Quarkyonic Matter

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Work in collaboration with
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Mass and radii of observed neutron stars and data from neutron star collisions give an excellent determination of the equation of state of strongly interacting matter

Such equations of state must be hard
The sound velocity squared is greater than or of the order of 1/3 at only a few times nuclear matter density
This is NOT what one expects from a phase transition
Relativistic degrees of freedom appear to be important

After a short review, will discuss a field theoretical method to include both quark and nucleon degrees of freedom in a consistent field theoretical formalism

Lecture I: Neutron Star Matter and Some Conjectures on Scale Invariance

From observations of neutron stars masses and radii, one gets very good information about the zero temperature equation of state of nuclear matter

One equates the outward force of matter arising from pressure inward force of gravity. This gives a general. relativistic equation of hydrostatic equilibrium.

For a specific equation of state, one obtains a relationship between radii and neutron star masses

Equations of state may be characterized by two dimensionless numbers

Sound velocity:

$$v_s^2 = \frac{dP}{de}$$

and the trace of the stress energy tensor scaled by the energy density

$$\Delta = \frac{1}{3} - \frac{P}{e}$$

P = - dE/dV

In a scale invariant theory at zero temperature:

$$E \sim (N/V)^{1/3} V \sim N^{4/3} V^{-1/3}$$

$$P = \frac{1}{3}\frac{E}{V} = \frac{1}{3} e$$

$$v_s^2 = \frac{dP}{de} = \frac{1}{3}$$

$$\Delta = \frac{1}{3} - \frac{p}{e} = 0$$

The trace of the stress energy tensor is taken to be a measure of scale invariance. It is anomalous in QCD.

$$T^\mu_\mu = -\beta(g)(E^2 - B^2) + m_q(1+\gamma_q)\overline{\psi}\psi$$

In this equation, the beta function of QCD is negative, and the fermion term is from quarks. It vanishes in the chiral limit.

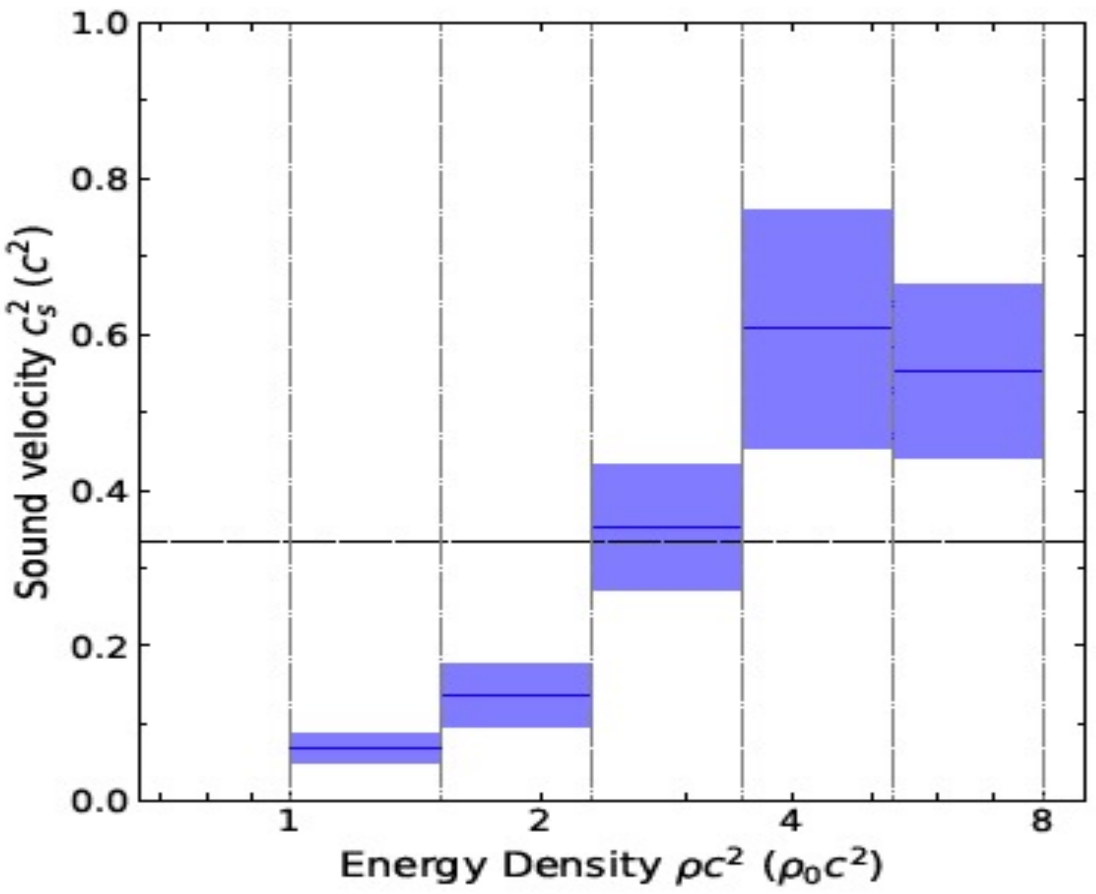
(Derivation will follow later)

