Equation of state of strongly interacting matter for hydrodynamical simulations of heavy-ion collisions

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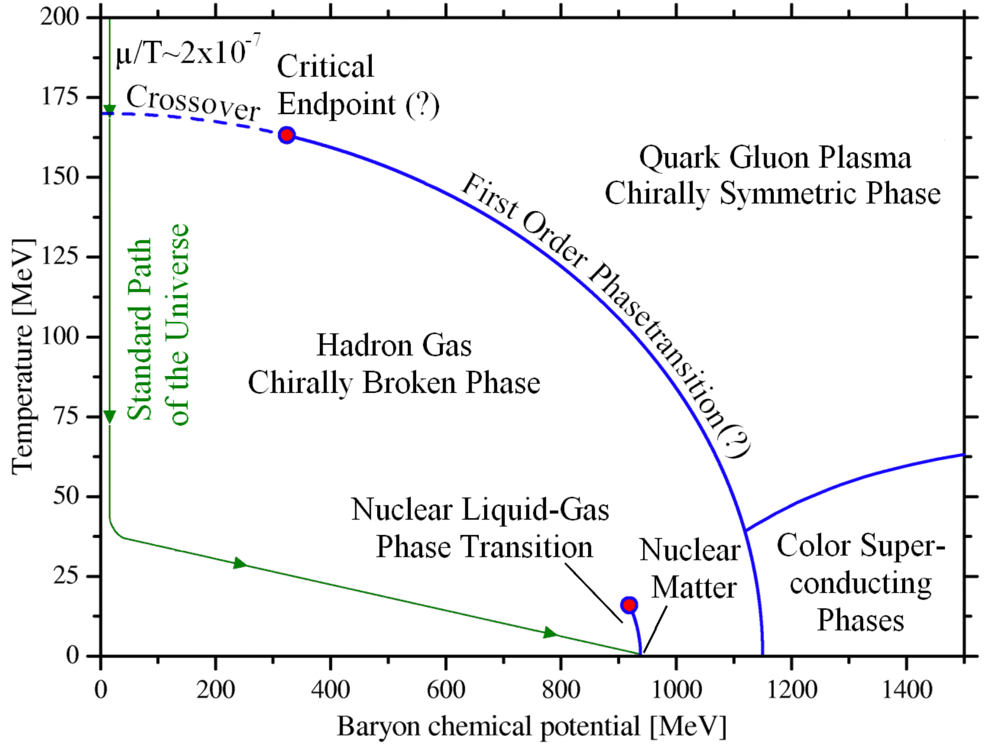
58 Karpacz Winter School of Theoretical Physics, Poland

Uniwersytet Wrocławski

Introduction



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Exploring of the QCD phase diagram:

- Detect signals of deconfinement PT
- Detect signals of (partial) chiral symmetry restoration
- Locate (tri)critical endpoint(s) if such exists

[Figures: Universe 4 (2018) 52 & PoS (KMI 2013) 025]

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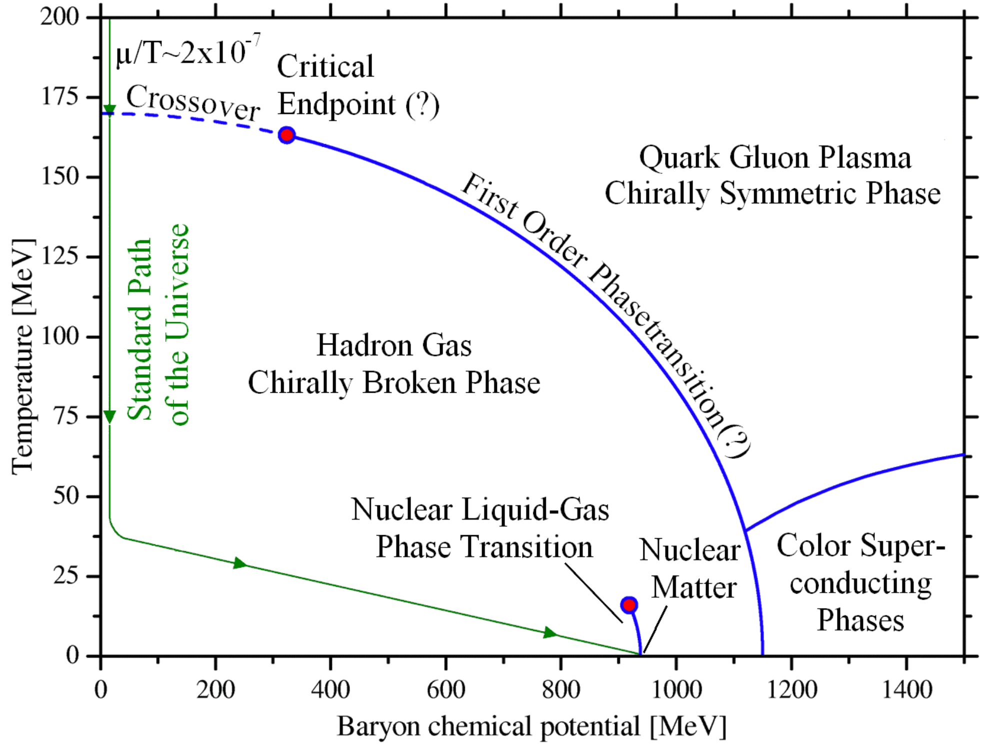

