Constraining the critical temperature for color superconductivity in heavy-ion collisions from neutron star phenomenology

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21.06.2022

OUTLINE

- Introduction
- Equation of State (EoS)
- Constructing a hybrid EoS I
- Preliminary results I
- Nambu-Jona-Lasinio (NJL) model
- Constructing a hybrid EoS II
- Preliminary results II
- Conclusion

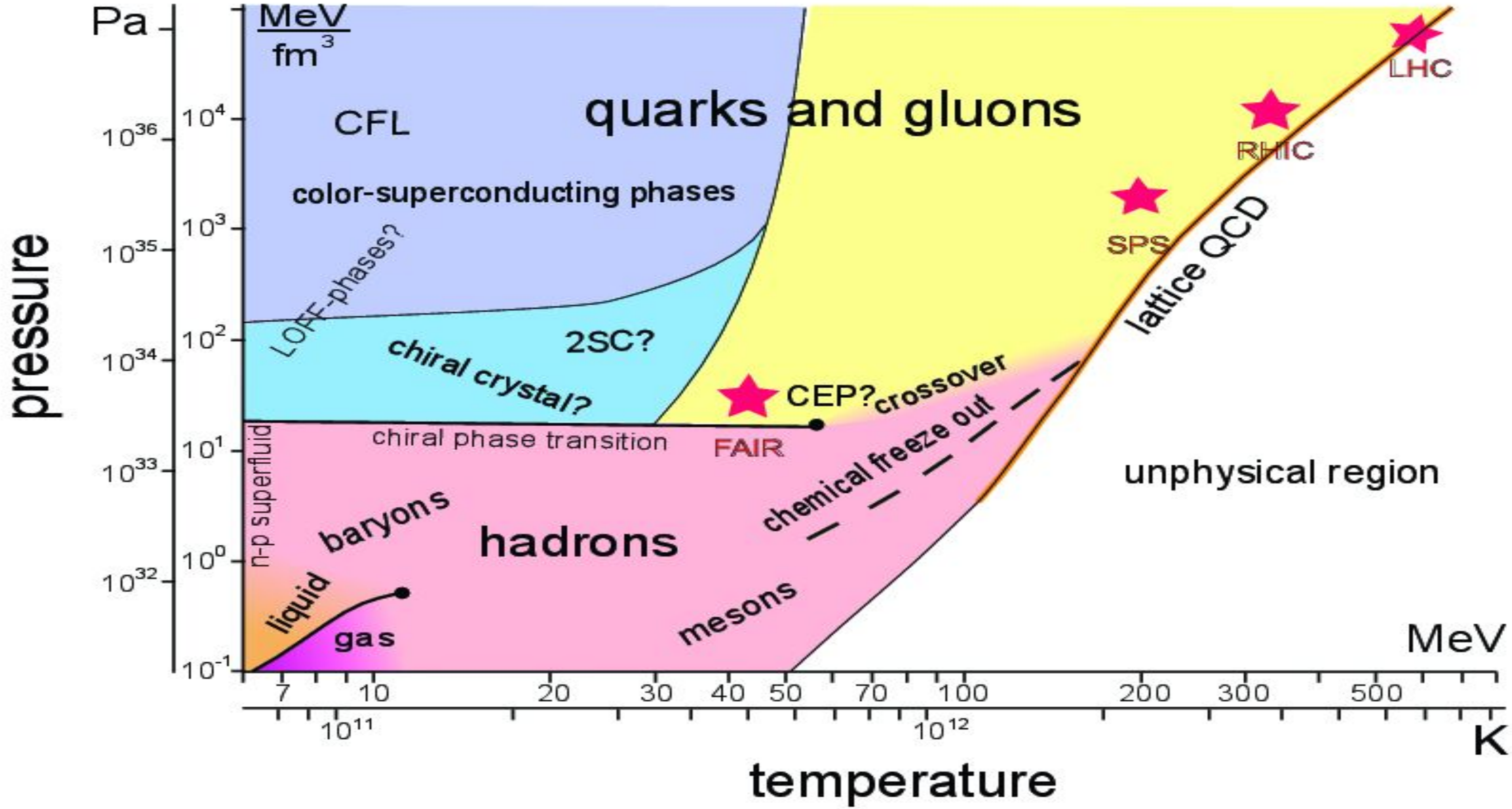
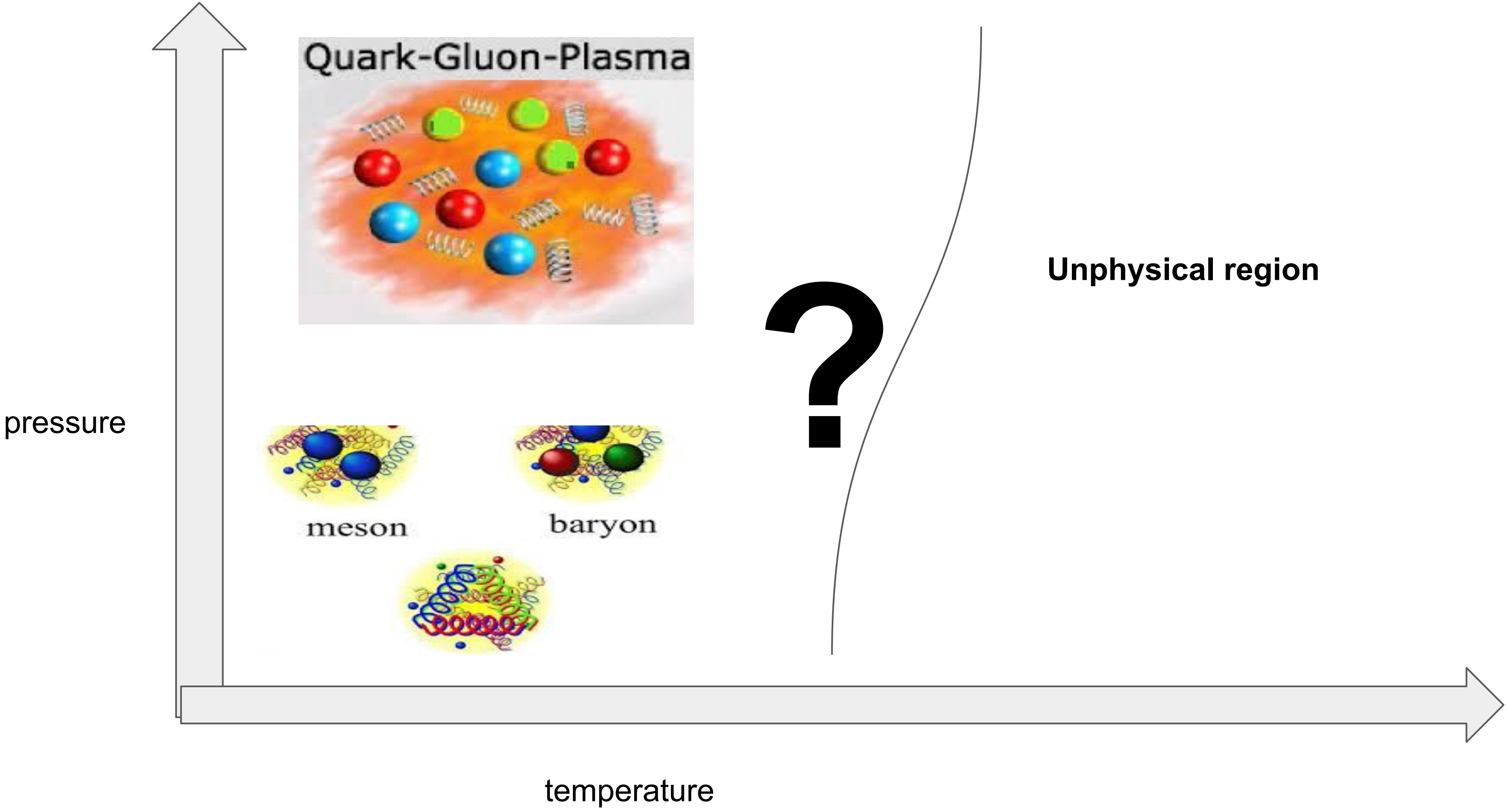


Figure courtesy: K. Heckmann (TU Darmstadt, 2012)

OPEN QUESTION



ADOPTED PRESCRIPTION

- M.Albright, J. Kapusta, C. Young, Phys. Rev. C 90, 024915 (2014)
- J. Kapusta, T. Welle, Phs. Rev. C 104, L012801 (2021)

