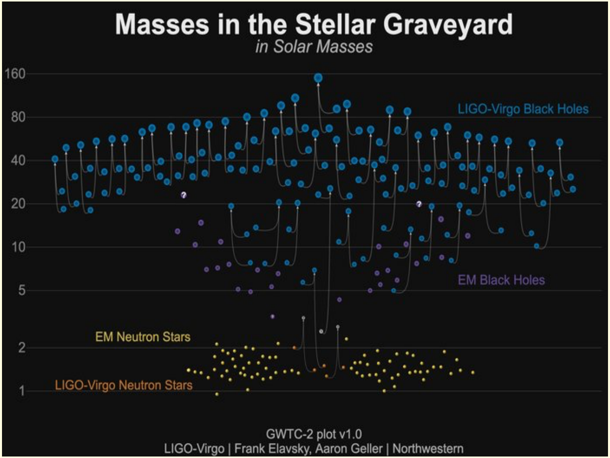
- Extreme mass asymmetric ratio created by a $22.2 - 24.3\, M_\odot$ black hole and a $2.50 - 2.67\, M_\odot$ compact object (no em counterpart).
- Light object's nature is enigmatic as it is in the mass gap $2.5\, M_\odot \lesssim M \lesssim 5\, M_\odot$ where no compact object had ever been observed before.

GW190814 - a compact object from pseudo-mass-gap for compact objects



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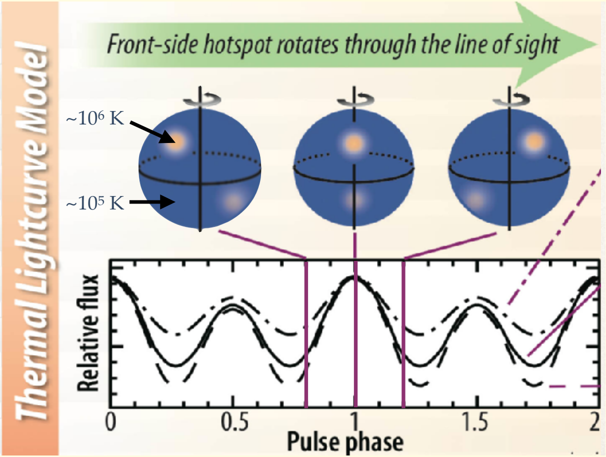

Pulse-profile modeling of the isolated 205.53 Hz millisecond pulsar PSR J0030+0451 observed in X-rays by NICER experiment:

- $1.44^{+0.15}_{-0.14} M_\odot$ –> $13.02^{+1.24}_{-1.06}$ km, Miller et al 2019.
- $1.34^{+0.15}_{-0.16} M_\odot$ –> $12.71^{+1.14}_{-1.19}$ km, Riley et al 2019.

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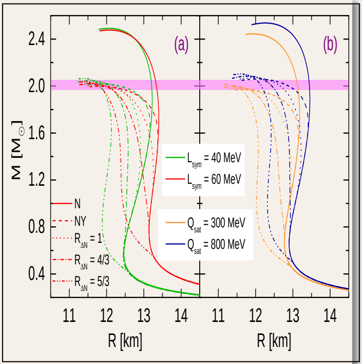
A robust prediction of gravity theory is the existence of maximum mass of neutron stars independent of the input equation of state:

- Einstein's field equations:

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = -8\pi T_{\mu\nu},$$

- Energy-momentum tensor:

$$T_{\mu\nu} = -P(r) g_{\mu\nu} + [P(r) + \epsilon(r)]\, u_\mu u_\nu$$



TOV equations describe static spherically symmetrical stars:

$$\frac{dP(r)}{dr} = -\frac{G\epsilon(r)M(r)}{c^2 r^2}\left(1 + \frac{P(r)}{\epsilon(r)}\right)\left(1 + \frac{4\pi r^3 P(r)}{M(r)c^2}\right)\left(1 - \frac{2GM(r)}{c^2 r}\right)^{-1}.$$

$$M(r) = 4\pi \int_0^r r^2 \epsilon(r) dr.$$

$$P[\epsilon] \quad \rightarrow \quad M, R, I, Q, \ldots$$

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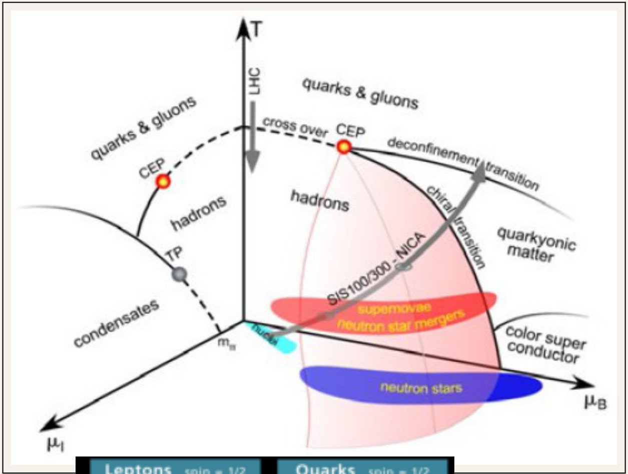
Phase diagram of strongly interacting matter



Leptons spin = 1/2			Quarks spin = 1/2		
Flavor	Mass GeV/c^2	Electric charge	Flavor	Approx. Mass GeV/c^2	Electric charge
ν_e electron neutrino	$<1\times10^{-8}$	0	u up	0.003	2/3
e electron	0.000511	−1	d down	0.006	−1/3
ν_μ muon neutrino	<0.0002	0	c charm	1.3	2/3
μ muon	0.106	−1	s strange	0.1	−1/3

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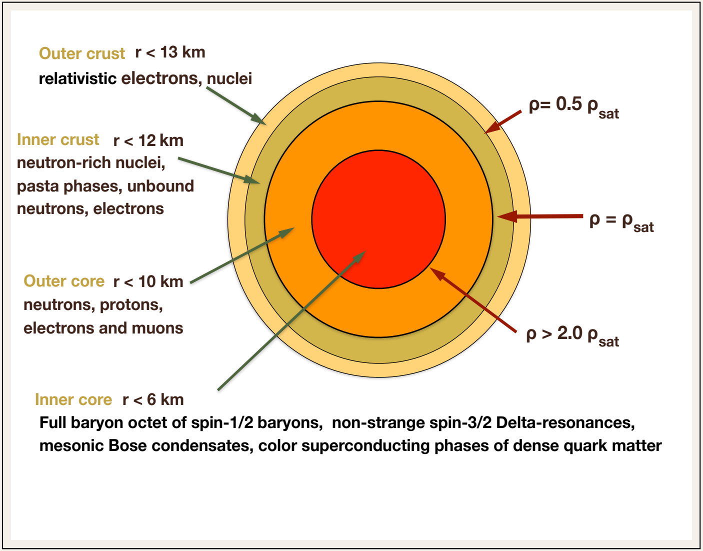
Internal structure of a compact star



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Goals:

- Construct an EoS in a form of density functional: the pressure of dense zero-temperature matter is a functional of energy-density: $P(\varepsilon(r))$
- The parameters of the functional are adjusted to the available data; in our case astrophysics and laboratory data.
- *Ab initio* calculations are data $\to$ check compatibility and adjust if required.
- DFT must be versatile enough to accommodate the baryon spin-1/2 octet and spin-3/2 decouplet.
- Fast in implementation to generate quickly families of EoS

DFT's :

- Non-relativistic DFTs (e.g. Skyrme or Gogny classes):
  (a) high accuracy at low-densities (+)
  (b) extensive tests on laboratory nuclei (+)
  (c) relativistic covariance is lost and high-density extrapolation is not obvious (-)
  (d) extensions to heavy baryons not straightforward (-)
- Relativistic mean-field models of nuclear matter reinterpreted as DFT:
  (a) relativistic covariance, causality is fulfilled automatically (+)
  (b) The Lorentz structure of interactions is maintained explicitly (+)
  (c) straightforward extension to the strange sector and resonances (+)
  (d) fast implementation (+)
  (e) the microscopic counterpart is unknown [not a QFT in the QED/QCD sense] (-)

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III. Equations of state of dense baryonic matter: non-relativistic DFTs

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Two-body Hamiltonian

$$\tilde{H} = \sum_i (\epsilon_i - \mu)\, a_i^\dagger a_i + \frac{1}{2}\sum_{ijkl} \langle ij|V|kl\rangle a_i^\dagger a_j^\dagger a_l a_k$$

Green's functions (GF) expressed through creation and annihilation operators

$$G_{12}(t-t') \equiv -i\Theta(t-t')\left[\left\langle a_1(t) a_2^\dagger(t')\right\rangle - \eta \left\langle a_2^\dagger(t')\, a_1(t)\right\rangle\right] \equiv \left\langle\left\langle a_1(t), a_2^\dagger(t')\right\rangle\right\rangle$$

Time evolution of the GF is built from the Heisenberg equation of motion

$$i\frac{d}{dt}G_{12}(t-t') = \delta_{12}\delta(t-t') + \left\langle\left\langle \left[a_1, \widetilde{H}\right]_t, a_2^\dagger(t')\right\rangle\right\rangle$$

which after substitution of $\tilde{H}$

$$\begin{aligned} i\frac{d}{dt}G_{12}(t-t') &= \delta_{12}\delta(t-t') + (\epsilon_1 - \mu)\, G_{12}(t-t') \\ &+\frac{1}{2}\sum_{ijkl}\left\{\delta_{i1}\langle ij|V|kl\rangle \left\langle\left(a_j^\dagger a_l a_k\right)_t, a_2^\dagger(t')\right\rangle\right\rangle - \delta_{j1}\langle ij|V|kl\rangle \left\langle\left(a_i^\dagger a_l a_k\right)_t, a_2^\dagger(t')\right\rangle\right\rangle\right\} \end{aligned}$$

where

$$(\cdots)_t \text{ stands for } (\exp(i\widetilde{H}))(\cdots)(\exp(-iHt))$$

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Mean-field approximation consists in breaking-up the correlator into pairs:

$$\left\langle\left\langle \left(a_j^\dagger a_l a_k\right)_t, a_2^\dagger\left(t'\right)\right\rangle\right\rangle \cong \delta_{jl}\left\langle a_j^\dagger a_l\right\rangle\left\langle\left\langle a_k(t), a_2^\dagger\left(t'\right)\right\rangle\right\rangle - \delta_{jk}\left\langle a_j^\dagger a_k\right\rangle\left\langle\left\langle a_l(t), a_2^\dagger\left(t'\right)\right\rangle\right\rangle$$

which leads to Hartree-Fock equation of motion for GF

$$\begin{aligned} i\frac{d}{dt}G_{12}\left(t-t'\right) &= \delta_{12}\delta\left(t-t'\right) + \left(\epsilon_1-\mu\right)G_{12}\left(t-t'\right) \\ &+ \sum_{jk}\left\{\langle 1j|V|kj\rangle - \langle 1j|V|jk\rangle\right\}\left\langle a_j^\dagger a_j\right\rangle G_{k2}\left(t-t'\right) \end{aligned}$$

In the momentum space this gives

$$\begin{aligned} i\frac{d}{dt}G_{p\alpha}\left(t-t'\right) &= \delta\left(t-t'\right) + \left(\epsilon_p-\mu\right)G_{p\alpha}\left(t-t'\right) \\ &+ \sum_{k\alpha'}\langle p,k|V|p,k\rangle\left\langle a_{k,\alpha'}^\dagger a_{k,\alpha'}\right\rangle G_{p\alpha}\left(t-t'\right) \\ &- \sum_{q\alpha'}\langle p,p-q|V|p-q,p\rangle\left\langle a_{p-q,\alpha'}^\dagger a_{p-q,\alpha'}\right\rangle G_{pa}\left(t-t'\right) \end{aligned}$$

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The integro-differential equation is brought to an algebraic form via a Fourier transform of GF

$$G_{AB}(E) = \frac{1}{2\pi}\int_{-\infty}^{\infty} G_{AB}(t)e^{iEt}dt, \qquad G_{p\alpha}(E) = \frac{1}{2\pi}\frac{1}{E-E_p},$$

where the pole of the GF explicitly reads

$$E_p = \epsilon_p - \mu + g\sum_k \langle p,k|V|p,k\rangle n_k - \sum_q \langle p,p-q|V|p-q,p\rangle n_{p-q},$$

where $g = (2s+1)(2\tau+1) = 4$ for nuclear matter and $g = (2s+1) = 2$ for neutron matter

$$n_k = \left\langle a^\dagger_{k,\alpha} a_{k,\alpha} \right\rangle = \frac{1}{e^{\beta E_k}+1}.$$

The statistical average of the Hamiltonian gives us the internal energy

$$U = \langle H\rangle = g\sum_k \epsilon_k n_k + \frac{g}{2}\sum_{kk'} \left\langle k,k'|V|k,k'\right\rangle n_k n_k + \frac{1}{2}\sum_{kq} \langle k,k-q|V|k-q,k\rangle n_k n_{k-q}.$$

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Skyrme interaction

$$V_{12} = t_0\delta\left(\mathbf{r}_1-\mathbf{r}_2\right)+\frac{1}{2}t_1\left[\delta\left(\mathbf{r}_1-\mathbf{r}_2\right)k^2+k'^2\delta\left(\mathbf{r}_1-\mathbf{r}_2\right)\right]+t_2\overline{\mathbf{k}}'\cdot\delta\left(\mathbf{r}_1-\mathbf{r}_2\right)\overrightarrow{\mathbf{k}}$$
$$+\frac{1}{6}t_3\rho^{\sigma}\delta\left(\mathbf{r}_1-\mathbf{r}_2\right)$$

Plane-wave matrix element

$$\langle \mathbf{q}_1\mathbf{q}_2\left|V_{12}\right|\mathbf{q}_3\mathbf{q}_4\rangle = \frac{1}{V_0}\delta_{\mathbf{K}_{12},\mathbf{K}_{34}}\left[t_0+\frac{t_3}{6}\rho^{\sigma}+\frac{t_1}{2}\left(\mathbf{k}_{12}-\mathbf{k}_{34}\right)^2+\left(t_1+t_2\right)\mathbf{k}_{12}\cdot\mathbf{k}_{34}\right]$$

Single-particle energies

$$E_p = (\epsilon_p-\mu)+\frac{g-1}{(2\pi)^3}\left(t_0+\frac{t_3}{6}\rho^{\sigma}\right)\int d^3k n_k$$
$$+\frac{1}{(2\pi)^3}\left[\frac{(g-1)t_1}{4}+\frac{(g+1)t_2}{4}\right]\int(\mathbf{p}-\mathbf{k})^2 n_k d^3k$$

The first and second terms are Hartree and Fock contributions respectively.