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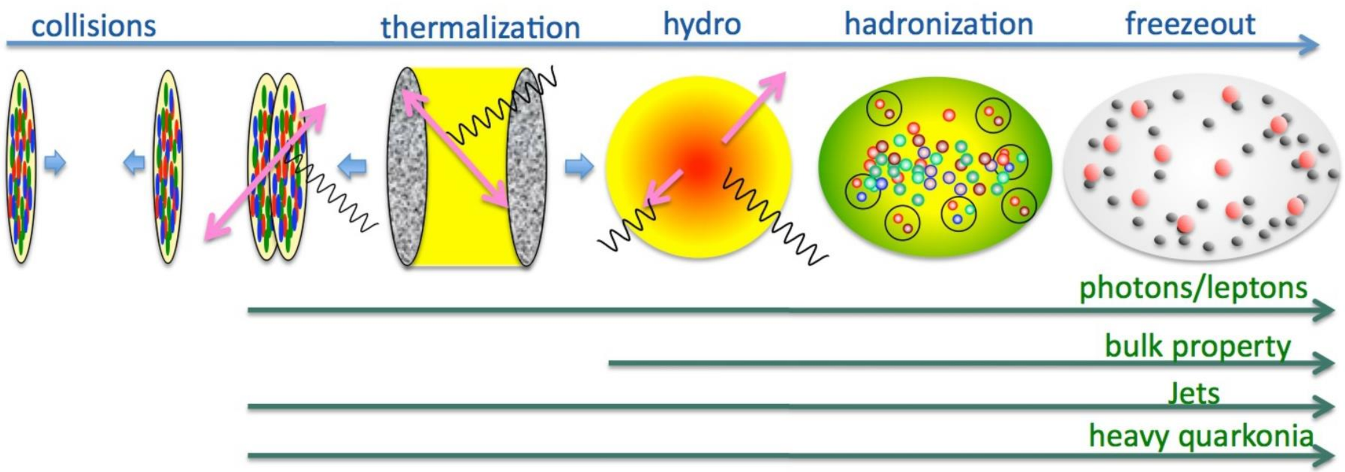

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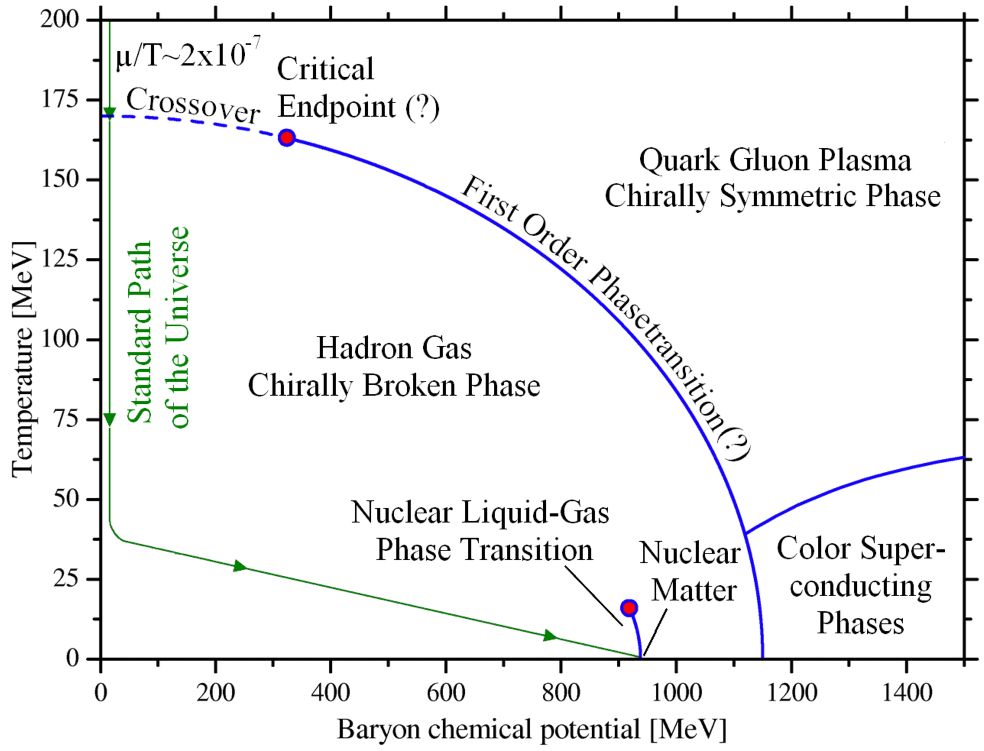

