Transport and Hybrid Approaches

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June 24th, 2022, Karpacz Summer School Lecture

The Plan for Lecture II

- Ingredients of Hybrid Approaches
  - Hydrodynamics and Transport
  - Initial conditions and Cooper-Frye surface
- Historical Overview
  - Development of hybrid approaches
  - Important results
- Interfaces between Hydrodynamics and Transport
  - Deviation from equilibrium in the initial stages
  - Negative contributions on Cooper-Frye surface
- One specific approach: SMASH-vHLLE hybrid
  - Application to beam energy scan energies
  - Results for hadronic rescattering

Hybrid Approaches: Ingredients

Time Evolution of Heavy-Ion Collisions

- Detailed dynamical modeling is essential to learn something about hot and dense QGP stage

1x 10^{-23} s

10 x 10^{-23} s

30 x 10^{-23} s

Nonequilibrium initial state dynamics → Relativistic Hydrodynamics → Hadron Transport

Hybrid approaches are current tool of choice

Hybrid approaches

Transport

Microscopic description of the whole phase-space distribution

Non-equilibrium evolution based on the Boltzmann equation

$$(p^{\mu}\partial_{\mu})f = I_{coll}$$

Partonic or hadronic degrees of freedom

Cross-sections are calculable using different techniques

Phase transition?

Hydrodynamics

Macroscopic description

Local equilibrium is assumed

$$\partial_{\mu} T^{\mu\nu} = 0 \qquad \partial_{\mu}(nu^{\mu}) = 0$$

+viscous corrections

Propagation according to conservation laws

Equation of state is an explicit input

Boundary conditions: Breakdown of equilibrium assumptions?

General Ingredients and Interfaces

- Initial conditions:
  - Saturation based, e.g. IP-Glasma, Magma, EKRT
  - Nucleon based, e.g. MC Glauber
  - Dynamic approaches, e.g. NEXUS, UrQMD, AMPT, SMASH
  - Generic parameterization Trento
- Non-equilibrium evolution
  - Free-streaming
  - Linearized kinetic theory e.g. Kompost
  - Dynamic approaches (see above)
- Viscous hydrodynamics (often with boost invariance)
- Cooper-Frye switching hypersurface and sampling
- Hadronic rescattering with UrQMD or SMASH

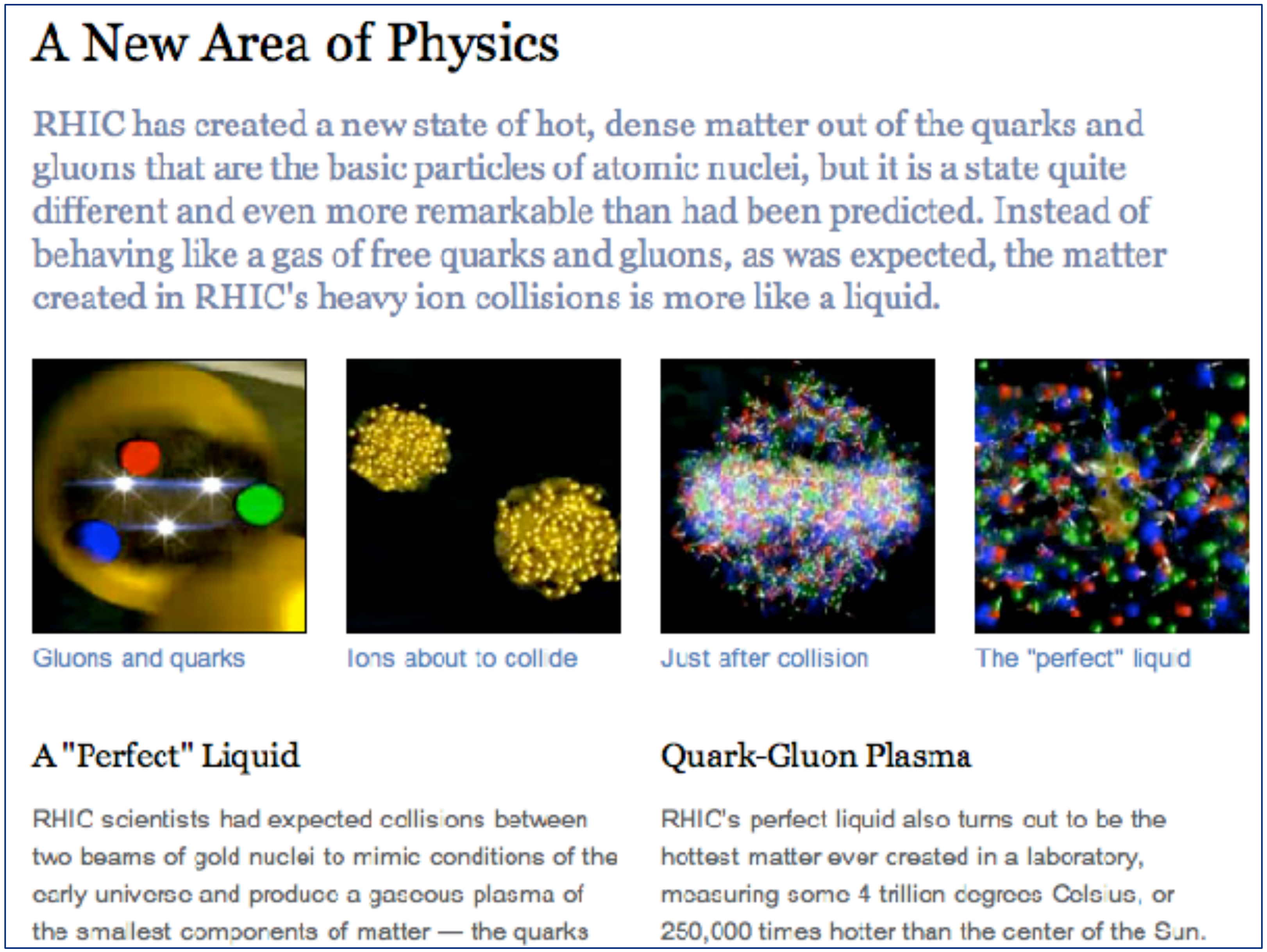
Applicability Limits

- Goal
  - Combine the two approaches in such a way, that switching is performed in region of joint applicability
  - Transport theory is appropriate off equilibrium and for dilute systems
  - Fluid dynamics is applicable, if energy-momentum tensor is close enough to a nearly ideal form
- Motivation
  - Hybrid approaches were developed to accommodate differential chemical and kinetic freeze-out for different particle species
  - Hydrodynamics breaking down for off-central collisions (small systems), low beam energies

The Perfect Liquid

- In 2005, Brookhaven National Lab announced in a press release that RHIC had created a new state of hot and dense matter which behaves like an ideal liquid

taken from website for RHIC at BNL



A New Area of Physics

RHIC has created a new state of hot, dense matter out of the quarks and gluons that are the basic particles of atomic nuclei, but it is a state quite different and even more remarkable than had been predicted. Instead of behaving like a gas of free quarks and gluons, as was expected, the matter created in RHIC's heavy ion collisions is more like a liquid.

Gluons and quarks | Ions about to collide | Just after collision | The "perfect" liquid

A "Perfect" Liquid

RHIC scientists had expected collisions between two beams of gold nuclei to mimic conditions of the early universe and produce a gaseous plasma of the smallest components of matter — the quarks

Quark-Gluon Plasma

RHIC's perfect liquid also turns out to be the hottest matter ever created in a laboratory, measuring some 4 trillion degrees Celsius, or 250,000 times hotter than the center of the Sun.

Google Image search for 'perfect liquid'

Here is the heavy ion reaction!

Hydrodynamics

- Ideal/Viscous relativistic fluid dynamics

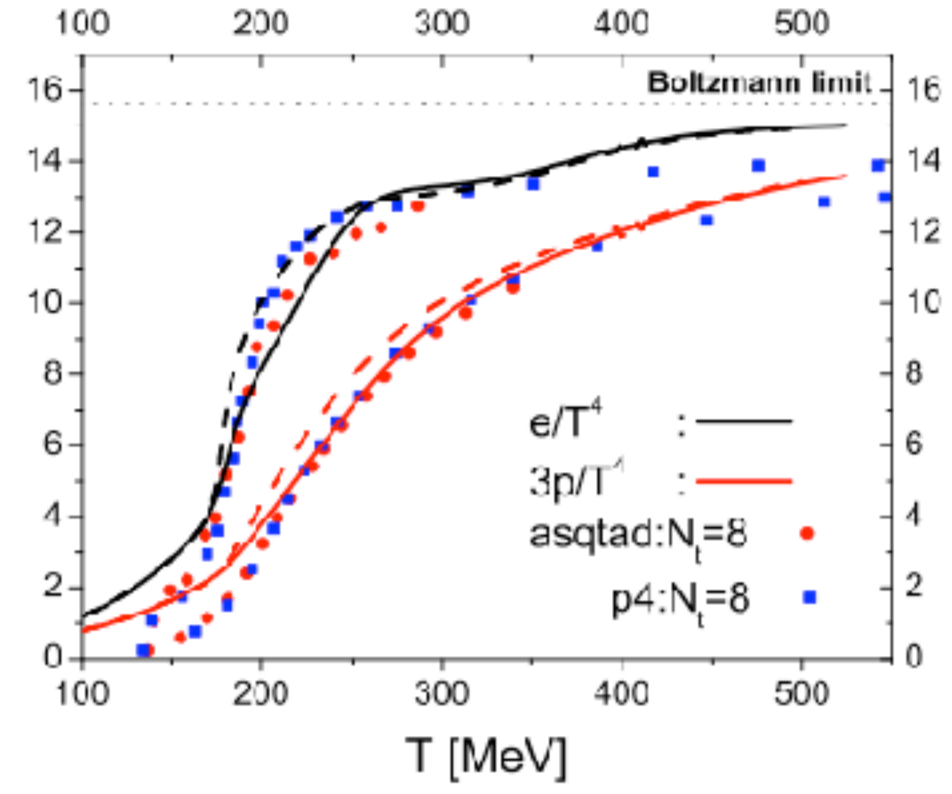
Conservation Laws

$$\partial_\mu T^{\mu\nu} = 0 \qquad \partial_\mu (n u^\mu) = 0$$

- Macroscopic quantities like energy density, temperature, pressure

$$T^{\mu\nu} = (\epsilon + P) u^\mu u^\nu - g^{\mu\nu} P$$

- Calculations on a grid with 8 Mio. cells that are evolved in time
- One can treat the **phase transition** via the equation of state (relation between pressure and temperature)

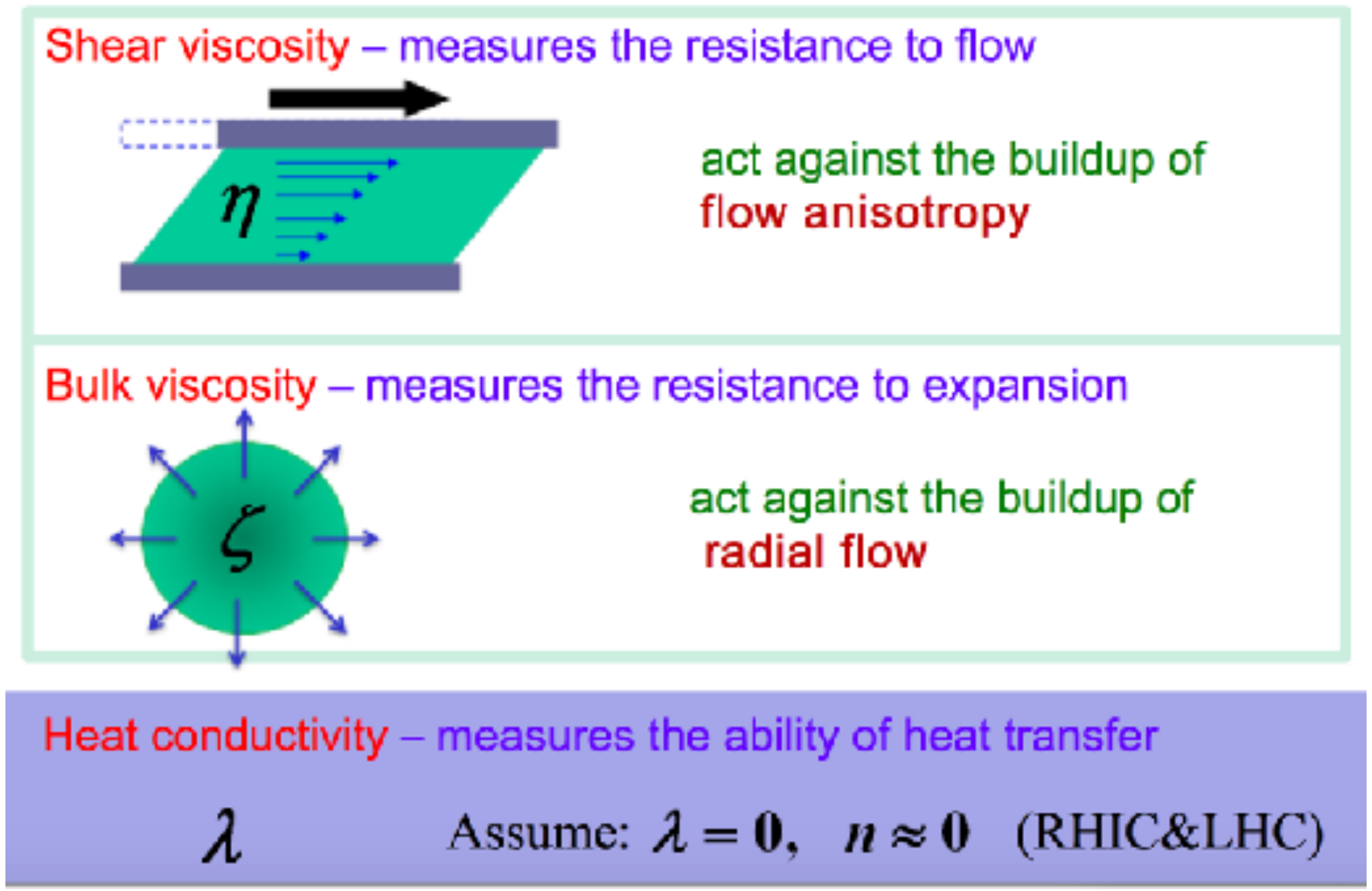


J. Steinheimer et al, J.Phys.G G38 (2011) 035001

Viscous Hydrodynamics

- 3+1d viscous hydrodynamics is applied by many groups
  - Stability against shocks is crucial for event-by-event calculations
- Equation of state that matches available lattice QCD data
- What about finite baryo-chemical potential?

