Quarkyonic Matter

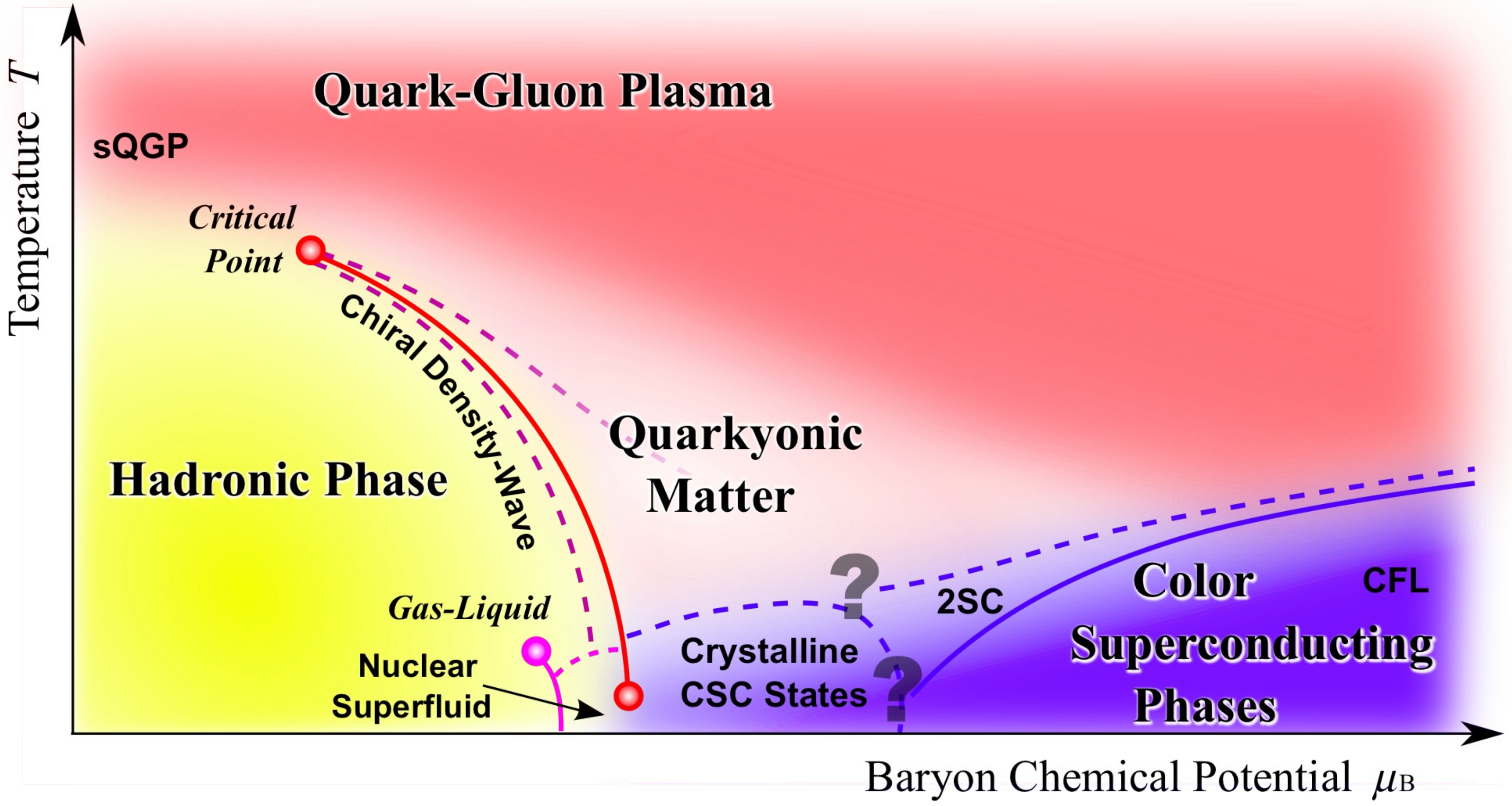
Larry McLerran
INT, University of Washington