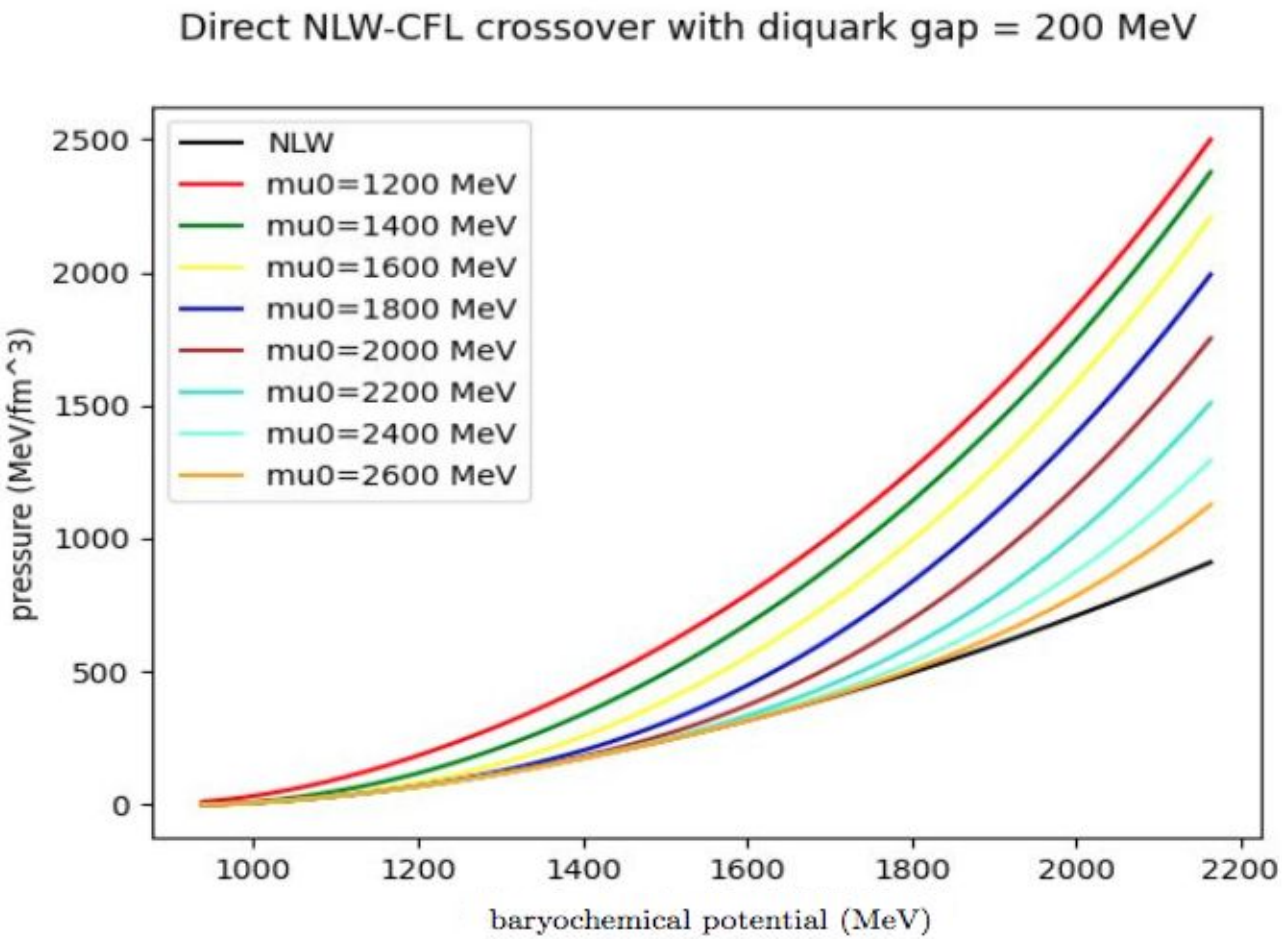
SWITCHING FUNCTION B/W HADRONS & QGP

$$S(\mu) = \exp\left\{-\left[\left(\frac{T}{T_0}\right)^r + \left(\frac{\mu}{\mu_0}\right)^r\right]^{-1}\right\}$$

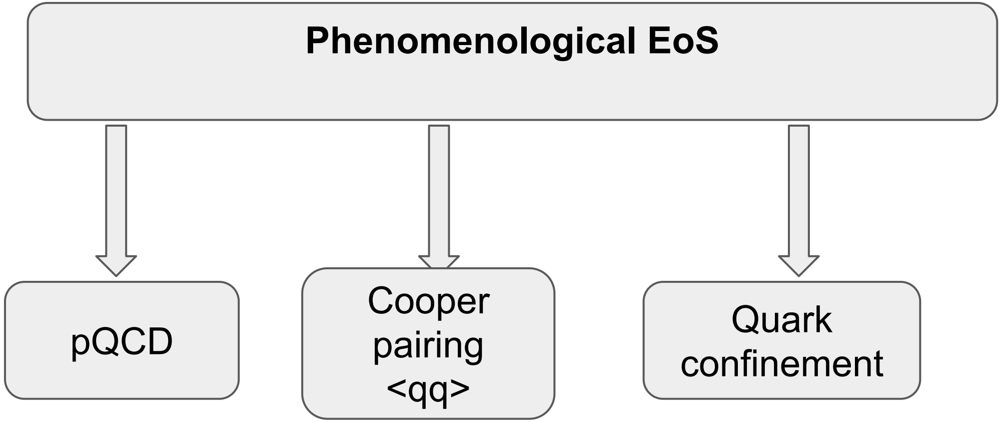
$$P(\mu, T) = S(\mu, T)P_q(\mu, T) + \left[1 - S(\mu, T)\right]P_h(\mu, T)$$

Unified EoS...?

PRELIMINARY RESULTS - I



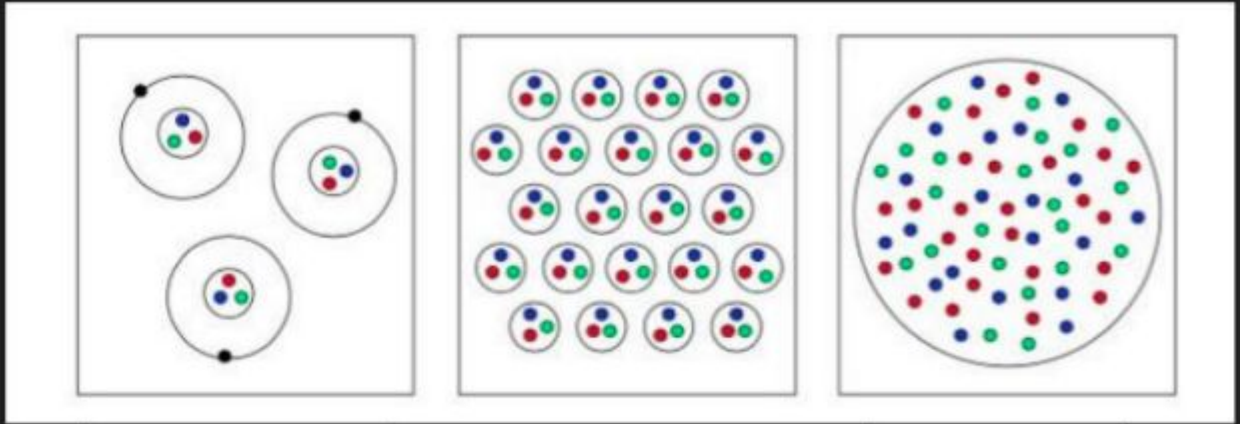
NLW = Non-linear Walecka



$$P_q(\mu) = \frac{3}{4\pi^2} a_4 \left(\frac{\mu}{3}\right)^4 + \frac{3}{\pi^2}\Delta^2 \left(\frac{\mu}{3}\right)^2 - B_{\rm eff}$$

C. Zhang, R. Mann, Phys.Rev.D 103 (2021), 6

M. Alford, M. Braby, M. Paris, S.Reddy, *Astrophys.J.* 629 (2005) 969-978



$$V_e(r) \sim \left\{ \frac{e_0^2}{r} \right\}$$

$$V_q(r) \sim r$$

Confinement? Asymptotic freedom

Lagrangian... Symmetries?

Degrees of freedom---> quarks, diquarks, multi-quark clusters...?