E.g. MUSIC code is available here
http://www.physics.mcgill.ca/music/



H. Song, CPOD 2014

- Temperature and net baryon density dependent transport coefficients? Cross-conductivities and diffusion matrix?

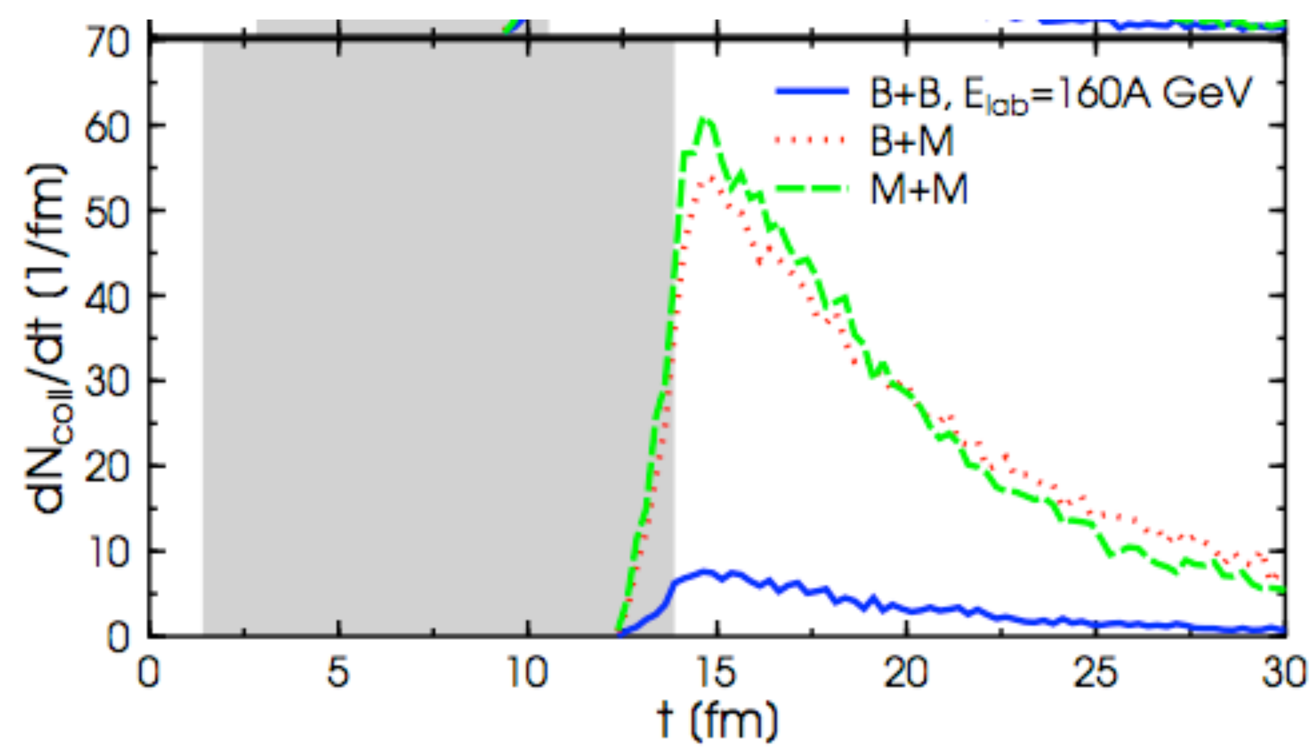
Freeze-out Procedure

- Deconfinement/Confinement transition happens through equation of state in hydrodynamics
- Transition from hydro to transport when temperature/energy density is smaller than critical value
- Particle distributions are generated according to the Cooper-Frye formula

$$E\frac{dN}{d^3p} = \int_\sigma f(x,p)p^\mu d\sigma_\mu$$

- Same EoS on both sides of the transition hypersurface
- Rescatterings and final decays calculated via hadronic cascade

> - Separation of chemical and kinetic freeze-out is taken into account
> - Large viscosity in hadron gas stage!



H.P. et al, Phys.Rev. C78 (2008) 044901

Hadronic Transport

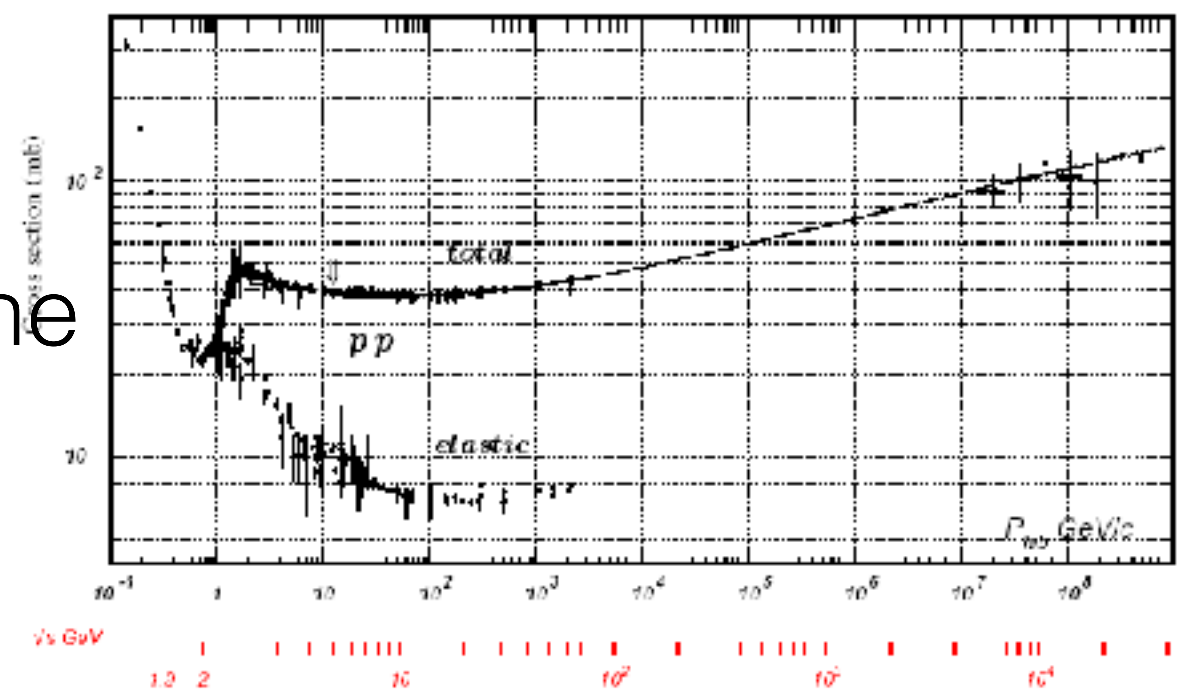
- Transport of a non-equilibrium system of microscopic particles
- All hadrons in the PDG up to masses of ~2.5 GeV
- Effective solution of Boltzmann equation

$$\left[\frac{\partial}{\partial t} + \frac{\vec{p}}{E} \times \frac{\partial}{\partial \vec{r}}\right] f_1(\vec{p}, \vec{r}, t) = \sum_{processes} C(\vec{p}, \vec{r}, t)$$

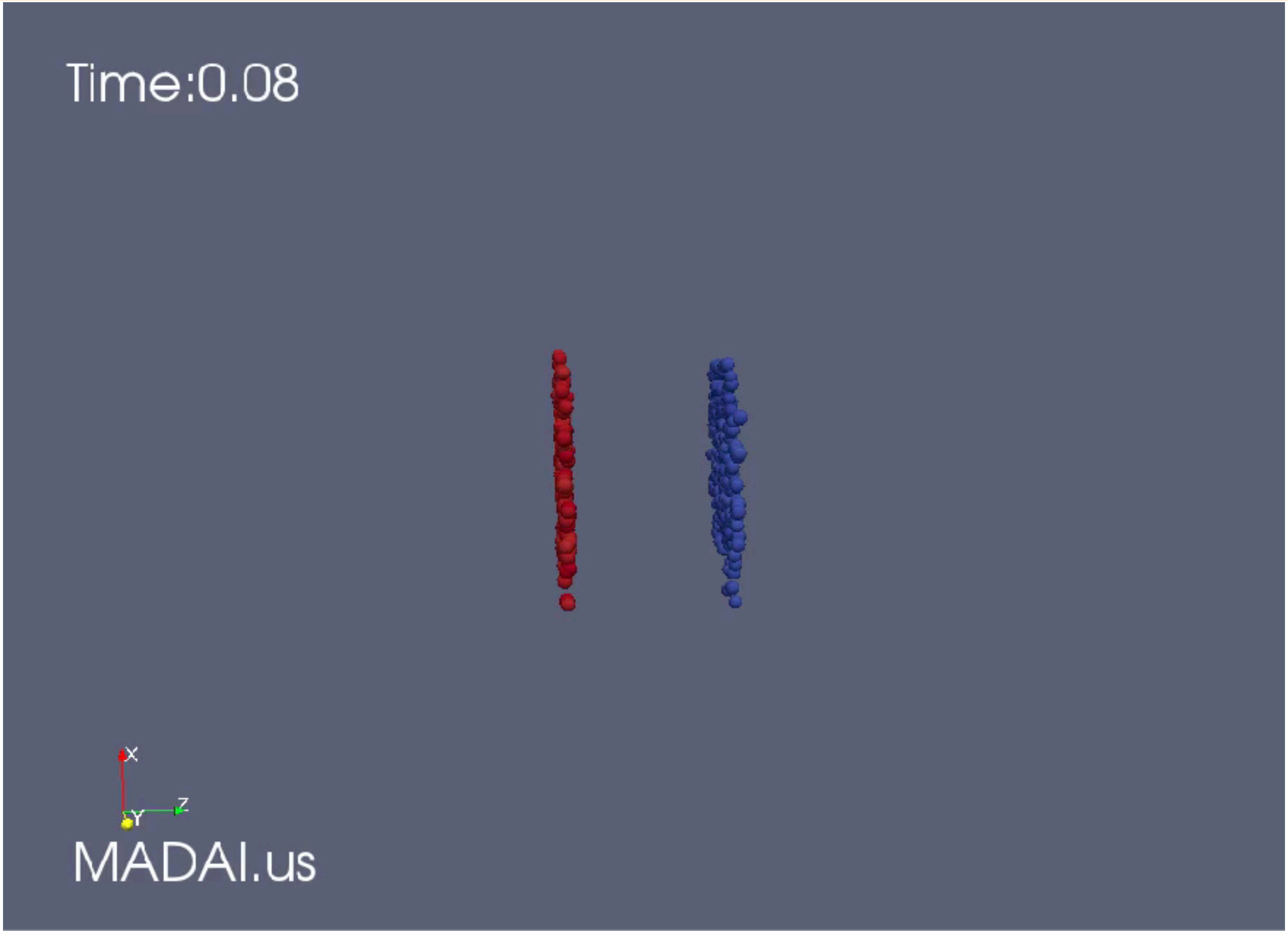
- Point particles, binary collisions
- Whole evolution and history of the evolution including full phase-space information

