

58. Karpacz Winter School
of Theoretical Physics



Relativistic fluid dynamics of multiple conserved charges

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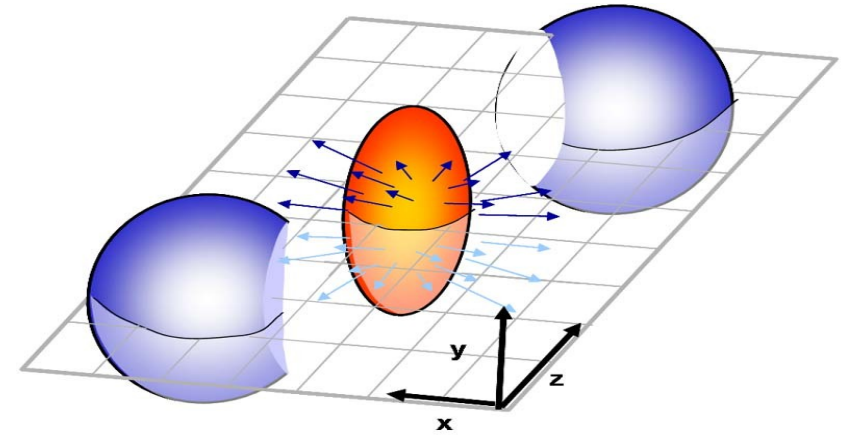
Harri Niemi, Etele Molnár, Moritz Greif, Gabriel Denicol, Dirk Rischke, Carsten Greiner

Motivation

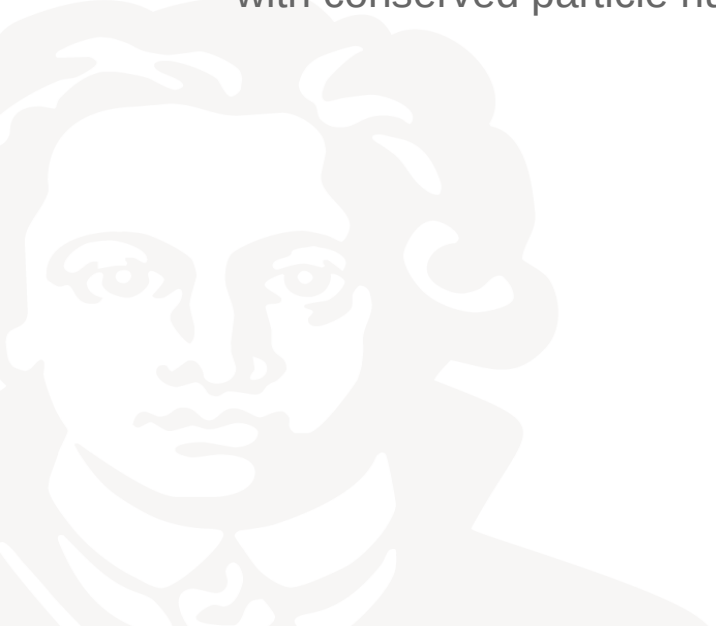
Traditionally:

Viewed as 'blob' of one type of matter (single component) with one velocity field

- usually 'blob' of energy
with conserved particle number



<https://www.quantumdiaries.org/wp-content/uploads/2011/02/FlowPr.jpg>

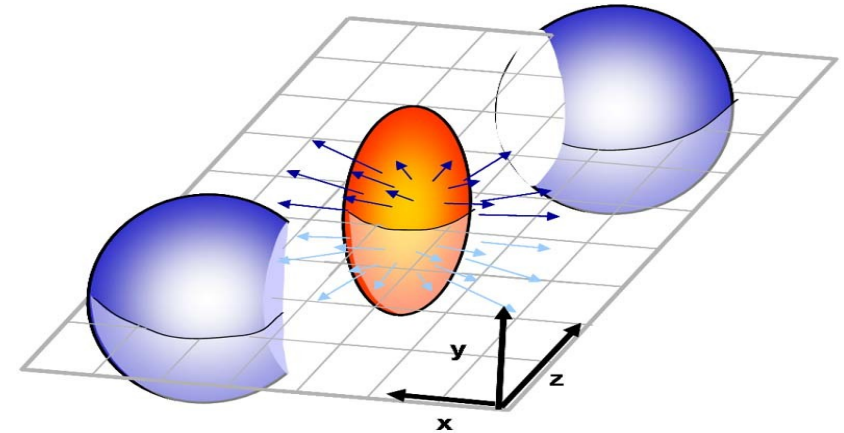


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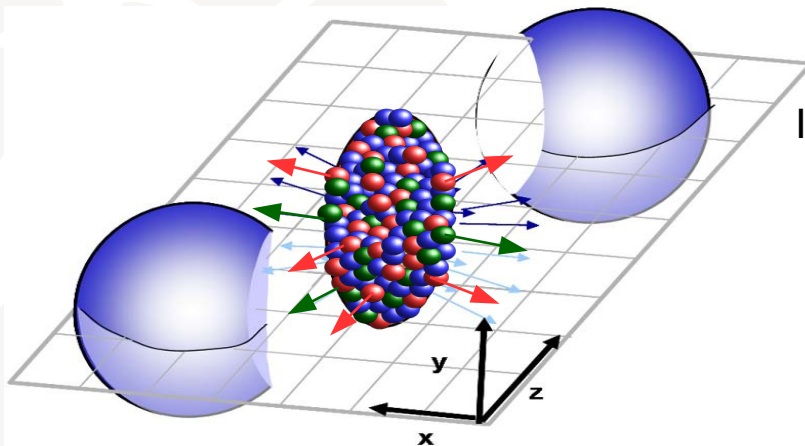
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In general:

Consists of multiple components with various properties with multiple velocity fields

- with **multiple conserved quantities** (e.g. energy, electric charge, baryon number, strangeness, ...)
- mixed chemistry → **coupled charge currents!**

Motivation

Increasing interest in the recent years ...

Simulation with baryon number

Denicol et al., PRC 98, 034916 (2018)

Li et al., PRC 98, 064908 (2018)

Du et al., Comp. Phys. Comm.
251 (2020) 107090

Diffusion coefficients with BQS

Greif et al., PRL 120, 242301 (2018)

Rose et al., PRD 101, 114028 (2020)

Fotakis et al., PRD 104, 034014 (2021)

Das et al., arXiv:2109.01543

Simulation with multiple charges

Fotakis et al., PRD 101, 076007 (2020)

Chen et al., arXiv:2203.04685

Theory with multiple conserved charges

Monnai et al., Nucl. Phys. A847:283-314 (2010)

Kikuchi et al., PRC 92, 064909 (2015)

Fotakis et al., arXiv:2203.11549

BQS equation of state

Noronha-Hostler et al., PRC 100, 064910 (2019)

Monnai et al., arXiv:2101.11591

On the QM2022 conference ...

Plaschke et al., Poster Session 1 T02/T03

Mishra et al., Poster Session 2 T03

Almaalol et al., Poster Session 2 T14_2

Pihan et al., Poster Session 2 T07_2

Weickgenannt., Plenary Session VII

... and many more!