Lecture at 2022 Karpacz Winter School of Theoretical Physics

Work in collaboration with
R. Pisarski, T. Kojo, Y. Hidaka, S. Reddy, Kiesang Jeon, Dyana Duarte, Saul Hernandez,
Yuki Fujimoto, Kenji Fukushima and. Michal Praszalowicz

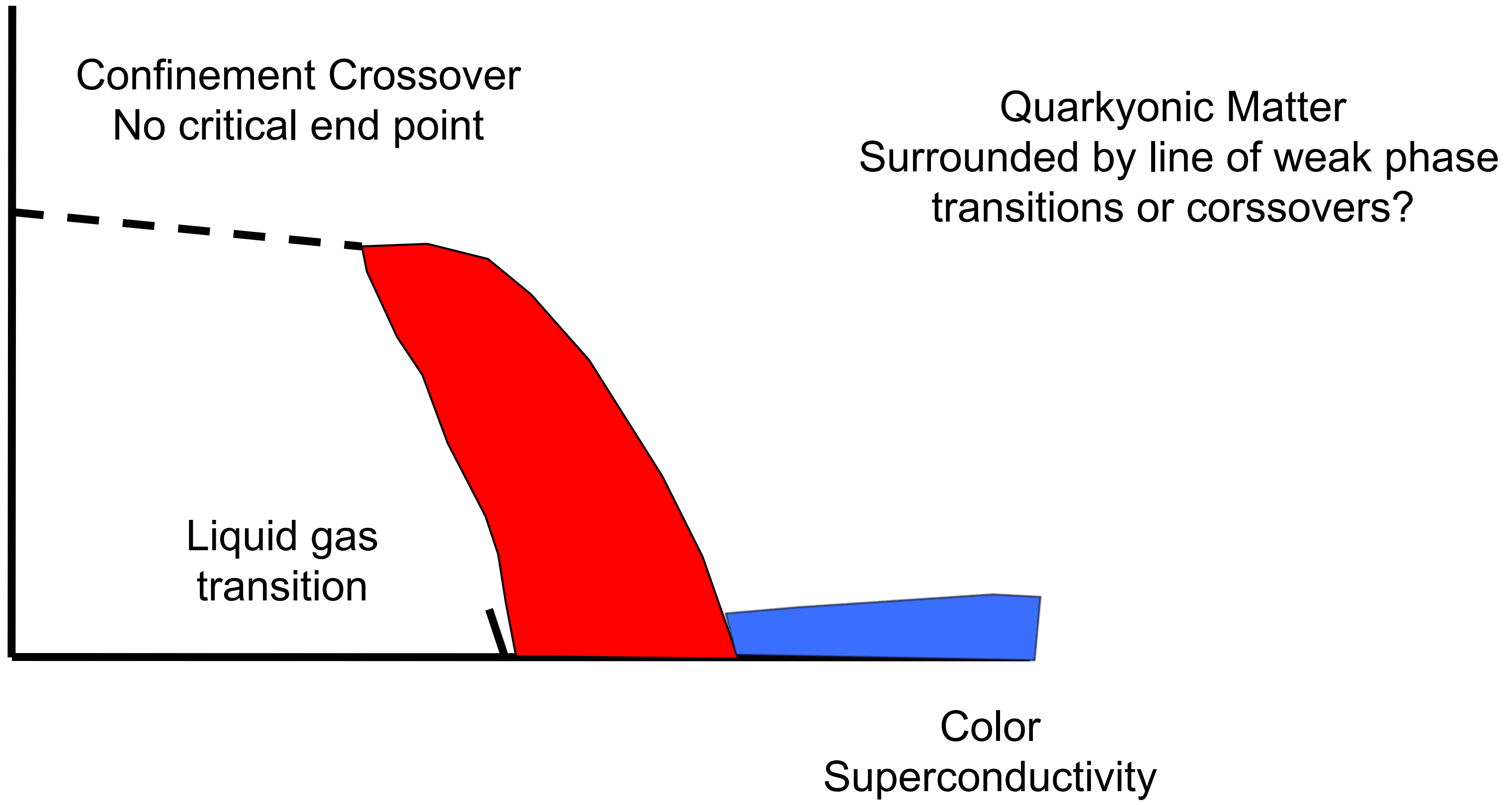
Lecture III: Quarkyonic Matter: Implications and Properties

