Equation of state for binary neutron star mergers and core-collapse supernovae:
Lecture 2: QCD phases

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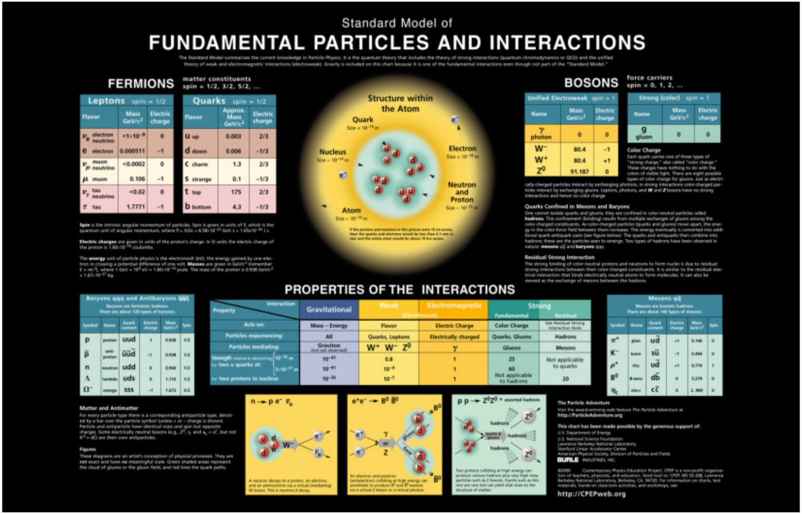
I. The standard model and QCD


Standard Model of
FUNDAMENTAL PARTICLES AND INTERACTIONS
FERMIONS
matter constituents spin = 1/2, 3/2, 5/2, ...
Leptons spin = 1/2
Quarks spin = 1/2
BOSONS
force carriers spin = 0, 1, 2, ...
Unified Electroweak spin = 1
Strong (color) spin = 1
Structure within the Atom
Quark
Nucleus
Electron
Neutron and Proton
Atom
PROPERTIES OF THE INTERACTIONS
Gravitational
Weak
Electromagnetic
Strong
http://CPEPweb.org

Quarks

symbol	name	electrical charge	mass
u	up	+2/3	0.31GeV
d	down	−1/3	0.31 GeV
c	charm	+2/3	1.6 GeV
s	strange	−1/3	0.5 GeV
t	top	+2/3	17.5 GeV
b	bottom	−1/3	4.6 GeV

Baryons

$$n = \langle ddu \rangle, \quad p = \langle uud \rangle, \tag{1}$$

$$\Sigma^0 = \langle uds \rangle, \quad \Sigma^+ = \langle uus \rangle, \quad \Sigma^- = dds \quad \Lambda = \langle uds \rangle, \tag{2}$$

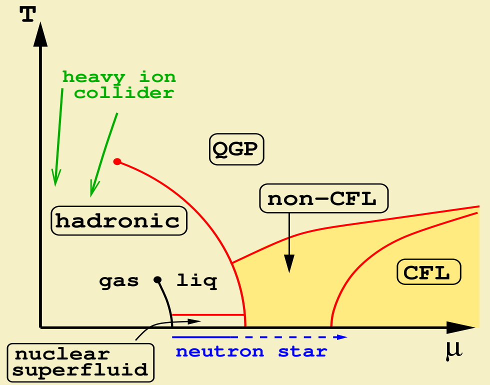
Mesons

$$\pi^- = \langle \bar{u}d \rangle, \quad \pi^+ = \langle u\bar{d} \rangle. \tag{3}$$

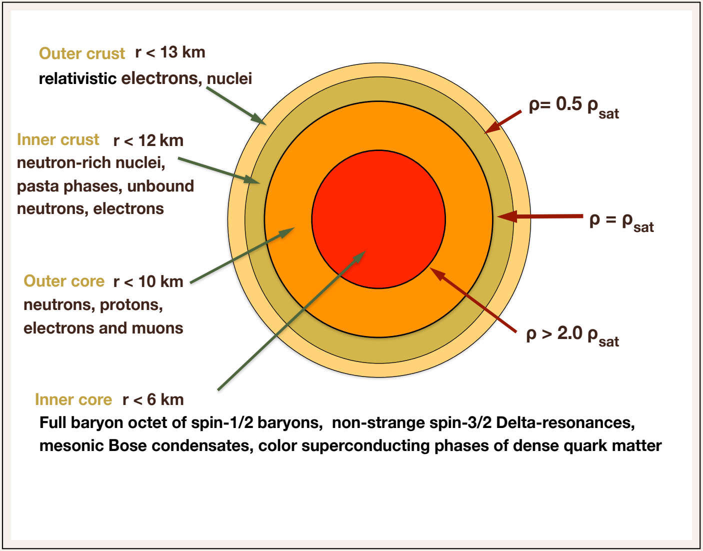
$$K^0 = \langle d\bar{s} \rangle, \quad K^+ = \langle u\bar{s} \rangle, \quad K^- = \langle \bar{u}s \rangle. \tag{4}$$

At sufficiently high density the nucleonic matter with make a transition to the quark matter state (deconfinement)

- High-temperature QGP phase is probed in heavy ion colliders
- Low-density low-temperature nucleonic matter in nuclei and low-densities of neutron stars
- Low-temperature high density phase of dense matter may be in the quark state (compact stars)



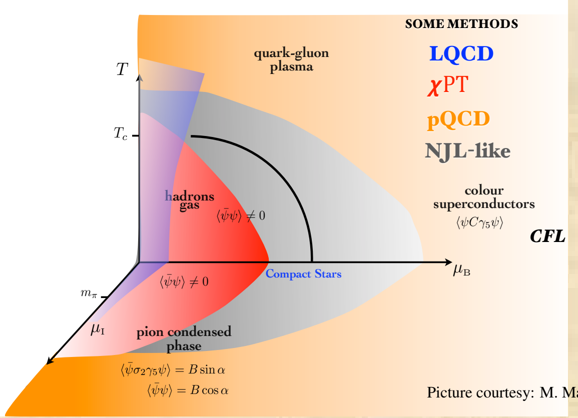
Internal structure of a compact star



The Lagrangian of QCD is written for $\psi_q = (\psi_{qR}, \psi_{qG}, \psi_{qB})^T$ as

$$\mathcal{L}_{QCD} = \underbrace{\bar{\psi}^i_q (i\gamma^\mu)(D_\mu)_{ij}\psi^j_q - m_q \bar{\psi}^i_q \psi_{qi}}_{\text{quarks}} - \underbrace{\frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu}}_{\text{gluons(Yang-Mills)}} ,$$

where $\underbrace{(D_\mu)_{ij} = \delta_{ij}\partial_\mu - i g_s t^a_{ij} A^a_\mu}_{\text{covarinat derivative}}$, and $\underbrace{F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu - 2q(A^\mu \times A^\nu)}_{\text{gluonic field (Yang-Mills) field tensor}}$