Book: The Physics of the Quark-Gluon Plasma, Edited by Sourav Sarkar, Helmut Satz, Bikash Sinha

Nambu-Jona-Lasinio (SU(2))

(with vector meson & diquark interactions)

D. Blaschke, M. Buballa, et. al, *Annals Phys.* 348 (2014) 228-255

Given (full) Lagrangian -

$$\mathscr{L} = \mathscr{L}_0 + \mathscr{L}_S + \mathscr{L}_V + \mathscr{L}_D$$

$$\mathscr{L}_0 = \bar{q}(i\partial\!\!\!/ - m_0 + \mu\gamma_0)q$$

$$\mathscr{L}_S = G_S[(\bar{q}q)^2 + (\bar{q}i\gamma_5\vec{\tau}q)^2]$$

$$\mathscr{L}_V = -G_V(\bar{q}\gamma_\mu q)^2$$

$$\mathscr{L}_D = G_D \sum_{A=2,5,7} (\bar{q}i\gamma_5\tau_2\lambda_A q^C)(\bar{q}^C i\gamma_5\tau_2\lambda_A q)$$

Partition function -

$$\mathscr{Z} = \int \mathscr{D}q\mathscr{D}\bar{q}\exp\left(\int d^4x_E\mathscr{L}\right)$$

Introducing Nambu-Gorkov bispinors $\Rightarrow \Psi \equiv \frac{1}{\sqrt{2}}\begin{pmatrix} q \\ q^C \end{pmatrix}$, and, $\bar{\Psi} \equiv \frac{1}{\sqrt{2}}\begin{pmatrix} \bar{q} & \bar{q}^C \end{pmatrix}$

And the charge-conjugation of a Dirac spinor (in position space) as -

$q_c(x) \equiv C\bar{q}^T(x)$

where, $C \equiv i\gamma^2\gamma^0$

THERMODYNAMIC POTENTIAL

$$\Downarrow$$

$$\Omega(T,\mu) = -\frac{T}{V}\ln\mathscr{Z}(T,\mu),$$

$$
\begin{aligned}
\Omega^{MFA} = & -\frac{T}{V}\ln \mathcal{Z}^{MFA} = \frac{\bar{\sigma}^2}{2G_S} + \frac{\bar{\Delta}^2}{2G_D} - \frac{\bar{\omega}^2}{2G_V} \\
& -2\int \frac{d^3\vec{p}}{(2\pi)^3}\,\xi(\vec{p}),
\end{aligned}
$$

where

$$
\begin{aligned}
\xi(\vec{p}) \quad & = \sum_{\kappa,s=\pm} 2\left\{\epsilon_r^\kappa/2 + T\ln\left[1 + e^{-\frac{\epsilon_r^\kappa + s\ \delta\tilde{\mu}_r}{T}}\right]\right\} \\
& + \sum_{\kappa,s=\pm}\left\{\bar{E}_b^\kappa/2 + T\ln\left[1 + e^{-\frac{\bar{E}_b^\kappa + s\ \delta\tilde{\mu}_b}{T}}\right]\right\}
\end{aligned}
$$

Gap equations

$$\bar{\sigma} = 2G_s \int \frac{d^3\vec{p}}{(2\pi)^3} \frac{g(\vec{p})\ M(\vec{p})}{E} \times \sum_{\kappa} \left\{ 1 - n_F(\frac{\bar{E}_b^\kappa + \delta\tilde{\mu}_b}{T}) - n_F(\frac{\bar{E}_b^\kappa - \delta\tilde{\mu}_b}{T}) + \frac{2\bar{E}_r^\kappa}{\epsilon_r^\kappa} \left[1 - n_F(\frac{\epsilon_r^\kappa + \delta\tilde{\mu}_r}{T}) - n_F(\frac{\epsilon_r^\kappa - \delta\tilde{\mu}_r}{T}) \right] \right\},$$

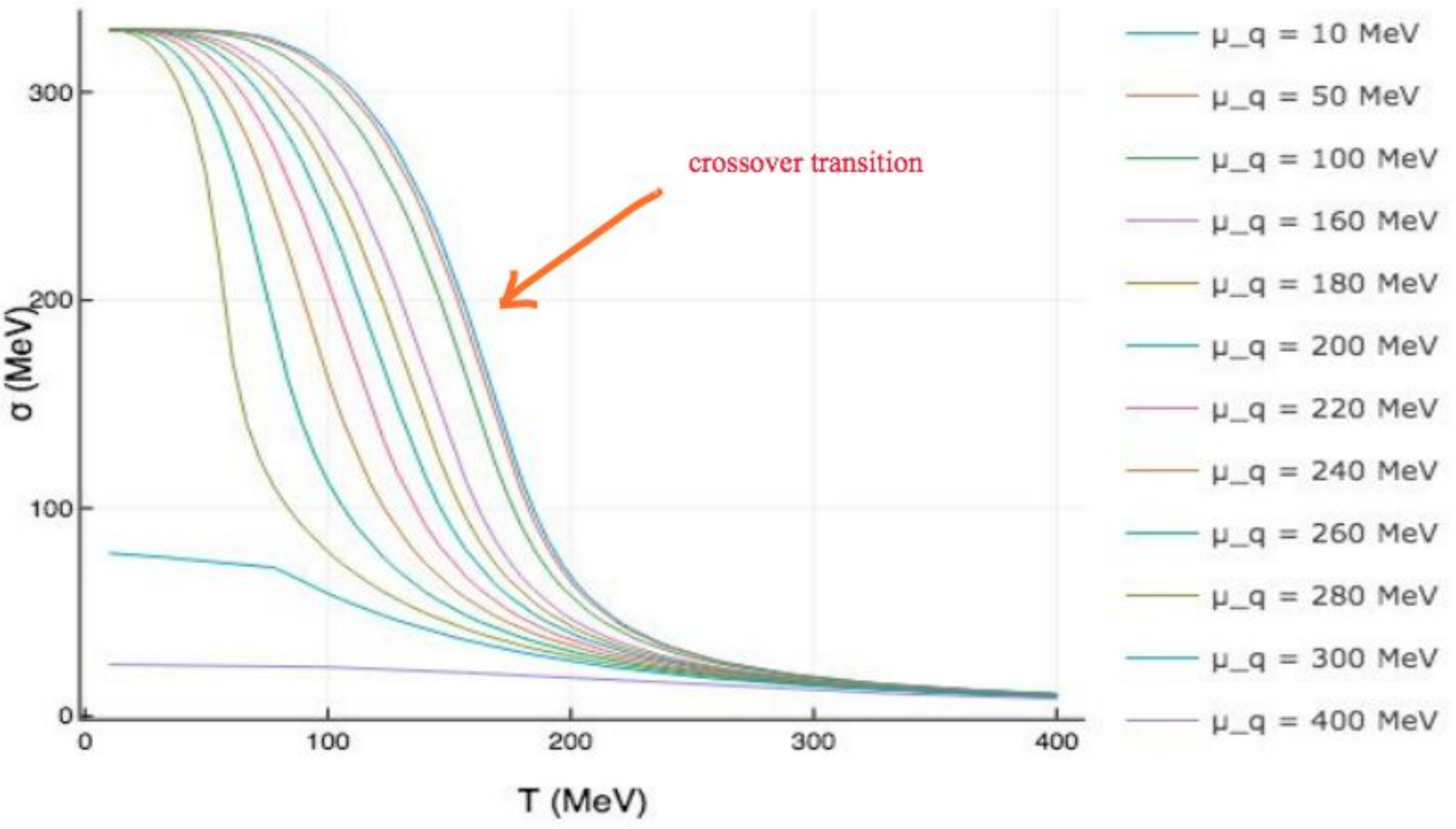
$$\bar{\Delta} = 2G_D \bar{\Delta} \int \frac{d^3\vec{p}}{(2\pi)^3} g^2(\vec{p}) \times \sum_{\kappa=\pm} \left\{ \frac{2}{\epsilon_r^\kappa} \left[1 - n_F(\frac{\epsilon_r^\kappa + \delta\tilde{\mu}_r}{T}) - n_F(\frac{\epsilon_r^\kappa - \delta\tilde{\mu}_r}{T}) \right] \right\},$$

