The Glueballonium

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Talk at 58 Karpacz Winter School of Theoretical Physics

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Young scientists' workshop and 58. Karpacz Winter School of Theoretical Physics "Heavy Ion Collision: From First to Last Scattering"

19-25 June 2022
Artus Hotel Karpacz

- Glueballs
- Scattering of two scalar glueballs
- Bound state of two glueballs: glueballonium
- Glueball resonance gas (work in progress)
- Conclusions

$$\textit{Hadrons} \begin{cases} \textit{Baryons} & [qqq] \\ \textit{Mesons} & [\bar{q}q] \end{cases}$$

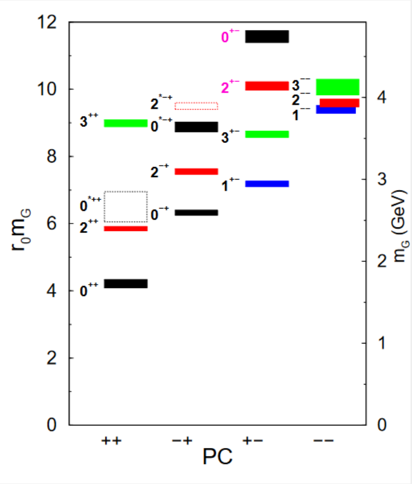
$q\bar{q}$ nonets	J^{PC}	Flavor states
Scalar (S)	0^{++}	$a_0^{\pm,0}$, $K_0^{*\pm,0,\bar{0}}$, σ_N, σ_S (f_0^L, f_0^H*)
Pseudoscalar (P)	0^{-+}	$\pi^{\pm,0}$, $K^{\pm,0,\bar{0}}$, η_N, η_S (η, η')
Vector (V_μ)	1^{--}	$\rho^{\pm,0}$, $K^{*\pm,0,\bar{0}}$, ω_N, ω_S (ω, ϕ).
Axial-vector (A_μ)	1^{++}	$a_1^{\pm,0}$, $K_{1,A}^{\pm,0,\bar{0}}$, $f_{1,A}^N$, $f_{1,A}^S$ (f_1, f_1').
Pseudovector (B_μ)	1^{+-}	$b_1^{\pm,0}$, $K_{1,B}^{\pm,0,\bar{0}}$, $f_{1,B}^N$, $f_{1,B}^S$ (h_1, h_1').

Table: Examples of nonets of conventional mesons, their spin and parity, and the flavor states. * means that the physical states are unknown

- QCD predicts also exotic mesons as tetraquarks ($\bar{q}\bar{q}qq$) and glueballs

- Gluons carry color charge and interact strongly with each other
  $\rightarrow$ bound states of gluons
- Widely studied on lattice QCD

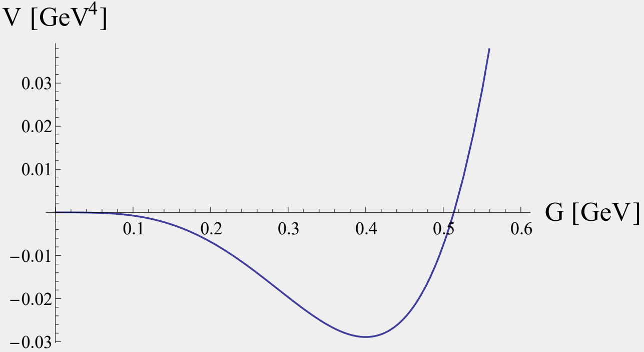
We worked with a single scalar glueball with $m_G = 1.7$ GeV.