Fluid dynamics of multi-component systems

Hydrodynamics: macroscopic effective field theory of thermal matter close to local equilibrium

Here: single fluid velocity field u^μ

Conservation of Energy and Momentum: $\partial_\mu T^{\mu\nu} = 0$

Conservation of charge: $\partial_\mu N_q^\mu = 0$



Hydrodynamics: macroscopic effective field theory of thermal matter close to local equilibrium

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$$T^{\mu\nu} = \sum_i T_i^{\mu\nu} = \epsilon u^\mu u^\nu - (P_0 + \Pi) \Delta^{\mu\nu} + \pi^{\mu\nu}$$

q-th conserved charge (eg. B,Q,S)

Conservation of charge: $\partial_\mu N_q^\mu = 0$

$$N_q^\mu = \sum_i q_i N_i^\mu = n_q u^\mu + V_q^\mu$$

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$10 + 4N_{\text{ch}}$ degrees of freedom, $4 + N_{\text{ch}}$ equations \rightarrow $6 + 3N_{\text{ch}}$ unknowns

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What needs to be known:

- Equation of state $P_0 = P_0(\epsilon, n_q)$, $T = T(\epsilon, n_q)$, $\alpha_q = \mu_q/T = \alpha_q(\epsilon, n_q)$
- Equations of motion for dissipative fields & transport coefficients $\Pi, V_q^\mu, \pi^{\mu\nu}$
- Initial state
- Final state: freeze-out and δf -correction

Deriving fluid dynamics from kinetic theory

Denicol et al., PRD 85, 114047 (2012)

On basis of DNMR theory: derivation from the Boltzmann equation with method of moments
Fotakis et al., arXiv:2203.11549

Also refer to: Monnai, Hirano, Nucl. Phys. A847:283-314 (2010) or Kikuchi et al., PRC 92, 064909 (2015)

relativistic Boltzmann eq.

$$k_i^\mu \partial_\mu f_{i,\mathbf{k}} = \mathcal{C}_i[f_i]$$



2nd-order (multi-component) hydro

$$\dot{\Pi}, \dot{V}_q^{\langle\mu\rangle}, \dot{\pi}^{\langle\mu\nu\rangle}$$

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$$\dot{\Pi}, \dot{V}_q^{\langle\mu\rangle}, \dot{\pi}^{\langle\mu\nu\rangle}$$

equilibrium

off-equilibrium

$$f_{i,\mathbf{k}} = f_{i,\mathbf{k}}^{(0)} + \delta f_{i,\mathbf{k}}$$

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2nd-order (multi-component) hydro

$$\dot{\Pi}, \dot{V}_q^{\langle\mu\rangle}, \dot{\pi}^{\langle\mu\nu\rangle}$$

Irreducible **off-equilibrium** moments
obey Boltzmann eq.:

Problem: infinitely many coupled PDEs.

Aim: Truncate in a well-defined manner (“perturbation theory”)

$$f_{i,\mathbf{k}} = \underbrace{f_{i,\mathbf{k}}^{(0)}}_{\text{equilibrium}} + \underbrace{\delta f_{i,\mathbf{k}}}_{\text{off-equilibrium}}$$
$$\rho_{i,n}^{\mu\nu} = \sum_{i=1}^{N_{\text{species}}} \int \frac{d^3\mathbf{k}_i}{(2\pi)^3 E_{i,\mathbf{k}}} E_{i,\mathbf{k}}^n k_i^{\langle\mu} k_i^{\nu\rangle} \delta f_{i,\mathbf{k}}$$

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Aim: Truncate in a well-defined manner (perturbation theory)

“Order-of-magnitude approximation”:

relate them to the dissipative fields with constituent’s transport coefficients

$$\rho_{i,n}^{\mu\nu} = \frac{\eta_{i,n}}{\eta} \pi^{\mu\nu} + \mathcal{O}(2)$$

Counting scheme:

Gradients in velocity, temperature etc. $\sigma^{\mu\nu} \sim \mathcal{O}(1), \mathcal{O}(Kn)$

Dissipative fields $\pi^{\mu\nu} \sim \mathcal{O}(1), \mathcal{O}(Rn^{-1})$

Equations of motion with multiple conserved charges

2nd-order (multi-component) hydro

$$\dot{\Pi}, \dot{V}_q^{\langle\mu\rangle}, \dot{\pi}^{\langle\mu\nu\rangle}$$

upcoming publication!

$$\begin{aligned} \tau_{\Pi} \dot{\Pi} + \Pi &= S_{\Pi} \\ \sum_{q'} \tau_{qq'} \dot{V}_{q'}^{\langle\mu\rangle} + V_q^{\mu} &= S_q^{\mu} \\ \tau_{\pi} \dot{\pi}^{\langle\mu\nu\rangle} + \pi^{\mu\nu} &= S_{\pi}^{\mu\nu} \end{aligned}$$

Relaxation equations
(Israel-Stewart-type
causal theory)



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$$\begin{aligned} S_q^{\mu} = & \sum_{q'} \kappa_{qq'} \nabla^{\mu} \alpha_{q'} - \sum_{q'} \tau_{qq'} V_{q',\nu} \omega^{\nu\mu} - \sum_{q'} \delta_{VV}^{(q,q')} V_{q'}^{\mu} \theta - \sum_{q'} \lambda_{VV}^{(q,q')} V_{q',\nu} \sigma^{\mu\nu} \\ & - \ell_{V\Pi}^{(q)} \nabla^{\mu} \Pi + \ell_{V\pi}^{(q)} \Delta^{\mu\nu} \nabla_{\lambda} \pi_{\nu}^{\lambda} + \tau_{V\Pi}^{(q)} \Pi \dot{u}^{\mu} - \tau_{V\pi}^{(q)} \pi^{\mu\nu} \dot{u}_{\nu} \\ & + \sum_{q'} \lambda_{V\Pi}^{(q,q')} \Pi \nabla^{\mu} \alpha_{q'} - \sum_{q'} \lambda_{V\pi}^{(q,q')} \pi^{\mu\nu} \nabla_{\nu} \alpha_{q'} \end{aligned}$$

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Mixed chemistry couples diffusion currents (coupled charge-transport); already present in 1st order term

2nd order terms: couples all currents to each other; depend on all gradients!

Explicit expressions for transport coefficients!

Equations of motion with multiple conserved charges

2nd-order (multi-component) hydro

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Diffusion
coefficients:
extensively
studied!