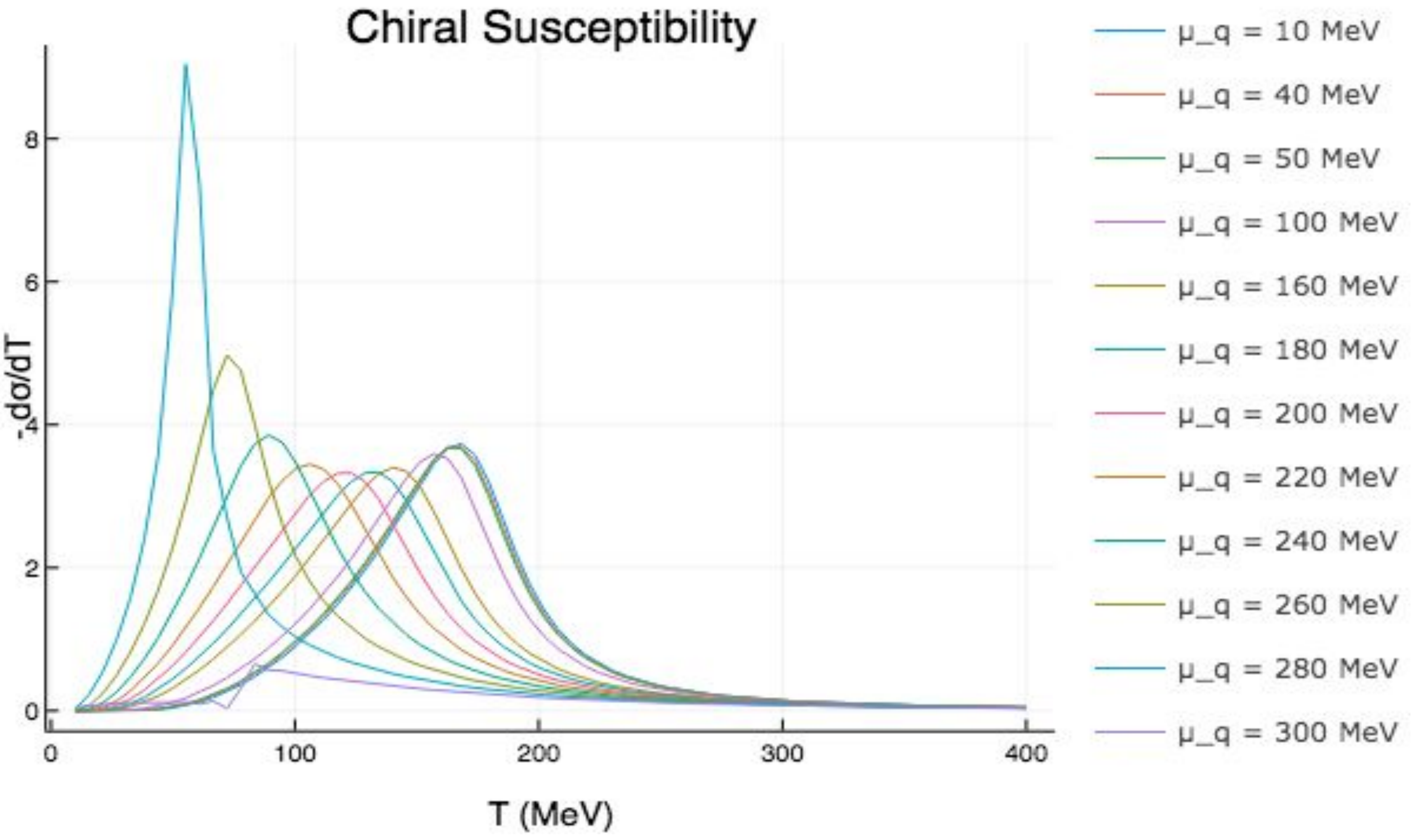
and

$$\bar{\omega} = 2G_V \int \frac{d^3\vec{p}}{(2\pi)^3} \times \sum_{\kappa=\pm} \kappa \left\{ 1 - n_F(\frac{\bar{E}_b^\kappa + \delta\tilde{\mu}_b}{T}) - n_F(\frac{\bar{E}_b^\kappa - \delta\tilde{\mu}_b}{T}) + \frac{2\ \bar{E}_r^\kappa}{\epsilon_r^\kappa} \left[1 - n_F(\frac{\epsilon_r^\kappa + \delta\tilde{\mu}_r}{T}) - n_F(\frac{\epsilon_r^\kappa - \delta\tilde{\mu}_r}{T}) \right] \right\},$$

PRELIMINARY RESULTS - II

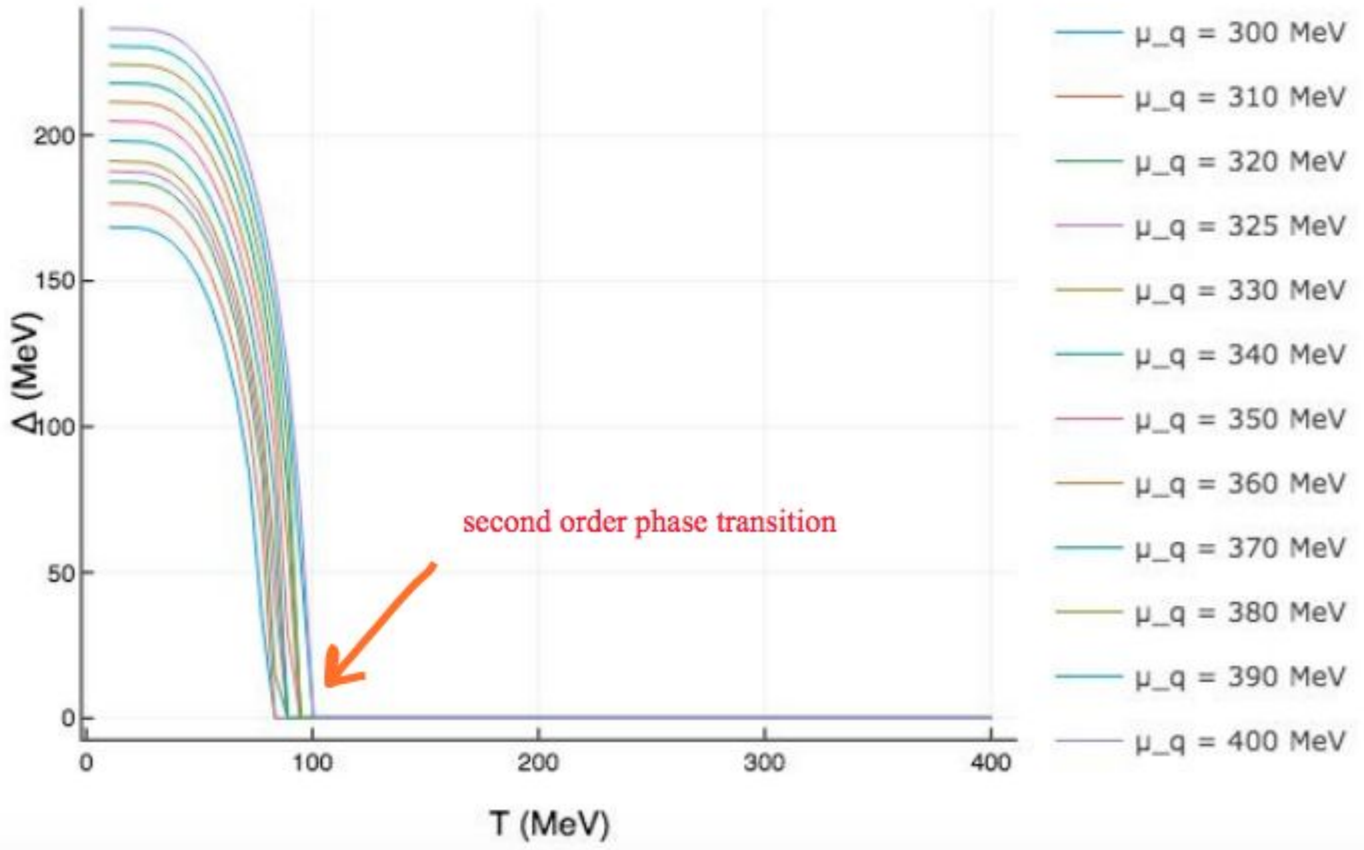
quark mass gap



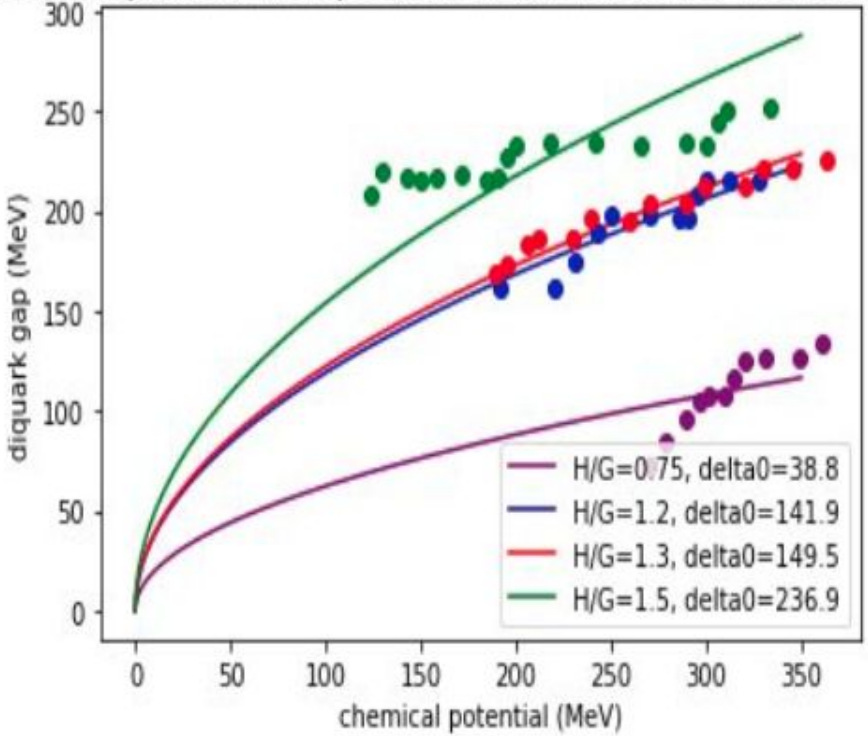


Chiral Susceptibility
-dσ/dT
T (MeV)
0
2
4
6
8
0
100
200
300
400
μ_q = 10 MeV
μ_q = 40 MeV
μ_q = 50 MeV
μ_q = 100 MeV
μ_q = 160 MeV
μ_q = 180 MeV
μ_q = 200 MeV
μ_q = 220 MeV
μ_q = 240 MeV
μ_q = 260 MeV
μ_q = 280 MeV
μ_q = 300 MeV

diquark pairing gap



Diquark gap as a function of quark chemical potential for different values of delta0 (fit to local NJL model data)



