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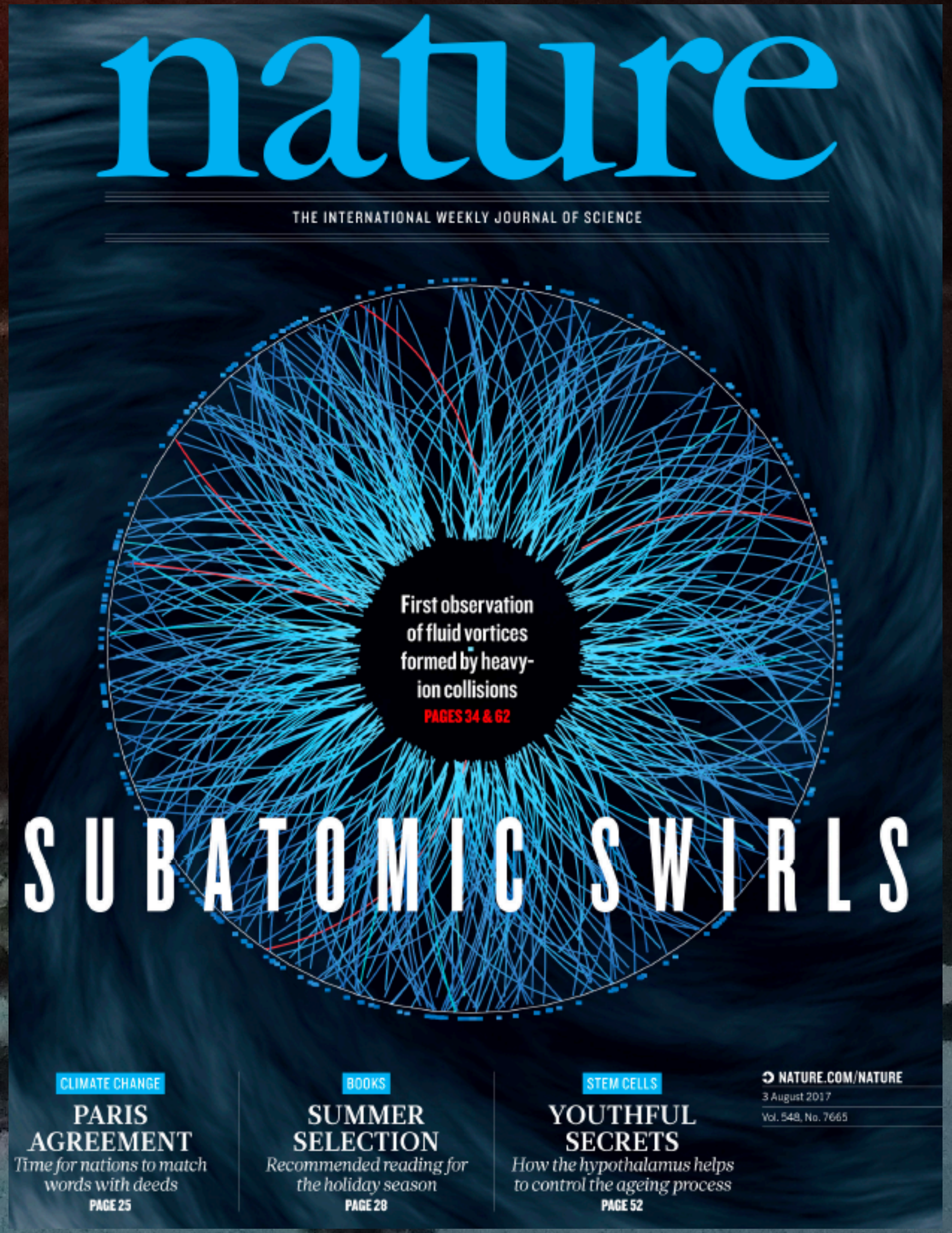
Relativistic perfect-fluid spin hydrodynamics based on GLW pseudogauge

Rajeev Singh

THE HENRYK NIEWODNICZAŃSKI
INSTITUTE OF NUCLEAR PHYSICS
POLISH ACADEMY OF SCIENCES



58th Karpacz Winter School

19 – 25 June 2022

Spin polarization in heavy-ion collisions: a new sensitive probe!

Non-central heavy-ion collisions create fireballs with large global orbital angular momenta

F. Becattini, F. Piccinini, J. Rizzo, PRC 77 (2008) 024906

$$L_{\text{initial}} \approx 10^5\,\hbar$$

Part of the angular momentum can be transferred from the orbital to the spin part

$$J_{\text{initial}} = L_{\text{initial}} = L_{\text{final}} + S_{\text{final}}$$

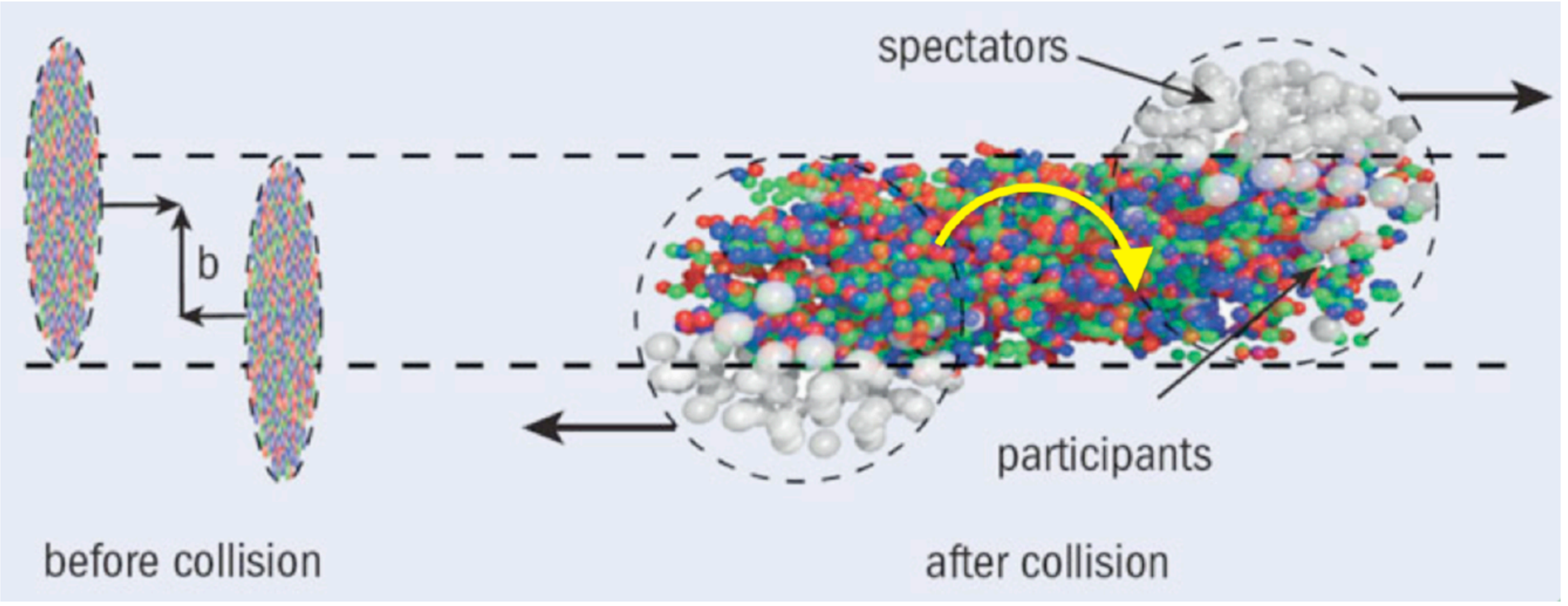
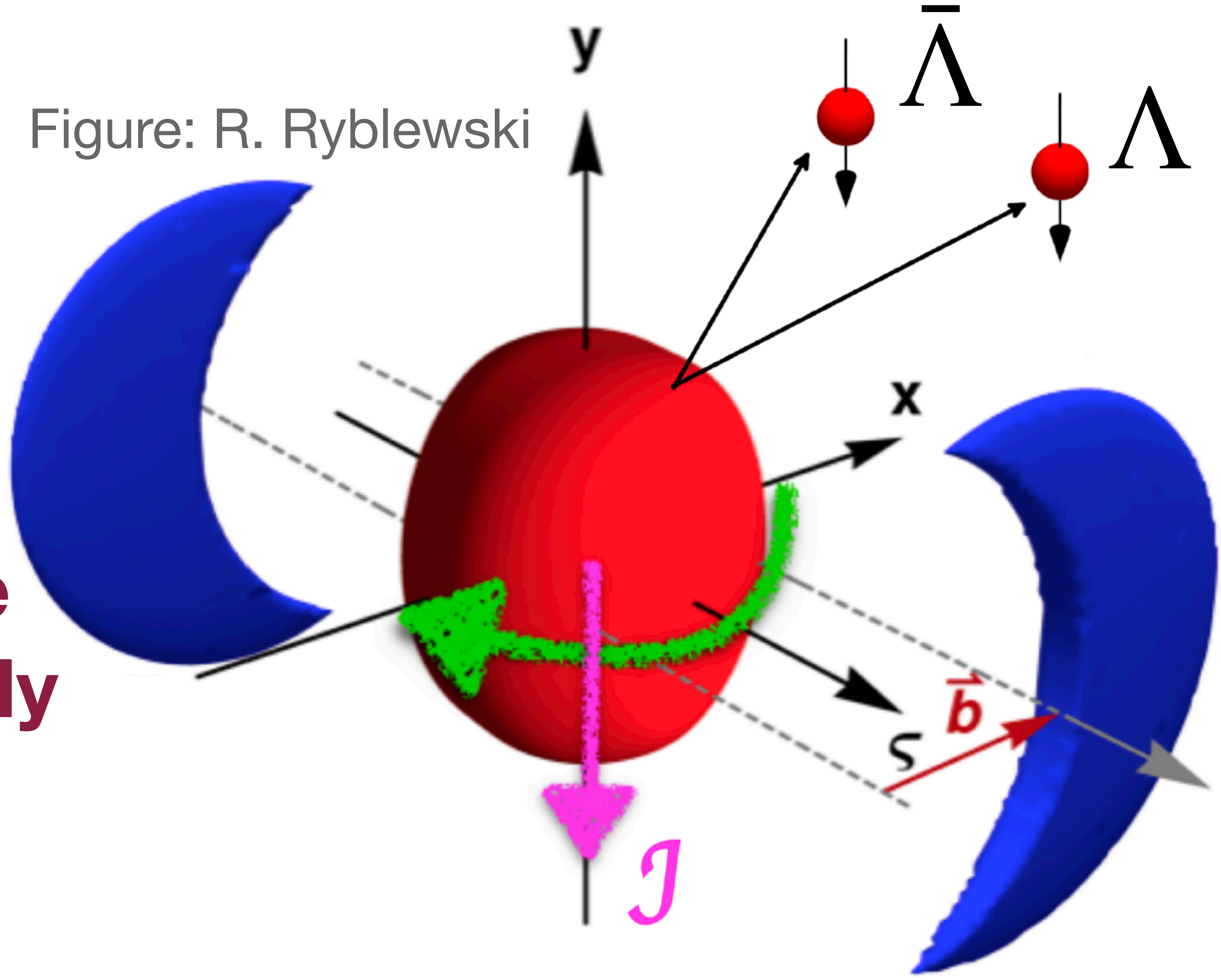
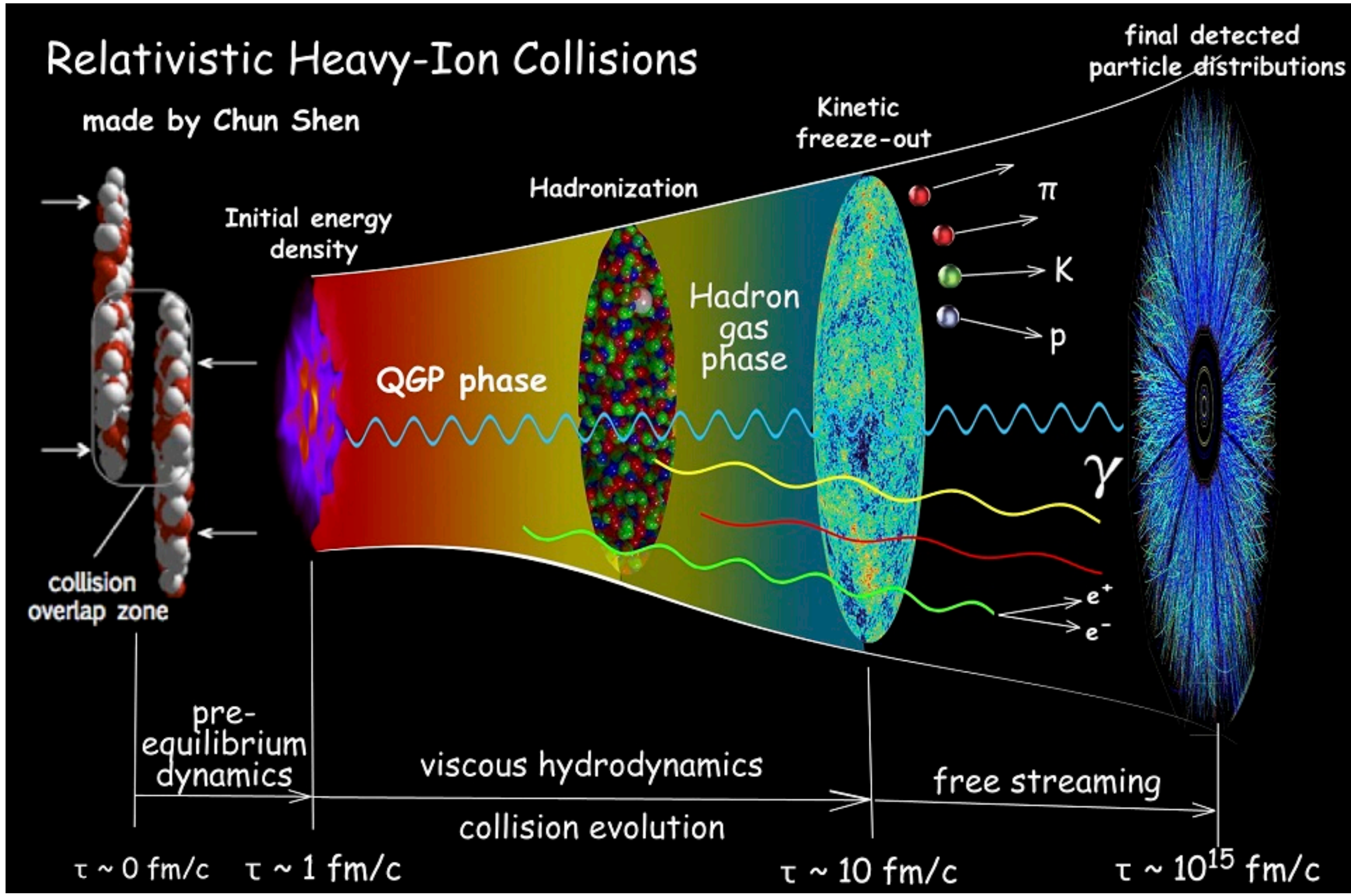