Conception by Hatsuda and Fukushima



Perhaps Quarkyonic matter breaks translational invariance and P.

Is the real phase diagram more like?



The Quarkyonic Chiral Spiral:

Near Fermi surface, theory dimensionally reduces to 1+1 D ‘t Hooft model

2Nf “Goldstone Bosons”

Translational non-invariant chiral condensate

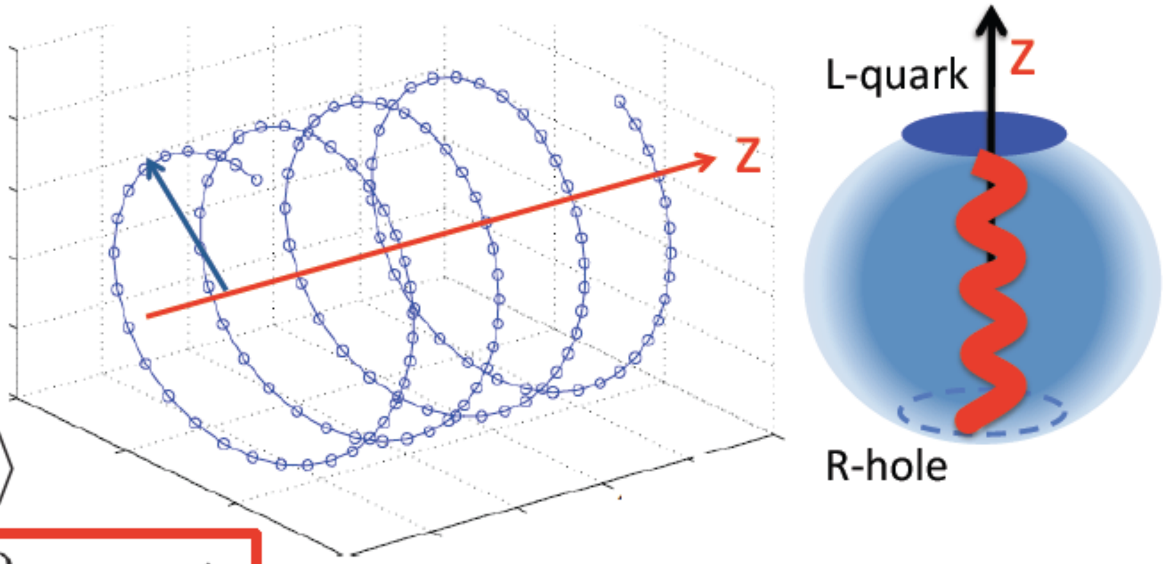
Condensate breaks parity and induces a periodic electric field

Is it True?

- Chiral rotation evolves in the longitudinal direction:

$\langle \bar{\Psi}\Psi \rangle$

$$\langle \bar{\psi}\psi \rangle$$



$\langle \bar{\Psi} i\Gamma_5 \Psi \rangle$

$$\langle \bar{\psi}\gamma^0\gamma^z\psi \rangle$$

(not conventional pion condensate in nuclear matter)

Chiral Spiral Formation

$$E = \mu_B + \Delta E, \; particle, \; k_F \sim \mu_B$$
$$E = \mu_B - \Delta E, \; hole, \; k_F \sim -\mu_B$$
$$antiparticle, \; E = \mu_B + \Delta E, k_F \sim \mu_B$$