DIQUARK GAP ANSATZ ⟹ $$\Delta(\mu) = \sqrt{\Delta_0 \mu / 3}$$

COMPRESSIBILITY ⟹ **SPEED OF SOUND** ⟹ **STIFFNESS OF EoS**

⇓

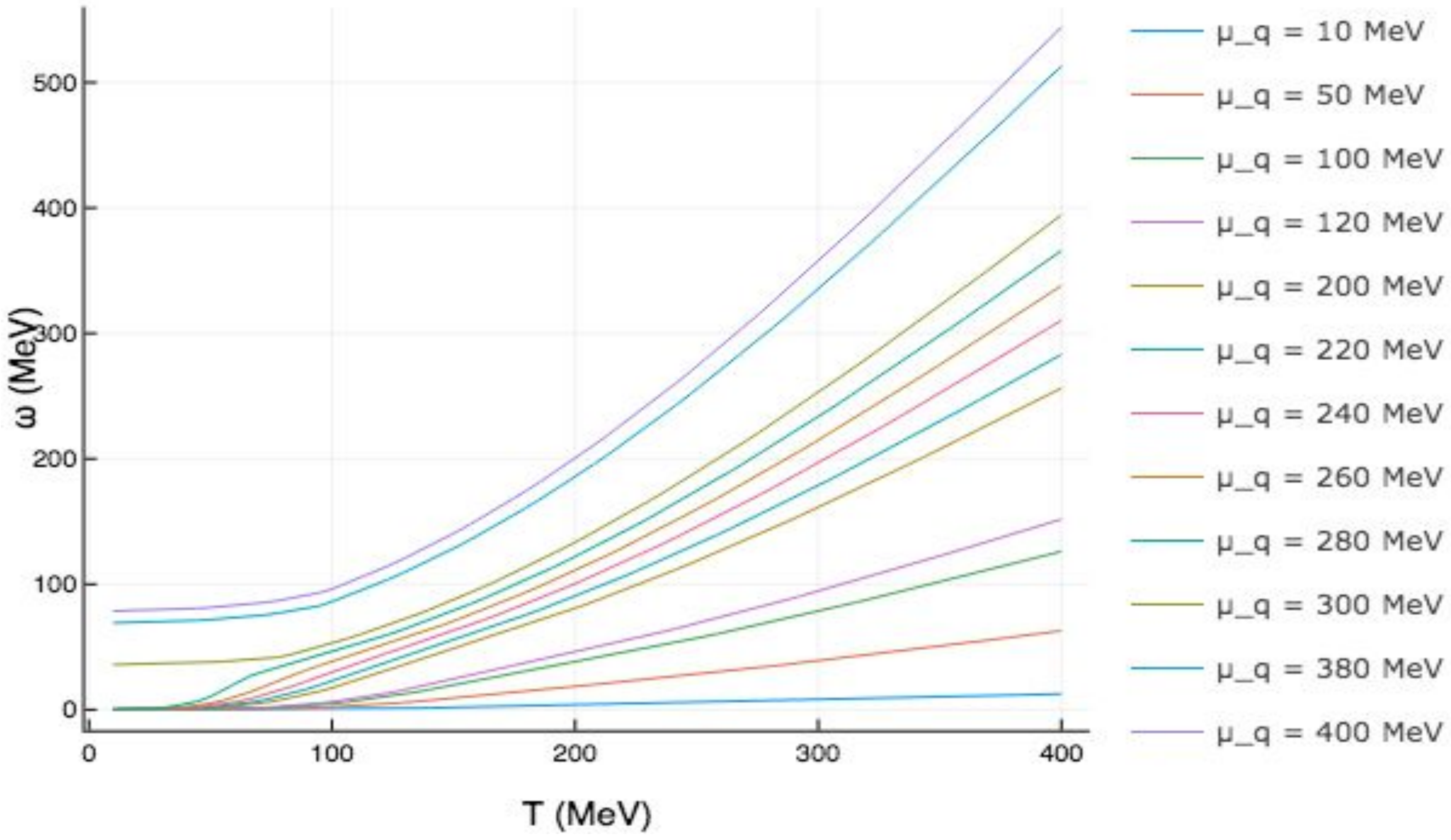
$$P_q(\mu) = \frac{3}{4\pi^2} a_4 \left(\frac{\mu}{3}\right)^4 + \frac{3}{\pi^2} \Delta^2 \left(\frac{\mu}{3}\right)^2 - B_{\rm eff}$$

⇓

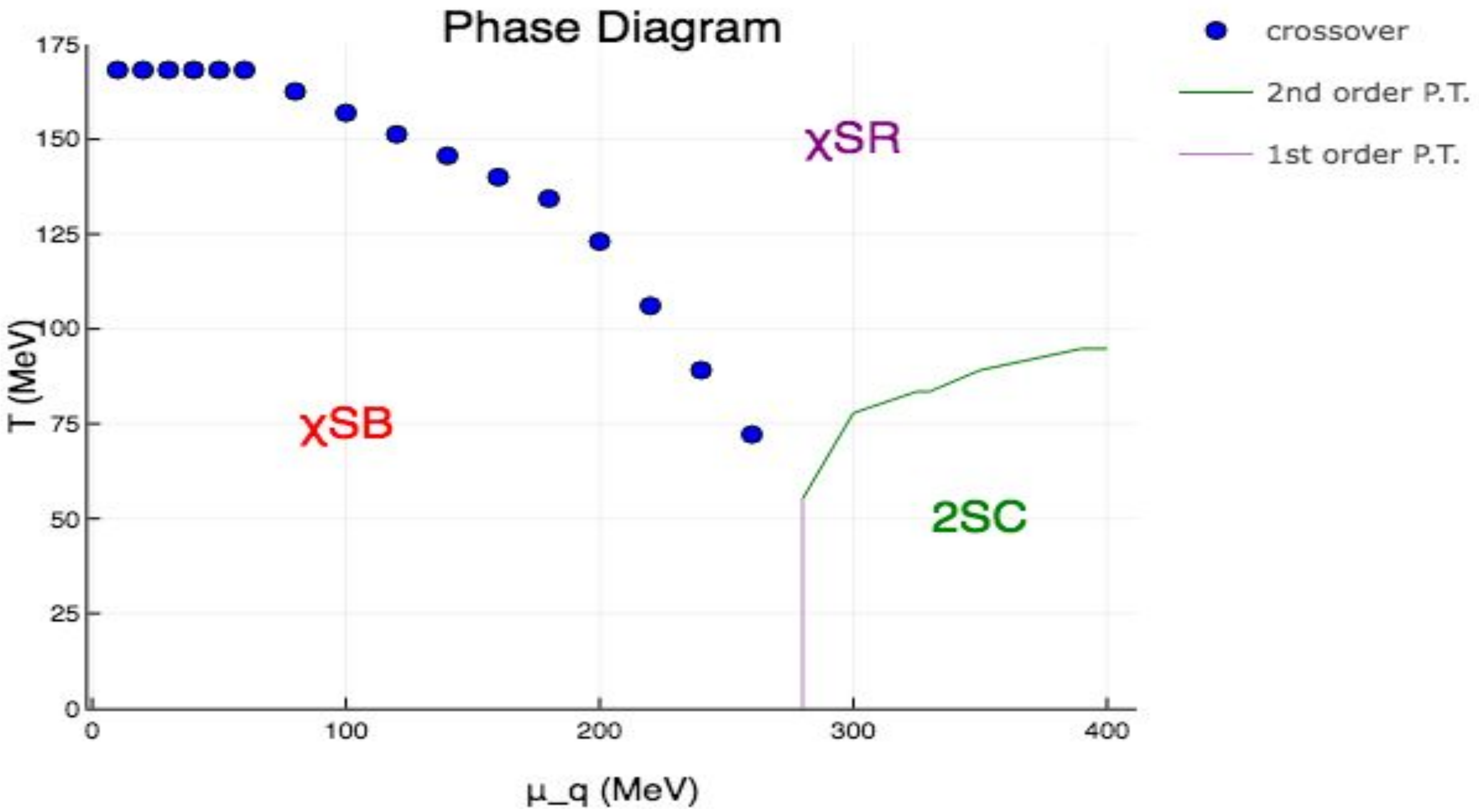
ANSATZ ⟹ $$\Delta(\mu) = \sqrt{\Delta_0 \mu / 3}$$

⇓

$$P_q(\mu) = \frac{3}{4\pi^2} a_4 \left(\frac{\mu}{3}\right)^4 + \frac{3}{\pi^2} \Delta_0 \left(\frac{\mu}{3}\right)^3 - B_{\rm eff}$$