Picture courtesy: M. Mannarelli

The Lagrangian of QCD is

$$\mathcal{L} = \bar{\psi}_q^i (i\gamma^\mu)(D_\mu)_{ij}\psi_q^j - m_q \bar{\psi}_q^i \psi_{qi} - \frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu} , \quad (5)$$

where ψ_q^i denotes a quark field with (fundamental) colour index i, $\psi_q = (\psi_{qR}, \psi_{qG}, \psi_{qB})^T$, γ^μ is a Dirac matrix that expresses the vector nature of the strong interaction, with μ being a Lorentz vector index, m_q allows for the possibility of non-zero quark masses (induced by the standard Higgs mechanism or similar), $F^a_{\mu\nu}$ is the gluon field strength tensor for a gluon with (adjoint) colour index a (i.e., $a \in [1, \ldots, 8]$), and D_μ is the covariant derivative in QCD,

$$(D_\mu)_{ij} = \delta_{ij}\partial_\mu - i g_s t^a_{ij} A^a_\mu , \quad (6)$$

with g_s the strong coupling (related to α_s by $g_s^2 = 4\pi\alpha_s$), A^a_μ the gluon field with colour index a, and t^a_{ij} proportional to the hermitean and traceless Gell-Mann matrices of $SU(3)$. The field tensor of the gluonic Yang-Mills field is given by

$$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu - 2q(A^\mu \times A^\nu) \quad (\boldsymbol{A} \times \boldsymbol{B})_i = \sum_{j,k=1}^{8} f_{ijk} A_j B_k \quad (7)$$

f_{ijk} are structure constant of $SU(3)$.

These generators are just the $SU(3)$ analogs of the Pauli matrices in $SU(2)$. By convention, the constant of proportionality is normally taken to be

$$t^a_{ij} = \frac{1}{2}\lambda^a_{ij}\,. \tag{8}$$

This choice in turn determines the normalization of the coupling g_s and fixes the values of the $SU(3)$ Casimirs and structure constants. t^a_{ij} proportional to the hermitean and traceless Gell-Mann matrices of $SU(3)$

$$\lambda^1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda^2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \lambda^3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\lambda^4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \quad \lambda^5 = \begin{pmatrix} 0 & -0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}$$

$$\lambda^6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \quad \lambda^7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix} \quad \lambda^8 = \frac{1}{\sqrt{3}}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix} \tag{9}$$

We have six replicas of Lagrangian for each quark flavor (differing by mass).
Each of the Lagrangians is invariant under $SU(3)$ gauge transformations and describes three equal mass fields of different color (say, red, green, blue).

In color space the one-gluon quark interaction has an attractive component

$$\sum_A^{N_c^2-1} t_{ki}t_{lj} = \underbrace{-\frac{N_c+1}{4N_c}(\delta_{jk}\delta_{il}-\delta_{ik}\delta_{jl})}_{\text{attractive}} + \underbrace{\frac{N_c-1}{4N_c}(\delta_{jk}\delta_{il}-\delta_{ik}\delta_{jl})}_{\text{repulsive}} \quad (10)$$

Because of attractive interaction quarks form Cooper-pairs:
The symmetry requires that for $J = 0$ (spin-zero pairs with zero angular momentum) need to quark of different flavor and different color

$$\boxed{\Delta_{ij}^{fg} = \epsilon_{ijk}\epsilon^{fg}\Delta_k, \quad \Phi_{ij}^{fg} = \gamma_5\Delta_{ij}^{fg}.} \quad (11)$$

This is the 2SC phase. Only red-green quarks are paired, blue quarks are unpaired. In a superconductor the quasiparticle spectrum is given by

$$E_{\boldsymbol{k}}^e = \sqrt{(\xi_{\boldsymbol{k}} - e\mu)^2 + \Delta^2}, \quad \Delta_k = \delta_{k3}\Delta. \quad (12)$$

In the three-flavor case color-flavor-locked (CFL) phase is realized

$$\boxed{\Delta_{ij}^{fg} = \epsilon_{ijk}\epsilon^{fgh}\Delta_k^h, \qquad \Delta_k^h = \delta_k^h\Delta \qquad \Phi_{ij}^{fg} = \gamma_5\Delta_{ij}^{fg}.} \quad (13)$$

To obtain the spectrum in a simple form allow for small sextet gaps:

$$\Delta_{ij}^{fg} = \Delta_{\bar{3},\bar{3}}(\delta_i^f\delta_j^g - \delta_i^g\delta_j^f) + \Delta_{6,6}(\delta_i^f\delta_j^g - \delta_i^g\delta_j^f), \quad \Delta_{ij}^{fg} = \Delta_1'\delta_i^f\delta_j^g + \Delta_2'\delta_i^g\delta_j^f \tag{14}$$

$$\Delta_1' = \Delta_{\bar{3},\bar{3}} + \Delta_{6,6} \qquad \Delta_2' = -\Delta_{\bar{3},\bar{3}} + \Delta_{6,6}. \tag{15}$$

Color projectors

$$[P_1]_{ij}^{fg} = \frac{1}{3}\delta_i^f\delta_j^g, \quad [P_2]_{ij}^{fg} = \frac{1}{2}(\delta_{ij}\delta^{fg} - \delta_i^g\delta_j^f), \tag{16}$$

$$[P_3]_{ij}^{fg} = \frac{1}{2}\delta_{ij}\delta^{fg}\delta_g^i\delta^{fg} + \frac{1}{2}\delta_i^g\delta_j^f - \frac{1}{3}\delta_i^f\delta_j^g, \quad \underbrace{\sum_n P_n = 1}_{\text{normalization}}, \quad \underbrace{P_iP_j = \delta_{ij}P_j}_{\text{orthogonality}}. \tag{17}$$

$$\boxed{\Delta_{ij}^{fg} = \sum_n^3 \Delta_n[P_n]_{ij}^{fg} = \frac{1}{3}(\Delta_1 + \Delta_2)\delta_i^f\delta_j^g - \Delta_2\delta_i^g\delta_f^j., \qquad \Delta_3 = -\Delta_2.} \tag{18}$$

II. Thermodynamics of QCD

Partition function of QCD at finite density/temperature:

$$\mathcal{L} = \overline{\psi}(i\gamma^\mu D_\mu + \hat{\mu}\gamma_0 - \hat{m})\psi - \frac{1}{4}F_a^{\mu\nu}G^a_{\mu\nu}, \quad F_a^{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + gf^{abc}A^b_\mu A^c_\nu, \quad (19)$$

ψ $4N_cN_f$-spinor, $D_\mu = \partial_\mu + igT_aA^a_\mu$, A^a_μ gauge fields and $T^a = \lambda^a/2$ $(a = 1, \dots, 8)$ generators of $SU(3)_c$.