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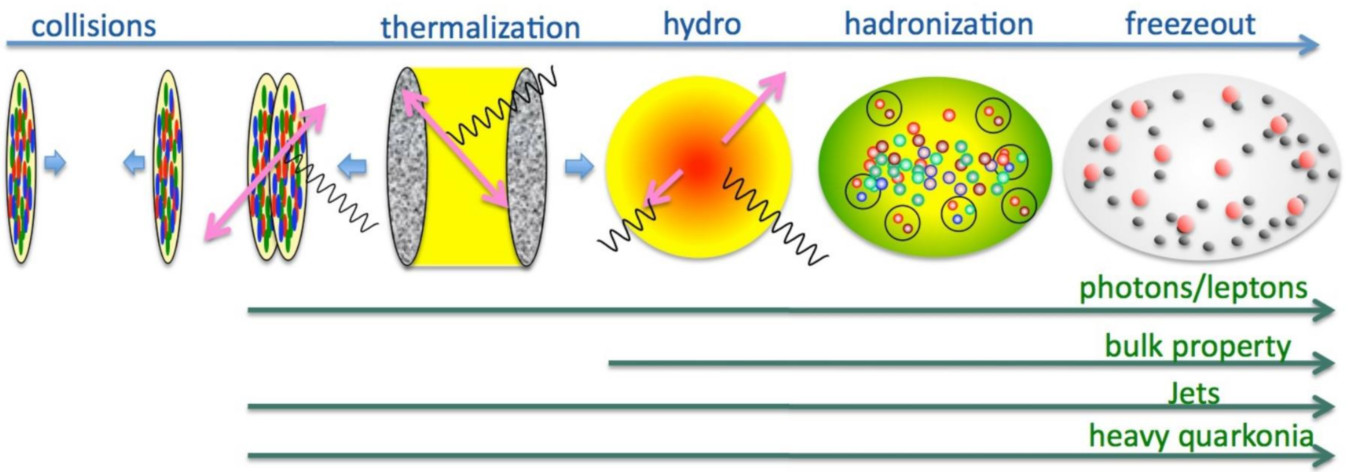

In order to resolve these tasks we need a very good observables and tools to analyse the data!

[Figures: Universe 4 (2018) 52 & PoS (KMI 2013) 025]

Hybrid Models & EoS

Physical Input into Hydro:
- Initial state
- Equation of State
- Transport coefficients
- Freezeout

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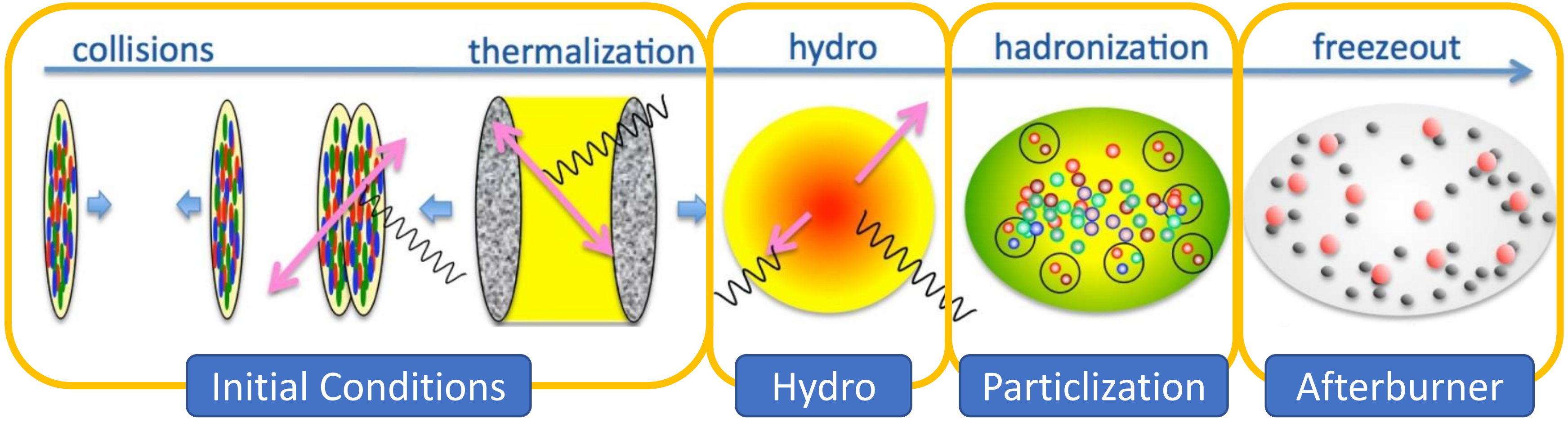