Morningstar, Peardon, doi:10.1103/PhysRevD.60.034509

Now only the YM sector of QCD will be considered.

$$\mathscr{L}_{dil} = \frac{1}{2}(\partial_\mu G)^2 - V(G) = \frac{1}{2}(\partial_\mu G)^2 - \frac{1}{4}\frac{m_G^2}{\Lambda_G^2}\left(G^4 ln\left|\frac{G}{\Lambda_G}\right| - \frac{G^4}{4}\right)$$

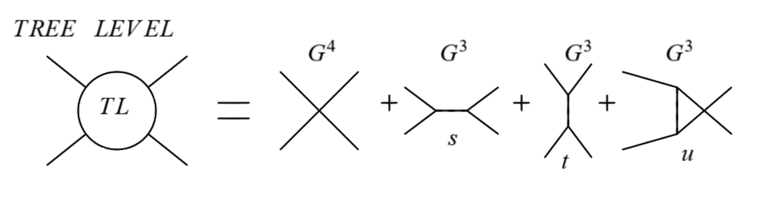


A. A. Migdal and M. A. Shifman, "Dilaton Effective Lagrangian In Gluodynamics," Phys. Lett.B114, 445 (1982)

Taylor expansion around min $V(G = \Lambda_G \approx 0.4$ GeV)

$V(G) = V(\Lambda_G) + \frac{1}{2} m_G^2 G^2 + \frac{1}{3!}\left(5\frac{m_G^2}{\Lambda_G}\right) G^3 + \frac{1}{4!}\left(11\frac{m_G^2}{\Lambda_G^2}\right) G^4 + \frac{1}{5!}\left(6\frac{m_G^2}{\Lambda_G^3}\right) G^5 + \ldots$

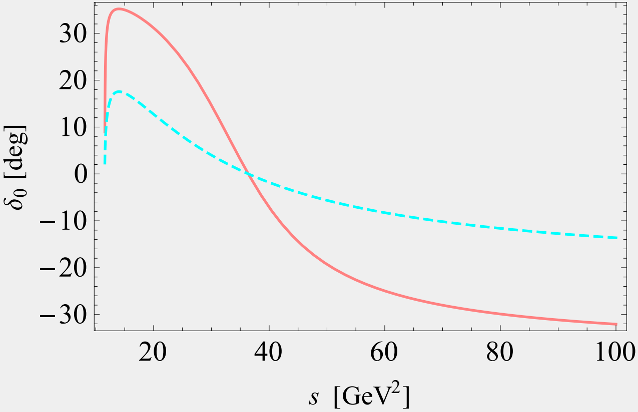
$A(s,t,u) = -11\frac{m_G^2}{\Lambda_G^2} - \left(5\frac{m_G^2}{\Lambda_G}\right)^2 \frac{1}{s-m_G^2} - \left(5\frac{m_G^2}{\Lambda_G}\right)^2 \frac{1}{t-m_G^2} - \left(5\frac{m_G^2}{\Lambda_G}\right)^2 \frac{1}{u-m_G^2}$



$$A_l(s) = \frac{1}{2}\int_{-1}^{+1} dcos\theta \; A(s, cos\theta) P_l(cos\theta),$$

$$A_0(s) = -11\frac{m_G^2}{\Lambda_G^2} - 25\frac{m_G^4}{\Lambda_G^2}\frac{1}{s - m_G^2} + 50\frac{m_G^4}{\Lambda_G^2}\frac{\log\left(1 + \frac{s-4m_G^2}{m_G^2}\right)}{s - 4m_G^2}$$

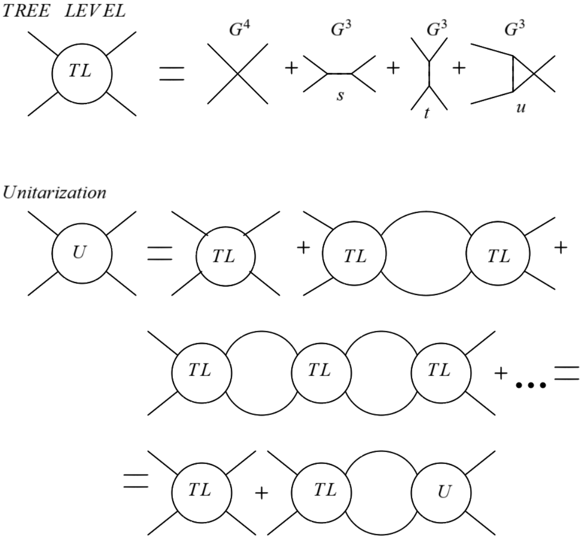
$$\delta_l(s) = \frac{1}{2} arg\left(1 + \frac{i\sqrt{s - 4m_G^2}}{16\pi\sqrt{s}} A_l\right).$$