ω (MeV)
T (MeV)
0
100
200
300
400
500
μ_q = 10 MeV
μ_q = 50 MeV
μ_q = 100 MeV
μ_q = 120 MeV
μ_q = 200 MeV
μ_q = 220 MeV
μ_q = 240 MeV
μ_q = 260 MeV
μ_q = 280 MeV
μ_q = 300 MeV
μ_q = 380 MeV
μ_q = 400 MeV



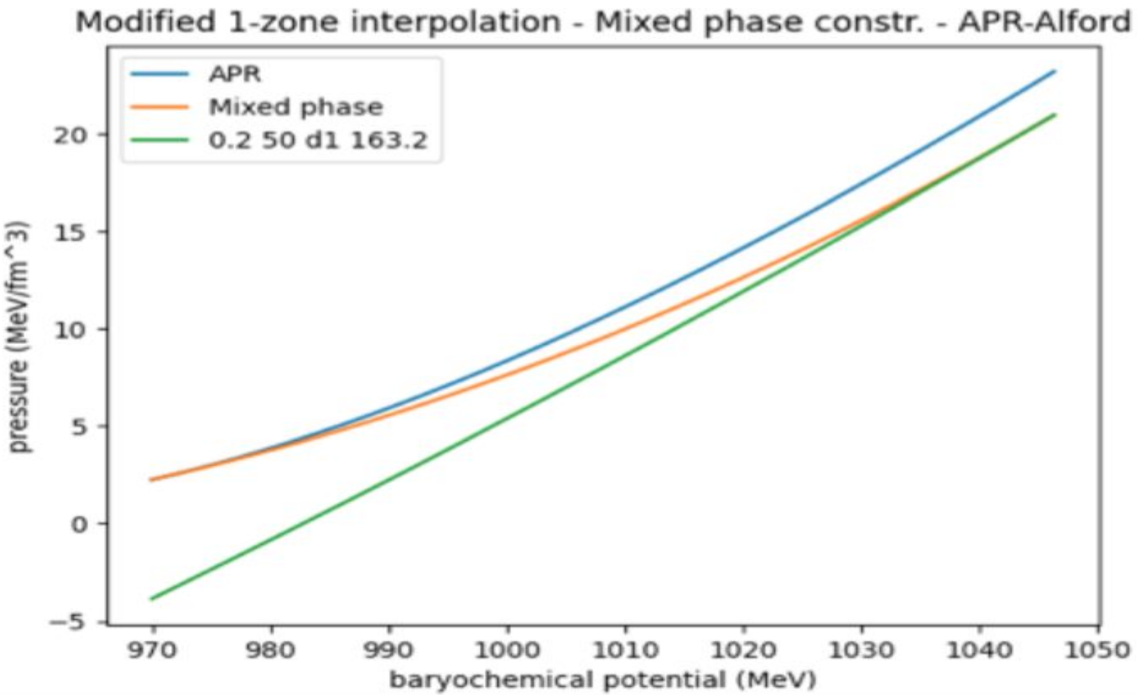
Phase Diagram
crossover
2nd order P.T.
1st order P.T.
χSR
χSB
2SC
T (MeV)
μ_q (MeV)
0
25
50
75
100
125
150
175
0
100
200
300
400

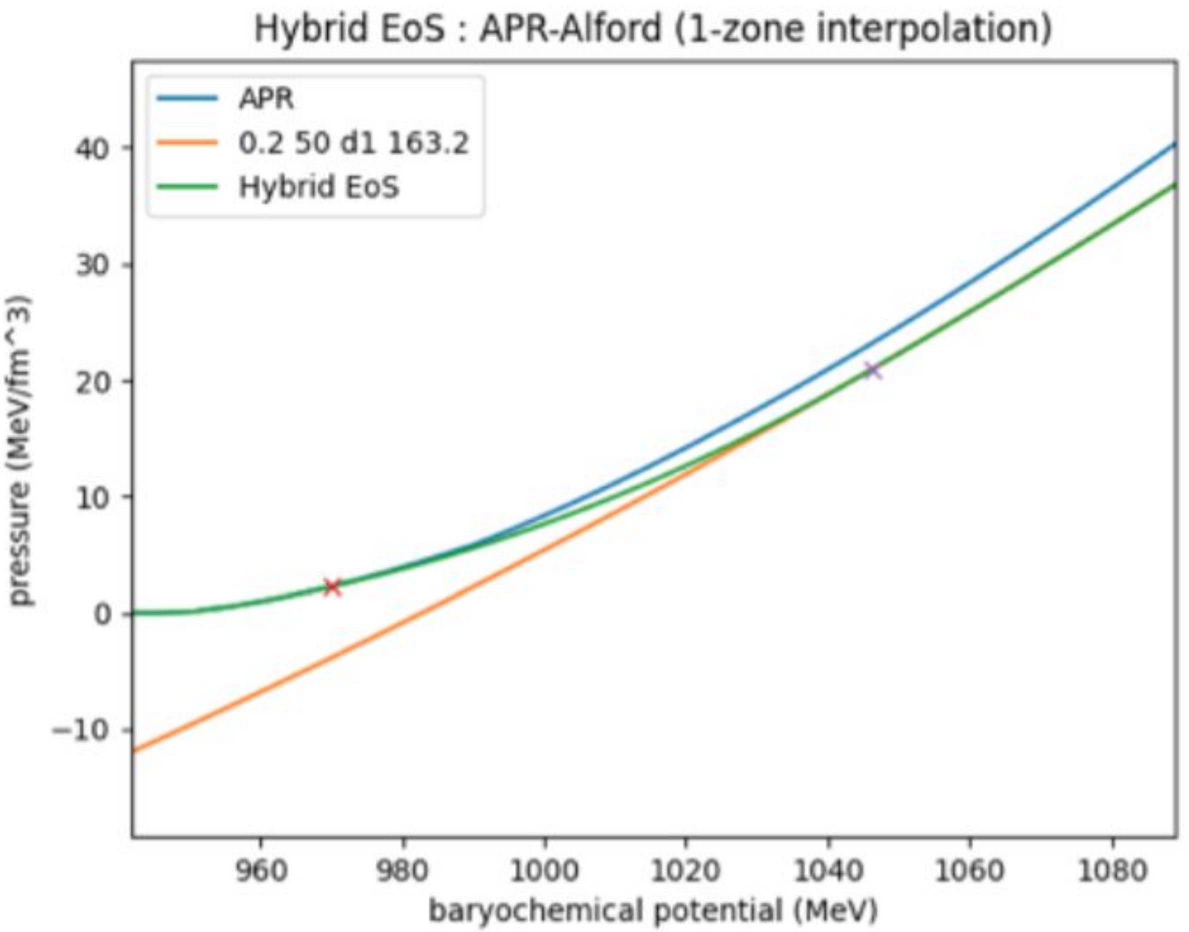
Mixed phase ansatz - A*mu^2 + B*mu + C

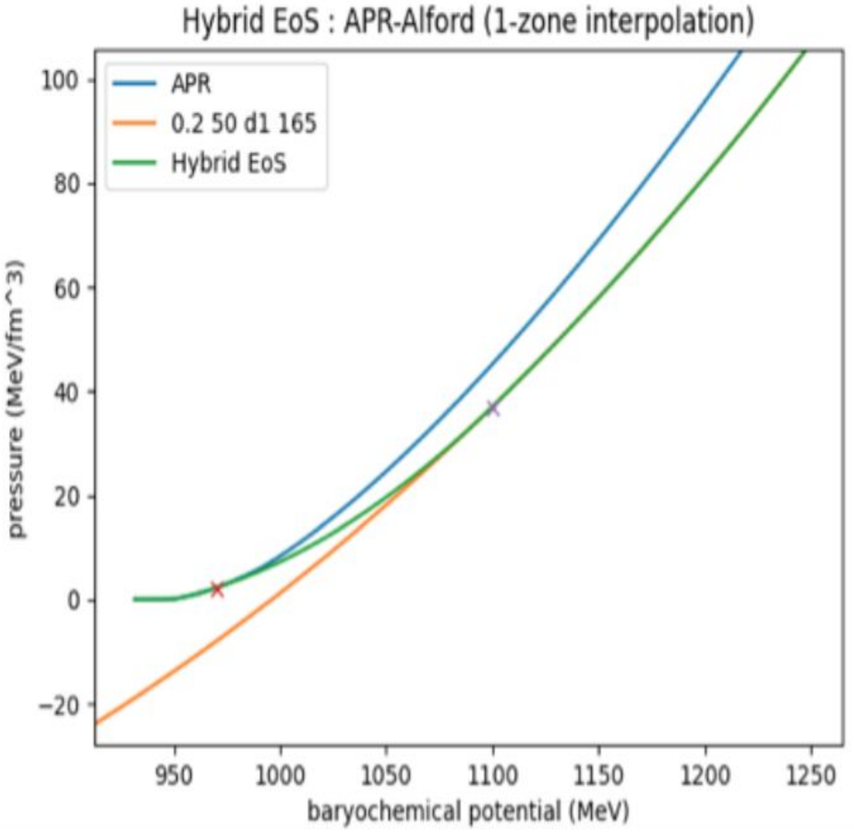
mu_H —> **fixed at saturation density** = 970 MeV

