Transport and Hybrid Approaches

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June 23rd, 2022, Karpacz Summer School Lecture

The Plan for Lecture I

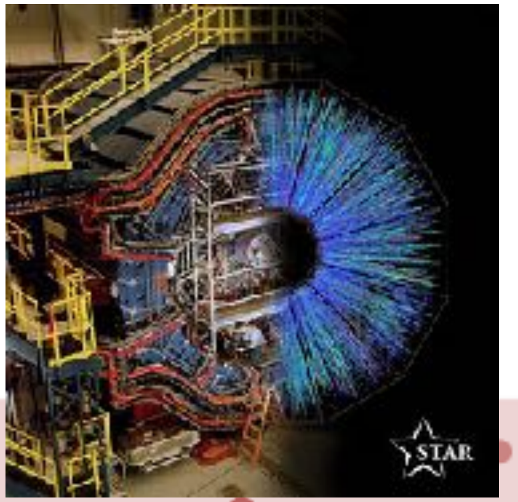
- Heavy-ion collision dynamics
- Transport approaches
  - Boltzmann equation
  - Degrees of freedom and their interactions
  - Resonances and string excitations
- A specific approach: SMASH
  - General ingredients and validation
  - Bulk observables and equation of state
  - Transport coefficients of the hadron gas
- Summary & questions (please ask also during the talk!)

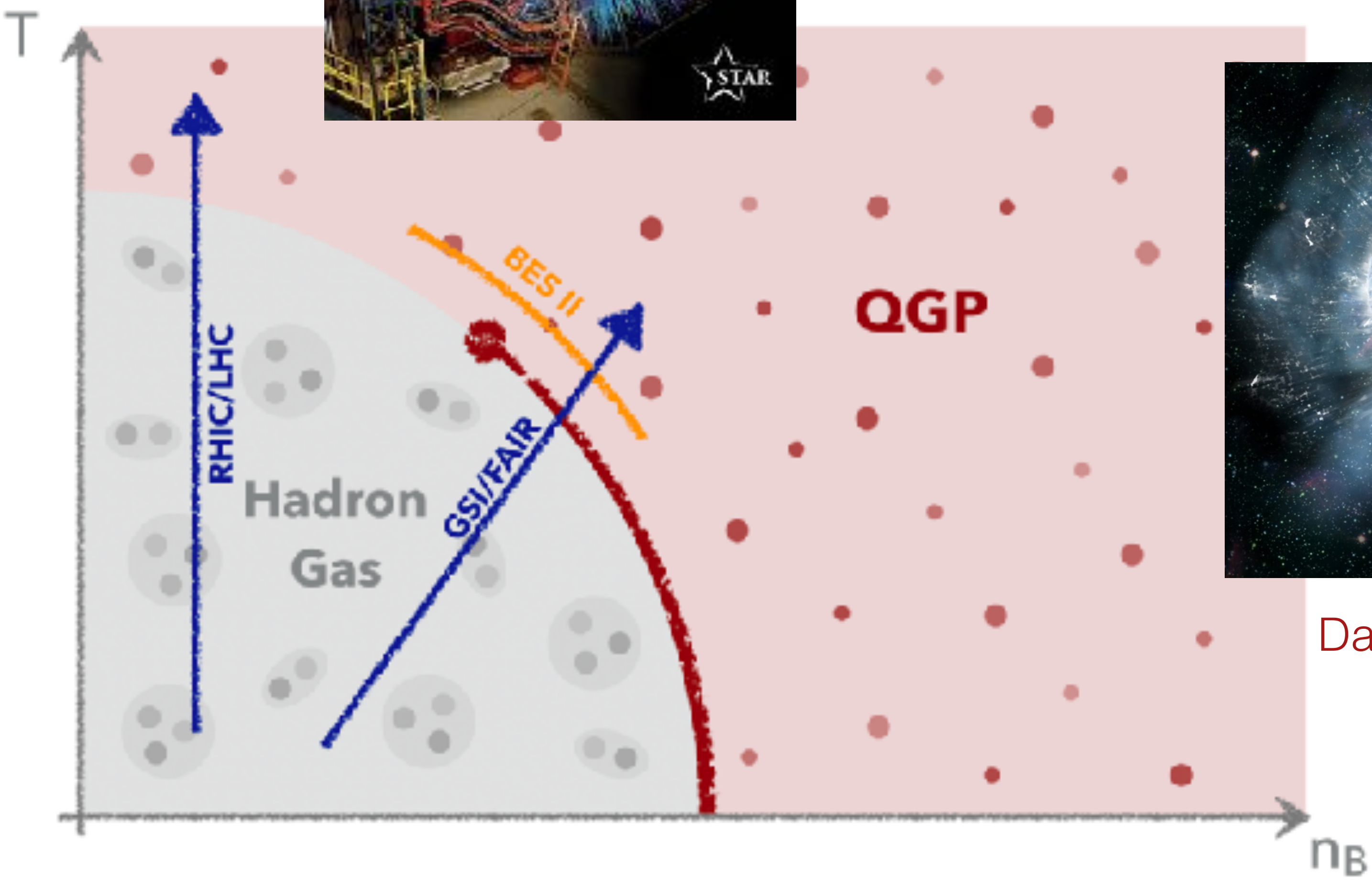
Introduction

The QCD Phase Diagram

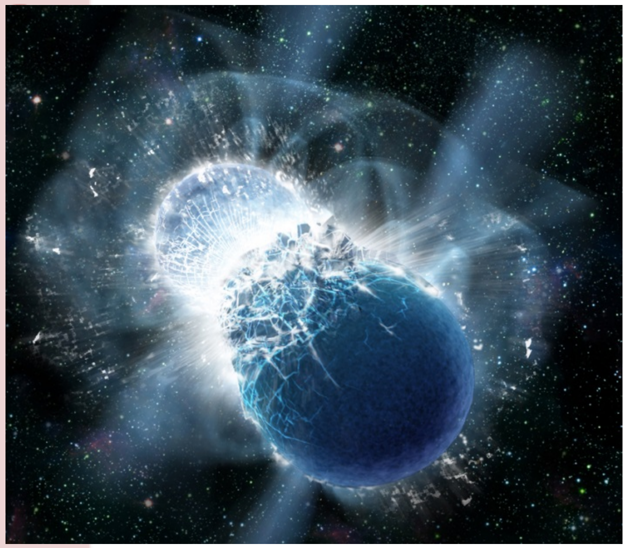
- **Main goals** of heavy-ion research:

STAR experiment at RHIC





- What are the relevant degrees of freedom at high densities?
- Phase transition, critical endpoint?
- Properties of neutron star mergers?

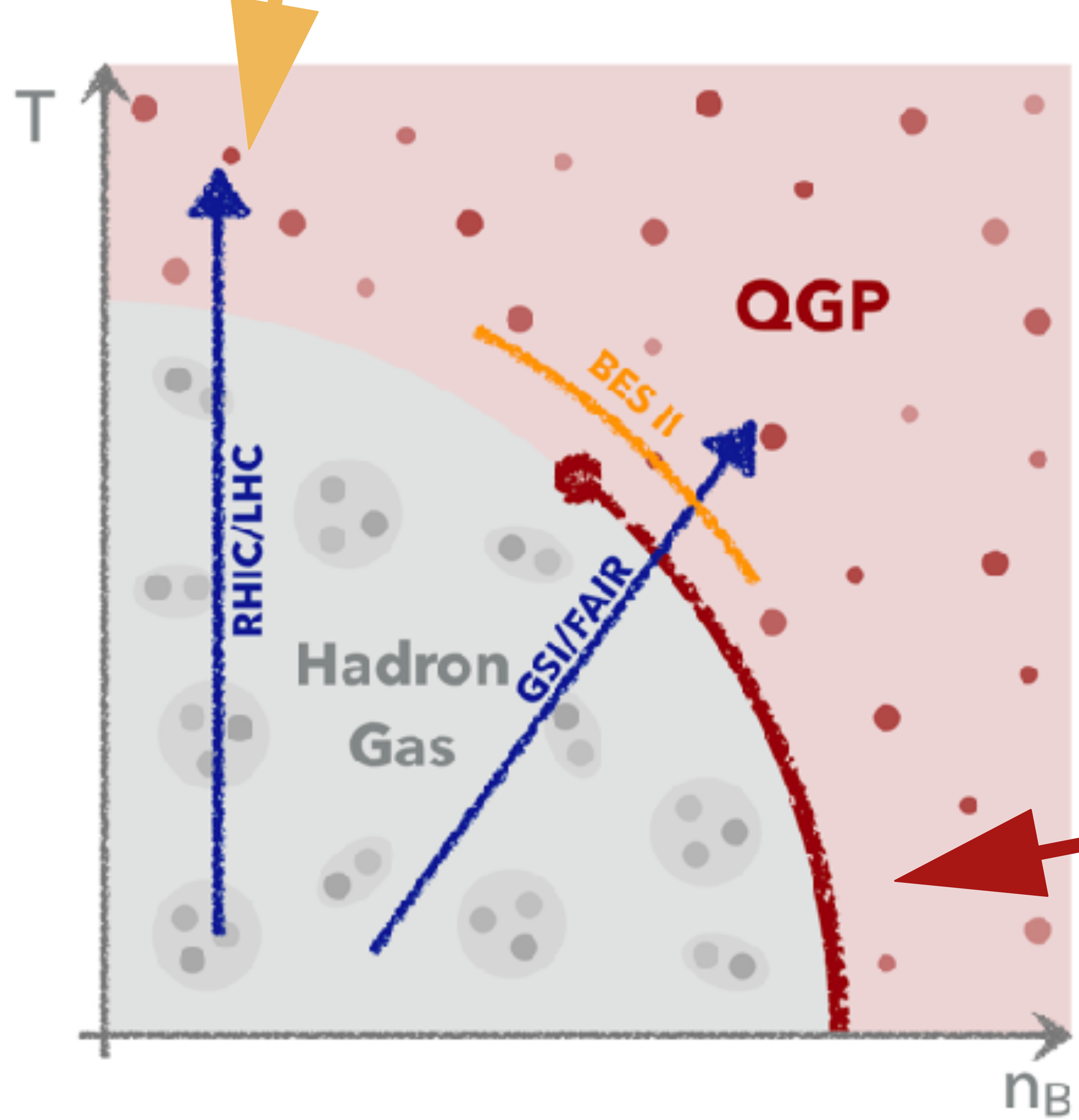


Dana Berry, SkyWorks Digital, Inc

Relevant for neutron star mergers as detected by gravitational waves (GW170817)

Dynamical Modeling

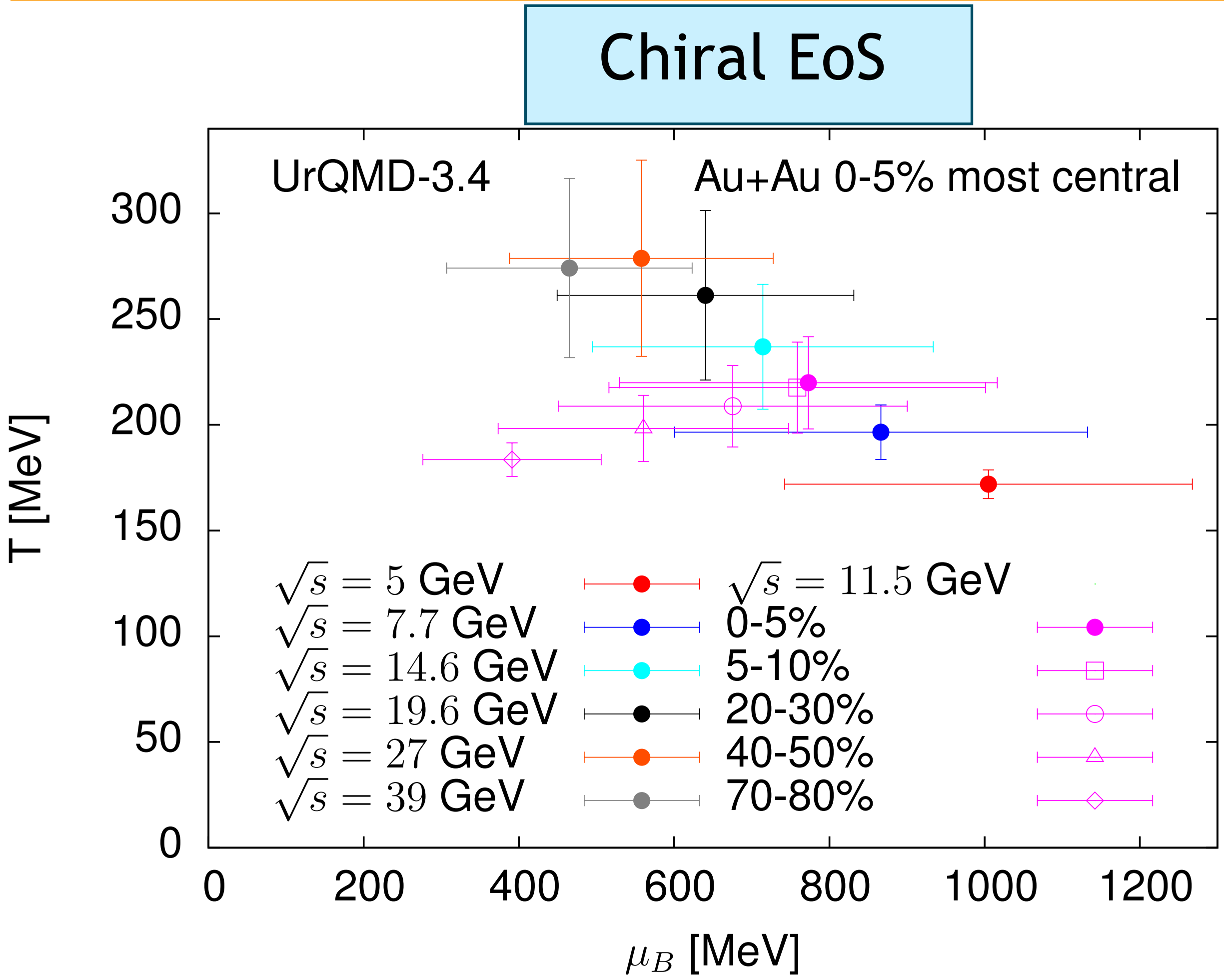
Standard approach at high energies
- Non-equilibrium initial evolution
- Viscous hydrodynamics
- Hadronic rescattering

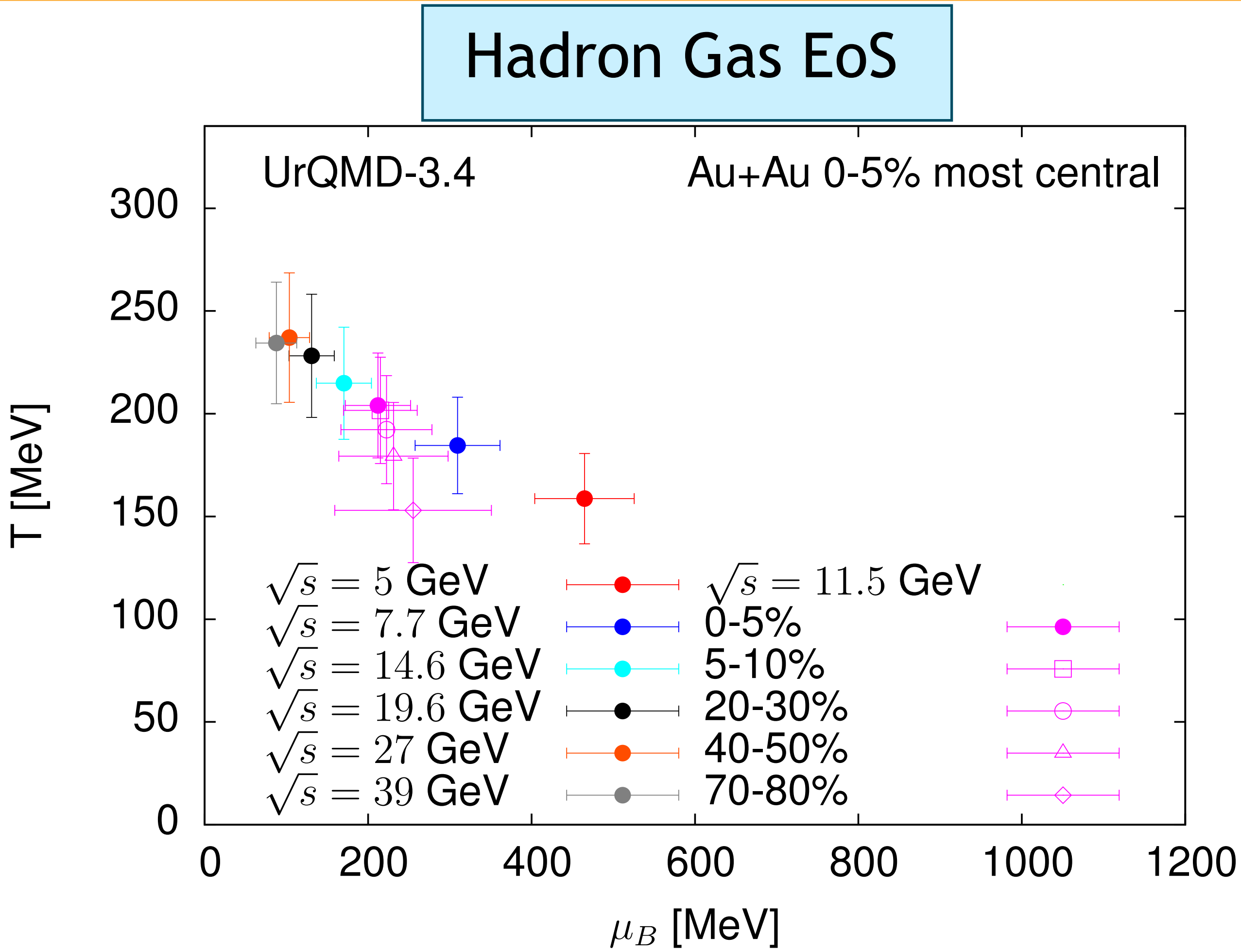


- ***Status***: Two regimes with well-established approaches
- Goals:
  - Constraint on the equation of state of nuclear matter
  - Limit of applicability of hadronic transport approaches
  - Qualitative signatures of first order phase transition and critical point

Standard approach at low beam energies
- Hadronic transport approaches
- Resonance dynamics
- Nuclear potentials

Experimental Access to Phase Diagram





G. Gräf, J. Steinheimer, UrQMD-3.4 on urqmd.org

- Event-by-event fluctuations negligible, but sizable spread in **single** events ➞ Different centralities increase spanned regions
- Absolute values are highly dependent on the Equation of State/ degrees of freedom
- Scan in rapidity windows might allow to divide spacing even more

FAIR Construction Site

- FAIR construction is in progress on GSI campus

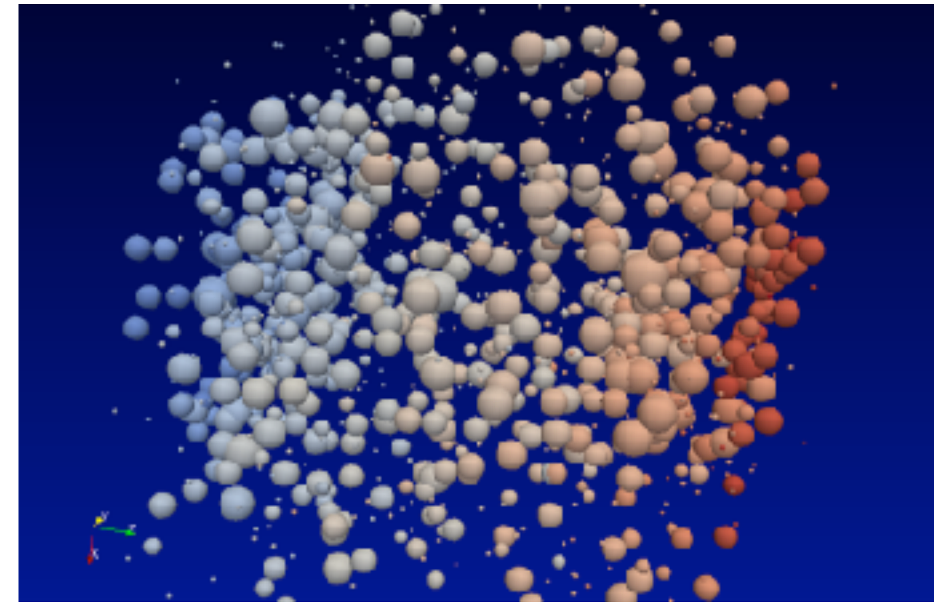
May 2020, SIS-100 tunnel,
GSI Helmholtzzentrum für Schwerionenforschung, D. Fehrenz/GSI/FAIR

Visualization of FAIR,
GSI Helmholtzzentrum für Schwerionenforschung, ion42

- High luminosity at beam energies up to Au+Au at 11A GeV

Transport Approaches

Transport Approaches

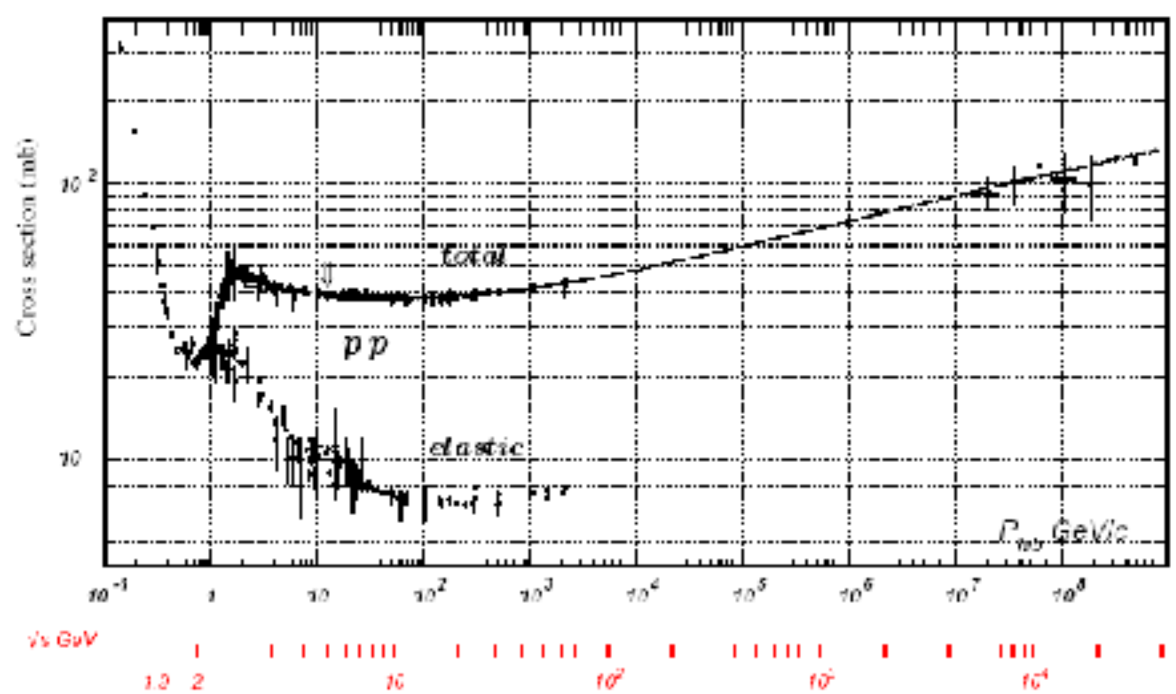


- Transport of a **non-equilibrium** system of microscopic particles
- Either hadronic **or** partonic degrees of freedom
- Effective solution of **Boltzmann equation**

Collision Criterion

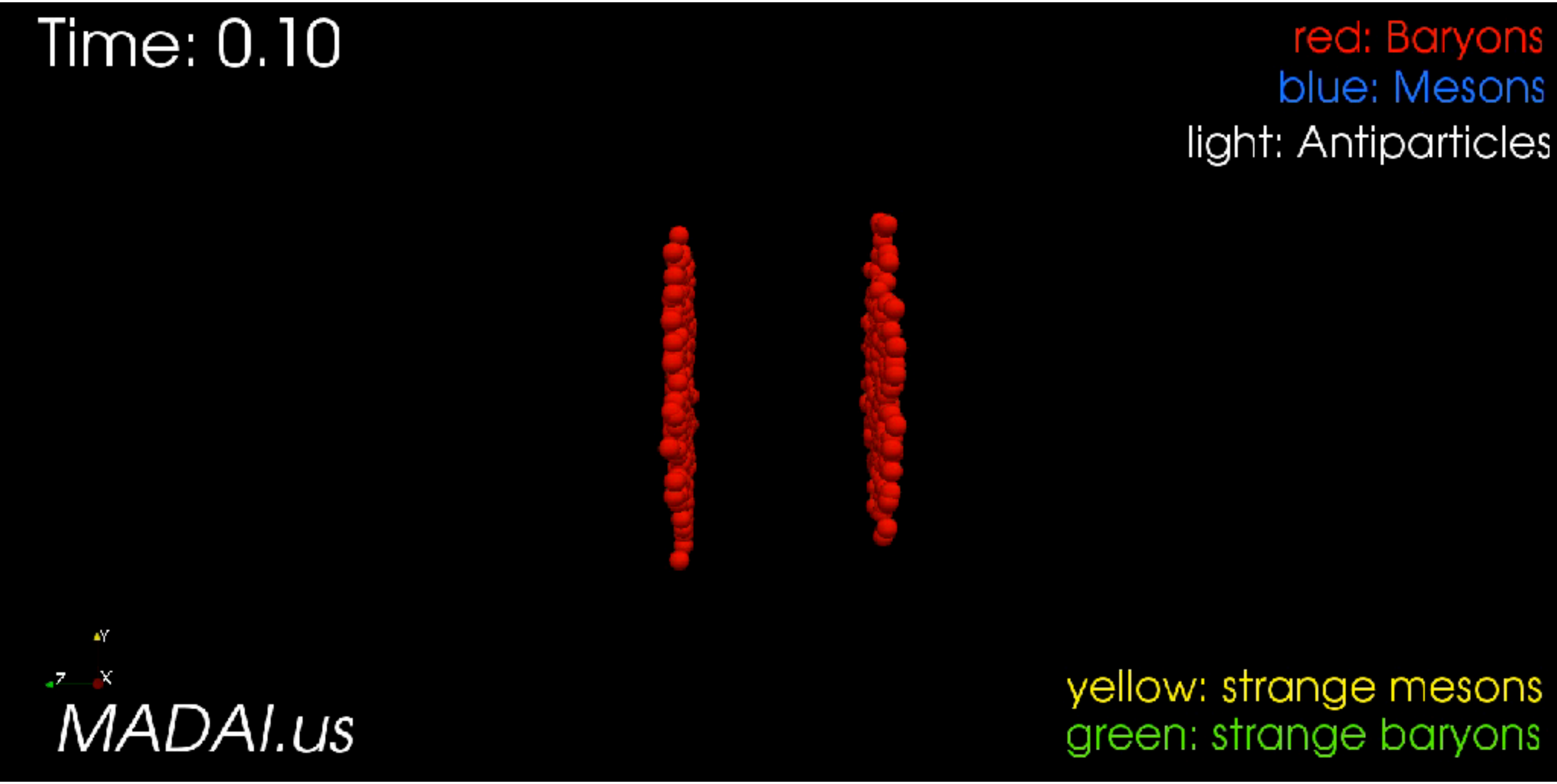
$$d_{\min} \le d_0 = \sqrt{\frac{\sigma_{tot}}{\pi}}$$

$$\left[\frac{\partial}{\partial t} + \frac{\vec{p}}{E} \times \frac{\partial}{\partial \vec{r}}\right] f_1(\vec{p},\vec{r},t) = \sum_{processes} C(\vec{p},\vec{r},t)$$



- Point particles, binary collisions
- Whole evolution and history of the evolution including **full phase-space information**

Wealth of Particles



- Different **species** are highlighted by color, the size indicates the mass of the particles

Highlight Interactions

- Light colors flash **interactions**, all the interactions are traced in this simulation

Boltzmann Equation

- Transport models are solving a (modified and/or complicated) version of the Boltzmann equation
- Non-relativistic:

$$\frac{df_i(x,p)}{dt} = \frac{\partial f}{\partial x}\frac{dx}{dt} + \frac{\partial f}{\partial p}\frac{dp}{dt} = C(f_i, f_j)$$

$$\frac{\partial f_i}{\partial x} v - \frac{\partial f_i}{\partial p} \nabla V = C(f_i, f_j)$$

- Solutions are $f_i(x,p,t)$, the single-particle distribution functions
- Coupled integro-differential equations for multiple particle species

A ‘real’ Equation for Low Energies

- Starting point is the Lagrangian:
  Free part:

$$
\begin{aligned}
\mathcal{L}_{\mathrm{F}} &= \bar{\psi}[\mathrm{i}\gamma_\mu\partial^\mu - M_N]\psi + \bar{\psi}^*[\mathrm{i}\gamma_\mu\partial^\mu - M_{N^*}]\psi^* \\
&\quad + \bar{\psi}_{\Delta\nu}[\mathrm{i}\gamma_\mu\partial^\mu - M_\Delta]\psi_\Delta^\nu + \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - U(\sigma) \\
&\quad - \frac{1}{4}\omega_{\mu\nu}\omega^{\mu\nu} + U(\omega) + \frac{1}{2}(\partial_\mu\boldsymbol{\pi}\partial^\mu\boldsymbol{\pi} - m_\pi^2\boldsymbol{\pi}^2)
\end{aligned}
$$

From G. Mao

+Interactions +Approximations and Transformations

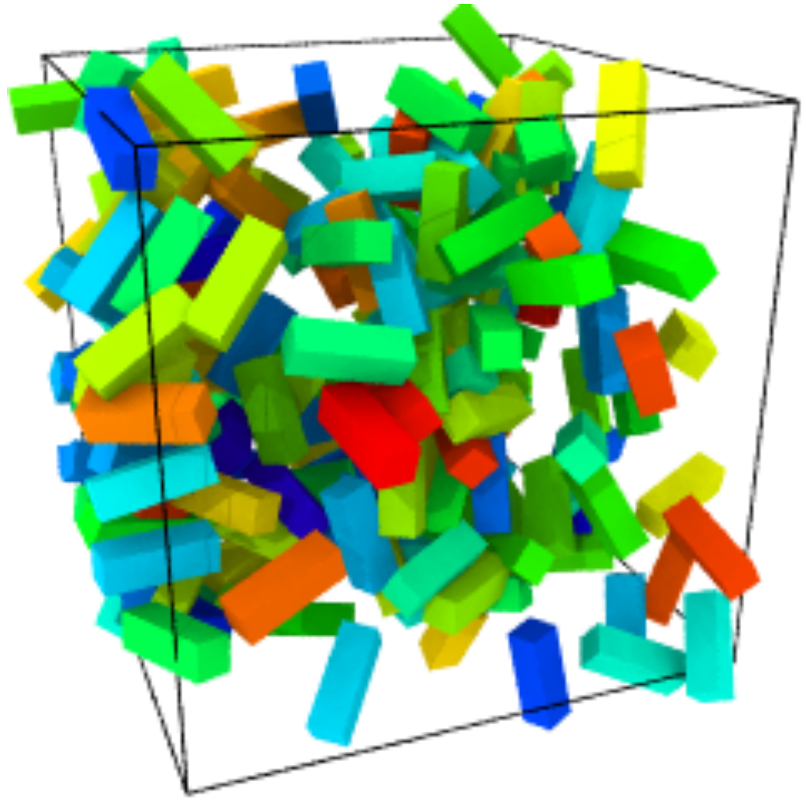
$$\begin{aligned}
\mathcal{L}_I &= \mathcal{L}_{NN} + \mathcal{L}_{N^*N^*} + \mathcal{L}_{\Delta\Delta} + \mathcal{L}_{NN^*} + \mathcal{L}_{\Delta N} + \mathcal{L}_{\Delta N^*} \\
&= g^{\sigma}_{NN}\bar{\psi}(x)\psi(x)\sigma(x) - g^{\omega}_{NN}\bar{\psi}(x)\gamma_\mu\psi(x)\omega^\mu(x) + g^{\pi}_{NN}\bar{\psi}(x)\gamma_\mu\gamma_5\boldsymbol{\tau}\cdot\psi(x)\partial^\mu\boldsymbol{\pi}(x) \\
&\quad + g^{\sigma}_{N^*N^*}\bar{\psi}^*(x)\psi^*(x)\sigma(x) - g^{\omega}_{N^*N^*}\bar{\psi}^*(x)\gamma_\mu\psi^*(x)\omega^\mu(x) - g^{\pi}_{N^*N^*}\bar{\psi}^*(x)\gamma_\mu\gamma_5\boldsymbol{\tau}\cdot\psi^*(x)\partial^\mu\boldsymbol{\pi}(x) \\
&\quad + g^{\sigma}_{\Delta\Delta}\bar{\psi}_{\Delta\nu}(x)\psi^\nu_\Delta(x)\sigma(x) - g^{\omega}_{\Delta\Delta}\bar{\psi}_{\Delta\nu}(x)\gamma_\mu\psi^\nu_\Delta(x)\omega^\mu(x) + g^{\pi}_{\Delta\Delta}\bar{\psi}_{\Delta\nu}(x)\gamma_\mu\gamma_5\mathbf{T}\cdot\psi^\nu_\Delta(x)\partial^\mu\boldsymbol{\pi}(x) \\
&\quad + [g^{\sigma}_{NN^*}\bar{\psi}^*(x)\psi(x)\sigma(x) - g^{\omega}_{NN^*}\bar{\psi}^*(x)\gamma_\mu\psi(x)\omega^\mu(x) + g^{\pi}_{NN^*}\bar{\psi}^*(x)\gamma_\mu\gamma_5\boldsymbol{\tau}\cdot\psi(x)\partial^\mu\boldsymbol{\pi}(x) \\
&\quad - g^{\pi}_{\Delta N}\bar{\psi}_{\Delta\mu}(x)\partial^\mu\boldsymbol{\pi}(x)\cdot\mathbf{S}^+\psi(x) - g^{\pi}_{\Delta N^*}\bar{\psi}_{\Delta\mu}(x)\partial^\mu\boldsymbol{\pi}(x)\cdot\mathbf{S}^+\psi^*(x) + H.c.] \\
&= g^{A}_{NN}\bar{\psi}(x)\Gamma^N_A\psi(x)\Phi_A(x) + g^{A}_{N^*N^*}\bar{\psi}^*(x)\Gamma^{N^*}_A\psi^*(x)\Phi_A(x) + g^{A}_{\Delta\Delta}\bar{\psi}_{\Delta\nu}(x)\Gamma^\Delta_A\psi^\nu_\Delta(x)\Phi_A(x) \\
&\quad + [g^{A}_{NN^*}\bar{\psi}^*(x)\Gamma^{N^*}_A\psi(x)\Phi_A(x) - g^{\pi}_{\Delta N}\bar{\psi}_{\Delta\mu}(x)\partial^\mu\boldsymbol{\pi}(x)\cdot\mathbf{S}^+\psi(x) \\
&\quad - g^{\pi}_{\Delta N^*}\bar{\psi}_{\Delta\mu}(x)\partial^\mu\boldsymbol{\pi}(x)\cdot\mathbf{S}^+\psi^*(x) + h.c.] \qquad (
\end{aligned}$$

Derived Transport (Boltzmann) equation:

$$\{p_\mu[\partial^\mu_x - \partial^\mu_x\Sigma^\nu_{N^*}(x)\partial^p_\nu + \partial^\nu_x\Sigma^\mu_{N^*}(x)\partial^p_\nu] + m^*_{N^*}\partial^\nu_x\Sigma^S_{N^*}(x)\partial^p_\nu\}\frac{f_{N^*}(\mathbf{x},\mathbf{p},t)}{E^*_{N^*}(p)}$$
$$= C^{N^*}(x,p).$$

How to Solve the Equation?

