Canonical Vs. Phenomenological Formulations of 1^{st} Order Dissipative Spin Hydrodynamics

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Fig:1 M.C. Escher, Contrast (Order and Chaos)

"We adore chaos because we love to produce order"
-M.C.Echer

Introduction

Experimental Observation

STAR experiment at RHIC observed an alignment between global angular momentum of the colliding system and the average spin of Λ^0 baryons emitted from it, indicating spin polarization of the produced matter along its vorticity direction.

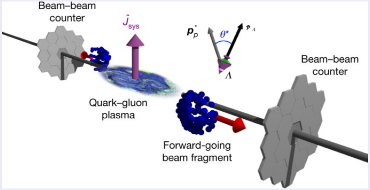

Fig:2 A sketch of a Au + Au collision [STAR, L. Adamczyk et al., Nature 548, 62 (2017)]

Theoretical Approach

Several groups are investigating such phenomena trying to understand the QGP's vortical structure using hydrodynamics with spin DoF.

[Florkowski et al.1811.04409, Rischke et al.2005.01506, Becattini et al.2103.14621]

Phenomenological Framework

Phenomenological spin-hydrodynamic framework [Hattori et al.1901.06615, Fukushima et al.2010.01608] is based on the conservation of the energy-momentum (EM) and total angular momentum (TAM) tensors

$$\partial_\mu T^{\mu\nu}_{\text{ph}} = 0 \quad , \quad \partial_\mu J^{\mu\alpha\beta}_{\text{ph}} = 0. \tag{1}$$

It commonly uses a simplified form of the spin tensor

$$T^{\mu\nu}_{\text{ph}} = \varepsilon u^\mu u^\nu - p\Delta^{\mu\nu} + T^{\mu\nu}_{\text{ph}(1)} \quad , \quad S^{\mu\alpha\beta}_{\text{ph}} = u^\mu S^{\alpha\beta} + S^{\mu\alpha\beta}_{\text{ph}(1)}. \tag{2}$$

Its entropy current is of the form

$$\partial_\mu \mathcal{S}^\mu_{\text{ph}} = T^{\mu\nu}_{\text{ph}(1s)}(\partial_\mu \beta_\nu) + T^{\mu\nu}_{\text{ph}(1a)}(\partial_\mu \beta_\nu + 2\beta\omega_{\mu\nu}) + \mathcal{O}(\partial^3). \tag{3}$$

where $\omega_{\mu\nu}$ is the $\mathcal{O}(\partial^1)$. Using the 2^{nd} law of thermodynamics $\partial_\mu S^\mu_{ph} \geq 0$, we can find all dissipative currents with their corresponding transport coefficients.

Canonical Framework

One can argue using Noether's theorem for a Dirac field having the Poincare group as its group of symmetries, that the spin tensor is totally anti-symmetric. For that the cornerstone of the 2 formalisms is the same

$$\partial_\mu T^{\mu\nu}_{\text{can}} = 0 \quad , \quad \partial_\mu J^{\mu\alpha\beta}_{\text{can}} = 0. \tag{4}$$

but the spin tensor is now totally anti-symmetric

$$S^{\mu\alpha\beta}_{\text{can}} = u^\mu S^{\alpha\beta} - u^\alpha S^{\mu\beta} + u^\beta S^{\mu\alpha} + S^{\mu\alpha\beta}_{\text{can}(1)}. \tag{5}$$

The main goal in this approach is also to find the transport coefficients using. The entropy divergence now is of the form:

$$\partial_\mu \mathcal{S}^\mu_{\text{can}} = 2\beta\omega_{\alpha\beta}\left[T^{\alpha\beta}_{\text{can}(1a)} + \frac{1}{2}\partial_\mu(-u^\alpha S^{\mu\beta} + u^\beta S^{\mu\alpha})\right] + \partial_\mu\beta_\nu T^{\mu\nu}_{\text{can}(1a)} + \partial_\mu\beta_\nu T^{\mu\nu}_{\text{can}(1s)}. \tag{6}$$

Having this additional term, we've shown that the canonical framework is not a well-defined initial value problem for an arbitrary set of hydrodynamics T, u^μ and $\omega^{\mu\nu}$.

