Quarkyonic Matter

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Lecture at 2022 Karpacz Winter School of Theoretical Physics

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Lecture II: Quarkyonic Matter and the Large N_c Limit

The Large N_c Limit

Useful to think in the large number of of colors limit

$$N_c \to \infty$$

In this limit, quark loops are suppressed by one over Nc

Quark pair production cannot short out the long range linear potential, so there is confinement.

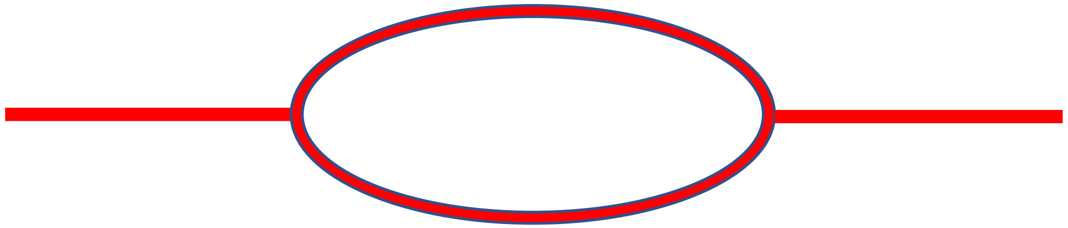
Baryons are heavy and made of N_c quark so their mass is of order N_c

Mesons are weakly interacting, with interaction strength of order 1/N_c

$g^2 N_c$ finite but $N_c \to \infty$

High Temperature World at Zero Baryon Density

The confining force is cutoff at a screening length associated with polarization of thermal gluons



$$1/r^2_{Debye} \sim g^2_{'tHooft} T^2$$

At some temperature the Debye length is less than the confinement scale, the potential can no longer become linear

$$T_D \sim \Lambda_{QCD}$$

$$\frac{1}{q^2} -> \frac{1}{q^2 + M_D^2}$$

$$\frac{1}{r} -> \frac{e^{-M_D r}}{r}$$

The Debye scale controls the distance at which interactions are cutoff. In confining theories. this controls the distance scale at this the linear potential cuts off

Debye screening at finite temperatures arises because on the average, gluons in matter which have octet color can screen a color singlet charge. This is not true at zero since there are no matter gluons present

Can also imagine it from thinking about hadrons

At low temperatures there is a gas of very weakly interacting hadrons, since if

$$T << \Lambda_{QCD}$$

The density of mesons is of order one in powers of N_c because the mesons are color singlet

If there was a gas of gluons, the density would be of order

$$N_c^2$$

This happens because there is an exponentially growing density of mesons that have cross sections of order 1/N_c^2 whose interactions become big when the density is of order N_c^2

The Hagedorn Model will generate soft equations of state

The entropy is singular at T_c

$$s \sim (T_c - T)^{-\gamma}$$

$$\frac{dP}{dT} = s$$

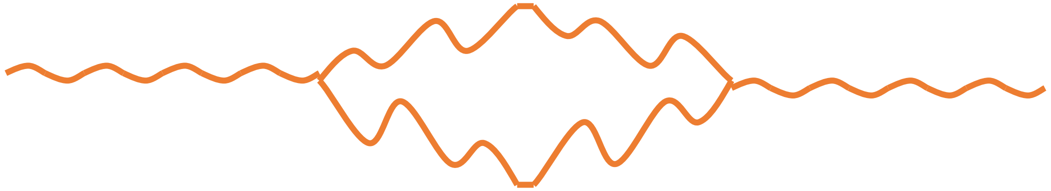
$$P \sim (T_c - T)^{1-\gamma}$$

$$P = -e + Ts$$

$$e \sim Ts \qquad lim_{T \to T_c} \frac{p}{e} \to 0$$

At the Hagedorn temperature, the degrees of freedom are non relativistic resonances

What about finite density at zero T:



$$g^2 N_c T^2 \sim \alpha_N T^2$$

Generates Debye Screening => Deconfinement at Tc

$$g^2 \mu_Q^2 \sim \alpha_N \mu_Q^2 / N_c$$

$$\mu_Q = \mu_B / N_c$$

Quark loops are always small by 1/N_c

For finite baryon fermi energy, confinement is never affected by the presence of quarks!