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The internal energy for the Skyrme functional is

$$
\begin{aligned}
U &= \frac{V_0}{(2\pi)^3}\int d^3k n_k \epsilon_k + \frac{g-1}{2}\frac{V_0}{(2\pi)^6}\int d^3k d^3k' n_k n_{k'}\left(t_0 + \frac{t_3}{6}\rho^\sigma\right) \\
&+ \frac{1}{2}\frac{V_0}{(2\pi)^6}\int d^3k d^3k' \frac{3t_1+5t_2}{4}\left(\mathbf{k}'-\mathbf{k}\right)^2 n_k n_{k'}
\end{aligned}
$$

The free-energy can be constructed at finite T

$$
F = U - TS
$$

using the expression for the entropy

$$
S = k_B \sum_k \left[n_k \ln n_k + (1-n_k)\ln(1-n_k)\right]
$$

In the zero-temperature limit from the internal energy we find the energy per particle

$$
\frac{E_0}{N} = \frac{3}{5}\epsilon_F + \frac{3}{8}\left(t_0 + \frac{t_3}{6}\rho^\sigma\right)\rho + \frac{3}{80}\left(3t_1+5t_2\right)\rho k_F^2
$$

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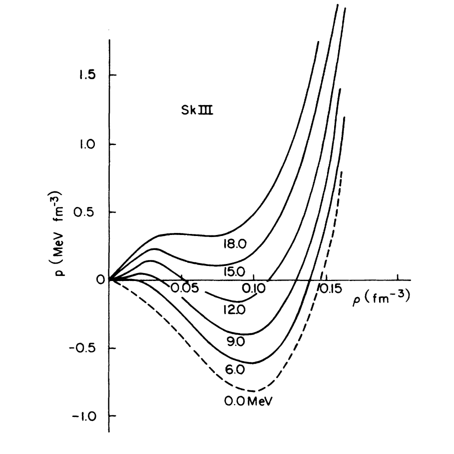
The isotherms of symmetrical nucelar matter for a Skyrme interaction.



Symmetrical nuclear matter EoS has a van-der-Waals gas equation of state (v is the volume per particle $v = n^{-1}$, n number density, b-repulsion, a-attraction)

$$P = \frac{RT}{v-b} - \frac{a}{v^2}$$

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- Symmetrical nuclear matter exhibits liquid-gas phase transition below the nuclear saturation density $\rho_s = 0.16\ \mathrm{fm}^{-3}$.
- The critical temperature can be read-off from the isotherm which has the property of $\partial P/\partial\rho = 0$ as the temperature is lowered.
- The isotherms should be corrected by “Maxwell construction” (see books on stat. phys.)
- one can introduce “order parameters” $m = \rho_l - \rho_c$ or $m = |\rho_g - \rho_c|$, where ρ_c is the critical density and $\rho_{l/g}$ are the liquid and gas densities.
- the external “field” is given by $h = P - P_c$.

Critical exponents:

- β-exponent $m_0 \sim (T_c - T)^\beta$, $h \to 0$ and $T \to T_c$
- γ exponents

$$\chi_0 \sim \left(\frac{\partial m}{\partial h}\right)\Big|_{T,h\to 0} \sim (T - T_c)^{-\gamma} \quad T > T_c \quad [(T_c - T)^{-\gamma'} \quad T < T_c]$$

- δ-exponent

$$|P - P_c|_{T=T_c} \sim |\rho - \rho_c|^\delta \quad T = T_c \quad P \to P_c$$

- α-exponents

$$c_V \sim (T - T_c)^{-\alpha} \quad T > T_c \quad [(T_c - T)^{-\alpha'} \quad T < T_c]$$

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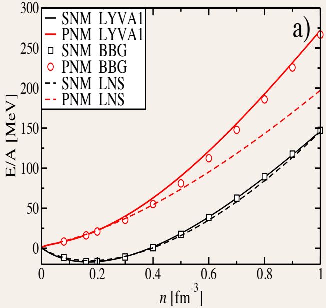
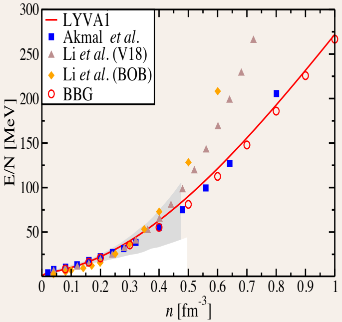
Left: Skyrme forces can be extended somewhat to have more parameters to fit the data. Extended models are able to reproduce accurately the microscopic calculations.





Left: Equation of state of symmetric nuclear matter (SNM) and pure neutron matter (PNM) in the case of Skyrme interaction, see arxiv: 1509.05744 by Davesne et al. A&A 585, A83 (2016). Right: Equation of state of PNM for various microscopic models (dots) and Skyrme force (solid line).

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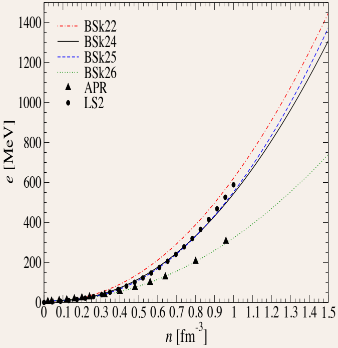
There is another class of models (Brussels-Montreal) of Skyrme models that have been extensively developed and tested on finite nuclei. The EoS can be compared to microscopic EoS.



EoSs for completely degenerate neutron matter with functionals from Pearson et al. MNRAS 486, 768 (2019). The points APR refer to the calculations of Akmal et al. (1998) and Li and Schulze (2008).

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- Stellar matter after few instance after birth settles in equilibrium with respect to the β-decay (Urca) reactions:

$$n \to p + e^- + \bar{\nu}_e \quad p + e^- \to n + \nu_e \tag{1}$$

which implies that β-equilibrium condition

$$\mu_n = \mu_p + \mu_e + \mu_\nu \tag{2}$$

- after initial seconds after birth cold neutron stars are transparent to neutrinos

$$\lambda_\nu \gg R \quad \to \quad \mu_\nu = 0, \tag{3}$$

where λ_ν is the neutrino mean free path, R is the radius.

- The matter must be charge neutral

$$\sum_i q_i n_i = 0, \quad i = n, p, e, \mu, \dots \tag{4}$$

- In hot matter neutrino-trapped matter $\lambda_\nu \ll R$, the matter is still charge neutral but no β-equilibrium, which requires to fix one additional quantity, typically electron faction Y_e.

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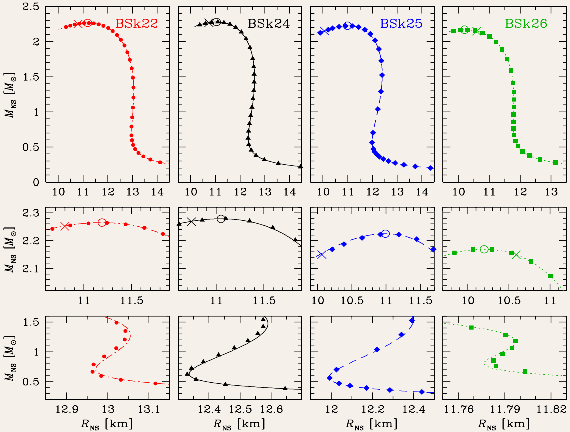

Solutions of TOV equations for the Skyrme functionals [from Pearson et al. MNRAS 486, 768 (2019)].

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IV. Equations of state of dense baryonic matter: relativistic DFTs

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Empirical parameters of nuclear matter

Our empirical knowledge about nuclear matter is based on the information coming from nuclei which are restricted to the densities around the nuclear saturation density

$$n_0 = 0.16\ \mathrm{fm}^{-3} \quad \rho_0 = m_B n_0 = 2.8 \times 10^{14} \mathrm{g\ \ cm}^{-3}.$$

The information on nuclei in their ground state is encoded in the semi-empirical liquid-drop formula:

$$M(A,Z) = A m_B c^2 b_1 + A \underbrace{m_B c^2 \frac{b_4}{4}}_{a_{\rm sym}} \left(\frac{N-Z}{A}\right)^2 + m_B c^2 \left[b_5 \frac{Z^2}{A^{1/3}} + b_2 A^{2/3} - b_3 Z \right].$$

We have introduced the symmetry energy coefficient, whose empirical value is

$$a_{\rm sym} = 32.5\ \mathrm{MeV}.$$

Nuclear matter – an infinitely extended collection of nucleons can be deduced from the mass formula when $A \to \infty$. In this limit the volume term $\propto A$ will survive. The *binding energy* of nucleus (which is always negative) is the difference between $M(A,Z) - Am_B$ - the energy of non-interacting collection of A nucleons at rest:

$$\frac{B}{A} = -16.3\mathrm{MeV}.$$

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Reminder on Lagrange theory

To formulate the theory of nuclear matter at and above the saturation density we need to use the field theory methods. The quantum field theories are commonly formulated in the language of Lagrangian dynamics. The key principle underlying these theories is the minimum of the action S

$$\delta S = 0, \tag{5}$$

where δ denotes the variation. The action is defined through the Lagrangian *density* as

$$S = \int_{t_1}^{t_2} dt \int d^3x \mathcal{L}\left[\phi(x), \partial_\mu \phi(x)\right] = \int_{t_1}^{t_2} d^4x \mathcal{L}\left[\phi(x), \partial_\mu \phi(x)\right], \tag{6}$$

where we assumed that the Lagrangian depends on the field $\phi(x)$ and its derivative $\partial_\mu \phi(x)$. Here x is the four (time and space) coordinate, the Greek indices run from 0 to 3 and denote the components of the vectors in the time and space. Please note that these two are treated as independent variable in the Lagrangian. The variation of the action leads to the *Euler-Lagrange equation*

$$\frac{\partial \mathcal{L}}{\partial \phi(x)} - \partial_\mu \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} = 0. \tag{7}$$

(For a derivation see any book on quantum field theory).

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For a scalar uncharged field $\sigma(x)$ the free-field Lagrangian is given by

$$\mathcal{L}_\sigma = \frac{1}{2}\left(\partial_\mu\sigma(x)\partial^\mu\sigma(x) - m_\sigma^2\sigma^2(x)\right). \tag{8}$$

The corresponding Euler-Lagrange equation is the well-known Klein-Gordon equation

$$(\Box + m_\sigma^2)\sigma(x) = 0, \quad \Box \equiv \frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial^2 \boldsymbol{x}} \tag{9}$$

where $\Box$ the d'Alambert operator. For a vector field, say $\omega_\mu(x)$ with a mass m_ω we need to define the strength tensor (as in the case of electromagnetism) which is related to the derivatives of the vector field

$$\omega_{\mu\nu} = \partial_\mu\omega_\nu - \partial_\nu\omega_\mu. \tag{10}$$

Then, in analogy to the electromagnetism, the Lagrangian of the theory can be written as

$$\mathcal{L}_\omega = -\frac{1}{4}\omega_{\mu\nu}\omega^{\mu\nu} + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu. \tag{11}$$

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The Lagrangian of free $\boldsymbol{\rho}$-field is given by

$$\mathcal{L}_\rho = -\frac{1}{4}\rho_{\mu\nu}\rho^{\mu\nu} + \frac{m_\rho^2}{2}\boldsymbol{\rho}_\mu \cdot \boldsymbol{\rho}^\mu. \tag{12}$$

As above the $\boldsymbol{\rho}$ field is divergenceless and its equation of motion has the form

$$(\Box + m_\rho^2)\boldsymbol{\rho}_\mu(x) = 0. \tag{13}$$

Finally all the fermions in the theory of nuclear matter are described by the Dirac Lagrangian

$$\mathcal{L} = \bar{\psi}(i\partial\!\!\!/ - m_f)\psi, \quad \bar{\psi} = \gamma^0\psi^\dagger, \tag{14}$$

where m_f is the fermion mass and we used the "Feynman slash" notation $\partial\!\!\!/ = \gamma^\mu\partial_\mu$, where γ^μ is the Dirac matrix (see any text on Quantum Field Theory). The corresponding Euler-Lagrange equation is the know as the Dirac equation

$$(i\partial\!\!\!/ - m_f)\psi = 0. \tag{15}$$

Nuclear matter is a strongly interacting system, i.e., we need to add to the free Lagrangians also interactions. The modeling of interactions is not unique - there exist many theories that seek to describe the nuclear matter. We will start with the simplest one and add some more details later on.