Greif, Fotakis et al., PRL 120, 242301 (2018)
Fotakis, Greif et al., PRD 101, 076007 (2020)
Fotakis, Soloveva et al., PRD 104, 034014 (2021)

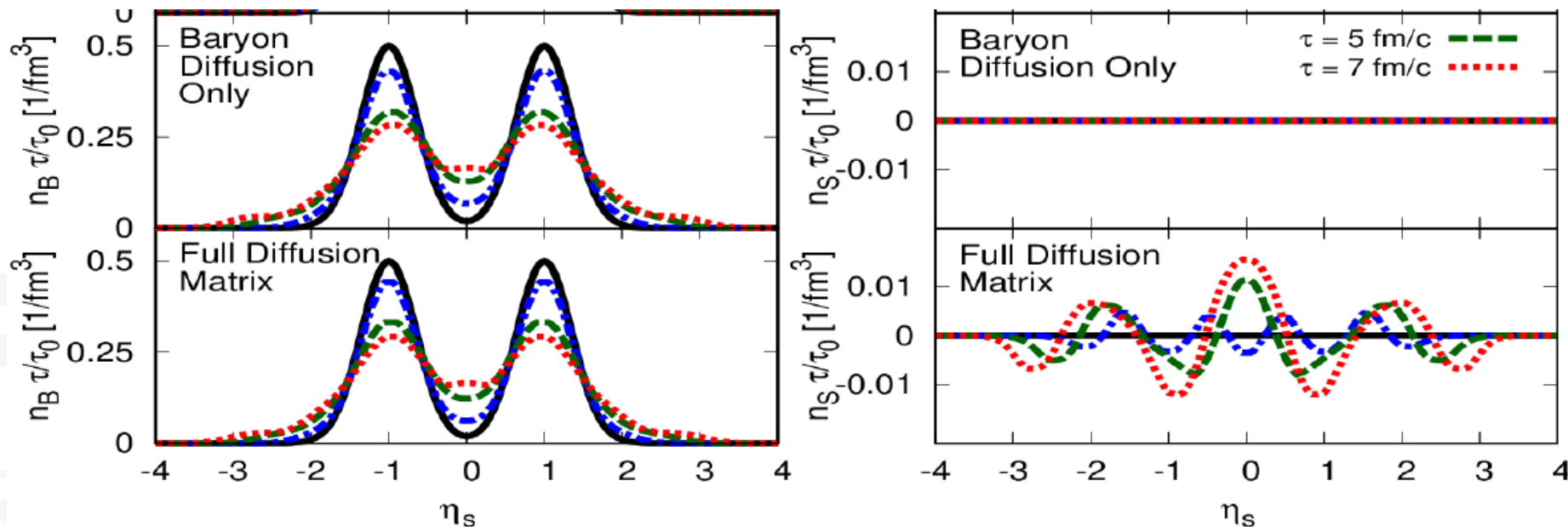
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Explicit expressions for transport coefficients!

Coupled charge-transport

Greif, Fotakis et al., PRL 120, 242301 (2018)
Fotakis, Greif et al., PRD 101, 076007 (2020)



Mixed chemistry couples diffusion currents and introduces charge-correlation through EoS



Generation of domains of non-vanishing local net charge (here net strangeness)!

e.g.: $\mu_S \equiv \mu_S(\epsilon, n_B, n_S)$
 $\nabla^\mu \alpha_S \sim \nabla^\mu n_B$

$$\Pi \equiv 0, \quad \pi^{\mu\nu} \equiv 0, \quad \tau_q \dot{V}_q^{\langle \mu} + V_q^\mu = \sum_{q'} \kappa_{qq'} \nabla^\mu \left(\frac{\mu_{q'}}{T} \right)$$

Conclusion

- Derived 2nd-order relativistic fluid dynamic theory for **multicomponent systems** from the Boltzmann equation
- **Transport coefficients given explicitly** containing all information about particle interactions
- Mixed chemistry couples diffusion currents to each other → **coupled charge-transport**
- **Consistency** of EoS, 1st and 2nd transport coefficients **is important!**
- Thermal features from LQCD can be adapted in transport coefficients with quasi-particle models
- Implemented derived fluid dynamic theory in **(3+1)D-hydro code**

Outlook

- Evaluate **2nd order transport coefficients** for more realistic systems
- Use more realistic **initial state** and **equation of state**
- Apply **freeze-out routines**, take δf -correction
- Find **observables** sensitive to charge-coupling → investigate impact

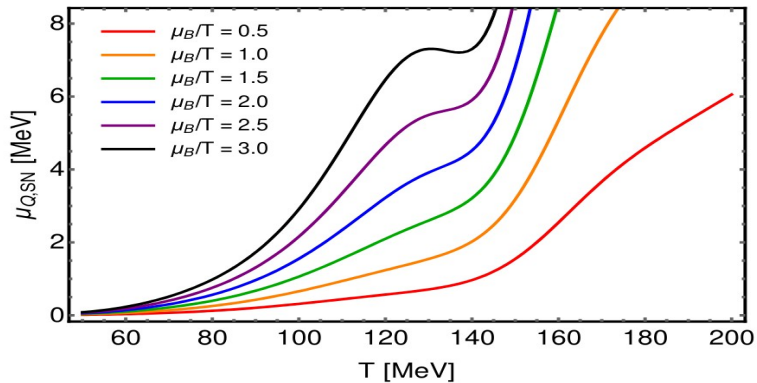
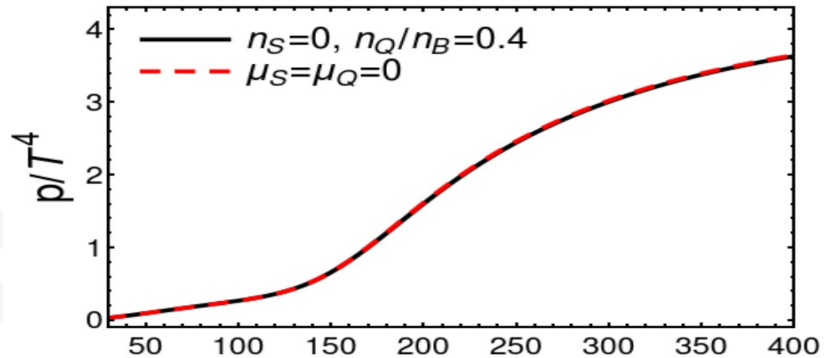
Backup



Equation of state with multiple conserved charges

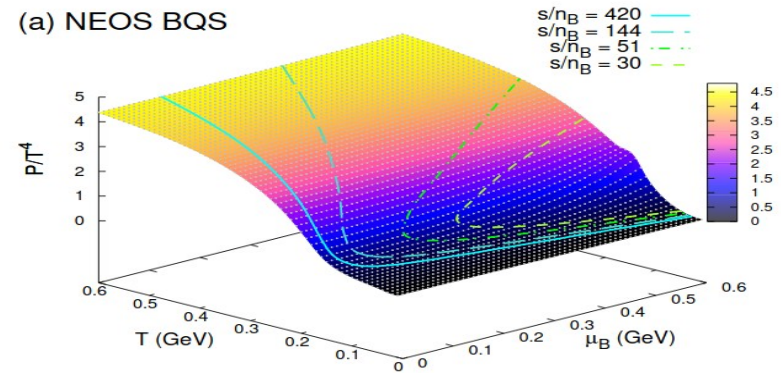
$$P_0(T) \rightarrow P_0(T, \mu_B, \mu_Q, \mu_S)$$

Noronha-Hostler et al., PRC 100, 064910 (2019)