If we take matrix elements of single particle states

$$< p \mid T^\mu_\mu \mid p > \sim p^2 = m^2 \geq = 0$$

In the chiral limit, this implies E > B, as we expect for massive quarks, except for the pion, which is very tightly. bound

For dilute systems, the trace anomaly is positive, as it is at high density for a quark gas. In general, we expect it to be positive, except possibly for small effects due to pion condensate, if they exist



Y. Fujimoto, K. Fukushima,
K. Murase

Tews, Carlso, Gandolfi and Reddy; Kojo; Anala, Gorda, Kurkela and Vorinen

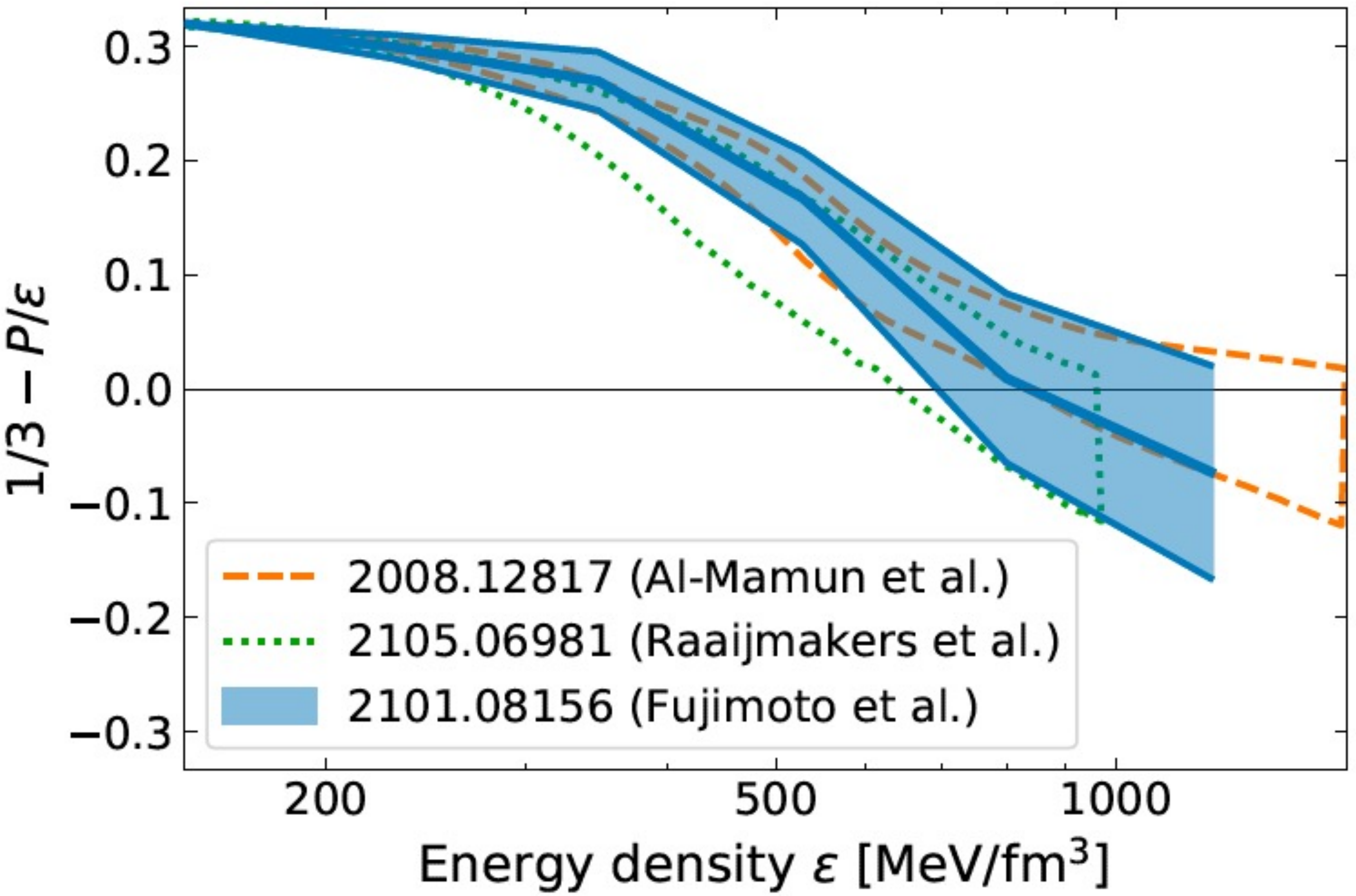
As a result of LIGO experiments, and more precise measurement of neutron star masses, the equation of state of nuclear matter at a few time nuclear matter density is tightly constrained