T_c does not depend upon baryon density!

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Quarkyonic Matter:

Confinement at finite temperature disappears because the Debye screening length become shorter than the confinement scale. Gluons give

$$1/r^2_{Debye} \sim g^2_{'tHooft} T^2$$

At finite density, there is a typical Fermi momentum or chemical potential associated with quark density

$$\mu_Q$$

The Debye screening length associated with quarks is very large

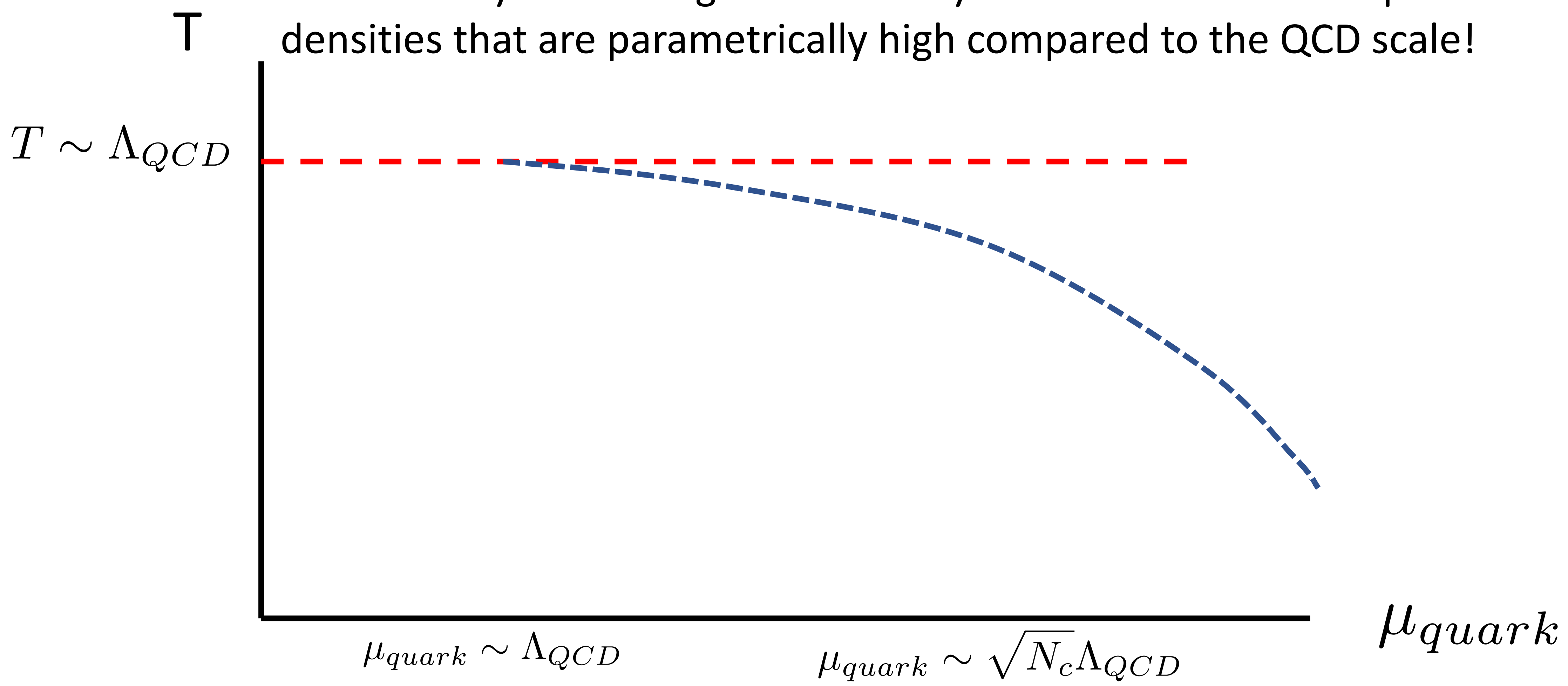
In the large N_c limit, at finite density and zero temperature limit, the deconfinement chemical potential is

$$1/r^{2\ quarks}_{Debye} \sim \mu^2_Q/N_c$$

In the large N_c limit, at finite density and zero temperature limit, the deconfinement chemical potential is