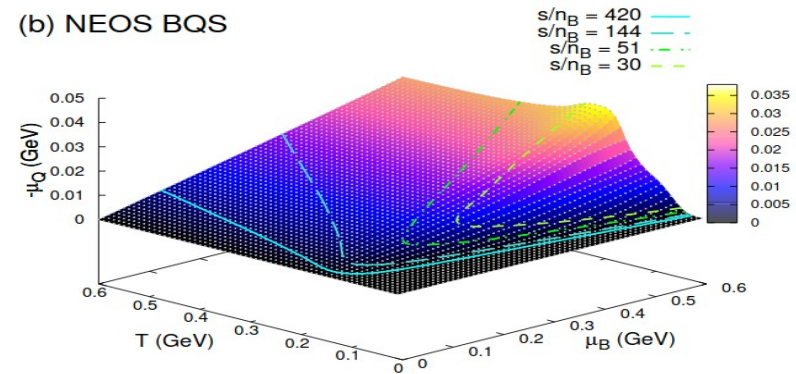


Monnai et al., PRC 100, 024907 (2019)

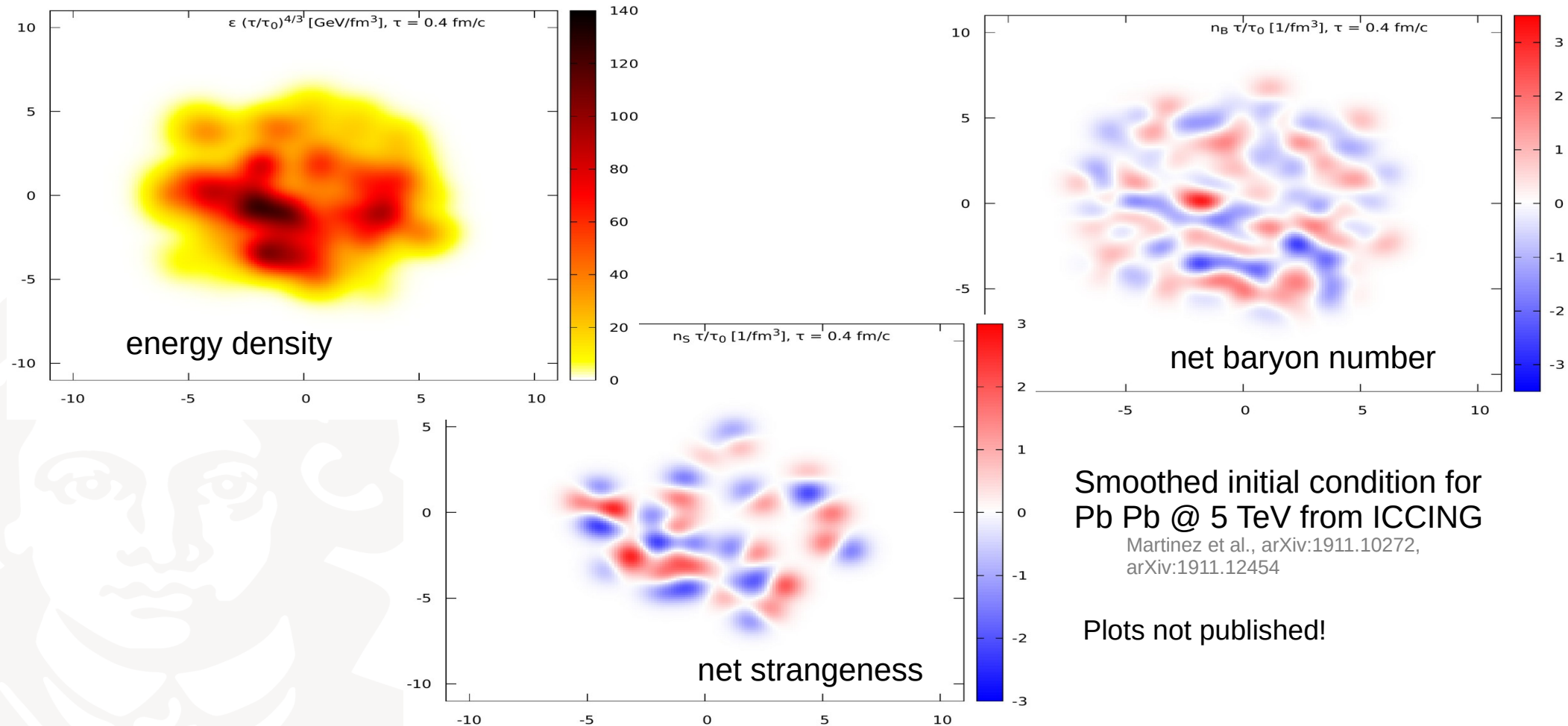
(a) NEOS BQS



(b) NEOS BQS



Initial state with multiple conserved charges



Computation of transport coefficients (Example: diffusion coefficients)

On basis of DNMR theory: derivation from the Boltzmann equation with method of moments

Fotakis et al., arXiv:2203.11549

relativistic Boltzmann eq.

$$k_i^\mu \partial_\mu f_{i,\mathbf{k}} = C_i[f_i]$$

2nd-order (multi-component) hydro

$$\dot{\Pi}, \dot{V}_q^{\langle\mu\rangle}, \dot{\pi}^{\langle\mu\nu\rangle}$$

$$C_{i,n-1}^{\langle\mu\rangle} \equiv \int \frac{d^3\mathbf{k}_i}{(2\pi)^3 E_{i,\mathbf{k}}} E_{i,\mathbf{k}}^{n-1} k_i^{\langle\mu\rangle} C_i[f_i]$$

$$= - \sum_{m=0}^{\infty} \sum_j C_{ij,nm}^{(1)} \rho_{j,m}^\mu + \text{non-linear terms}$$

Entries of „collision matrix“ (for diffusive moments)

$$\kappa_{qq'} = \sum_{n=0}^{\infty} \sum_{i,j=1}^{N_{\text{species}}} \left(C^{(1)}\right)_{ij,0n}^{-1} q_i \left(q'_j J_{j,n+1,1} - \frac{n_{q'}}{\epsilon + P_0} J_{j,n+2,1} \right)$$

Diffusion coefficient matrix! (equivalent to our PRL and PRD expression)