$\Lambda_G = 0.4$ (pink) and 0.8 (blue) GeV

$\Delta\delta_0 := \delta_0(s \to \infty) - \delta_0(4m_G^2) \neq n\pi \to$ unitarization

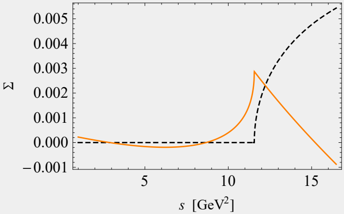
SCHEMATIC REPRESENTATION OF THE UNITARIZATION THROUGH LOOPS



$$U = TL + (TL)\sum U$$

Loop function $\Sigma(s)$ in such a way to:
-preserve the pole corresponding to $s = m_G^2$
-$A_l^{unit}(s = 3m_G^2) = \infty$: the branch point $s = 3m_G^2$ is generated by the single particle pole for m_G^2 along the t and u channels.
It follows that:

$$\Sigma(s) = \frac{(s - m_G^2)(s - 3m_G^2)}{\pi} \int_{4m_G^2}^{\infty} \frac{\frac{\frac{s'}{4} - m_G^2}{16\pi\sqrt{s'}}}{(s' - s)(s' - 3m_G^2)(s' - m_G^2)} ds'$$



$Re\Sigma$ (orange) vs. $Im\Sigma$ (black)

The unitarized amplitude is so given by:

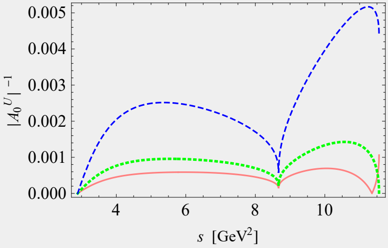
$$A_l^{unit}(s) = [A_l^{-1}(s) - \Sigma(s)]^{-1}$$

Now:

$$\delta_l^{unit}(s) = \frac{1}{2} arg\left[1 + 2i\frac{\sqrt{\frac{s}{4} - m_G^2}}{16\pi\sqrt{s}} A_l(s)^{unit}\right]$$

With our value of m_G:

$$a_0(s_{th}) = \infty \leftrightarrow \Lambda_G = \Lambda_{G,crit} \approx 0.504\ GeV$$



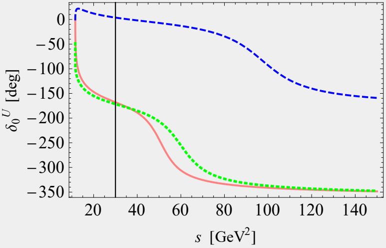
$|A_0^U|^{-1}(s)$ for:

▷ $\Lambda_G = 0.4$ *GeV* (pink)

▷ $\Lambda_G = \Lambda_{G,crit} \approx 0.504$ *GeV* (green)

▷ $\Lambda_G = 0.8$ *GeV* (blue).

$\Lambda_{G,crit} : a_0^U(s = s_{th})|_{\Lambda_G=\Lambda_{G,crit}} = \infty$



▷ $\Lambda_G = 0.4$ *GeV* (pink)

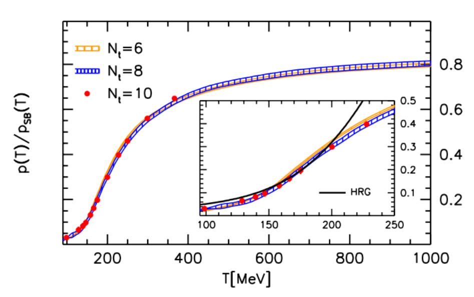
▷ $\Lambda_G = \Lambda_{G,crit} \approx 0.504$ *GeV* (green)

▷ $\Lambda_G = 0.8$ *GeV* (cyan).