Collision Criterion

$$d_{\min} \leq d_0 = \sqrt{\frac{\sigma_{tot}}{\pi}}$$

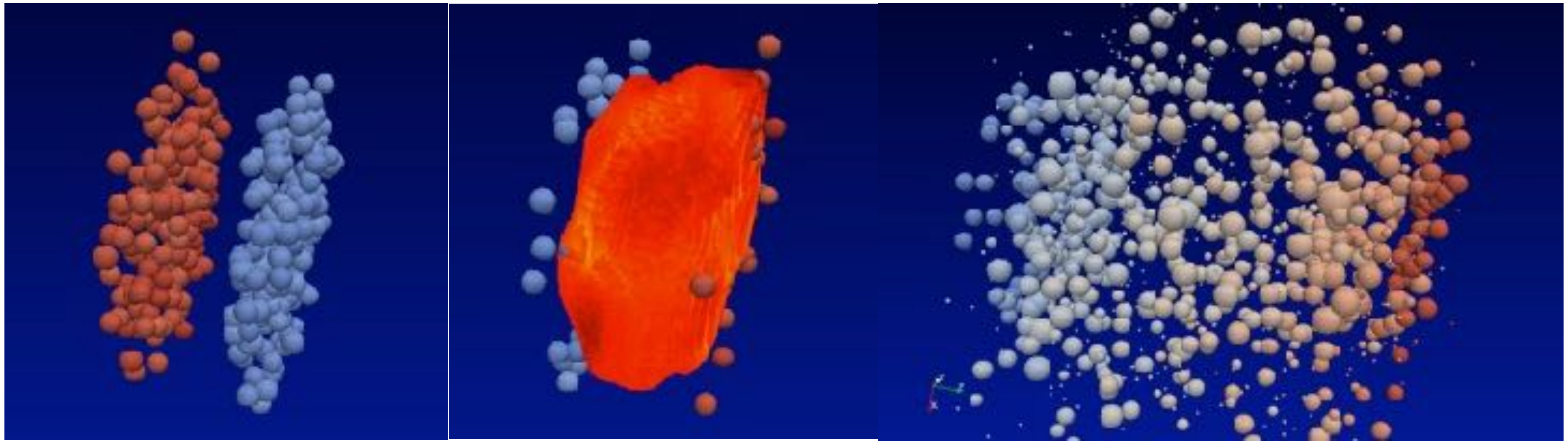


One Event at RHIC Energies



Status of Theoretical Modeling

Ultimate Goal: Calculate two colliding nuclei at the speed of light as a dynamical many-body problem from the QCD Lagrangian



- Elements for a consistent description of the whole heavy ion reaction at highest RHIC and LHC energies:
  - Initial Conditions
  - Pre-equilibrium Evolution
  - Hydrodynamics
  - Hadronization
  - Hadronic Rescattering and Freeze-out

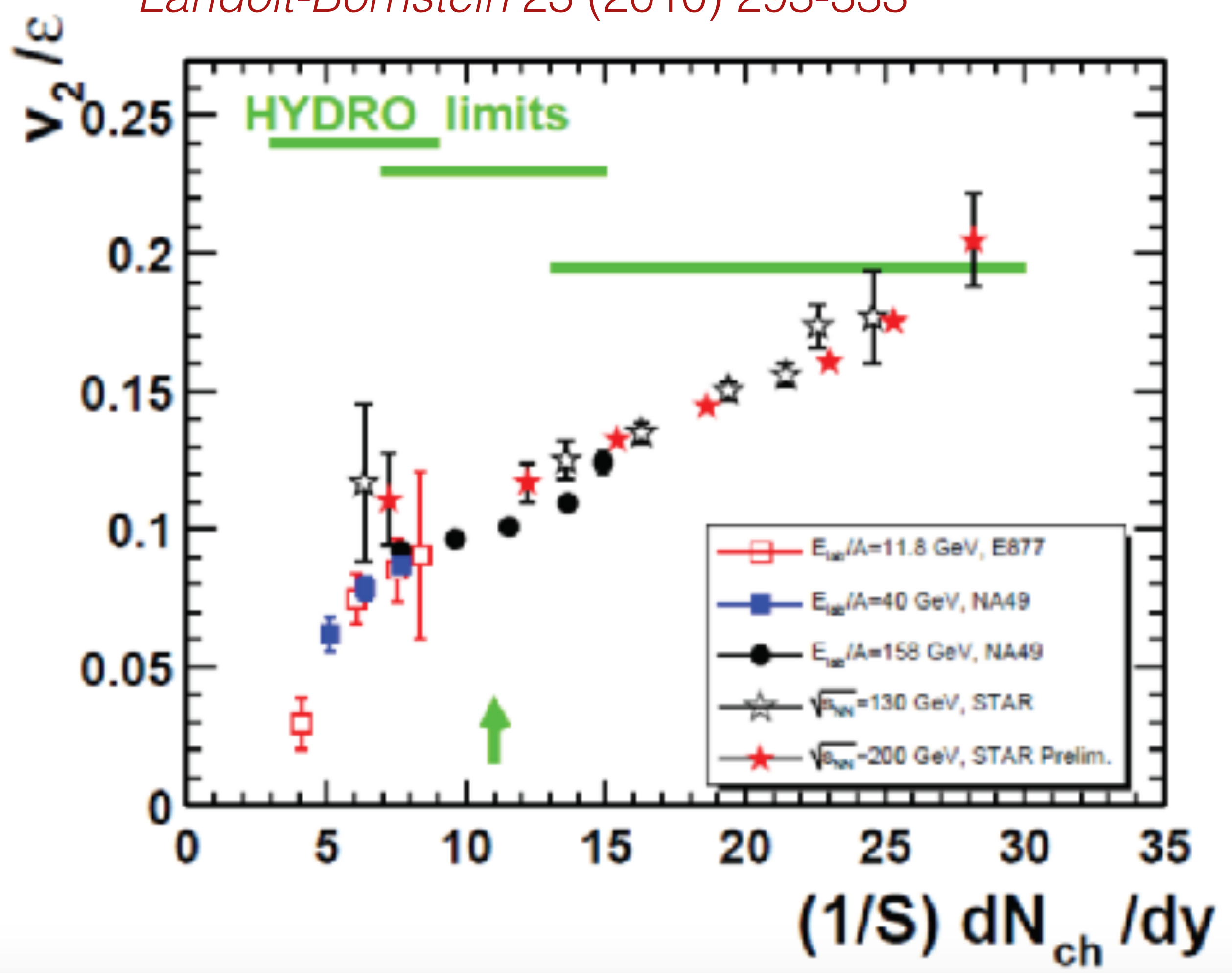
- Understanding of bulk properties, fluctuation observables,
- Background for hard probes, especially correlations

Historical Overview

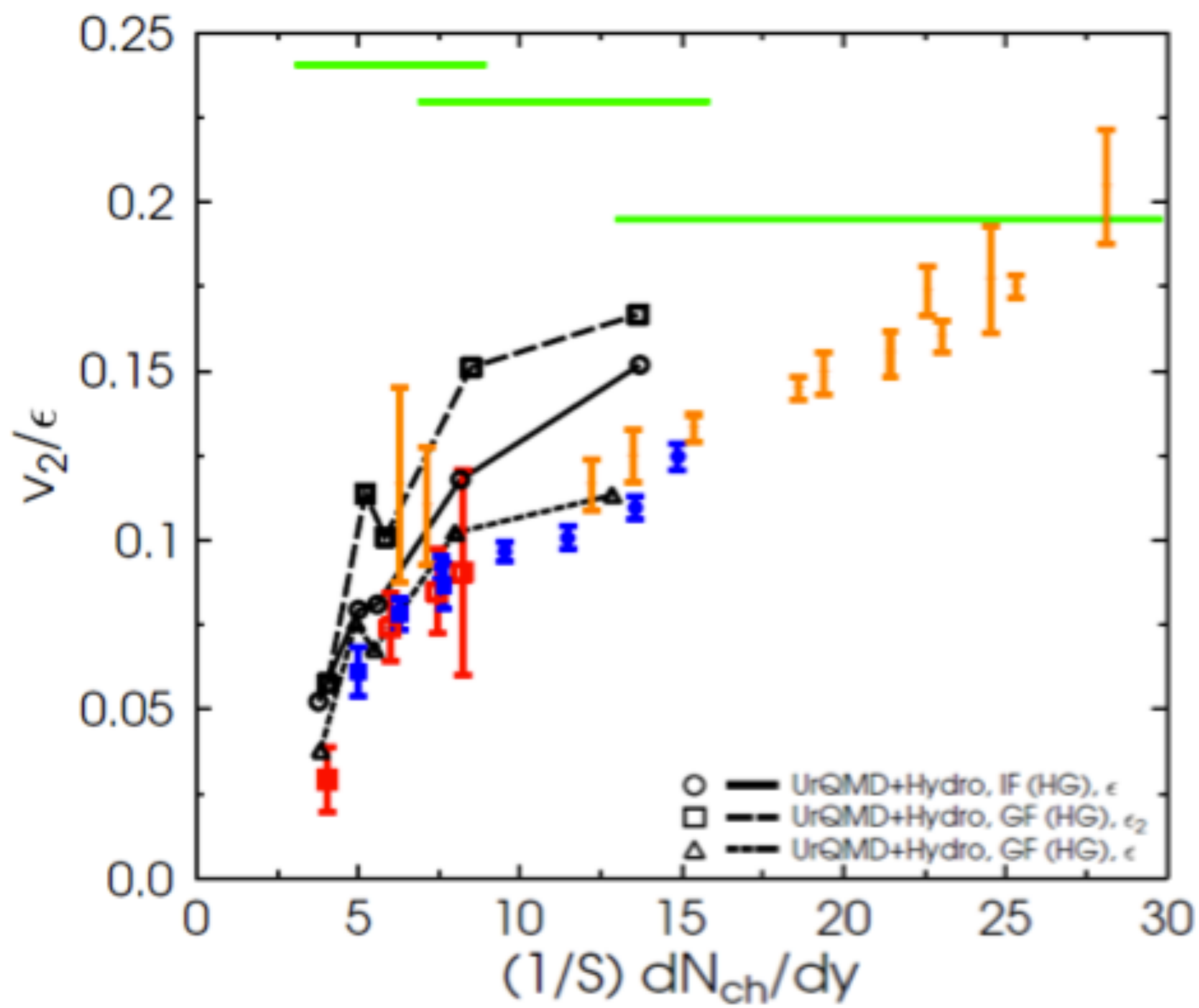
Based on HP J.Phys.G 41 (2014) 12, 124005

3+1 D Ideal Event-by-Event Hydro+Cascade

Voloshin, Snellings, Poskanzer, arXiv: 0809.2949, *Landolt-Bornstein* 23 (2010) 293-333



HP et al, Phys.Rev. C79 (2009) 054904



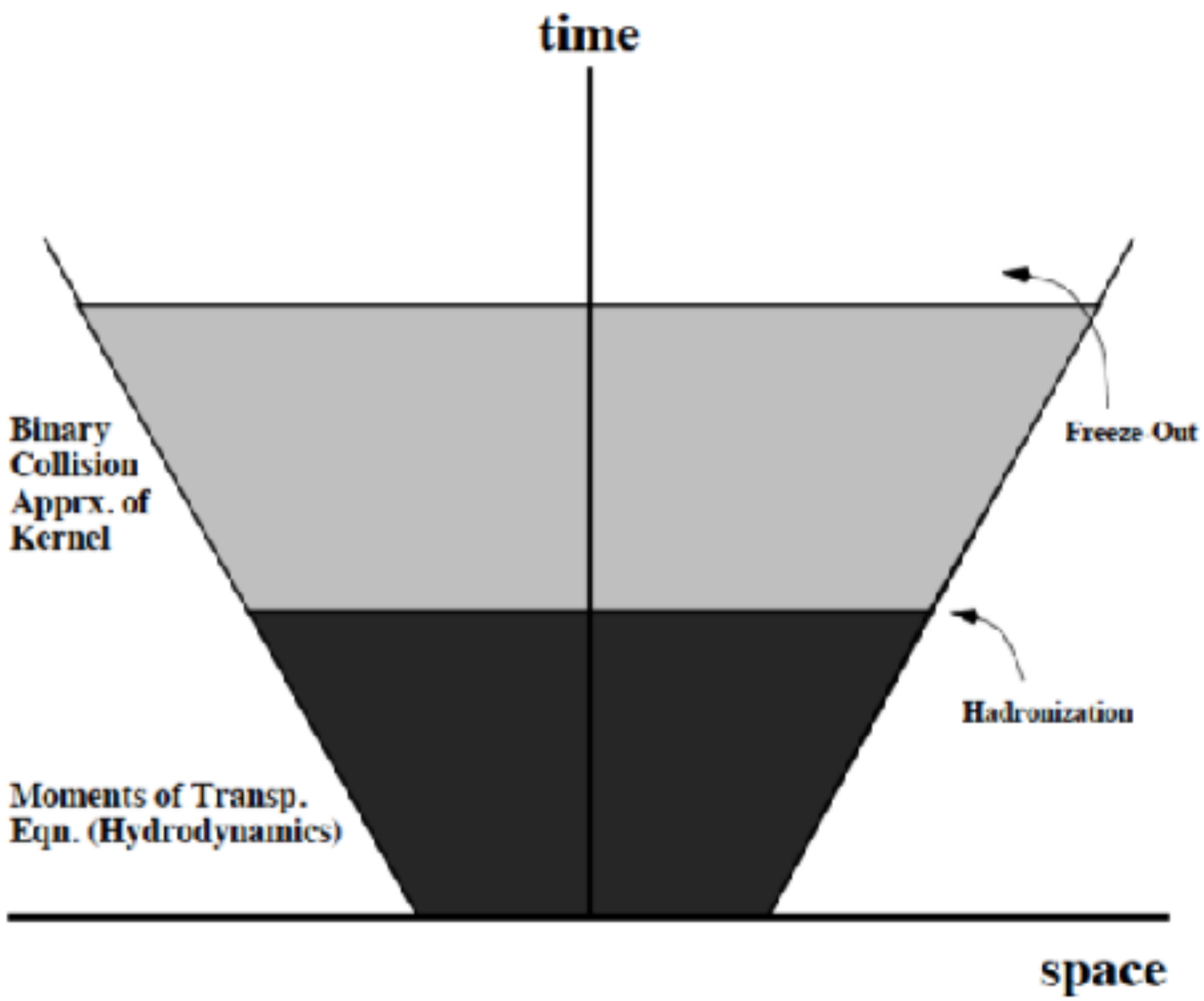
Elliptic flow scaled by the initial eccentricity vs. density in the overlap region allows to compare different centralities and energies