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Here we describe a minimal model of relativistic nuclear dynamics where the degrees of freedom are nucleons (i.e. neutrons and protons) which interact with each other by an exchange of σ scalar meson and ω_μ vector meson; (here μ is a Dirac index).
The Lagrangians of free nucleons and can be written as

$$\mathcal{L}_{N,free} = \bar{\psi}\left(i\gamma_\mu\partial^\mu - m\right)\psi. \tag{16}$$

$$\mathcal{L}_{m,free} = \frac{1}{2}\left(\partial_\mu\sigma\partial^\mu\sigma - m_\sigma^2\sigma^2\right) - \frac{1}{2}\left(\frac{1}{2}\omega_{\mu\nu}\omega^{\mu\nu} - m_\omega^2\omega_\mu\omega^\mu\right). \tag{17}$$

where the first term in braces represent the contribution of the σ-meson and the second term - the contribution of ω_μ-meson. The interactions between the nucleon and meson fields are described by the interaction Lagrangian ($x \equiv x^\mu = (t, x, y, z)$)

$$\mathcal{L}_{\rm int} = g_\sigma\sigma(x)\bar{\psi}(x)\psi(x) - g_\omega\omega_\mu(x)\bar{\psi}(x)\gamma^\mu\psi(x). \tag{18}$$

The total Lagrangian is

$$\mathcal{L} = \bar{\psi}[i\gamma_\mu\underbrace{\left(\partial^\mu + ig_\omega\omega^\mu\right)}_{D^\mu} - \underbrace{(m - g_\sigma\sigma)}_{m_D^*}]\psi + \frac{1}{2}\left(\partial_\mu\sigma\partial^\mu\sigma - m_\sigma^2\sigma^2\right) - \frac{1}{4}\omega_{\mu\nu}\omega^{\mu\nu} + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu. \tag{19}$$

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The Euler-Lagrange equations for this Lagrangian are given by

$$(\Box + m_\sigma^2)\sigma(x) = g_\sigma \bar{\psi}(x)\psi(x), \tag{20}$$

$$(\Box + m_\omega^2)\omega_\mu(x) - \partial_\mu\partial^\nu\omega_\nu(x) = g_\omega \bar{\psi}(x)\gamma_\mu\psi(x), \tag{21}$$

$$\{\gamma_\mu\,[i\partial^\mu - g_\omega\omega^\mu(x)] - [m - g_\sigma\sigma(x)]\}\,\psi(x) = 0, \tag{22}$$

These equations represent a set of coupled non-linear differential equations. They can be solved in a so-called mean-field approximation, which implies that the meson fields can be replaced by their mean values in the ground state denoted as $\langle\sigma\rangle$ and $\langle\omega_\mu\rangle$. Assuming static, uniform matter we can drop the x-dependence of the fields. In that case the differentiation operators acting on field will give zero and (20)-(22) will reduce to

$$m_\sigma^2\langle\sigma\rangle = g_\sigma\langle\bar{\psi}(x)\psi(x)\rangle, \tag{23}$$

$$m_\omega^2\langle\omega_0\rangle = g_\omega\langle\psi^\dagger(x)\psi(x)\rangle, \tag{24}$$

$$m_\omega^2\langle\omega_{\boldsymbol{k}}\rangle = g_\omega\langle\bar{\psi}(x)\gamma_{\boldsymbol{k}}\psi(x)\rangle, \tag{25}$$

$$\{\gamma_\mu\,[i\partial^\mu - g_\omega\langle\omega^\mu\rangle] - (m - g_\sigma\langle\sigma\rangle)\}\,\psi(x) = 0. \tag{26}$$

The quantities on the right-hand side of Eqs. (23)-(26) are called nucleonic currents and we shall discuss in a moment how to evaluate their expectation values denoted as $\langle\ldots\rangle$. From now on we shall denote the constant values of the meson fields using the same notation as before, i.e., $\sigma \equiv \langle\sigma\rangle$, etc.

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We first work out some results that are not directly related to the theory we are discussing, but are necessary to make further progress. Nucleon wave functions are momentum eigenstates in uniform static matter, therefore we can write them as plane waves $\psi(x) = \psi(k)\exp(-ik \cdot x)$, where the four-scalar product is defined as $k \cdot x = k_0 t - \boldsymbol{k} \cdot \boldsymbol{x}$. With this definition the Dirac equation reads

$$\{\gamma_\mu \left[k^\mu - g_\omega \omega^\mu\right] - (m - g_\sigma \sigma)\}\, \psi(\boldsymbol{k}) = (\gamma_\mu K^\mu - m^*)\, \psi(\boldsymbol{k}) = 0. \tag{27}$$

where $K^\mu = k^\mu - g_\omega \omega^\mu, \quad m^* = m - g_\sigma \sigma$. We can also use the "Feynman slash" notation $\not{k} = \gamma_\mu k^\mu$ to write the Dirac equation in a compact form

$$(\not{K} - m^*)\psi(K) = 0. \tag{28}$$

We now find the eigenvalues of the Dirac operator corresponding to the plane waves. To this end we multiply Eq. (74) by a factor $(\not{K} + m^*)$

$$\begin{aligned}
(\not{K} + m^*)(\not{K} - m^*) &= \not{K}\not{K} - m^{*2} = \gamma_\alpha K^\alpha \gamma_\beta K^\beta - m^{*2} \\
&= K_\alpha K_\beta \frac{\gamma^\alpha \gamma^\beta + \gamma^\beta \gamma^\alpha}{2} - m^{*2} \\
&= K_\alpha K_\beta g^{\alpha\beta} - m^{*2} = K_\alpha K^\alpha - m^{*2},
\end{aligned} \tag{29}$$

where we used the property of the Dirac matrices that $\{\gamma^\alpha, \gamma^\beta\} = 2g^{\alpha\beta}$, where $\{a, b\} = ab + ba$ denotes the anti-commutator of operators a and b.

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The Dirac equation now reads

$$(K_\alpha K^\alpha - m^{*2})\psi(K) = 0, \quad \rightarrow \quad K_\alpha K^\alpha - m^{*2} = 0. \tag{30}$$

Writing the last condition explicitly, i.e., $K_0 K^0 - \boldsymbol{K}\cdot\boldsymbol{K} - m^{*2} = 0$, we obtain for the absolute value

$$K_0^{\pm} = \pm\sqrt{\boldsymbol{K}^2 + m^{*2}} = \pm\sqrt{(\boldsymbol{k} - g_\omega \boldsymbol{\omega})^2 + (m - g_\sigma \sigma)^2}, \tag{31}$$

where the upper sign corresponds to the particle and the lower sign to anti-particles. Here we inserted back the expressions for some of the renormalized quantities. Now we recall that per definition $K^0 = k^0 - g_\omega \omega^0$, so that the substitution in Eq. (31)

$$k_0^{\pm} = g_\omega \omega^0 \pm \sqrt{(\boldsymbol{k} - g_\omega \boldsymbol{\omega})^2 + (m - g_\sigma \sigma)^2}, \tag{32}$$

Thus we conclude that the eigenvalues of the Dirac equation corresponding to particles and anti-particles are

$$\epsilon(\boldsymbol{k}) \equiv +\sqrt{(\boldsymbol{k} - g_\omega \boldsymbol{\omega})^2 + (m - g_\sigma \sigma)^2} + g_\omega \omega_0, \tag{33}$$

$$\bar{\epsilon}(\boldsymbol{k}) \equiv -\sqrt{(\boldsymbol{k} - g_\omega \boldsymbol{\omega})^2 + (m - g_\sigma \sigma)^2} + g_\omega \omega_0. \tag{34}$$

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Our next task is the computation of the expectation values of the currents on the right-hand side of Eqs. (23)- (25). We denote by α all the internal degrees of freedom of the nucleons, i.e., spin, and isospin (neutron's isospin is -1/2, proton's isospin is 1/2).
The first step is to compute the currents and we start by identifying the so-called *Dirac Hamiltonian*. Let us rewrite the Dirac equation (27) as

$$\left(\gamma_\mu k^\mu - \gamma_\mu g_\omega \omega^\mu - m^*\right)\psi_\alpha(k) = \left(\gamma_0 k_0 - \boldsymbol{\gamma}\cdot\boldsymbol{k} - g_\omega \gamma_\mu \omega^\mu - m^*\right)\psi_\alpha(k) = 0. \quad (35)$$

After multiplying this equation by γ_0 from left and recalling that $\gamma_0^2 = 1$

$$\left[k_0 - \gamma_0\left(\boldsymbol{\gamma}\cdot\boldsymbol{k} + g_\omega\gamma_\mu\omega^\mu + m^*\right)\right]\psi_\alpha(k) = 0, \quad \rightarrow \quad H_D\psi(k) = k_0\psi_\alpha(k) \quad (36)$$

where we defined the Dirac Hamiltonian as

$$H_D = \gamma_0\left(\boldsymbol{\gamma}\cdot\boldsymbol{k} + g_\omega\gamma_\mu\omega^\mu + m^*\right). \quad (37)$$

The expectation value of the Dirac Hamiltonian for particle states is given by (see Eq. (31))

$$\left(\psi^\dagger H_D \psi\right)_{\boldsymbol{k},\alpha} = \epsilon(\boldsymbol{k}) = \sqrt{(\boldsymbol{k} - g_\omega\boldsymbol{\omega})^2 + (m - g_\sigma\sigma)^2} + g_\omega\omega_0, \quad (38)$$

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Consider the derivative of the left-hand side of this equation with respect to any parameter entering H_D (say, η),

$$\frac{\partial}{\partial \eta}\left(\psi^\dagger H_D \psi\right)_{\boldsymbol{k},\alpha} = \left(\psi^\dagger \frac{\partial H_D}{\partial \eta}\psi\right)_{\boldsymbol{k},\alpha} + \epsilon(\boldsymbol{k})\frac{\partial}{\partial \eta}\left(\psi^\dagger \psi\right)_{\boldsymbol{k},\alpha} \tag{39}$$

The derivative in the second term vanishes because the quantity $\left(\psi^\dagger \psi\right)_{\boldsymbol{k},\alpha}$ is normalized to some number (typically 1). Since we already computed the bracket on the left hand side, see (38), we can further write

$$\frac{\partial}{\partial \eta}\left(\sqrt{(\boldsymbol{k} - g_\omega \boldsymbol{\omega})^2 + (m - g_\sigma \sigma)^2} + g_\omega \omega_0\right) = \left(\psi^\dagger \frac{\partial H_D}{\partial \eta}\psi\right)_{\boldsymbol{k},\alpha} \tag{40}$$

Let us use this result in several cases:

- $\eta = \omega_0$, which implies

$$g_\omega = \left(\psi^\dagger \frac{\partial H_D}{\partial \omega_0}\psi\right)_{\boldsymbol{k},\alpha} = g_\omega\left(\psi^\dagger \gamma_0^2 \psi\right)_{\boldsymbol{k},\alpha} \quad \rightarrow \quad \left(\psi^\dagger \psi\right)_{\boldsymbol{k},\alpha} = 1. \tag{41}$$

which is just the normalization of states.

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- $\eta = \boldsymbol{k}$, which implies

$$\frac{\partial}{\partial \boldsymbol{k}}\sqrt{(\boldsymbol{k}-g_\omega\boldsymbol{\omega})^2+(m-g_\sigma\sigma)^2} = \left(\psi^\dagger\gamma_0\boldsymbol{\gamma}\psi\right)_{\boldsymbol{k},\alpha} = (\bar{\psi}\boldsymbol{\gamma}\psi)_{\boldsymbol{k},\alpha} \tag{42}$$

which corresponds to the nucleonic current.

- $\eta = m$, which implies

$$\frac{\partial}{\partial \eta}\left(\sqrt{(\boldsymbol{k}-g_\omega\boldsymbol{\omega})^2+(m-g_\sigma\sigma)^2}\right) = \left(\psi^\dagger\gamma_0\psi\right)_{\boldsymbol{k},\alpha} = (\bar{\psi}\psi)_{\boldsymbol{k},\alpha} \tag{43}$$

or after taking the derivative explicitly

$$\frac{m-g_\sigma\sigma}{\sqrt{(\boldsymbol{k}-g_\omega\boldsymbol{\omega})^2+(m-g_\sigma\sigma)^2}} = (\bar{\psi}\psi)_{\boldsymbol{k},\alpha}\,. \tag{44}$$

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Now we need to compute the statistical averages of the currents derived above. For any Dirac matrix Γ these are defined as

$$\langle\bar{\psi}\Gamma\psi\rangle = \sum_\alpha \int \frac{d^3k}{(2\pi)^3} \left(\bar{\psi}\Gamma\psi\right)_{\boldsymbol{k},\alpha} f_\alpha(\boldsymbol{k}). \tag{45}$$

where $\left(\bar{\psi}\Gamma\psi\right)_{\boldsymbol{k},\alpha}$ means the expectation value in the quantum mechanical sense computed above, $f_\alpha(\boldsymbol{k})$ is the Fermi distribution function for nucleons with internal quantum number α. The degeneracy factor $g = 2$ for purely neutron matter and 4 in symmetrical nuclear matter (equal number of protons and neutrons). Thus we find that $m^* = m - g_\sigma\sigma$

$$\rho_S \equiv \langle\bar{\psi}(x)\psi(x)\rangle = g\int \frac{d^3k}{(2\pi)^3} \left(\bar{\psi}\psi\right)_{\boldsymbol{k}} f(\boldsymbol{k}) = g\int \frac{d^3k}{(2\pi)^3} \frac{m^* f(\boldsymbol{k})}{\sqrt{(\boldsymbol{k} - g_\omega\boldsymbol{\omega})^2 + m^{*2}}} \tag{46}$$

$$\rho_V \equiv \langle\psi^\dagger(x)\psi(x)\rangle = g\int \frac{d^3k}{(2\pi)^3} \left(\psi^\dagger\psi\right)_{\boldsymbol{k}} f(\boldsymbol{k}) = g\int \frac{d^3k}{(2\pi)^3} f(\boldsymbol{k}), \tag{47}$$

$$\begin{aligned} \vec{j} = \langle\bar{\psi}(x)\boldsymbol{\gamma}\psi(x)\rangle &= g\int \frac{d^3k}{(2\pi)^3} \left(\bar{\psi}\boldsymbol{\gamma}\psi\right)_{\boldsymbol{k}} f(\boldsymbol{k}) \\ &= g\int \frac{d^3k}{(2\pi)^3} \frac{\partial}{\partial\boldsymbol{k}} \sqrt{(\boldsymbol{k} - g_\omega\boldsymbol{\omega})^2 + (m - g_\sigma\sigma)^2} f(\boldsymbol{k}) \\ &= g\int \frac{d^3k}{(2\pi)^3} \frac{\partial}{\partial\boldsymbol{k}} [\epsilon(\boldsymbol{k}) - g_\omega\omega_0] f(\boldsymbol{k}) = g\int \frac{d^3k}{(2\pi)^3} \frac{\partial\epsilon(\boldsymbol{k})}{\partial\boldsymbol{k}} f(\boldsymbol{k}) \end{aligned} \tag{48}$$

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Note that the last term is simply proportional to the velocity of the quasiparticle $\boldsymbol{v} \equiv \partial\epsilon(\boldsymbol{k})/\partial\boldsymbol{k}$ (as is natural to expect from the definition of the current). Since for homogeneous fluids the velocity is along the momentum we concluded that the integral is zero. But this quantity acts as a source in the equation of motion of the $\omega_{\boldsymbol{k}}$ field (see Eq. (25)); therefore we can conclude that $\omega_{\boldsymbol{k}} \equiv 0$. It is seen that the vector density ρ_V is simply the density of baryonic matter. The scalar density ρ_S has no immediate physical interpretation, except than it acts as the source of the σ-field. Returning to the equations of motions in the mean-field approximation (23) and (24) we obtain

$$m_\sigma^2\sigma = g_\sigma\rho_S, \quad m_\omega^2\omega_0 = g_\omega\rho_V, \quad \boldsymbol{\omega} = 0. \tag{49}$$

Note that the first relation represents an integral equation for the scalar field σ. Indeed as see from the explicit expression

$$g_\sigma\sigma = \left(\frac{g_\sigma}{m_\sigma}\right)^2 g\int\frac{d^3k}{(2\pi)^3}\frac{m - g_\sigma\sigma}{\sqrt{(\boldsymbol{k} - g_\omega\boldsymbol{\omega})^2 + (m - g_\sigma\sigma)^2}}f(\boldsymbol{k}), \tag{50}$$

the variable $g_\sigma\sigma$ appears on both side of the equation. The solution can be found by an iterative procedure.