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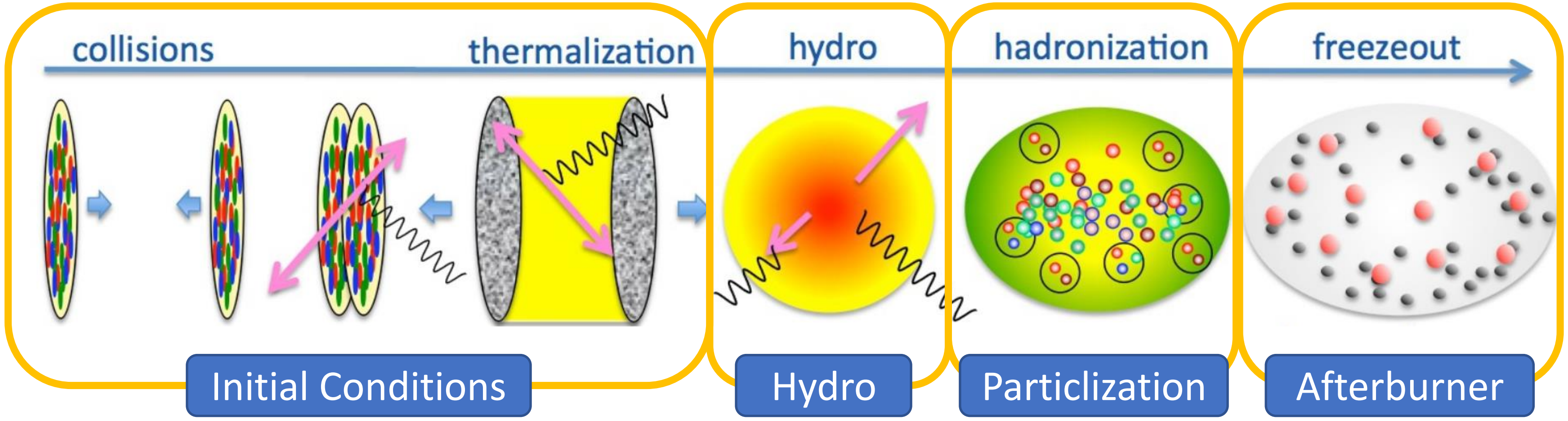

EoS
- Is needed to build closed system of equations
- Contains all the information about the thermodynamic properties of the system that affect its evolution
- Modeling heavy-ion collisions with different EoS, one can perform a-posteriori analyses of the QCD matter properties in the unsolvable regions of the phase diagram

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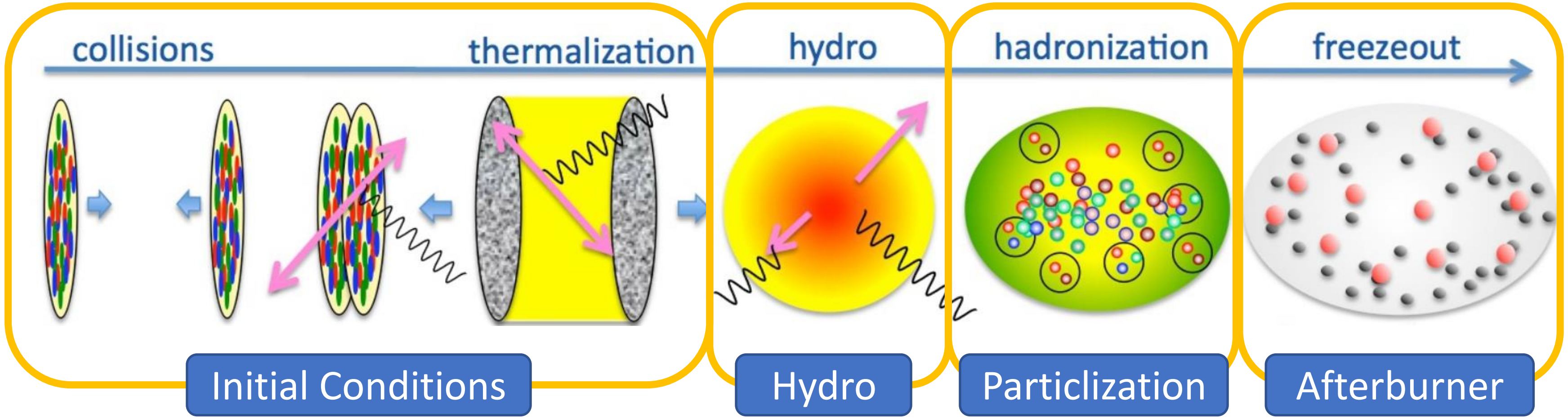