- Simplification: omit the potential (cascade mode)
  - Straight line trajectories between collisions
  - At point of closest approach interactions are possible as given by cross section
- Other methods:
  - Test particles, transition probabilities
- Problems:
  - Multi-particle collisions
  - Consistency between ‘potential’ and cross section
  - Memory effects (Markovian limit),…



Reaction Stages

- Initialization of projectile and target (Lorentz-contracted Woods-Saxon)
- Generate table with collision/decay sequence from

$$d_{min} = \sqrt{\frac{\sigma_{tot}}{\pi}} \quad , \sigma_{tot} = \sigma_{tot}(\sqrt{s}, |h_1\rangle, |h_2\rangle)$$

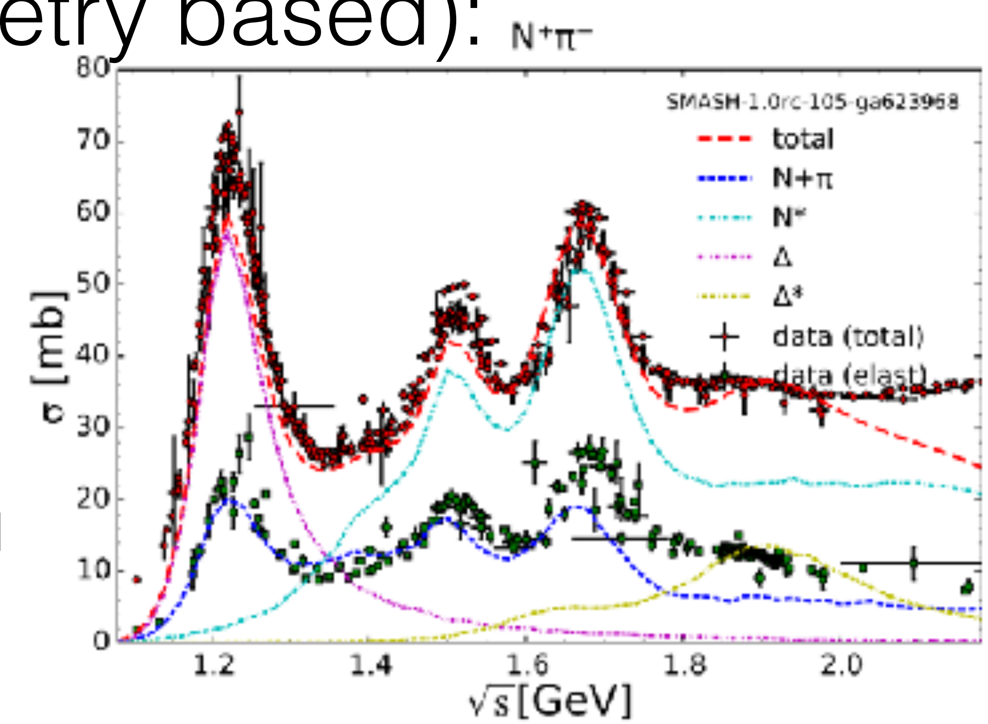
- Propagate to next collision
- Perform collision according to cross sections
  - Elastic scattering
  - Inelastic scattering
    - Resonance production
    - Soft string formation and fragmentation
    - pQCD hard scattering / fragmentation
- Update particle arrays, update collision table, perform next collisions

Monte Carlo methods

Transport/Cascade Models

- QMD, IQMD, BQMD (non-relativistic models, limited particle species)
- RQMD (the grandfather of relativistic transport models) development stopped around 2000
- UrQMD (development started 1996 at Frankfurt)
- (P)HSD (Giessen, Cassing), GiBUU (Giessen, Mosel)
- Parton cascades (ZPC, MPC, GPC, VNI/B, BAMPS….)
- NOT transport/cascade models (geometry based):
  - HIJING
  - PYTHIA/FRITIOF
  - NEXUS, VENUS
  - DPM

SMASH (C++, FFM)
= Simulating Many Accelerated Strongly-Interacting Hadrons



Included Particles

N	Δ	Λ	Σ	Ξ	Ω
938	1232	1116	1192	1317	1672
1440	1600	1405	1385	1530	
1520	1620	1520	1660	1690	
1535	1700	1600	1670	1820	
1650	1900	1670	1790	1950	
1675	1905	1690	1775	2025	
1680	1910	1800	1915		
1700	1920	1810	1940		
1710	1930	1820	2030		
1720	1950	1830			
1990†		2100			
2080		2110			
2190					
2200					
2250					

0^{-+}	1^{--}	0^{++}	1^{++}
π	ρ	a_0	a_1
K	K^*	K_0^*	K_1^*
η	ω	f_0	f_1
η'	ϕ	f_0^*	f_1'

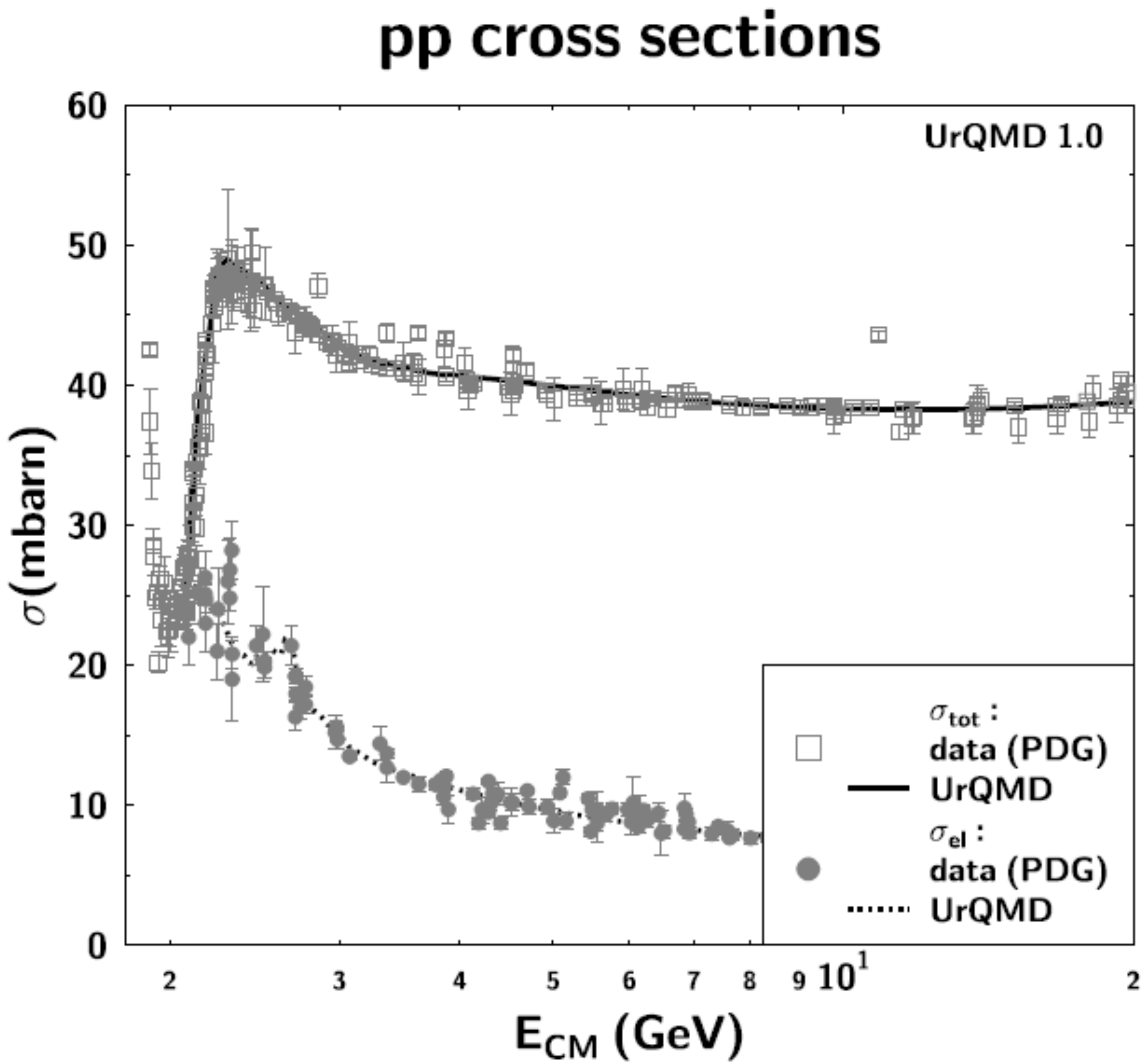
1^{+-}	2^{++}	$(1^{--})^*$	$(1^{--})^{**}$
b_1	a_2	ρ_{1450}	ρ_{1700}
K_1	K_2^*	K_{1410}^*	K_{1680}^*
h_1	f_2	ω_{1420}	ω_{1662}
h_1'	f_2'	ϕ_{1680}	ϕ_{1900}

UrQMD particle list

- Problem: at least 60x60 cross sections needed →need to group into classes

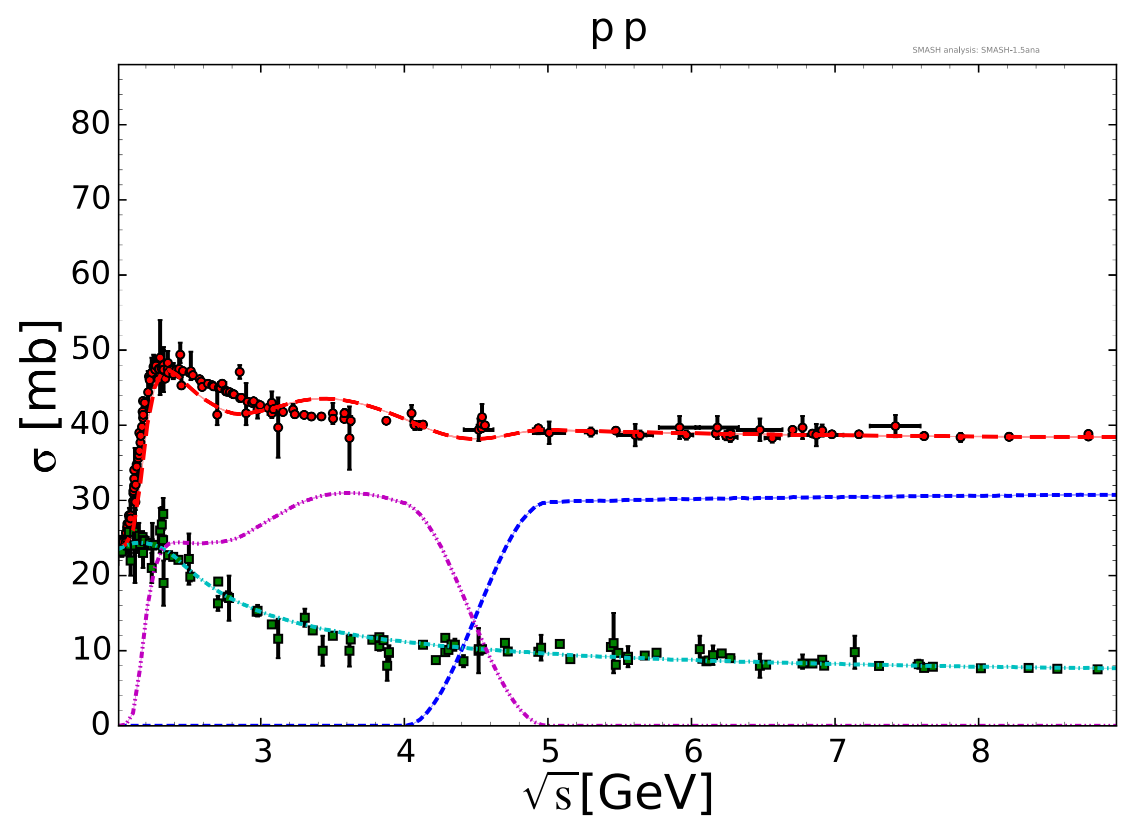
Cross-Sections and Processes

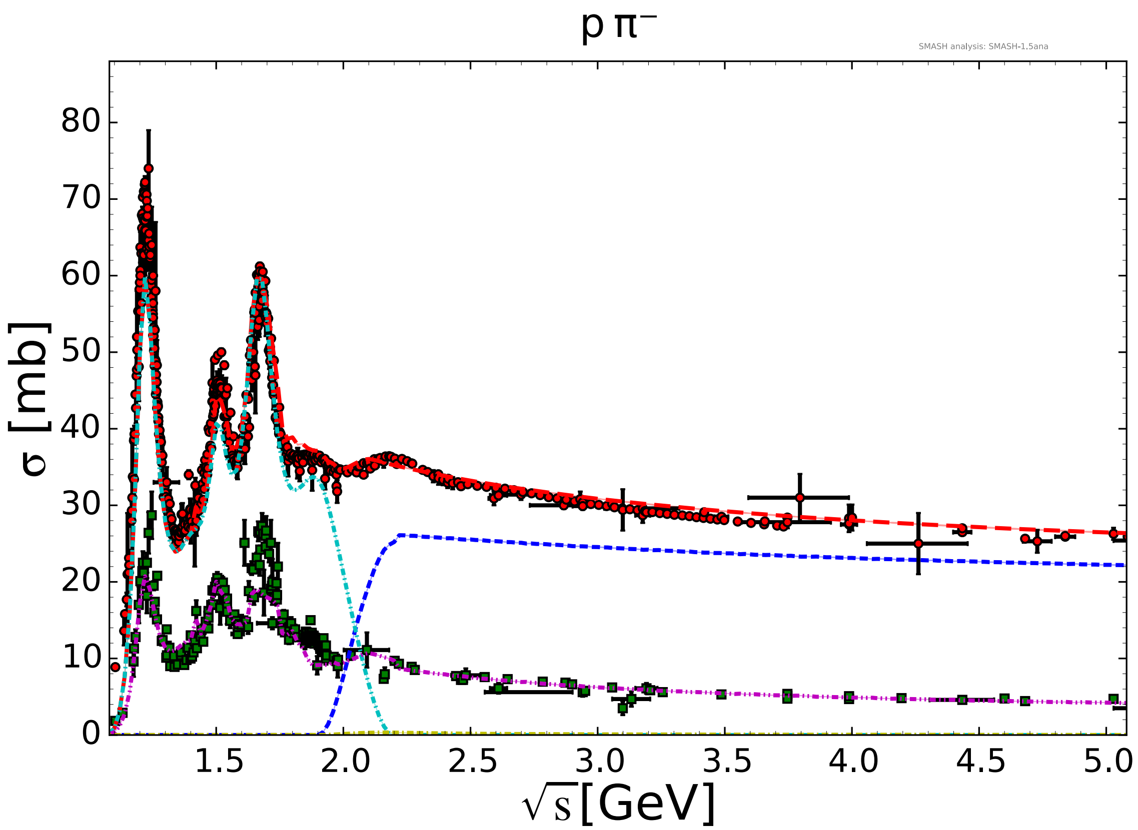
Cross-Sections



fitted to data

Elementary Cross Sections





- Total cross section for pp/pπ collisions
- Parametrized elastic cross section
- Many resonance contributions to inelastic cross section
- Reasonable description of experimental data
- Strings at higher energies

J. Weil et al, PRC 94 (2016), updated SMASH-1.5

Scattering Cross Section

- Phase space x Matrix element

$$d\sigma_{a\to b} = \frac{|\mathcal{M}|^2}{64\pi^2 s}\frac{\vec{p_b}}{\vec{p_a}}d\Omega\prod_{i=3}^{4}d\mu_i$$

$$d\mu_i = \begin{cases} \delta(p_i^2 - m_i^2)\mathrm{d}p_i^2 & : \\ \int \frac{\Gamma}{(m-m_i)^2 + \Gamma^2/4}\frac{\mathrm{d}m}{2\pi} & : \end{cases}$$

hard mass shell condition (stable particles)

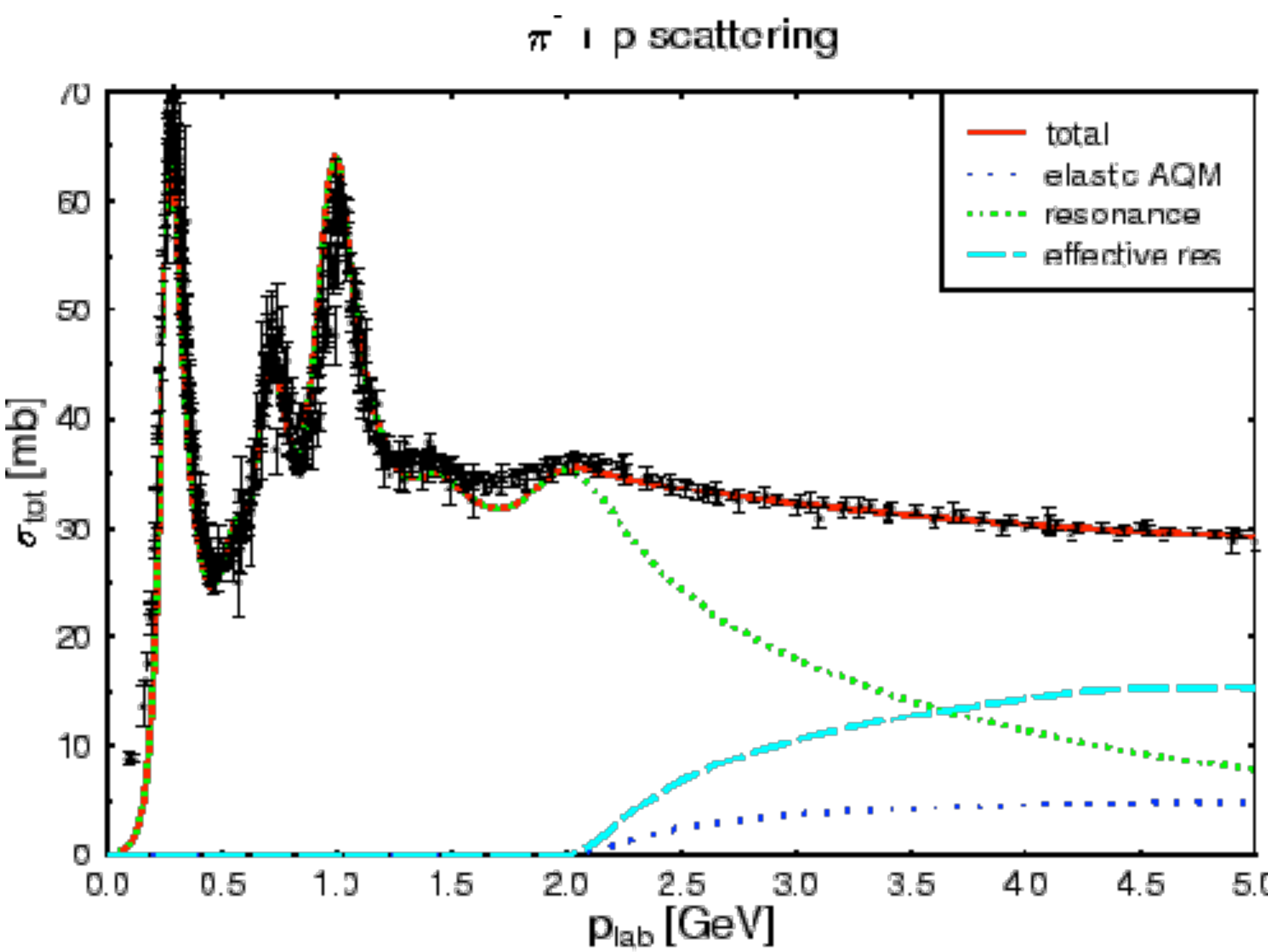
soft mass shell condition (resonances)

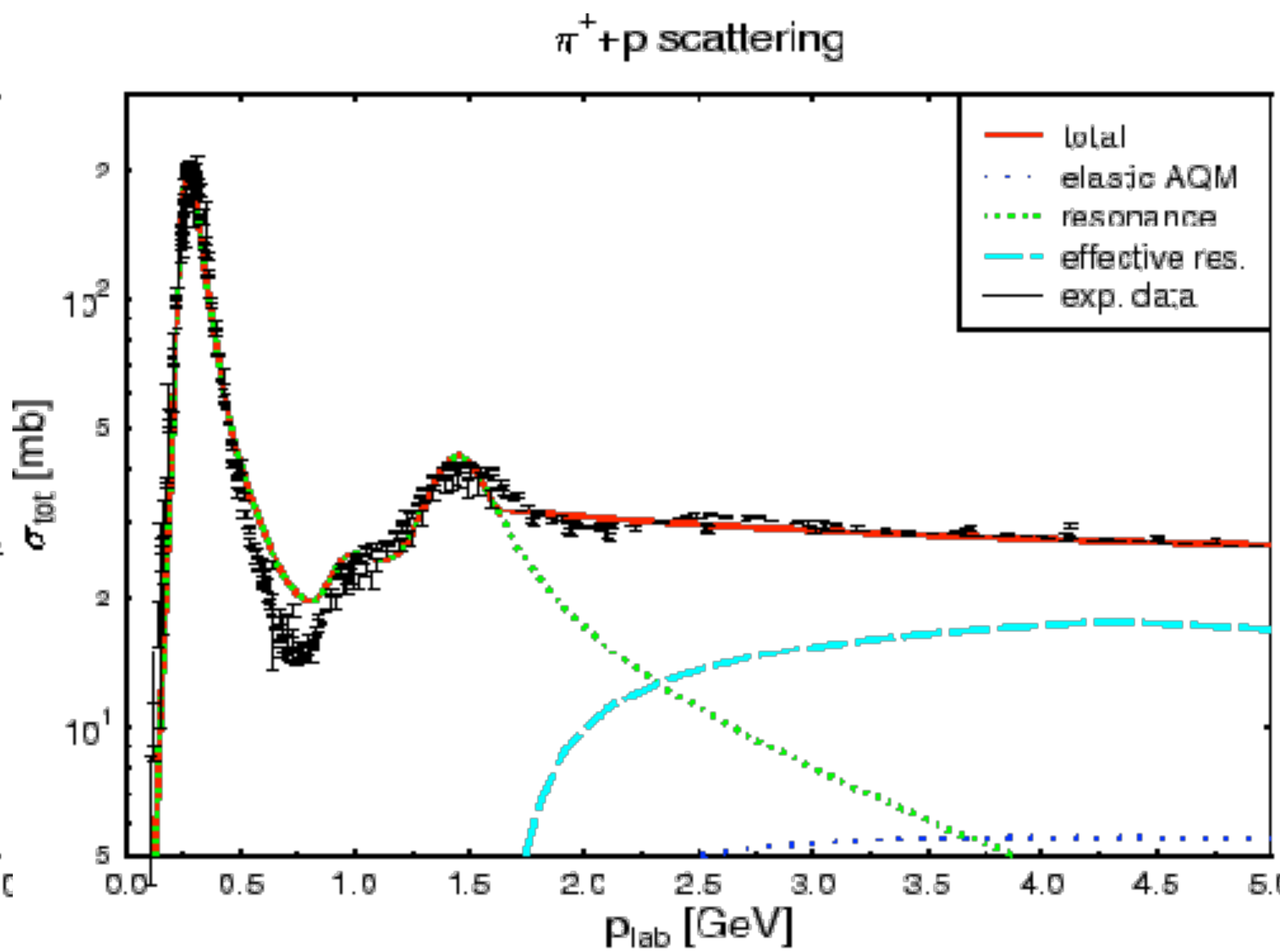
$$\frac{\mathrm{d}\sigma_{b\to a}}{\mathrm{d}\Omega} = \frac{\langle p_a^2\rangle}{\langle p_b^2\rangle}\frac{(2S_1+1)(2S_1+1)}{(2S_3+1)(2S_4+1)}\sum_{J=J_-}^{J_+}\frac{\langle j_1 m_1 j_2 m_2 \| JM\rangle^2}{\langle j_3 m_3 j_4 m_4 \| JM\rangle^2}\frac{d\sigma_{a\to b}}{d\Omega}.$$

take care of Clebsch-Gordans

- Physics is in Matrix elements

Resonance Cross Sections

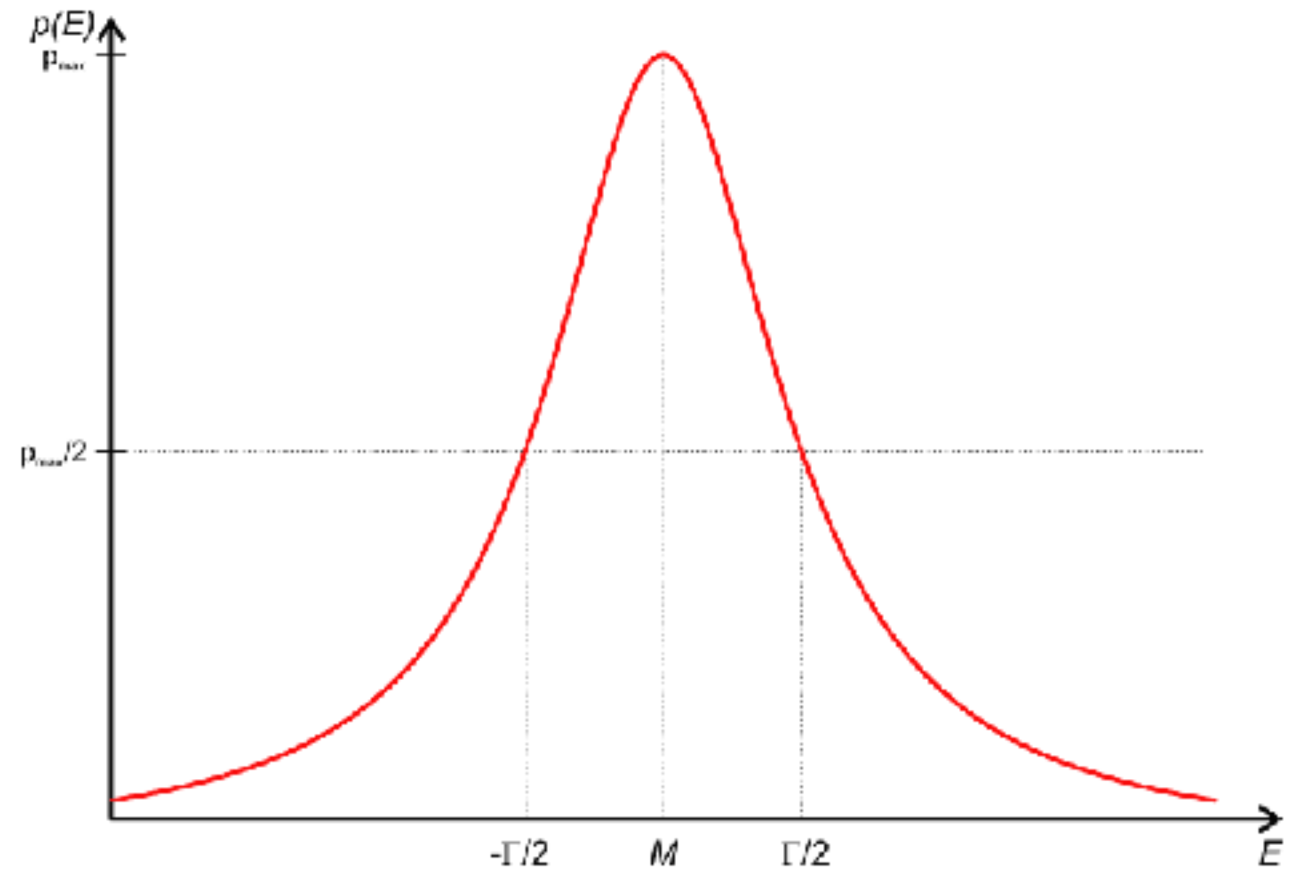




$$\sigma_{tot}^{MB}(\sqrt{s}) = \sum_{R=\Delta,N^*} \langle j_B, m_B, j_M, m_M \| J_R, M_R \rangle \frac{2S_R+1}{(2S_B+1)(2S_M+1)} \times \frac{\pi}{p_{CMS}^2} \frac{\Gamma_{R\to MB}\Gamma_{tot}}{(M_R-\sqrt{s})^2+\frac{\Gamma_{tot}^2}{4}}$$