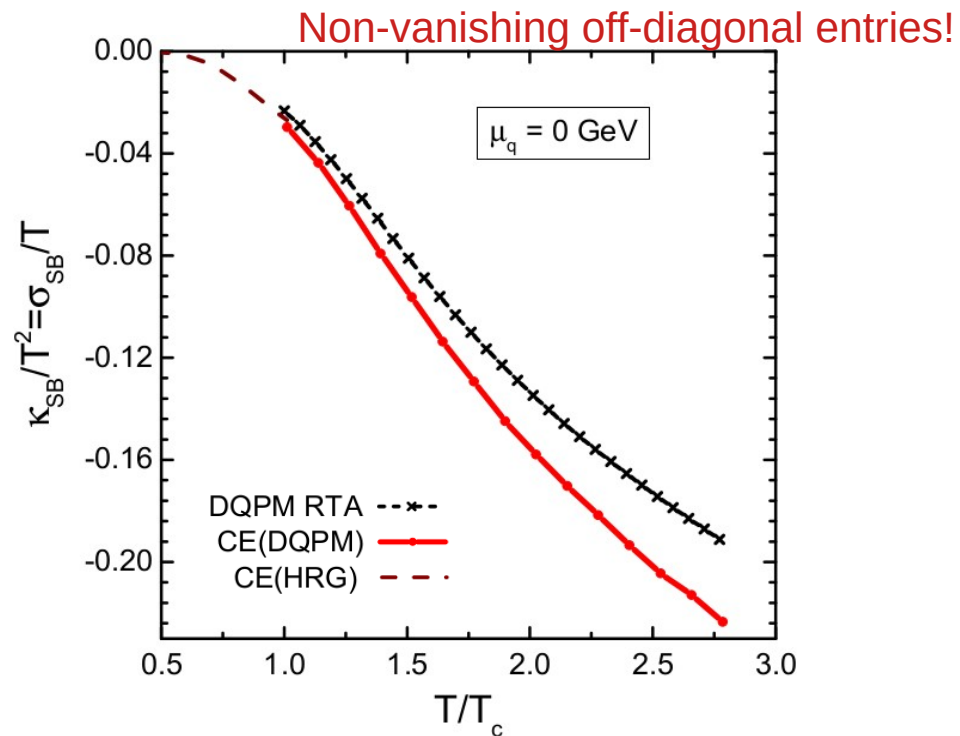
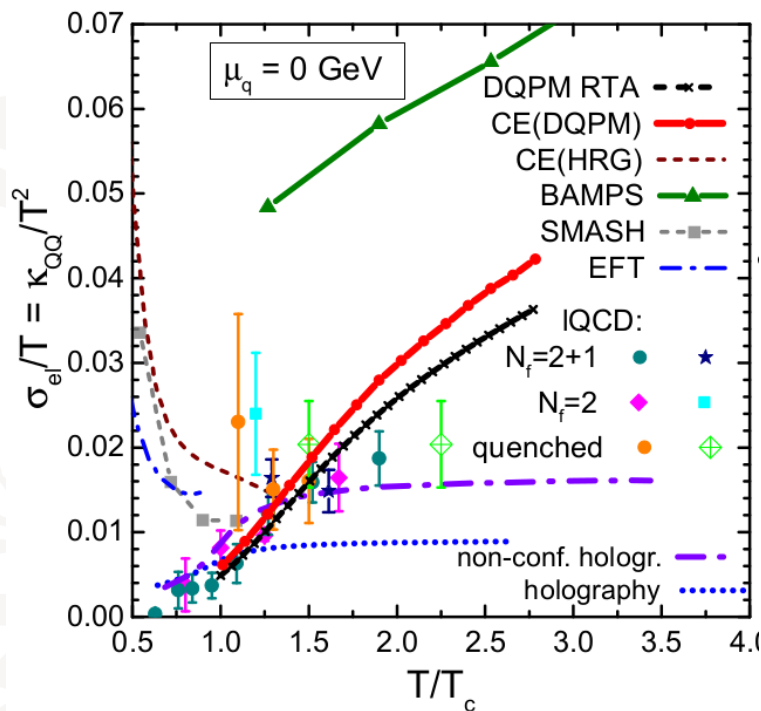
Greif, Fotakis et al., PRL 120, 242301 (2018)
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Coupled charge-transport

$$\kappa_{qq'} = \sum_{n=0}^{\infty} \sum_{i,j=1}^{N_{\text{species}}} \tau_{ij,0n}^{(1)} q_i \left(q'_j J_{j,n+1,1} - \frac{n q'}{\epsilon + P_0} J_{j,n+2,1} \right)$$

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Fotakis, Greif et al., PRD 101, 076007 (2020)
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Example: introduction of features from LQCD via the usage of DQPM



- Hadronic system including lightest 19 species

$$\pi^\pm, \pi^0, K^\pm, K^0, \bar{K}^0, p, \bar{p}, n, \bar{n}, \Lambda^0, \bar{\Lambda}^0, \Sigma^0, \bar{\Sigma}^0, \Sigma^\pm, \bar{\Sigma}^\pm$$

- Assume classical statistics and non-interacting limit

$$P_0(T, \{\mu_q\}) \equiv \frac{1}{3} \sum_{i=1}^{N_{\text{species}}} \int \frac{dp^3}{(2\pi)^3 E_{i,p}} (E_{i,p}^2 - m_i^2) g_i \exp(-E_{i,p}/T + \sum_q q_i \alpha_q)$$

- Only assume **baryon number** and **strangeness**, **neglect electric charge**
- Tabulate state variables over energy density and net charge densities

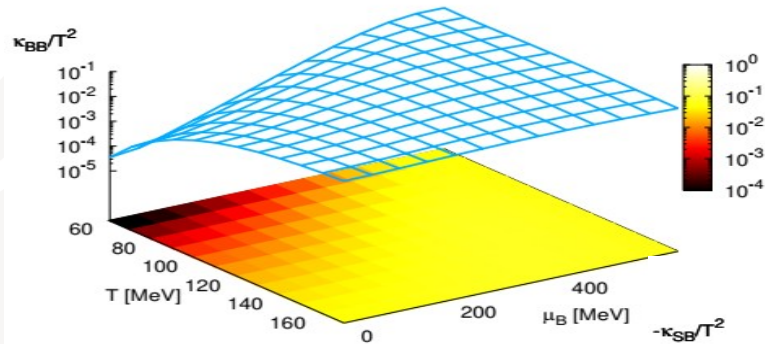
$$T \equiv T(\epsilon, \{n_q\}), \quad \mu_q \equiv \mu_q(\epsilon, \{n_q\}), \quad P_0 \equiv P_0(\epsilon, \{n_q\})$$

Diffusion coefficient matrix - details

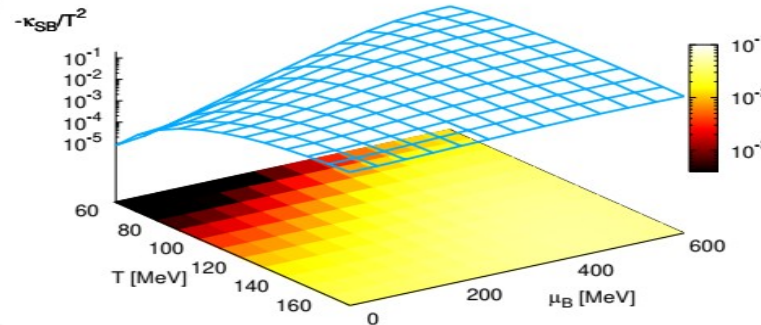
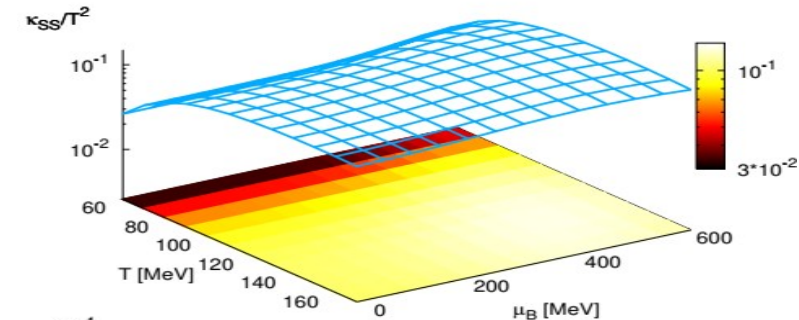
$$\begin{pmatrix} V_B^\mu \\ V_S^\mu \end{pmatrix} \sim \begin{pmatrix} \kappa_{BB} & \kappa_{BS} \\ \kappa_{SB} & \kappa_{SS} \end{pmatrix} \begin{pmatrix} \nabla^\mu \alpha_B \\ \nabla^\mu \alpha_S \end{pmatrix}$$