Sound velocity approaches and perhaps exceeds

$$v_s^2 = 1/3$$

at a few times nuclear matter density

Work with Y. Fujimoto, K. Fukushima and M. Praszalowicz



1/3 – P/e approaches zero from above. By about 5 times nuclear matter density, the system is approximately scale invariant.

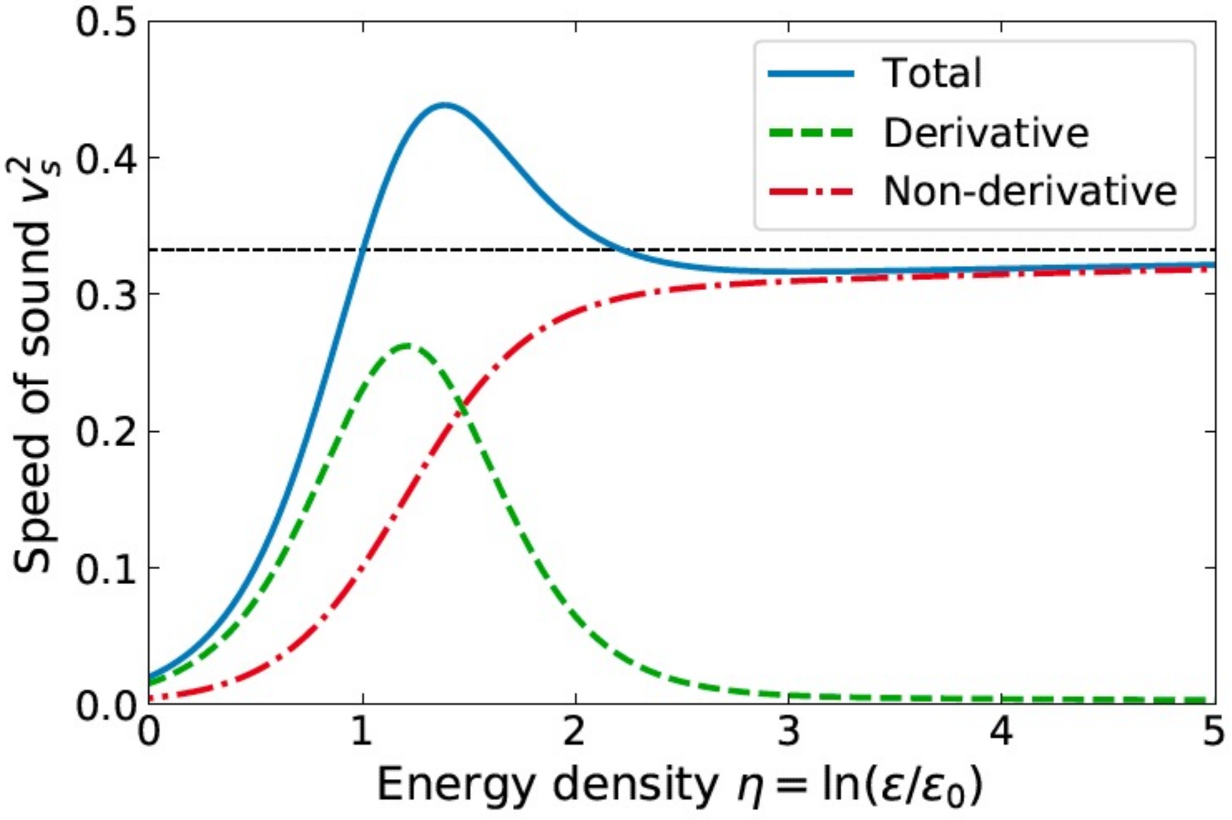
Suggests degrees of freedom are strongly interacting relativistic quarks