Resonances

- Approximation to spectral function with Breit-Wigner distribution
- Peak corresponds to mass
- Lifetime is inverse width
- Are propagated explicitly in many transport approaches



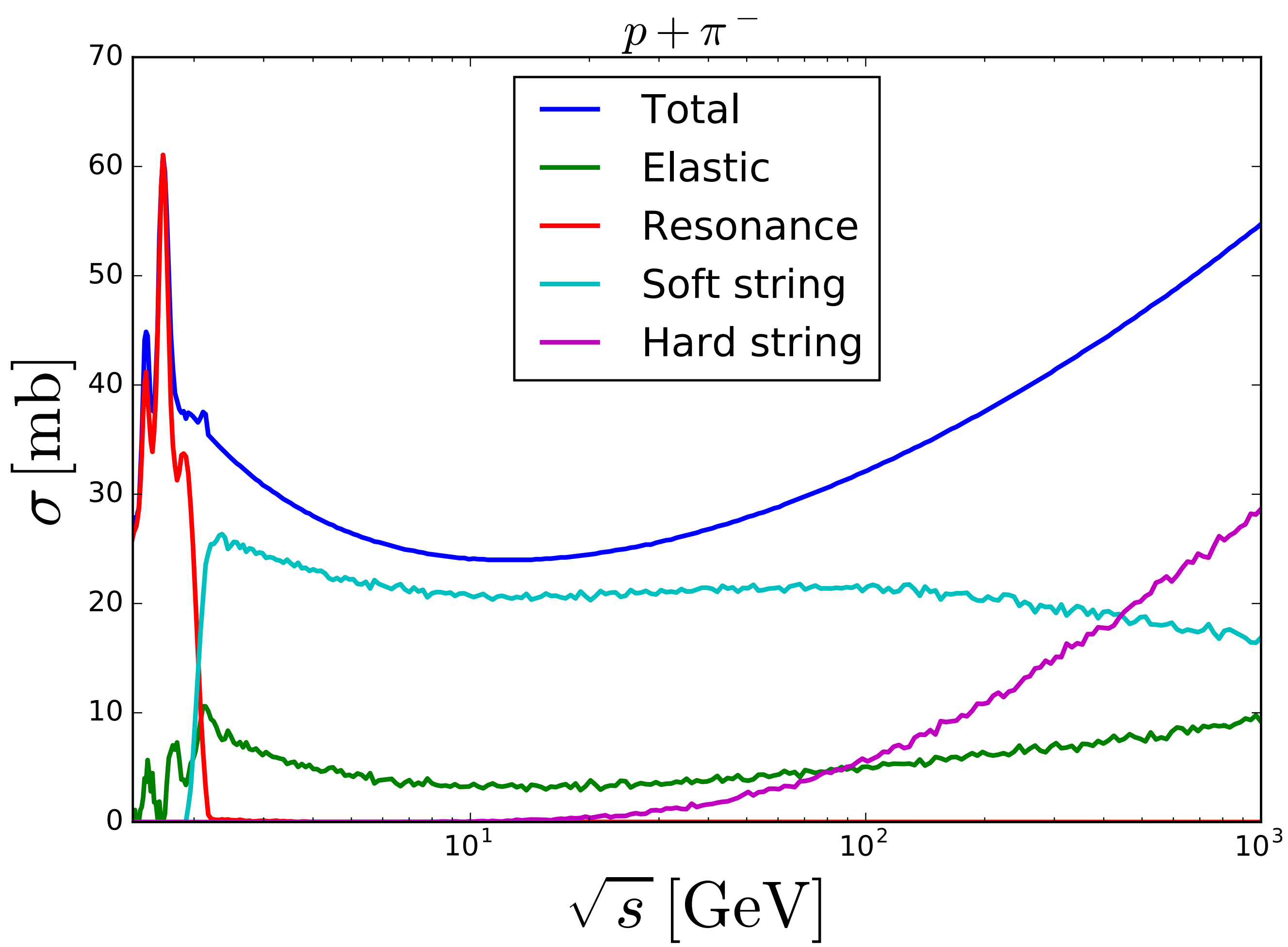
- Important to differentiate:
  - Vacuum parameters from PDG or fits to data
  - Dynamical collisional broadening, rescattering of resonances
  - Actual medium modifications of spectral functions

Moving to Higher Energies

- High energy cross-section is dominated by string excitation and fragmentation

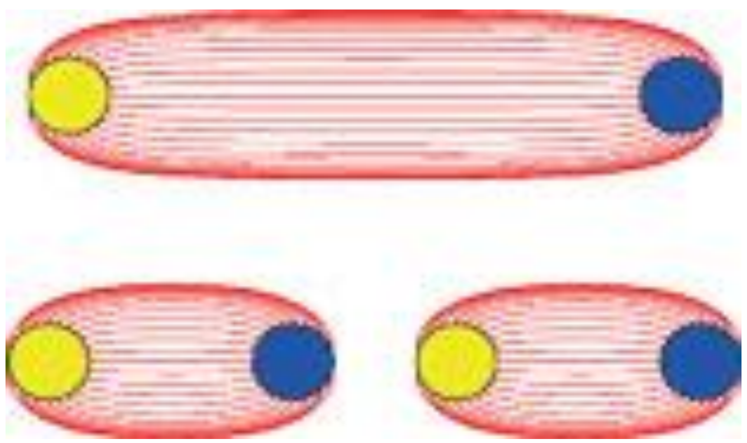
J. Mohs, S. Ryu and HE, *J.Phys.G* 47 (2020)

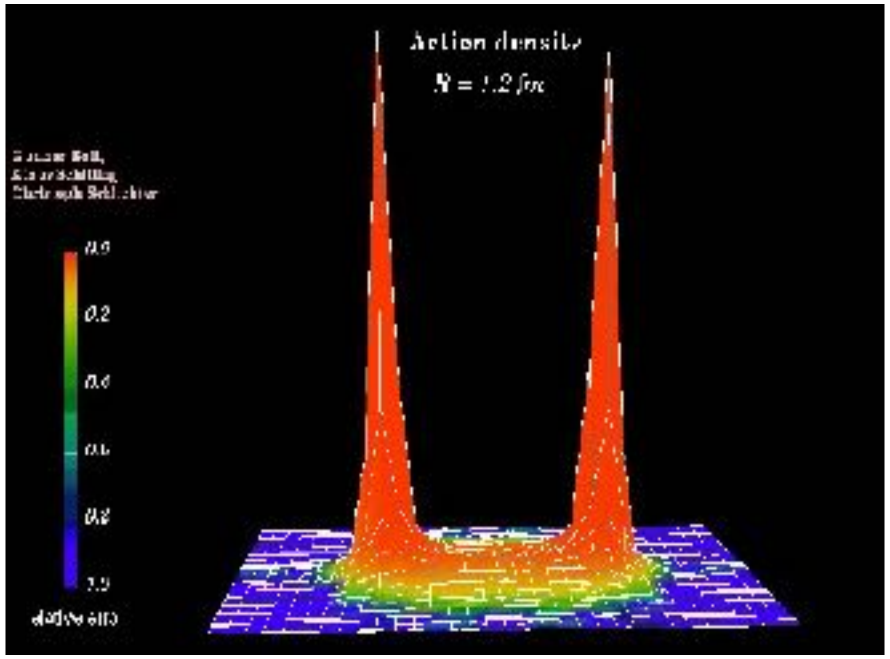
- Soft strings
  - Pythia is only employed for fragmentation
  - Single-diffractive, double diffractive and non-diffractive processes
- Hard strings
  - Fully treated by Pythia
  - All species mapped to pions and nucleons

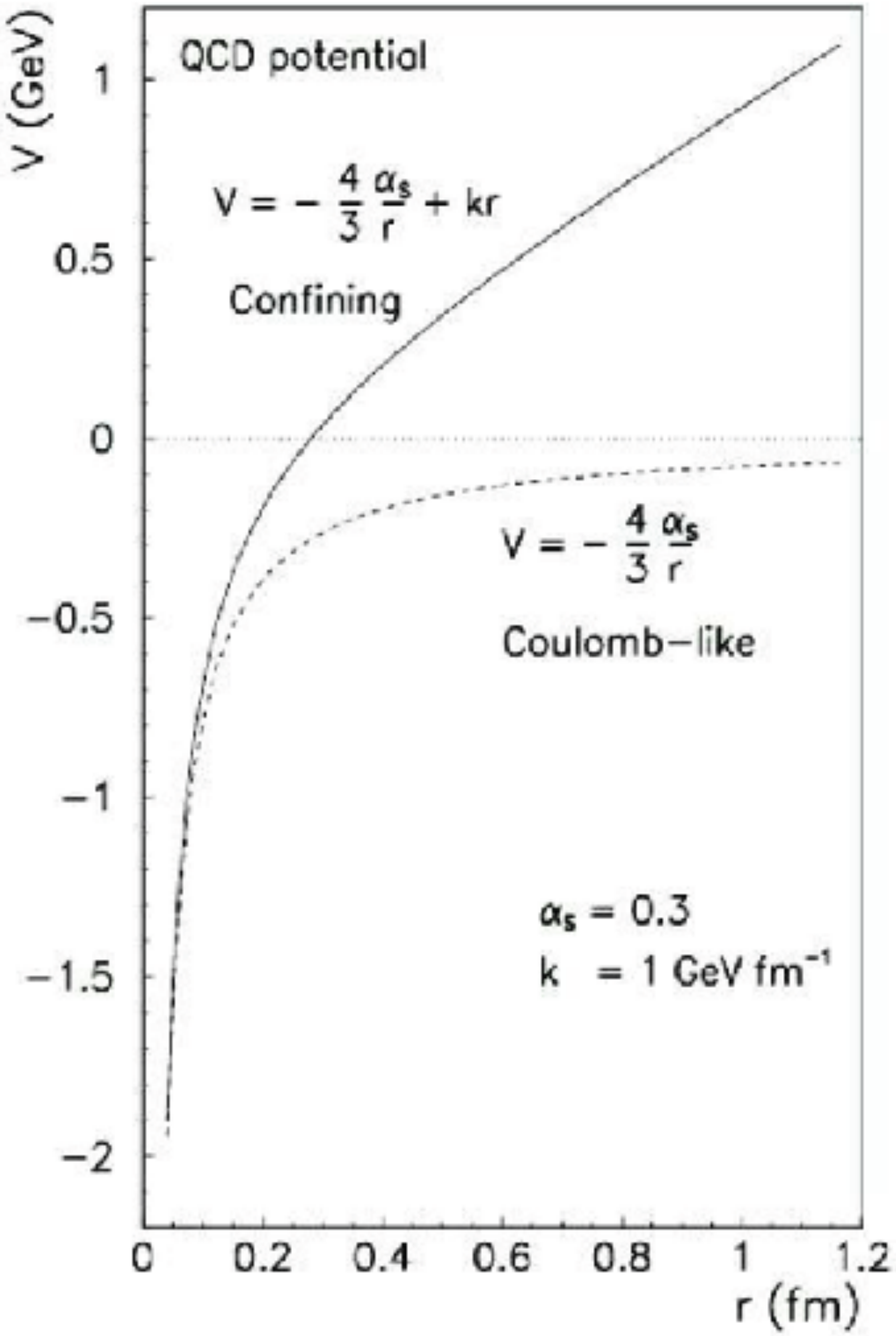


QCD Potential

- If the quarks travel away from each other the QCD potential leads to particle production in the critical field
- Color flux tube (gluon fields)



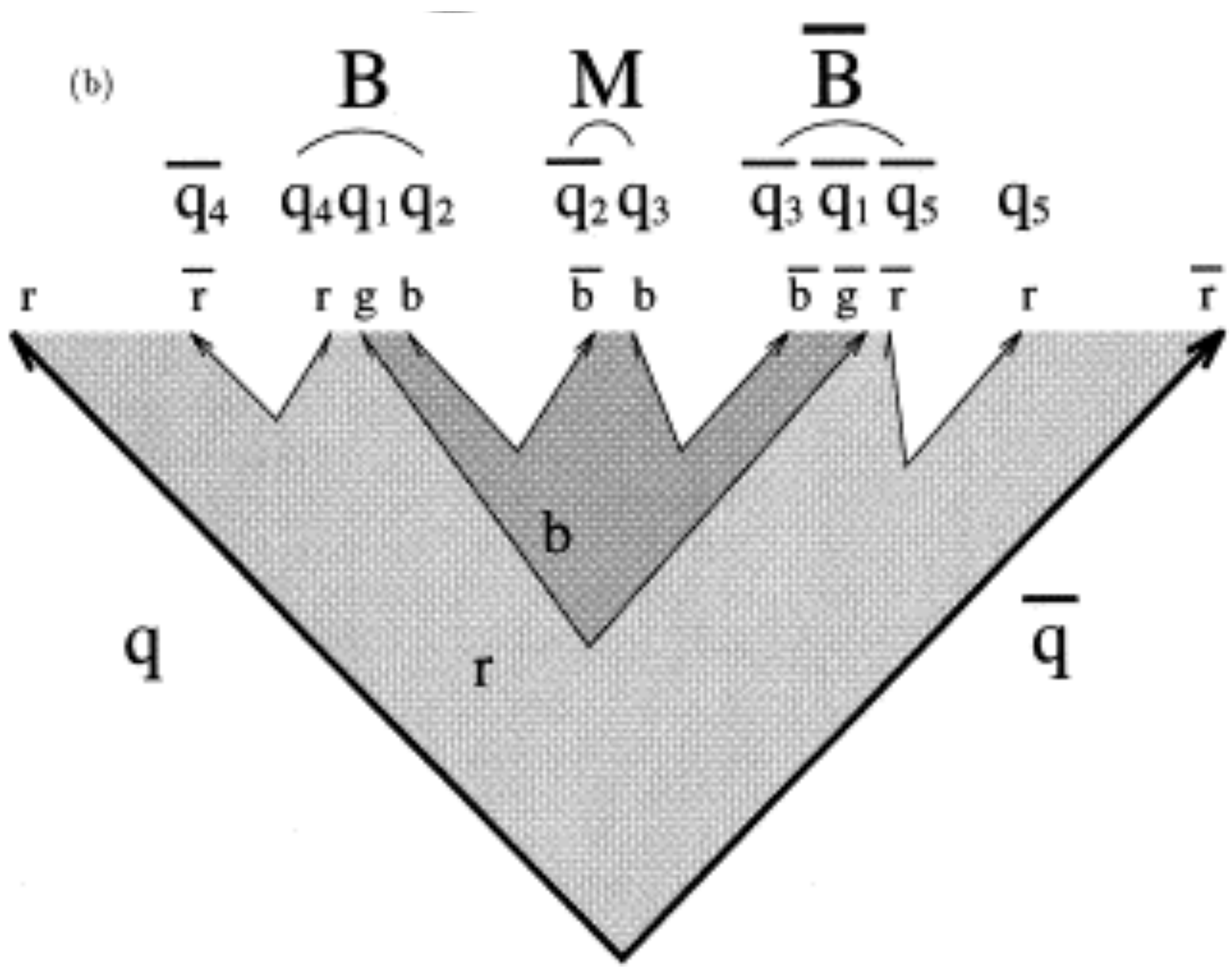




How are Hadrons Formed?

- We expect the production of new q-qbar pairs out of the decay of the critical vacuum between the quarks

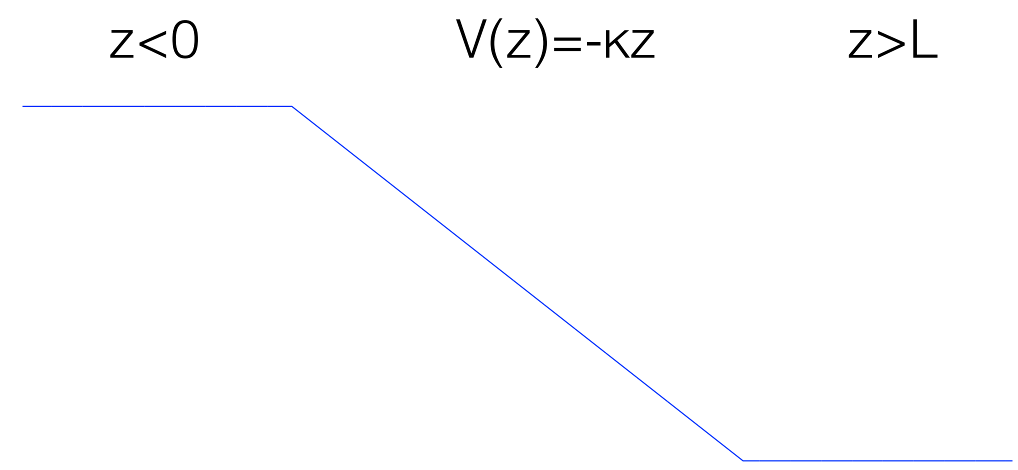
← **q**….qbar-q….qbar-q….qbar-q….qbar-q…**qbar** →



Pair Production

- The form of the potential is

$$V(z) = \begin{cases} 0 & z < 0 \\ -\kappa z & 0 < z < L \\ -\kappa L & L < z \end{cases}$$



- Solving the Klein-Gordon equation

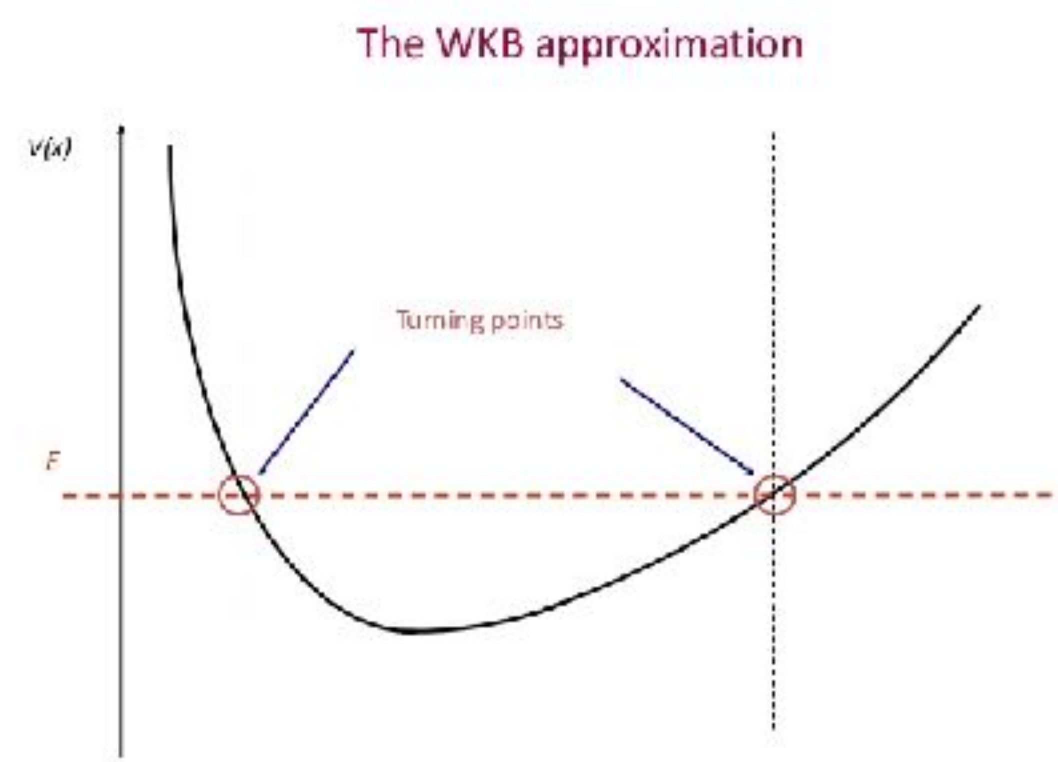
$$[(\hat{p} - A)^2 - m^2]\Psi = 0$$

leads to Schrödinger equation

$$\left[\frac{\hat{p_z}^2}{2m_\perp} + V_{eff} - E_{eff}\right] f(z) = 0$$

- Applying the WKB approximation,
we obtain the penetrability

$$P = \exp\left(-\frac{\pi m_\perp}{\kappa}\right)$$



Suppression factors

- Integration over transverse momentum yields

$$\frac{dN}{dV\,dt} = g_s \frac{\kappa^2}{(2\pi)^3} \exp\left(-\frac{\pi m^2}{\kappa}\right)$$

- This is similar to the full QED Schwinger result

$$\frac{dN}{dV\,dt} = g_s \frac{\kappa^2}{(2\pi)^3} \sum_{n=1}^{\infty} \frac{1}{n^2} \exp\left(-\frac{n\pi m^2}{\kappa}\right)$$

- The string model predicts strangeness suppression

$$\frac{N_s + N_{\bar{s}}}{N_u + N_{\bar{u}}} = \frac{\text{production rate for } s\bar{s}}{\text{production rate for } u\bar{u}} = \exp\left(-\frac{\pi(m_s^2 - m_u^2)}{\kappa}\right) = 0.214$$

κ = 1 GeV/fm, m_u = 325 MeV, m_s = 450 MeV

- It also predicts di-quark suppression

$$\frac{\text{rate for } \overline{uu}uu}{\text{rate for } \bar{u}u} = \exp\left(-\frac{\pi(4m_u^2 - m_u^2)}{\kappa}\right) = 6.9 \cdot 10^{-3}$$

BAMPS

- Boltzmann Approach of MultiParton Scatterings

Z. Xu and C. Greiner *Phys.Rev.C* 71 (2005) 064901

– A transport algorithm solving the Boltzmann equations for on-shell partons with pQCD interactions

$$p^{\mu}\partial_{\mu}f(x,p) = C_{gg \rightarrow gg}(x,p) + C_{gg \leftrightarrow ggg}(x,p)$$

(Z)MPC, VNI/BMS, AMPT, PACIAE

New Development

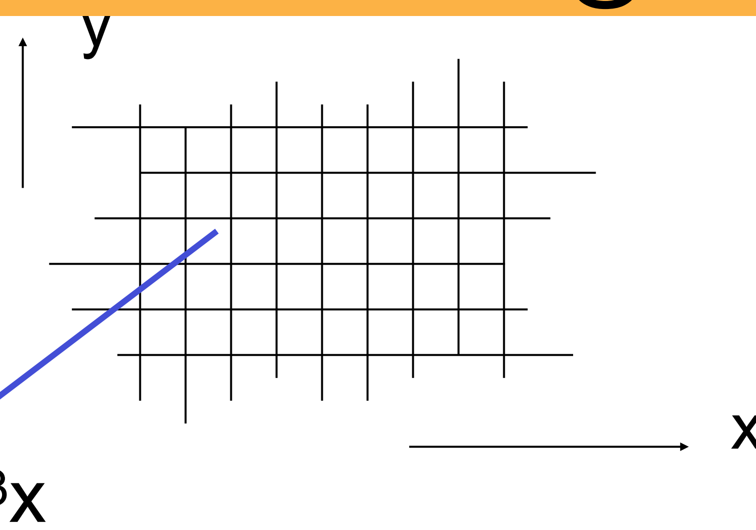
– Elastic scatterings are ineffective in thermalization

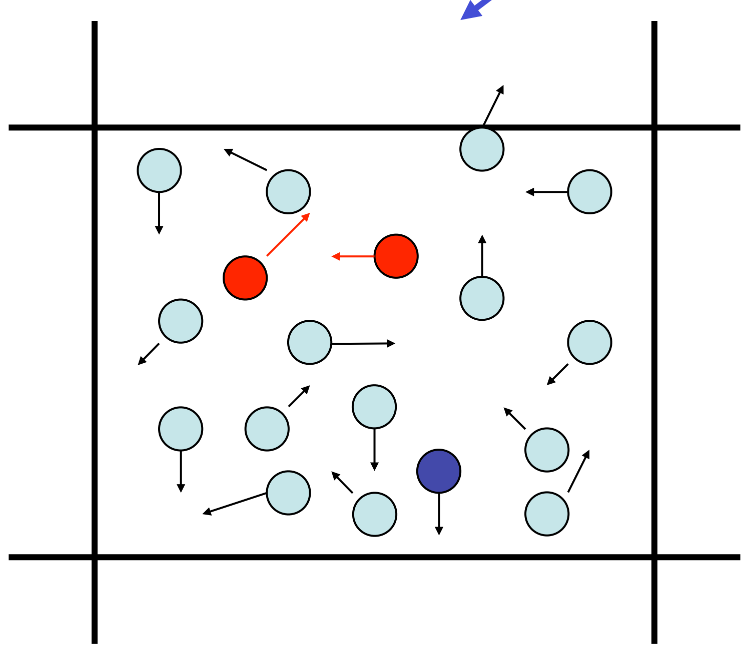
– Inelastic interactions are needed

– Detailed balance

Stochastic Algorithm

Space is divided into small cells !





collision probability -- stochastic

for $2 \rightarrow 2$ $\quad P_{22} = v_{rel} \dfrac{\sigma_{22}}{N_{test}} \dfrac{\Delta t}{\Delta^3 x}$

for $2 \rightarrow 3$ $\quad P_{23} = v_{rel} \dfrac{\sigma_{23}}{N_{test}} \dfrac{\Delta t}{\Delta^3 x}$

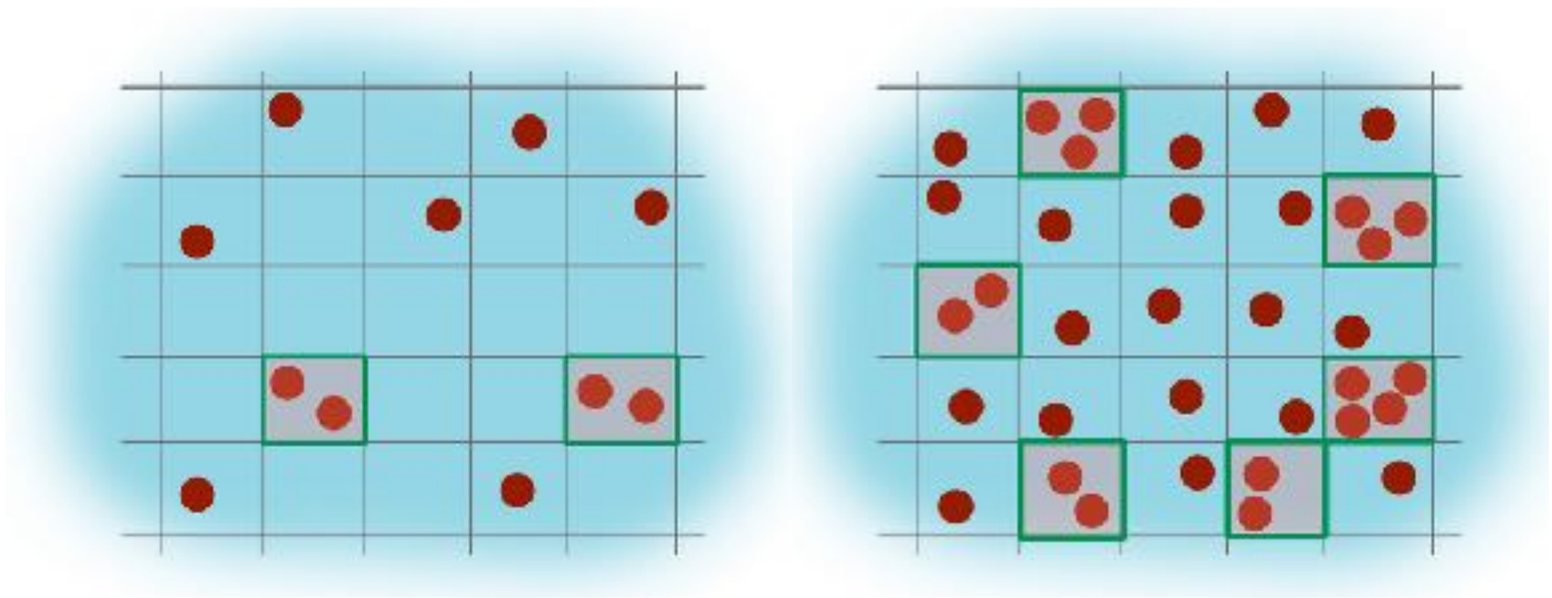
for $3 \rightarrow 2$ $\quad P_{32} = \dfrac{1}{8E_1E_2E_3} \dfrac{I_{32}}{N_{test}^2} \dfrac{\Delta t}{(\Delta^3 x)^2}$

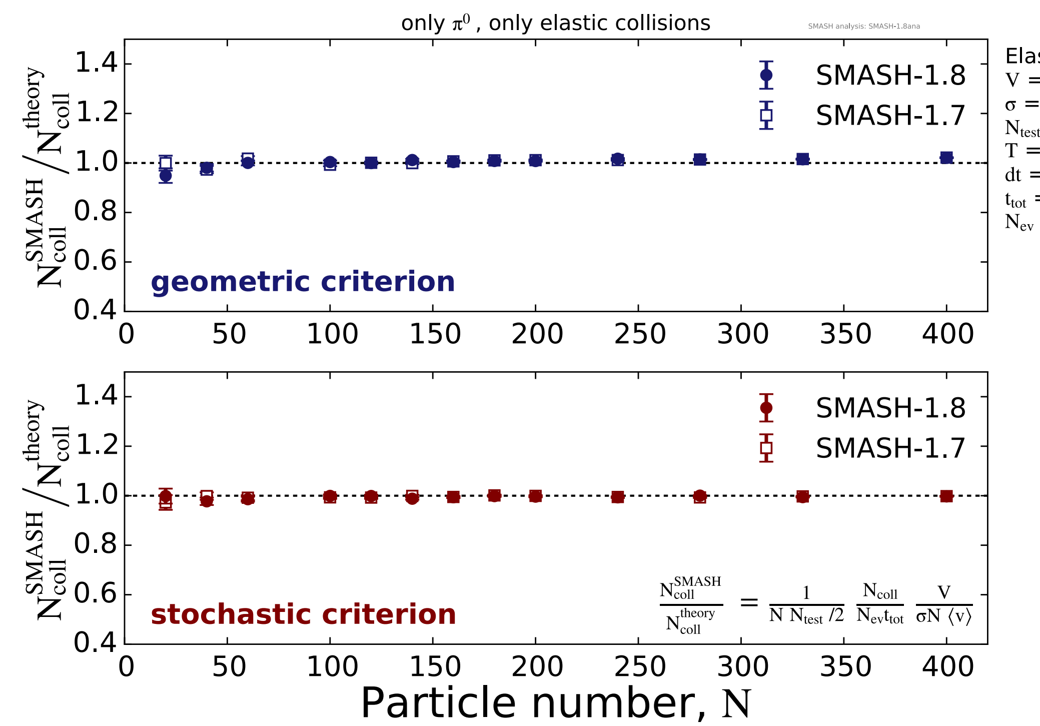
$$I_{32} = \frac{1}{2}\int \frac{d^3p'_1}{(2\pi)^3 2E'_1} \frac{d^3p'_2}{(2\pi)^3 2E'_2} \left|M_{\text{123®1'2'}}\right|^2 (2\pi)^4 \delta^{(4)}(p_1 + p_2 + p_3 - p'_1 - p'_2)$$

Multi-particle Interactions

- At high densities multiparticle interactions will become relevant also in hadronic transport approaches