Partition function in terms of fields $\psi(x)$

$$\mathcal{Z} = \int \mathcal{D}\bar{\psi}\mathcal{D}\psi \exp\left[S_0[\bar{\psi},\psi] + S_I[\bar{\psi},\psi]\right], \quad (20)$$

Free part

$$S_0[\bar{\psi},\psi] = \int dx\, dy\, \bar{\psi}(x)\left[G_0^+\right]^{-1}(x,y)\psi(y), \quad (21)$$

Interaction part

$$S_I[\bar{\psi},\psi] = \frac{g^2}{2}\int dx\, dy \sum_{a,b} \bar{\psi}(x)\Gamma^\mu_a\psi(x)D^{ab}_{\mu\nu}(x,y)\bar{\psi}(y)\Gamma^\nu_b\psi(y). \quad (22)$$

Consider $N_f = 2$ and $N_c = 3$; define the basis

$$\bar{\psi} = (\bar{\psi}^u_r, \bar{\psi}^d_r, \bar{\psi}^u_g, \bar{\psi}^d_g, \bar{\psi}^u_b, \bar{\psi}^d_b). \tag{23}$$

Consider a local transformation on quark fields given by

$$\psi' \to \psi e^{-i\theta(x)}, \quad \bar{\psi}' \to \bar{\psi} e^{i\theta(x)}, \tag{24}$$

and special case $\theta(x) = Q_\mu x^\mu/2$, $Q = (0, \boldsymbol{Q})$. Quark Green's function becomes

$$\left[\tilde{G}^+_0\right]^{-1}(x,y) = [G^+_0]^{-1}(x,y) + \gamma^\mu \partial_\mu \theta(y). \tag{25}$$

Momentum space

$$[\tilde{G}^\pm_0]^{-1} = \gamma^\mu (\pm Q_\mu/2 + k_\mu) \pm \mu\gamma_0 - m. \tag{26}$$

Bosonize the action

$$\Delta^+(x,y) = g^2 \sum_{a,b} \bar{\Gamma}^\mu_a \langle \psi_C(x)\bar{\psi}(y)\rangle \Gamma^\nu_b \mathcal{D}^{ab}_{\mu\nu}(x,y), \tag{27}$$

Nambu-Gorkov spinor fields

$$\Psi = \begin{pmatrix} \psi \\ \psi_C \end{pmatrix}, \qquad \bar{\Psi} = \left(\bar{\psi}, \bar{\psi}_C\right), \tag{28}$$

Full propagator/self-energy

$$\mathcal{G} = \begin{pmatrix} \tilde{G}_0^+ & F^- \\ F^+ & \tilde{G}_0^- \end{pmatrix}, \qquad \Omega = \begin{pmatrix} \Sigma^+ & \Delta^- \\ \Delta^+ & \Sigma^- \end{pmatrix}. \tag{29}$$

Schwinger-Dyson equations

$$[G^\pm]^{-1} = \left[G_0^\pm\right]^{-1} + \Sigma^\pm - \Delta^\mp \left(\left[G_0^\mp\right]^{-1} + \Sigma^\mp\right)^{-1} \Delta^\pm, \tag{30}$$

$$F^\pm = -\left(\left[G_0^\mp\right]^{-1} + \Sigma^\mp\right)^{-1} \Delta^\pm G^\pm, \tag{31}$$

Partition function in the mean field approximation:

$$\begin{aligned} \mathcal{Z}_{\rm MF} &= \left[\det_k(\beta\mathcal{G}^{-1})\right]^{1/2} \\ &\times \exp\left[\frac{g^2}{2\beta V}\int\frac{d^4k d^4p}{(2\pi)^8}\sum_{a,b}\mathrm{Tr}\left[\tilde{G}^-(k)\bar{\Gamma}^\mu_a\tilde{G}^+(p)\Gamma^\nu_b\right]D^{ab}_{\mu\nu}(k-p)\right], \\ G_0^\pm &= \mathrm{diag}\left(G_{0\,r}^{\pm u}, G_{0\,r}^{\pm d}, G_{0\,g}^{\pm u}, G_{0\,g}^{\pm d}, G_{0\,b}^{\pm u}, G_{0\,b}^{\pm d}\right). \end{aligned} \tag{32}$$

The gap functions, in the basis (37), are given by

$$\Delta^\pm = \begin{pmatrix} 0 & 0 & 0 & \Delta_1^\pm & 0 & 0 \\ 0 & 0 & \Delta_2^\pm & 0 & 0 & 0 \\ 0 & \Delta_2^\pm & 0 & 0 & 0 & 0 \\ \Delta_1^\pm & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}. \tag{33}$$

$$\Delta^+_{1,2}(k) = \sum_e \eta^e_{1,2}(k)\Lambda^e(\boldsymbol{k}), \tag{34}$$

The full anomalous propagator

$$F^{\pm} = \begin{pmatrix} 0 & 0 & 0 & F^{\pm ud}_{rg} & 0 & 0 \\ 0 & 0 & F^{\pm du}_{rg} & 0 & 0 & 0 \\ 0 & F^{\pm ud}_{gr} & 0 & 0 & 0 & 0 \\ F^{\pm du}_{gr} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \qquad (35)$$

$$F^{\pm ud}_{rg} = -G^{\mp u}_{0\ r}\Delta^{\pm}_{1}G^{\pm d}_{0\ g}, \quad F^{\pm du}_{rg} = -G^{\mp d}_{0\ r}\Delta^{\pm}_{2}G^{\pm u}_{0\ g}, \qquad \text{etc} \qquad (36)$$

The quasiparticle spectrum is determined from

$$\left(G^{-\,f}_{0\ i}\right)^{-1}\left(G^{+\,g}_{0\ j}\right)^{-1} - \sum_{e}|\eta^{efg}_{ij}|^{2}\Lambda^{e}(\boldsymbol{k}) = 0, \quad E^{\pm}_{e}(\eta^{e}_{1,2}) = E_{A,e} \pm \sqrt{E^{2}_{S,e} + |\eta^{e}_{1,2}|^{2}},$$

$$E_{S,e}(|\boldsymbol{k}|, |\boldsymbol{Q}|, \theta, \bar{\mu})^{2} = (|\boldsymbol{k}| - e\bar{\mu})^{2}, \quad E_{A,e}(|\boldsymbol{Q}|, \theta, \delta\mu) = \delta\mu + e|\boldsymbol{Q}|\cos\theta, \qquad (37)$$

Integrate out gluons:

$$D^{ab}_{\mu\nu} = \delta^{ab}\frac{g_{\mu\nu}}{\Lambda^2}, \tag{38}$$

where Λ is a characteristic momentum scale. The partition function becomes

$$\ln\mathcal{Z}_{\rm MF} = \ln\mathcal{Z}^{\Delta}_{\rm MF} + \ln\mathcal{Z}^{0}_{\rm MF}, \tag{39}$$

the first term red-green quark condensate:

$$\ln\mathcal{Z}^{\Delta}_{\rm MF} = \frac{3}{8}\frac{\Lambda^2}{g^2}\beta V\sum_n(|\eta^+_n|^2 + 3\sum_{n',n\neq n'}\eta^+_n\eta^+_{n'}) \tag{40}$$

$$+\frac{1}{2}\sum_{e,n}\int\frac{d^3k}{(2\pi)^3}\Big\{\beta(E^+_e(\eta^e_n) - E^-_e(\eta^e_n))$$

$$+2\ln\left[f^{-1}\left(-E^+_e(\eta^e_n)\right)\right] + 2\ln\left[f^{-1}\left(-E^-_e(\eta^e_n)\right)\right]\Big\}, \tag{41}$$