- Matrix is symmetric

L. Onsager, Phys. Rev. 37, 405 (1931) & Phys. Rev. 38, 2265 (1931)



- Elastic isotropic cross sections from PDG, SMASH, GiBUU, UrQMD



κ_{SB} is **negative** and has **similar magnitude** as κ_{BB}

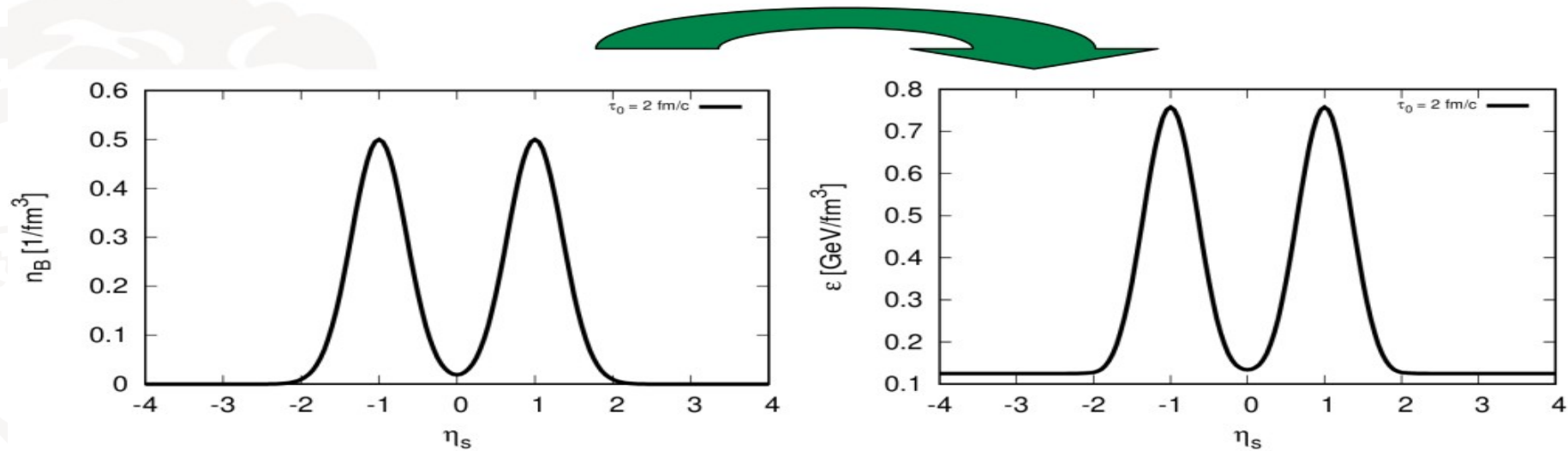
⇒ significant coupling?

- Tabulate coefficient matrix over T, μ_B, μ_S

- $\mu_Q = 0$

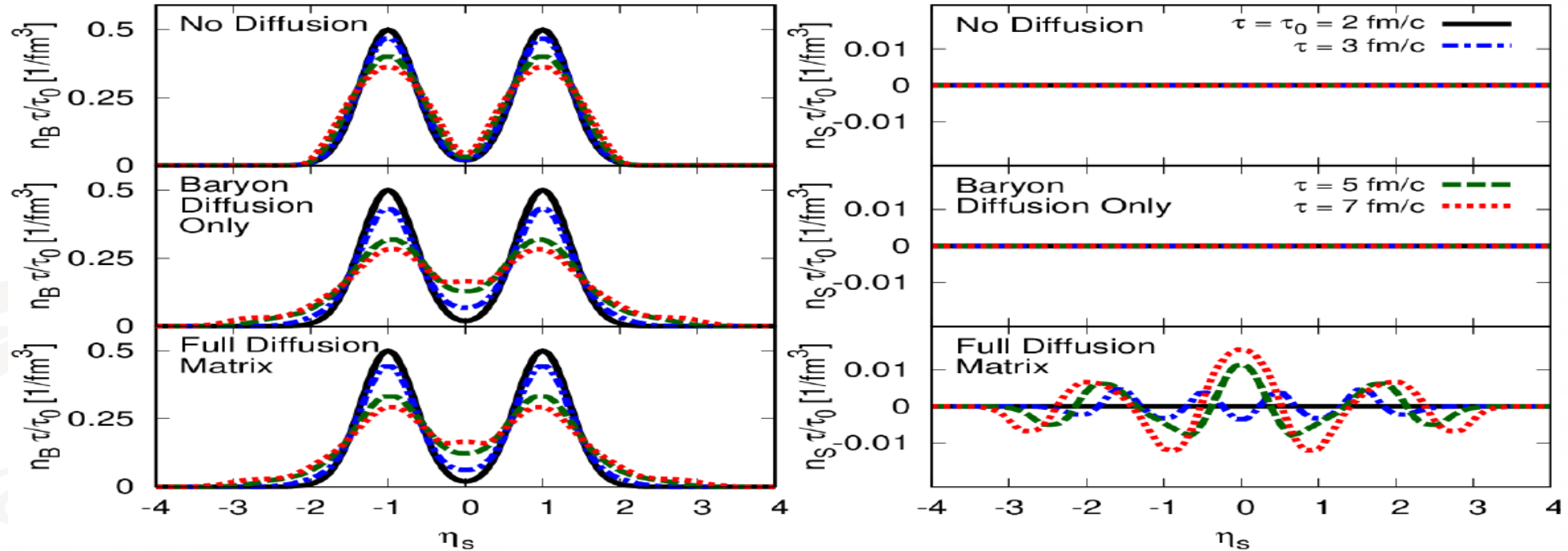
Initial conditions - details

- $\tau_0 = 2 \text{ fm}/c$
- Initially: no dissipation and only **Bjorken scaling flow**
- Temperature = 160 MeV
- Double-gaussian profile in net baryon number
- From **EoS**: get energy density



Coupled charge-transport

Greif, Fotakis et al., PRL 120, 242301 (2018)
Fotakis, Greif et al., PRD 101, 076007 (2020)