$\Delta\delta_0^U \to -2\pi \to$ Levinson theorem fulfilled

Hadron Resonance Gas model:

- All baryons and mesons ($m < 2.5$ GeV) from PDG [Borsnayi et al.JHEP11(2010)077]



- Can we do in pure YM (GRG)?

Pressure and energy density

$$\hat{p} = -T \sum_J (2J+1) \int_0^\infty \frac{k^2}{2\pi^2} \ln\left(1 - e^{-\frac{\sqrt{k^2+m^2}}{T}}\right) dk$$

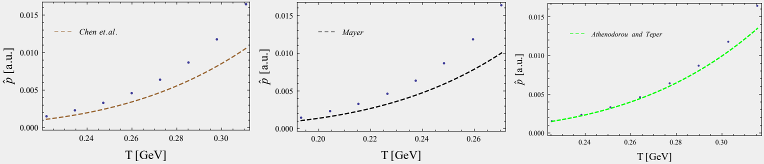
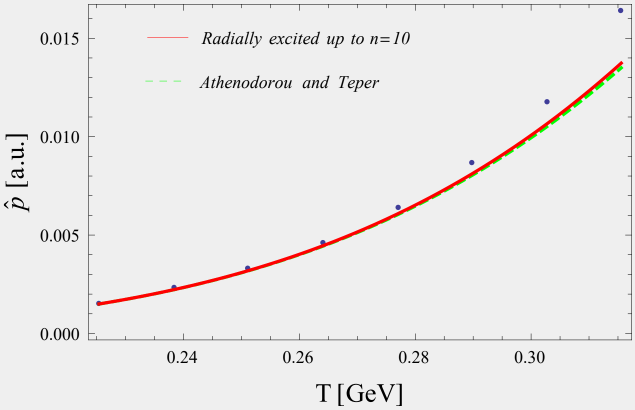
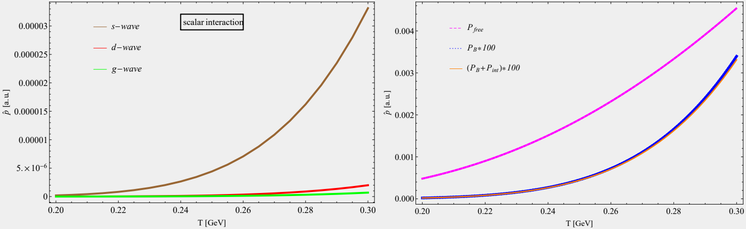


Figure: Free glueball pressure as function of the temperature for three different sets of lattice masses within GRG model, compared with the values given in **Borsanyi et al. arXiv:1204.6184 [hep-lat]**.
Panels:
left **Chen et al. arXiv:hep-lat/051007**,
center **Mayer arXiv:hep-lat/050800**,
right **Athenodorou, Teper arXiv:2007.06422 [hep-lat]**.


Radially excited up to n=10
Athenodorou and Teper
0.015
0.010
0.005
0.000
$\hat{p}$ [a.u.]
0.24
0.26
0.28
0.30
T [GeV]

$$P^{J^{PC}}_{tot} = P_{int} + P_{free} + P_B$$

$$P^{\rm int} + P_B = -T \sum_{l=0}^{\infty} \int_0^{\infty} dx \frac{2l+1}{\pi} \frac{d\delta_l^U(s=x^2)}{dx} \int \frac{d^3k}{(2\pi)^3} \ln\left(1 - e^{-\beta \frac{\sqrt{k^2+x^2}}{T}}\right)$$



- TL is not enough.
- Unitarized amplitudes for the l-th waves (l=0,2,4).
- The evaluated scattering lengths and phase shifts could be simulated on the lattice, hence a comparison is possible.
- Emergence of a 2 scalar glueball bound state, that we named "glueballonium", could be found on the lattice and/or in experiments (PANDA).
- $m_B(\Lambda_G)$. $\exists$ bound state if $\Lambda_G < \Lambda_{G,crit} \simeq 0.504$ GeV if $m_G = 1.7$ GeV
- FOR FUTURE: scattering of other glueballs or using other unitarization schemes.