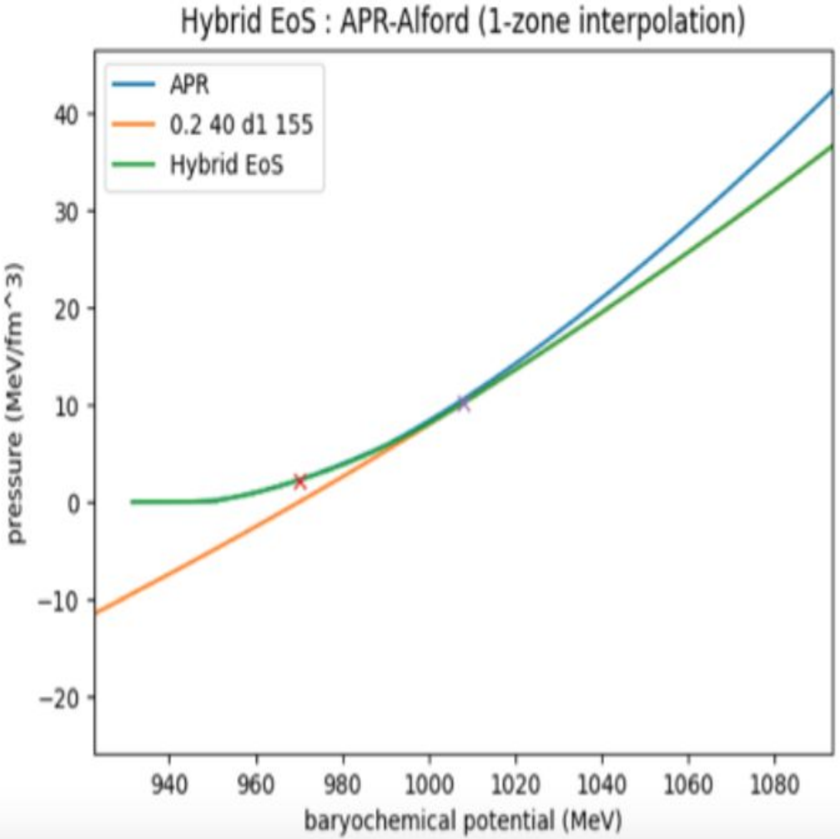
Unknowns - mu_Q, A, B, C (solved via four continuity equations)

1) Quark EoS (Alford) - 0.2 50 d1 163.2 - (no crossing in the acceptable range of mu)




Hybrid EoS : APR-Alford (1-zone interpolation)
APR
0.2 50 d1 163.2
Hybrid EoS
pressure (MeV/fm^3)
40
30
20
10
0
−10
960
980
1000
1020
1040
1060
1080
baryochemical potential (MeV)


Hybrid EoS : APR-Alford (1-zone interpolation)
APR
0.2 50 d1 165
Hybrid EoS
pressure (MeV/fm^3)
baryochemical potential (MeV)
100
80
60
40
20
0
−20
950
1000
1050
1100
1150
1200
1250


Hybrid EoS : APR-Alford (1-zone interpolation)
APR
0.2 40 d1 155
Hybrid EoS
pressure (MeV/fm^3)
baryochemical potential (MeV)
40
30
20
10
0
−10
−20
940
960
980
1000
1020
1040
1060
1080

Mass-Radius (observational constraint)

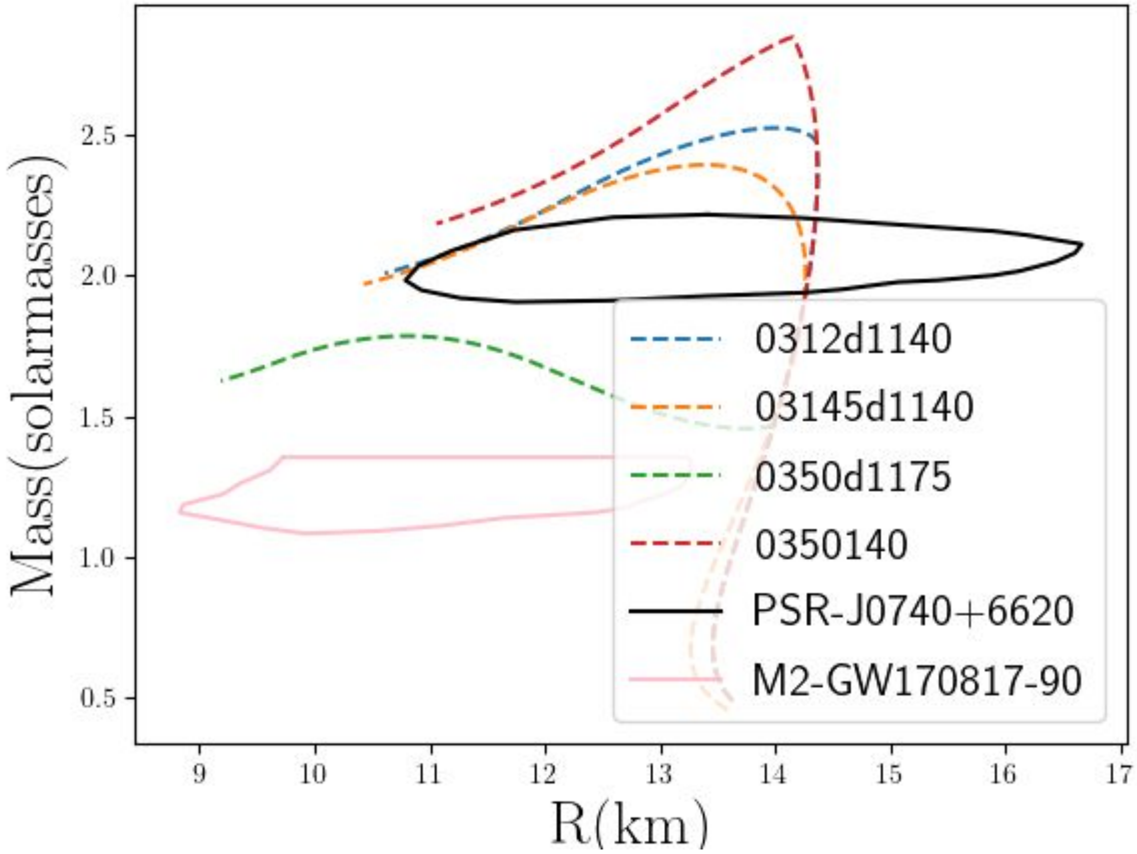


Figure courtesy : S. Liebing

Tidal Deformability (observational constraint)

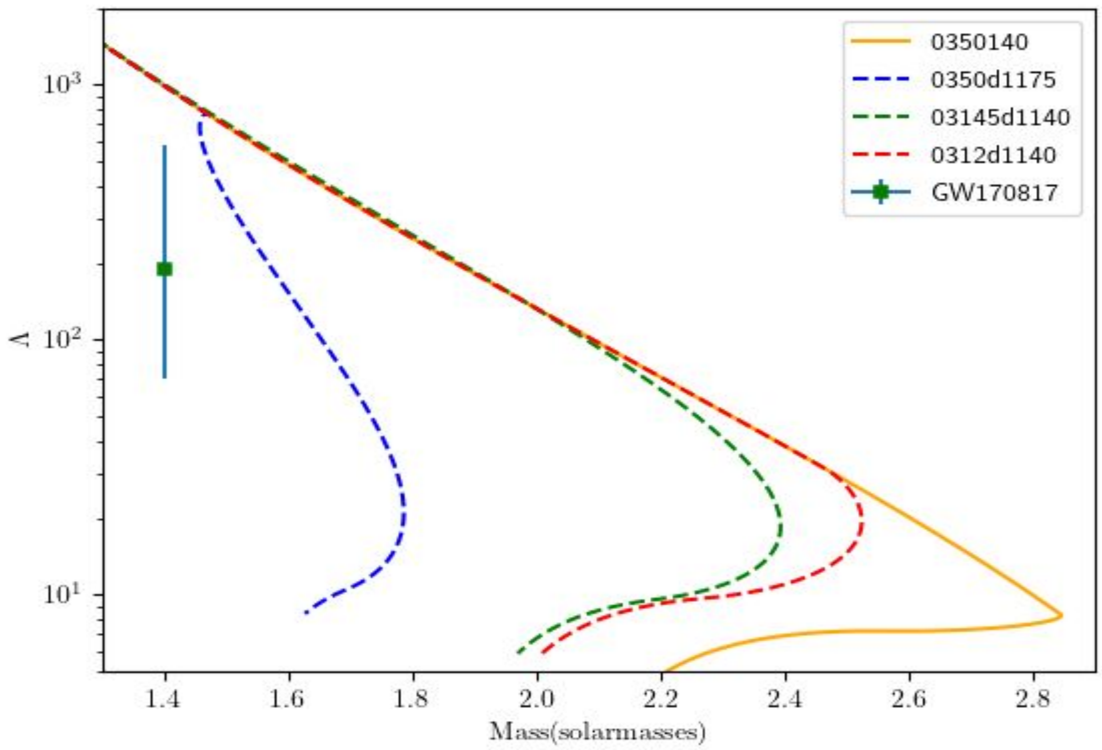


Figure courtesy : S. Liebing

OUTLOOK

- D. Blaschke, M. Buballa, A. Dubinin, G. Ropke, D. Zablocki , Annals of Physics, 348 (2014)
- D. Blaschke, D. Zablocki, Phys. Part. Nucl. 39.1010 (2008)