If form a bound state with negative biding energy =>

Chiral condensate

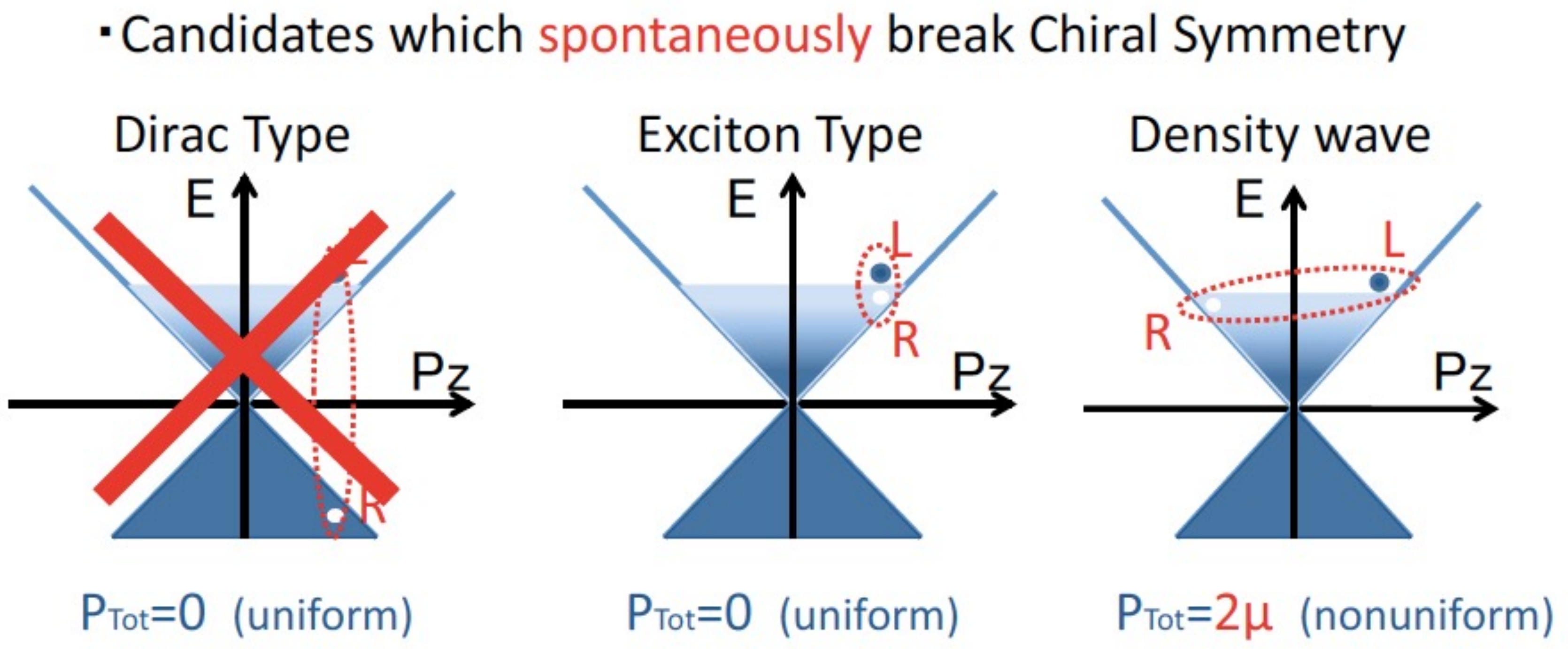
Condensate breaks translational invariance => crystal