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With the Lagrangian theory of fields the energy (the 00 component of the energy and momentum tensor) and the pressure (the sum of the remaining diagonal components divided by 3) are given by

$$\epsilon = -\langle\mathcal{L}\rangle + \langle\bar{\psi}\gamma_0 k_0\psi\rangle, \tag{51}$$

$$p = \langle\mathcal{L}\rangle + \frac{1}{3}\langle\bar{\psi}\boldsymbol{\gamma}\cdot\boldsymbol{k}\psi\rangle. \tag{52}$$

The contribution of the Dirac fields to $\langle\mathcal{L}\rangle$ drops out by the Dirac equation, whereas in the mean-field approximation the terms with time and space derivatives are zero. Then

$$\langle\mathcal{L}\rangle = -\frac{1}{2}m_\sigma^2\sigma^2 + \frac{1}{2}m_\omega^2\omega_0^2. \tag{53}$$

The second term in Eq. (51) originates from the expectation value $\langle\bar{\psi}\gamma_0 k_0\psi\rangle = \langle\psi^\dagger\epsilon(\boldsymbol{k})\psi\rangle$ therefore we find

$$\begin{aligned}\langle\psi^\dagger(x)\epsilon(\boldsymbol{k})\psi(x)\rangle &= g\int\frac{d^3k}{(2\pi)^3}\epsilon(\boldsymbol{k})f(\boldsymbol{k}) = g\int\frac{d^3k}{(2\pi)^3}\left(g_\omega\omega_0 + \sqrt{k^2+m^{*2}}\right)f(\boldsymbol{k})\\ &= g_\omega\omega_0\rho + g\int\frac{d^3k}{(2\pi)^3}\sqrt{k^2+m^{*2}}f(\boldsymbol{k}) = m_\omega^2\omega_0^2 + g\int\frac{d^3k}{(2\pi)^3}\sqrt{k^2+m^{*2}}f(\boldsymbol{k}).\end{aligned}$$

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Further we have

$$\begin{aligned}\langle\bar{\psi}(x)\boldsymbol{\gamma}\cdot\boldsymbol{k}\psi(x)\rangle &= g\int\frac{d^3k}{(2\pi)^3}\frac{\boldsymbol{k}\cdot(\boldsymbol{k}-g_\omega\boldsymbol{\omega})}{\sqrt{(\boldsymbol{k}-g_\omega\boldsymbol{\omega})^2+(m-g_\sigma\sigma)^2}}f(\boldsymbol{k})\\ &= g\int\frac{d^3k}{(2\pi)^3}\frac{k^2}{\sqrt{k^2+(m-g_\sigma\sigma)^2}}f(\boldsymbol{k}).\end{aligned} \tag{54}$$

Thus, summarizing the result we can write that

$$\epsilon = \frac{1}{2}m_\sigma^2\sigma^2+\frac{1}{2}m_\omega^2\omega_0^2+g\int\frac{d^3k}{(2\pi)^3}(\sqrt{k^2+m^{*2}})\,f(\boldsymbol{k}), \tag{55}$$

$$p = -\frac{1}{2}m_\sigma^2\sigma^2+\frac{1}{2}m_\omega^2\omega_0^2+\frac{1}{3}g\int\frac{d^3k}{(2\pi)^3}\frac{k^2}{\sqrt{k^2+(m-g_\sigma\sigma)^2}}f(\boldsymbol{k}). \tag{56}$$

In the zero temperature limit $f(\boldsymbol{k})=\theta(k_F-k)$. This constrains the momenta of particle to those within the Fermi-sphere. Then we can write

$$g\int\frac{d^3k}{(2\pi)^3}\theta(k_F-k)\cdots = g\int_0^{k_F}\frac{dk\,k^2}{2\pi^2}\int\frac{d\Omega}{4\pi}\cdots = g\int_0^{k_F}\frac{dk\,k^2}{2\pi^2}\dots \tag{57}$$

where . . . stand for any particular function in the integrand.

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Equation of state

Thus the equation of state in the zero temperature limit reads

$$\epsilon = \frac{1}{2}m_\sigma^2\sigma^2 + \frac{1}{2}m_\omega^2\omega_0^2 + g\int_0^{k_F}\frac{dk\, k^2}{2\pi^2}\left(\sqrt{k^2+m^{*2}}\right), \quad (58)$$

$$p = -\frac{1}{2}m_\sigma^2\sigma^2 + \frac{1}{2}m_\omega^2\omega_0^2 + \frac{1}{3}g\int_0^{k_F}\frac{dk\, k^2}{2\pi^2}\frac{k^2}{\sqrt{k^2+(m-g_\sigma\sigma)^2}}. \quad (59)$$

As in the case of an ideal gas the Fermi momentum and density of protons (p) and neutrons (n) are related to their Fermi momenta by

$$\rho_n = \frac{k_{Fn}^3}{3\pi^2}, \qquad \rho_p = \frac{k_{Fp}^3}{3\pi^2}, \quad (60)$$

where we have carried out the summation over the two spin direction, but have kept the isospin explicit. Note that we have a parametric form of the equation of state $p(\epsilon)$ where the parameter which relates these two quantities are the densities.

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Extension of the model to include scalar self-interactions

Since the field equations for the meson field depend on two parameters, the ratios of the coupling constant to the mass of the corresponding meson g_σ/m_σ, and g_ω/m_ω we can fix the magnitude of the binding energy and the saturation density. The remaining observables of nuclear matter are predictions: the model predicts $K = 550$ MeV and $m^*/m = 0.5$ in disagreement with the empirical data. These parameters can be fixed if one assigns some self-interaction to the scalar mesons, i.e., the Lagrangian of the theory changes to

$$\mathcal{L} = \mathcal{L}_{\sigma-\omega} - \frac{1}{3} b m_B (g_\sigma \sigma)^3 - \frac{1}{4} c m_B (g_\sigma \sigma)^4, \tag{61}$$

where c and b are two additional parameters that can be used to fit K and m^*/m. (Subscript $\sigma - \omega$ refers to the two-parameter $\sigma - \omega$ model.) Clearly, the additional term will change the equation determining the σ field to

$$g_\sigma \sigma = \left(\frac{g_\sigma}{m_\sigma}\right)^2 \left[\frac{2}{\pi^2}\int_0^{k_f} dk k^2 \frac{m - g_\sigma\sigma}{\sqrt{k^2 + (m - g_\sigma \sigma)^2)}} - bm(g_\sigma\sigma)^2 - c(g_\sigma\sigma)^2\right]. \tag{62}$$

The energy and pressure of the model with self-interactions is

$$\epsilon = \frac{1}{3} b m_B (g_\sigma\sigma)^3 + \frac{1}{4} c m_B (g_\sigma\sigma)^4 + \epsilon_{\sigma-\omega} \tag{63}$$

$$p = -\frac{1}{3} b m_B (g_\sigma\sigma)^3 - \frac{1}{4} c m_B (g_\sigma\sigma)^4 + p_{\sigma-\omega} \tag{64}$$

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The inclusion of the $\boldsymbol{\rho}$-meson is needed to describe the isospin asymmetrical nuclear matter. We already discussed the free Lagrangian of the $\boldsymbol{\rho}$-meson field (13). The vector ρ_μ is must be invariant with respect to the rotations in the isospin space. The invariance follows from fact that the Lagrangian consists of scalar products in the isospin space. The rotation in the isospin space can be written as

$$\boldsymbol{\rho}_\mu \to \boldsymbol{\rho}_\mu + \boldsymbol{\Lambda} \times \boldsymbol{\rho}_\mu = \boldsymbol{\rho}_\mu + \epsilon_{ijk}\Lambda_j \rho_k, \tag{65}$$

where $\boldsymbol{\Lambda}$ is an infinitesimal vector in isospin space. The isospin current according to the theorem on Noether currents can be written as

$$I_j^\nu = \frac{\partial \mathcal{L}}{\partial(\partial_\nu \rho_{i\mu})}\rho_{k\mu}\epsilon_{ijk} = \boldsymbol{\rho}_\mu \times \frac{\partial \mathcal{L}}{\partial \partial_\nu \boldsymbol{\rho}_\mu} \tag{66}$$

$$= -\boldsymbol{\rho}_\mu \times \frac{\partial}{\partial \partial_\nu \boldsymbol{\rho}_\mu}\left[\frac{1}{4}\rho_{\mu\nu}\rho^{\mu\nu} - \frac{m_\rho}{2}\boldsymbol{\rho}_\mu \cdot \boldsymbol{\rho}^\mu\right] = \boldsymbol{\rho}_\mu \times \boldsymbol{\rho}^{\nu\mu}. \tag{67}$$

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The interactions lead to addition term in the ρ-meson Lagrangian of the type

$$\mathcal{L}_{\rho,int} = -g_\rho \boldsymbol{\rho}^\nu \cdot (\boldsymbol{\rho}^\mu \times \boldsymbol{\rho}_{\mu\nu}) = -g_\rho (\boldsymbol{\rho}^\nu \times \boldsymbol{\rho}^\mu) \cdot \boldsymbol{\rho}_{\nu\mu}. \tag{68}$$

The contribution of this term to the isospin current is evaluated in the same manner

$$\begin{aligned} \tilde{I}_j^\nu &= -\boldsymbol{\rho}_\mu \times \frac{\partial \mathcal{L}_{\rho,int}}{\partial \partial_\nu \boldsymbol{\rho}_\mu} = +g_\rho \boldsymbol{\rho}_\mu \times \frac{\partial}{\partial \partial_\nu \boldsymbol{\rho}_\mu} \left[(\boldsymbol{\rho}^\alpha \times \boldsymbol{\rho}^\beta) \cdot (\partial_\alpha \boldsymbol{\rho}_\beta - \partial_\beta \boldsymbol{\rho}_\alpha) \right] \\ &= 2 g_\rho \boldsymbol{\rho}_\mu \times \boldsymbol{\rho}^{\nu\mu}. \end{aligned} \tag{69}$$

The total isospin current is then obtained by summing the meson contribution together with nucleonic contribution, which is simply the Dirac current multiplied by the isospin vector for nucleons $\boldsymbol{\tau}$:

$$\boldsymbol{I}^\nu = \frac{1}{2}\bar{\psi}\gamma^\nu \boldsymbol{\tau}\psi + \boldsymbol{\rho}_\mu \times \boldsymbol{\rho}^{\nu\mu} + 2 g_\rho (\boldsymbol{\rho}^\nu \times \boldsymbol{\rho}^\mu) \times \boldsymbol{\rho}_\mu \tag{70}$$

where the first term is the isospin current of nucleons, the second is the isospin current of the free $\boldsymbol{\rho}$-field, whereas the last term is the contribution Eq. (69).

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The interaction of a ρ-meson with the matter is described by coupling the isospin current to the meson-field, i.e.,

$$\mathcal{L}_\rho = -g_\rho \boldsymbol{\rho}_\nu \cdot \boldsymbol{I}^\nu . \tag{71}$$

As a consequence of this interaction the Dirac equation will change; this change follows from

$$\frac{\partial \mathcal{L}_{int}}{\partial \bar{\psi}} = -\frac{g_\rho}{2}\gamma_\nu \boldsymbol{\rho}^\nu \cdot \boldsymbol{\tau}\psi. \tag{72}$$

- Finite isospin in baryonic matter induces non-vanishing component of the 3 component of baryon isospin current.
- Baryon isospin current in the direction of 1 and 2 components their expectation values will vanish.
- Matter is neutral $\rightarrow$ only the neutral ρ meson will survive.
- Only non-vanishing component of the ρ-meson is the ρ_3^0, which will couple to the 3-component of baryon isospin current.
- Similar to the ω-meson the spatial component of the ρ meson field will vanish.