Figure: M. Lisa, talk @ “Strangeness in Quark Matter 2016”

Figure: R. Ryblewski

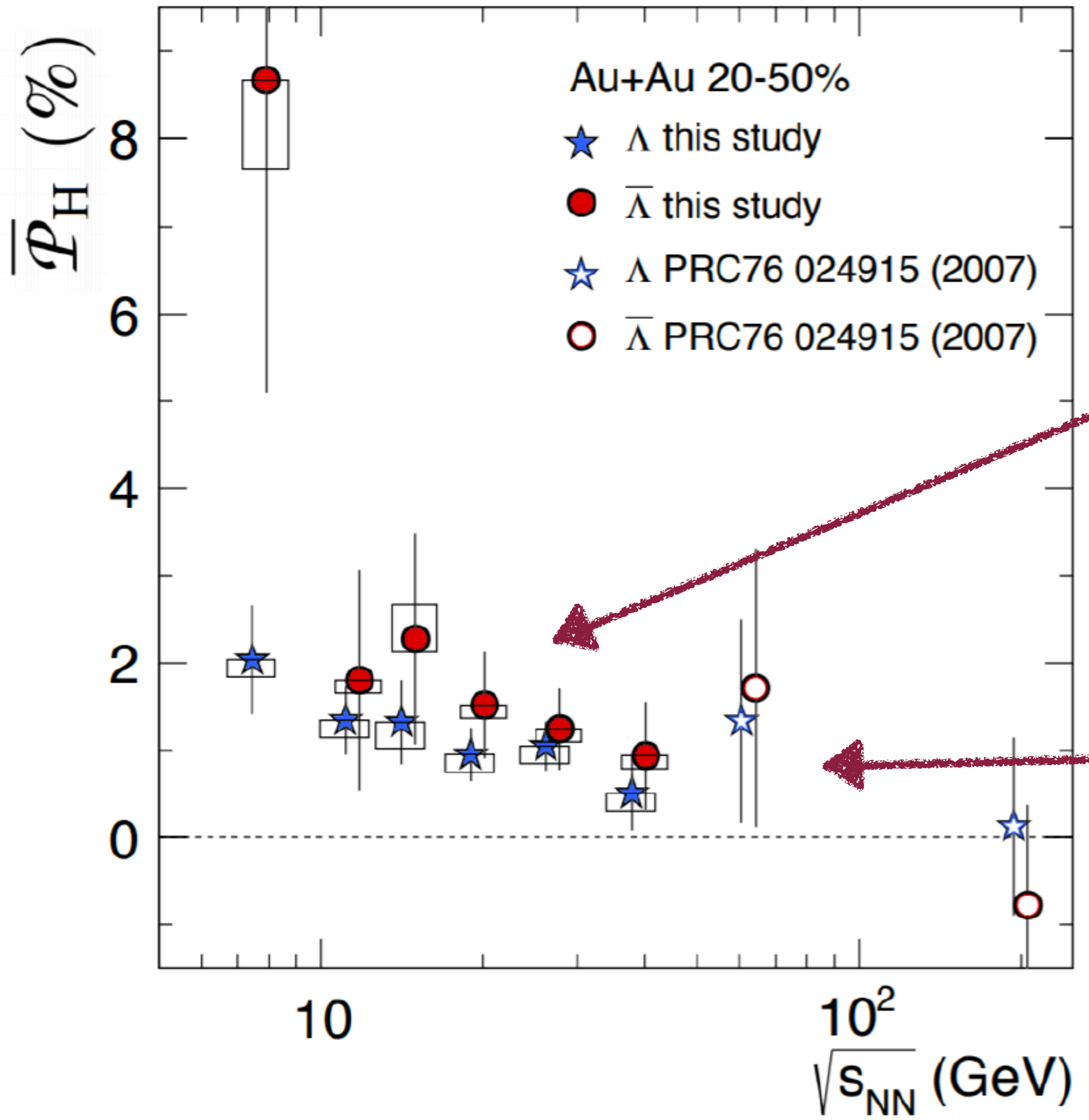


Emitted particles are expected to be globally polarized along the system’s angular momentum



Experimental measurement of $\Lambda(\bar{\Lambda})$ spin polarization in heavy-ion collisions

L. Adamczyk et al. (STAR) (2017), Nature 548 (2017) 62-65



~2% - small but measurable effect

Small difference in the magnitude of Λ & $\bar{\Lambda}$ possibly due to initial magnetic field

Self-analysing parity-violating hyperon weak decay allows to measure polarization of Λ