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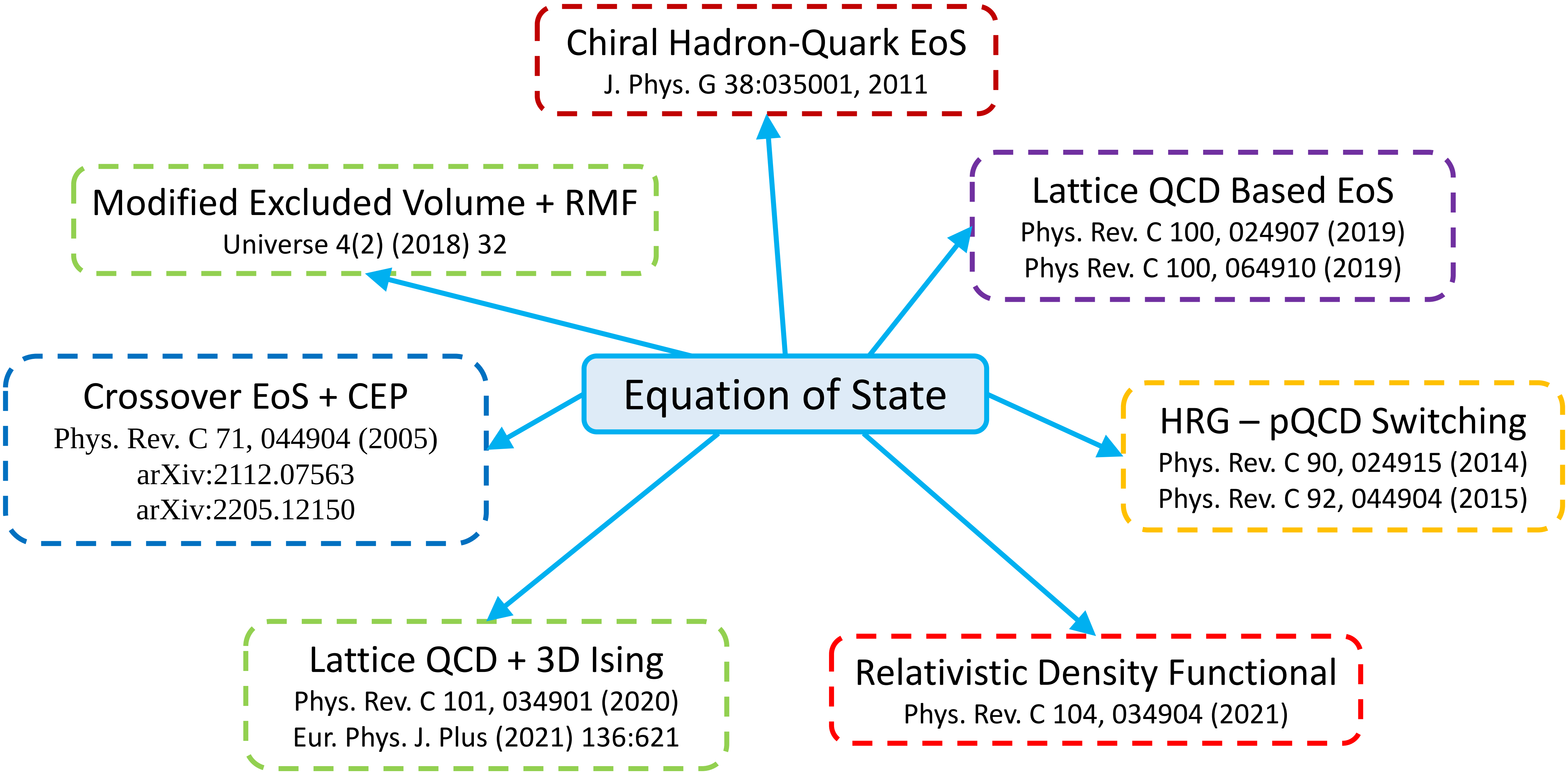
Construction of the family of equation of states with and without 1st order PT → Heavy-ion collision simulations; Computation of observables