Qualitative behaviour nicely reproduced in 3+1d transport+hydro approach

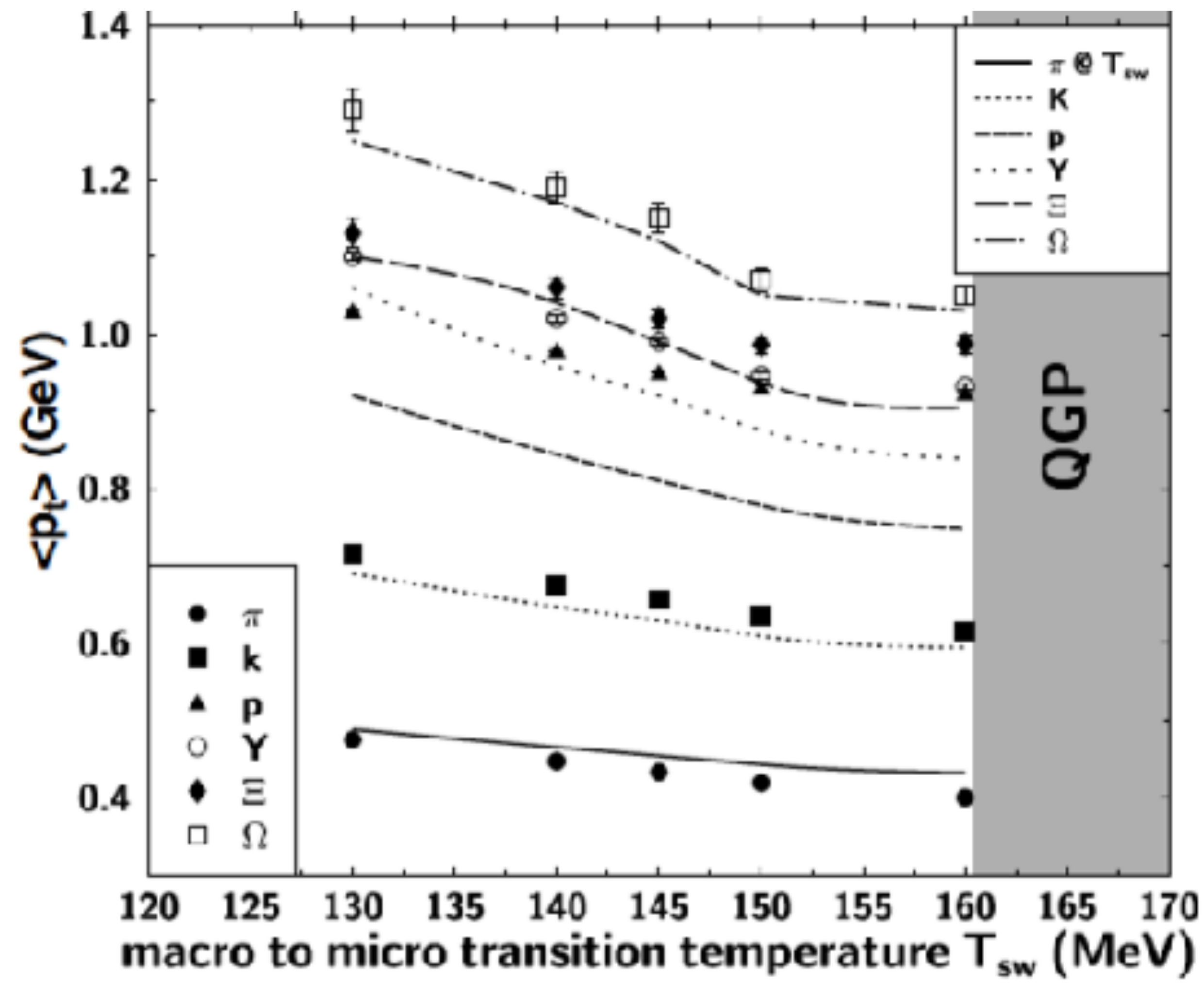
(Uncertainty due to **eccentricity** calculation)

First Hybrid Approach

Early Sketch ~2000



Lines: Hydro alone
Symbols: Hydro +Transport

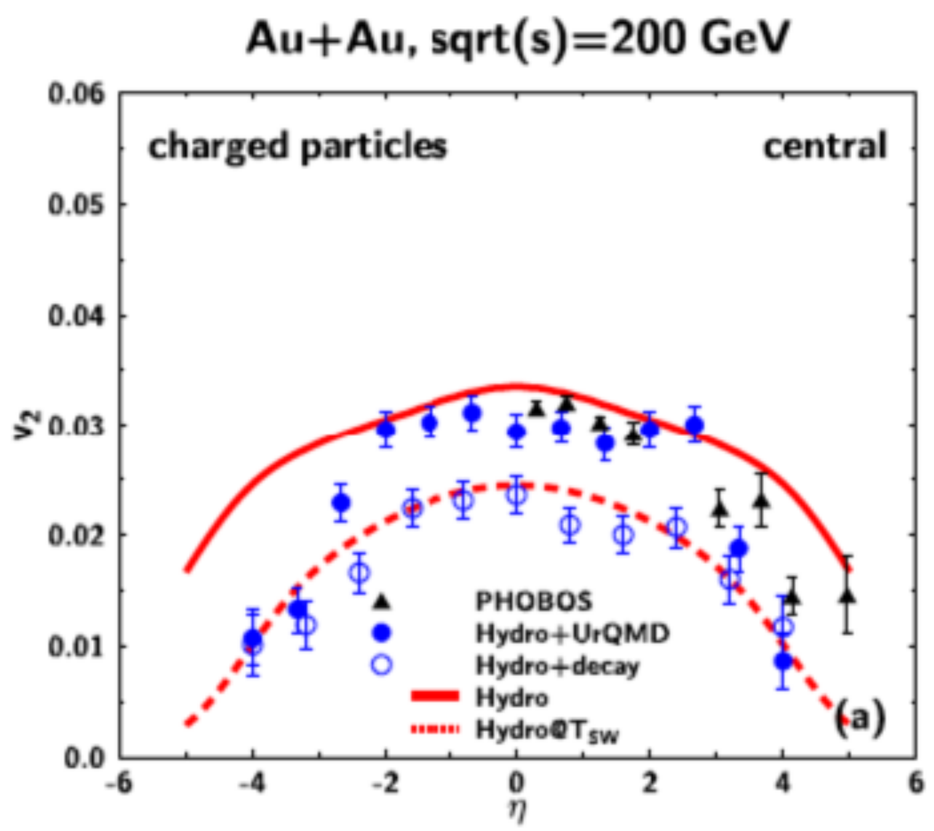


- Mainly the mean transverse momentum of protons is affected by hadronic rescattering

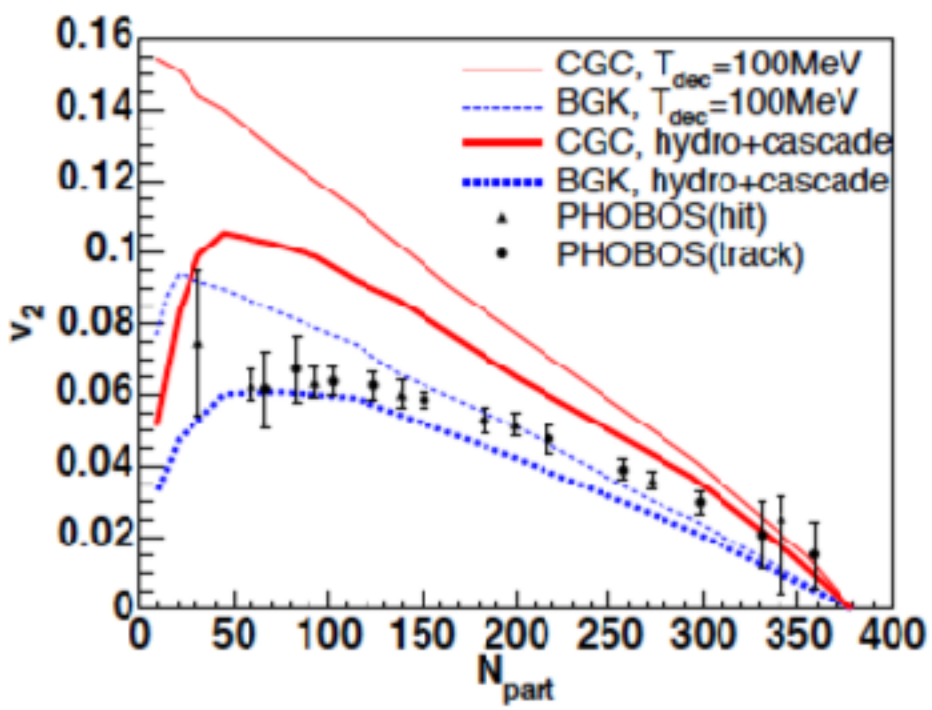
S. A. Bass and A. Dumitru, Phys. Rev. C 61 (2000) 064909, see also D. Teaney, J. Lauret and E. Shuryak nucl-th/0110037

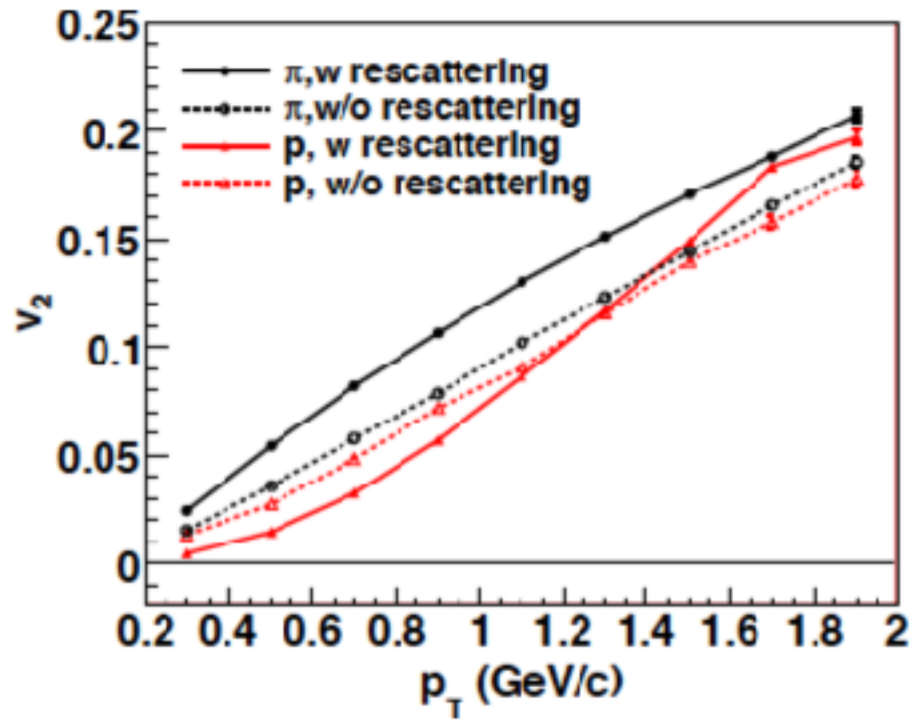
3+1D Ideal Hydro+Cascade

- Transport evolution helps at forward rapidities and in peripheral collisions
- Mass splitting is increased during hadronic transport



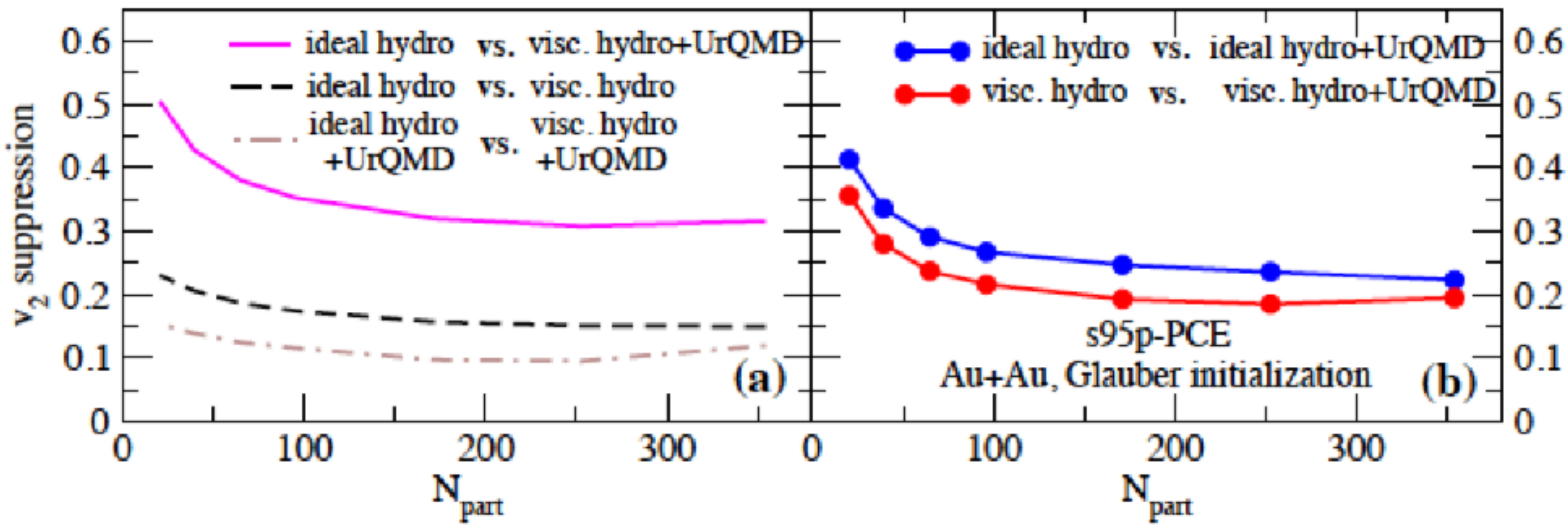
C. Nonaka and S. A. Bass, PRC 75 (2007) 014902