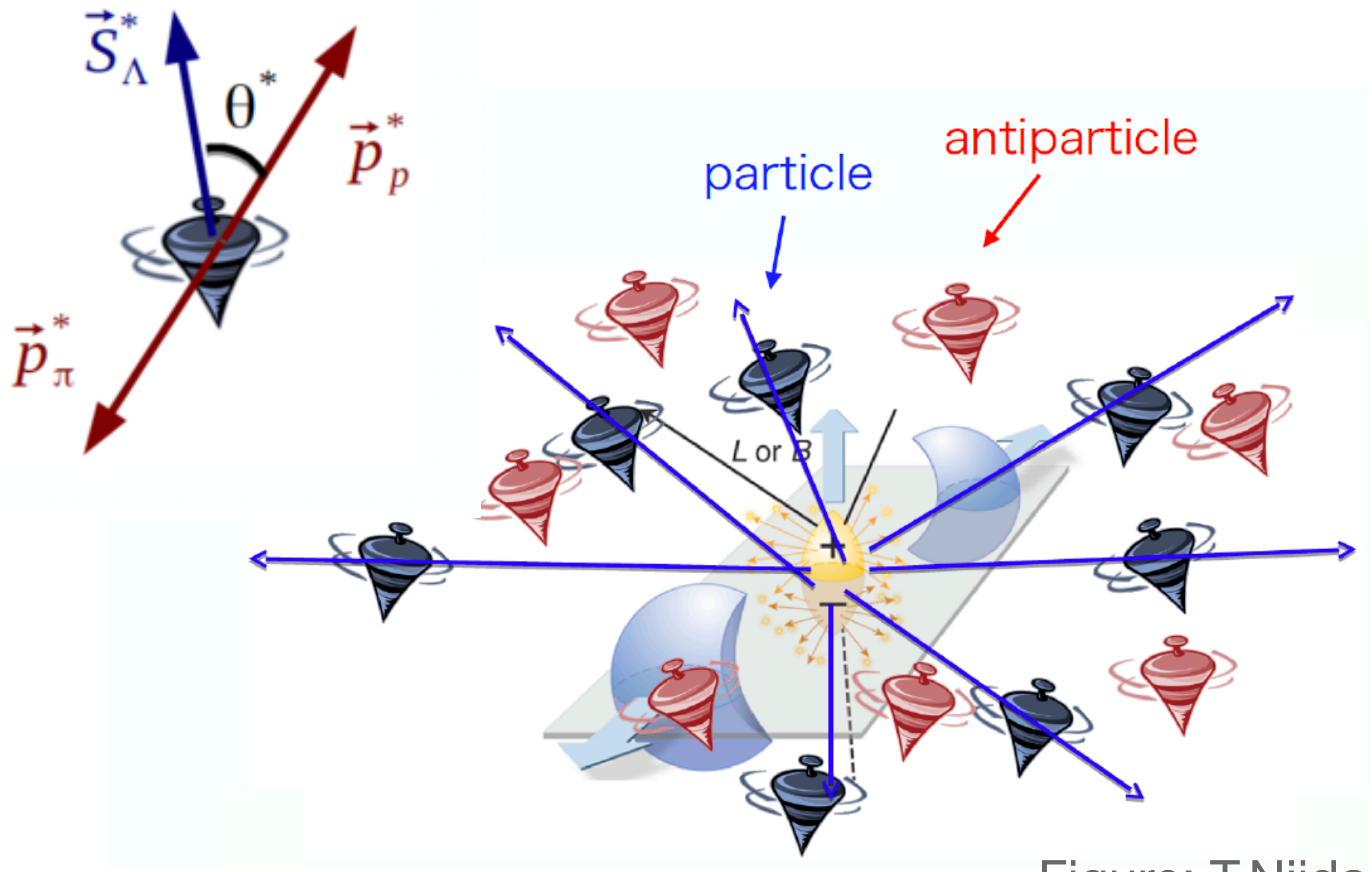


Figure: T.Niida

$$\frac{dN}{d\Omega^*} = \frac{1}{4\pi}\left(1 + \alpha_H \mathbf{P}_H \cdot \mathbf{p}^*_\mathbf{p}\right)$$

QGP **is the** ***hottest***, ***least viscous,*** **and** ***most vortical*** **fluid ever produced**

$$\omega = (P_\Lambda + P_{\bar{\Lambda}})k_B T/\hbar \sim 0.6 - 2.7 \times 10^{22}\ \mathrm{s}^{-1}$$

$$P_\Lambda \approx \frac{1}{2}\frac{\omega}{T} + \frac{\mu_\Lambda B}{T} \qquad P_{\bar{\Lambda}} \approx \frac{1}{2}\frac{\omega}{T} - \frac{\mu_\Lambda B}{T}$$

$P_\Lambda \approx P_{\bar{\Lambda}}$ ➡ **first direct observation of spin**

Spin polarization in equilibrated QGP - spin-thermal approach

In local thermodynamic equilibrium at $\mathcal{O}((\varpi^{\mu\nu})^2)$ one can establish a link between spin and thermal vorticity

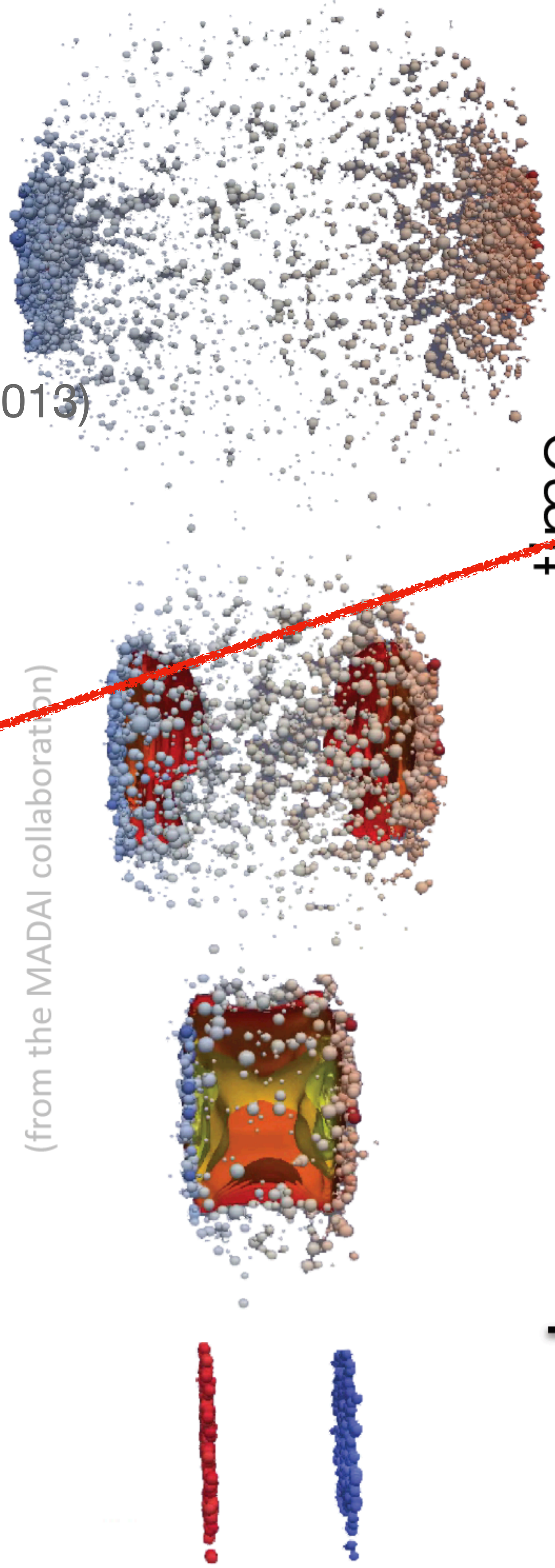
Becattini F, Piccinini F. Ann. Phys. 323:2452 (2008)
Becattini F, Chandra V, Del Zanna L, Grossi E. Ann. Phys. 338:32 (2013)
Fang R, Pang L,Wang Q,Wang X. Phys. Rev. C 94:024904 (2016)

$$S^\mu(p) = -\frac{1}{8m}\epsilon^{\mu\rho\sigma\tau}p_\tau \frac{\int d\Sigma_\lambda p^\lambda n_F (1-n_F)\varpi_{\rho\sigma}}{\int d\Sigma_\lambda p^\lambda n_F}$$

$$\varpi_{\mu\nu} = -\frac{1}{2}\left(\partial_\mu\beta_\nu - \partial_\nu\beta_\mu\right) \qquad \beta^\mu = \frac{u^\mu}{T}$$

$$n_F = (1+\exp[\beta\cdot p - \mu Q/T])^{-1}$$

Allows to extract polarisation at the freeze-out hypersurface in any model which provides u^μ, T and μ