and comparison with existing data; Model calibration & constraints on EoS → Prediction for basic observables for future experiments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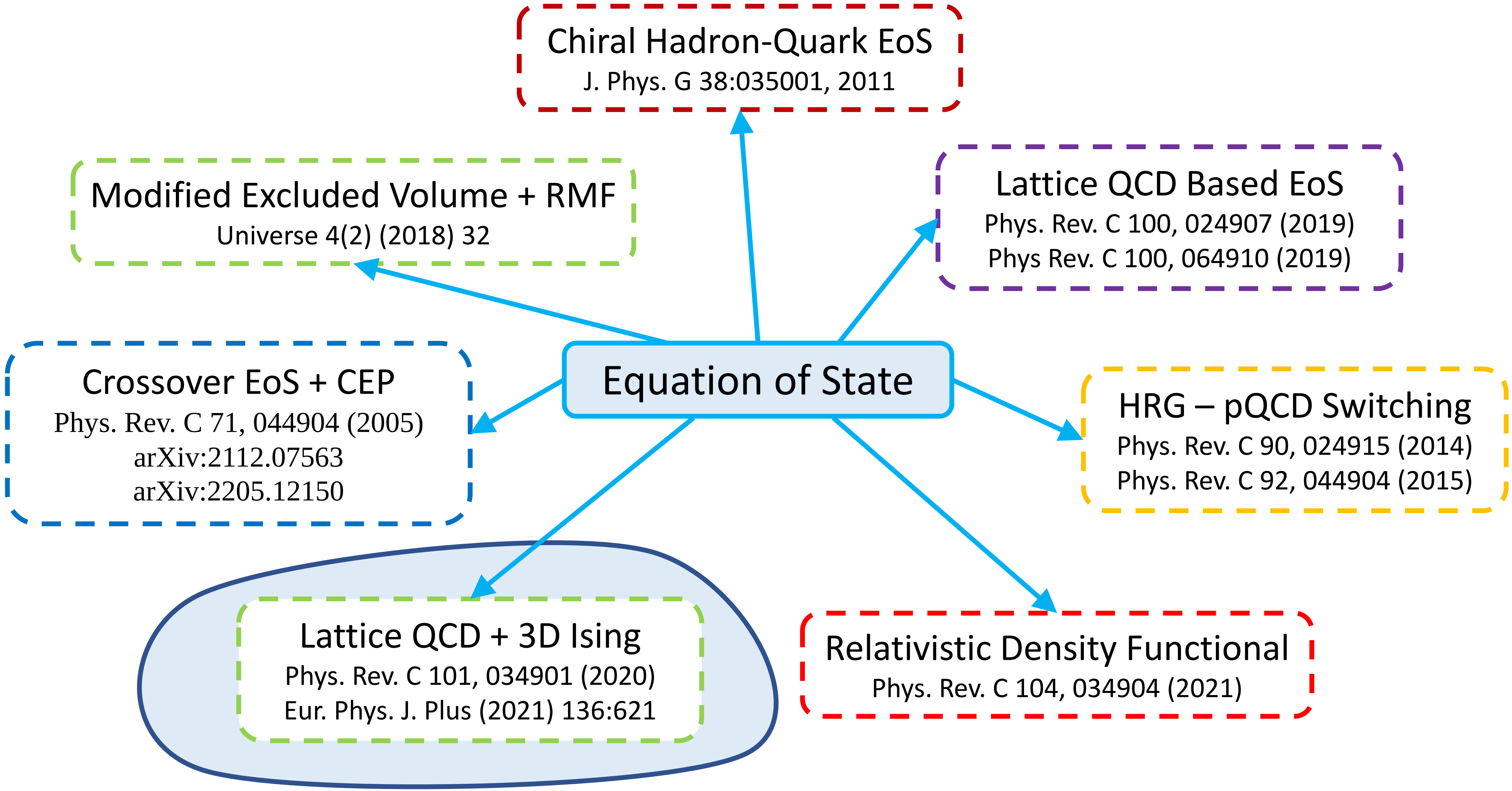