NJL → PNJL → CONFINEMENT

- M.A. Kaltenborn, N.U.F. Bastian, D. Blaschke, Phys. Rev, D, 96, 056024, (2017)

Relativistic Density Functional (RDF) → String-flip potential model (**2nd order)** → CONFINEMENT

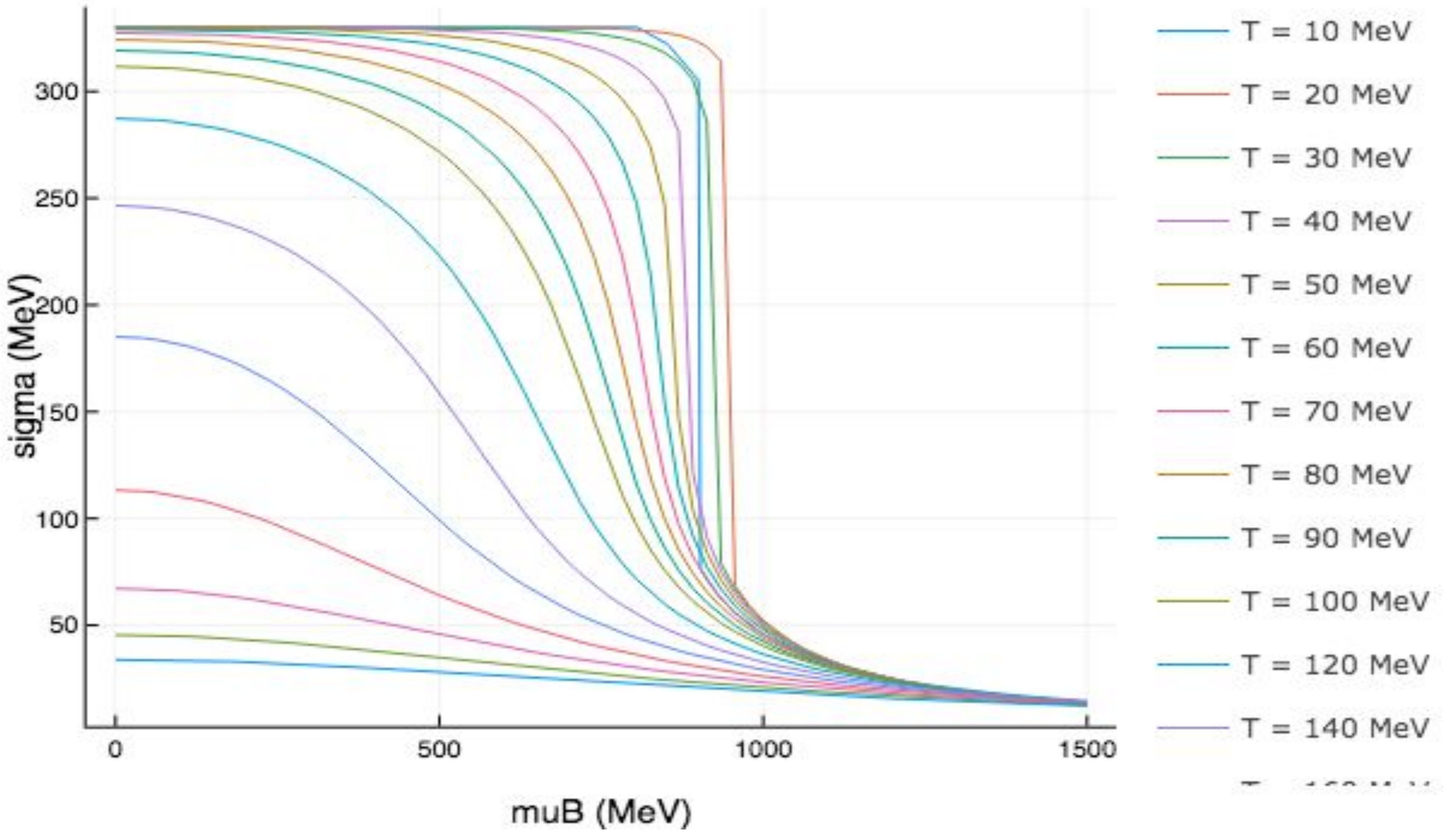
- M.Albright, J. Kapusta, C. Young, Phys. Rev. C 90, 024915 (2014)
- J. Kapusta, T. Welle, Phs. Rev. C 104, L012801 (2021)
- Including better npEoS , eg. density-dependent couplings by S. Typel

Unified EoS

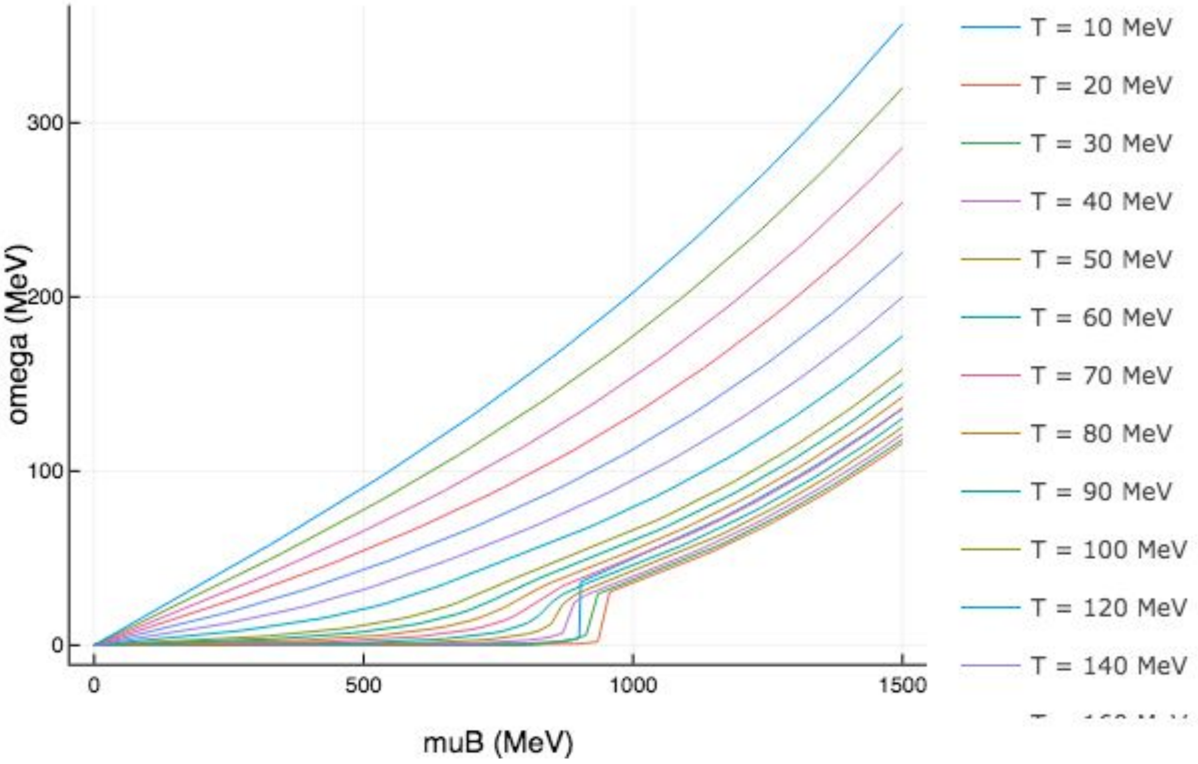
THANK YOU FOR YOUR ATTENTION!

BACKUP SLIDES

3DFF NJL



300
250
200
150
100
50
sigma (MeV)
0
500
1000
1500
muB (MeV)
T = 10 MeV
T = 20 MeV
T = 30 MeV
T = 40 MeV
T = 50 MeV
T = 60 MeV
T = 70 MeV
T = 80 MeV
T = 90 MeV
T = 100 MeV
T = 120 MeV
T = 140 MeV


omega (MeV)
0
100
200
300
0
500
1000
1500
muB (MeV)
T = 10 MeV
T = 20 MeV
T = 30 MeV
T = 40 MeV
T = 50 MeV
T = 60 MeV
T = 70 MeV
T = 80 MeV
T = 90 MeV
T = 100 MeV
T = 120 MeV
T = 140 MeV