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As a consequence of the items above we have

$$g_\rho \rho_3^0 = \frac{1}{2}\left(\frac{g_\rho}{m_\rho}\right)^2 \langle \bar{\psi}\gamma^0\tau_3\psi\rangle = \left(\frac{g_\rho}{m_\rho}\right)^2 \frac{1}{2}\left(\rho_p - \rho_n\right),$$

$$g_\rho \rho_3^k = \frac{1}{2}\left(\frac{g_\rho}{m_\rho}\right)^2 \langle \bar{\psi}\gamma^k\tau_3\psi\rangle = 0. \qquad (73)$$

The Dirac equation (27) now becomes

$$\left\{\gamma_\mu\left(k^\mu - g_\omega\omega^\mu - \frac{1}{2}g_\rho\tau_3\rho_3^\mu\right) - (m - g_\sigma\sigma)\right\}\psi(\boldsymbol{k}) = 0. \qquad (74)$$

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The eigenvalues of this Dirac equation will acquire new terms due to the ρ meson contribution

$$\epsilon(\boldsymbol{k}) \equiv \sqrt{k^2 + (m - g_\sigma \sigma)^2} + g_\omega \omega_0 + g_\rho \rho_3^0 I_3, \quad (75)$$

$$\bar{\epsilon}(\boldsymbol{k}) \equiv -\sqrt{(k^2 + (m - g_\sigma \sigma)^2} + g_\omega \omega_0 + g_\rho \rho_3^0 I_3, \quad (76)$$

Finally we need to construct the energy and pressure of asymmetrical matter. The statistical average of the Lagrangian of the theory will acquire an additional term

$$\langle \mathcal{L} \rangle \to \langle \mathcal{L} \rangle + \frac{1}{2} m_\rho^2 \rho_{03}^2. \quad (77)$$

To evaluate the energy density we need

$$\langle \bar{\psi} \gamma_0 k_0 \psi \rangle = g \int \frac{d^3k}{(2\pi)^3} (\bar{\psi} \gamma_0 k_0 \psi)_{\boldsymbol{k}} f(k) \quad (78)$$

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We can carry out the spin and isospin summations now. Because of the isospin degeneracy we will have different contributions for neutrons and protons

$$
\begin{aligned}
\langle \bar{\psi}\gamma_0 k_0 \psi\rangle &= \frac{1}{\pi^2}\int [g_\omega \omega^0 + \frac{1}{2} g_\rho \rho_3^0 + \sqrt{k^2+m^{*2}}] n_{F,p}(k) k^2 dk \\
&+ \frac{1}{\pi^2}\int [g_\omega \omega^0 - \frac{1}{2} g_\rho \rho_3^0 + \sqrt{k^2+m^{*2}}] n_{F,n}(k) k^2 dk, \qquad (79)
\end{aligned}
$$

where the indexes p and n refer to neutrons and protons. The parts of the integrand which do not depend on k variable can be easily integrated and will give the densities of neutrons and protons respectively. Further using the field equations we obtain a simpler form of the equation above

$$
\begin{aligned}
\langle \bar{\psi}\gamma_0 k_0 \psi\rangle &= m_\omega^2 \omega_0^2 + m_\rho^2 \rho_{03}^2 \\
&+ \frac{1}{\pi^2}\int \sqrt{k^2+m^{*2}} n_{F,p}(k) k^2 dk + \frac{1}{\pi^2}\int \sqrt{k^2+m^{*2}} n_{F,n}(k) k^2 dk \qquad (80)
\end{aligned}
$$

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Collecting all the terms we obtain

$$\begin{aligned} \epsilon &= \frac{1}{3}bm_B(g_\sigma\sigma)^3 + \frac{1}{4}cm_B(g_\sigma\sigma)^4 + \frac{1}{2}m_\sigma^2\sigma^2 + \frac{1}{2}m_\omega^2\omega_0^2 + \frac{1}{2}m_\rho^2\rho_{03}^2 \\ & +2\sum_{\alpha=n,p}\int\frac{d^3k}{(2\pi)^3}(\sqrt{k^2+m^{*2}})f(k), \end{aligned} \tag{81}$$

$$\begin{aligned} p &= -\frac{1}{3}bm_B(g_\sigma\sigma)^3 - \frac{1}{4}cm_B(g_\sigma\sigma)^4 - \frac{1}{2}m_\sigma^2\sigma^2 + \frac{1}{2}m_\omega^2\omega_0^2 + \frac{1}{2}m_\rho^2\rho_{03}^2 \\ &+ \frac{2}{3}\sum_{\alpha=n,p}\int\frac{d^3k}{(2\pi)^3}\frac{k^2}{\sqrt{k^2+m^{*2}}}f(k), \end{aligned} \tag{82}$$

where we carried out the spin summations (factor 2) and α summation is over the neutrons and protons. In the $\sigma - \omega$ model and its extension that included the self-interactions the asymmetry between the neutrons and protons was induced by the fact that they were filling different Fermi spheres, but there was no energy cost of converting the neutron to the proton. As a consequence the symmetry energy was not reproduced by these models (this was about the half of its empirical value). Including the ρ meson we were able to account for the missing part of the symmetry energy. It provides the missing energy that favors isospin symmetrical nuclear matter. This term, as expected, is proportional to ρ_{03}^2 term in the energy density which rises the energy of the system quadratically with the asymmetry.

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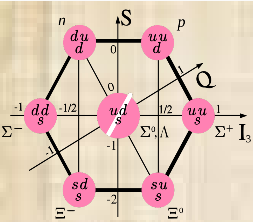
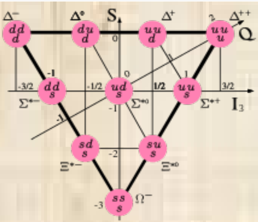
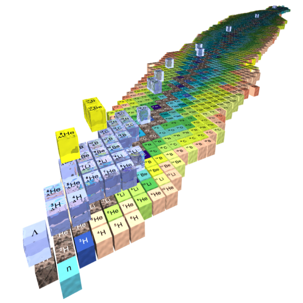
Beyond nucleons: Baryon octet $J^p = 1/2^+$ and baryon decuplet $J^p = 3/2^+$

Strangeness carrying baryons + resonances (excitations of a nucleon)







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Hyper-Nuclear matter Lagrangian:

$$\begin{aligned}
\mathcal{L}_{NM} &= \underbrace{\sum_B \bar{\psi}_B \left[\gamma^\mu \left(i\partial_\mu - g_{\omega BB}\omega_\mu - \frac{1}{2} g_{\rho BB}\boldsymbol{\tau}\cdot\boldsymbol{\rho}_\mu\right) - (m_B - g_{\sigma BB}\sigma)\right]\psi_B}_{\text{baryons}} \\
&+ \underbrace{\frac{1}{2}\partial^\mu\sigma\partial_\mu\sigma - \frac{1}{2}m_\sigma^2\sigma^2 - \frac{1}{4}\omega^{\mu\nu}\omega_{\mu\nu} + \frac{1}{2}m_\omega^2\omega^\mu\omega_\mu}_{\text{mesons}} \\
&- \underbrace{\frac{1}{4}\boldsymbol{\rho}^{\mu\nu}\boldsymbol{\rho}_{\mu\nu} + \frac{1}{2}m_\rho^2\boldsymbol{\rho}^\mu\cdot\boldsymbol{\rho}_\mu}_{\text{mesons}} + \underbrace{\sum_\lambda \bar{\psi}_\lambda(i\gamma^\mu\partial_\mu - m_\lambda)\psi_\lambda}_{\text{leptons}} - \underbrace{\frac{1}{4}F^{\mu\nu}F_{\mu\nu}}_{\text{electromagnetism}},
\end{aligned}$$

- B-sum is over the baryonic octet $B \equiv p, n$
- Meson fields include σ meson, $\boldsymbol{\rho}_\mu$-meson and ω_μ-meson
- Leptons include electrons, muons and neutrinos for $T \neq 0$

Two types of relativistic density functionals based on relativistic Lagrangians

- linear mesonic fields, density-dependent couplings (DDME2, DD2, etc.)
- non-linear mesonic fields; coupling constant are just numbers (NL3, GM1-3, etc.)

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Hyper-Nuclear + Delta matter Lagrangian:

$$\mathcal{L}_{NM} = \underbrace{\sum_B \bar\psi_B\left[\gamma^\mu\left(i\partial_\mu - g_{\omega BB}\omega_\mu - \frac{1}{2}g_{\rho BB}\boldsymbol{\tau}\cdot\boldsymbol{\rho}_\mu\right) - (m_B - g_{\sigma BB}\sigma)\right]\psi_B}_{\text{baryons}}$$
$$+ \underbrace{\frac{1}{2}\partial^\mu\sigma\partial_\mu\sigma - \frac{1}{2}m_\sigma^2\sigma^2 - \frac{1}{4}\omega^{\mu\nu}\omega_{\mu\nu} + \frac{1}{2}m_\omega^2\omega^\mu\omega_\mu}_{\text{mesons}}$$
$$- \underbrace{\frac{1}{4}\boldsymbol{\rho}^{\mu\nu}\boldsymbol{\rho}_{\mu\nu} + \frac{1}{2}m_\rho^2\boldsymbol{\rho}^\mu\cdot\boldsymbol{\rho}_\mu}_{\text{mesons}} + \underbrace{\sum_\lambda \bar\psi_\lambda(i\gamma^\mu\partial_\mu - m_\lambda)\psi_\lambda}_{\text{leptons}} - \underbrace{\frac{1}{4}F^{\mu\nu}F_{\mu\nu}}_{\text{electromagnetism}},$$

- B-sum is over the baryonic octet $B \equiv p, n, \Delta^+, \Delta^{++}, \Delta^0, \Delta^-, \Lambda, \Sigma^{\pm,0}, \Xi^{-,0}$.
- Meson fields include σ meson, $\boldsymbol{\rho}_\mu$-meson and ω_μ-meson
- Leptons include electrons, muons and neutrinos for $T \neq 0$

Nucleonic excitations from $J^p = 3/2^+$ decouplet have couplings nearly the same as for nucleons

$$\boxed{\Delta^+, \quad \Delta^{++}, \quad \Delta^0, \quad \Delta^-}$$

Changes the charge neutrality conditions and β-equilibrium

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Fixing the couplings: nucleonic sector

$$g_{iN}(\rho_B) = g_{iN}(\rho_0) h_i(x), \qquad i = \sigma, \omega, \qquad h_i(x) = a_i \frac{1 + b_i(x + d_i)^2}{1 + c_i(x + d_i)^2}$$

$$g_{\rho N}(\rho_B) = g_{\rho N}(\rho_0) \exp[-a_\rho(x - 1)].$$

DD-ME2 parametrization of D. Vretenar, P. Ring et al. Phys. Rev. C 71, 024312 (2005).

	σ	ω	ρ
m_i [MeV]	550.1238	783.0000	763.0000
$g_{Ni}(\rho_0)$	10.5396	13.0189	3.6836
a_i	1.3881	1.3892	0.5647
b_i	1.0943	0.9240	—
c_i	1.7057	1.4620	—
d_i	0.4421	0.4775	—

Total number of parameters 8: boundary conditions on $h(x)$ at $x = 1$.

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Fixing the couplings: hyperonic sector

$R_{\alpha Y} = g_{\alpha Y}/g_{\alpha N}$ and $\kappa_{\alpha Y} = f_{\alpha Y}/g_{\alpha Y}$ for hyperons in SU(6) spin-flavor model

$R\backslash Y$	Λ	Σ	Ξ
$R_{\sigma Y}$	2/3	2/3	1/3
$R_{\sigma^* Y}$	$-\sqrt{2}/3$	$-\sqrt{2}/3$	$-2\sqrt{2}/3$
$R_{\omega Y}$	2/3	2/3	1/3
$\kappa_{\omega Y}$	-1	$1 + 2\kappa_{\omega N}$	$-2 - \kappa_{\omega N}$
$R_{\phi Y}$	$-\sqrt{2}/3$	$-\sqrt{2}/3$	$-2\sqrt{2}/3$
$\kappa_{\phi Y}$	$2 + 3\kappa_{\omega N}$	$-2 - \kappa_{\omega N}$	$1 + 2\kappa_{\omega N}$
$R_{\rho Y}$	0	2	1
$\kappa_{\rho Y}$	0	$-3/5 + (2/5)\kappa_{\rho N}$	$-6/5 - (1/5)\kappa_{\rho N}$
$f_{\pi Y}$	0	$2\alpha_{ps}$	$-(1/2)\alpha_{ps}$

$\alpha_{ps} = 0.40$. κ is the ratio of the tensor to vector couplings of the vector mesons.

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Breaking the SU(6) symmetry: hypernuclei

Λ-hypernuclei

Single-particle energies of the Λ $1s_{1/2}$ states, binding energies, and rms radii of the Λ-hyperon, neutron, and proton of ${}^{17}_{\Lambda}$O, ${}^{41}_{\Lambda}$C, and ${}^{49}_{\Lambda}$Ca are presented for optimal model. In addition, single-particle energies of the Λ $1s_{1/2}$ states, i.e. separation energies of the Λ-particle, obtained from the mass formula of Levai et al (1998) are given for these Λ-hypernuclei. Furthermore, the properties of ^{16}O, ^{40}Ca, and ^{48}Ca are given for the optimal model. The optimal model obtained in this way has $x_{\sigma\Lambda} = 0.6164$.

	$E_{Mass}[\Lambda\ 1s_{1/2}]$ [MeV]	$E[\Lambda\ 1s_{1/2}]$ [MeV]	E/A [MeV]	r_p [fm]	r_n [fm]	r_Λ [fm]
${}^{17}_{\Lambda}$O	−12.109	−11.716	−8.168	2.592	2.562	2.458
^{16}O	–	–	−8.001	2.609	2.579	–
${}^{41}_{\Lambda}$C	−17.930	−17.821	−8.788	3.362	3.309	2.652
^{40}Ca	–	–	−8.573	3.372	3.320	–
${}^{49}_{\Lambda}$Ca	−19.215	−19.618	−8.858	3.379	3.562	2.715
^{48}Ca	–	–	−8.641	3.389	3.576	–

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Breaking the SU(6) symmetry: hyperonic potentials

The depth of hyperonic potentials in symmetric nuclear matter are used as a guide the range of hyperonic couplings:

- Λ particle: $V_{\Lambda}^{(N)}(\rho_0) \simeq -30$ MeV
- Ξ particle: $V_{\Xi}^{(N)}(\rho_0) \simeq -14$ MeV
- Σ particle: $V_{\Xi}^{(N)}(\rho_0) \simeq +30$ MeV

These ranges capture the most interesting regions of the parameter space of masses and radii.