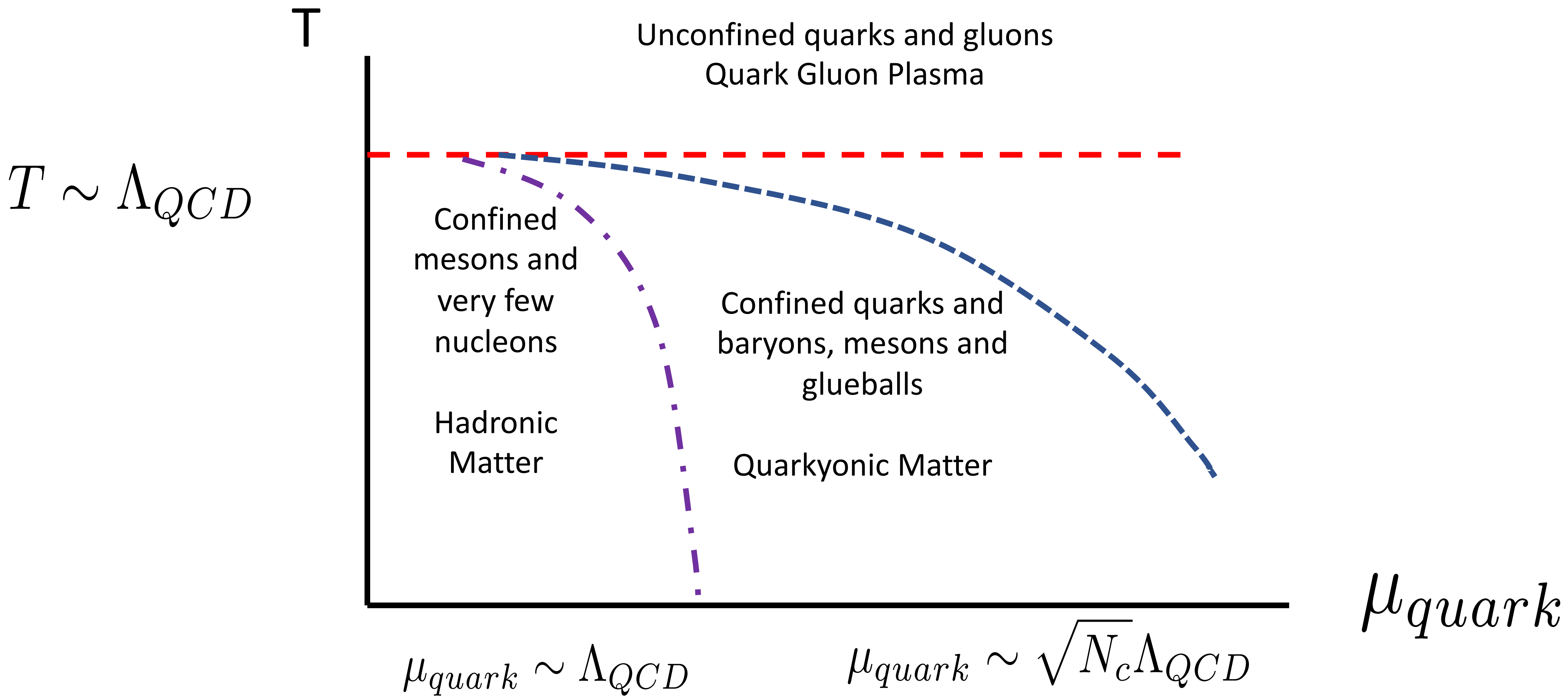
$$\mu_{quark} \sim \sqrt{N_c}\Lambda_{QCD} >> \Lambda_{QCD}$$

This is a very interesting result: the system is confined until quark densities that are parametrically high compared to the QCD scale!