T. Hirano, U. Heinz, D. Karzheev, R. Lacey and Y. Nara, PLB 636 (2006) 299 and PRC 77 (2008) 044909

2+1D Viscous Hydro+UrQMD



H. Song, S. A. Bass and U. Heinz, PRC 83 (2011), 024912

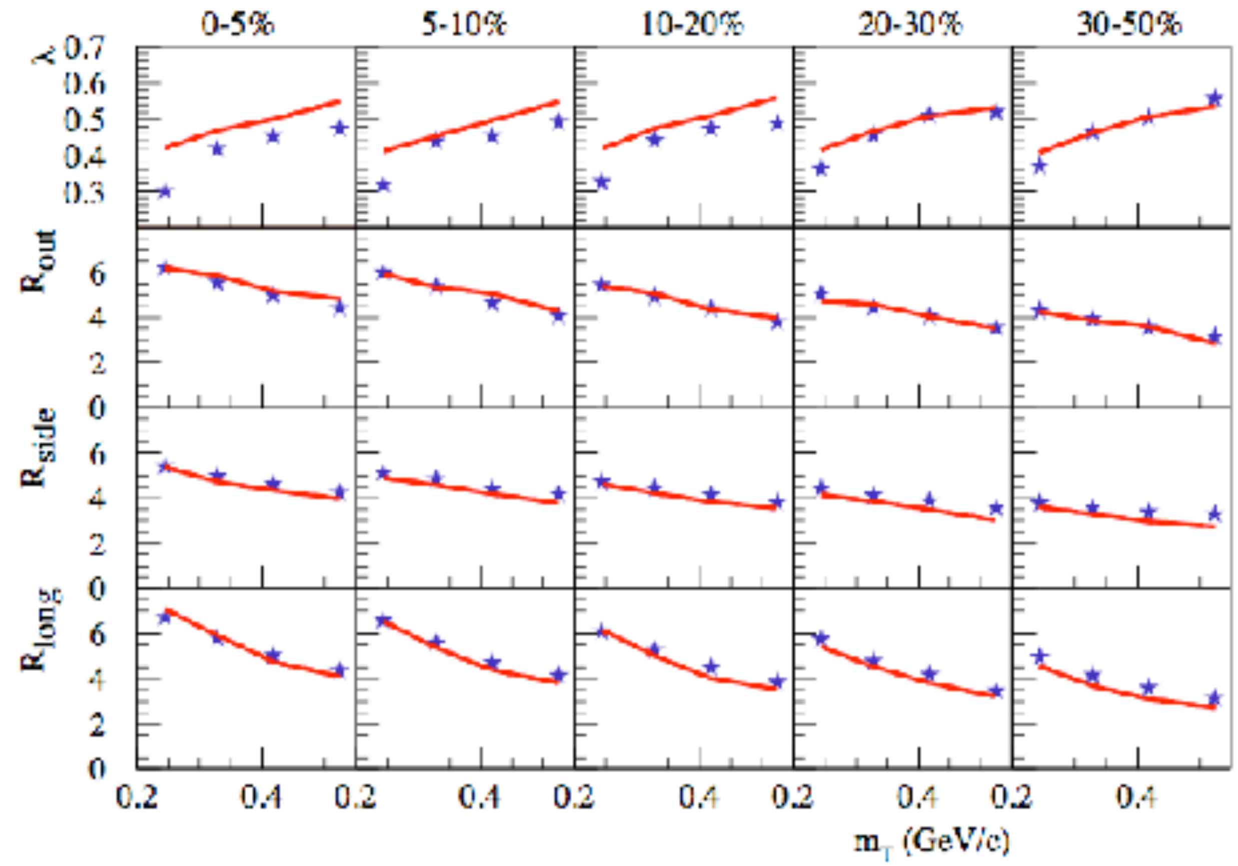
- Suppression factor quantifies v2 reduction

$$v_2^{\mathrm{supp.}} = \frac{v_2^A - v_2^B}{v_2^A}$$

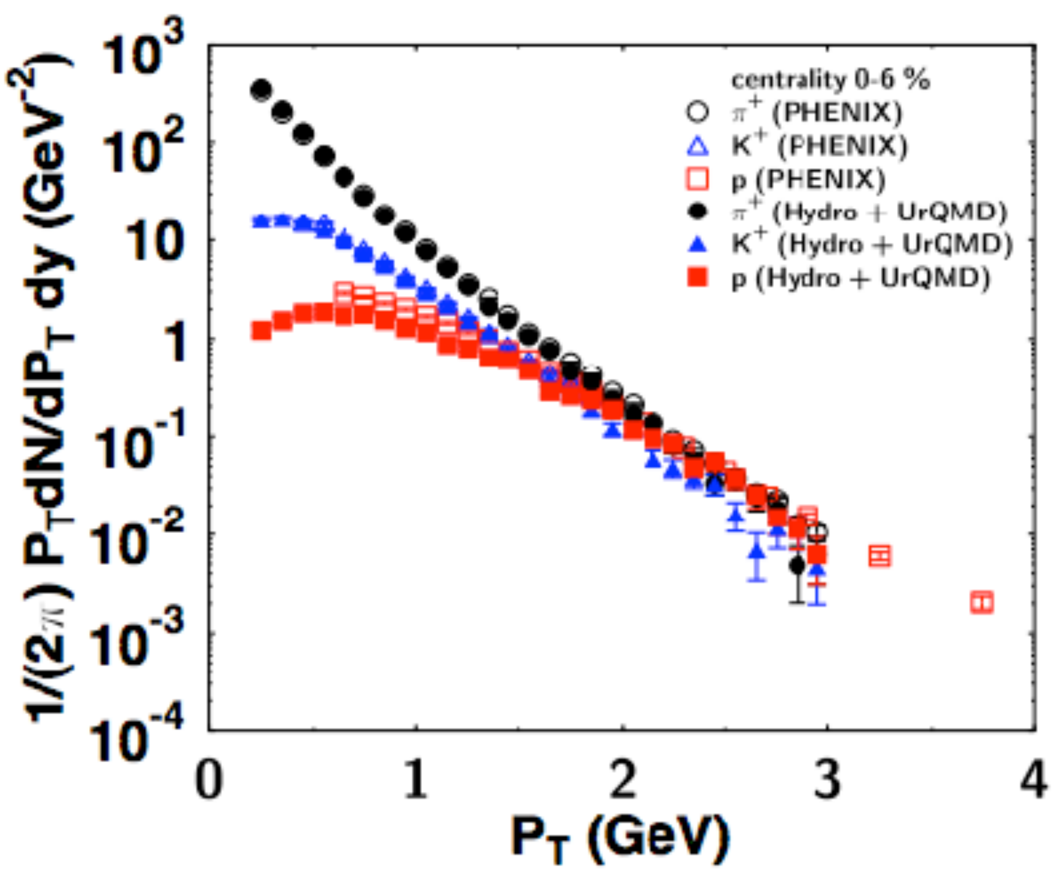
- Hadronic rescattering yields more suppression of elliptic flow than viscous hydro (with η/s = 0.08) compared to ideal hydro

Hydrodynamics + Afterburner at RHIC

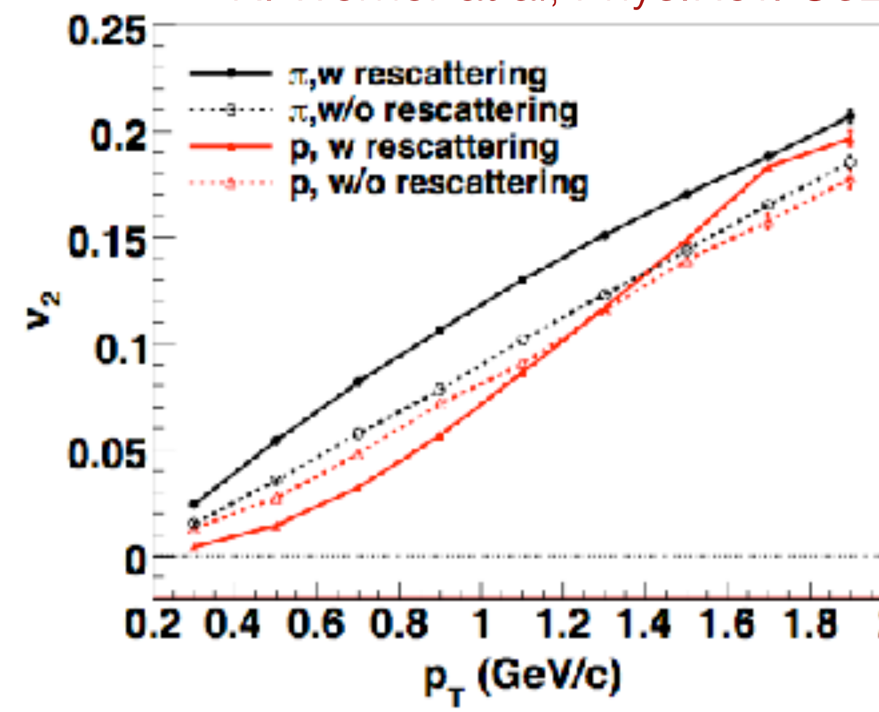
- Hybrid models describe bulk properties of the quark gluon plasma at RHIC
- Hadronic rescattering has effect on spectra and elliptic flow