The depth of Δ-potentials in symmetric nuclear matter are used as a guide the range the couplings:

- Electron and pion scattering: $-30 \text{ MeV} + V_{\Delta}^{(N)}(\rho_0) \le V_{\Delta}(\rho_0) \le V_N(\rho_0)$
- Use instead $R_{m\Delta} = g_{m\Delta}/g_{mN}$ for which the typical range used is

$$R_{\rho\Delta} = 1, \quad 0.8 \le R_{\omega\Delta} \le 1.6, \quad R_{\sigma\Delta} = R_{\omega\Delta} \pm 0.2.$$

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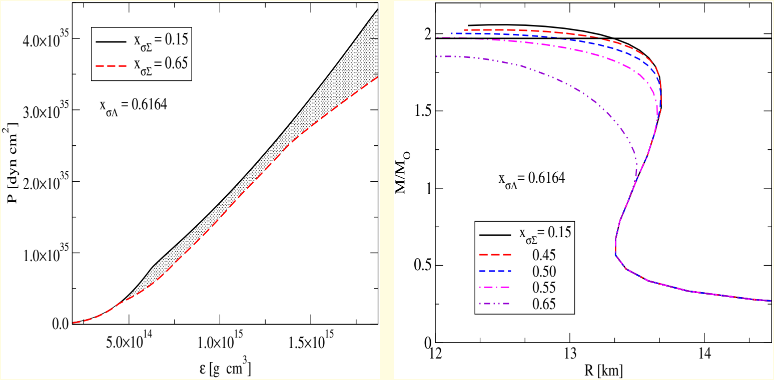
EoS of hypernuclear matter and TOV solutions



- EoS of β-equilibrated and charge neutral hypernuclear matter
- Hyperon puzzle: Hyperons tend to soften the EoS –> it can become incompatible with the data on the masses of massive pulsars.
- GW170817 modeling indicates upper limit on the maximum mass of NS $2.17 M_{\odot}$ —> required for DD models to push the maximum mass to lower values.

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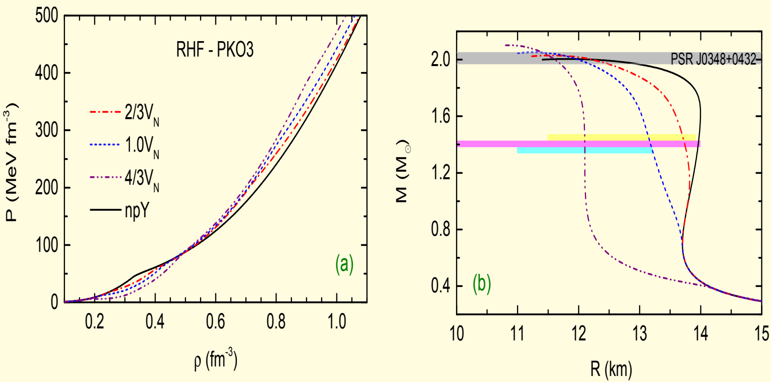
EoS of hypernuclear + Δ matter and TOV solutions



- Softens the EoS at intermediate densities and hardens it at high densities
- Decrease in the radius of the stars; no significant changes in the maximum mass

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Symmetry energy and higher order characteristics

— How robust are the previous results?
— Check the sensitivity on the modeling of the EoS and inclusion of Δ-resonance matter

Model study – varying parameters

$$\frac{E}{A}(\rho,\beta) = E_{\rm sat} + \frac{1}{2!}K_{\rm sat}\chi^2 + \frac{1}{3!}Q_{\rm sat}\chi^3 + E_{\rm sym}\delta^2 + L_{\rm sym}\delta^2\chi + \mathscr{O}(\chi^4,\chi^2\delta^2)$$

where $\delta = (n_n - n_p)/(n_n + n_p)$, $\chi = (\rho - \rho_0)/3\rho_0$ and

$E_{\rm sat}$	saturation energy
$K_{\rm sat}$	compressibility
$Q_{\rm sat}$	skewness
$E_{\rm sym}$	symmetry energy
$L_{\rm sym}$	slope of symmetry energy

Values of Δ-potential are changed in a range

$$V_N \le V_\Delta \le \frac{5}{3}V_N \quad \rightarrow \quad V_\Delta = \left[V_N;\ \frac{4}{3}V_N;\ \frac{5}{3}V_N\right]$$

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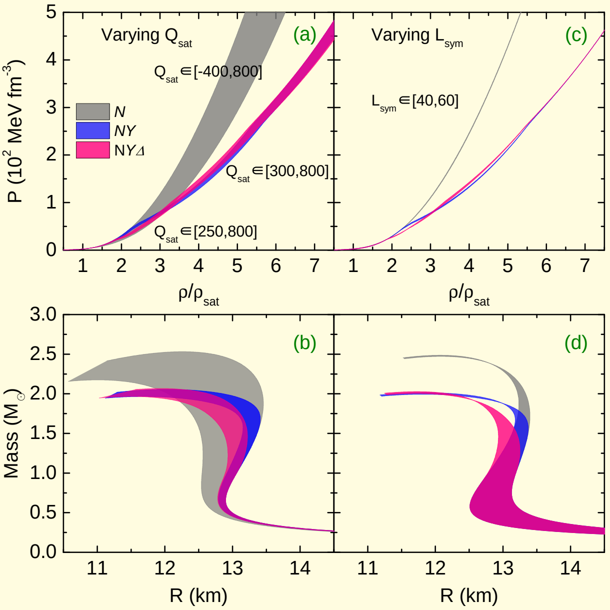

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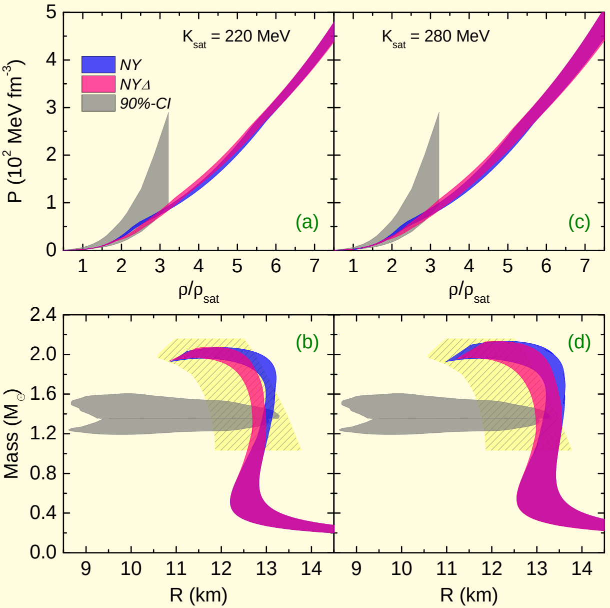

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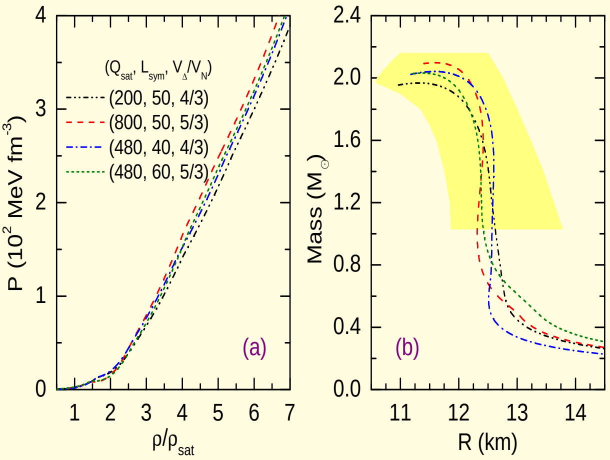

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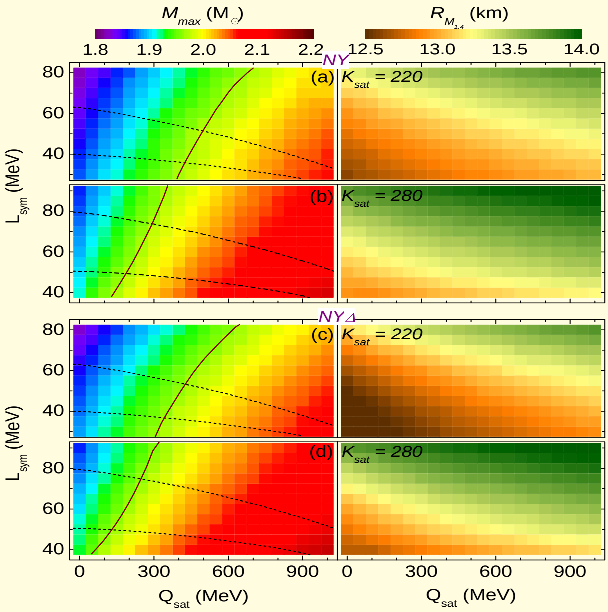

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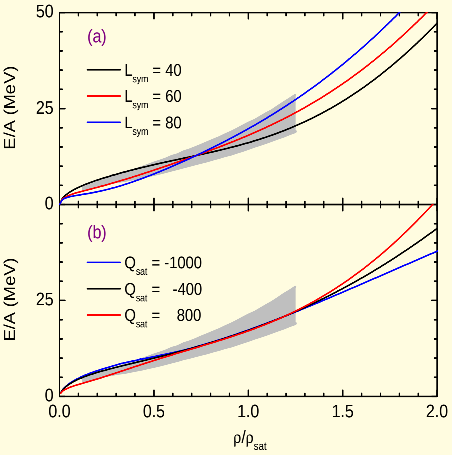
Gauging the low-density limit of DFT with chiral interactions



The low-density EoS: comparison between microscopic EoS and DFT.

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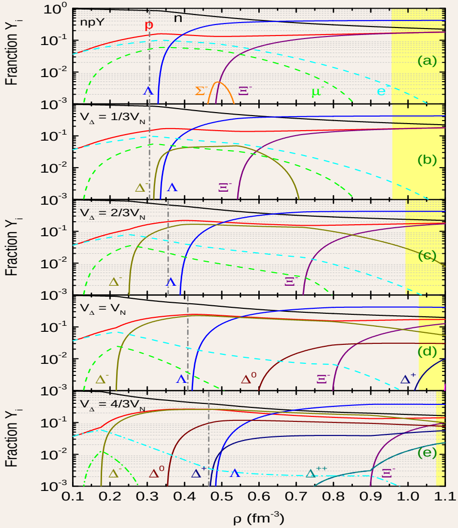
Compositions of hypernuclear matter



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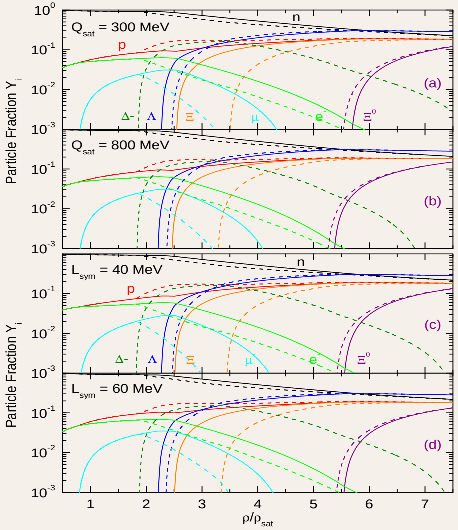
Dependence of characteristics



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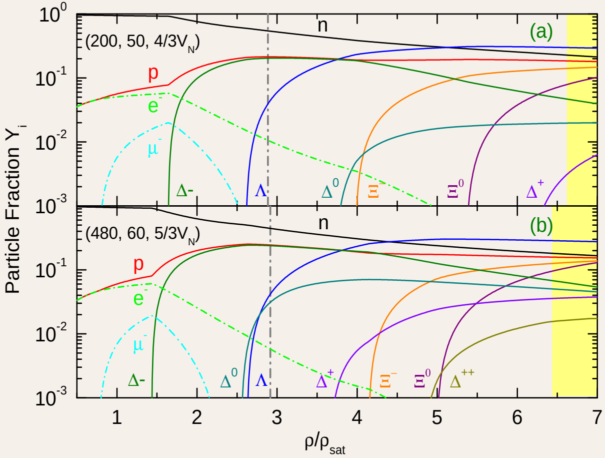
Dependence of Δ-potential depth



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Tidal deformabilities

The tidal deformability λ can be expressed in terms of the dimensionless tidal Love number k_2 and star's radius R as

$$\lambda = \frac{2}{3} k_2 R^5. \tag{83}$$

The tidal Love number k_2 is calculated along with the solution of the Tolman-Oppenheimer-Volkov (TOV) equations.
More convenient to work with the dimensionless tidal deformability Λ, which is related to the Love number k_2 and the compactness parameter $C = M/R$

$$\Lambda = \lambda/M^5 = \frac{2}{3}\frac{k_2}{C^5}. \tag{84}$$

The total tidal effect of two compact stars in an inspiraling binary system is given by the mass-weighted tidal deformability

$$\tilde{\Lambda} = \frac{16}{13}\left[\frac{(M_1 + 12M_2)M_1^4\Lambda_1}{(M_1 + M_2)^5} + 1 \leftrightarrow 2\right]. \tag{85}$$

where $\Lambda_1(M_1)$ and $\Lambda_2(M_2)$ are the tidal deformabilities of the individual binary components.