Thank you for the attention

Decay $G \to \pi\pi$ takes the form (inserting a nonzero pion mass):

$$\Gamma_{G\to\pi\pi} = 6\frac{\sqrt{\frac{m_G^2}{4} - m_\pi^2}}{8\pi m_G^2}\left(\frac{m_\sigma^2}{2\Lambda_G}\right)^2 .$$

For $m_G \simeq 1.7$ GeV, $m_\sigma \simeq 1.3$ GeV ($\simeq f_0(1370)$), $\Lambda_G \simeq 0.4$ GeV: $\Gamma_{G\to\pi\pi} \simeq 0.310$ GeV.
$SU(3)$ extension:

$$\Gamma_{G\to KK} = 8\frac{\sqrt{\frac{m_G^2}{4} - m_K^2}}{8\pi m_G^2}\left(\frac{m_\sigma^2}{2\Lambda_G}\right)^2 , \qquad \Gamma_{G\to\eta\eta} = 2\frac{\sqrt{\frac{m_G^2}{4} - m_\eta^2}}{8\pi m_G^2}\left(\frac{m_\sigma^2}{2\Lambda_G}\right)^2 .$$

$\Gamma_{G\to KK} \simeq 0.340$ GeV and $\Gamma_{G\to\eta\eta} \simeq 0.080$ GeV.
The sum of the 3 pseudoscalar channel: 0.729 GeV.
$\oplus$ $G \to \rho\rho \to 4\pi$, expected sizable,cannot be determined within this simple approach (using $V(G, \sigma, \boldsymbol{\pi})$).
Such a glueball would have a total decay of about 1 GeV: would not be observable.

$$\mathcal{L}_{\text{YM}} = -\frac{1}{4} G^a_{\mu\nu} G^{a,\mu\nu}, \qquad G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g_0 f^{abc} A^b_\mu A^c_\nu$$

gluonic quantum fluctuation: ~~dilatatation symmetry~~

$$g_0 \xrightarrow[\text{anomaly}]{\text{trace}} g(\mu)$$

$$\Rightarrow \partial_\mu J^\mu_{\text{dil}} = (T^\mu_\mu)_{\text{YM}} = \frac{\beta(g)}{2g} G^a_{\mu\nu} G^{a,\mu\nu} \neq 0, \beta(g) = \partial g / \partial \ln \mu$$

At one loop ($\beta(g) = -bg^3$ & $b = 11N_c/(48\pi^2)$):

$$g^2(\mu) = \frac{1}{2b \ln(\mu/\Lambda_{\text{YM}})}, \Lambda_{\text{YM}} \approx 250 \text{ MeV}$$

Nonvanishing expectation value of the trace anomaly:

$$\langle T^\mu_\mu \rangle_{\text{YM}} = -\frac{11N_c}{24} \left\langle \frac{\alpha_s}{\pi} G^a_{\mu\nu} G^{a,\mu\nu} \right\rangle = -\frac{11N_c}{24} C^4$$

Gluons $\xrightarrow[\text{confinement}]{}$ not the asymptotic states of the theory

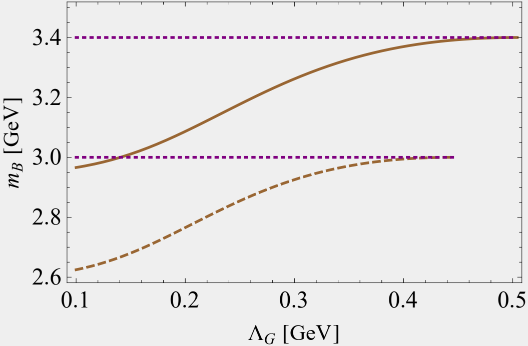
Scalar field G describing scalar glueball & trace anomaly at the composite level