Blue quarks:

$$\ln\mathcal{Z}^0_{\rm MF} = 2V\int\frac{d^3k}{(2\pi)^3}\sum_{e,f}\Big\{\ln\left[f^{-1}\left(-\xi^+_e(\boldsymbol{k},\mu_{b,f})\right)\right] + \ln\left[f^{-1}\left(-\xi^-_e(\boldsymbol{k},\mu_{b,f})\right)\right]\Big\}.$$

The thermodynamic potential is obtained from the log of the partition function as

$$\Omega_{\rm MF} = -\frac{1}{V\beta}\ln\, \mathcal{Z}_{\rm MF}. \tag{42}$$

The stationary point(s) of the thermodynamic potential determine the equilibrium values of the order parameters

$$\frac{\partial\Omega_{\rm MF}}{\partial\eta_1^e} = 0, \quad \frac{\partial\Omega_{\rm MF}}{\partial\eta_2^e} = 0, \quad \frac{\partial\Omega_{\rm MF}}{\partial|\boldsymbol{Q}|} = 0. \tag{43}$$

The direction of the vector $\boldsymbol{Q}$ is chosen by the superconductor spontaneously. Densities of quarks

$$\frac{\partial\Omega_{\rm MF}}{\partial\mu_u} = n_u, \quad \frac{\partial\Omega_{\rm MF}}{\partial\mu_d} = n_d. \tag{44}$$

The pressure is obtained then from the thermodynamics formula

$$p = \frac{1}{V}\Omega \tag{45}$$

Under stellar conditions impose color and electric charge neutrality and conservation of baryon number.
The total densities of up and down quarks are

$$n_d = n_d^{rg} + n_d^b, \quad (46)$$

$$n_u = n_u^{rg} + n_u^b. \quad (47)$$

The baryon density is

$$n_B = \frac{n_u + n_d}{3}. \quad (48)$$

The electrical and charge neutrality require, respectively,

$$\sum Q_e = 0, \quad \rightarrow \quad \frac{2}{3}n_u - \frac{1}{3}n_d - n_{\text{elec}} = 0, \quad (49)$$

$$\sum Q_c = 0, \quad \rightarrow \quad n_d^{rg} + n_u^{rg} - 2n_d^b - 2n_u^b = 0. \quad (50)$$

III. NJL model of QCD and hybrid stars

Color-superconductivity within the NJL model

$$
\begin{aligned}
\mathcal{L}_{NJL} \quad = \quad & \underbrace{\bar\psi(i\gamma^\mu\partial_\mu - \hat m)\psi}_{\text{free quarks}} + \underbrace{G_V(\bar\psi i\gamma^\mu\psi)^2}_{\text{vector}} + \underbrace{G_S\sum_{a=0}^{8}[(\bar\psi\lambda_a\psi)^2 + (\bar\psi i\gamma_5\lambda_a\psi)^2]}_{\text{scalar}-\text{pseudoscalar}} \\
+ \quad & \underbrace{G_D\sum_{\gamma,c}[\bar\psi^a_\alpha i\gamma_5\epsilon^{\alpha\beta\gamma}\epsilon_{abc}(\psi_C)^b_\beta][(\bar\psi_C)^r_\rho i\gamma_5\epsilon^{\rho\sigma\gamma}\epsilon_{rsc}\psi^s_\sigma]}_{\text{pairing}} \\
- \quad & \underbrace{K\left\{\det_f[\bar\psi(1+\gamma_5)\psi] + \det_f[\bar\psi(1-\gamma_5)\psi]\right\}}_{\text{t}'\text{Hooft interaction}},
\end{aligned}
$$

- quarks: ψ^a_α, color $a = r, g, b$, flavor ($\alpha = u, d, s$); mass matrix: $\hat m = \mathrm{diag}_f(m_u, m_d, m_s)$;
- other notations: λ_a, $a = 1, ..., 8$, $\psi_C = C\bar\psi^T$ and $\bar\psi_C = \psi^T C$, $C = i\gamma^2\gamma^0$.

Parameters of the model:

- G_S the scalar coupling and cut-off Λ are fixed from vacuum physics
- G_D is the di-quark coupling $\simeq 0.75 G_S$ (via Fierz) but free to change
- G_V and ρ_{tr} are treated as free parameters

QCD interactions pairing interactions and gaps

$$\Delta \propto \langle 0|\psi^a_{\alpha\sigma}\psi^b_{\beta\tau}|0\rangle$$

- Symmetric in space wave function (isotropic interaction)
- Antisymmetry in colors a, b for attraction
- Antisymmetry in spins σ, τ (Cooper pairs as spin-0 objects)
- Antisymmetry in flavors α, β

2SC phase:

Low densities, large m_s (strange quark decoupled)

$$\Delta(2SCs) \propto \Delta\epsilon^{ab3}\epsilon_{\alpha\beta} \qquad \delta\mu \ll \Delta,$$

Crystalline or gapless phases:

Intermediate densities, large m_s (strange quark decoupled)

$$\Delta(\text{cryst.}) \propto \epsilon_{\alpha\beta}\Delta_0\, e^{i\vec{Q}\cdot\vec{r}} \qquad \delta\mu \geq \Delta,$$

CFL phase:

High densities nearly massless u, d, s quarks

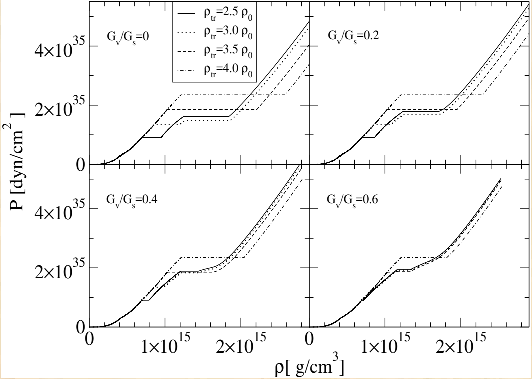
$$\Delta(CFL) \propto \langle 0|\psi^a_{\alpha L}\psi^b_{\beta L}|0\rangle = -\langle 0|\psi^a_{\alpha R}\psi^b_{\beta R}|0\rangle = \Delta\epsilon^{abC}\Delta\epsilon_{\alpha\beta C}.$$

EOS including (hyper)nuclear, 2SC and CFL phases of matter

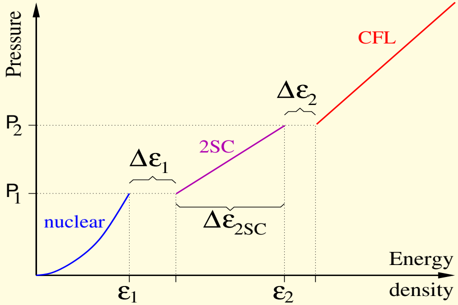
Choose Maxwell (large surface tension) or Glendenning (low surface tension) constructions. Matching condition for Maxwell is simply

$$P_N(\mu_B) = P_Q(\mu_B),$$

i.e., with low-density nuclear and high-density quark phases



Synthetic equations of state with constant speed of sound



- Instead of full NJL-model EoS with 2SC-CFL transition use synthetic EoS
- Realistic DD-ME2 EoS below the deconfinement (Colucci-Sedrakian EoS)
- Parametrize synthetic EoS via Constant Speed of Sound (CSS) parameterization (Alford-Han-Prakash 2013), also Haensel-Zdunik (2012).