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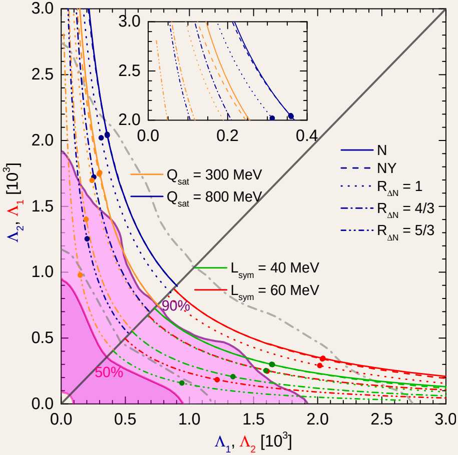
Tidal deformabilities hypernuclear + Δ-resonance matter and GW170817 constraints



Astrophys. J. Lett, 874 (2019) L22

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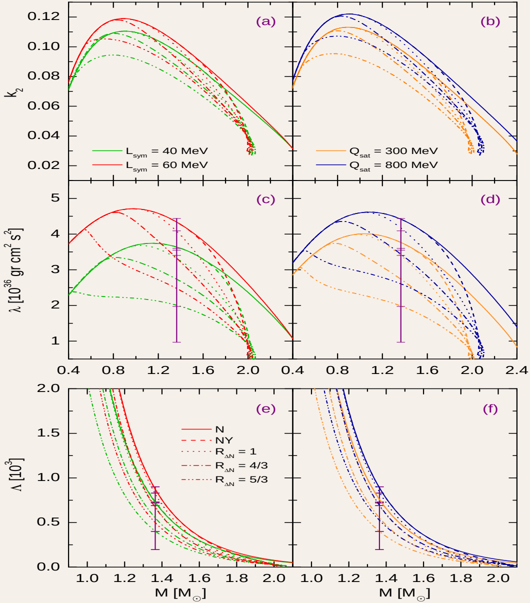
Mass-dependence of tidal deformabilities $NY\Delta$ matter



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VI. Rapidly rotating compact stars

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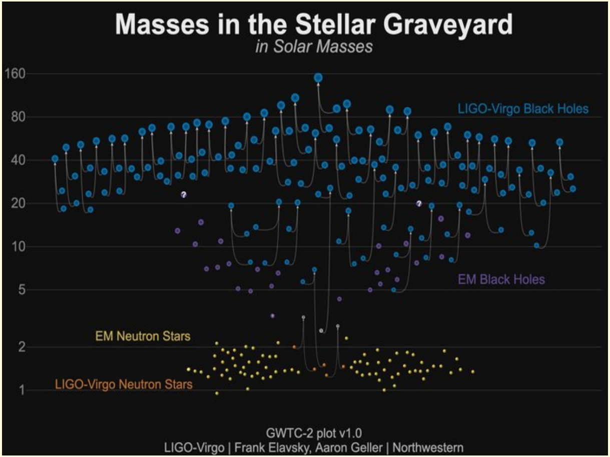
– GW190814 event: extreme mass asymmetric ratio created by a $22.2 - 24.3\, M_\odot$ black hole and a $2.50 - 2.67\, M_\odot$ compact object (no em counterpart).
– Light object's nature is uncertain as it is in the mass gap $2.5\, M_\odot \lesssim M \lesssim 5\, M_\odot$ where no compact object had ever been observed before.

GW190814 - a compact object from pseudo-mass-gap for compact objects



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– Light object's nature is uncertain as it is in the mass gap $2.5\, M_\odot \lesssim M \lesssim 5\, M_\odot$ where no compact object had ever been observed before.

Speculations:

- Simply a stellar-mass black hole (formation mechanism from SN explosion is questionable)
- The formation path via stellar evolution - hierarchical triple systems - remnants of previous mergers?
- Static neutron star - purely nucleonic with stiff EoS - tension with GW170817
- Rapidly rotating stars (Keplerian star) but with softer nucleonic equation of state compatible with GW170817
- Non-nucleonic degrees - hyperons, Δ-resonances, quarks
- Hyperonization in tension with the compact star interpretation.

Hyperons and Δ-resonances:

- Li, Jia Jie, Sedrakian, Armen; Weber, Fridolin; Physics Letters B, 810, 135812, (2020)
- Sedrakian, Armen; Weber, Fridolin; Li, Jia Jie, Physical Review D, 102, 041301 (2020)

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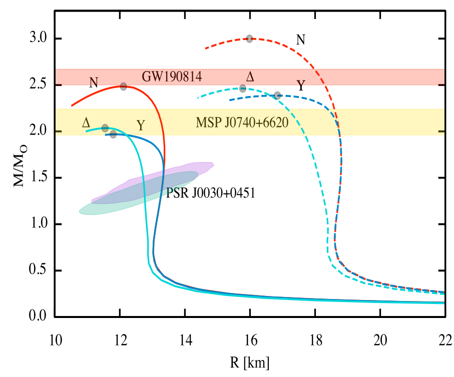

Solid curves – static solutions; dashed curves - maximally rotating (Keplerian) solutions.

— Nucleonic models are compatible with both event GW170817 and GW190814 and NICER *with and without rotation*
— Hyperonic models are compatible with GW170817 and NICER *with and without rotation* but are inconsistent with GW190814 being a compact star *even assuming Keplerian rotation* – Physics Letters B, 810, 135812, (2020)

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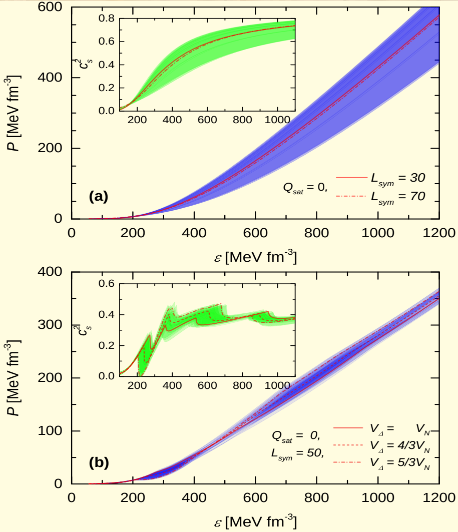

EoS and the corresponding speed-of-sound squared for (a) N and (b) $NY\Delta$ matter. In (a) $Q \in [-600, 900]$ and $L_{\rm sym} \in [30, 70]$. EoS with $Q = 0$, $L_{\rm sym} = 30$ and 70 are shown by solid and dash-dotted lines for illustration. In (b) $Q \in [300, 900]$, $_{\rm sym} \in [30, 70]$ $V_D/V_N = 1, 4/3$ and $5/3$ EoSs with $Q = 600$, $L_{\rm sym} = 50$ and three indicated values of V_D are shown for illustration.

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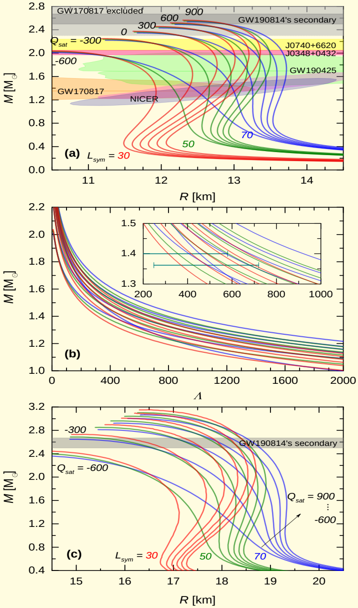

Mass-radius (a) mass-tidal deformability (b) for static N-stars. (c) Mass-radius for maximally rotating (Keplerian) sequences.

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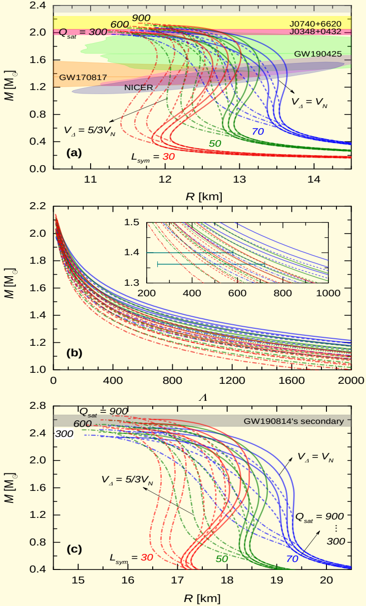

Mass-radius (a) mass-tidal deformability (b) for static $NY\Delta$-stars. (c) Mass-radius for maximally rotating (Keplerian) sequences.

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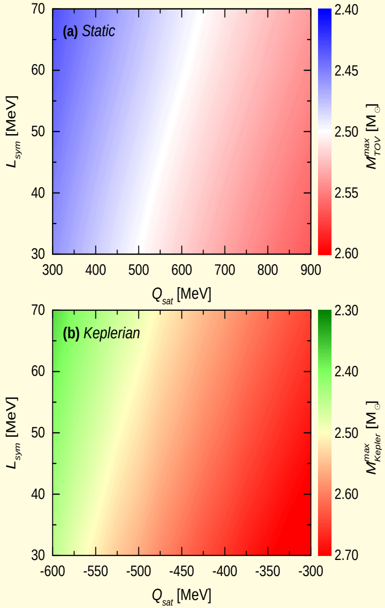

Maximum masses of (a) static and (b) Keplerian N-stars.

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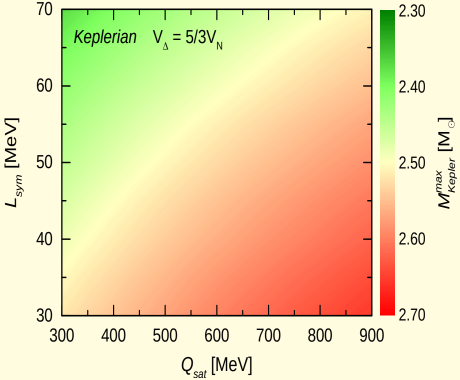

Maximum masses of (a) static and (b) Keplerian $NY\Delta$-stars. The Δ potential $V_\Delta = 5/3 V_N$, the maximal value studied.

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Conclusions on GW190814:

- A rapidly, uniformly rotating compact star made of purely nucleonic matter could have been the secondary stellar object involved in the GW190814 event.
- Taking hyperon and Δ-resonance populations into account our EoS models reduces the masses of compact stars. In particular, the maximal masses of non-rotating stars are reduced to $2.0\,M_\odot \lesssim M \lesssim 2.2\,M_\odot$ if $Q_{\rm sat} \gtrsim 300$ MeV. So none of these models comes even close to the $2.5\,M_\odot$ constraint set by GW190814.
- In this case, the stellar models computed for a strongly attractive Δ-potential in nuclear matter of $V_\Delta/V_N = 5/3$ reach the $2.5\,M_\odot$ mass limit rather comfortably. The situation is strongly depending on the $Q_{\rm sat}$ and $L_{\rm sym}$ values.
- We note that all the valid EoS models in this figure lead to $R_{1.4}$ and $\Lambda_{1.4}$ values that are in agreement with observation.
- The combinations required for $Q_{\rm sat}$ and $L_{\rm sym}$ lie outside the range covered by presently known non-relativistic and relativistic nuclear density functionals. A few exceptions to this are the functionals with values $Q_{\rm sat} \gtrsim 500$ MeV and the possibility of $M^{\rm max}_{\rm Kepler}/M_\odot \gtrsim 2.5$.
- a neutron star interpretation cannot be excluded at this time, but would require a range of extreme assumptions:
  (a) rapid (Keplerian) rotation, which may not be reached due to various instabilities that may set in at lower rotation frequencies;
  (b) strongly attractive Δ-resonance potential in symmetric nuclear matter;
  (c) large, positive value of the isoscalar skewness $Q_{\rm sat}$ parameter.

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VII. Universalities of global characteristics at finite T

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Universalities of TOV solutions:

- Universal (independent of the underlying EoS) relations among the global properties of compact stars - $I-L-Q$ relations. (Yagi and Yunes 2013a; Maselli et al. 2013; Breu and Rezzolla 2016; Yagi and Yunes 2017)
  Well established for:
  (a) zero temperature slowly rotating stars
  (b) rapidly rotating cold star
  (c) magnetized cold star
- Finite temperature stars (proto-neutron stars, BNS remnants) - universalities, $I-L-Q$ relations and $I(C)$ are broken (Martinon et al. 2014; Marques et al. 2017; Lenka et al. 2019). Both S =Const and S-gradients
- But if one considers fixed values of $(S/A, Y_{L,e})$ universal relations hold - accuracy comparable to cold compact stars; A. Raduta, M. Oertel, A. S., arXiv:2008.00213
- Universalities also hold for rapidly rotating hot stars for fixed values of $(S/A, Y_{L,e})$ S. Khadkikar, A. Raduta, M. Oertel, A. S. arXiv:2102.00988
- Universality can be used to extract the maximum mass of hot static compact stars from GW170817. arXiv:2102.00988

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Universal relations that are tested

- Normalize moment of inertia $\tilde{I} = I/\left(M_G R^2\right)$ as a function of compactness $C = M_G/R$:

$$\tilde{I} = c_0 + c_1 C + c_2 C^2 + c_3 C^3 + c_4 C^4. \tag{86}$$

- Another normalization $\bar{I} = I/\left(M_G^3\right)$

$$\bar{I} = a_1 C^{-1} + a_2 C^{-2} + a_3 C^{-3} + a_4 C^{-4}. \tag{87}$$

- Tidal deformability $\bar{\lambda} = \lambda/M_G^5$ vs compactness

$$C = b_1 + b_2 \ln \bar{\lambda} + b_3 \left(\ln \bar{\lambda}\right)^2. \tag{88}$$

- Quadrupole moment $\bar{Q} = Q M_G/J^2$ (J is angular momentum)

$$\bar{Q} = e_0 + e_1 C^{-1} + e_2 C^{-2} + e_3 C^{-3}. \tag{89}$$

- Binding energy of the star $E_B = M_B - M_G$:

$$\frac{E_B}{M_G} = \frac{d_1 C}{1 - d_2 C}. \tag{90}$$

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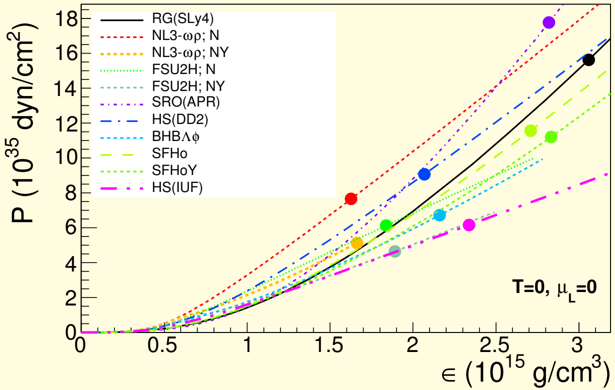

Pressure of cold, β-equilibrated neutron star matter as function of its energy density according to the EoS models. The symbols indicate the central energy density of the maximum mass configuration for cold, β-equilibrated matter.