K. Werner at al, Phys.Rev. C82 (2010) 044904

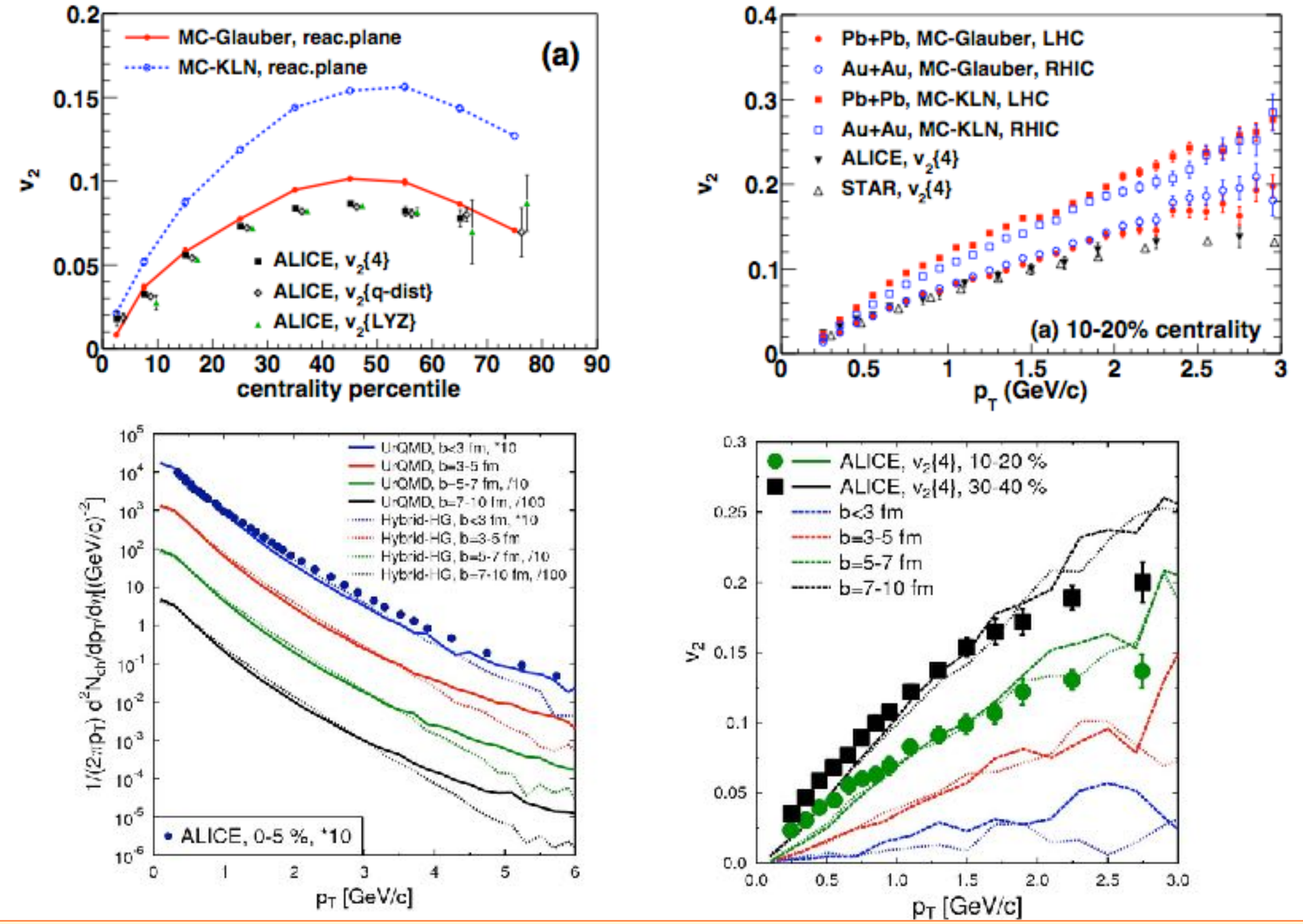


C. Nonaka, S. A. Bass, Phys.Rev. C75 (2007) 014902



Hirano et al, Phys.Rev. C77 (2008) 044909

Hybrid Results at LHC



Hirano et al, Phys.Rev. C84 (2011) 011901

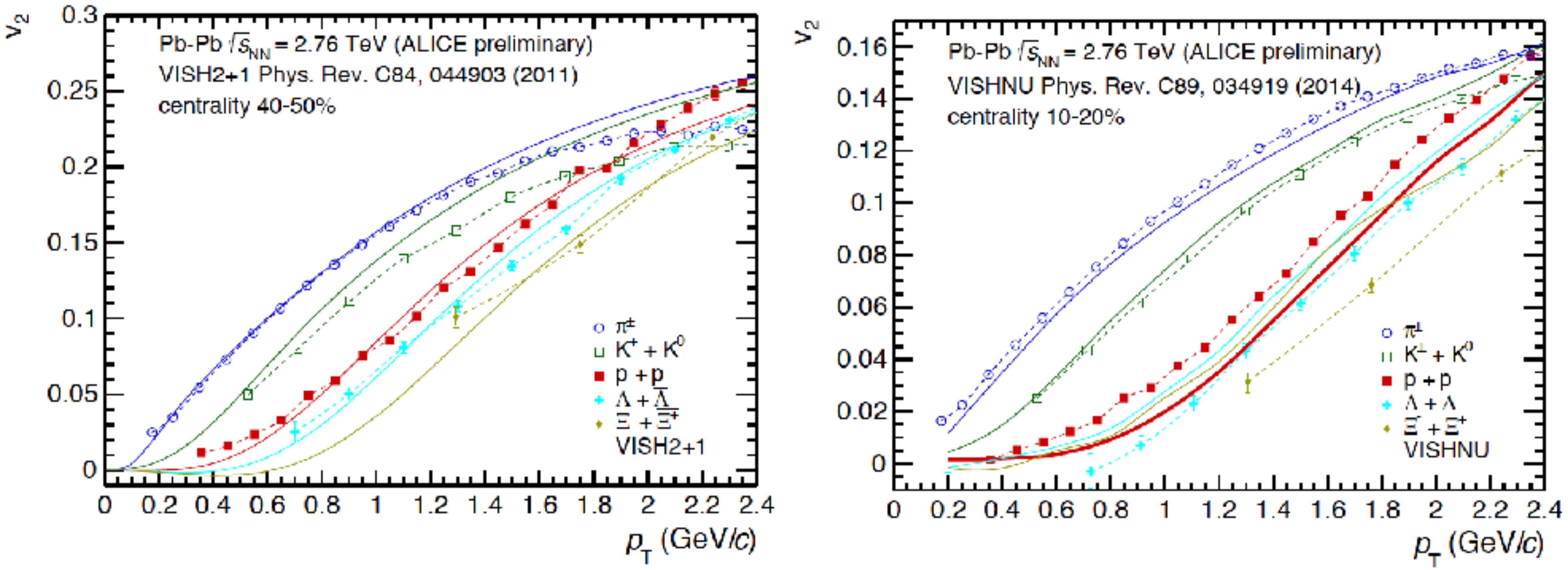
H.P., Phys.Rev. C84 (2011) 034912

Bulk properties well-described by hybrid approaches also at the LHC

$v_2(p_T)$ for Identified Particles

- VISHNU compared to ALICE data on identified hadron elliptic flow

Snellings, arXiv: 1411.7690

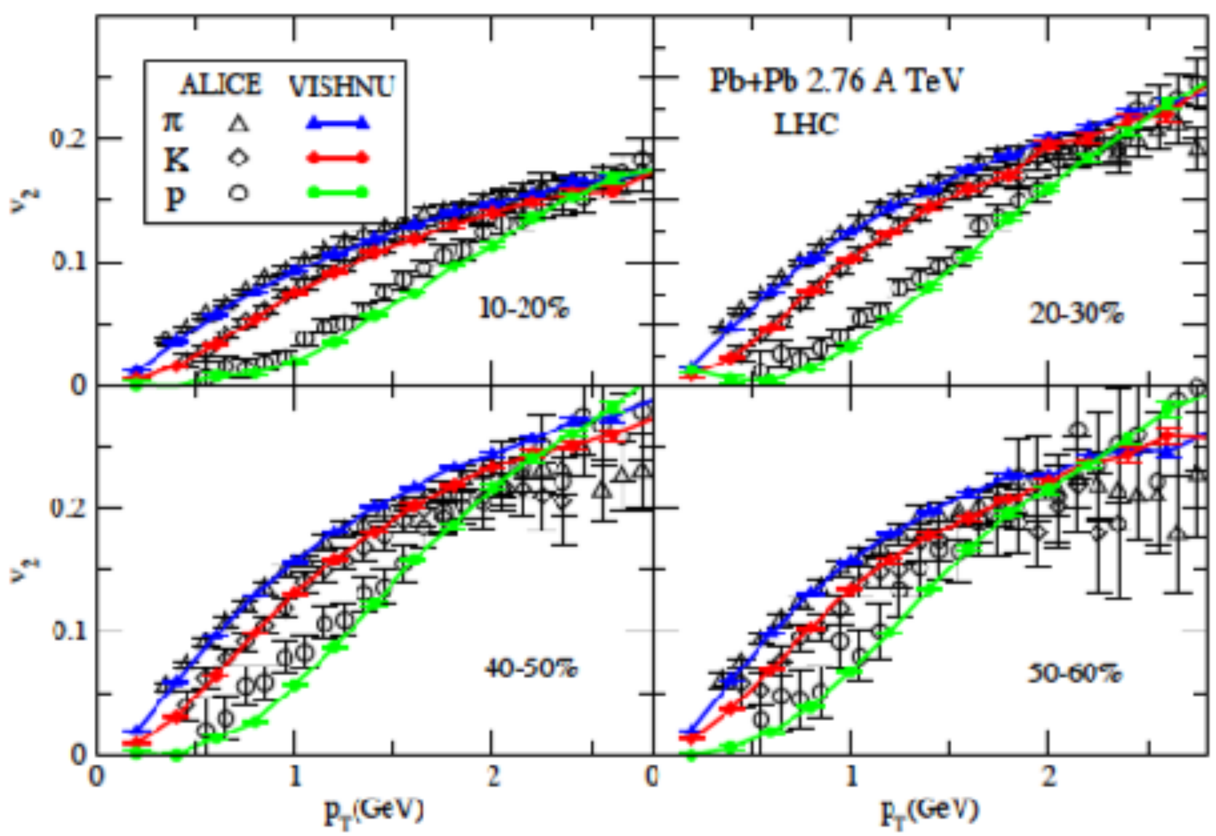


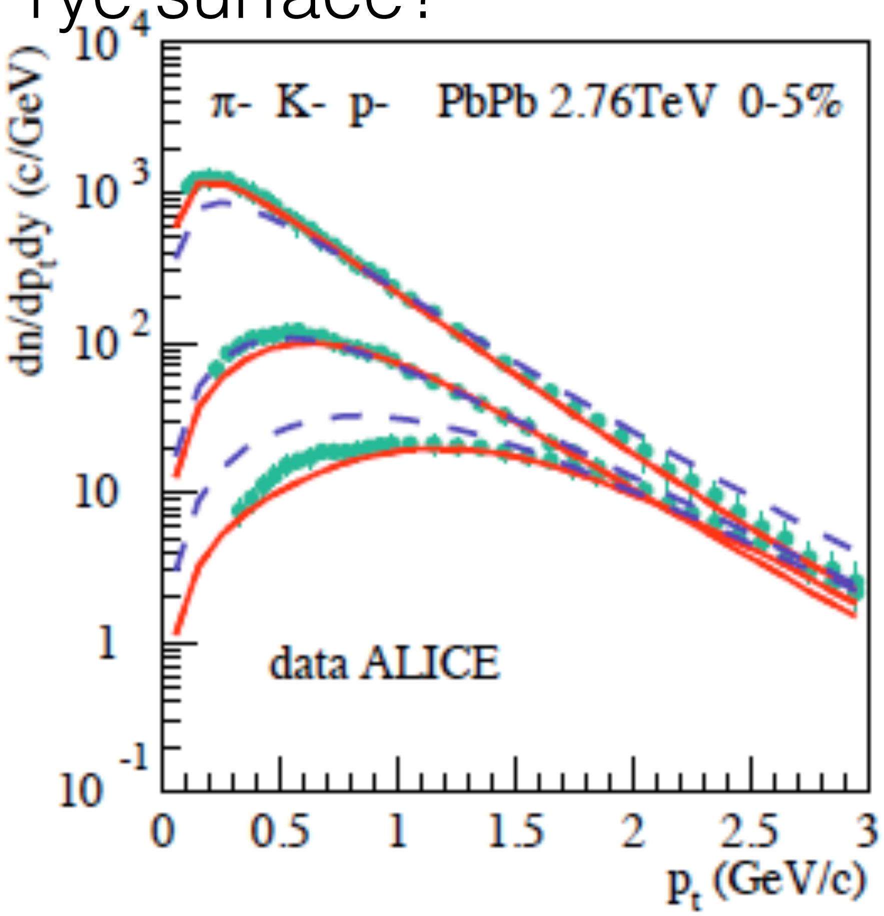
- In central collisions the rescattering destroys the mass ordering from original hydro calculation

Final State Rescattering

- Why it matters:
  - Separation of chemical and kinetic freeze-out
  - Influences the dynamics of identified particles:
    - Increase of mean transverse momentum by up to 30%
    - Mass splitting for anisotropic flow
- How large is the effect on higher harmonics? What is the viscous correction to the distribution functions on the Cooper-Frye surface?

Song et al, arXiv:1311.0157

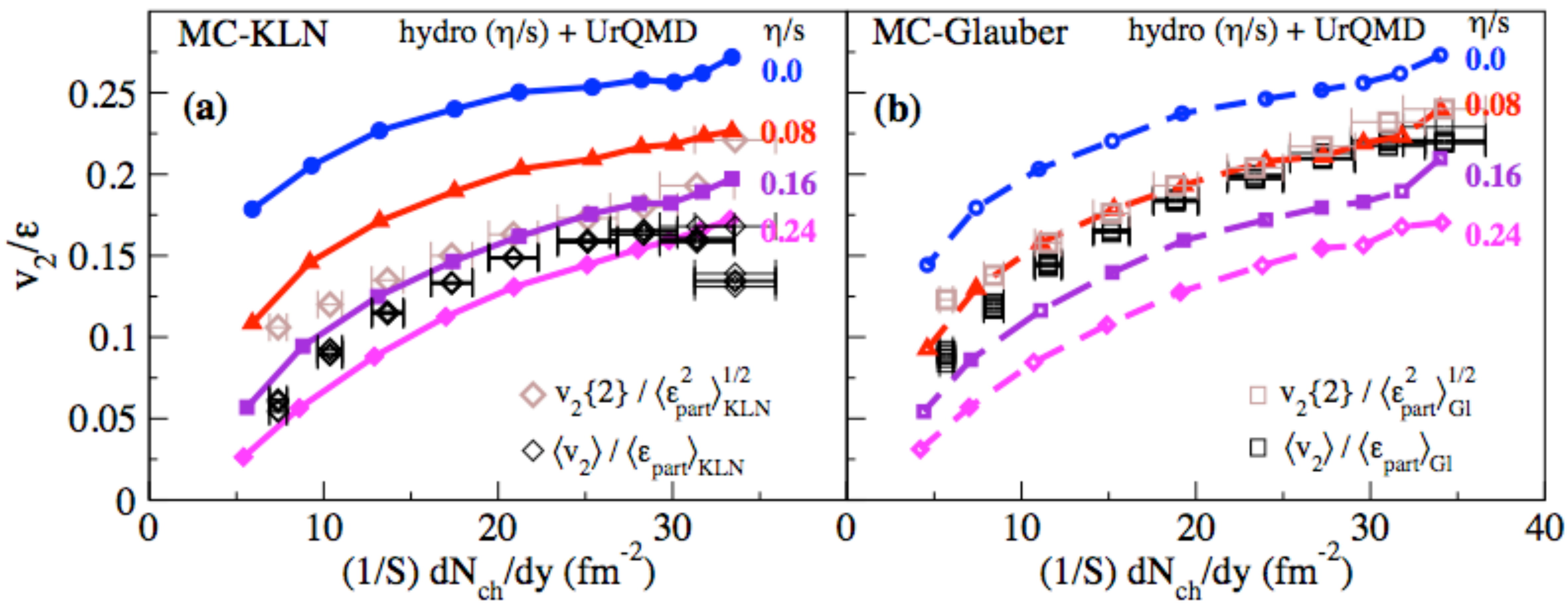




Werner et al, Phys.Rev. C85 (2012) 064907

Viscous Hybrid

- Hybrid calculation confirms previously found low values of viscosity during the hot and dense stage of the evolution
- Initial Conditions yield a factor of 2 uncertainty