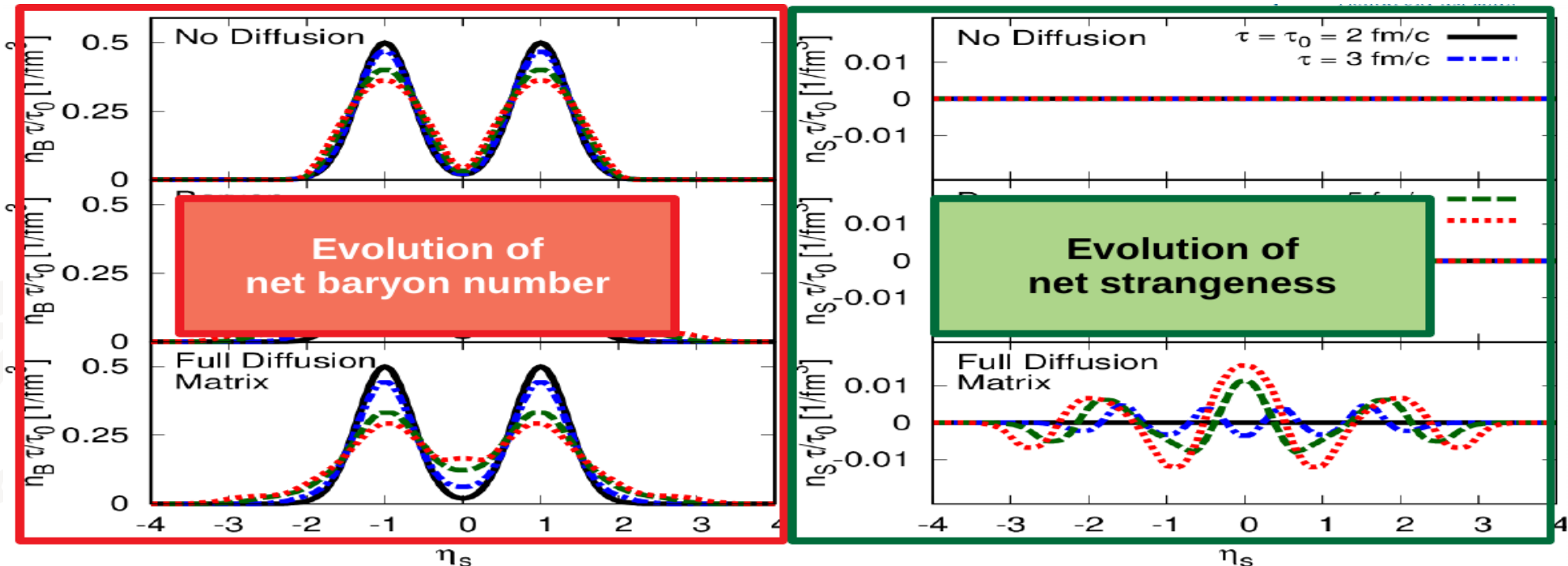
Simplistic case study: no viscosity, diffusion only, no 2nd-order terms, conserved B and S charge, classical, hadronic system (19 species), realistic binary elastic cross sections

Hydrodynamic (1+1)D-simulation with SHASTA

$$\Pi \equiv 0, \quad \pi^{\mu\nu} \equiv 0, \quad \tau_q \dot{V}_q^{\langle\mu} + V_q^{\mu} = \sum_{q'} \kappa_{qq'} \nabla^{\mu} \left(\frac{\mu_{q'}}{T} \right)$$

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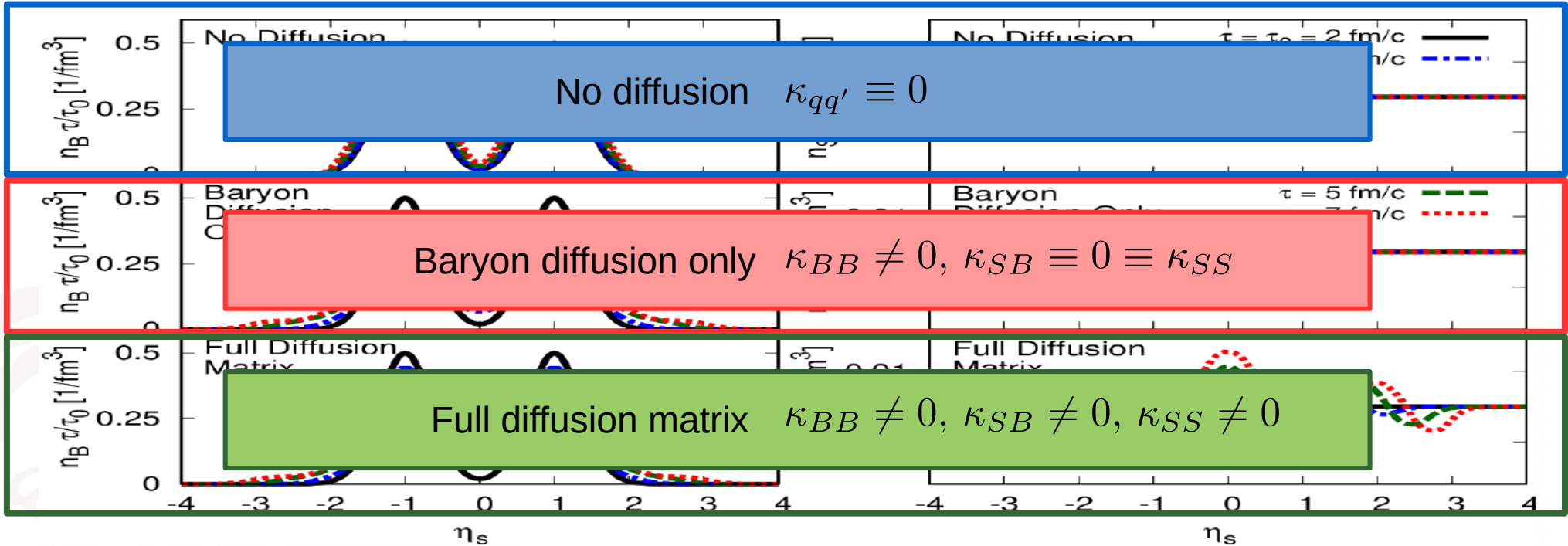
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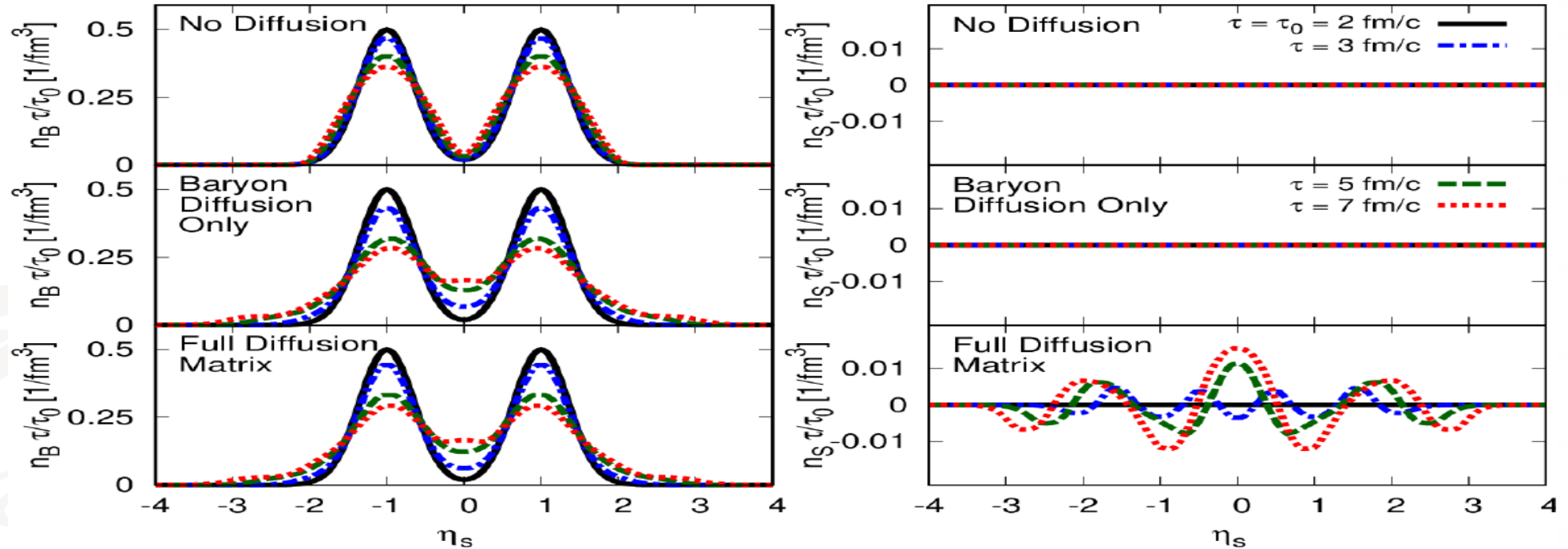
Simplistic case study: no viscosity, diffusion only, no 2nd-order terms, conserved B and S charge, classical, hadronic system (19 species), realistic binary elastic cross sections