Results

Improved Canonical

Such a problem can be solved by a proper modification of the EM tensor resulting in an improved canonical framework

$$T^{\mu\nu}_{\rm can} = T^{\mu\nu}_{(0)} + T^{\mu\nu}_{{\rm can}(1s)} + T^{\mu\nu}_{{\rm can}(1a)} + \partial_\lambda(u^\nu S^{\mu\lambda}), \tag{7}$$

$$S^{\mu\alpha\beta}_{\rm can} = u^\mu S^{\alpha\beta} - u^\alpha S^{\mu\beta} + u^\beta S^{\mu\alpha} + S^{\mu\alpha\beta}_{{\rm can}(1)}. \tag{8}$$

where $\partial_\lambda(u^\nu S^{\mu\lambda})$ is not due to a pseudo-gauge transformation. The entropy-current then is of the form

$$\partial_\mu \widetilde{\mathcal{S}}^\mu_{\rm can} = \partial_\mu(\beta u_\nu) T^{\mu\nu}_{{\rm can}(1s)} + \left[\partial_\alpha(\beta u_\beta) + 2\beta\omega_{\alpha\beta}\right] T^{\alpha\beta}_{{\rm can}(1a)}, \tag{9}$$

which is already known to be positive. This means we can reproduce the dissipative currents along with the transport coefficients.

Results

One to One Correspondence

On top of that, we were able to show that it is possible to construct explicitly a pseudo-gauge transformation [Florkowski et al.1811.04409] leading directly from the improved canonical framework to the phenomenological one

$$T^{\mu\nu}_{ph} = \tilde{T}^{\mu\nu}_{can} + \frac{1}{2}\partial_\lambda(\Sigma^{\lambda\mu\nu} - \Sigma^{\mu\lambda\nu} - \Sigma^{\nu\lambda\mu}), \tag{10}$$

$$S^{\lambda\mu\nu}_{ph} = \tilde{S}^{\lambda\mu\nu}_{can} - \Sigma^{\lambda\mu\nu}, \tag{11}$$

$$\Sigma^{\lambda\mu\nu} = 2\Phi^{\lambda\mu\nu}_{can(0)} + \Sigma^{\lambda\mu\nu}_1, \tag{12}$$

$$\Phi^{\lambda\mu\nu}_{can(0)} = u^{[\mu} S^{\nu]\lambda}. \tag{13}$$

In addition, starting from the phenomenological formulation we can re-obtain the improved canonical framework.

Recap

Recap

To stay on the same track till further advancements, we will just recap what we have done until now:

1. We've discussed the already used spin hydrodynamics formalism by other colleagues

$$T^{\mu\nu}_{\text{ph}} = \varepsilon u^\mu u^\nu - p\Delta^{\mu\nu} + T^{\mu\nu}_{\text{ph}(1)}, \tag{14}$$

$$S^{\mu\alpha\beta}_{\text{ph}} = u^\mu S^{\alpha\beta} + S^{\mu\alpha\beta}_{\text{ph}(1)}. \tag{15}$$

2. Introduced the canonical framework and showed how it is different from the phenomenological one

$$T^{\mu\nu}_{\text{can}} = \varepsilon u^\mu u^\nu - p\Delta^{\mu\nu} + T^{\mu\nu}_{\text{can}(1)}, \tag{16}$$

$$S^{\mu\alpha\beta}_{\text{can}} = u^\mu S^{\alpha\beta} - u^\alpha S^{\mu\beta} + u^\beta S^{\mu\alpha} + S^{\mu\alpha\beta}_{\text{can}(1)}. \tag{17}$$

Recap

Recap

1. Then we showed that there is a problem with canonical entropy current, and we proposed the improved canonical framework as a solution

$$T^{\mu\nu}_{\rm can} = T^{\mu\nu}_{(0)} + T^{\mu\nu}_{{\rm can}(1s)} + T^{\mu\nu}_{{\rm can}(1a)} + \partial_\lambda(u^\nu S^{\mu\lambda}), \tag{18}$$

$$S^{\mu\alpha\beta}_{\rm can} = u^\mu S^{\alpha\beta} - u^\alpha S^{\mu\beta} + u^\beta S^{\mu\alpha} + S^{\mu\alpha\beta}_{{\rm can}(1)}. \tag{19}$$

2. Finally, we've illustrated how one can go back and forth between the improved canonical and the phenomenological formalism using a proper pseudo-gauge.

Thank You!