relativistic heavy-ion collision

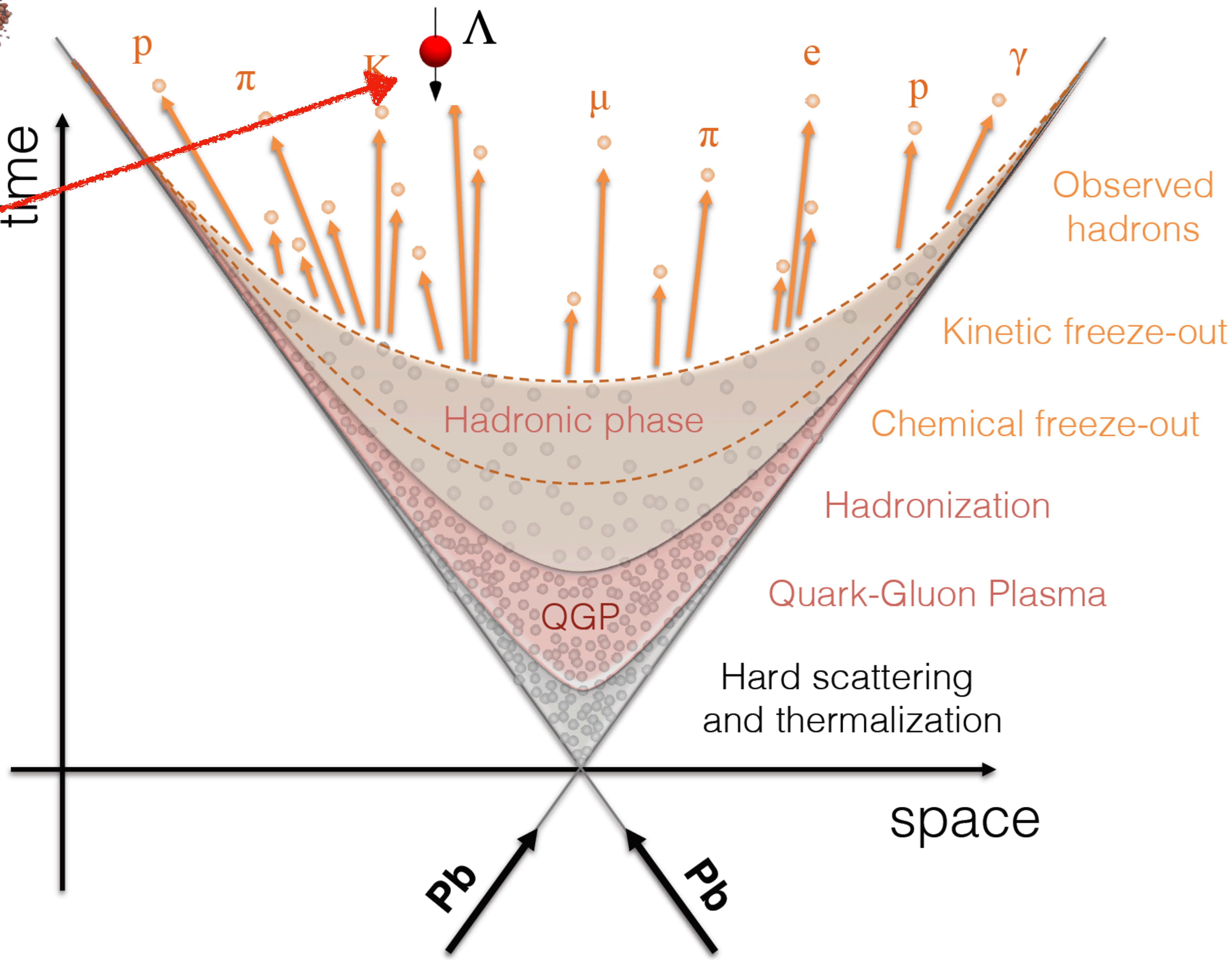


figure: D.D. Chinellato

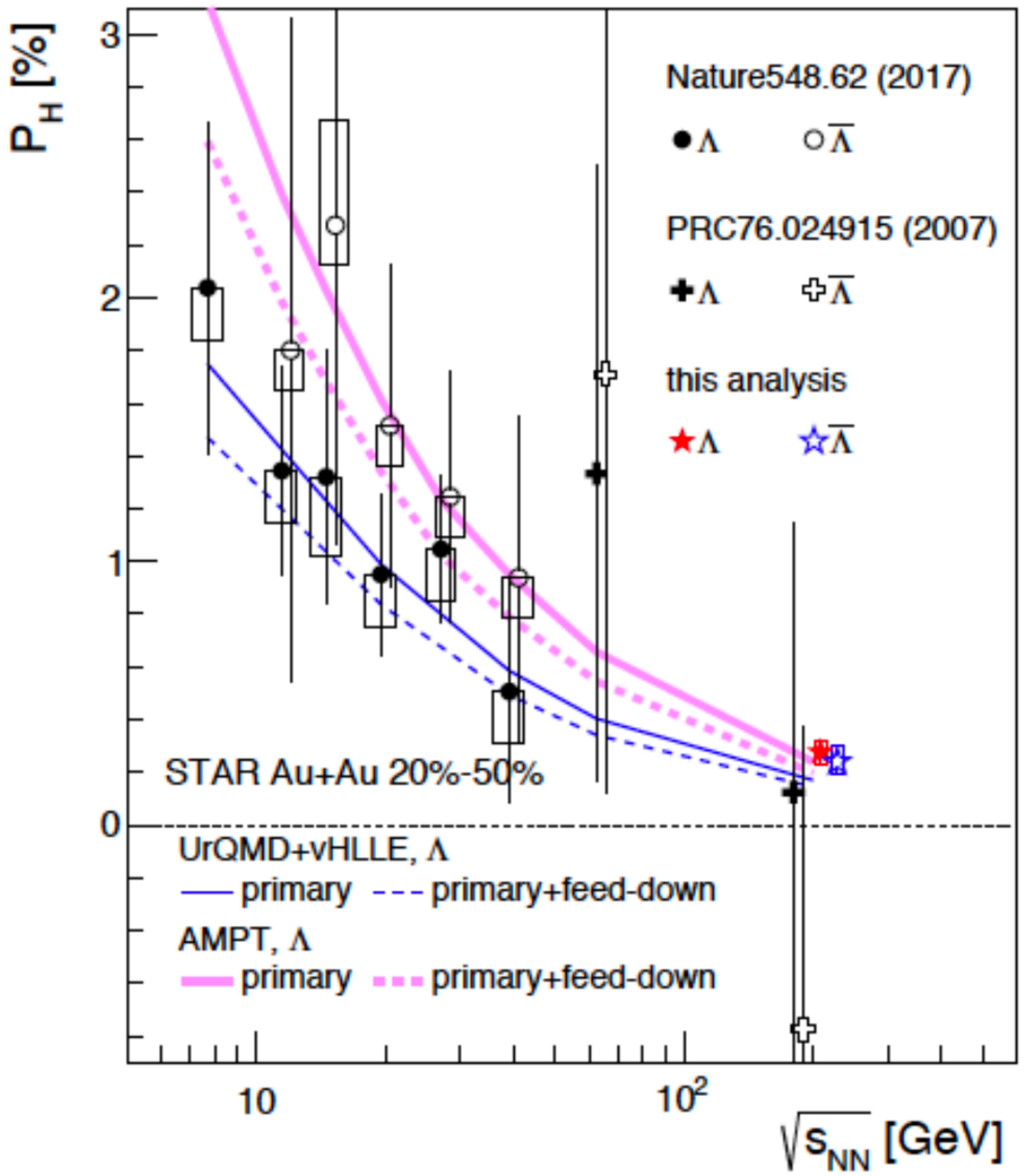
Global polarization

Global polarization data supports the spin-thermal approach

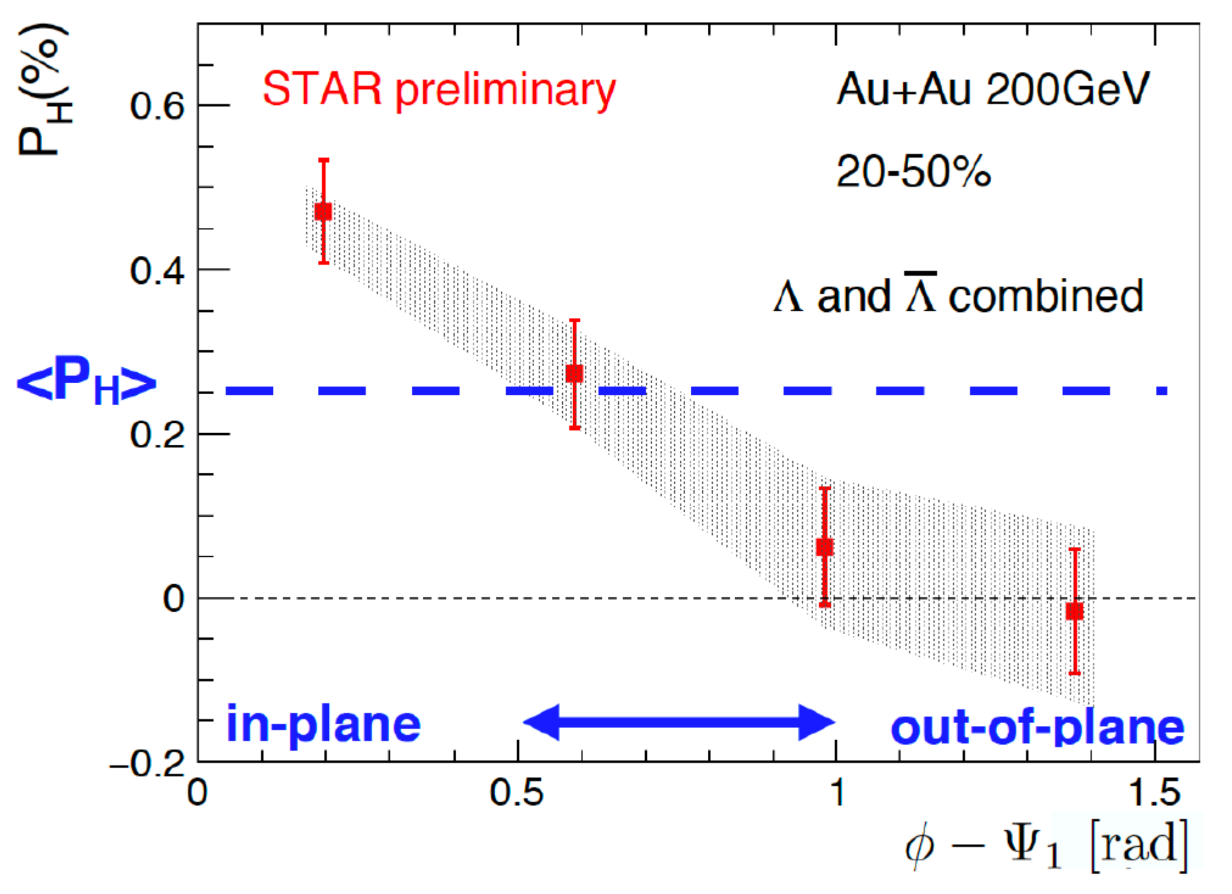
Signal is pretty robust and agrees for both multiphase transport model (AMPT) and viscous hydrodynamics (UrQMD+vHLLE)