Equilibria of compact objects

150 The Equilibrium and Stability of Fluid Configurations

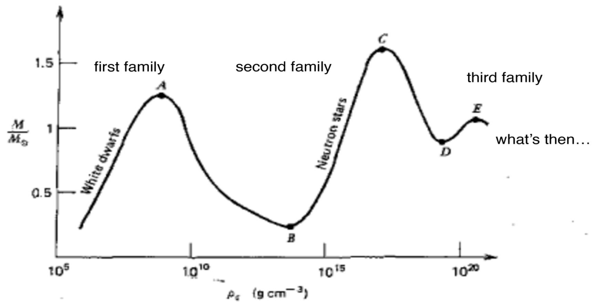


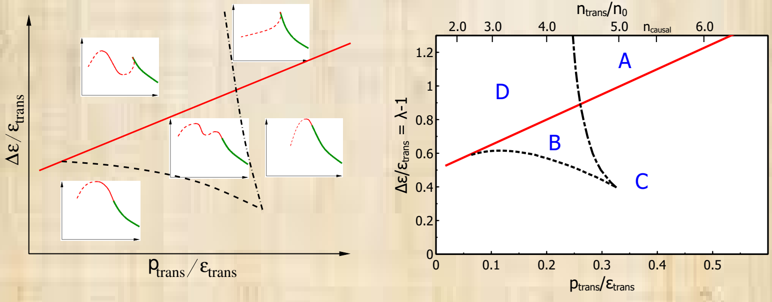
Figure 6.2 Schematic diagram showing the turning points in the mass versus central density diagram for equilibrium configurations of cold matter.

S. Shapiro, S. Teukolsky, "Black holes, White dwarfs and Neutron Stars"

-*White dwarfs* -first family, $M \leq 1.5 M_\odot$, [S. Chandrasekhar, L. Landau (1930-32)]
-*Neutron Stars* - second family, $M \leq 2 M_\odot$, [Oppenhimer-Volkoff (1939)]
-*Hybrid Stars* - third family, $M \leq 2 M_\odot$, [Gerlach (1968), Glendenning-Kettner (2000)]
- *Fourth Family?* M. Alford and A. Sedrakian, Phys. Rev. Lett. 119, 161104 (2017).

Phase diagram in M-R space

Phase diagram for hybrid star branches in the mass-radius relation of compact stars. The left panel shows schematically the possible topological forms of the mass-radius relation in each region of the diagram. [M. Alford, S. Han, M. Prakash, Phys. Rev. D 88, 083013 (2013).]



Criterium for having a twin confirguation

$$\frac{\Delta\varepsilon_{\rm crit}}{\varepsilon_{\rm trans}} = \frac{1}{2} + \frac{3}{2}\frac{p_{\rm trans}}{\varepsilon_{\rm trans}} .$$

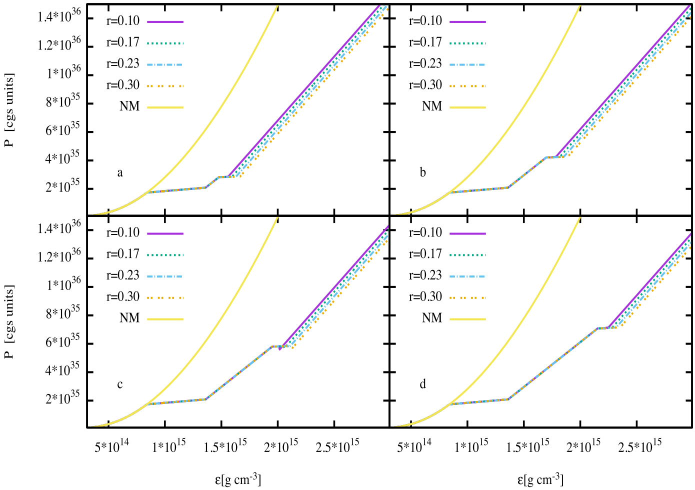
The EoS is analytically given

$$p(\varepsilon) = \begin{cases} p_1, & \varepsilon_1 < \varepsilon < \varepsilon_1 + \Delta\varepsilon_1 \\ p_1 + s_1\left[\varepsilon - (\varepsilon_1 + \Delta\varepsilon_1)\right], & \varepsilon_1 + \Delta\varepsilon_1 < \varepsilon < \varepsilon_2 \\ p_2, & \varepsilon_2 < \varepsilon < \varepsilon_2 + \Delta\varepsilon_2 \\ p_2 + s_2\left[\varepsilon - (\varepsilon_2 + \Delta\varepsilon_2)\right], & \varepsilon > \varepsilon_2 + \Delta\varepsilon_2. \end{cases}$$

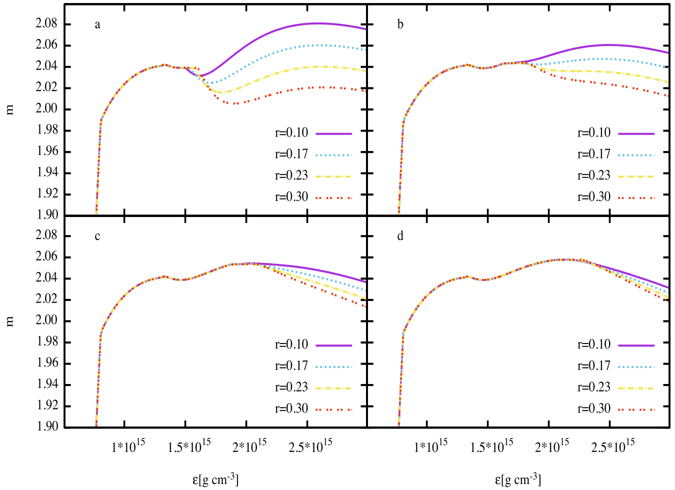
Need to specify:

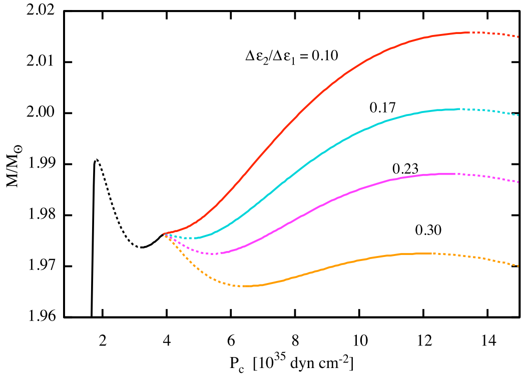
- the two speeds of sounds: s_1 and s_2
- the point of transition from NM to QM ε_1, P_1
- the magnitude of the first jump $\Delta\varepsilon_1$
- the size of the 2SC phase, i.e, the second transition point ε_2, P_2
- the size of the second jump $\Delta\varepsilon_2$

Varying parameters of EoS with sequential phase transition



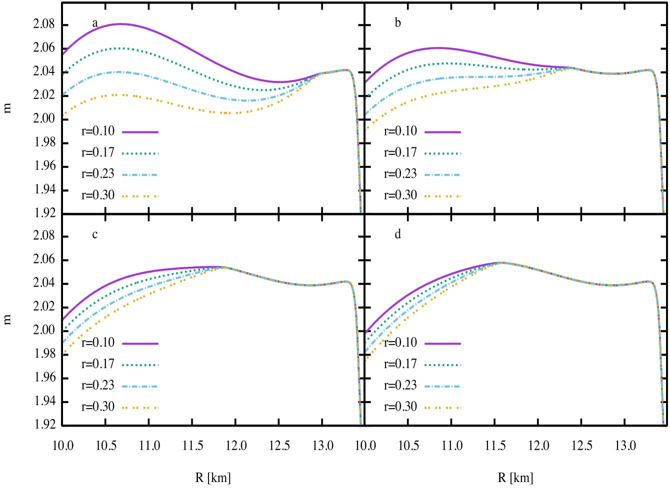
.... and resulting topologies of sequences

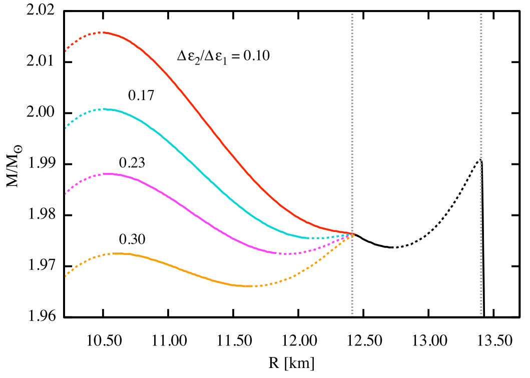




The stellar mass as a function of the star's central pressure for four different values of $\Delta\varepsilon_2$. The other parameters of the EOS are fixed at $P_1 = 1.7 \times 10^{35}$ dyn cm^{-2}, $s_1 = 0.7$, $\Delta\varepsilon_{2SC}/\varepsilon_1 = 0.27$, $\Delta\varepsilon_1/\varepsilon_1 = 0.6$, and $s_2 = 1$. The vertical dotted lines mark the two phase transitions at P_1 and P_2. Stable branches are solid lines, unstable branches are dashed lines. We see the emergence of separate 2SC and CFL hybrid branches along with the occurrence of triplets.

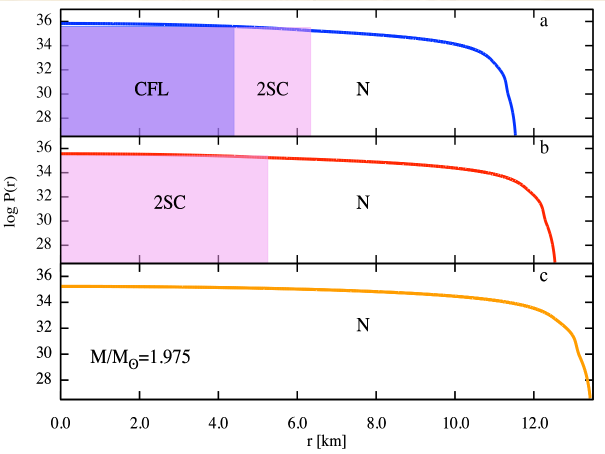
.... and resulting topologies of mass-radius relations





The M-R relations for the parameter values defined above . We have fixed the properties of the nuclear $\to$ 2SC transition and the speed of sound in 2SC and CFL matter. For the 2SC $\to$ CFL transition we have fixed the critical pressure and we vary the energy-density discontinuity $\Delta\varepsilon_2$. The separate 2SC and CFL hybrid branches are clearly visible, along with the occurrence of triplets.

Profiles of triplets stars (same mass)



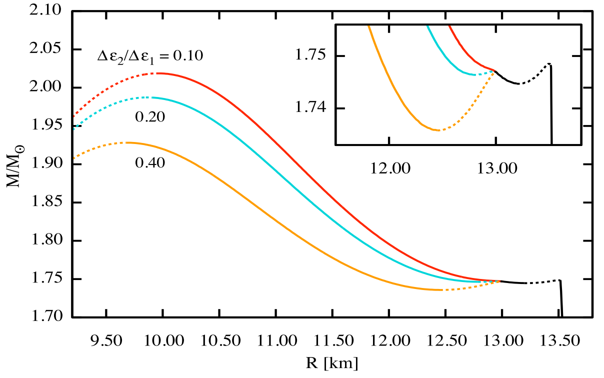
The profiles (here the log of pressure as a function of the internal radius) of the three members of a triplet with masses $M = 1.975\ M_{\odot}$. Here "N" means the nuclear phase. The parameter values are as above, with $\Delta\varepsilon_2/\Delta\varepsilon_1 = 0.23$.

Stability range

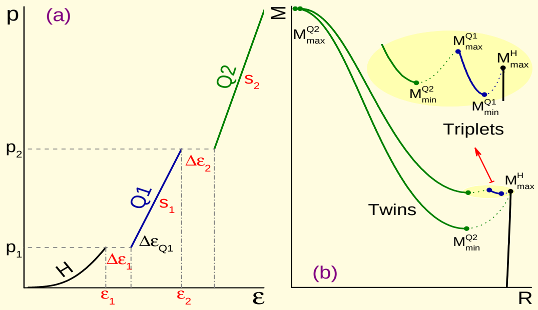
$\Delta\varepsilon_2/\Delta\varepsilon_1$	$\Delta\varepsilon_1/\varepsilon_1$ 0.4	0.5	0.6	0.7
0.1	s, s	s, s	$\underbrace{us, s}_{\text{N-2SC}}$	$\underbrace{u, us}_{\text{N-CFL}}$
0.2	s, s	s, s	$\underbrace{us, us}_{\text{triplet}}$	$\underbrace{u, us}_{\text{N-CFL}}$
0.3	s, s	s, s	$\underbrace{us, us}_{\text{N-2SC;N-CFL}}$	$\underbrace{u, us}_{\text{N-CFL}}$
0.4	s, s	$\underbrace{s, us}_{\text{2SC-CFL}}$	$\underbrace{us, u}_{\text{N-2SC}}$	u, u
0.5	s, s	$\underbrace{s, us}_{\text{2SC-CFL}}$	$\underbrace{us, u}_{\text{N-2SC}}$	u, u

In each entry stable/unstable branches are referred by s/u, the 2SC and CFL phases are separated by comma, and the pressure increases from left to right. The presence of twin hybrid configurations or triplet configurations is marked by the underbraces with information about the involved phases (“N” means nuclear).