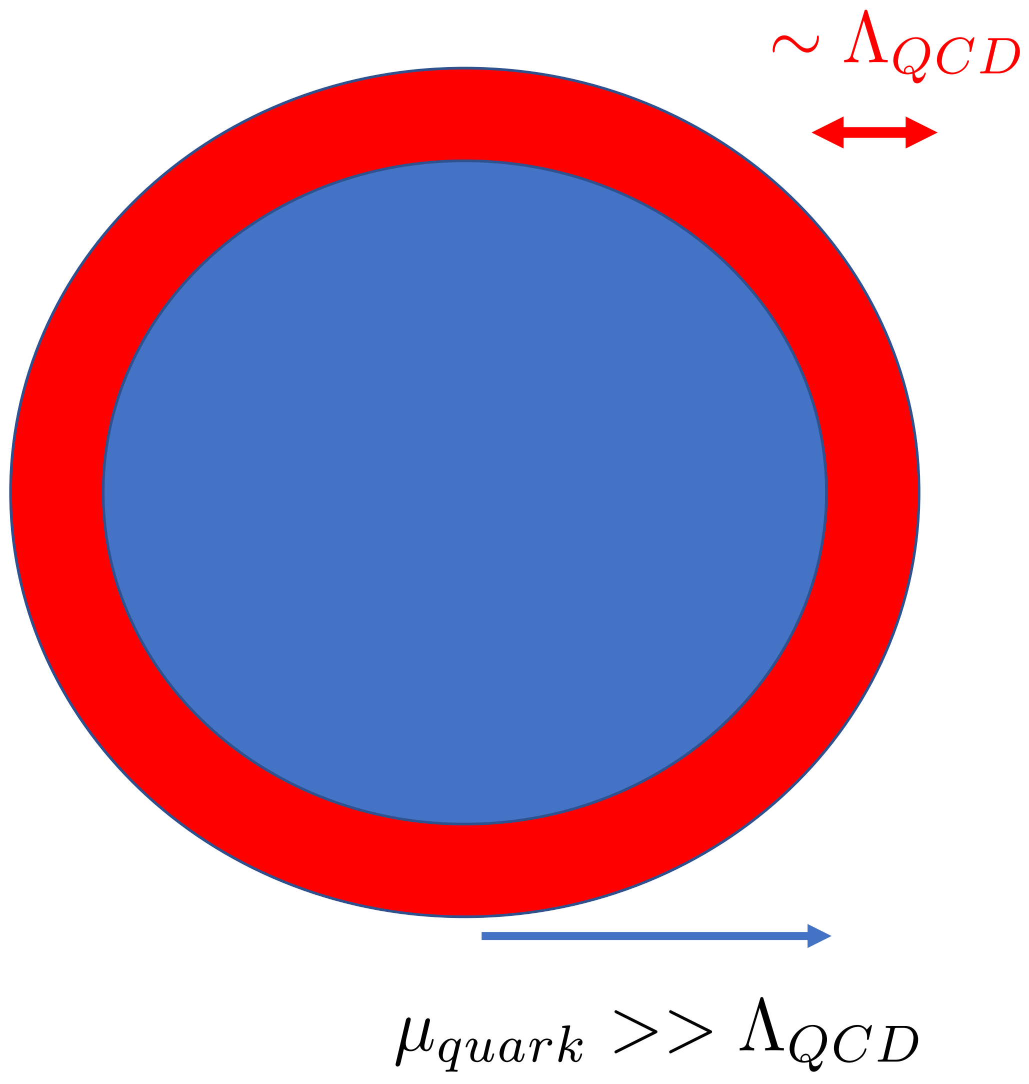


$$n_{baryon} \sim e^{(\mu_B - E)/T} \sim e^{N_c(\mu_q - E_q)/T}$$

No baryons for $\mu_{quark} < M_{nucleon}/N_c \sim \Lambda_{QCD}$



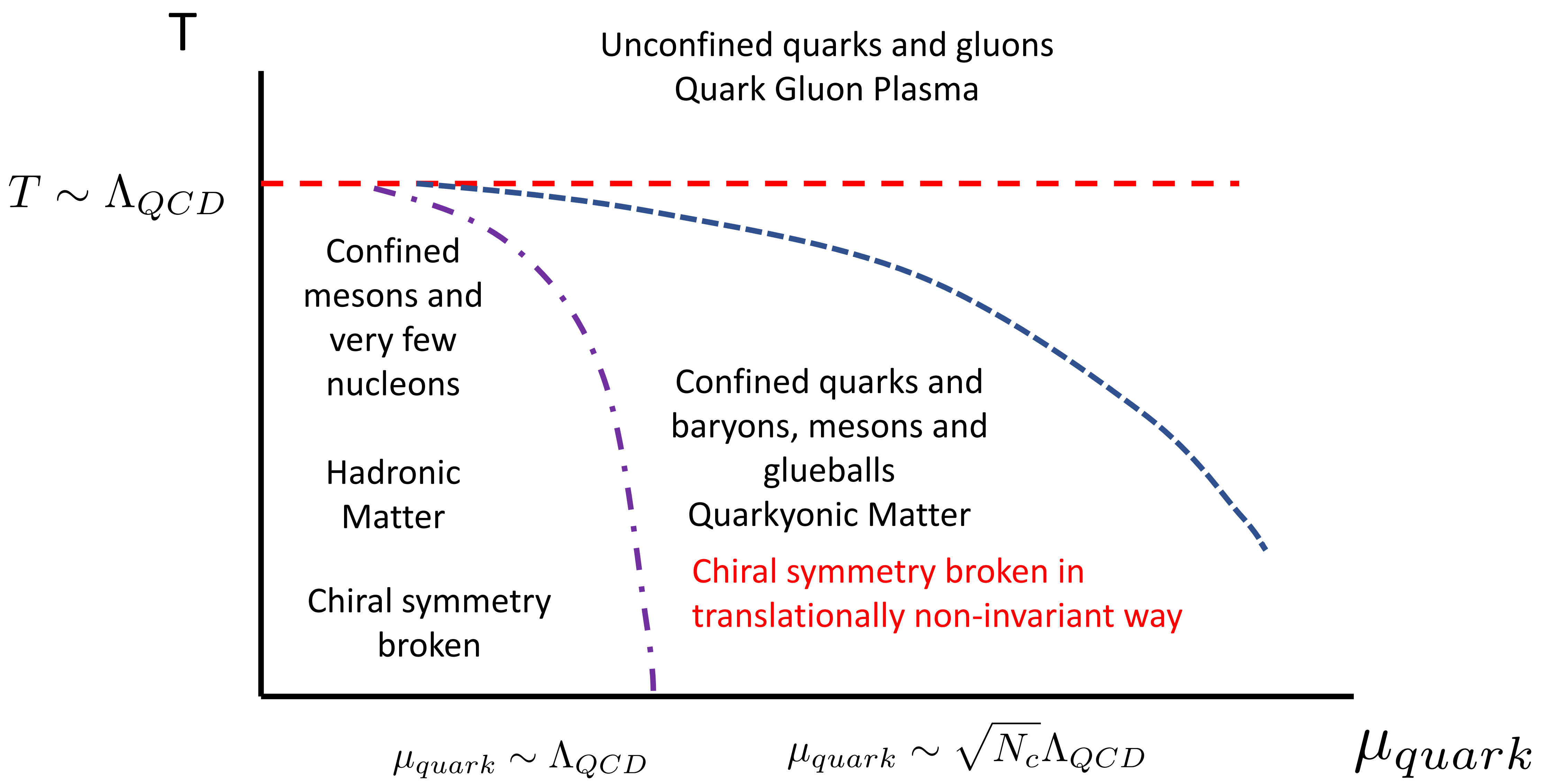
Fermi Surface is Non-perturbative



Fermi Surface
Interactions sensitive to infrared
Degrees of freedom: baryons, mesons and glueballs

Fermi Sea: Dominated by exchange interactions which are less sensitive to IR.
Degrees of freedom are quarks

Chiral symmetry remains broken in quarkyonic matter



Simple large N_c considerations

Near nuclear matter density

$$k_F \sim \Lambda_{QCD}$$

$$\epsilon/n - M_N \sim \Lambda_{QCD}^2/2M_N \sim \Lambda_{QCD}/N_c$$

On the other hand, short distance QCD interactions are of order N_c

$$\epsilon/n - M_N \sim N_c\Lambda_{QCD}$$

But the density of hard cores is also parametrically of order

$$\Lambda_{QCD}^3$$

Sound velocity of order one has important consequences

For zero temperature Fermi gas:

$$\frac{n_B}{\mu_B dn_B/d\mu_B} = v_s^2$$

where the baryon chemical potential includes the effects of nucleon mass

$$\frac{\delta\mu_B}{\mu_B} \sim v_s^2 \ \frac{\delta n_B}{n_B}$$

So if the sound velocity is of order one, an order one change in the baryon density generates a change in the baryon number chemical potential of order the nucleon mass