Dilute System   Dense System



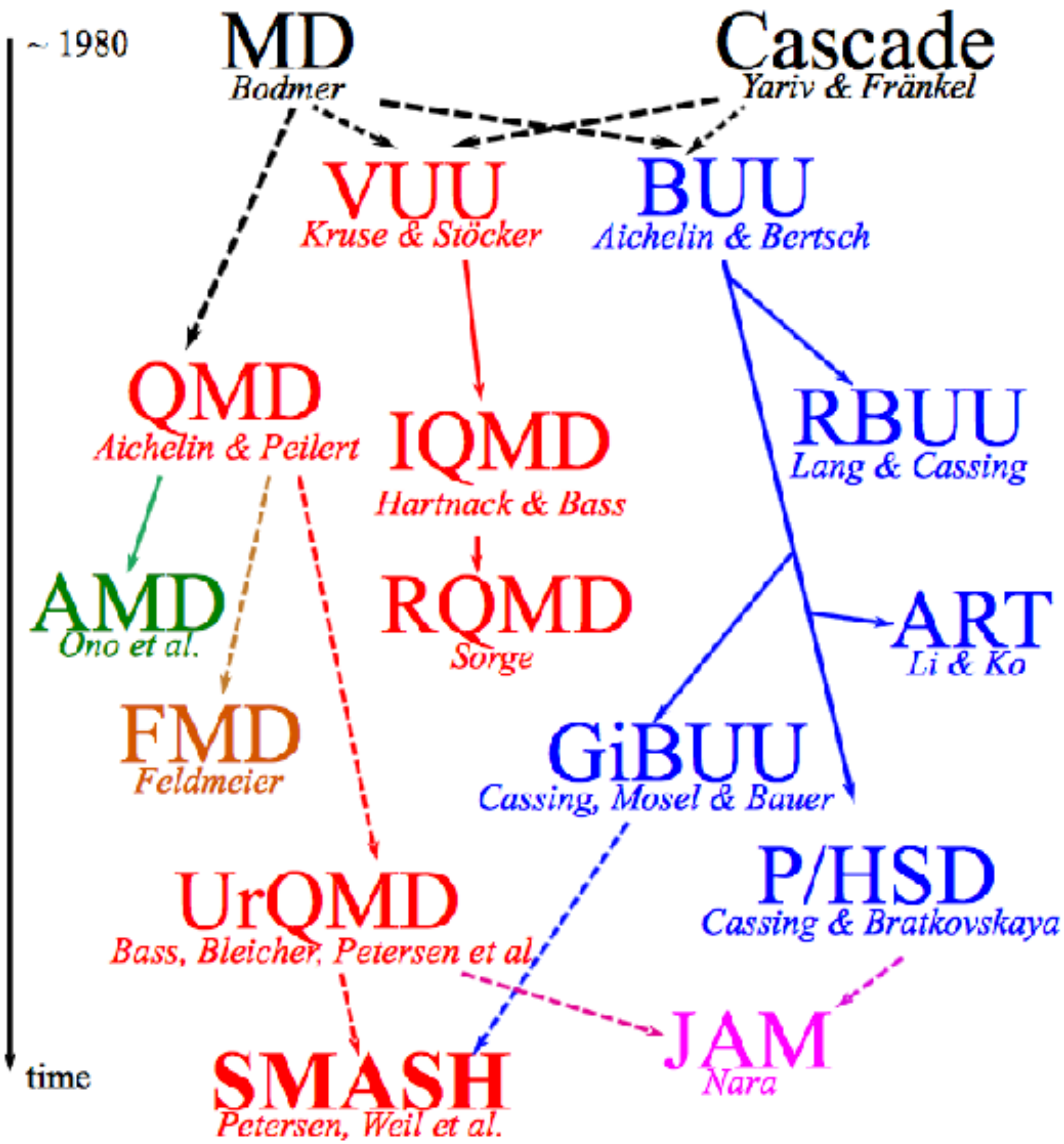


- $\omega \leftrightarrow 3\pi,\ B\bar{B} \leftrightarrow 5\pi,\ M \leftrightarrow N$

- 2<->2, 2<->1, 3<->1 and 2<->3 is implemented
- Application to interesting physics will be shown tomorrow

SMASH - A Specific Example

History of Transport Models



thanks to Steffen Bass

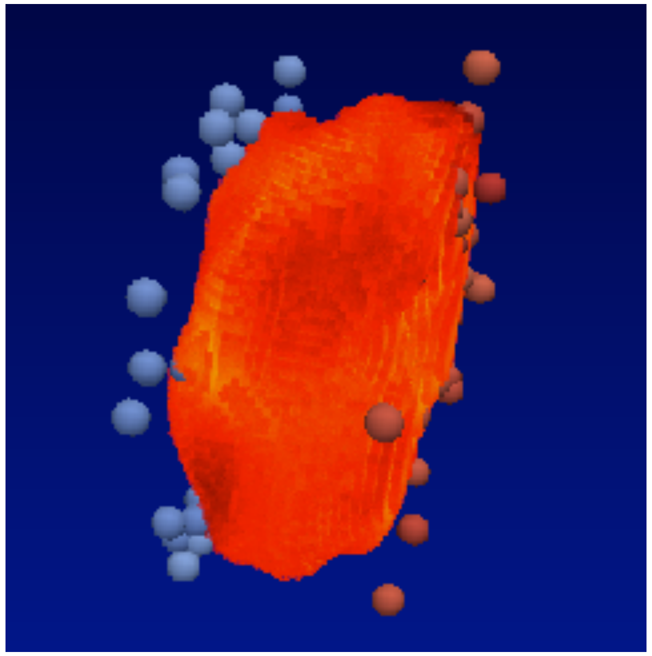
- Hadronic transport approaches are **successfully** applied for decades
- Goals for new code:
  - **Reference** for hadronic system with vacuum properties
  - Modeling of **non-equilibrium** phase transition

BUU vs QMD

- Boltzmann Uhling Uhlenbeck equation governs single-particle distribution function
  - Potentials are density dependent
  - Density and forces (gradients) are calculated with test particles
  - Parallel ensembles allow to recover some event-by-event fluctuations
- Quantum Molecular Dynamics
  - Each particle is represented by a Gaussian wave packet
  - Potentials are sums of two-particle forces
- Difference only relevant when mean fields are important, for cascade calculations (higher beam energies, afterburner) both are identical

Why a new Approach?

- Hadronic transport approaches are successfully applied for the dynamical evolution of heavy ion collisions
- Hadronic non-equilibrium dynamics is crucial for
  - Full/partial evolution at low/intermediate beam energies
  - Late stage rescattering at high beam energies (RHIC/LHC)



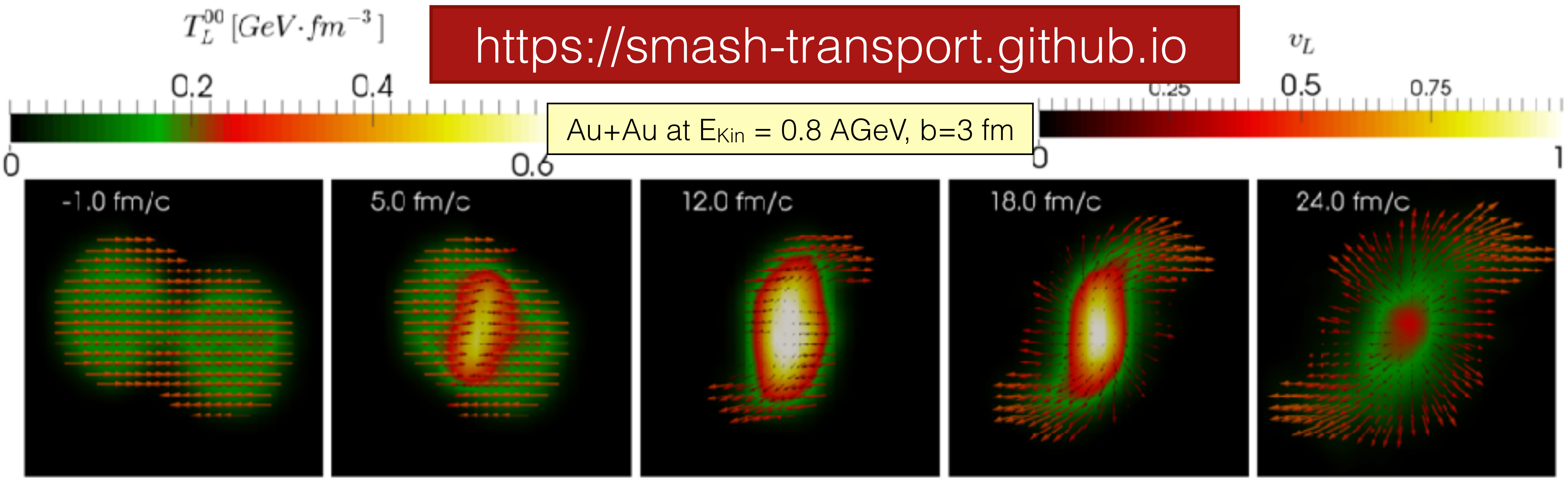
- New experimental data for cross-sections and resonance properties is available (e.g. COSY, GSI-SIS18 pion beam etc)
- Philosophy: Flexible, modular approach condensing knowledge from existing approaches
- Goal: Baseline calculations with hadronic vacuum properties essential to identify phase transition

SMASH*

J. Weil et al, PRC 94 (2016)

- Hadronic transport approach:
  - Includes all mesons and baryons up to ~2 GeV
  - Geometric collision criterion
  - Binary interactions: Inelastic collisions through resonance/string excitation and decay
  - Infrastructure: C++, Git, Doxygen, (ROOT)

https://smash-transport.github.io



* Simulating Many Accelerated Strongly-Interacting Hadrons

The SMASH Team

- In Frankfurt:
  - Oscar Garcia-Montero
  - Gabriele Inghirami
  - Alessandro Sciarra
  - Jan Staudenmaier
  - Justin Mohs
  - Jan Hammelmann
  - Niklas Götz
  - Renan Hirayama
  - Nils Saß
  - Jonas Rongen
  - Antonio Bozic
  - Orhan Özel
  - Lucas Constantin
  - Julia Gröbel
  - Branislav Balinovic

- In US:
  - Dmytro Oliinychenko
  - Agnieszka Sorensen

Group excursion in May 2022

General Setup

- Transport models provide an effective solution of the relativistic Boltzmann equation

$$p^\mu \partial_\mu f_i(x,p) + m_i F^\alpha \partial_\alpha^p f_i(x,p) = C_{\rm coll}^i$$

- Particles represented by Gaussian wave packets for density calculations
- Geometric collision criterion

$$d_{\rm trans} < d_{\rm int} = \sqrt{\frac{\sigma_{\rm tot}}{\pi}}$$

$$d_{\rm trans}^2 = (\vec{r_a} - \vec{r_b})^2 - \frac{((\vec{r_a} - \vec{r_b}) \cdot (\vec{p_a} - \vec{p_b}))^2}{(\vec{p_a} - \vec{p_b})^2}$$

As in UrQMD

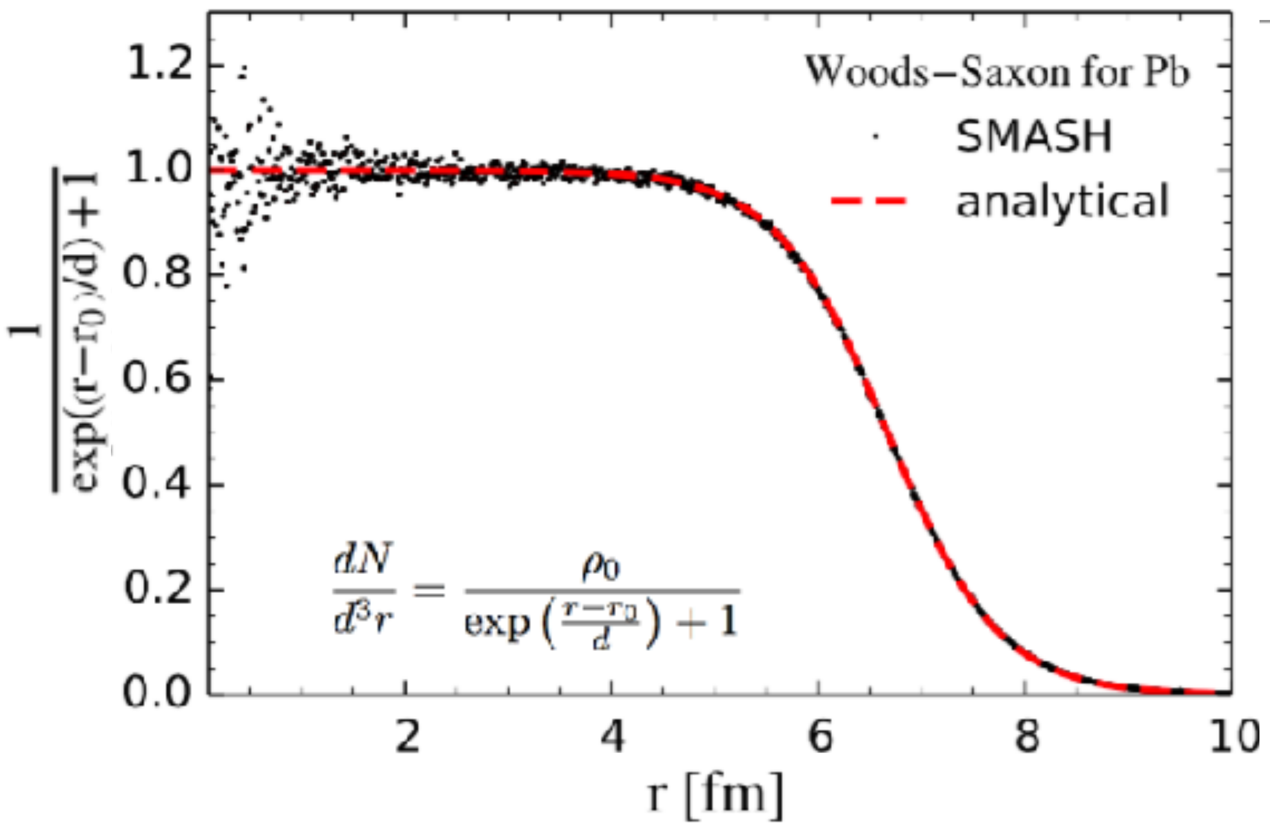
- Test particle method

$$\sigma \mapsto \sigma \cdot N_{\rm test}^{-1}$$

$$N \mapsto N \cdot N_{\rm test}$$

Initial Conditions

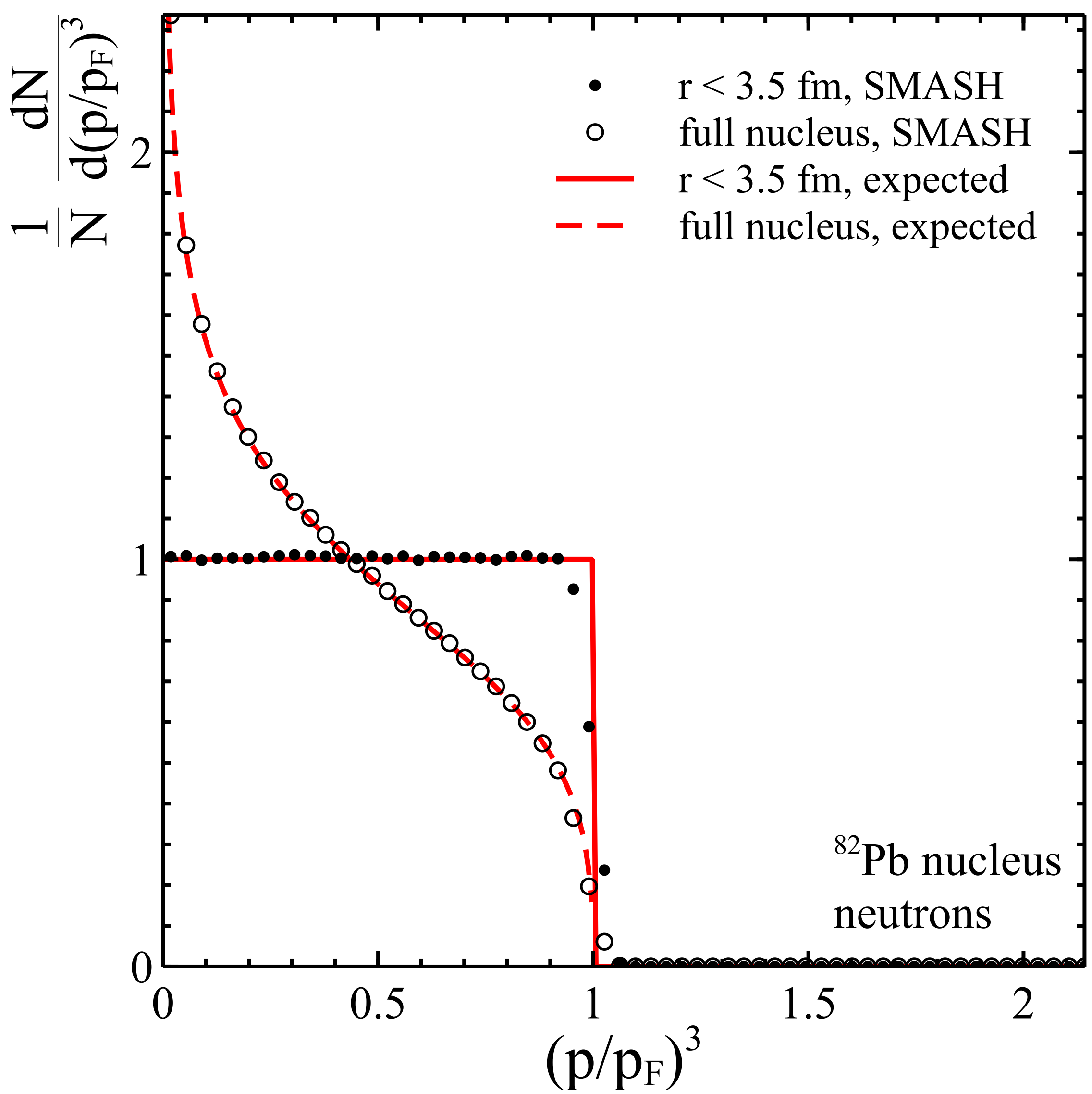
- Nuclear Collisions
  - Woods-Saxon distribution in coordinate space



  - *optional:* deformed nuclei and (frozen) Fermi motion
  - *optional*: read-in of more realistic initial states with correlations, neutron skin

Fermi Motion

- Fermi motion is randomly assigned to each nucleon depending on the local density $p_F(\vec{r}) = \hbar c(3\pi^2\rho(\vec{r}))^{1/3}$



J. Weil et al., PRC 78, 2016

- Fermi motion would lead to unstable nuclei
- Attractive part of mean field has to balance Fermi motion
- For higher beam energies where mean fields are not required
- -> Frozen Fermi motion
  - Fermi momentum is not taken into account for propagation only for collisions

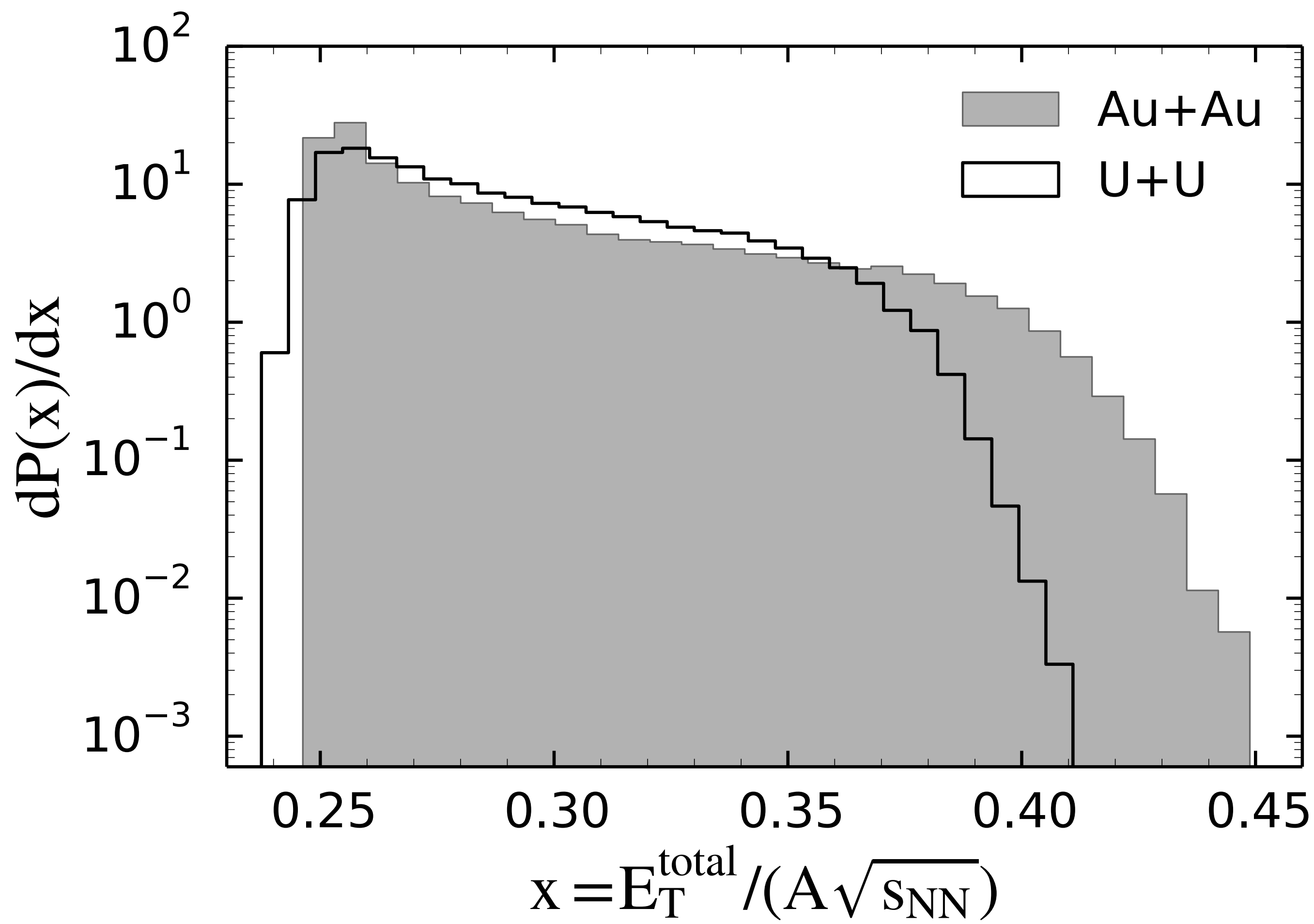
Deformation of Nuclei

- SMASH includes the most basic deformation parameters
- After sampling nucleons, the whole configuration is rotated by random Euler angles to provide independent initial states
- Specific values for certain known nuclei are provided

J. Weil et al., PRC 78, 2016

$$\rho(r,\theta,\varphi) = \frac{\rho_0}{1+\exp\left(\frac{r-r(\theta,\varphi)}{d}\right)}$$

$$r(\theta,\varphi) = r_0\left(1+\sum_{l=1}^{\infty}\sum_{m=-l}^{l}\beta_{lm}Y_l^m\right)$$



- BSc student works on extending this by β_3 and γ

Degrees of Freedom

N	Δ	Λ	Σ	Ξ	Ω	Unflavored				Strange
N_{938}	Δ_{1232}	Λ_{1116}	Σ_{1189}	Ξ_{1321}	Ω^-_{1672}	π_{138}	$f_{0\,980}$	$f_{2\,1275}$	$\pi_{2\,1670}$	K_{494}
N_{1440}	Δ_{1620}	Λ_{1405}	Σ_{1385}	Ξ_{1530}	Ω^-_{2250}	π_{1300}	$f_{0\,1370}$	$f'_{2\,1525}$		K^*_{892}
N_{1520}	Δ_{1700}	Λ_{1520}	Σ_{1660}	Ξ_{1690}		π_{1800}	$f_{0\,1500}$	$f_{2\,1950}$	$\rho_{3\,1690}$	$K_{1\,1270}$
N_{1535}	Δ_{1900}	Λ_{1600}	Σ_{1670}	Ξ_{1820}			$f_{0\,1710}$	$f_{2\,2010}$		$K_{1\,1400}$
N_{1650}	Δ_{1905}	Λ_{1670}	Σ_{1750}	Ξ_{1950}		η_{548}		$f_{2\,2300}$	$\phi_{3\,1850}$	K^*_{1410}
N_{1675}	Δ_{1910}	Λ_{1690}	Σ_{1775}	Ξ_{2030}		η'_{958}	$a_{0\,980}$	$f_{2\,2340}$		$K_0^*{}_{1430}$
N_{1680}	Δ_{1920}	Λ_{1800}	Σ_{1915}			η_{1295}	$a_{0\,1450}$		$a_{4\,2040}$	$K_2^*{}_{1430}$
N_{1700}	Δ_{1930}	Λ_{1810}	Σ_{1940}			η_{1405}		$f_{1\,1285}$		K^*_{1680}
N_{1710}	Δ_{1950}	Λ_{1820}	Σ_{2030}			η_{1475}	ϕ_{1019}	$f_{1\,1420}$	$f_{4\,2050}$	$K_{2\,1770}$
N_{1720}		Λ_{1830}	Σ_{2250}				ϕ_{1680}			$K_3^*{}_{1780}$
N_{1875}		Λ_{1890}				σ_{800}		$a_{2\,1320}$		$K_{2\,1820}$
N_{1900}		Λ_{2100}					$h_{1\,1170}$			$K_4^*{}_{2045}$
N_{1990}		Λ_{2110}				ρ_{776}		$\pi_{1\,1400}$		
N_{2060}		Λ_{2350}				ρ_{1450}	$b_{1\,1235}$	$\pi_{1\,1600}$		
N_{2080}						ρ_{1700}				
N_{2100}							$a_{1\,1260}$	$\eta_{2\,1645}$		
N_{2120}						ω_{783}				
N_{2190}						ω_{1420}		$\omega_{3\,1670}$		
N_{2220}						ω_{1650}				
N_{2250}										

As of SMASH-1.7

- + corresponding antiparticles
- Perturbative treatment of photons and dileptons
- Isospin symmetry

- Mesons and baryons according to particle data group
- Isospin multiplets and anti-particles are included

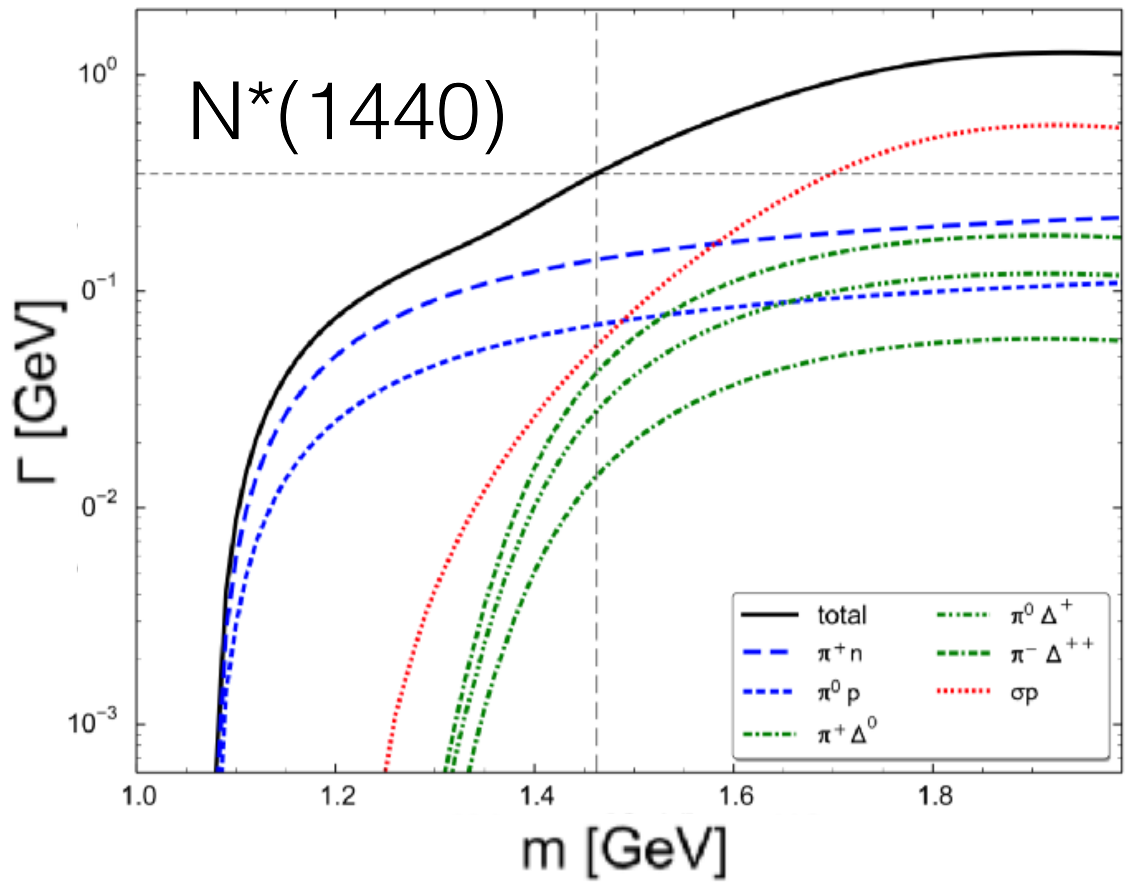
Resonances

- Spectral function
  - All unstable particles („resonances“) have relativistic Breit-Wigner spectral functions

$$\mathcal{A}(m) = \frac{2\mathcal{N}}{\pi} \frac{m^2\Gamma(m)}{(m^2 - M_0^2)^2 + m^2\Gamma(m)^2}$$

- Decay widths
  - Particles stable, if width < 10 keV $(\pi,\ \eta,\ K, ...)$
  - Treatment of Manley et al

$$\Gamma_{R\to ab} = \Gamma^0_{R\to ab} \frac{\rho_{ab}(m)}{\rho_{ab}(M_0)}$$



D. M. Manley and E. M. Saleski, Phys. Rev. D 45, 4002 (1992)

As in GiBUU

Treatment of Manley

- Scaling of on-shell decay width:

D. M. Manley and E. M. Saleski, Phys. Rev. D 45, 4002 (1992)

$$\Gamma_{R\to ab} = \Gamma^0_{R\to ab}\frac{\rho_{ab}(m)}{\rho_{ab}(M_0)}$$

- Definition of rho-function:

$$\rho_{ab}(m) = \int dm_a dm_b \mathcal{A}_a(m_a)\mathcal{A}_b(m_b) \times \frac{|\vec{p}_f|}{m} B_L^2(|\vec{p}_f|R)\mathcal{F}_{ab}^2(m)$$

Blatt Weisskopf functions

$$B_0^2 = 1$$

$$B_1^2(x) = x^2/(1+x^2)$$

$$\ldots$$

- Hadronic Form Factor:

M. Post, S. Leupold, U. Mosel, Nucl. Phys. A 741, 81 (2004)

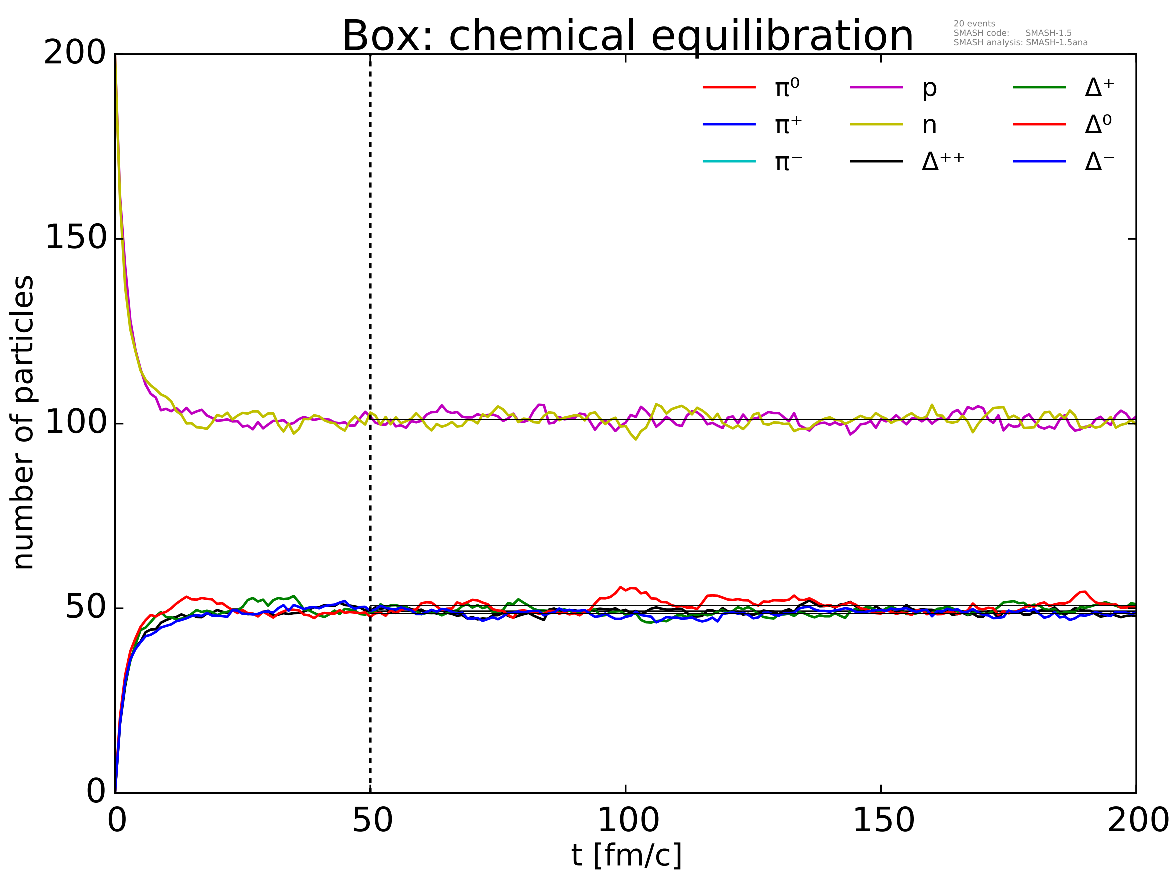
$$\mathcal{F}_{ab}(m) = \frac{\lambda^4 + 1/4(s_0 - M_0^2)^2}{\lambda^4 + \left(m^2 - 1/2(s_0 + M_0^2)\right)^2}$$

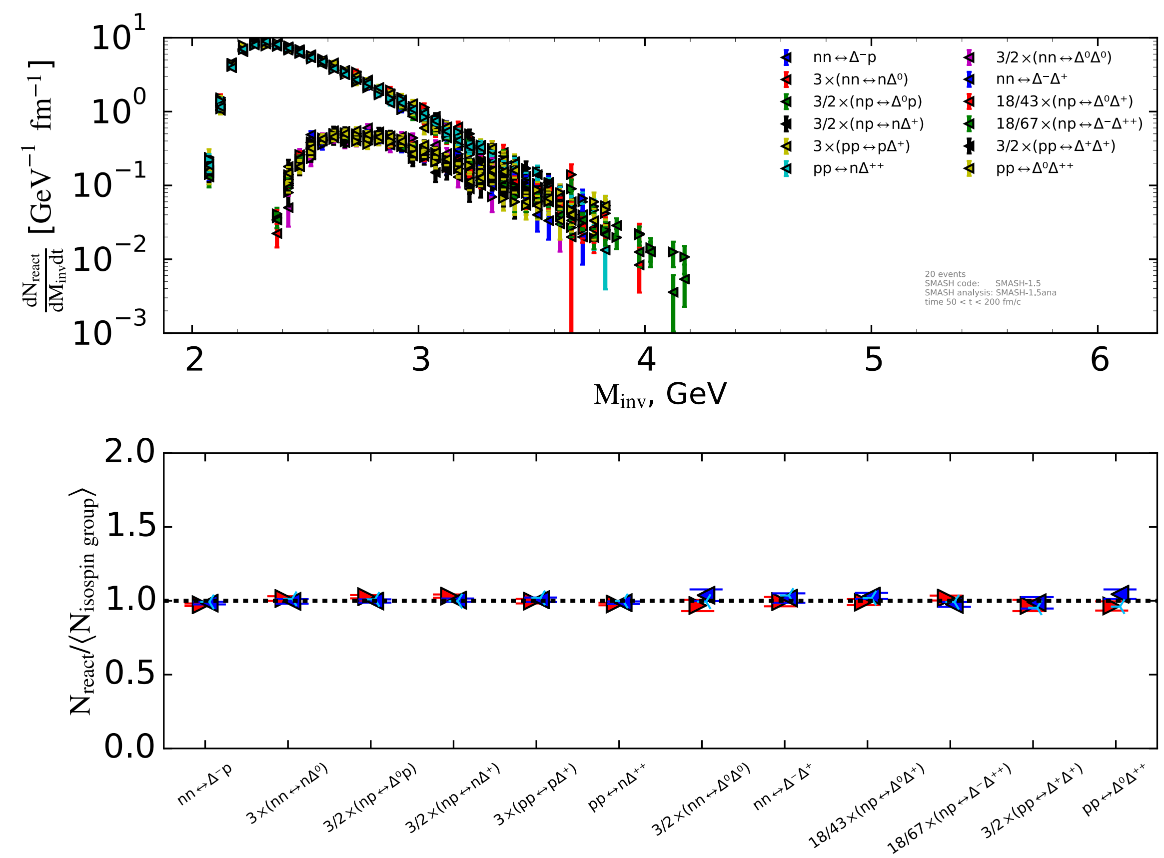
decay	λ [GeV]
$\pi\rho$	0.8
unstable mesons (e.g. ρN, σN)	1.6
unstable baryons (e.g. $\pi\Delta$)	2.0
two unstable daughters (e.g. $\rho\rho$)	0.6

Detailed Balance

- Inverse absorption cross section calculated from production cross section
- Conservation of detailed balance (only 1 <—> 2 or 2 <—> 2 processes)

J. Weil et al, PRC 94 (2016), updated SMASH-1.5

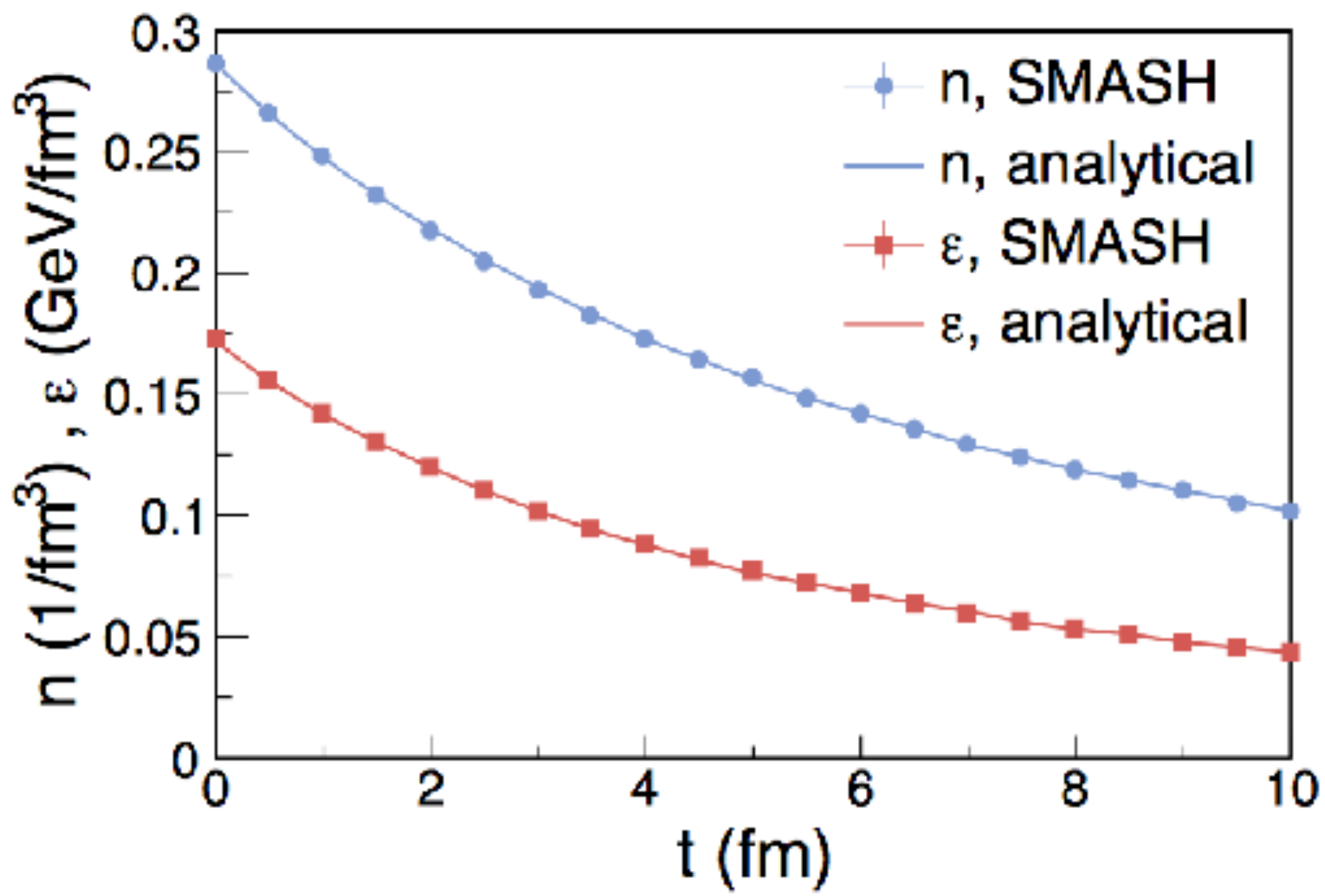


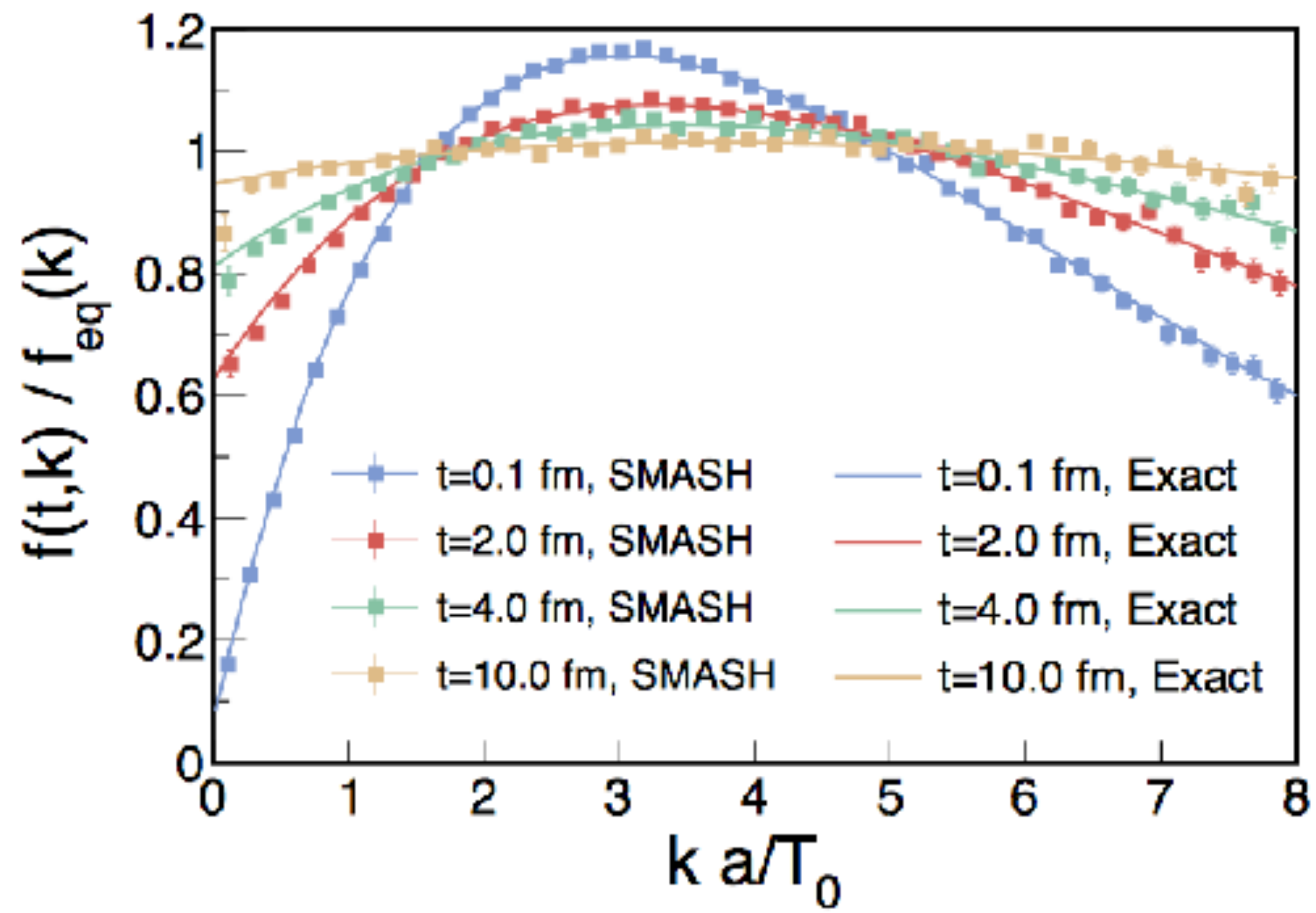


- Infinite matter calculations —> Important cross-check

Analytic Solution

- Comparison to analytic solution of Boltzmann equation within expanding metric





- Perfect agreement proves correct numerical implementation of collision algorithm

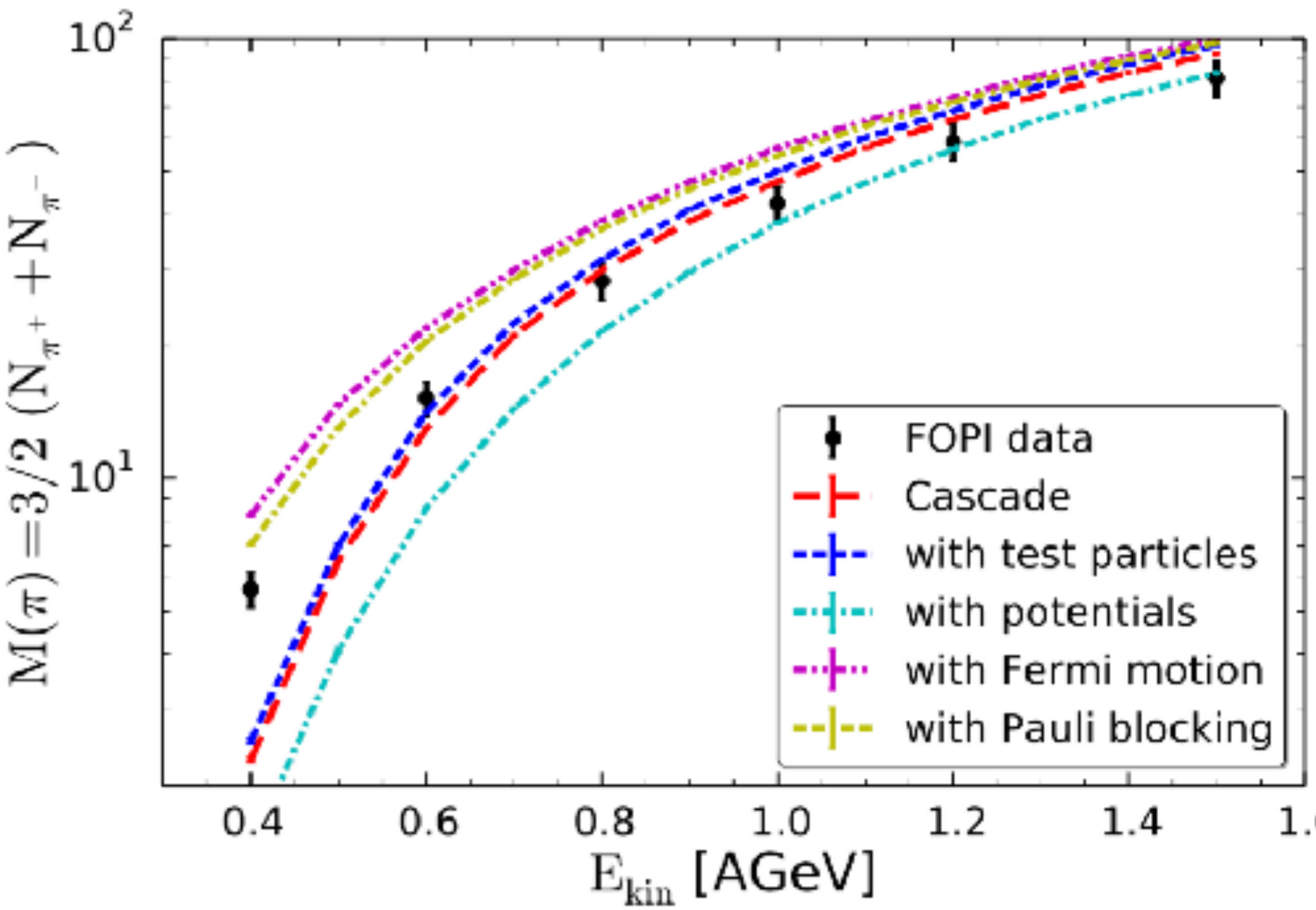
D. Bazow et al., PRL 116 (2016) and PRD 94 (2016)

J. Tindall et al., PLB 770 (2017)

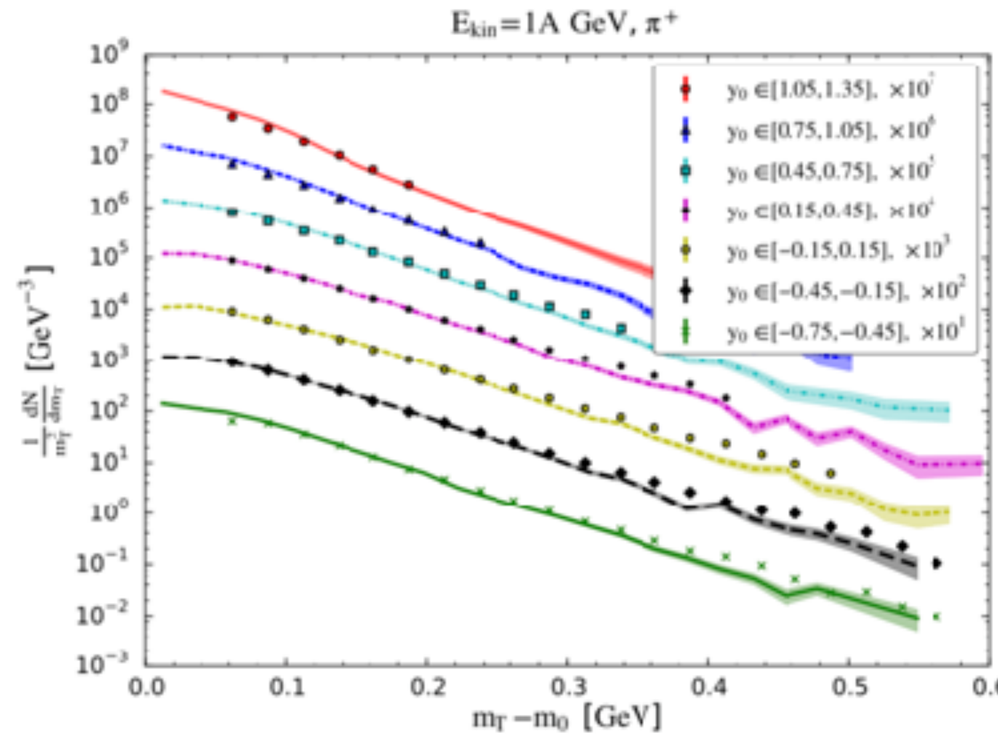
Bulk Observables and Equation of State

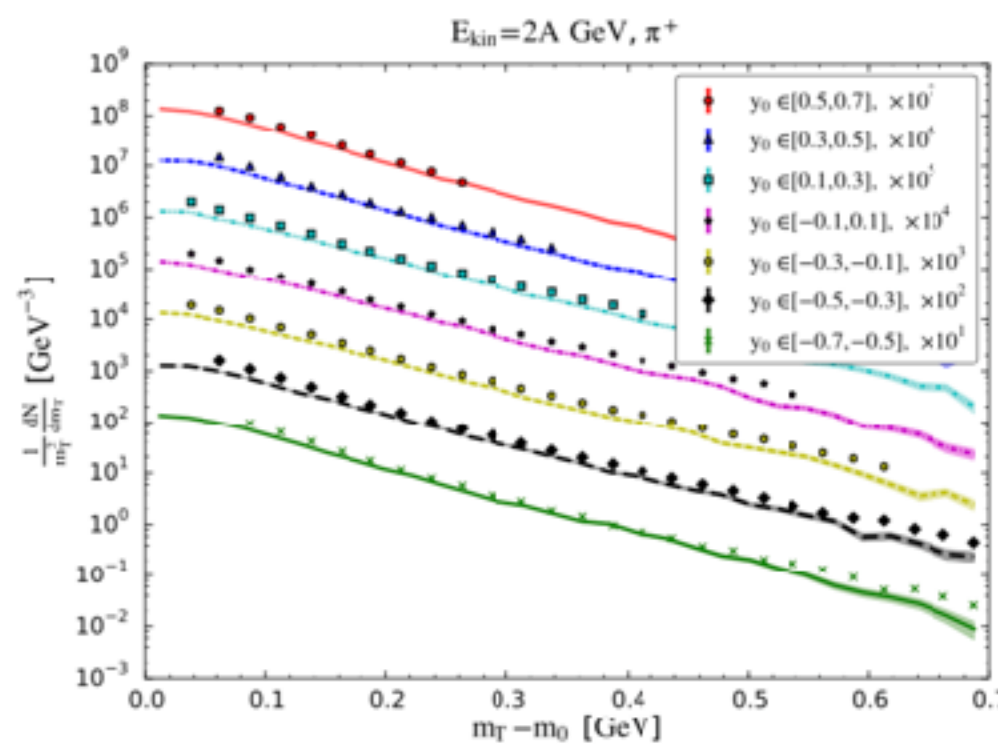
Pion Production in Au+Au

- Potentials decrease pion production, while Fermi motion increases yield
- Nice agreement with SIS experimental data



Note: consecutive addition of features

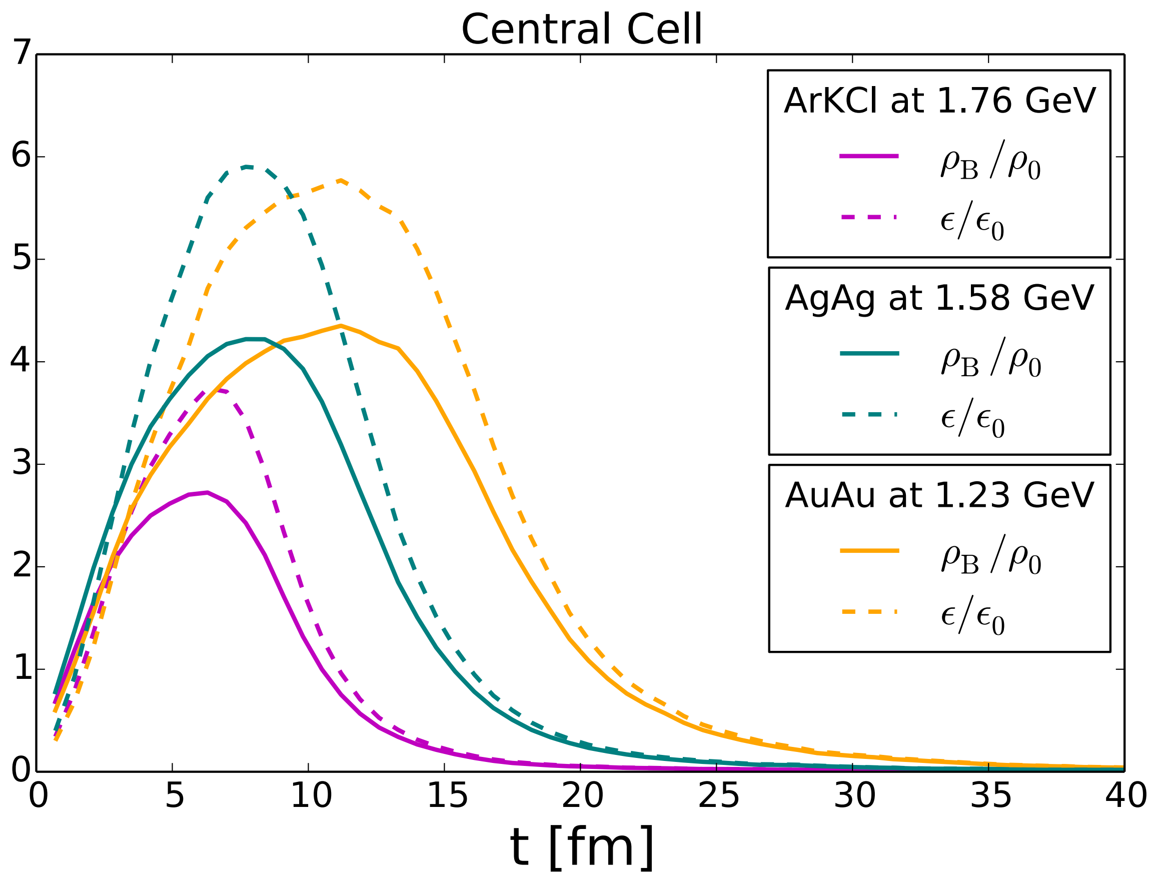


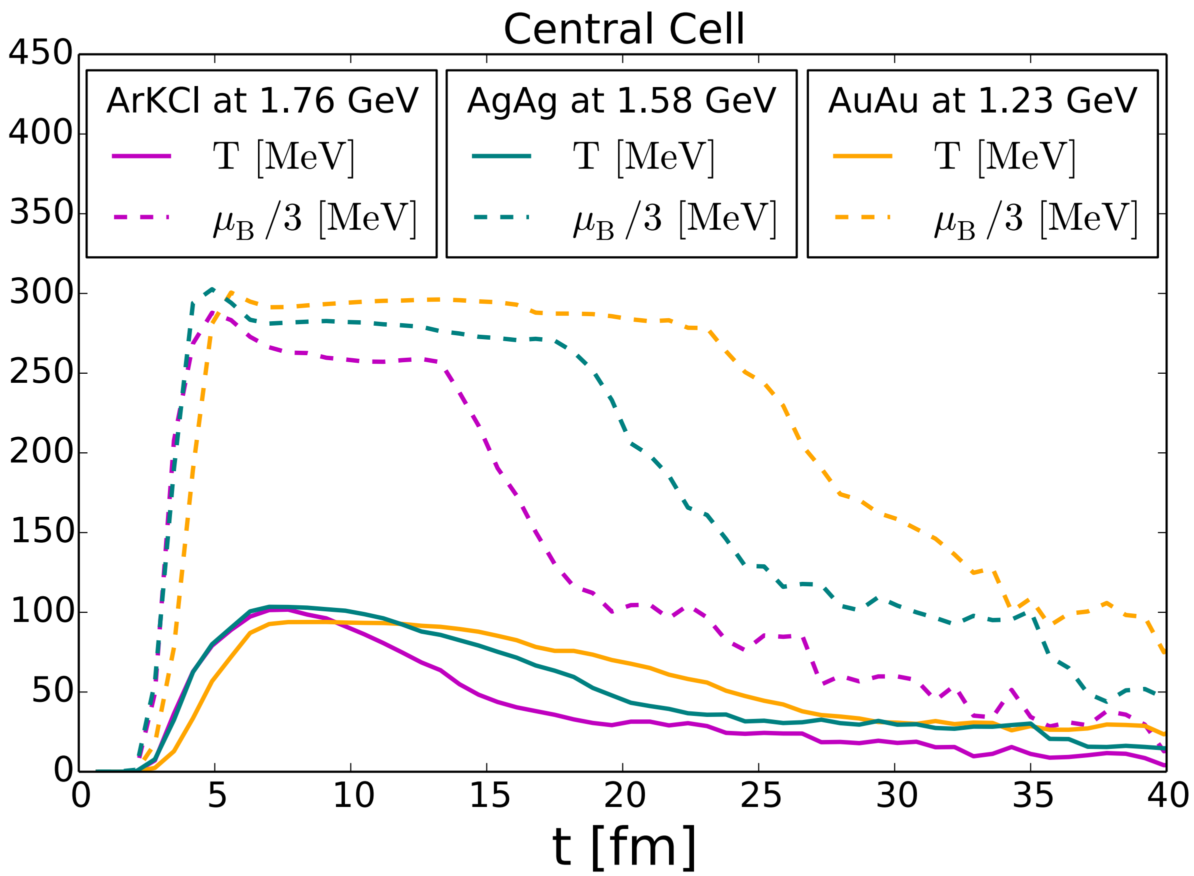


J. Weil et al, PRC 94 (2016)

Time Evolution

- Density and temperature in a central cell for heavy ion collisions at SIS-18 energies





J. Staudenmaier, N. Kübler and HE, arXiv:2008.05813

- 2-4 times nuclear ground state density reached

see also, T. Galatyuk et al. and S. Endres et al

Collective Behaviour

- Potentials in SMASH
  - Basic Skyrme and symmetry potential

$$U_{\text{Skyrme}} = \alpha(\rho/\rho_0) + \beta(\rho/\rho_0)^\tau \qquad U_{\text{Symmetry}} = \pm 2 S_{\text{Pot}} \frac{\rho_{I_3}}{\rho_0}$$

  - Describes interactions between nucleons, repulsive at high densities

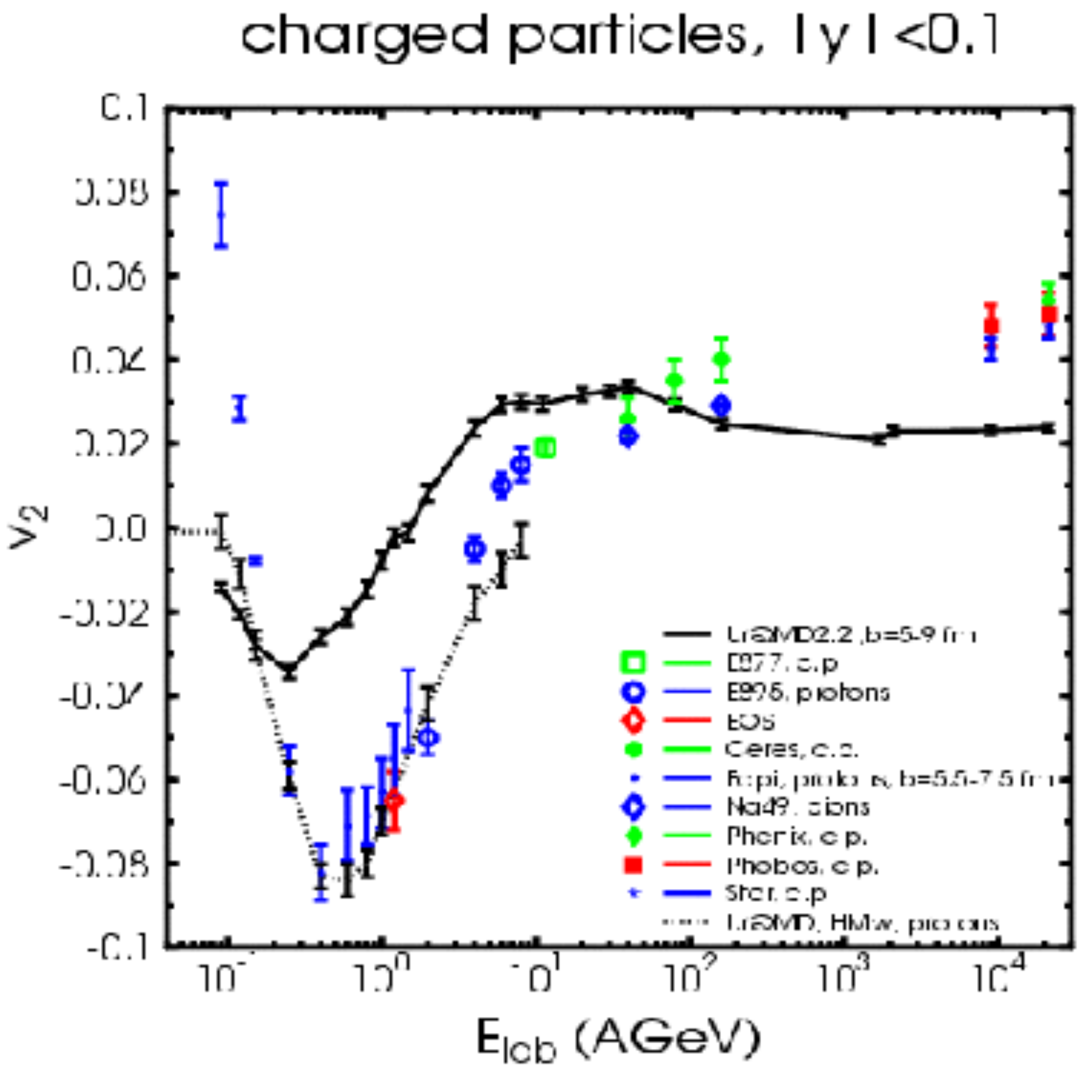
	soft EoS	default EoS	hard EoS
α	-356.0 MeV	-209.2 MeV	-124.0 MeV
β	303.0 MeV	156.4 MeV	71.0 MeV
τ	1.17	1.35	2.00
κ	200 MeV	240 MeV	380 MeV