Azimuthal modulation is not captured

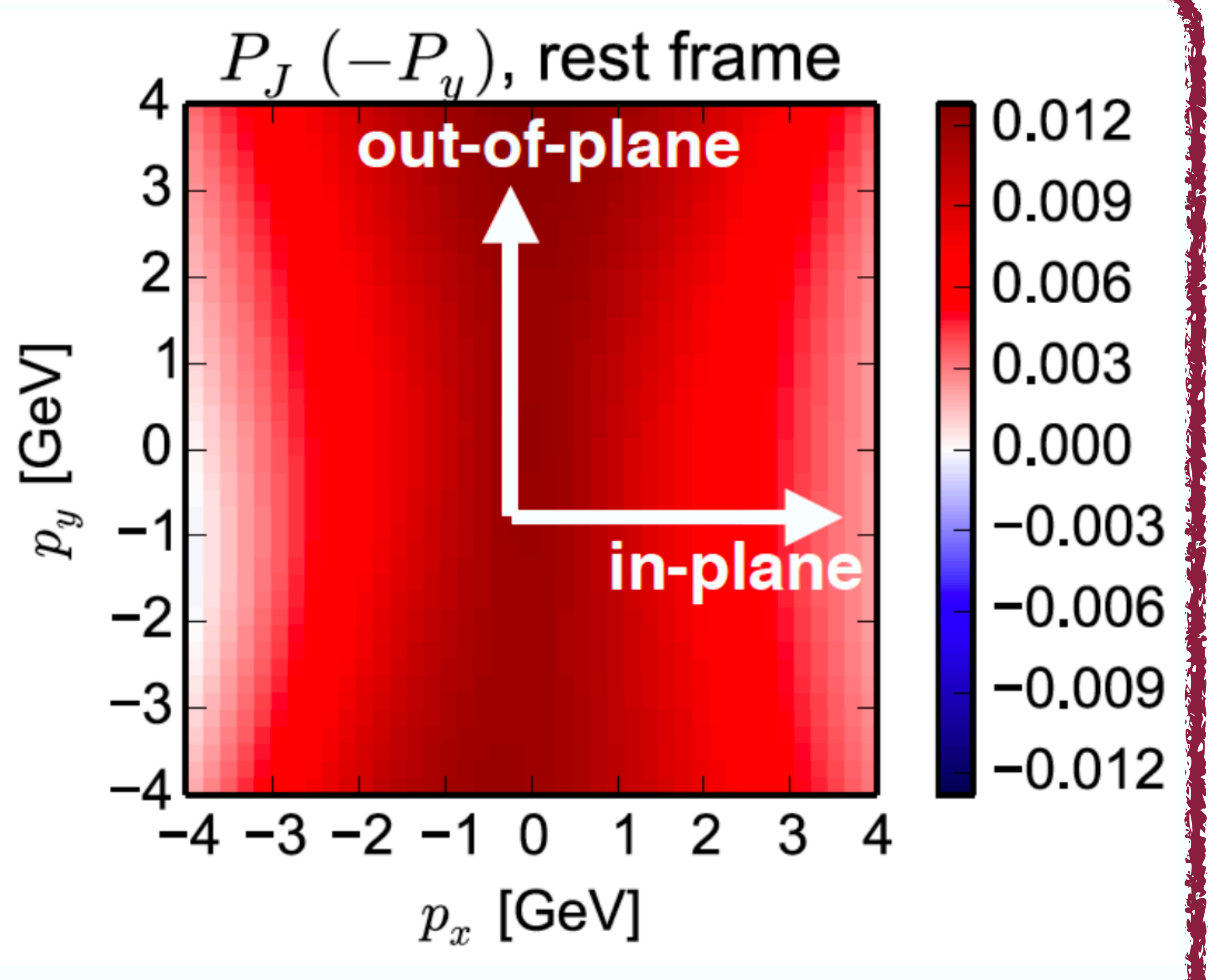
J. Adam, et al., Phys. Rev. C 98, 014910 (2018)



UrQMD+vHLLE: I. Karpenko, F. Becattini, EPJC 77, 213 (2017)
AMPT: H. Li, L. Pang, Q. Wang, and X. Xia, PRC 96, 054908 (2017)

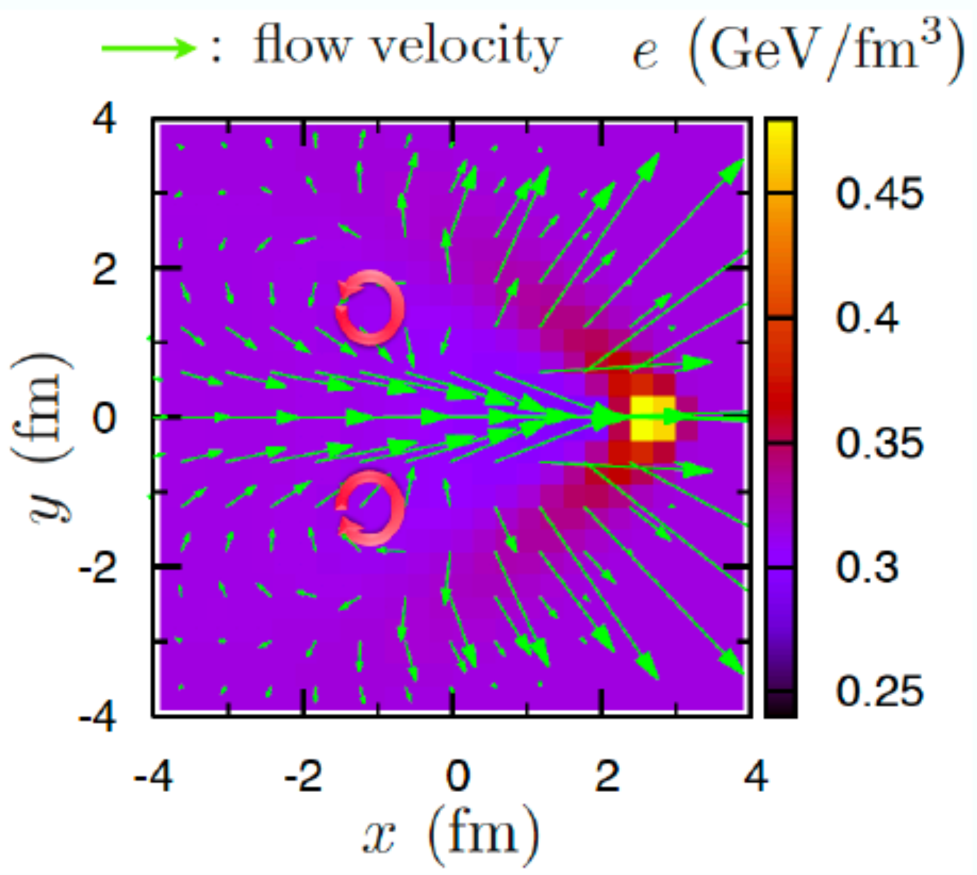


Credit: T.Niida, The 5th Workshop on Chirality, Vorticity and Magnetic Field in Heavy Ion Collisions, 2019

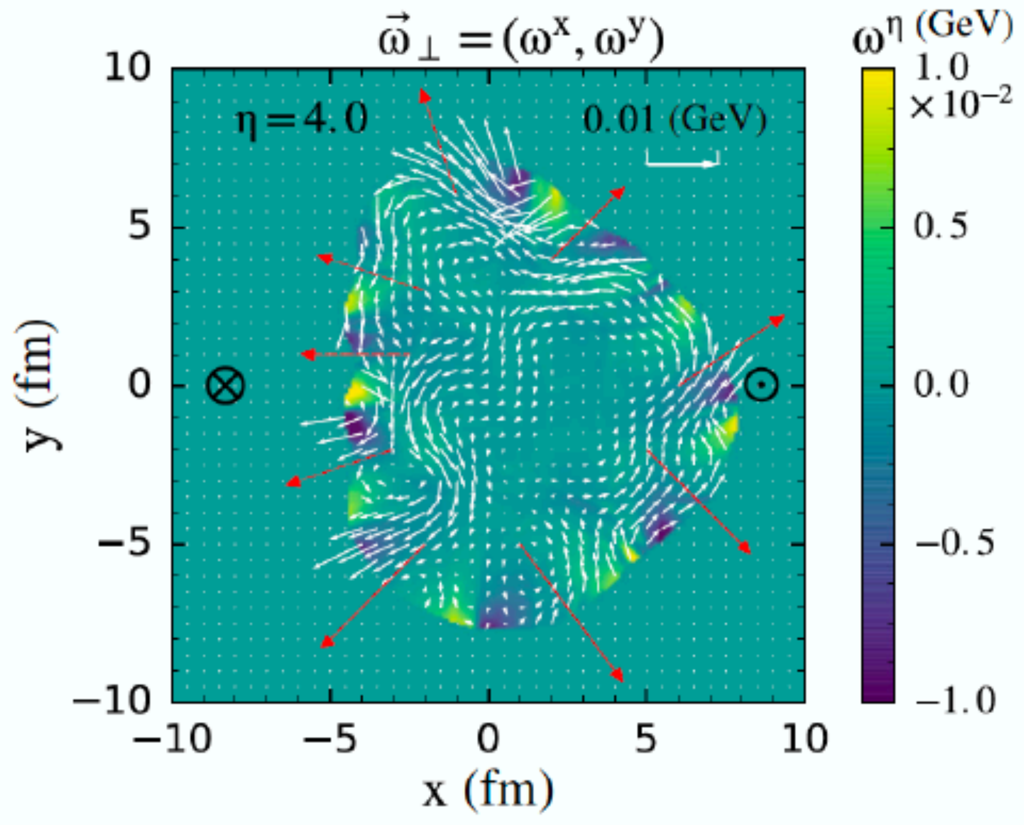


I. Karpenko, F. Becattini, EPJC 77, 213 (2017)

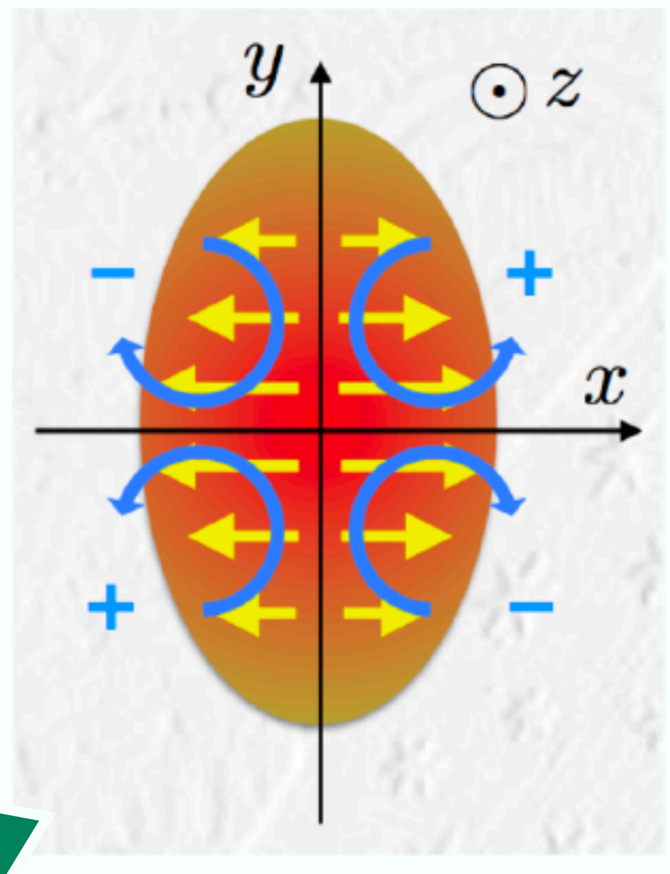
Local (momentum-differential) polarization