H.Song et al, PRL 106, 192301 (2011)

Elliptic flow from viscous hydrodynamics+hadron transport

Status at QM 12

- Summary table from J.-Y. Ollitrault (plenary at QM12)

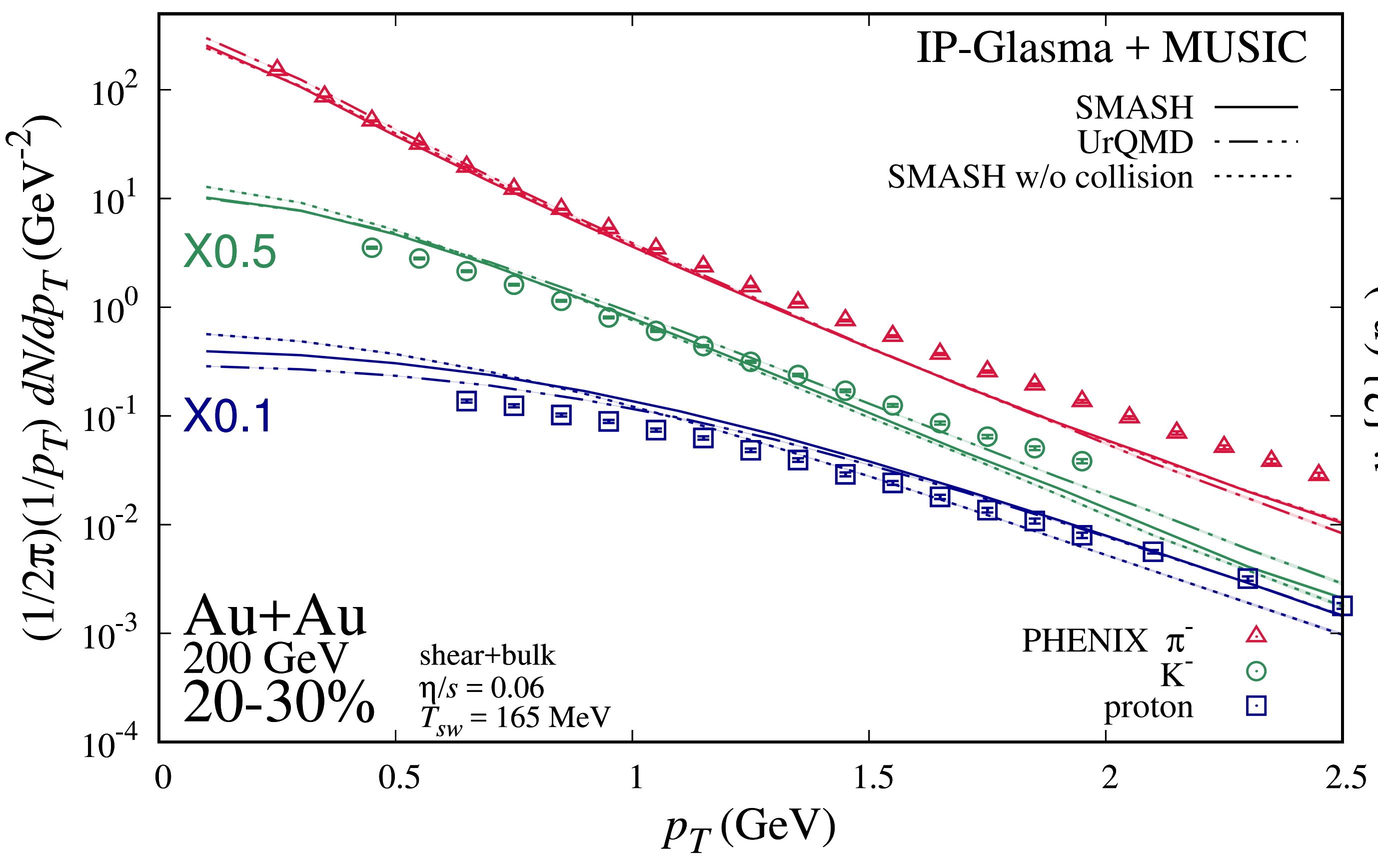
Author/Presenter	QM2012	arXiv	initial fluctuations	3+1d	viscous	afterburner
Huichao Song	1D	1207.2396			✓	✓
Teaney/Yan	1A	1206.1905			✓	
Chun Shen	1A	1202.6620			✓	
Sangyong Jeon	2A		✓	✓	✓	✓
Matt Luzum	2A				✓	
Piotr Bozek	2C	1204.3580	✓	✓	✓	
Björn Schenke	3A	1109.6289	✓	✓	✓	
Dusling/Schaefer	3A	1109.5181			✓	
Chiho Nonaka	3A	1204.4795	✓	✓	✓	
Ryblewski/Florkowski	3D	1204.2624		✓		
Longgang Pang	4D	1205.5019	✓	✓		
Hannah Petersen	VA	1201.1881	✓	✓		✓
Fernando Gardim	6D	1111.6538	✓	✓		
Zhi Qiu	29	1208.1200	✓		✓	
Gardim/Grassi	52	1203.2882	✓	✓		
Katya Retinskaya	57	1203.0931			✓	
Hirano/Murase	255	1204.5814	✓	✓		✓
Holopainen/Huovinen	284	1207.7331	✓			
Asis Chaudhuri		1112.1166	✓		✓	
Iurii Karpenko		1204.5351		✓		✓
Yu-Liang Yan		1110.6704		✓		✓
Josh Vredevoogd		1202.1509		✓	✓	
Ron Soltz		1208.0897			✓	✓
Rafael Derradi de Souza		1110.5698	✓	✓		

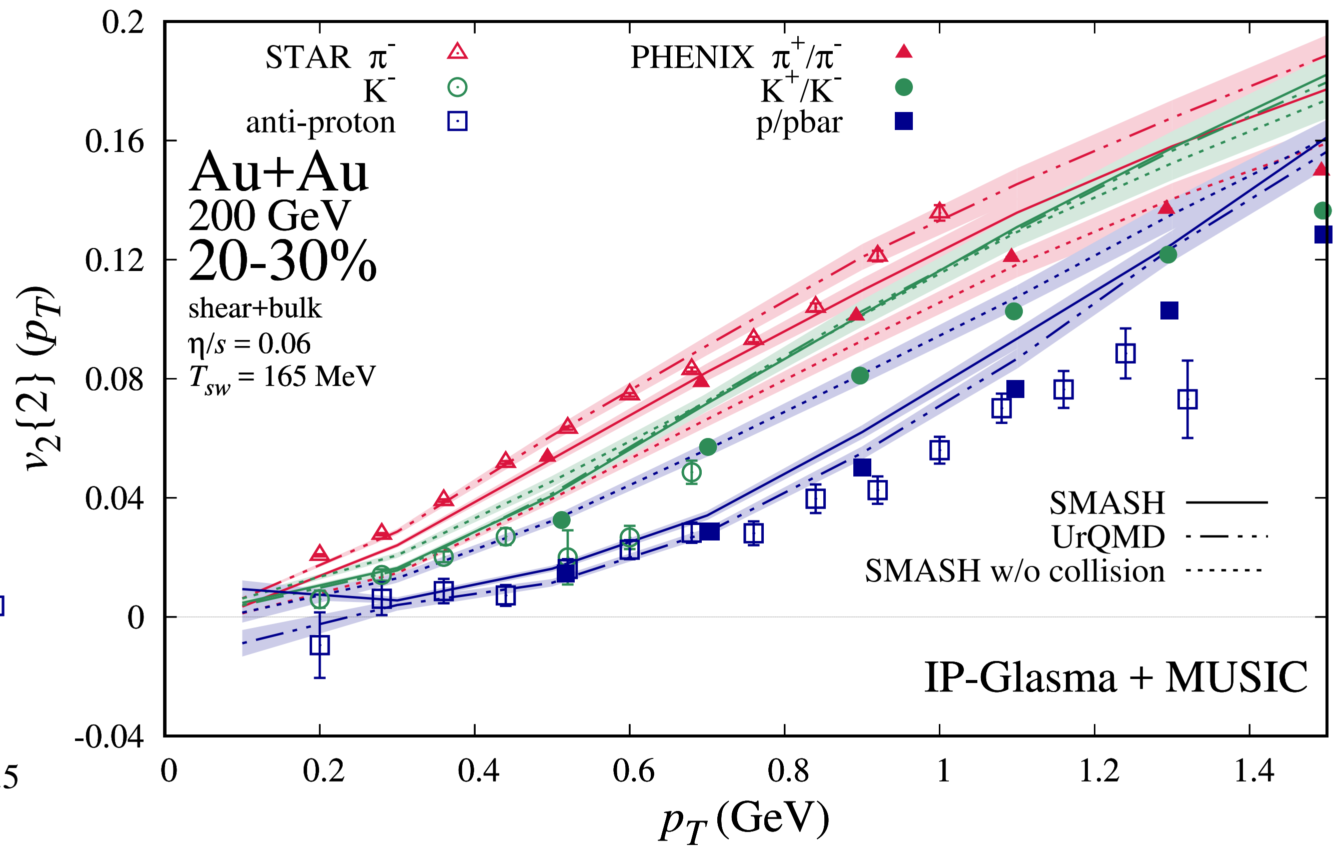
- + 10 more years of development support the ‚standard model for heavy-ion physics‘ as hybrid approaches

MUSIC+SMASH Hybrid

- Optional: Global conservation laws and broad mass distributions for resonances at particlization

MUSIC+UrQMD from S. Ryu et al, PRC 97 (2018)

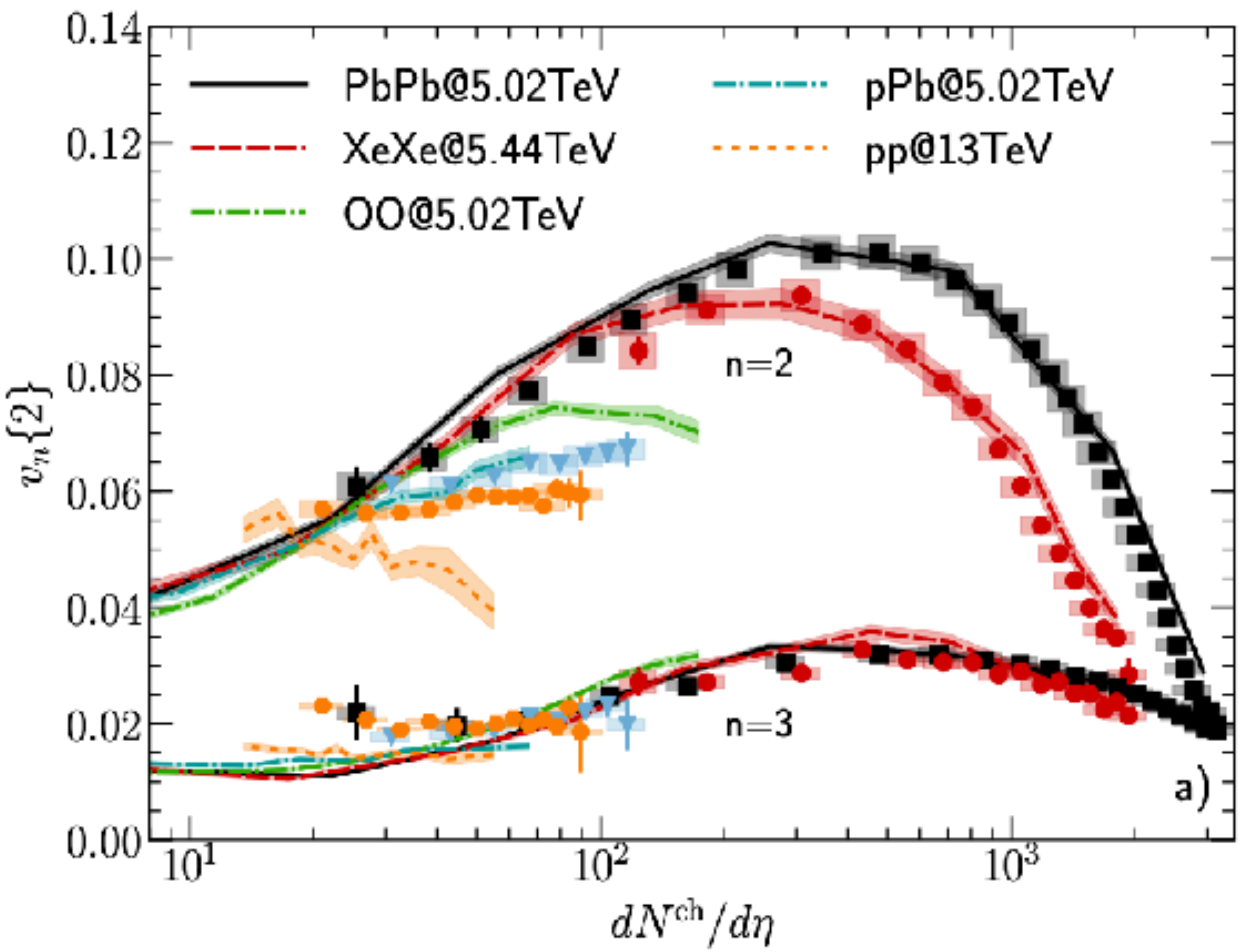




- Results for bulk observables are similar as within UrQMD
- Effects on spectra and mass splitting of elliptic flow consistent with prior findings