Lower mass triplets



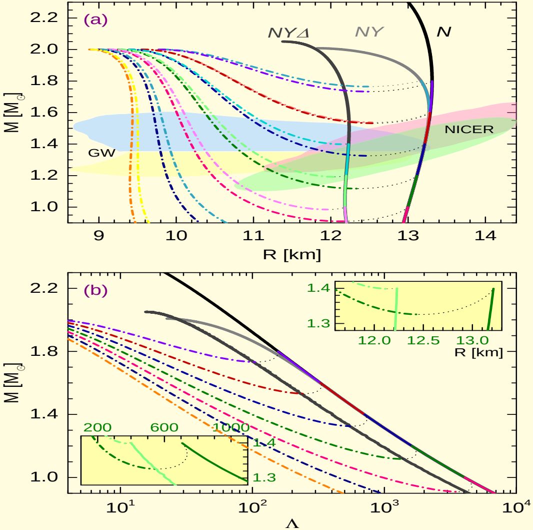
- Low-mass triplets via early transition NM$\rightarrow$ QM
- Still 2-solar mass members possible but only with the NM-2SC-CFL composition



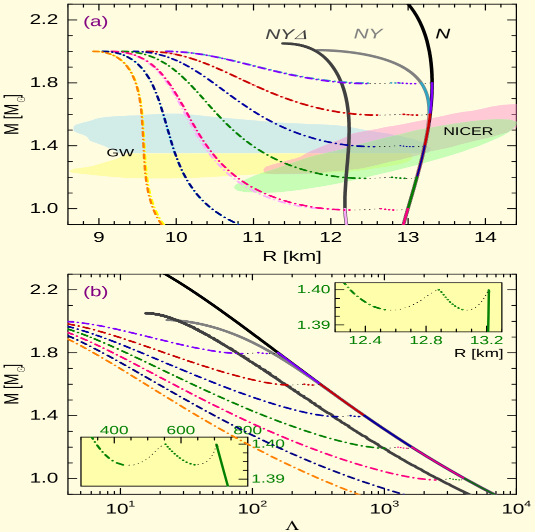
Left: EoS with two sequential phase transitions. *Right:* Mass-radius relationships, emergences of minima in the function $M(R)$.

Case when $NY\Delta$-matter makes a first order phase *sequential* transitions to various *generic new phases* (we had in mind phases of color superconducting phases).

$$p(\varepsilon)=\begin{cases} p_1, & \varepsilon_1<\varepsilon<\varepsilon_1+\Delta\varepsilon_1\\ p_1+s_1\left[\varepsilon-(\varepsilon_1+\Delta\varepsilon_1)\right], & \varepsilon_1+\Delta\varepsilon_1<\varepsilon<\varepsilon_2\\ p_2, & \varepsilon_2<\varepsilon<\varepsilon_2+\Delta\varepsilon_2\\ p_2+s_2\left[\varepsilon-(\varepsilon_2+\Delta\varepsilon_2)\right], & \varepsilon>\varepsilon_2+\Delta\varepsilon_2. \end{cases}$$



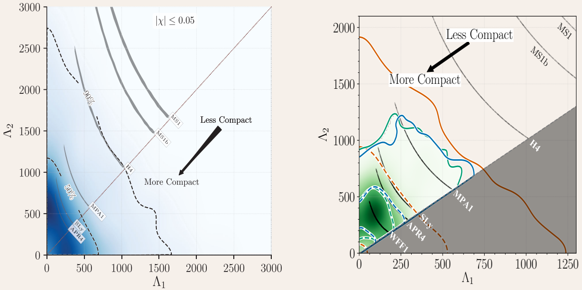
MR relation (a) and deformabilities (b) for hybrid stars with a single phase transition(s).

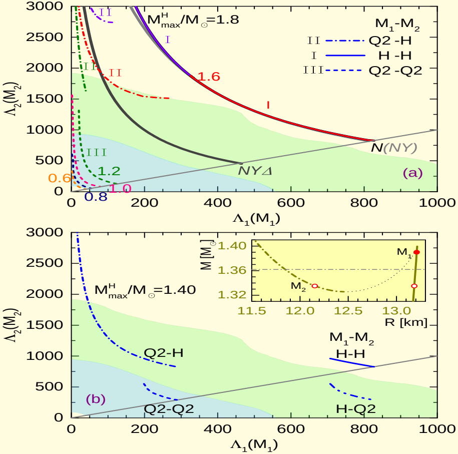


MR relation (a) and deformabilities (b) for hybrid stars with a double phase transition(s).

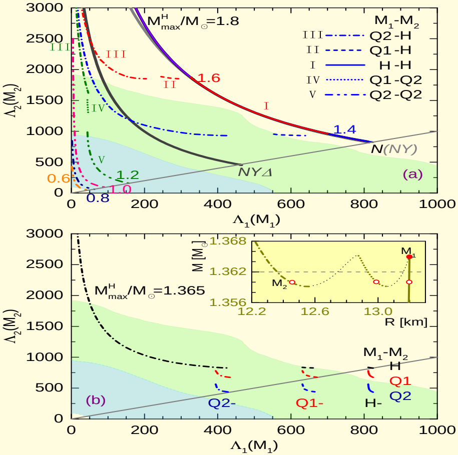
TABLE I. Source properties for GW170817: we give ranges encompassing the 90% credible intervals for different assumptions of the waveform model to bound systematic uncertainty. The mass values are quoted in the frame of the source, accounting for uncertainty in the source redshift.

	Low-spin priors ($\vert\chi\vert \leq 0.05$)	High-spin priors ($\vert\chi\vert \leq 0.89$)
Primary mass m_1	$1.36–1.60\ M_\odot$	$1.36–2.26\ M_\odot$
Secondary mass m_2	$1.17–1.36\ M_\odot$	$0.86–1.36\ M_\odot$
Chirp mass $\mathcal{M}$	$1.188^{+0.004}_{-0.002} M_\odot$	$1.188^{+0.004}_{-0.002} M_\odot$
Mass ratio m_2/m_1	0.7–1.0	0.4–1.0
Total mass $m_{\rm tot}$	$2.74^{+0.04}_{-0.01} M_\odot$	$2.82^{+0.47}_{-0.09} M_\odot$
Radiated energy $E_{\rm rad}$	$> 0.025 M_\odot c^2$	$> 0.025 M_\odot c^2$
Luminosity distance $D_{\rm L}$	40^{+8}_{-14} Mpc	40^{+8}_{-14} Mpc
Viewing angle Θ	$\leq 55°$	$\leq 56°$
Using NGC 4993 location	$\leq 28°$	$\leq 28°$
Combined dimensionless tidal deformability $\tilde{\Lambda}$	≤ 800	≤ 700
Dimensionless tidal deformability $\Lambda(1.4M_\odot)$	≤ 800	≤ 1400