Y. Tachibana and T. Hirano, NPA904-905 (2013) 1023

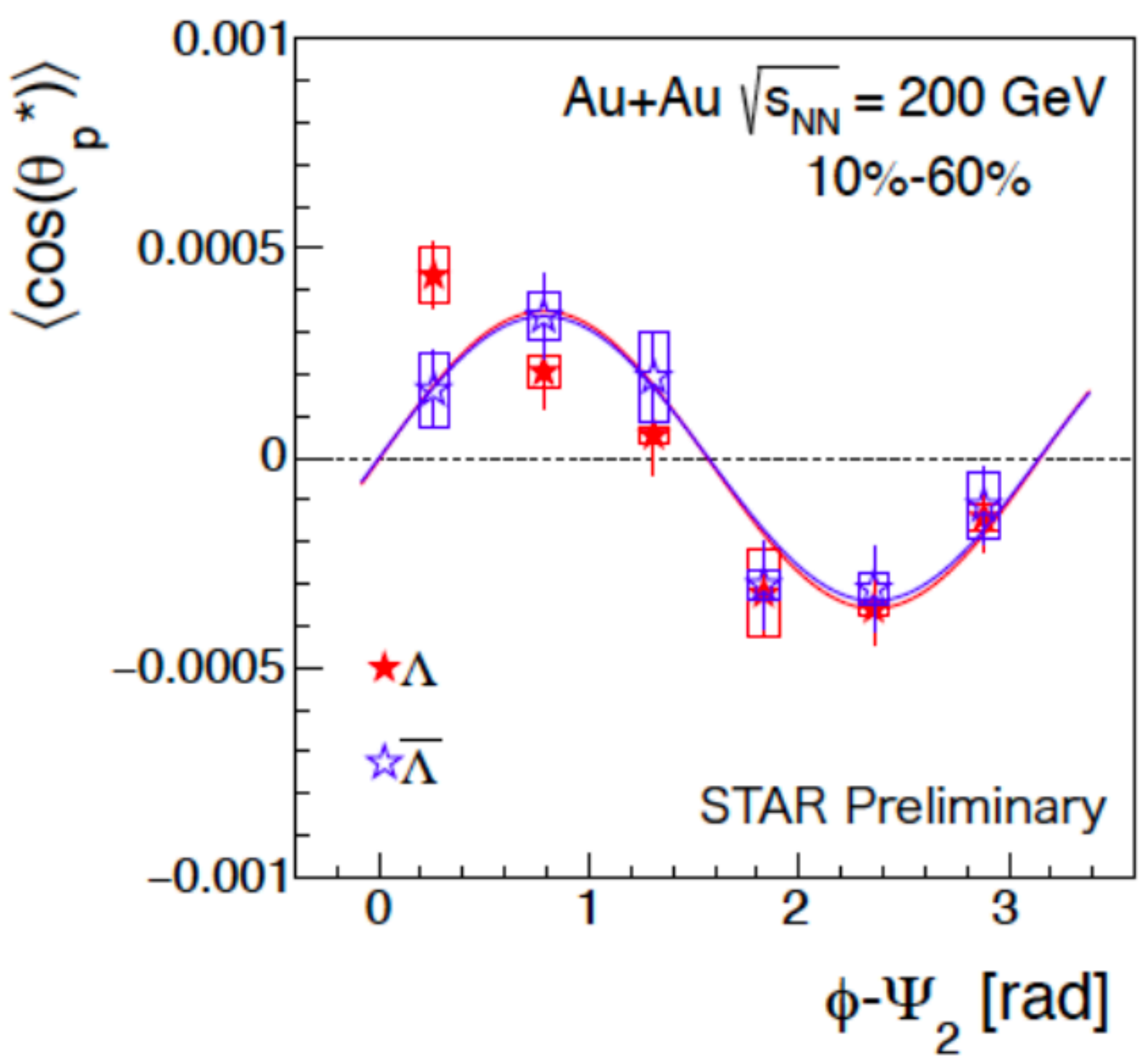


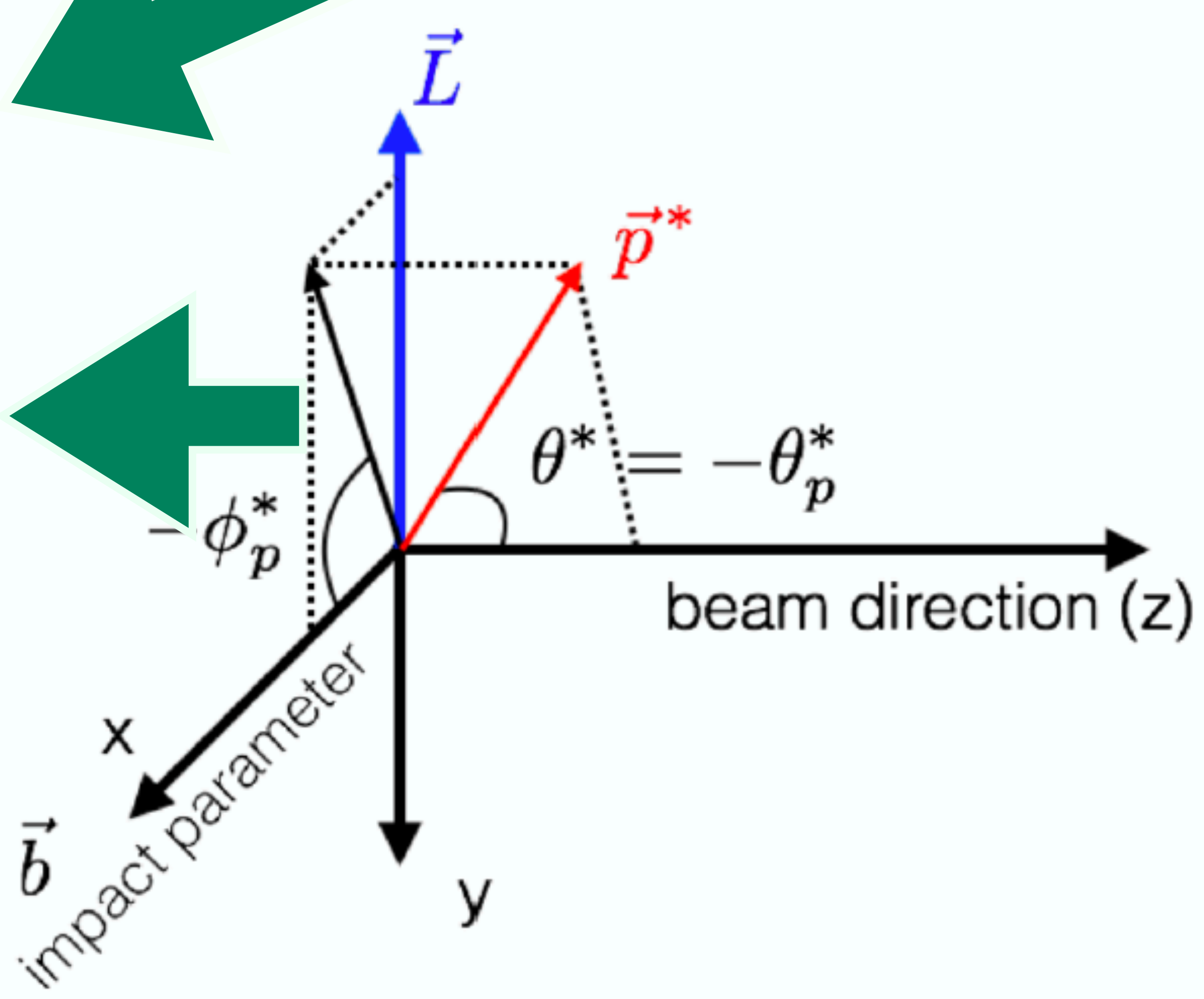
L.-G. Pang, H. Peterson, Q. Wang, and X.-N. Wang, PRL117, 192301 (2016)



F. Becattini and I. Karpenko, PRL120.012302 (2018)
S. Voloshin, EPJ Web Conf.171, 07002 (2018)

Flow structure in the transverse plane (jet, ebe fluctuations etc.) may generate longitudinal polarization





$$\frac{dN}{d\Omega^*} = \frac{1}{4\pi}(1 + \alpha_H \mathbf{P_H} \cdot \mathbf{p}_p^*)$$

$$\langle\cos\theta_p^*\rangle = \int \frac{dN}{d\Omega^*}\cos\theta_p^* d\Omega^*$$

$$= \alpha_H P_z \langle(\cos\theta_p^*)^2\rangle$$

$$\therefore\ P_z = \frac{\langle\cos\theta_p^*\rangle}{\alpha_H\langle(\cos\theta_p^*)^2\rangle}$$

$$= \frac{3\langle\cos\theta_p^*\rangle}{\alpha_H}$$ (if perfect detector)