$\mathcal{L}_{\text{dil}} = \frac{1}{2}(\partial_\mu G)^2 - V(G)$

$\partial_\mu J^\mu_{\text{dil}} = 4V - G\,\partial_G V \propto G^4$ only if $V(G) = \dfrac{1}{4}\dfrac{m_G^2}{\Lambda_G^2}\left(G^4 ln\left|\dfrac{G}{\Lambda_G}\right| - \dfrac{G^4}{4}\right)$

$\Lambda_G \simeq 0.4$ GeV: $\left[\partial_\mu J^\mu_{\text{dil}} = -\frac{1}{4}\,\frac{m_G^2}{\Lambda_G^2}\,G^4\right] \ominus \left\langle T^\mu_\mu \right\rangle_{YM}$: Dilaton field saturates the trace of the dilatation current.

- Twice-subtracted loop ($s = m_G^2, 3m_G^2$): single-particle pole and branch point fixed to the ones of TL amplitude.
  Once-subtracted loop ($s = m_G^2$): ghost with negative norm
- Results depend not only on the chosen subtraction, but also on the applied unitarization scheme (overall phenomenology expected similar)
- Multiparticle states contribute to the imaginary part: $Im\Sigma$ not valid up to $\infty \rightarrow$ Cutoff function (form not known) interpretable as G wf overlap: this would introduce a model dependence on the result.



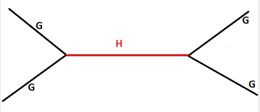
Mass of the glueballonium as function of Λ_G

For $m_G = 1.5$ and 1.7 GeV:
$\Lambda_{G,crit} \sim 0.445$ and 0.504 GeV.

For $\Lambda_G > \Lambda_{G,crit}$, $\nexists$ bound state.

Other glueballs effect: heavy scalar

H: other glueball entering the s-channel (heavy scalar and non-scalar)



$$\mathcal{L} = \mathcal{L}_{\text{dil}} + \frac{1}{2}(\partial_\mu H)^2 - \frac{\alpha}{2} G^2 H^2 - \beta H^4$$

No 3-leg interaction terms ($H^2 G$,...): break dilatation invariance (only $ln\left|\frac{G}{\Lambda_G}\right|$ does it in the proper way).
No $ln\left|\frac{H}{\Lambda_H}\right|$: another scale would be needed (not done here).
The interaction term proportional to $G^2 H^2$ affects the scattering via the an intermediate HH loop, relevant at $(2m_H)^2 \sim 36$ GeV^2, far from threshold.

NON SCALAR J^{PC}

No $J > 2$ for dilatation invariance.

	P	C
0^{-+}	✗	✓
0^{+-}	✗	✗
0^{--}	✓	✗
1^{++}	✗	✓
1^{-+}	✓	✓
1^{+-}	✗	✗
1^{--}	✓	✗

Table: Symmetries for XG^2 terms involving a heavy nonscalar glueball X and two scalar glueballs G.

The term in $\mathscr{L}$ corresponding to the coupling XG^2, $X \equiv 1^{-+}$, is the only acceptable. However, vector current for a neutral scalar vanishes: $G\partial^\mu G - (\partial^\mu G)G = 0$.

HRG

Hadron Resonance Gas model

$$\frac{p^{HRG}}{T^4} = \frac{1}{VT^3}\left(\sum_{i\in \text{mes}} \log \mathcal{Z}^M(T,V,\mu_{X^a},m_i) + \sum_{i\in \text{bar}} \log \mathcal{Z}^B(T,V,\mu_{X^a},m_i)\right)$$

(Fugacities: $z_i := \exp\left\{\frac{1}{T}\sum_a X_i^a \mu_{X^a}\right\}$. X^a: all possible conserved charges.)

$$\log \mathcal{Z}^{M,B}(T,V,\mu_{X^a},m_i) := \mp V d_i \int_0^\infty \frac{dk}{2\pi^2} k^2 \log\left(1 \mp z_i e^{-\frac{\sqrt{k^2+m_i^2}}{T}}\right)$$