For nuclear matter densities

$$\mu_B - M \sim \frac{\Lambda_{QCD}^2}{2M} \sim 100 \ MeV$$

Large sound velocities will require very large intrinsic energy scales, and a partial occupation of available nucleon phase space because density is not changing much while Fermi energy changes a lot

How can the density stay fixed if the Fermi momentum goes up.? The nucleon must sit on a shell of varying thickness as the density increases. The added baryon number has to come in the form of new degrees of freedom: quarks

This can be understood in large N_c arguments:

$k_f \sim \Lambda_{QCD}$ requires $\mu_B - M_N \sim \Lambda_{QCD}/N_c$

Quarks should become important when

$$\mu_Q = \mu_B/N_c \sim \Lambda_{QCD}$$

The hypothesis of quarkyonic matter implies there need be no first order phase transition, The quarkyonic hypothesis requires a transition when the baryon Fermi energy is very close to the nucleon mass, so the transition may in principle occur quite close to nuclear matter density.

Relation between quark and nucleon Fermi momenta

$$m_q = m_N / N_c$$

$$\mu_q = \mu_N / N_c$$

$$k_q^2 = \mu_q^2 - m_q^2 = \mu_n^2 / N_c^2 - M_N^2 / N_c^2 = k_N^2 / N_c^2$$

For 2 flavors of nucleons

$$n_B^N = 4 \int^{k_N} \frac{d^3k}{(2\pi)^3} = \frac{2}{3\pi^2} k_N^3$$

For two flavors of quarks

$$n_q^N = \frac{1}{N_c} 4N_c \int^{k_q} \frac{d^3k}{(2\pi)^3} = \frac{2}{3\pi^2} k_q^3$$

To get any baryons from the quarks at the bottom of the shell of nucleons, need need the quark fermi momentum to be of the order of the QCD scale, so that the nucleon in the shell are relativistic. Naturally driven to the conformal behaviour. Also we see to absorb the baryons, that have been eaten, the chemical potential of the quarks must jump up from a small value to a typical QCD scale

If there is a continuous transition then the baryon density will have to remain fixed, so the chemical potential will change by of order N_c. The sound velocity is changing for a very non-relativistic system to a very relativistic one.

$$\epsilon_B = M_N \Lambda_{QCD}^3 \sim N_c \Lambda_{QCD}^4$$

$$\epsilon_Q \sim \Lambda_{QCD} n_q \sim N_c \Lambda_{QCD}^4$$

$$P \sim \frac{k_F}{M_B} \epsilon_N$$

The pressure on the other hand must jump by order N_c squared

$$P \sim \epsilon_q$$

Energy density and density fixed, but pressure and chemical potential jump.

A first order phase transition has pressure and chemical potential fixed, but energy density and density jump

Kiesang Jeon,
Srimoyee Sen and LM

A simple model for the quarkyonic transition:

Suppose we assume that the chemical potential for nucleons diverge as we approaches critical density,. The density of hard core overlap

$$\mu_N - M = \kappa \frac{M}{N_c^2} \left\{ (1 - n_N^N/n_0)^{-\gamma} - 1 \right\}$$

At low density

$$\lim_{n \to 0} (\mu_N - M) \sim \kappa\gamma \frac{M}{N_c^2} \frac{n_N^N}{n_0}.$$

So the sound velocity is

$$v_s^2 = \frac{n_N^N}{\mu_N \; dn_N^N/d\mu_N} \sim \frac{\kappa\gamma}{N_c^2} \frac{n_N^N}{n_0}$$