α_H: hyperon decay parameter
θ_p^*: θ of daughter proton in Λ rest frame

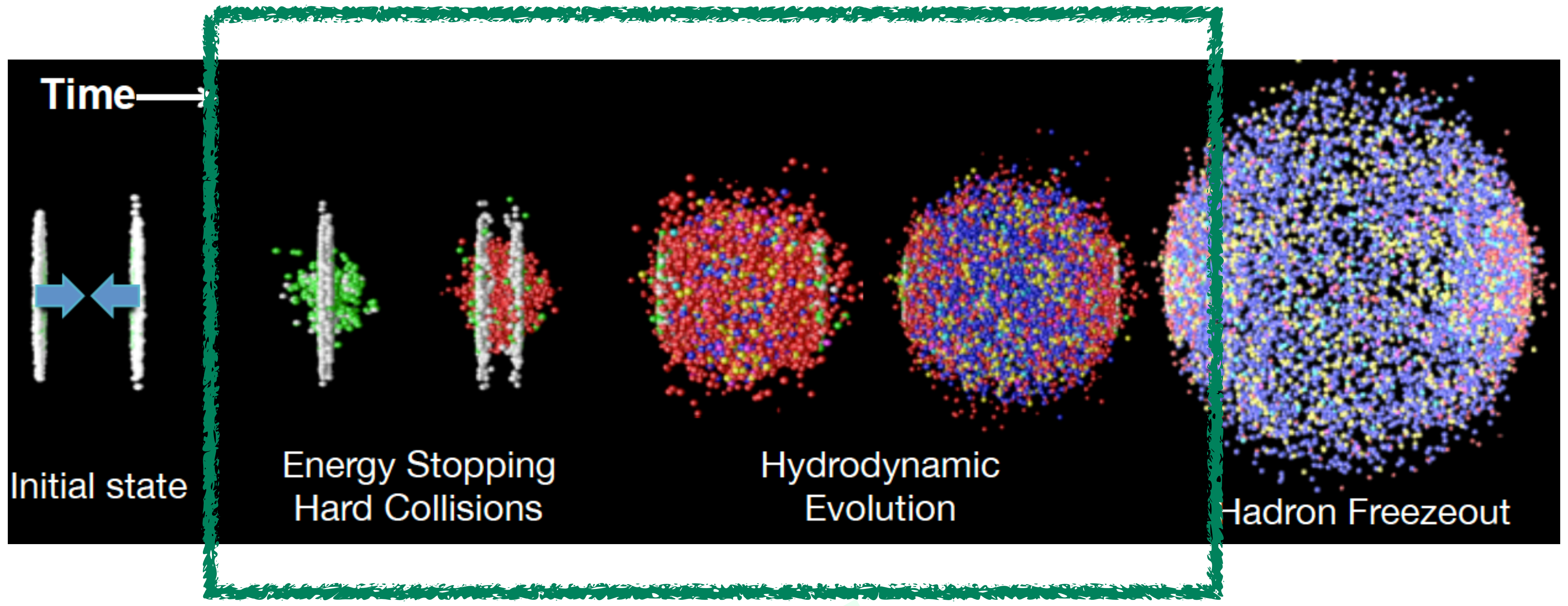
How to describe dynamics of spin?

Spin-thermal approach does not capture differential observables

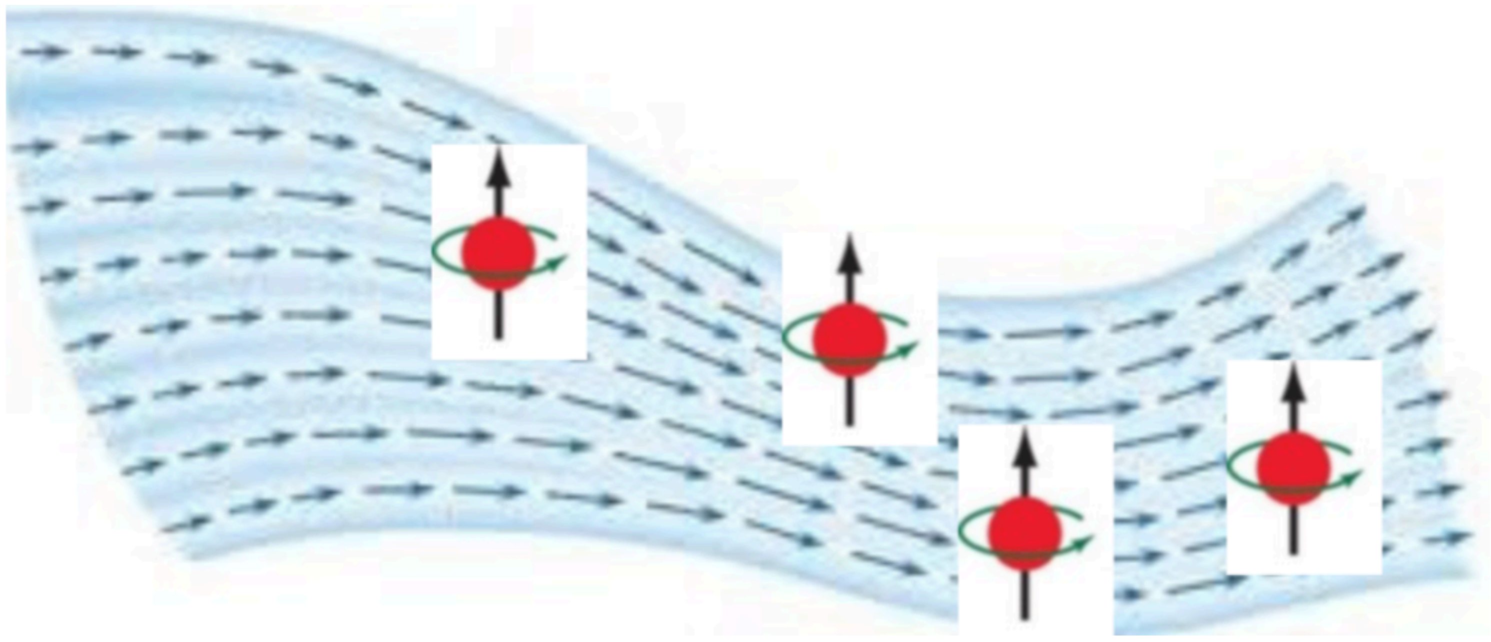
Is spin polarization always enslaved to thermal vorticity?

Non-trivial space-time dynamics of spin?

Relativistic fluid dynamics forms the basis of HIC models

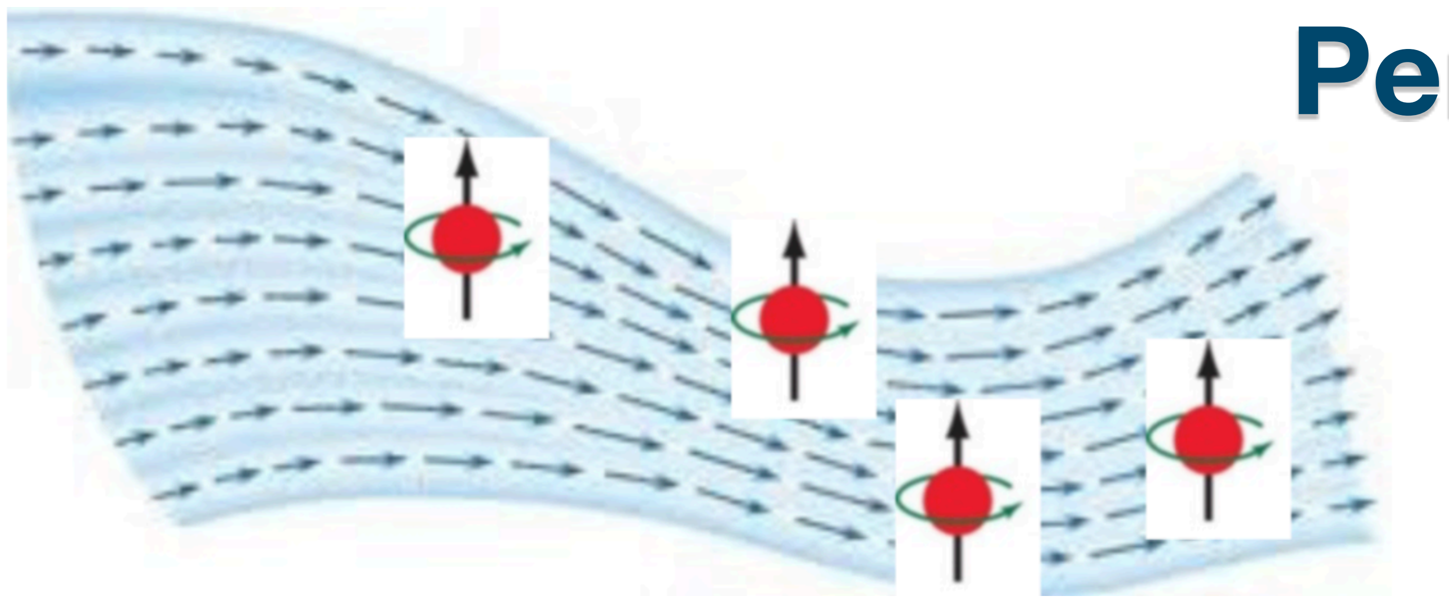


Fluid dynamics with spin?