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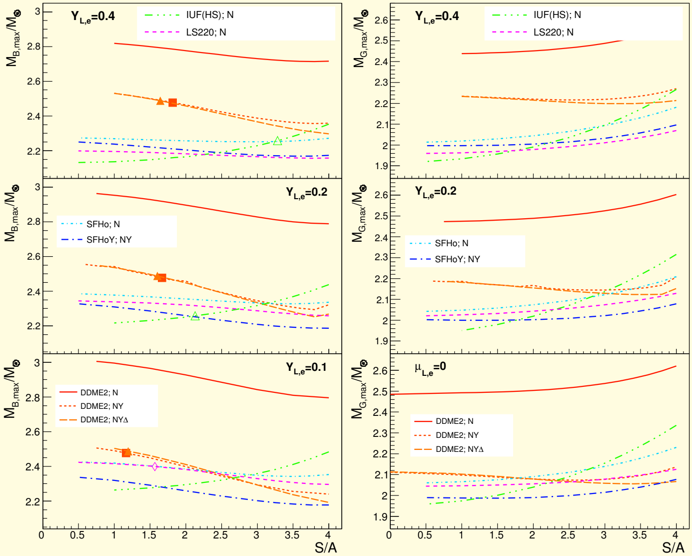

Maximum baryonic (left) and gravitational (right) mass as a function of S/A for different equations of state.

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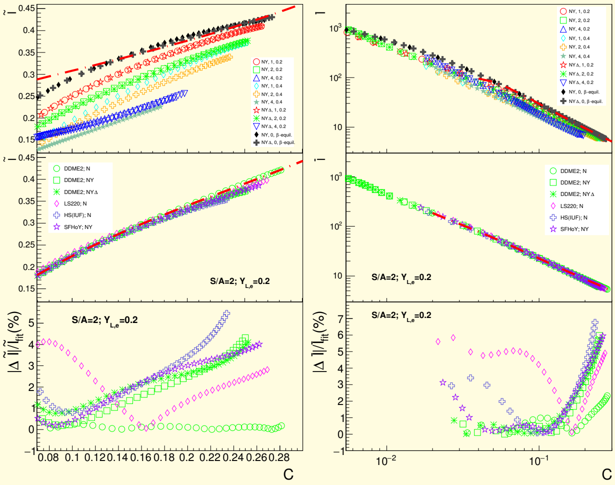

Normalized moments of inertia (upper panel - different S, middle and lower panel fixed S) as function of compactness $C = M_G/R$.

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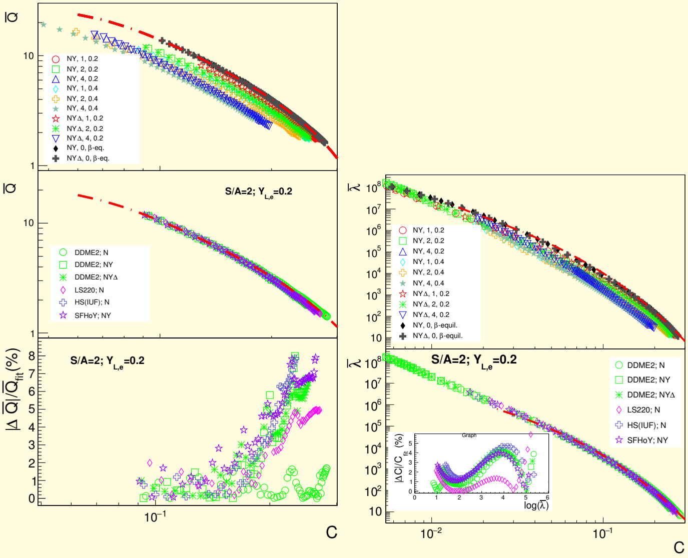

Normalized quadrupole moment $\bar{Q}$ and deformability $\bar{\lambda}$ of slowly and rigidly rotating stars as function of compactness, C.

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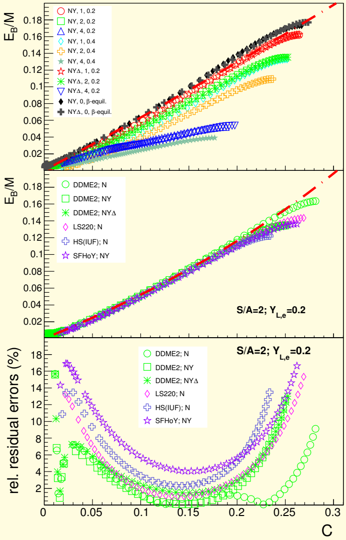

Binding energy per unit of gravitational mass E_B/M_G of non-rotating spherically-symmetric stars as a function of compactness C.

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Conclusions on universality of static stars:

- Martinon et al. 2014; Marques et al. 2017; Lenka et al. 2019 have argued that thermal effects induce deviations from the universal relations. These findings were confirmed.
- When the universal relations are studied using various EoS at the same entropy per baryon and the same lepton fraction, universality is recovered.
- I-Love-Q relations are valid if equations of states are tested under the same thermodynamics conditions, for example S/A and Y_e.

Relating static (TOV) and Keplerian (K) star properties

- value of the maximal mass of Keplerian configuration

$$M_K^\star = C_R^\star M_{\rm TOV}^\star,$$

- circumferential radius of the maximum mass static configuration *($\star$ refers to maximum mass star)*

$$R_K^\star = C_R^\star R_{\rm TOV}^\star,$$

- rotation frequency of this maximum mass configuration

$$f_K^\star = C_f^\star x_{\rm TOV}^\star, \quad x_{\rm TOV}^\star = (M_{\rm TOV}^\star/M_\odot)^{1/2} \cdot (10\,{\rm km}/R_{\rm TOV}^\star)^{3/2}$$

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Relating the static and Keplerian properties of hot stars

- Maximum mass

$$M_K^\star(S/A, Y_e) = C_M^\star(S/A, Y_e) M_S^\star(S/A, Y_e) ,$$

- Radius of maximum mass star

$$R_K^\star(S/A, Y_e) = C_R^\star(S/A, Y_e) R_S^\star(S/A, Y_e) ,$$

- Rotation frequency of maximum mass star

$$f_K^\star(S/A, Y_e) = C_f^\star(S/A, Y_e) x_S^\star(S/A, Y_e) ,$$

- And not only for the maximum of a sequence, but for same M_G and fixed S/A and Y_e

$$R_K(M) = C_R R_S(M) ,$$

- $$f_K(M) = C_f x_S(M) , \quad x_S = \left[(M/M_\odot) \cdot (10\,\mathrm{km}/R_S(M))^3 \right]^{1/2} .$$

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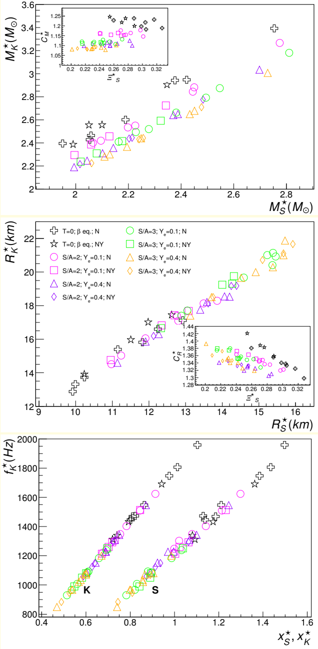

Universality for maximum mass objects.

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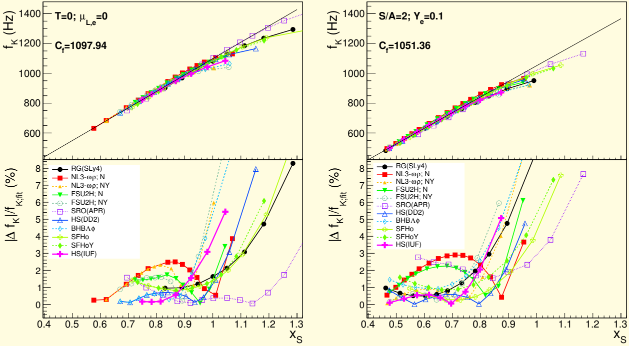

Universality of frequency for various masses (not only maximum mass).

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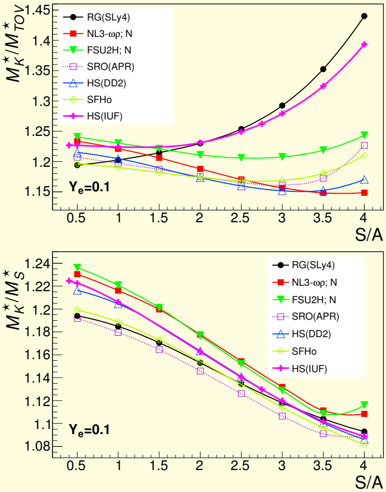

The different normalizations (cold vs hot) show that the universality is broken when normalized to the cold TOV mass and is maintained if normalized by static hot star.

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Maximum TOV mass from GW170817

Previous work by used universalities for cold stars to extract the upper limit on the TOV mass

$$M^{\star}_{\rm TOV} \leq 2.16^{+0.17}_{-0.15} M_{\odot} \quad M^{\star}_{\rm TOV} \leq 2.3 M_{\odot}$$

Scenario supported by observations and numerical simulations:

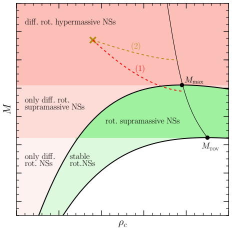

Figure from Rezzolla et al. ApJ Lett. 852, L25 (2018)

- the merger leaves TOV behind a hypermassive neutron star (HMNS)
- the internal dissipation leads to vanishing internal shears and eventually to uniform rotation.
- the star enters the region of stability of supramassive neutron stars close to the maximum mass
- the star crosses the stability line beyond which it is unstable to collapse

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Maximum TOV mass from GW170817

- The extraction of the upper limit circumvents the full dynamical study and uses the *baryon mass* conservation at *at merger* $t = 0$ *and collapse* $t = t_c$

$$M_B(t_c, S/A, Y_e) = M_B(0) - M_{\rm out} - M_{\rm ej},$$

- Go over from the baryon to the gravitational mass

$$M_B(t_c, S/A, Y_e) = \eta(S/A, Y_e) M(t_c, S/A, Y_e) = \eta(S/A, Y_e) M_K^\star(S/A, Y_e),$$

The last step assumes that the star is Keplerian (Shibata et al 2019 relax this assumption).

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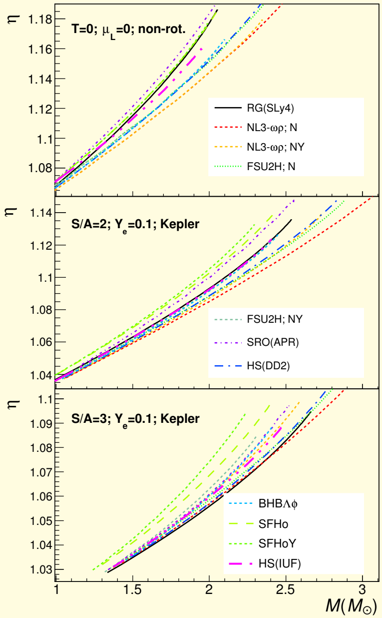

Dependence of the η parameter on the gravitational mass.

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Solce the mass conservation for hot Keplerian mass

$$M_K^\star(S/A, Y_e) = \frac{1}{\eta(S/A, Y_e)} \left[\eta(0)M(0) - M_{\rm out} - M_{\rm ej}\right]. \quad (91)$$

The analysis of GW170817 gives us the values:

- $M(0) = 2.73 M_\odot \; {}^{+0.04}_{-0.01}$
- $M_{\rm ej} = 0.04 \pm 0.01 M_\odot \to M_{\rm out} + M_{\rm ej} = 0.1 \pm 0.041$
- In GW170817 the primary/secondary masses lie in the range $1.35 \leq M/M_\odot \leq 1.6$
- for cold compact stars based on our collection of EoS we have $\eta(0) \simeq 1.120 \pm 0.002$ for $M = 1.6 M_\odot$ and $\eta(0) \simeq 1.085 \pm 0.001$ for $M = 1.2 M_\odot$
- For our estimates we adopt the value $\eta(0) \simeq 1.1004^{+0.0014}_{-0.0003}$ leading to $M_B(0) = 3.00^{+0.05}_{-0.01} M_\odot$.
- Assuming that the star is rotating at the Keplerian frequency. We then find that $\eta(2, 0.1) \simeq 1.139 \pm 0.004$ and $\eta(3, 0.1) \simeq 1.099 \pm 0.003$. For the quantity $(M_{\rm out} + M_{\rm ej})/\eta(S/A, Y_e)$ we obtain 0.087 ± 0.036 and 0.091 ± 0.037 for $S/A = 2$ and 3 and $Y_e = 0.1$,
- Substituting the numerical values we find

$$\boxed{M_K^\star(2, 0.1) = 2.55^{+0.06}_{-0.04}, \quad M_K^\star(3, 0.1) = 2.64^{+0.06}_{-0.04}.}$$

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Next step gives us *maximum mass of non-rotating hot compact stars*, using *universality*

$$M_S^\star(2,0.1) = 2.19^{+0.05}_{-0.03}, \quad M_S^\star(3,0.1) = 2.36^{+0.05}_{-0.04}.$$

The universality is broken, but we can deduce an upper limit on *the maximum mass of cold compact stars*. The average values $C_M^\star = 1.19 \pm 0.04$ for $S = 2$ and $C_M^\star = 1.18 \pm 0.11$ for $S = 3$ can now be used to obtain, respectively,

$$\boxed{M_{\rm TOV}^\star = 2.15^{+0.09+0.16}_{-0.07-0.16}, \qquad M_{\rm TOV}^\star = 2.24^{+0.10+0.44}_{-0.07-0.44}.}$$

(2σ standard deviation)

- Constant entropy per baryon and constant electron fraction star (?)
- $S/A = 2$ and 3 – average values for the inner part of the merger remnant.
- more precise result would require a profile for S

Conclusion: Accounting for the finite temperature of the merger remnant relaxes the derived constraints on the maximum mass of the cold, static compact star, obtained in by Margalit-Metzger 2017, Rezzolla et al 2018, Ruiz et al (2018), Shibata et al (2019a).

Universality is lost and the final upper limit becomes EoS dependent due to the EoS dependence of $C_M^\star$.

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Summary of topics covered in Lecture 1

- Astrophysical constraints
- Density functionals for non-relativistic nuclear matter (Skyrme)
- Density functionals for relativistic hypernuclear matter with Δ resonances
- Mass, radius, tidal deformabilities for static stars
- Rapidly rotating stars
- Universalities of global parameters