  - Default values according to transport code comparison

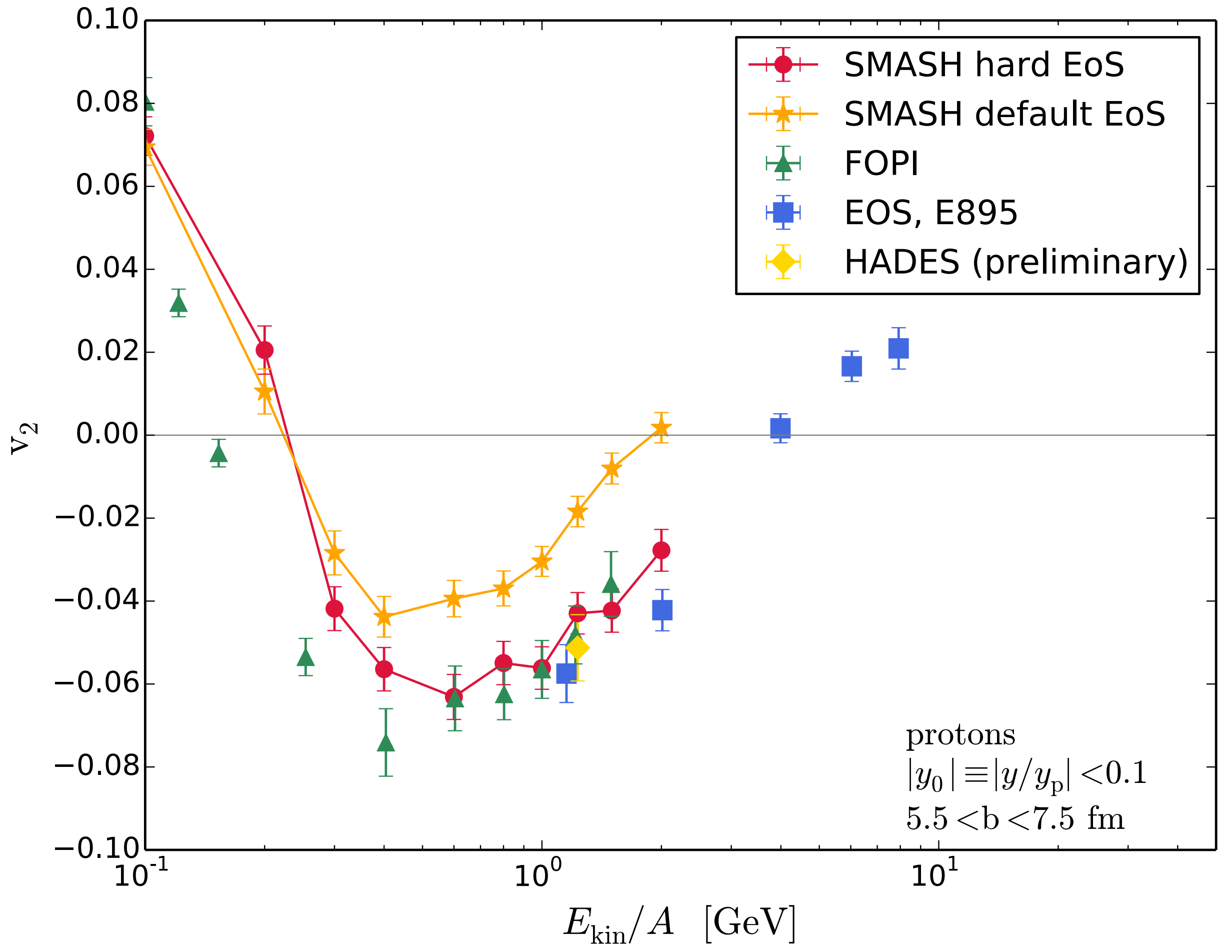
J. Xu et al., PRC 93 (2016)

v_2 Excitation Function

- Transition from squeeze-out at low energies to in-plane flow at high energies-> hadron transport underestimates flow



H. Petersen (now Elfner) et al, PRC 74, 2006

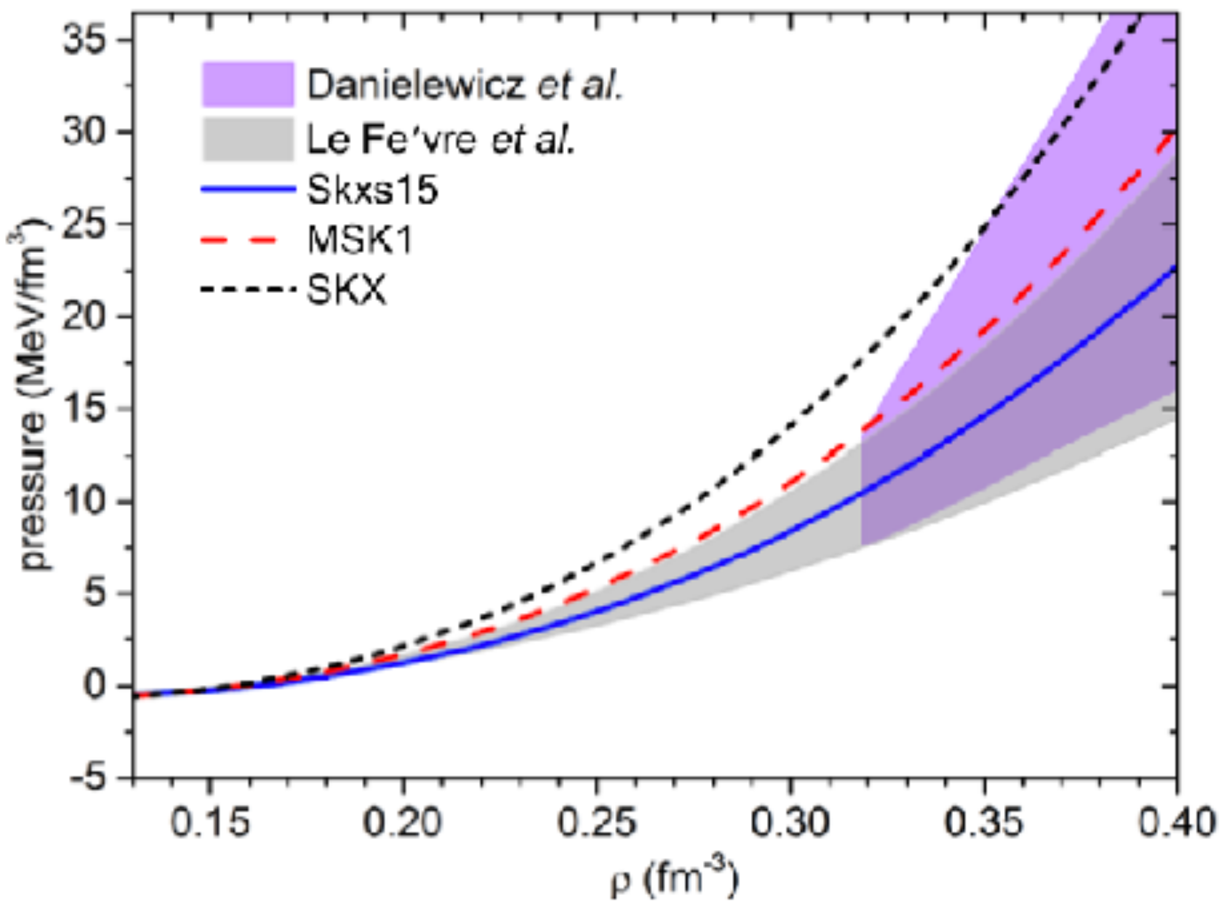


H.Petersen (now Elfner) et al, NPA 982, 2019

- Sensitive to equation of state of nuclear matter, here in terms of a mean field between nucleons

Nuclear Matter Equation of State

- ‚Danielewicz constraint' and newer constraints ->varying mean field parameters to fit flow observables best



Y. Wang et al, arXiv:1804.04293

- Open issues: Results are dependent on details of transport code and in particular mean field properties

P. Danielewicz at EMMI Workshop 2019, https://indico.gsi.de/event/8242/timetable/#20190213.detailed

Issues

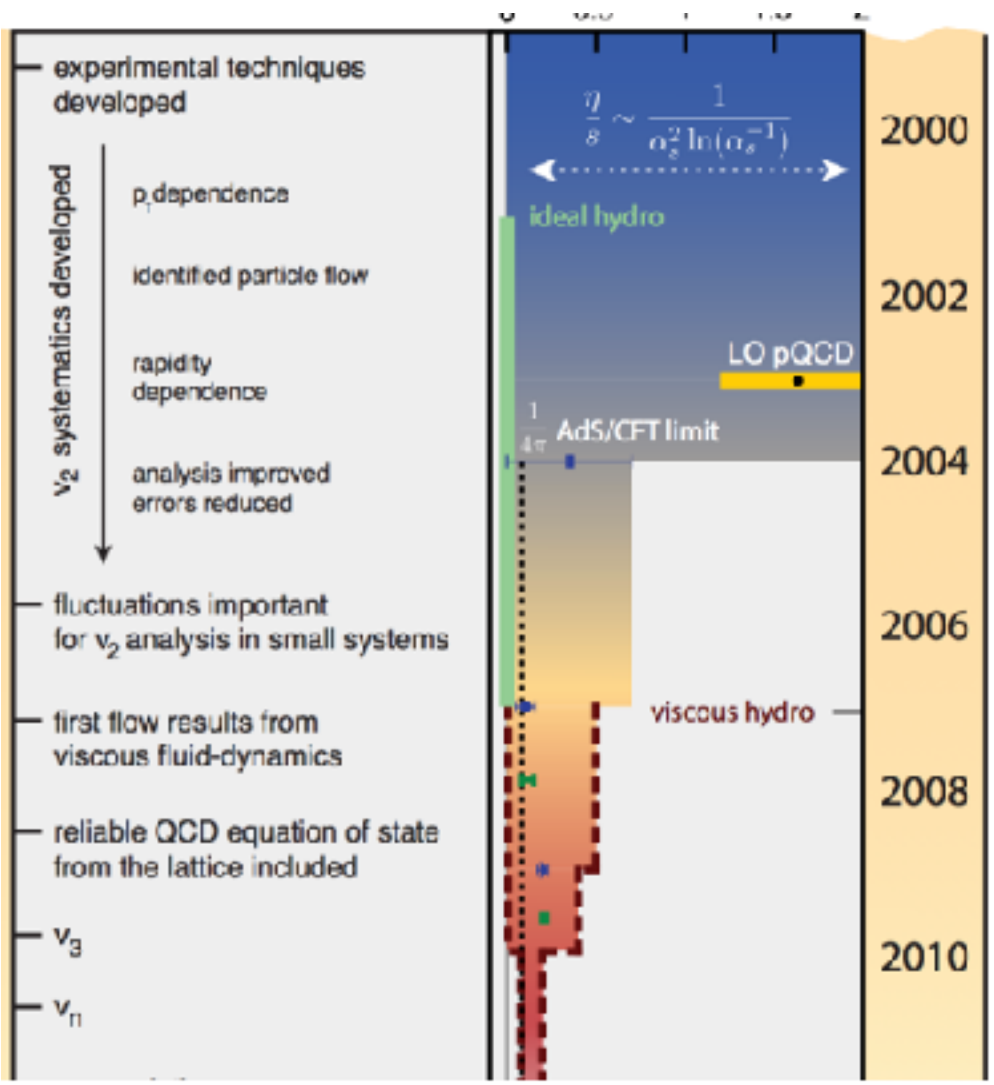
- Challenging to describe multiple data sets within one approach consistently (FOPI, HADES, v1/v2, protons/ deuterons/pions)
- Degrees of freedom matter and resonance dynamics influences mean field parameters
- Medium-modified cross-sections
- Influence of Coulomb potential
- Clustering and light nuclei production
- Spectators and mixing of forward and central dynamics
- To do: Bayesian analysis to see how well data can constrain EoS/mean field
  - Add future data to study sensitivities
  - Assess systematic uncertainties in modeling

Transport Coefficients of Hadron Gas

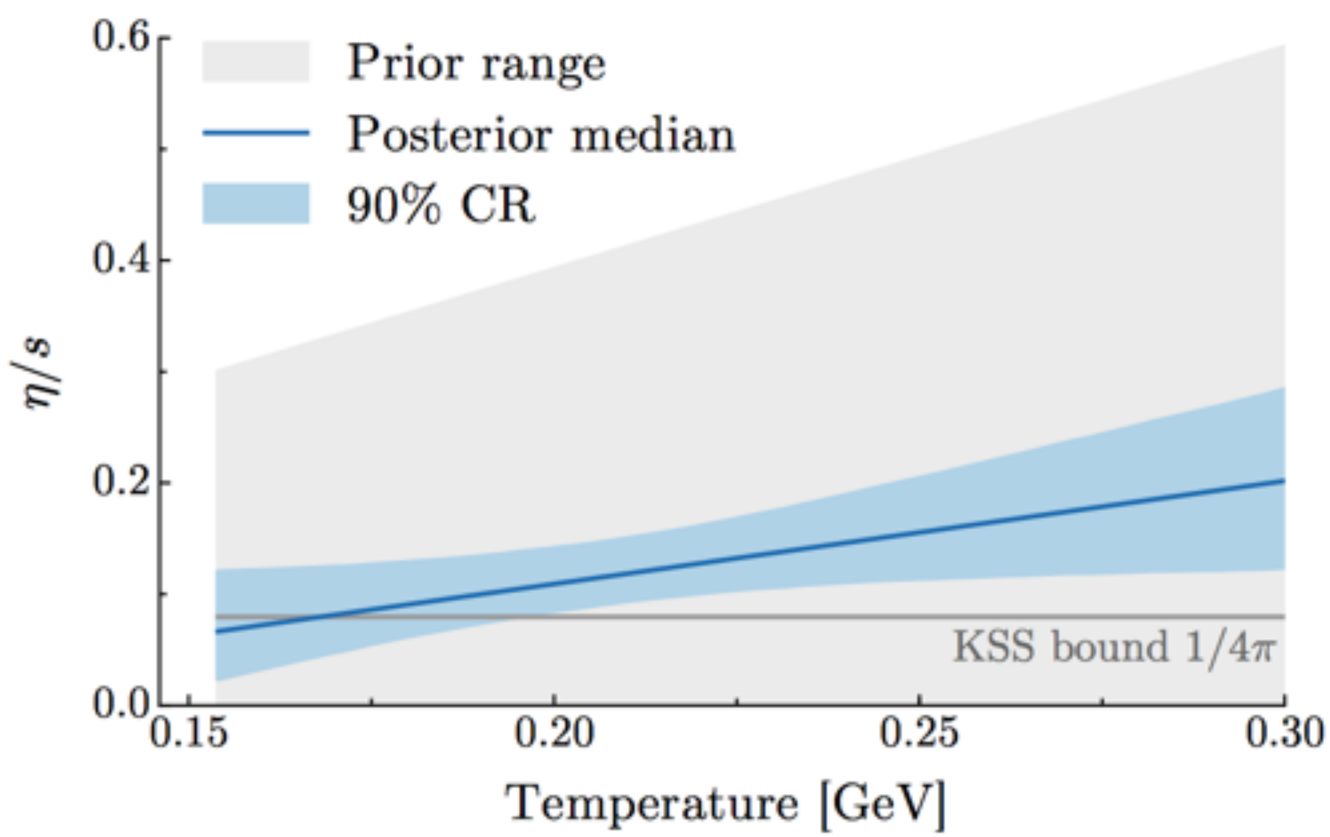
Transport Coefficients

- Within hydrodynamics/hybrid approaches the shear viscosity is an input parameter

RHIC White paper, 2012



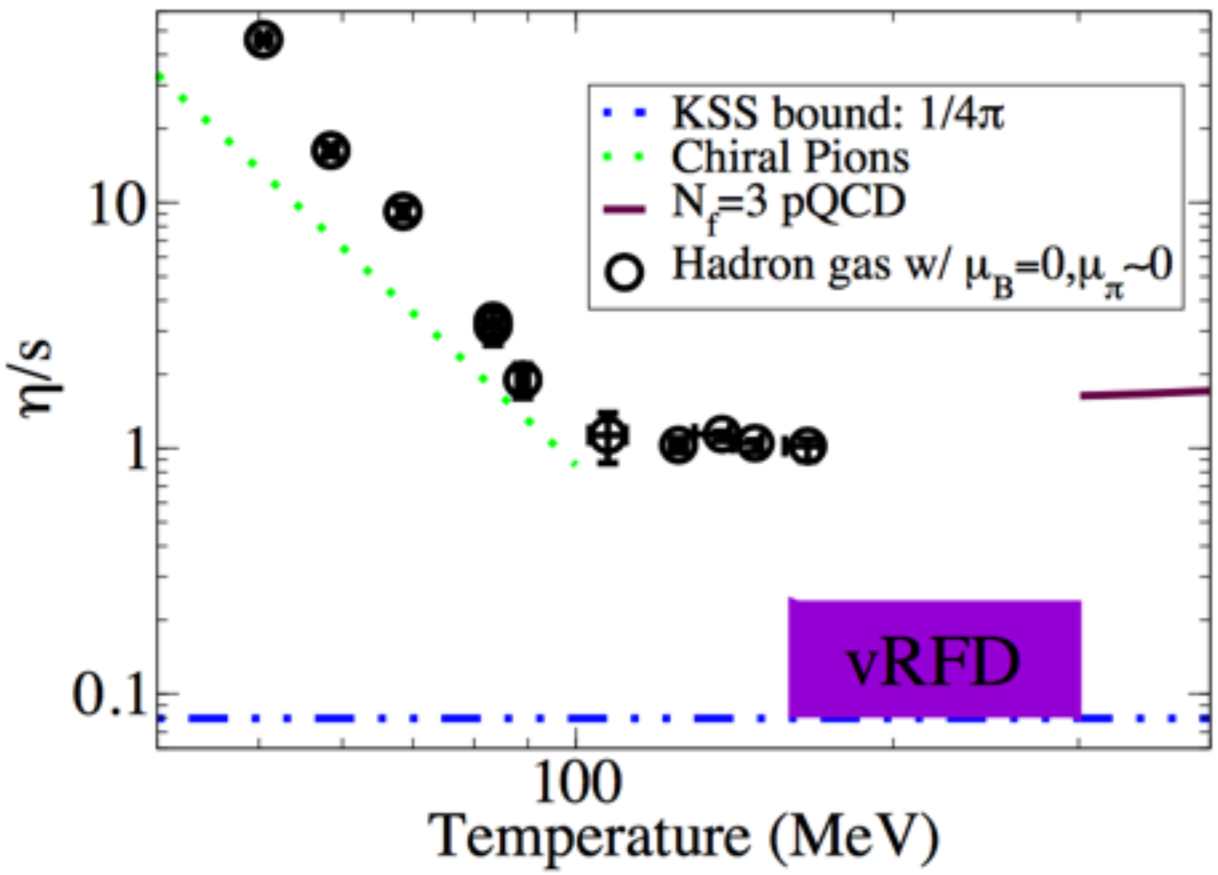
J. Bernhard et al, Phys.Rev. C94 (2016)



- The low temperature part corresponds to a hadron gas with its interactions

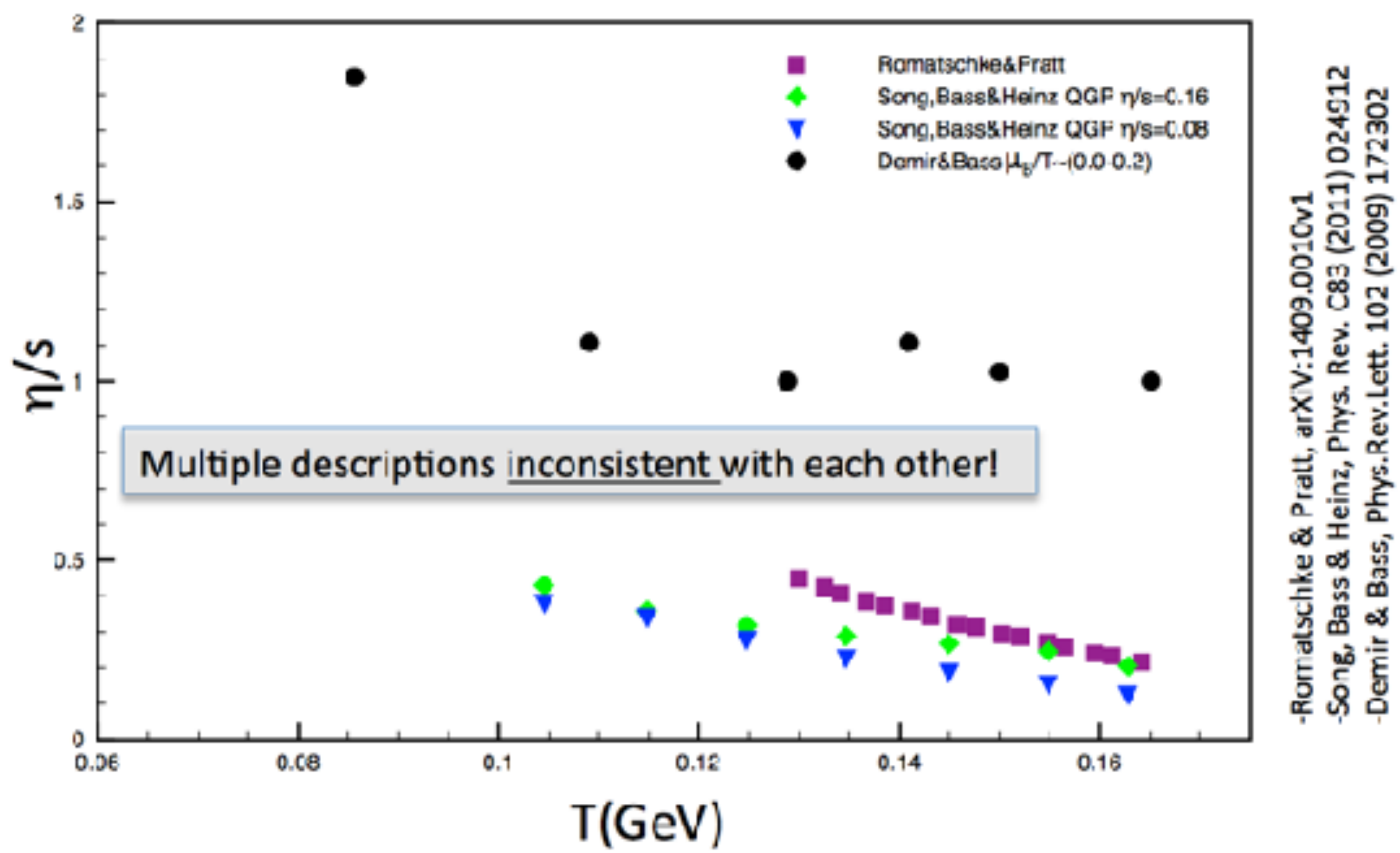
Shear Viscosity of the Hadron Gas

Green-Kubo formalism
UrQMD



N. Demir and S.A. Bass, PRL 102 (2009)

Discrepancy with
hydro-inspired B3D and VISHNU



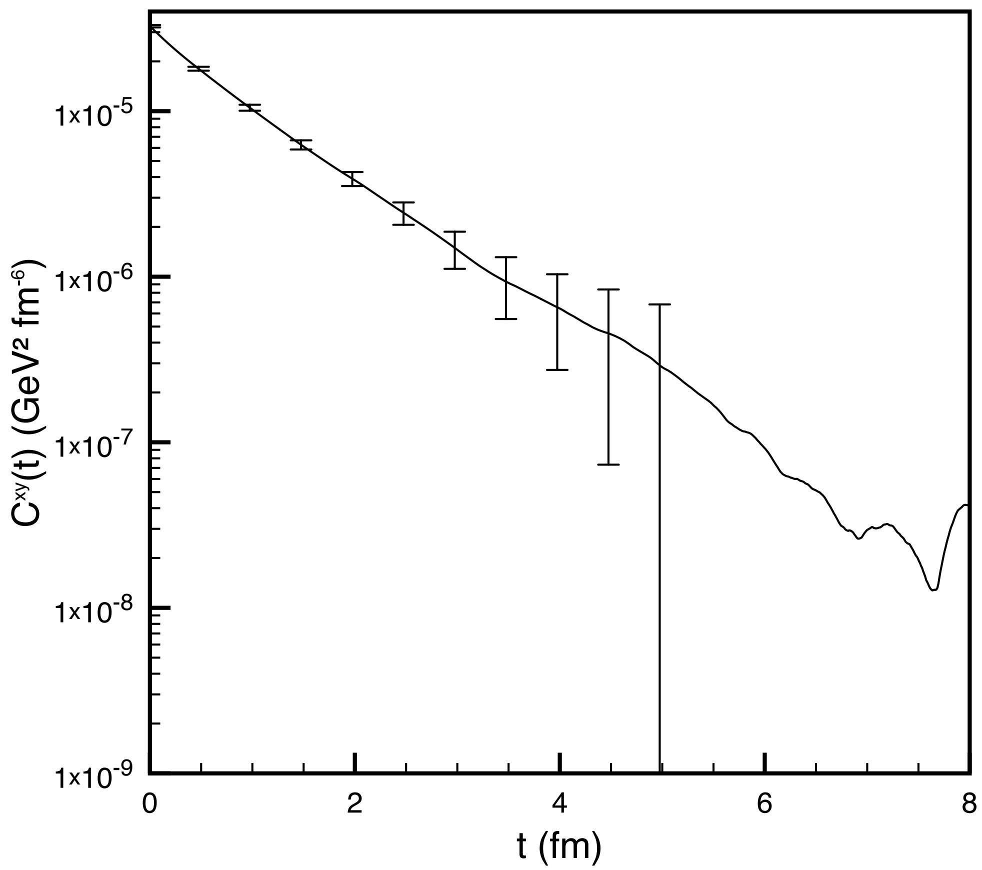
N. Demir and S.A. Bass, PRL 102 (2009)

- Long standing question: Why are the results so different from each other?