$$v_s^2 = v^2_{s,\,(\mathrm{deriv})} + v^2_{s,\,(\mathrm{non\text{-}deriv})} \equiv \varepsilon \frac{d}{d\varepsilon}\left(\frac{P}{\varepsilon}\right) + \frac{P}{\varepsilon}$$

Work with Y. Fujimoto, K. Fukushima and M. Praszalowicz

Peak in sound velocity is because system rapidly approaches scale invariant limit

The maximum measures the energy density of transition between nucleon and quark degrees of freedom



Quarkyonic matter has precisely the properties needed to describe this behaviour

Derivation of the Trace Anomaly (following Preskill)

Consider first pure Yang Mils Theory

$$S = \int d^4x \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

Generalize to general curved background

$$S = \int d^4x \sqrt{g}\ \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

$$S = \int d^4x \sqrt{g}\ \frac{1}{4}\ F_{\mu\nu} F_{\lambda\sigma}\ g^{\mu\nu} g^{\lambda\sigma}$$

Perturb the metric by

$$g^{\mu\nu} \to g^{\mu\nu} + \delta g^{\mu\nu}$$

$$\sqrt{g} \to \sqrt{g}(1 + \frac{1}{2}\ g\delta g)$$

$$\delta S = \frac{1}{2} \int \ d^4x\ \sqrt{g}\ \theta_{\mu\nu}\delta g^{\mu\nu}$$

Where the stress energy tensor is

$$\theta^{\mu\nu} = F^{\mu\lambda}F^{\nu}_{\lambda} - \frac{1}{4}\ g^{\mu\nu}\ F_{\lambda\sigma}F^{\lambda\sigma}$$

Now consider a scale change

$$g^{\mu\nu} \to \lambda^2 g^{\mu\nu}$$

Scale invariance implies for the classical theory therefore that

$$\theta^{\mu}_{\mu} = 0$$

What happens in the quantum theory?

We need to write the action with a scale dependent coupling.
The theory is only if we rescale the coupling along with the scale change

The action is

$$S = \int d^4x \sqrt{g} \frac{1}{4e^2} F_{\mu\nu} F^{\mu\nu}$$

Let the metric change as

$$\eta^{\mu\nu} \to \lambda^2 \eta^{\mu\nu}$$

$$e_{\Lambda}^2 \to e_{\Lambda/\lambda}^2$$

This gives that

$$\theta_{\mu}^{\mu} \frac{1}{e^2} - \frac{1}{2e^3} \left(\Lambda \frac{d}{d\Lambda} e \right) F_{\mu\nu} F^{\mu\nu} = 0$$

So the trace anomaly of pure Yang-Mills theory is

$$\theta^{\mu}_{\mu} = -\beta(e)(E^2 - B^2)$$

When we include massive quarks, the quarks break the scale invariance explicitly, and there is a contribution

$$m\overline{\psi}\psi$$

And when we vary the quark mass dependence with respect to the QCD scale

$$m_q\overline{\psi}\psi \rightarrow (1+\gamma_q)m_q\overline{\psi}\psi$$

In effective theories of mesons, pion condensation can induce a negative trace anomaly.

How does this occur?

Can it affect neutron. Star matter?

The vanishing of the trace anomaly for relatively large coupling is somewhat mysterious. Is AdSCFT model building reasonable to describe this?