For such a nucleon gas we can determine the energy density from

$$\frac{d\epsilon_N^N}{dn_N^N} = \mu_N = M + \kappa \frac{M}{N_c^2} \left\{ (1 - n_N^N/n_0)^{-\gamma} - 1 \right\}.$$

$$\frac{\epsilon_N^N - M n_N^N}{n_0 M} = \frac{\kappa}{(\gamma - 1) N_c^2} \left\{ (1 - n_N^N/n_0)^{1-\gamma} + (1-\gamma) n_N^N/n_0 \right\}$$

and the pressure from

$$\frac{dP_N^N}{d\mu_N} = n_N^N.$$

$$\frac{n_N^N}{n_0} = 1 - \left\{ \frac{(\mu_N - M) N_c^2}{\kappa M} + 1 \right\}^{-1/\gamma}$$

The sound velocity is

$$v_s^2 = \gamma \frac{n_N^N/n_0}{1 - n_N^N/n_0} \left\{ 1 + \frac{N_c^2}{\kappa} (1 - n_N^N/n_0)^\gamma \right\}^{-1}$$

So for a pure nucleon gas, the sound velocity would diverge

This is fixed by adding in a quark term

$$\epsilon^N = \epsilon_N^N(n_N^N) + \epsilon_Q^N(n_Q^N)$$

$$n^N = n_N^N + n_Q^N$$

Minimizing the energy density with respect to the quark and nucleon number density at fixed total baryon number density gives

$$\frac{d\epsilon_Q^N}{dn_Q^N} = \frac{d\epsilon_N^N}{dn_N^N} \equiv \mu_N$$

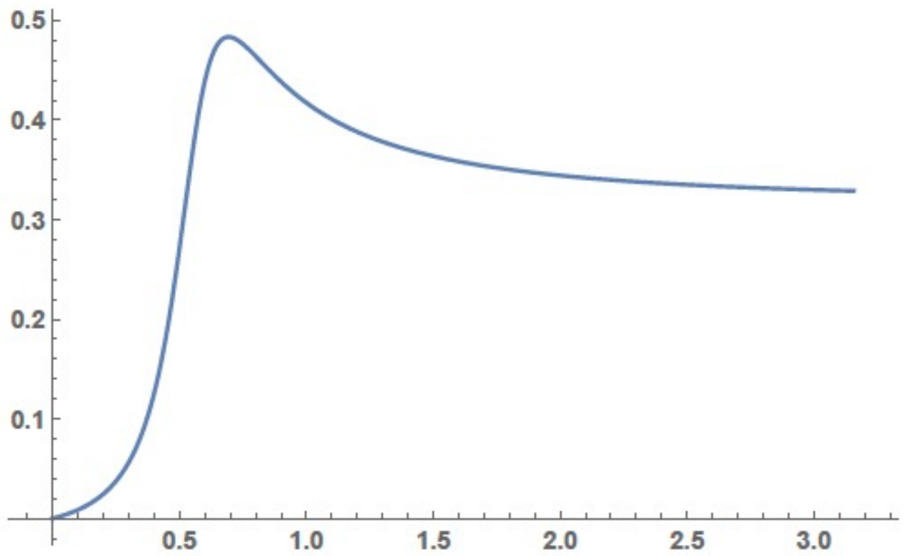
Which also implies

$$\mu = \frac{d\epsilon^N}{dn^N} = \frac{d\epsilon_N^N}{dn_N^N}\frac{dn_N^N}{dn^N} + \frac{\epsilon_Q^N}{dn_Q^N}\frac{dn_Q^N}{dn^N}$$
$$= \frac{d\epsilon_N^N}{dn_N^N}\left(1 - \frac{dn_Q^N}{dn^N}\right) + \frac{d\epsilon_Q^N}{dn_Q^N}\frac{dn_Q^N}{dn^N}$$
$$= \frac{d\epsilon_N^N}{dn_N^N} = \mu_N.$$

The sound velocity becomes

$$v_s^2 = \frac{n_N^N + n_Q^N}{\mu_N(dn_N^N/d\mu_N + dn_Q^N/d\mu_N)}$$

At lowish baryon densities, the quark contribution is very small , but when the nucleon densities stalls at the hard core density, then the quark density term begins to dominate



$$\gamma = 0.7 \;\; n_0^N = 0.6 \; Fm^{-3}$$

Such a model does not solve the problem that nucleons cannot coexist with a closed Fermis sea. This can be fixed by putting the nucleons on a thin shell that surround the filled Fermis sea of quarks. There are some small modification required to implement this, but it can be done and does not much change the computation of the sound velocity