J.-B. Rose, J. M. Torres-Rincon, A. Schäfer, D. Oliinychenko and HP, PRC97 (2018) and JPCS 1024 (2018)

Shear Viscosity over Entropy Density

- Hadron gas in thermodynamic equilibrium realised by box with periodic boundary conditions
- Entropy is calculated via Gibbs formula from thermodynamic properties
- The shear viscosity is extracted following the Green-Kubo formalism:



$$T^{\mu\nu} = \frac{1}{V} \sum_i^{N_{part}} \frac{p_i^\mu p_i^\nu}{p_i^0}$$

$$C^{xy}(t) = \frac{1}{N} \sum_s^N T^{xy}(s) T^{xy}(s+t)$$

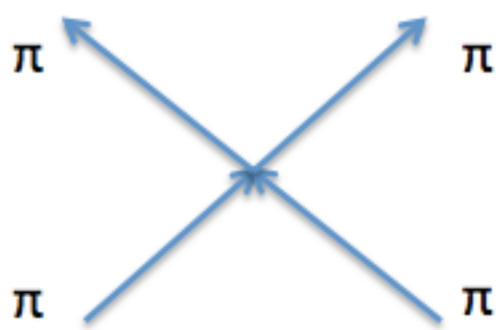
$$C^{xy}(t) \simeq C^{xy}(0) \exp\left(-\frac{t}{\tau}\right)$$

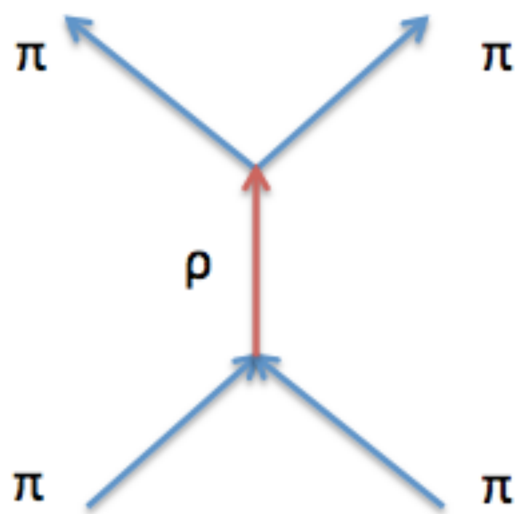
$$\eta = \frac{V C^{xy}(0) \tau}{T}$$

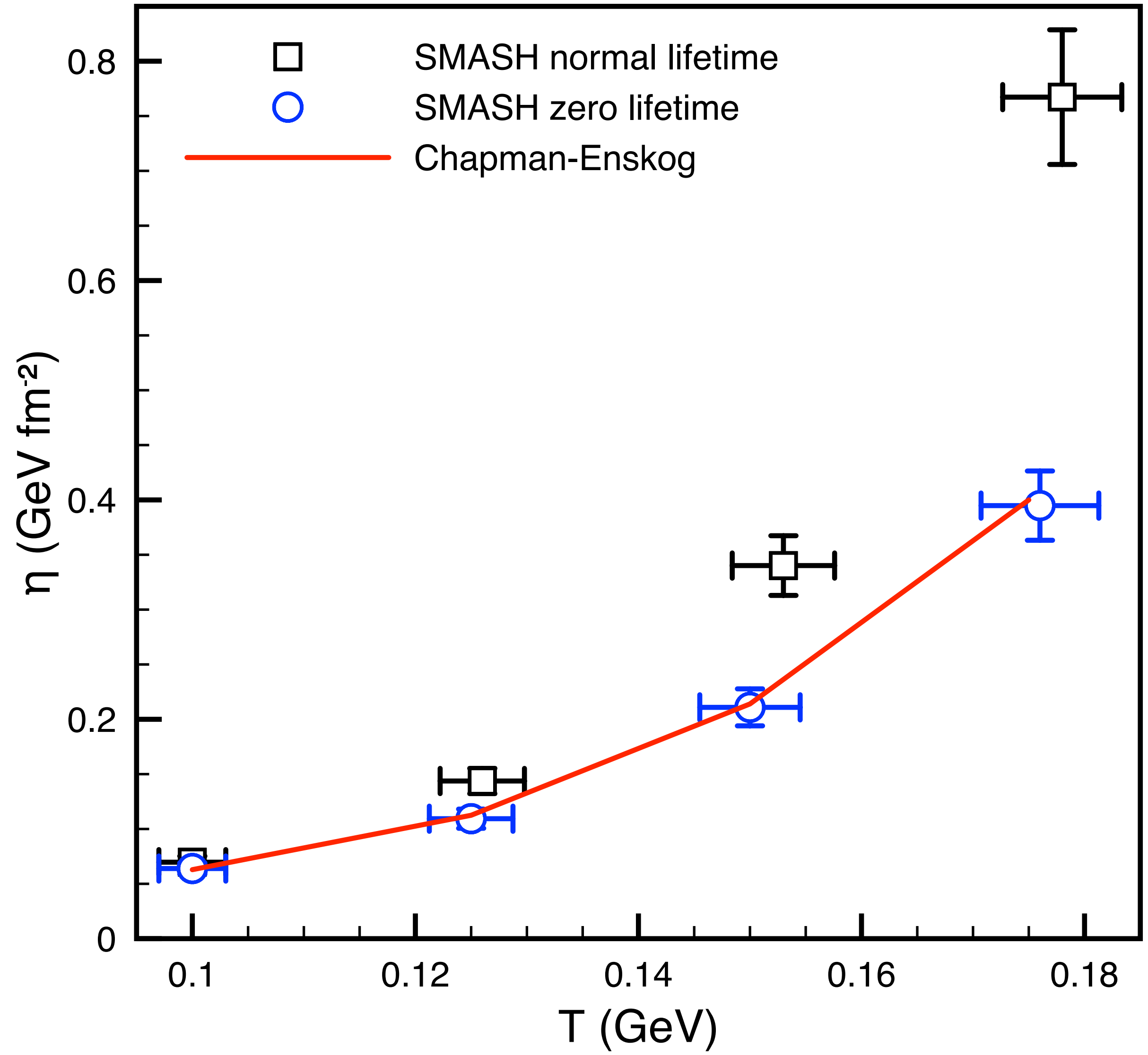
J.-B. Rose, J. M. Torres-Rincon, A. Schäfer, D. Oliinychenko and HP, PRC97 (2018) and JPCS 1024 (2018)

Resonance Dynamics

- Energy-dependence of cross-sections is modelled via resonances
- Point-like in analytic calculation and finite lifetime in transport approach





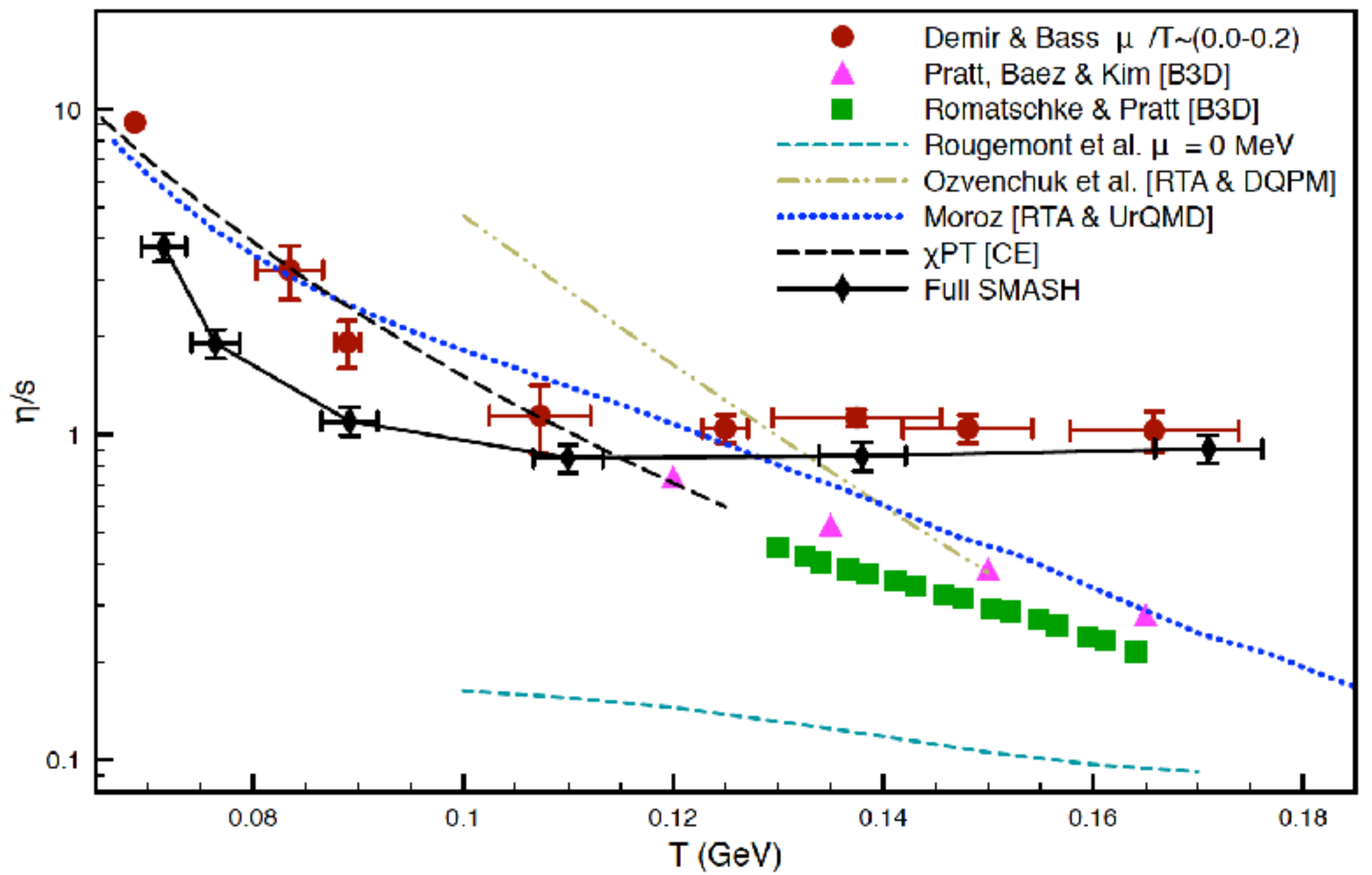


- Agreement recovered by decreasing ρ meson lifetime

J.-B. Rose, J. M. Torres-Rincon, A. Schäfer, D. Oliinychenko and HP, PRC97 (2018) and JPCS 1024 (2018)

Comparison to Literature

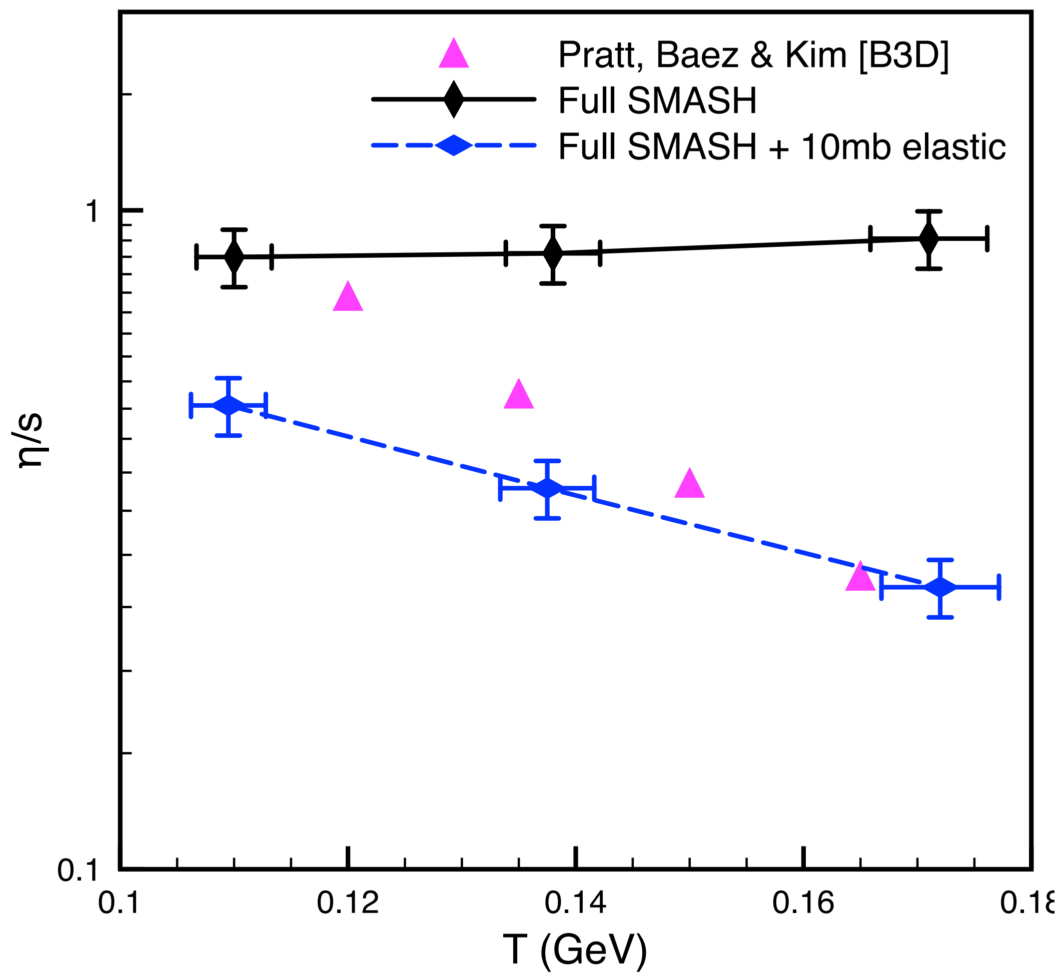
- Closest similarity to Bass/Demir result as expected

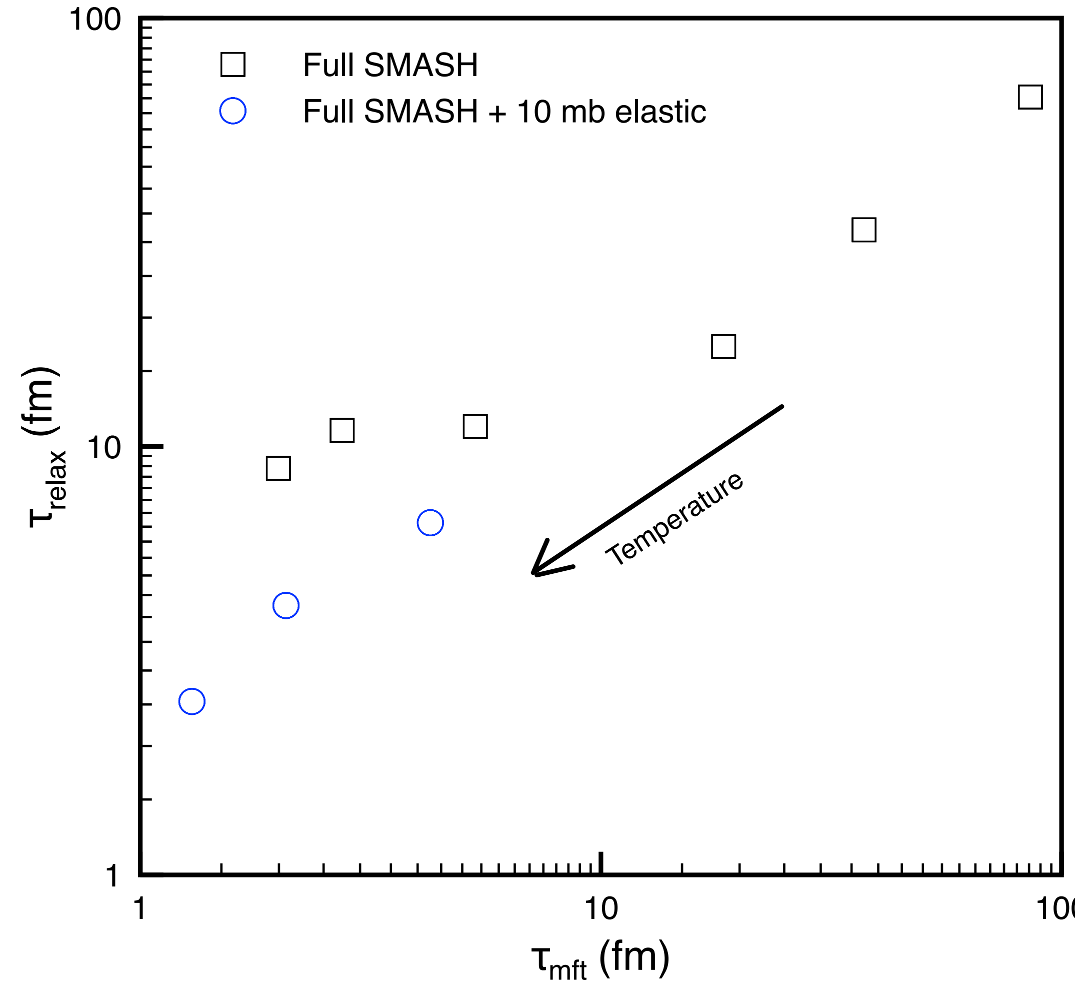


J.-B. Rose, J. M. Torres-Rincon, A. Schäfer, D. Oliinychenko and HP, PRC97 (2018) and JPCS 1024 (2018)

Point-like Interactions

- Adding a constant elastic cross section leads to agreement with B3D result

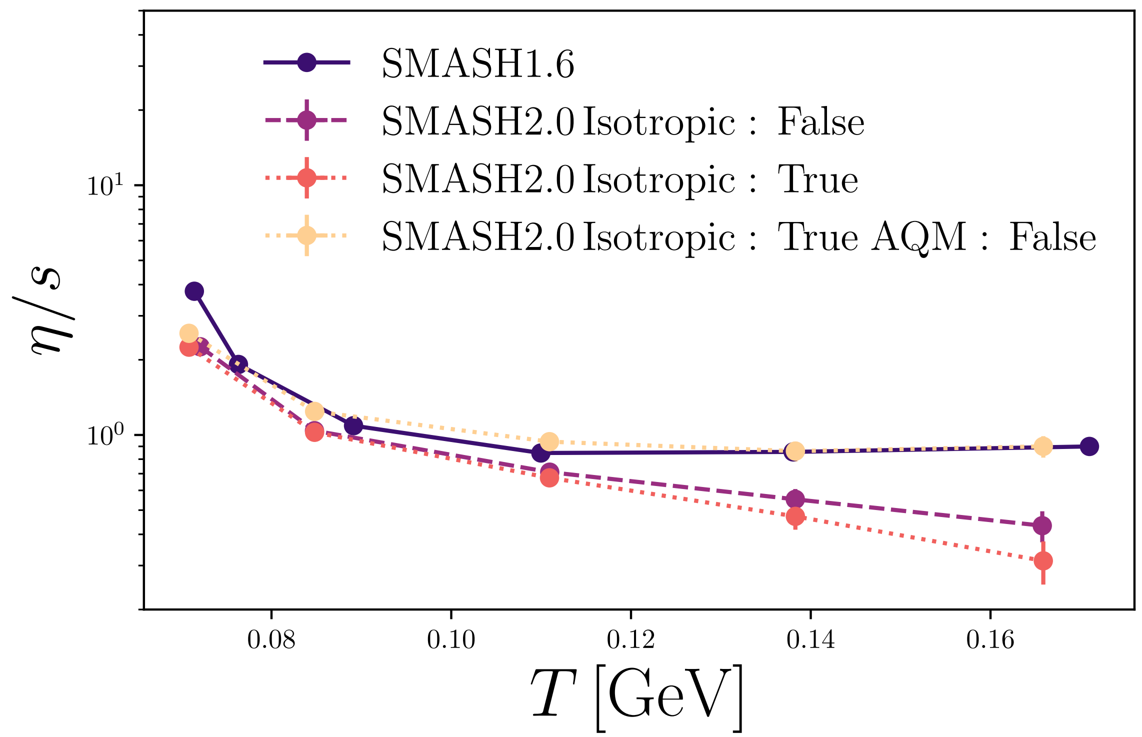


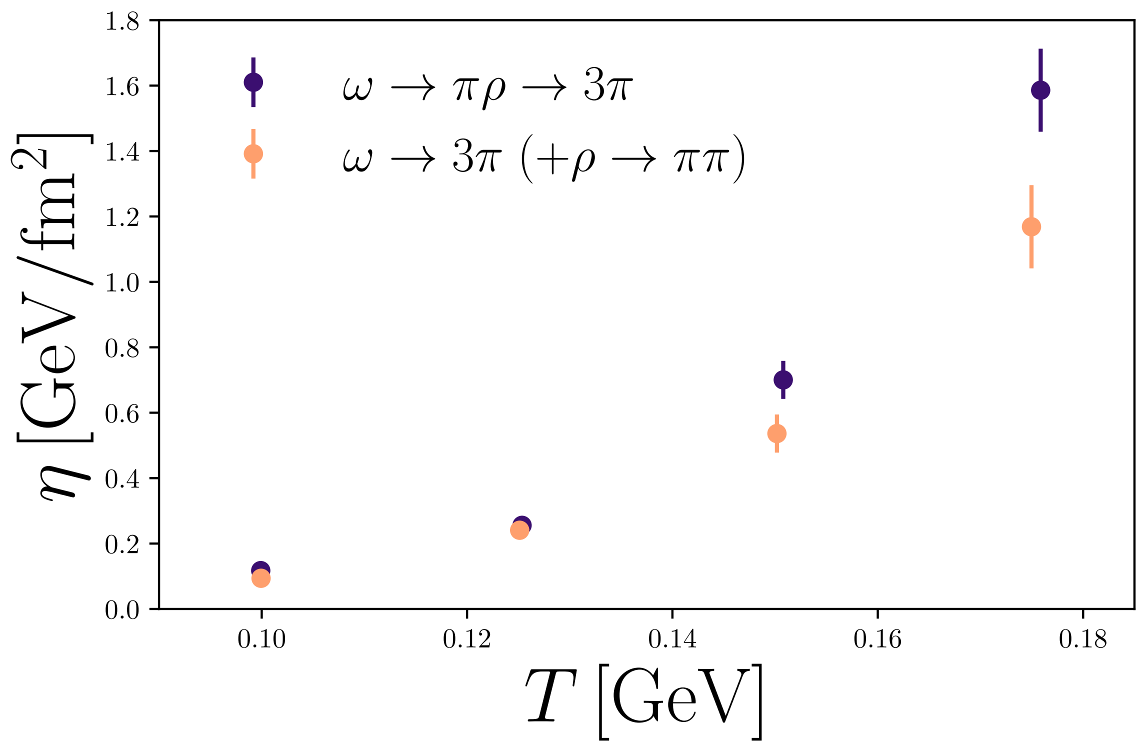


- Approximately linear relationship between relaxation time and mean free time is recovered
- Viscosity constrains the hadronic interactions

J.-B. Rose, J. M. Torres-Rincon, A. Schäfer, D. Oliinychenko and HP, PRC97 (2018) and JPCS 1024 (2018)

Update in SMASH





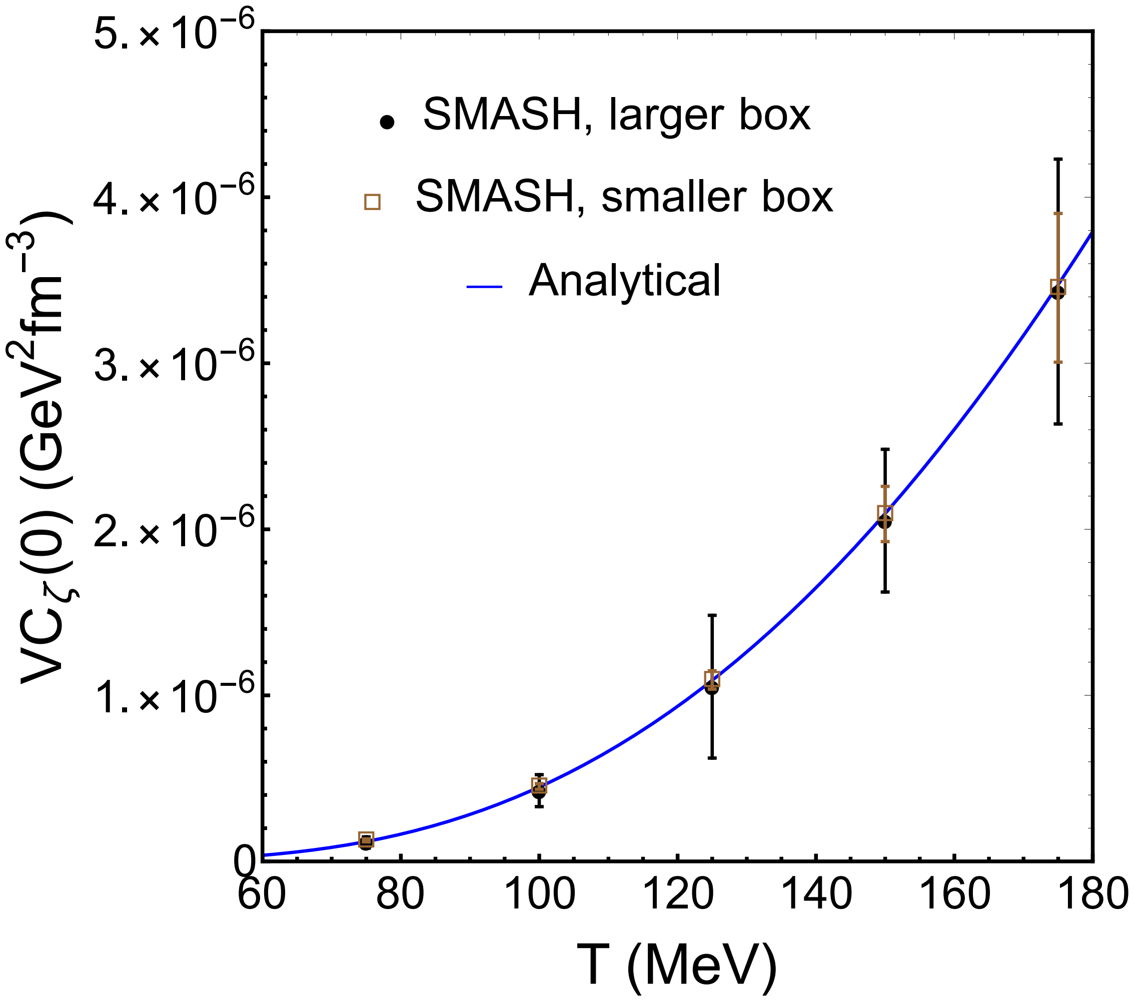
J. Hammelmann, work in progress

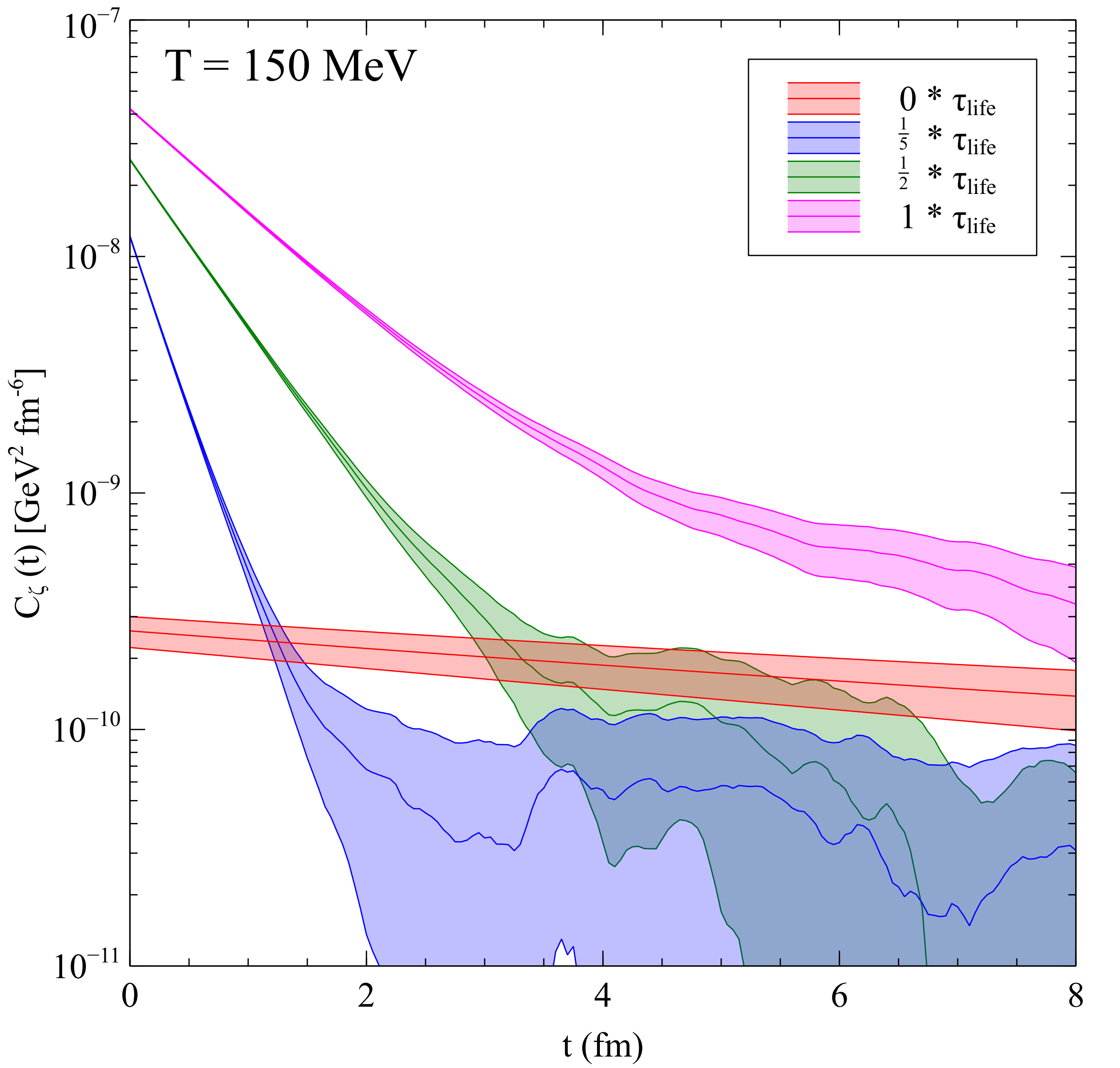
- Addition of elastic cross-sections leads to decrease of shear viscosity to entropy ratio
- The anisotropy of elastic scatterings has only small effect
- Direct decay of ω into 3 pions leads to smaller viscosity than the process via an intermediate ρ meson
- Enabled by multi-particle reactions

Bulk Viscosity of the Hadron Gas

- Bulk viscosity is very sensitive to the resonances and their dynamics

J.-B. Rose, J. M. Torres-Rincon and HE, J.Phys.G 48 (2020)

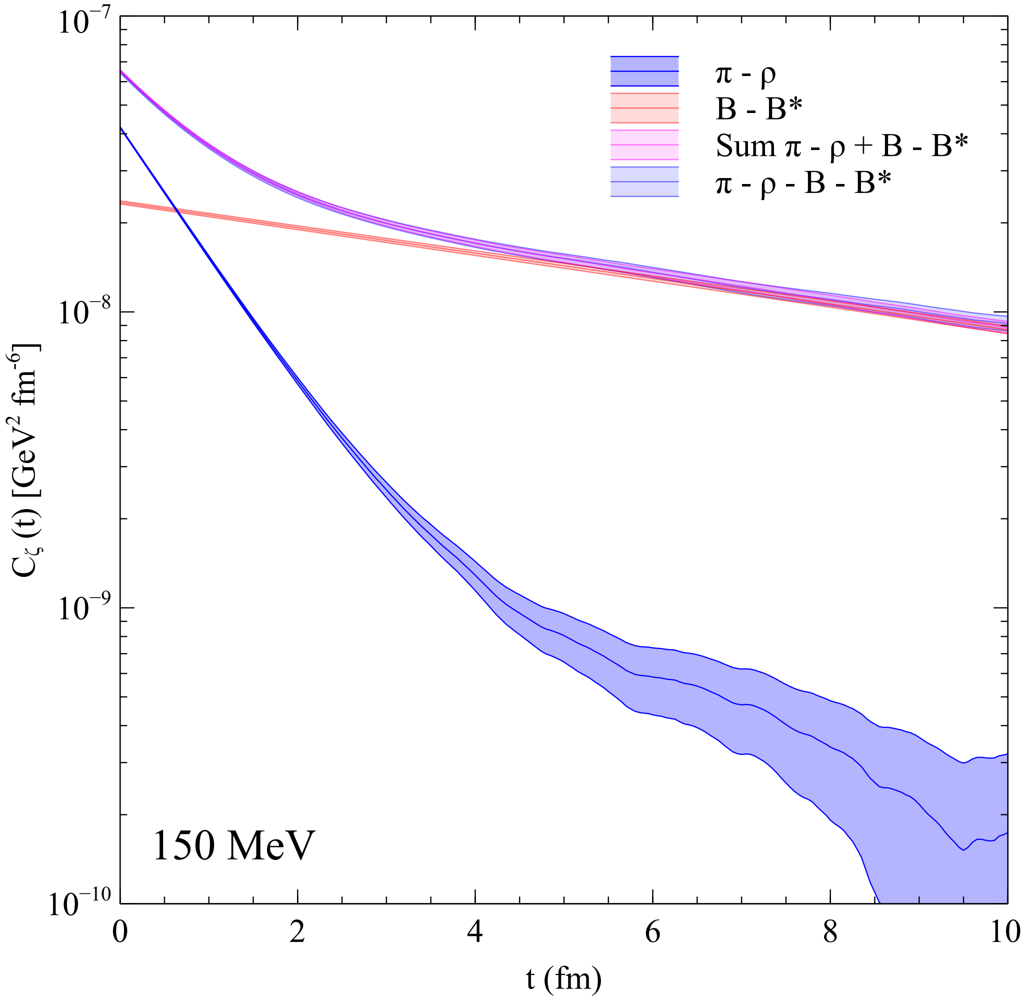


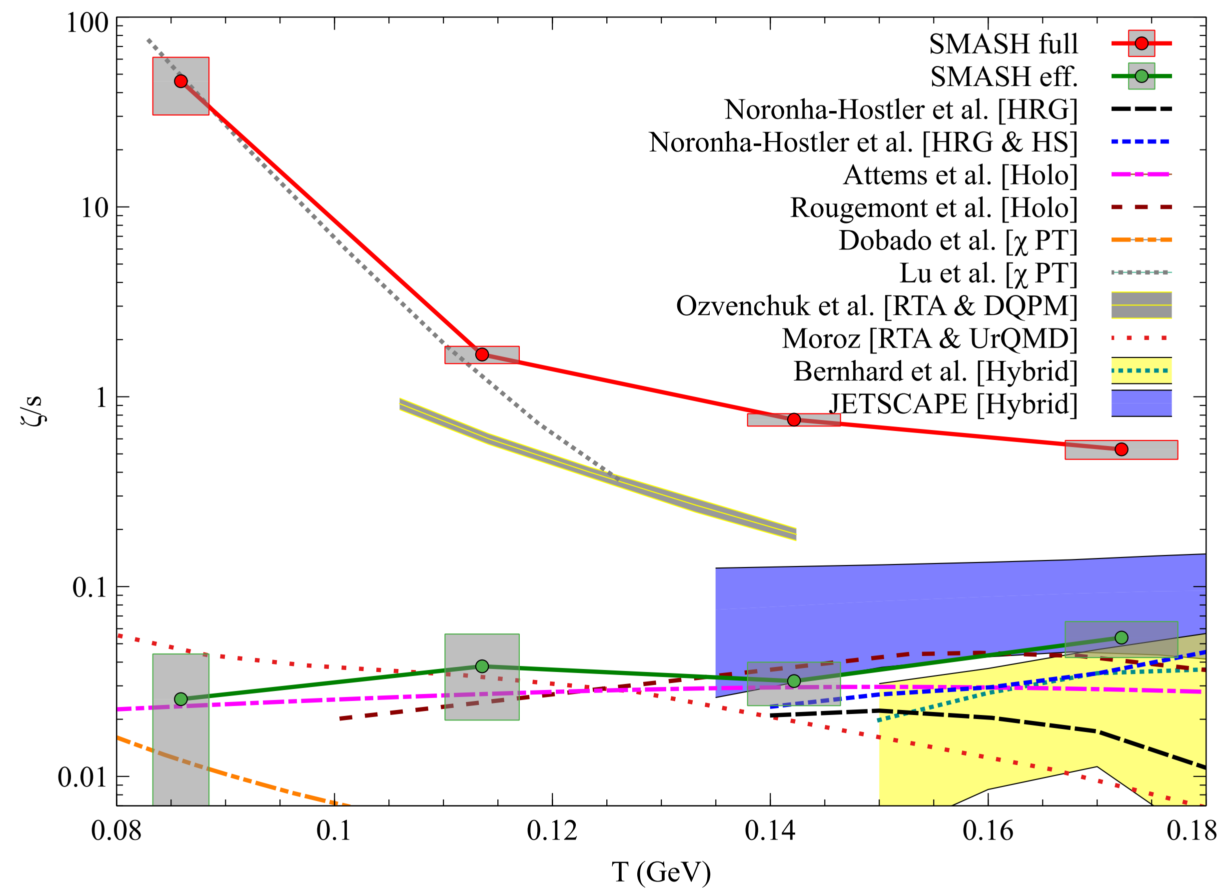


- Comparison to analytic solution successful, but correlation function is very sensitive to life time assumptions