Chiral symmetry breaking of order

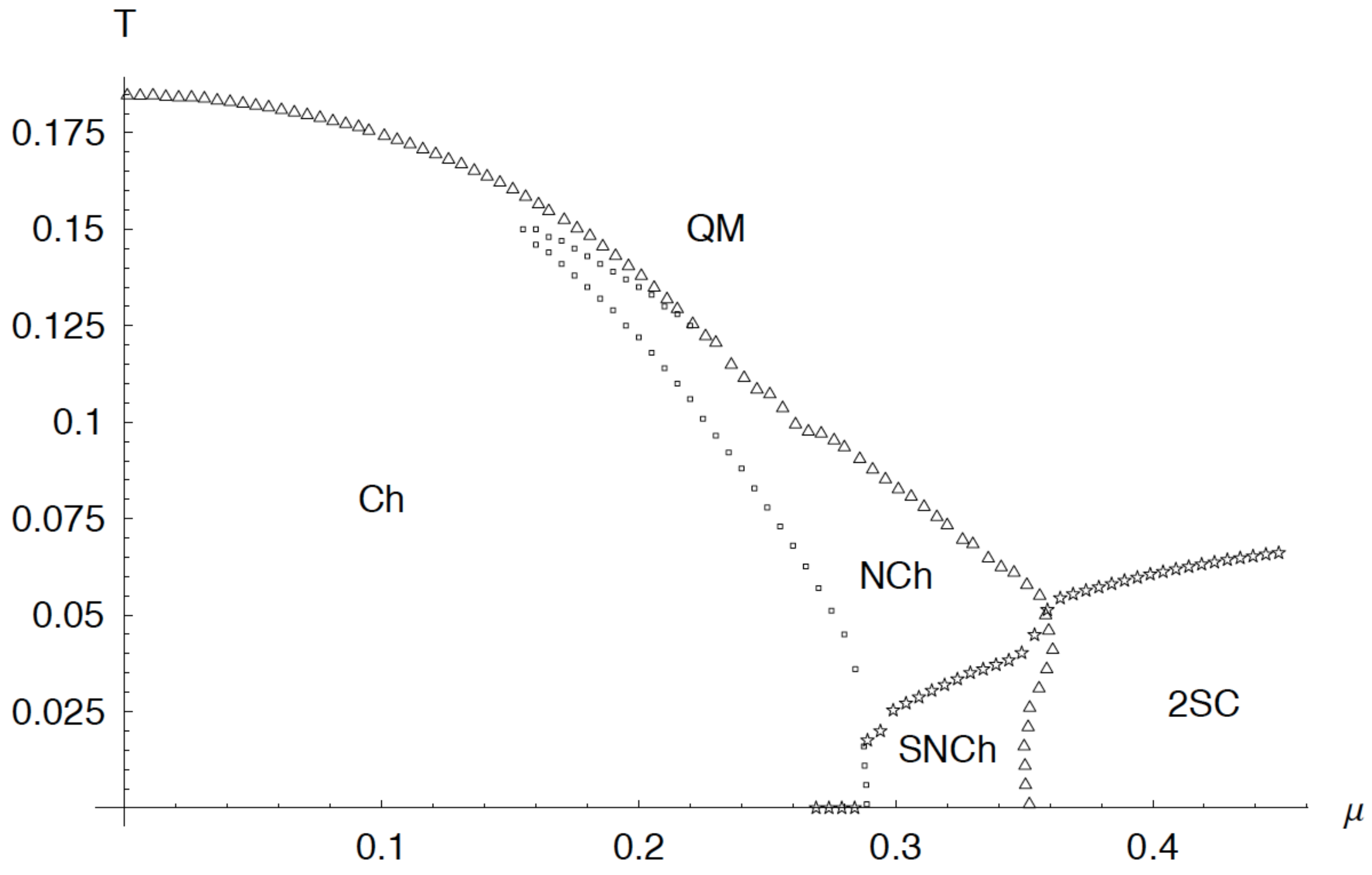
$$\Lambda^2_{QCD}/\mu^2_Q$$

Hidaka, Kojo, McLerran, Pisarski

Quarkyonic phase weakly breaks chiral symmetry



M. Sadzikowski, Phys. Lett. B642, (2006), 2006
with pion condensates

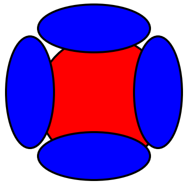


Inside the Quarkyonic Region:
Are there a large number of phase transitions corresponding to different nestings of chiral density waves on the Fermi surface?