Hydrodynamic (1+1)D-simulation with SHASTA

$$\Pi \equiv 0, \quad \pi^{\mu\nu} \equiv 0, \quad \tau_q \dot{V}_q^{\langle\mu} + V_q^\mu = \sum_{q'} \kappa_{qq'} \nabla^\mu \left(\frac{\mu_{q'}}{T} \right)$$

Coupled charge-transport

Greif, Fotakis et al., PRL 120, 242301 (2018)
Fotakis, Greif et al., PRD 101, 076007 (2020)



Mixed chemistry couples diffusion currents and introduces charge-correlation through EoS



Generation of domains of non-vanishing local net charge (here net strangeness)!

e.g.: $\mu_S \equiv \mu_S(\epsilon, n_B, n_S)$
 $\nabla^\mu \alpha_S \sim \nabla^\mu n_B$

Single-component vs. Multi-component system

A potentially problematic term in single-component systems $S_q^\mu = (\dots) + \ell_{V\pi}^{(q)} \Delta^{\mu\nu} \nabla_\lambda \pi_\nu^\lambda + (\dots)$

Ultrarelativistic, classical system with hard-sphere interactions:

Denicol et al., PRD 85, 114047 (2012)

TABLE I. The coefficients for the particle diffusion for a classical gas with constant cross section in the ultrarelativistic limit, in the 14-moment approximation. The transport coefficient $\tau_{n\pi}$ was incorrectly listed as being zero in Ref. [1]

κ	$\tau_n[\lambda_{\text{mfp}}]$	$\delta_{nn}[\tau_n]$	$\lambda_{nn}[\tau_n]$	$\lambda_{n\pi}[\tau_n]$	$\ell_{n\pi}[\tau_n]$	$\tau_{n\pi}[\tau_n]$
$3/(16\sigma)$	$9/4$	1	$3/5$	$\beta_0/20$	$\beta_0/20$	$\beta_0/80$

Used in simulations of heavy-ion collisions!

Multi-component system:

$$\ell_{V\pi}^{(q)} = \frac{9}{80\sigma P} \sum_{i=1}^{N_{\text{spec}}} q_i \frac{P_i}{P} \xrightarrow{\text{single}} \ell_{n\pi} = \frac{\tau_n}{20T}$$

Fotakis et al., arXiv:2203.11549

The problem: system with **conserved net-charge** and if **each** constituent has **anti-particle partner** then at **vanishing** chemical potential it is:

$$\ell_{V\pi}^{(q)} = 0 \neq \ell_{n\pi} = \frac{\tau_n}{20T}$$

Single-component vs. Multi-component system

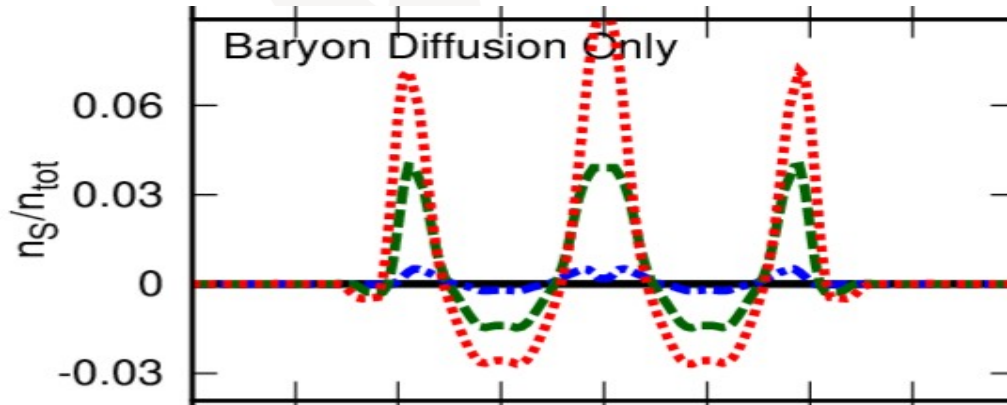
Run simulation of system with conserved baryon number and strangeness with **shear viscosity** and **diffusion**; account for **second-order terms**

$$\frac{\eta}{s} = \frac{1}{4\pi}$$

Second-order coefficients in **ultrarelativistic, single-component limit** from Denicol (2012)

$$\tau_n \dot{V}_q^{\langle \mu} + V_q^\mu = \sum_{q'} \kappa_{qq'} \nabla^\mu \alpha_{q'} - V_{q,\nu} \omega^{\nu\mu} - \tau_n V_{q'}^\mu \theta - \frac{3\tau_n}{5} V_{q,\nu} \sigma^{\mu\nu} + \frac{\tau_n}{20T} \Delta^{\mu\nu} \nabla_\lambda \pi_\nu^\lambda - \frac{\tau_n}{20T} \pi^{\mu\nu} \dot{u}_\nu - \frac{\tau_n}{20T} \pi^{\mu\nu} \nabla_\nu \alpha_q$$

Here (plot): baryon diffusion only: $\kappa_{BB} \neq 0$, $\kappa_{SS} = 0 = \kappa_{SB}$



Second-order transport coefficients **not consistent** with assumed system

→ generation of unphysical charge currents

Consistency is important in charge transport!
Use multi-component expressions.