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General form: $P(T,\mu_B) = P_0(T,\mu_B) + F(T,\mu_B)G(T,\mu_B)$

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Parametric representation of 3D Ising EoS

[Nucl. Phys. B 489 (1997) 626]

$M = M_0 R^{\beta}\theta$

$h = h_0 R^{\beta\delta}\theta(1 + a\theta^2 + b\theta^4)$

$r = R(1-\theta^2)$

$G = -F_{Sing}(R,\theta) = h_0 M_0 R^{2-\alpha}\tilde{g}(\theta)$

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One more mapping is needed!

$(R,\theta) \rightarrow (r,h) \leftarrow (T,\mu_B)$

$T = T_c + a_{11} r + a_{12} h$

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$$T = \frac{1}{2}\big(2T_c(h+1) - \mu_c(1-r)\big)$$
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$$T = \frac{T_{pc}}{c}(h+1)\sqrt{c^2 - \mu_{pc}^2(1-r)^2}$$
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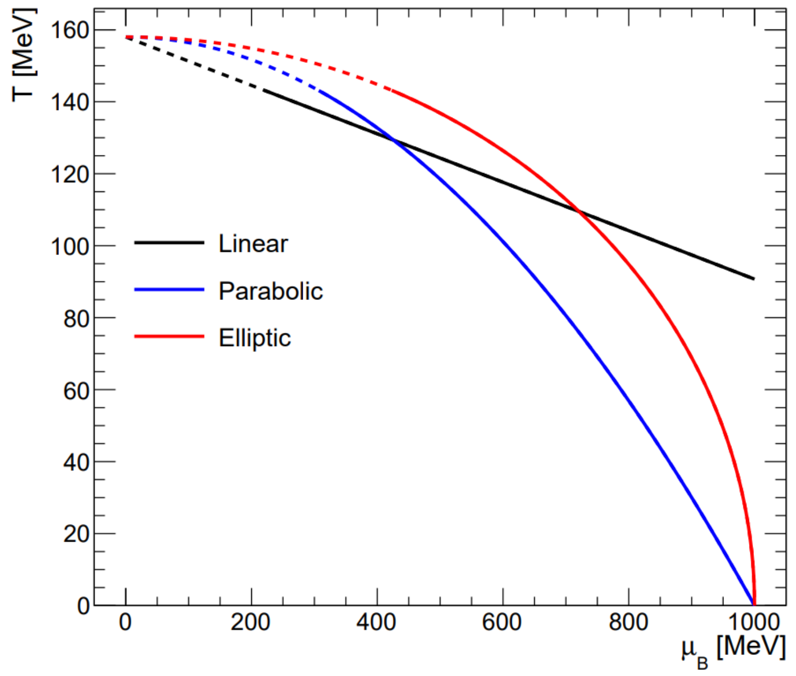