Width of patch

$\sim \Lambda_{QCD}$

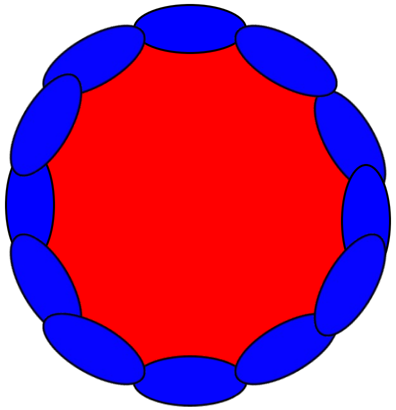
Kojo, Pisarski, Tsvelik



$\mu_Q \sim \Lambda_{QCD}$

Each change in number of patches is a phase transition. What does it mean for structure of Quarkyonic Crystal?
How do we interlace spirals in 3-d?

Are the phase changes first order?



$\mu_Q \sim \sqrt{N_c}\Lambda_{QCD}$

$N_{patches} \sim N_c$

When number of patches is of order Nc, one is approaching the edge of the quarkyonic region, where the transition should be weak, and the number of patches large so that contnuous translational symmetry is restored. In infinite Nc limit, is upper boundary of quarkyonic region a line of second order transitions?
Gentle steps from the beach becoming big steps near the cliffs

Note: Picture is for 2 spatial dimensional Fermi sea for visualization reasons. In reality patches cover surfaces of 3-d Fermi sphere

How should we think about quarkyonic matter?

At very high density a fermi sea of quarks surrounded by a fermi surface of confined nucleons is sensible

But what about when this sea first form and its fermi momentum is not so big?

Imagine the lowest mass nucleons are present and then we increase fermi energy. There will be higher nucleonic resonance that appear, but because of the mass, the fermi momentum will be lower than that of the nucleon. This happens successively until all the quark states associated with these resonances are occupied. This fully occupied Fermis sea we interpret as the quark fermi sea, since all quark fermi levels are occupied and it is in fact a degenerate but non perturbative gas of nucleon.
T. Kojo

For this to happen, nucleons resonance masses must come down perhaps by the mechanism of Marczenko, Redlich and Sasaki?

How to model the formation of quarkyonic matter?

Nucleons have very strong interactions among themselves and quarks do not?

Make a theory with both quark degrees of freedom and nucleon degrees of freedom.

Can model nucleon interaction by making an excluded volume model for the nucleons. Nucleon cores are exluded volume. When density approaches that of nucleonic cores, then the energy density is singular. Nucleons can have no higher density. High density is achieved by letting nucleon fill the Fermi surface until the critical density is achieved. Then a sea of quarks is filled to go to higher density
Kiesang Jeon, LM, Srimoyee Sen

Based on excluded volume considerations of Gorenstein et al.

Has a reasonable phenomenology.

How to construct a theory of with both nucleon and quark degrees of freedom?

Quarks and nucleons exist in differing regions of momentum space. When states are fully occupied, quarks block nucleons. Consider a theory with nucleon, quarks and ghosts:

$$T, \mu_N, \mu_Q, \mu_G$$

The physical nucleonic baryons are basically the N minus the G states

μ_N is the top of the Fermi sea for nucleons

μ_G is the bottom of the Fermi sea for nucleons

μ_Q is the top of the quark Fermi sea

Interactions of N and G field must be identical to cancel the nucleons in the occupied phase space of the quarks. For the quark interactions among themselves there is the action of QCD. Nucleon-quark interactions?