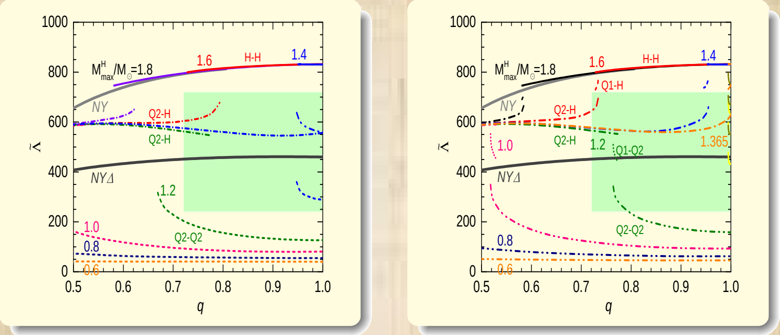




a) Tidal deformabilities of compact objects in the binary with chirp mass $\mathcal{M} = 1.186 M_\odot$ (b) Prediction by an EoS with maximal hadronic mass $M^H_{max} = 1.365 M_\odot$. The inset shows the mass-radius relation around the phase transition region. The circles M_2 are two possible companions for circle M_1, generating two points in the Λ_1-Λ_2 curves while one point is located below the diagonal line.

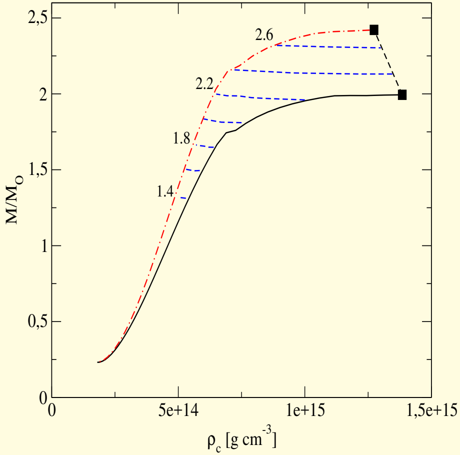


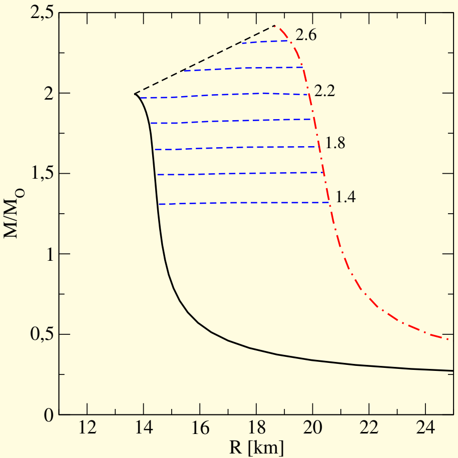
The case of double phase transition a) Tidal deformabilities of compact objects in the binary with chirp mass $\mathcal{M} = 1.186 M_\odot$ (b) Prediction by an EoS with maximal hadronic mass $M^{\rm H}_{\rm max} = 1.365 M_\odot$. The inset shows the mass-radius relation around the phase transition region. The circles M_2 are two possible companions for circle M_1, generating two points in the Λ_1-Λ_2 curves while one point is located below the diagonal line.

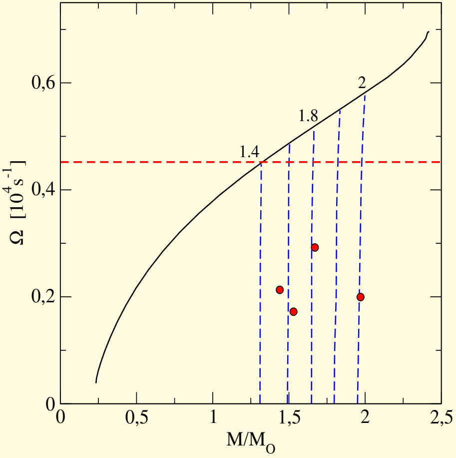


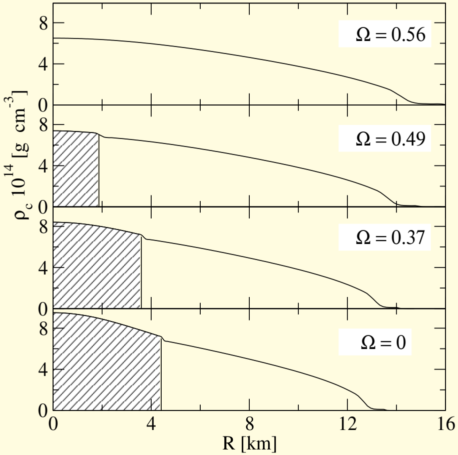
Mass weighted deformability vs. mass asymmetry for a binary system with fixed chirp mass $\mathcal{M} = 1.186 M_\odot$ predicted by a range of hybrid EoS with single phase transition and various values of $M^{\rm H}_{\rm max}$. The error shading indicates the constraints estimated from the GW170817 event and the electromagnetic transient AT2017gfo.

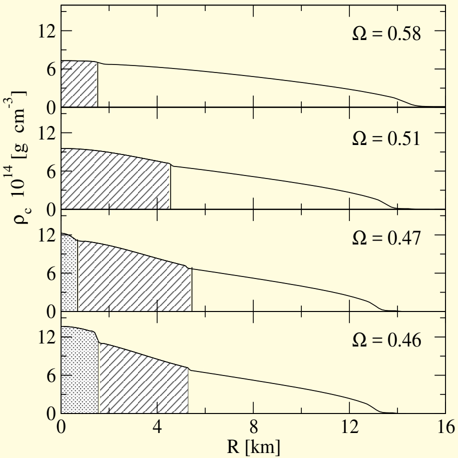
IV. Rapidly rotating hybrid stars


2,5
2
1,5
1
0,5
0
M/M_O
0
5e+14
1e+15
1,5e+15
ρ_c [g cm^{-3}]
2.6
2.2
1.8
1.4


2,5
2
1,5
1
0,5
0
12
14
16
18
20
22
24
R [km]
M/M_O
2.6
2.2
1.8
1.4


0,6
0,4
0,2
0
Ω [10^4 s^-1]
0
0,5
1
1,5
2
2,5
M/M_O
1.4
1.8
2


Ω = 0.56
Ω = 0.49
Ω = 0.37
Ω = 0
$\rho_c\ 10^{14}$ [g cm^{-3}]
R [km]
0
4
8
12
16


Ω = 0.58
Ω = 0.51
Ω = 0.47
Ω = 0.46
ρc 10^14 [g cm^-3]
12
6
0
0
4
8
12
16
R [km]

Summary of topics covered in Lecture 2

- QCD phases at large densities and low temperatures
- QCD partition function and thermodynamics
- Constructing EoS with QCD phases
- Mass and radius relation, twins and triplets
- Tidal deformabilities of QCD matter
- Rapidly rotating stars with quark cores