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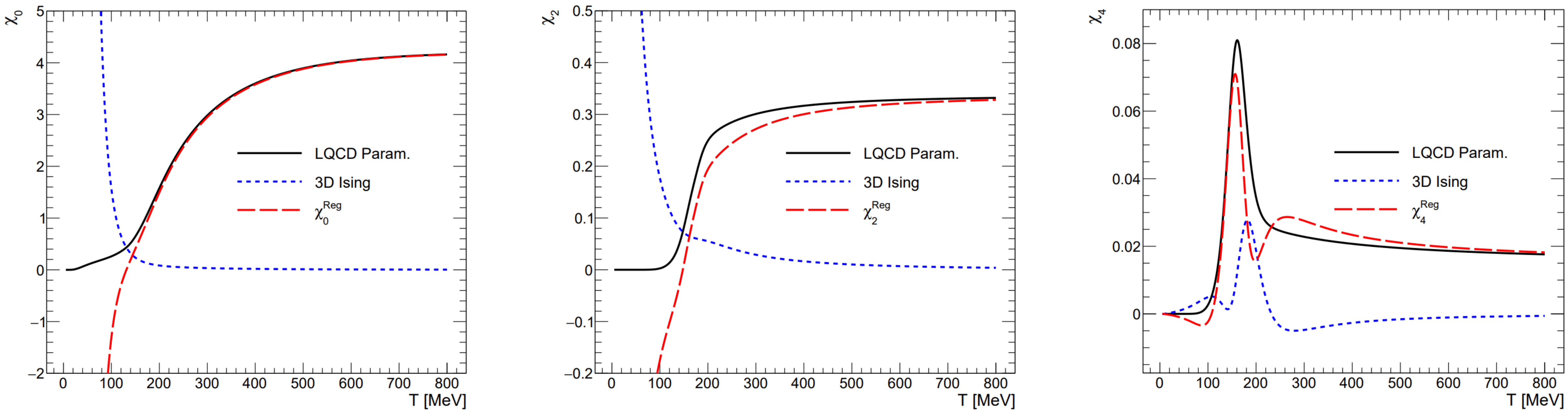
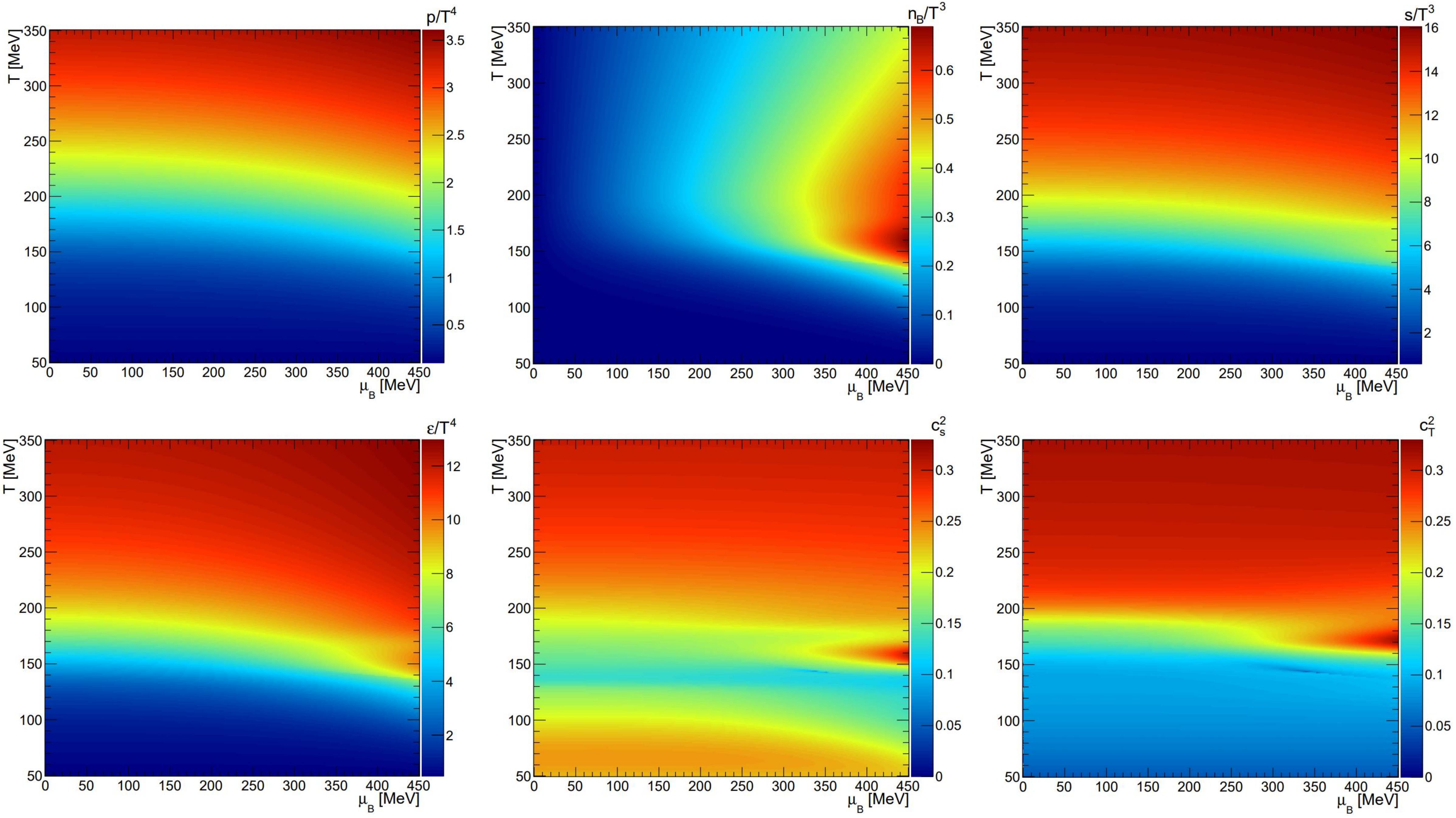


$$P \Rightarrow fP + (1-f)P^{HRG}, \text{ where } f(T,\mu_B) = \frac{1}{2}\left[1 + \tanh\left(\frac{T - T_0(\mu_B)}{\Delta T}\right)\right]$$



p/T^4
n_B/T^3
s/T^3
ε/T^4
c_s^2
c_T^2
T [MeV]
μ_B [MeV]

Conclusions & Outlook

Conclusions:

- We examined effective EoS construction with the incorporated 3D Ising model singularity
- Such class of EoS constructions allows for the full control on the shape of the phase transition line and the critical endpoint location
- However, it is hard to construct causal and stable EoS
- Other approach is needed

Outlook:

- Try another constructions & provide own one
- Start hydrodynamic simulations to determine observables which are sensitive to different EoS features
- Use Bayesian analyses to constrain parameter space