There is a relation between the ghost chemical potential and that of the quarks. In the additive quark parton model

$$\mu_G = N_c \mu_Q$$

With a mean field, in the additive quark parton model

$$\mu_G - gV = N_c \mu_Q$$

More generally, we expect that at the Fermi surface:

$$\frac{dN^B_G}{d^3k} = N_c^3 \frac{dN^B_Q}{d^3k}$$

The factor of N_c^3 $\frac{2\pi j N_c}{L}$ $\frac{2\pi (j+1) N_c}{L}$

misses states $\frac{2\pi (j N_c + k)}{L}$ Which are composed of quark states of slightly different momenta.

Such states remain blocked

Finally, there is the quark chemical potential. This is determined by extremizing the pressure, or at zero temperature, the energy per baryon at fixed total baryon number. This means that this chemical potential is dynamically determined. Similar to what occurs in the excluded volume models considered by Duarte, Jeong. Hernandiz-Ortiz and LM.

Advantages of this technique are that one can have an effective field theories for nucleons in combination with underlying dynamics for quarks, and a smooth continuation between such theories. Allows to match onto nuclear mean field theories. Dynamical generation of quarkyonic matter

$$
\begin{aligned}
S_E \;=\; & \int_0^\beta dt \int_V d^3x \left\{ \overline{N} \left(\frac{1}{i}\gamma \cdot \partial - i\mu_N \gamma^0 + M_N \right) N \right. \\
& + \overline{G} \left(\frac{1}{i}\gamma \cdot \partial - i\mu_G \gamma^0 + M_N \right) G \\
& \left. + \overline{Q} \left(\frac{1}{i}\gamma \cdot \partial - i\gamma^0 \mu_Q + M_Q \right) Q \right\} .
\end{aligned}
$$

Kieang Jeon,Dyana Duarte, Saul Hernandiz-Ortiz, LM

Kinetic energy term. Can include meson nucleon interactions, and QCD for quarks. Nucleon-quark interactions?

How to find quarkyonic matter:

$$\epsilon(\rho_N = \rho_B + \rho_G - \rho_Q, \rho_G, \rho_Q)$$

Minimize with respect to the quark density at fixed total baryon density.

$$d\rho_G/d\rho_Q = N_c^3$$

The determine:

$$d\epsilon/dn_B = \mu_B$$

And require that the pressure be maximum at any minima found for the energy density, including the end point minimum at zero quark density. If there are two possible values with equal pressure, then do a Maxwell construction

A. Sorensen, D. Oliinychunko, V. Koch, LM

Cummulents and the Speed of Sound: Very Low T

$$\kappa_1 = V n_B \ , \qquad \kappa_2 = \frac{V T n_B}{\left(\frac{dP}{dn_B}\right)_T} \ ,$$

$$\kappa_3 = \frac{V T^2 n_B}{\left(\frac{dP}{dn_B}\right)_T^2} \left[1 - \frac{n_B}{\left(\frac{dP}{dn_B}\right)_T} \left(\frac{d^2 P}{dn_B^2}\right)_T \right]$$

At low temperature:

$$\left(\frac{d \ln c_T^2}{d \ln n_B}\right)_T + c_T^2 \approx 1 - \frac{\kappa_3 \kappa_1}{\kappa_2^2}$$

Summary:

How do we include finite temperature?

How do we construct a transport theory that allows the production of quarkyonic matter?

Can we construct realistic theories with nucleon, ghosts and quarks? How to determine the ghost fermi energy?

Can we see evidence for a large sound velocity in low energy heavy ion collisions?
Sorensen, Oliinuchenko, Koch LM

Are there alternative viable methods of achieving large sound velcotiy and small trace anomaly? Bose condensation al a Son and Stephanov, Hadron models al a Marczenko, Redlich and Sasaki?

Relationship to phase transitions and critical end point? Blaschke.