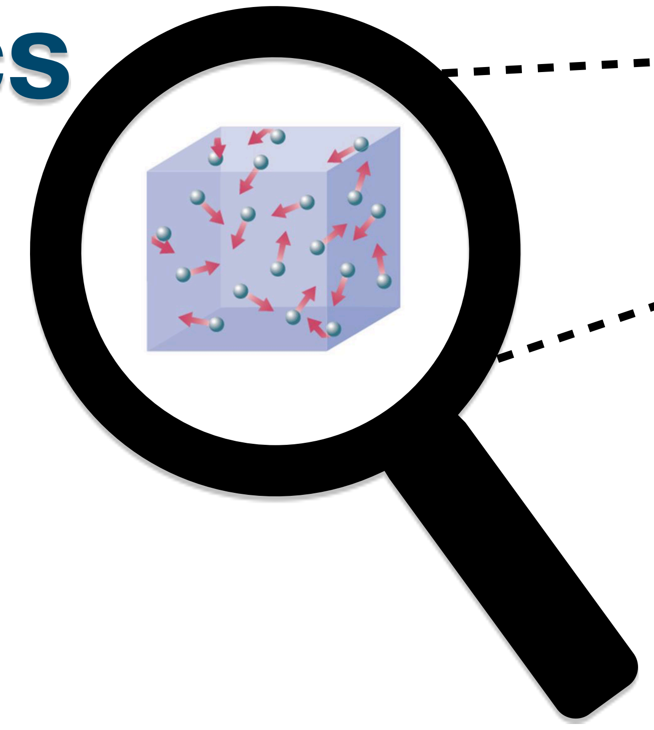


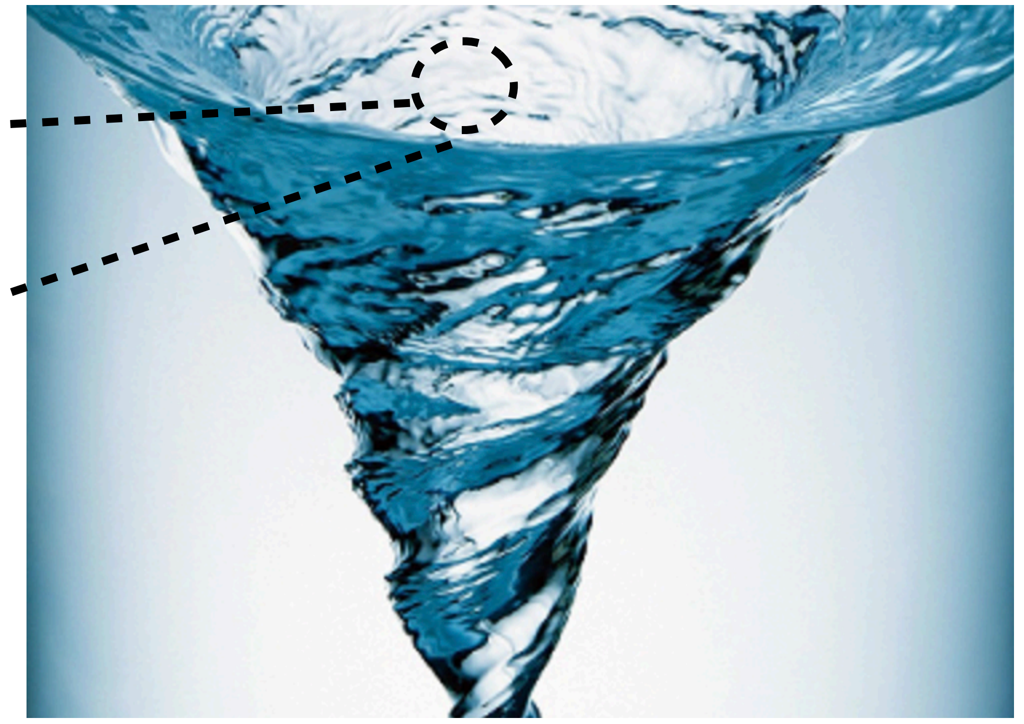
Most of the time close to equilibrium but the dissipation is also important

Perfect-fluid spin hydrodynamics



Relativistic kinetic theory
formulation of ideal fluid





For dilute systems, the derivation of fluid dynamics can be done starting from the underlying kinetic theory

Quantum RKT

$$\left(\gamma_\mu K^\mu - m\right) \mathscr{W}(x,k) = C[\mathscr{W}(x,k)]$$

$$K^\mu = k^\mu + \frac{i}{2}\left(\hbar\partial^\mu\right)$$

$$k^\mu \partial_\mu \mathscr{F}_{\rm eq}(x,k) = 0$$

$$k^\mu \partial_\mu \mathscr{A}^\nu_{\rm eq}(x,k) = 0$$

Moments method

Semi-classical expansion

$$\partial_\mu N^\mu = 0 \quad \partial_\mu T^{\mu\nu} = 0 \quad \partial_\lambda S^{\lambda,\mu\nu} = 0$$

Conservation laws

Classical spin treatment - perfect fluid

W. Florkowski, R. Ryblewski, A. Kumar, Prog. Part. Nucl. Phys. 108 (2019) 103709
J.-W. Chen, J.-y. Pang, S. Pu, Q. Wang, PRD 89 (9) (2014) 094003

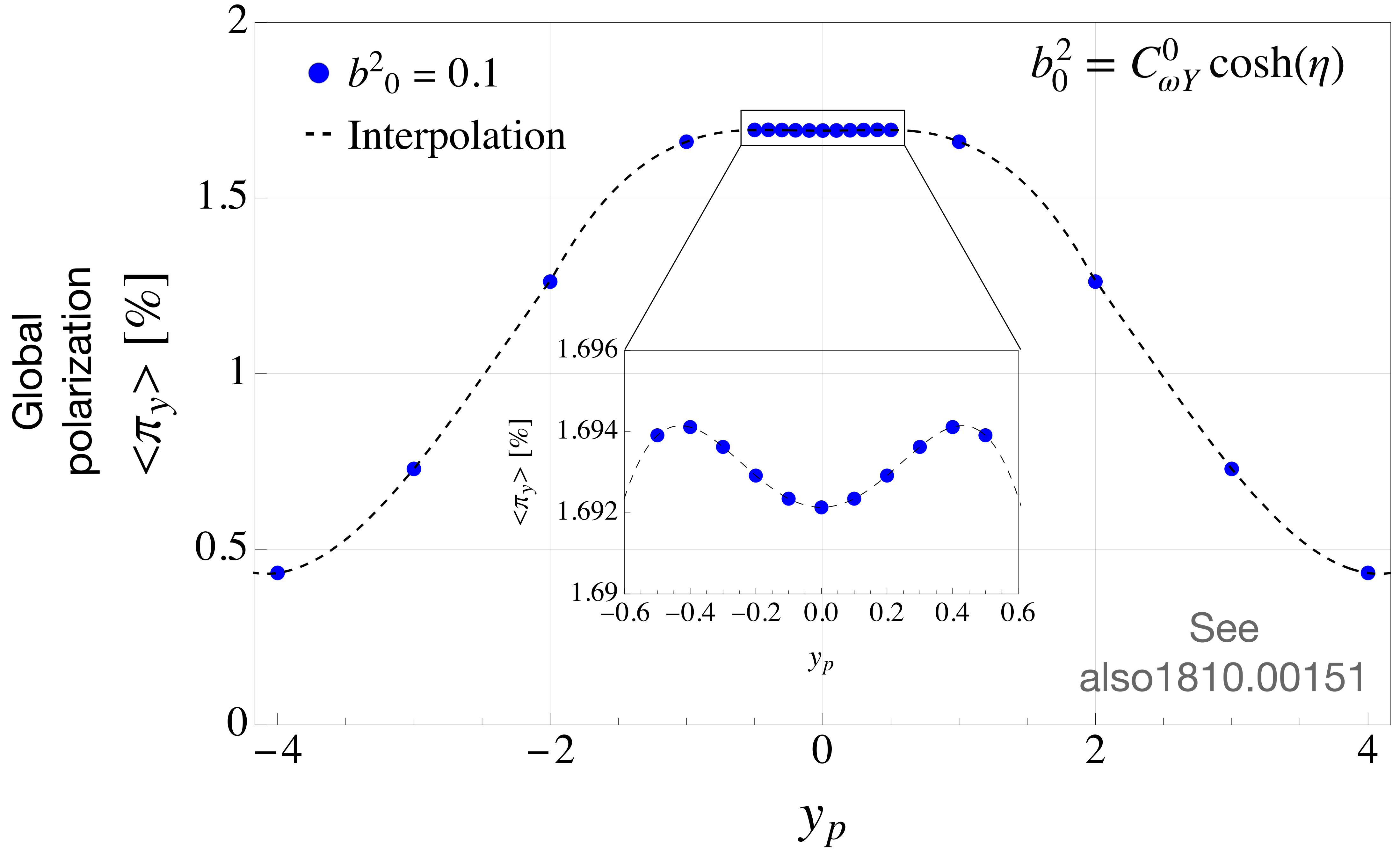
$$f^{\pm}_{\rm eq}(x,p,s) = \exp\left(-p\cdot\beta(x) \pm \xi(x) + \frac{1}{2}\omega_{\alpha\beta}(x)\, s^{\alpha\beta}\right)$$

$$\int dS = \frac{m}{\pi \mathfrak{s}} \int d^4s\, \delta(s\cdot s + \mathfrak{s}^2)\, \delta(p\cdot s)$$

$$N^{\mu}_{\rm eq} = \int dP \int dS\, p^{\mu} \left[f^{+}_{\rm eq}(x,p,s) - f^{-}_{\rm eq}(x,p,s)\right]$$

$$T^{\mu\nu}_{\rm eq} = \int dP \int dS\, p^{\mu}p^{\nu} \left[f^{+}_{\rm eq}(x,p,s) + f^{-}_{\rm eq}(x,p,s)\right]$$

$$S^{\lambda\mu\nu}_{\rm eq} = \int dP \int dS\, p^{\lambda} s^{\mu\nu} \left[f^{+}_{\rm eq}(x,p,s) + f^{-}_{\rm eq}(x,p,s)\right]$$



W. Florkowski, A. Kumar, R. Ryblewski, R. S., *Phys.Rev.C* 99 (2019) 4, 044910
R. S., G. Sophys, R. Ryblewski, *Phys.Rev.D* 103 (2021) 7, 074024
R. S., M. Shokri, R. Ryblewski, *Phys.Rev.D* 103 (2021) 9, 094034
W. Florkowski, R. Ryblewski, R. S., G. Sophys, *Phys.Rev.D* 105 (2022) 5, 054007

Explicit constitutive relations

$$N^{\alpha}_{\rm eq} = n u^{\alpha}$$

$$T^{\alpha\beta}_{\rm eq}(x) = \varepsilon u^{\alpha}u^{\beta} - P\Delta^{\alpha\beta}$$

$$S^{\lambda,\mu\nu}_{\rm eq} = S^{\lambda,\mu\nu}_{\rm GLW} = \mathcal{C}\left(n_0(T)u^{\lambda}\omega^{\mu\nu} + S^{\lambda,\mu\nu}_{\Delta\rm GLW}\right)$$

$$S^{\alpha,\beta\gamma}_{\Delta\rm GLW} = \mathcal{A}_0\, u^{\alpha}u^{\delta}u^{[\beta}\omega^{\gamma]}_{\ \delta} + \mathcal{B}_0\left(u^{[\beta}\Delta^{\alpha\delta}\omega^{\gamma]}_{\ \delta} + u^{\alpha}\Delta^{\delta[\beta}\omega^{\gamma]}_{\ \delta} + u^{\delta}\Delta^{\alpha[\beta}\omega^{\gamma]}_{\ \delta}\right)$$

$$\langle\pi_{\mu}\rangle = \frac{\int dP \langle\pi_{\mu}\rangle_p E_p \frac{d\mathcal{N}(p)}{d^3p}}{\int dP E_p \frac{d\mathcal{N}(p)}{d^3p}} \equiv \frac{\int d^3p\, \frac{d\Pi^{\star}_{\mu}(p)}{d^3p}}{\int d^3p\, \frac{d\mathcal{N}(p)}{d^3p}}$$

For $|\omega_{\mu\nu}| < 1$ one obtains the formalism that agrees with that based on the quantum description of spin (in the GLW version).

Thank you for listening!

Looking forward to more discussions!