Effective Bulk Viscosity

- Tail of correlation function is dominated by slow modes that are not relevant for the evolution of heavy-ion collisions





J.-B. Rose, J. M. Torres-Rincon and HE, J.Phys.G 48 (2020)

- The effective bulk viscosity excluding slow modes is in accordance with findings from Bayesian analysis

Electric Conductivity

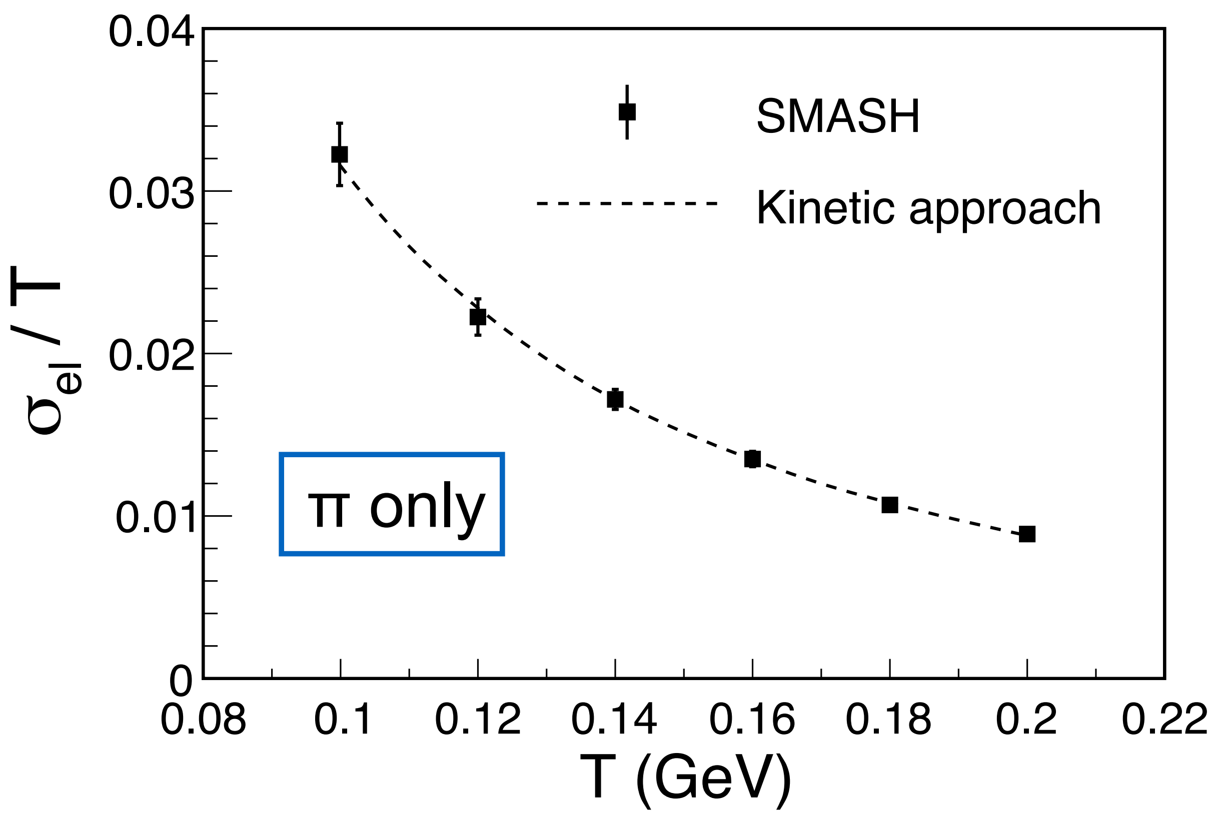
- Comparison to linear response kinetic theory to validate our approach

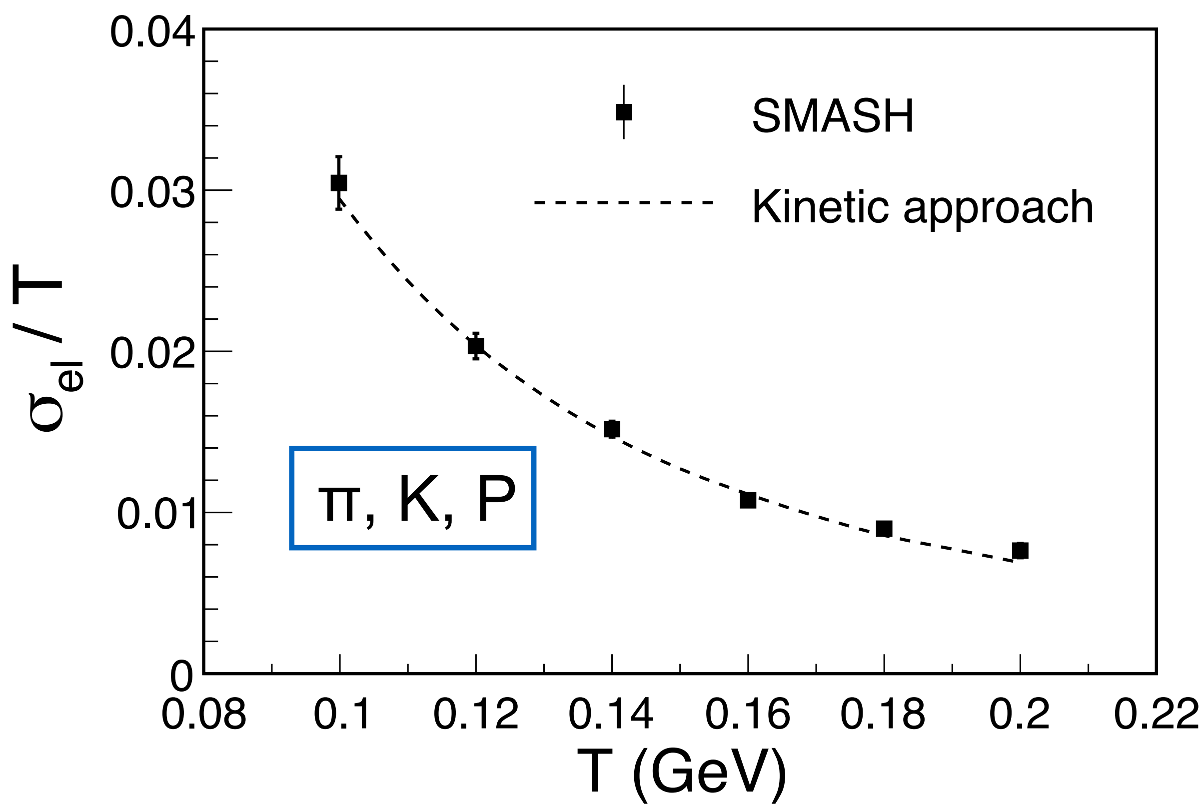
$$\sigma_{el} = \frac{V}{T}\int_0^\infty \langle j_i(0) j_i(t)\rangle dt \qquad\qquad \sigma_{el} = \frac{VC(0)\tau}{T}$$

- Infinite matter with constant σ = 30 mb

J. Hammelmann et al, Phys.Rev. D99 (2019) no.7, 076015

Greif et al, Phys.Rev. D93 (2016)

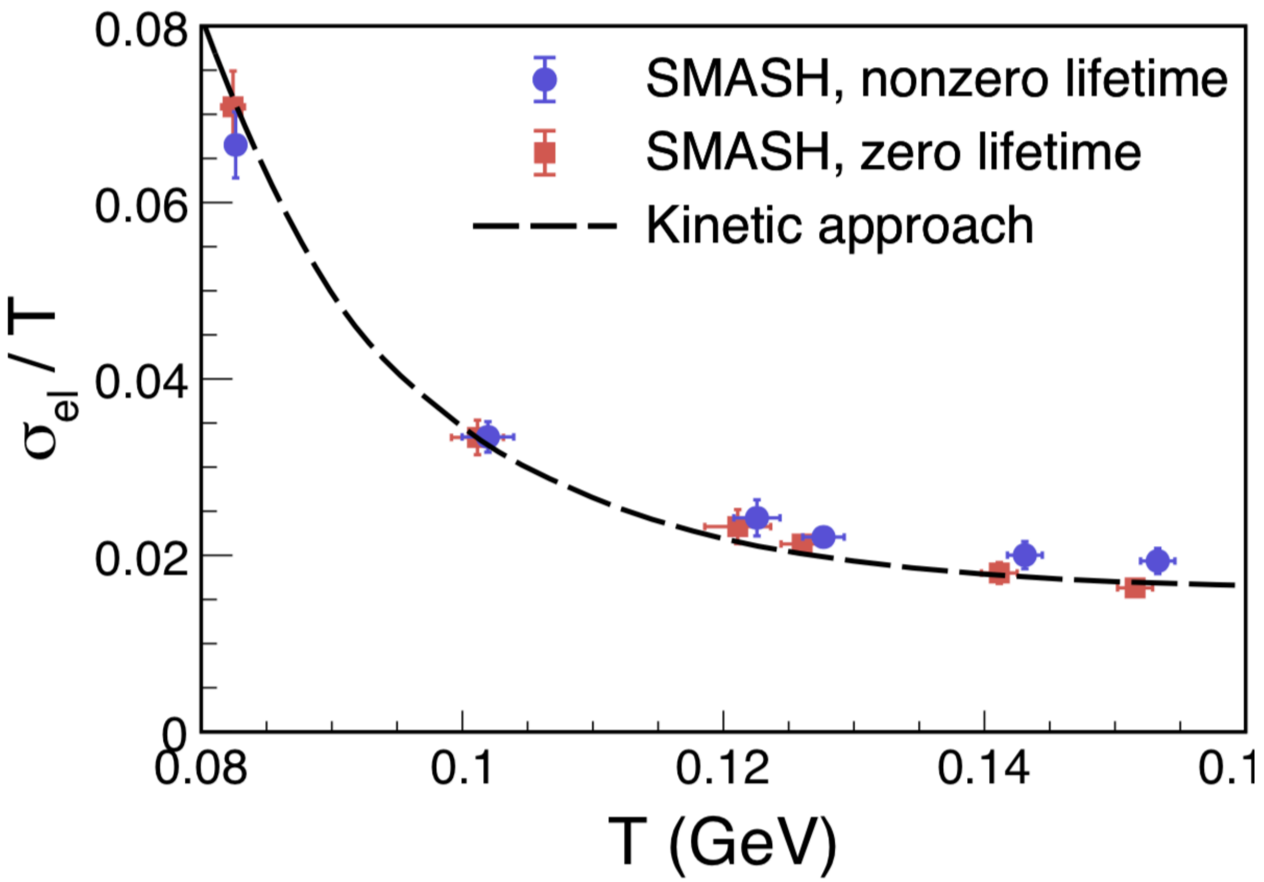


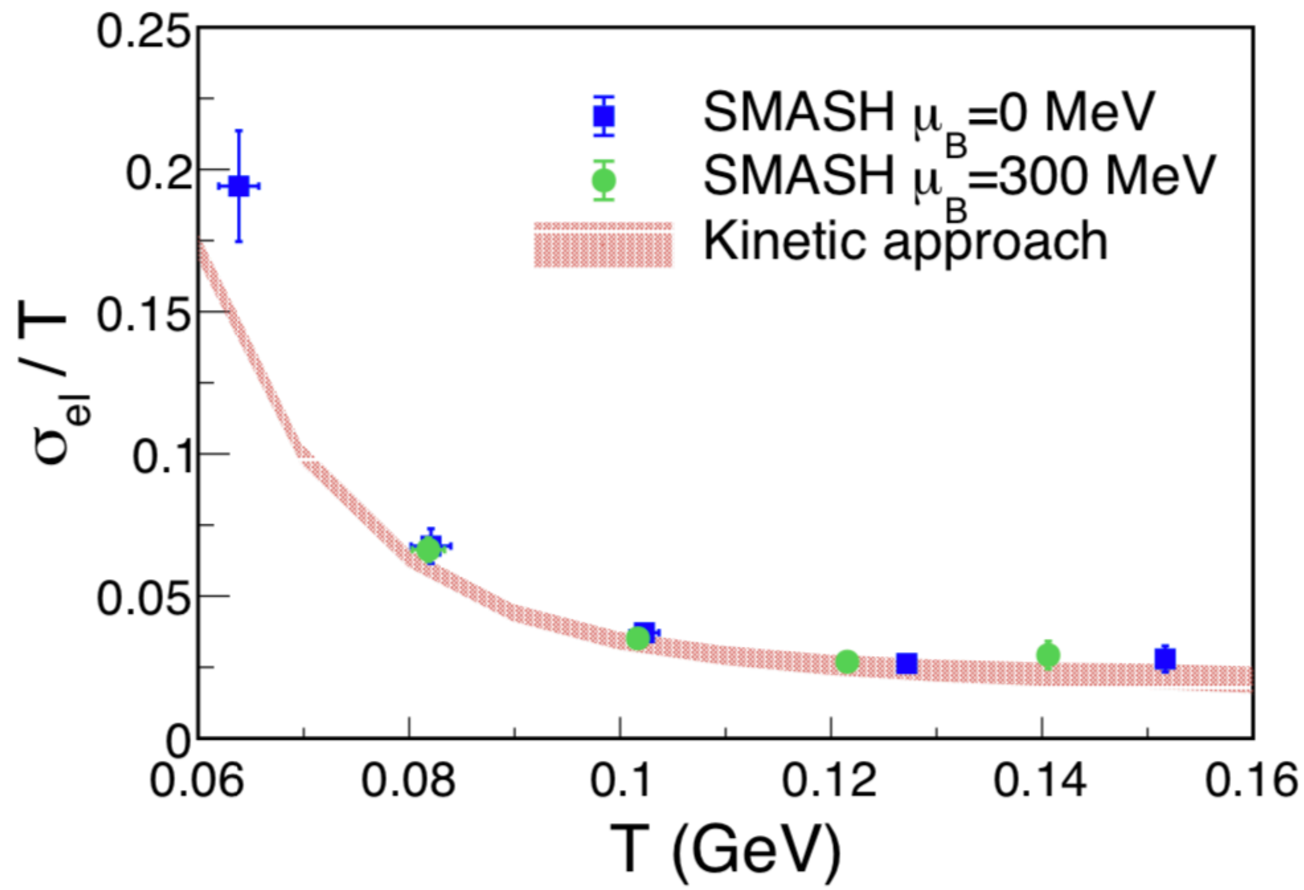


Influence of Lifetime

- Results for electric conductivity are independent of resonance lifetimes

J. Hammelmann et al, Phys.Rev. D99 (2019) no.7, 076015



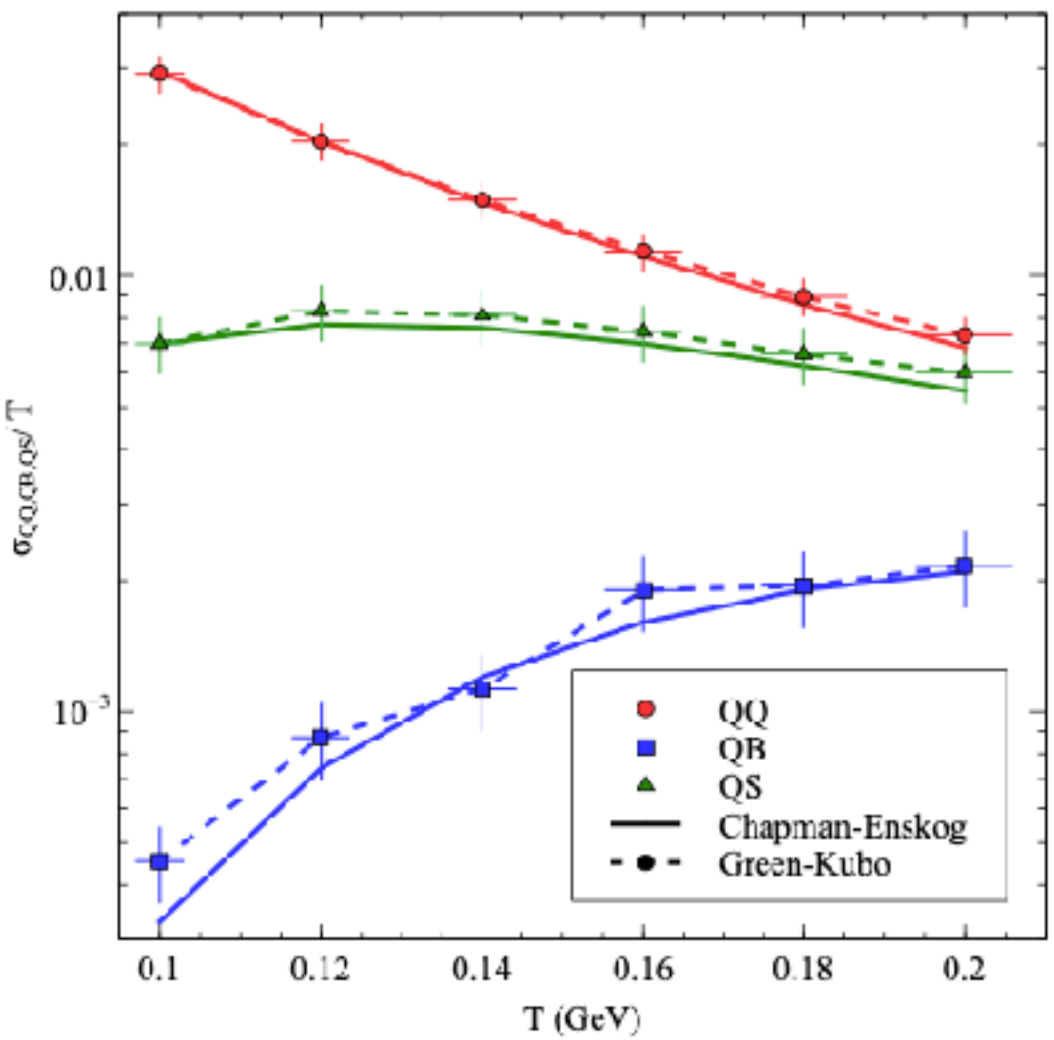


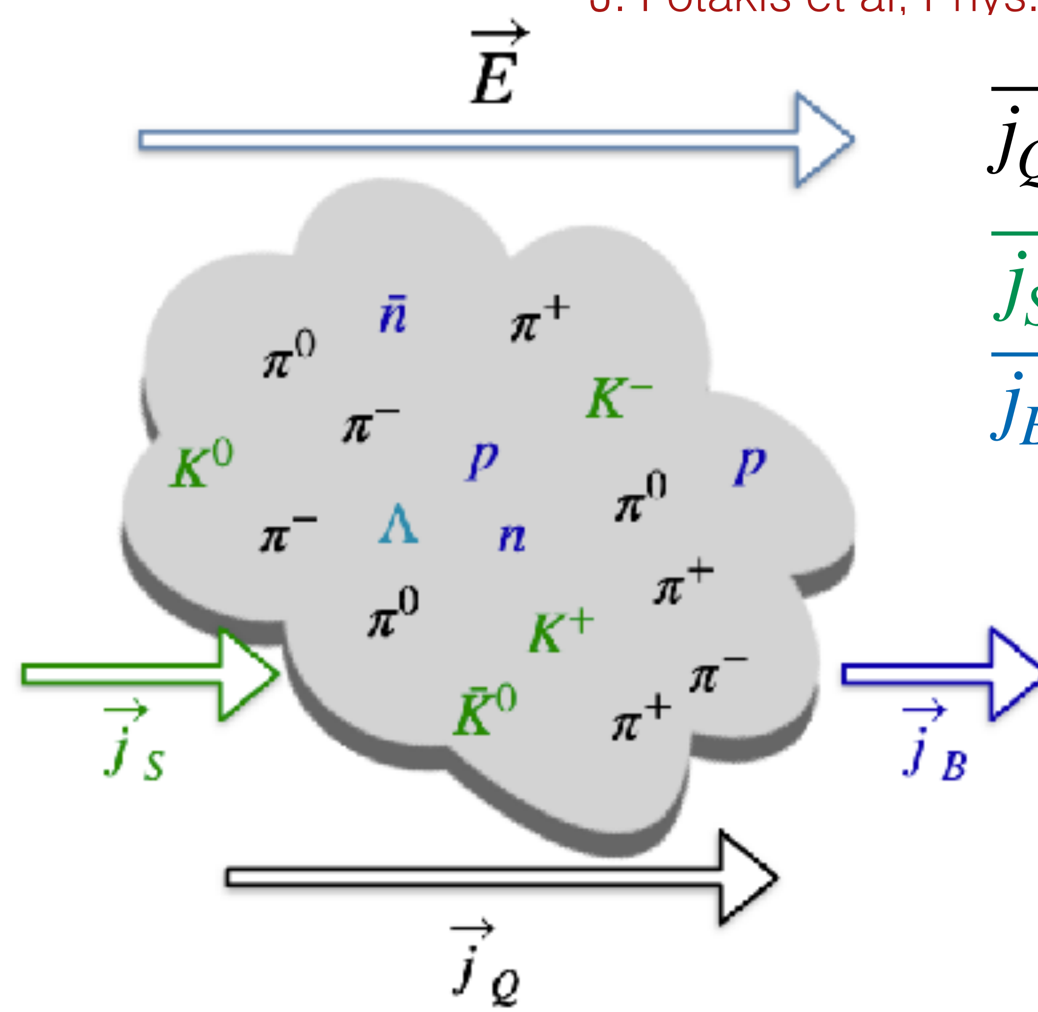
- Electric current relaxes already at formation of resonances and not only at the decay (full momentum exchange)

Cross-Conductivities

- The different species in a hadron gas have mixed quantum numbers, currents develop and mix

M. Greif et al, Phys.Rev.Lett. 120 (2018)
J. Fotakis et al, Phys.Rev.D 101 (2020)





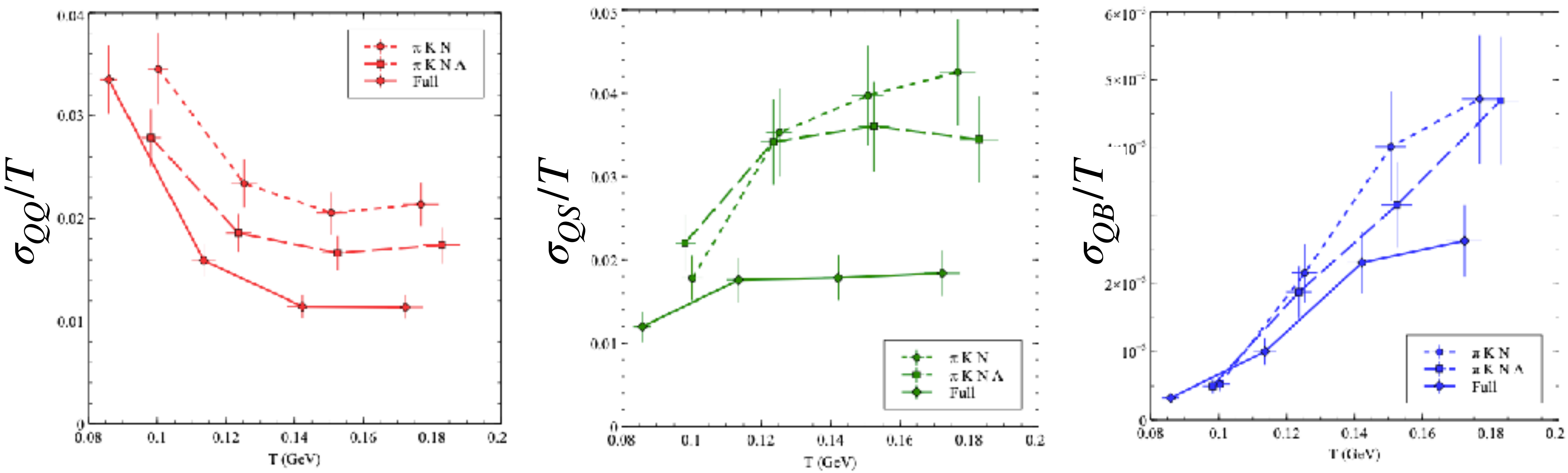
$$\vec{j}_Q = \sigma_{QQ}\vec{E}_Q$$
$$\vec{j}_S = \sigma_{QS}\vec{E}_Q$$
$$\vec{j}_B = \sigma_{QB}\vec{E}_Q$$

J.-B. Rose et al, Phys.Rev.D 101 (2020)

- Comparison in simple π-K-P system to analytic calculation successful

Cross-Conductivities

- All conductivities are highly dependent on the degrees of freedom employed in the calculation



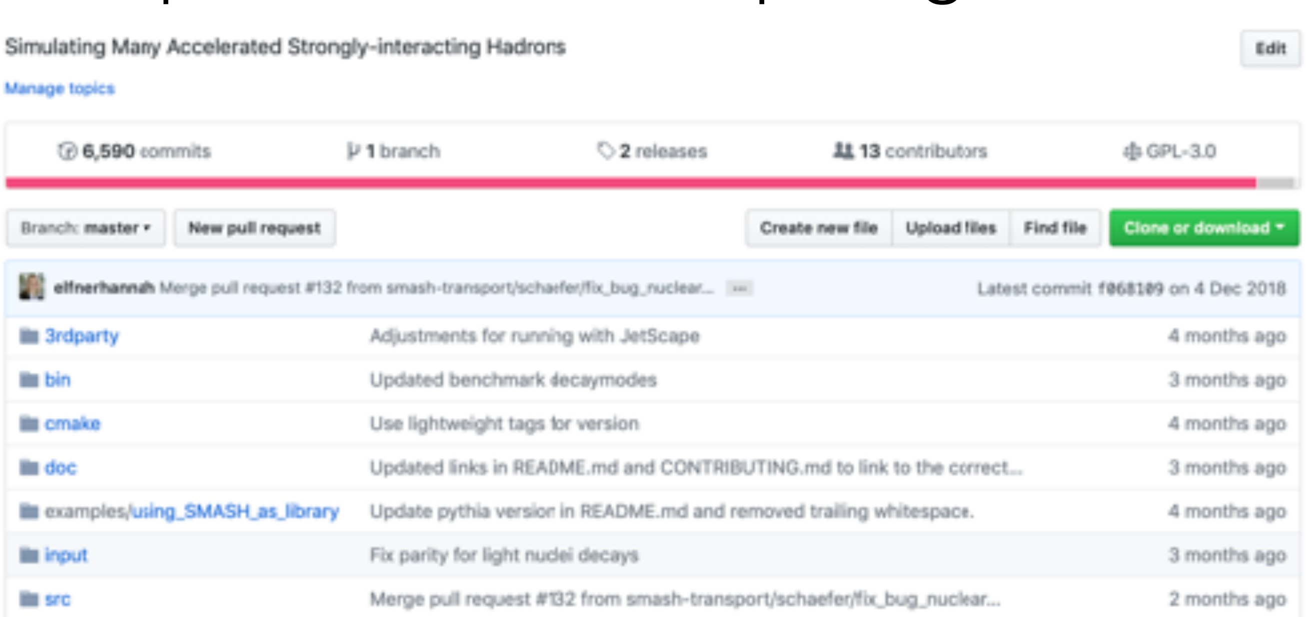
J.-B. Rose et al, Phys.Rev.D 101 (2020)

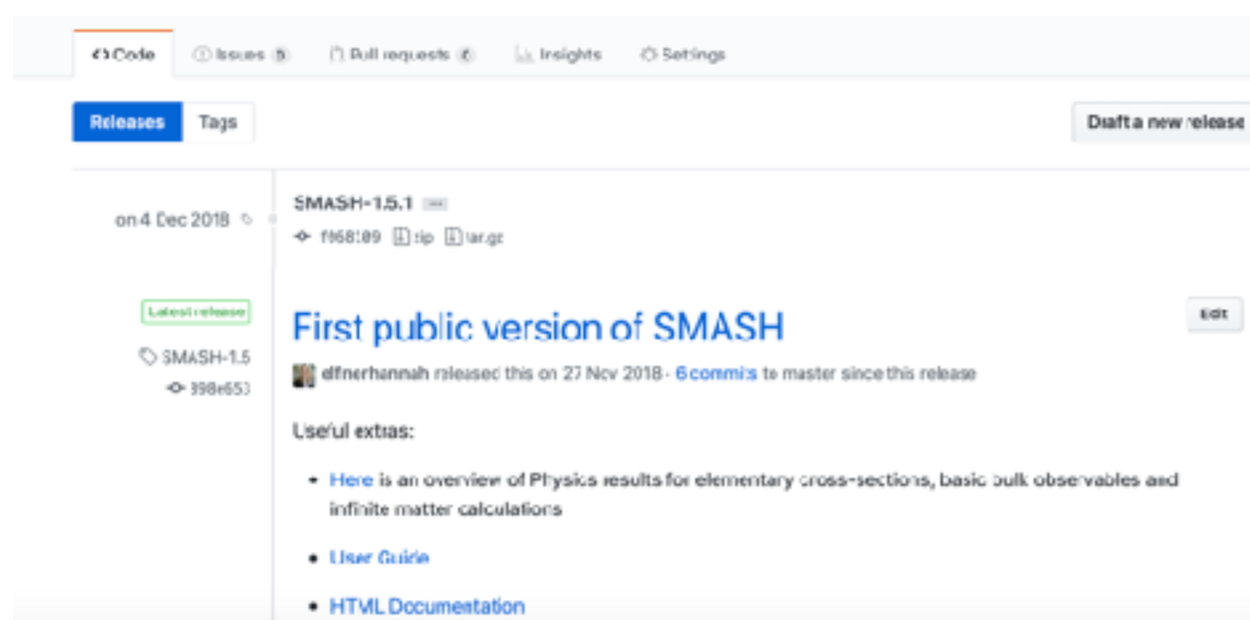
- Potential to constrain active degrees of freedom in the hadron gas by comparison to future lattice results

How to Use SMASH?

- Visit the webpage to find publications and link to SMASH-2.2 results
  https://smash-transport.github.io
- Download the code at
  https://github.com/smash-transport/smash
- Checkout the Analysis Suite at
  https://github.com/smash-transport/smash-analysis
- Find user guide and documentation at
  https://github.com/smash-transport/smash/releases
- Animations and Visualization Tutorial under
  https://smash-transport.github.io/movies.html

SMASH-2.2 has HepMC and RIVET





Summary

- Transport approaches are an indispensable tool to understand observables from heavy-ion reactions
- In contrast to thermal and fluid dynamic models they allow for insights on the microscopic level
- Coupled Boltzmann equations need to be solved numerically
- SMASH as a specific example for hadronic transport
  - Bulk observables are sensitive to equation of state
  - Transport coefficients of hadron gas
- Source code is public and ready to use!
- In general: Transport codes have to be thoroughly validated and are the only way to describe the non-equilibrium evolution properly

Questions

- What is the difference between geometrical cross sections and transition probabilities in transport approaches?
- How can one describe the phase transition in microscopic transport approaches?
- What is a hadronic resonance?
- How are the inputs for hadronic transport approaches, the degrees of freedom and their interactions, constrained?
- How are the transport coefficients in a box calculation related to the ones in a full dynamic calculation?
- Which other transport properties would be interesting?
- For which stages of the reaction are transport approaches applicable?