IP-Glasma+MUSIC+UrQMD

- Hybrid approach applied to many systems at different energies



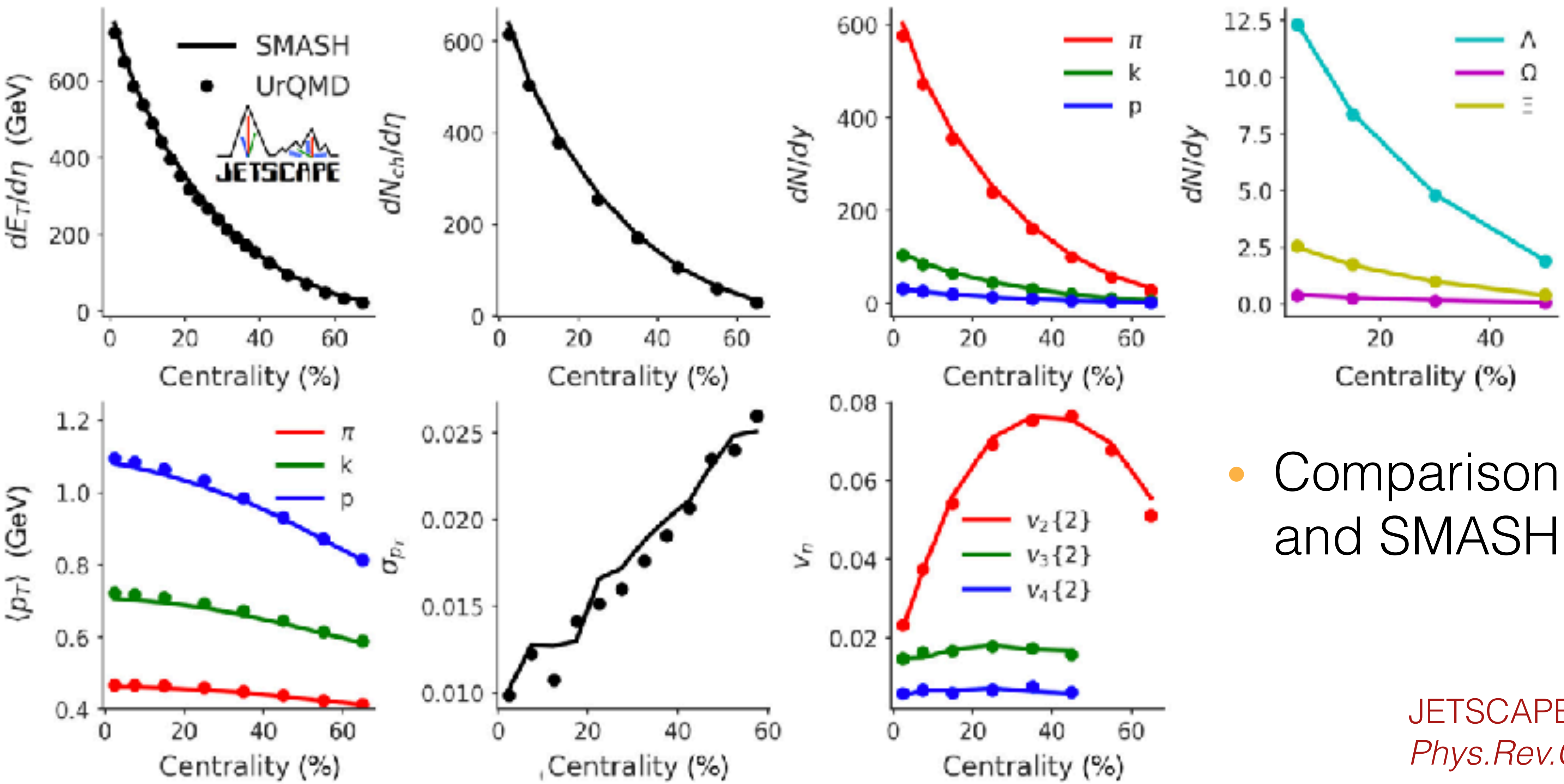
B. Schenke et al, *Nucl.Phys.A* 1005 (2021)

- $v_{2/3}$ are very well described

JETSCAPE

https://jetscape.org

- Open source framework with modules for all stages of collision (including energy loss modules)



- Comparison of UrQMD and SMASH afterburner

JETSCAPE collaboration, *Phys.Rev.C* 103 (2021) 5

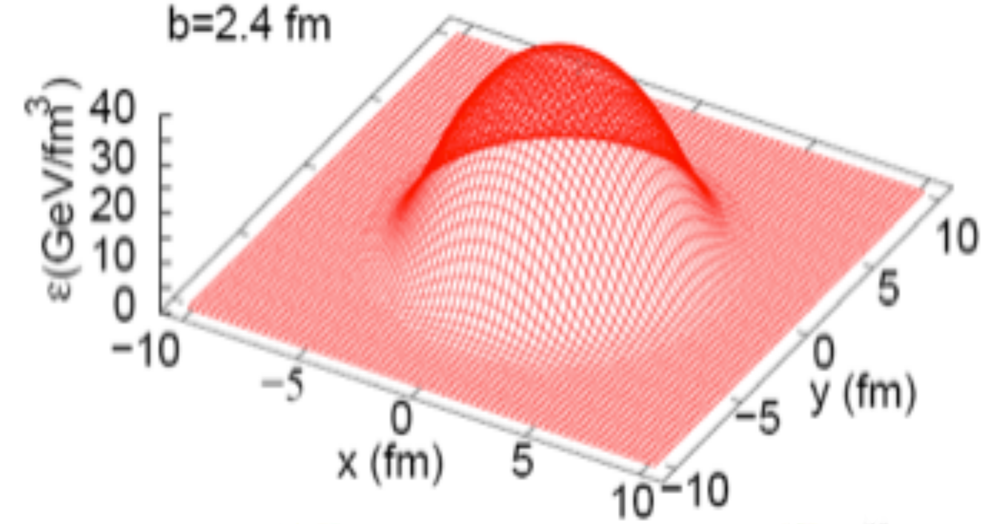
- JETSCAPE summer school from July 25-August 5 2022 https://indico.cern.ch/event/1162218/

Initial State Interface

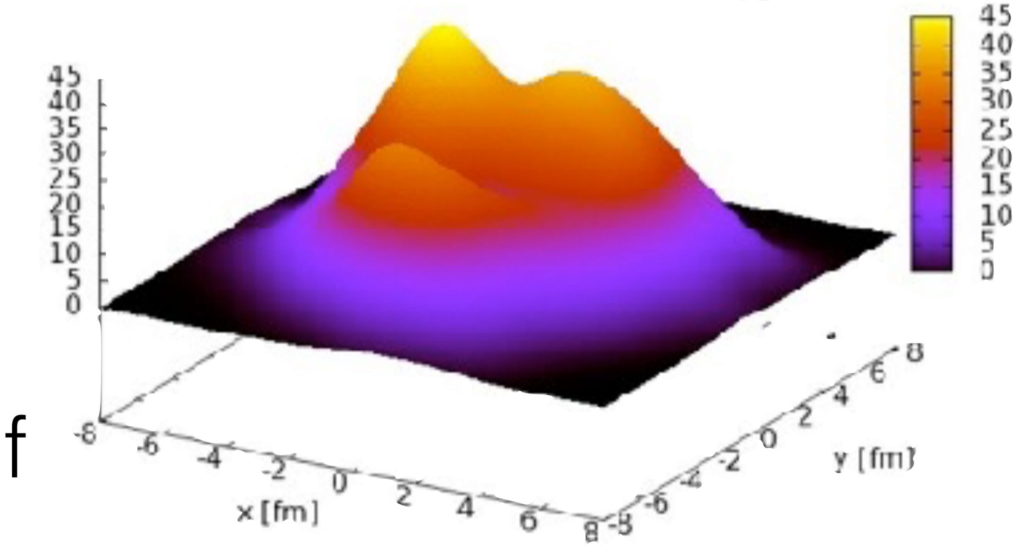
Sources of Fluctuations

- Density profiles are not smooth, but there are local peaks in transverse and longitudinal direction
- Impact parameter fluctuations within one specific centrality class, multiplicity fluctuations and differences in initial geometry
- Event plane rotation with respect to reaction plane in the laboratory

—> All these effects are averaged out if assuming a smooth symmetric initial density profile



C. Nonaka and S.A. Bass, PRC 75, 014902, 2007



J.Steinheimer et al., PRC 77,034901,2008

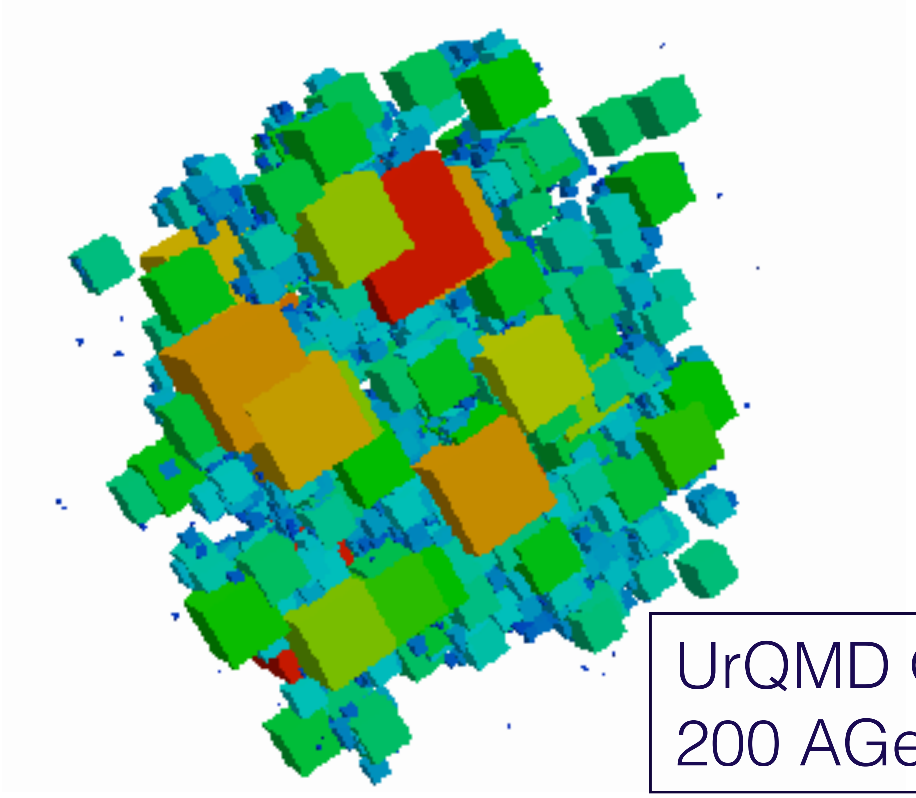
Included in dynamical models of the initial state (e.g. a parton cascade, NEXUS/EPOS, UrQMD) or in Glauber or CGC Monte Carlo approaches

Initial Conditions from Dynamical Approaches

- The initial $T^{\mu\nu}$ for hydrodynamics has to be given via:

$$\epsilon(x,y,z),\ p(x,y,z)\ \text{and}\ n(x,y,z)$$

- Energy deposition model needs to describe final dE_T/dy in p-p and A-A correctly
- Granularity is influenced by
  - Shape of the incoming nuclei
  - Distribution of binary collisions
  - Interaction mechanism
  - Degree of thermalization



UrQMD @ 200 AGeV

- Differences in shape and fluctuations need to be quantified
  – Challenge: How is local equilibrium reached so fast?

What is Usually Done?

- To calculate the energy-momentum tensor and four-current from particles a smearing kernel (Gaussian) is used:

$$T^{\mu\nu}_{init}(r) = \sum_i \frac{p_i^\mu p_i^\nu}{p_i^0} K(\boldsymbol{r} - \boldsymbol{r_i}, \boldsymbol{p})$$

$$j^{\mu}_{init}(r) = \sum_i \frac{p_i^\mu}{p_i^0} K(\boldsymbol{r} - \boldsymbol{r_i}, \boldsymbol{p})$$

- Assuming that the resulting tensor has the form for relativistic ideal fluid dynamics, the following equations are solved iteratively

$$\begin{cases} T^{00} = (\epsilon + p)\gamma^2 - p \\ T^{0i} = (\epsilon + p)\gamma^2 \boldsymbol{v} \\ j^0_B = n\gamma \\ p = p_{EoS}(n, \epsilon) \end{cases}$$

- The other option: Solve the eigenvalue problem and decompose the tensor in the Landau frame

Different Approaches

Model	Initial condition	Hydro	Switching criterion	Smearing kernel	Getting $T^{\mu\nu}_{ideal}$
UrQMD hybrid [12]	UrQMD cascade	ideal 3+1D, SHASTA	t_{CM}[fm/c] = $max(2R\sqrt{\frac{E_{lab}}{2m_N}}, 1.0)$	Gaussian z-contracted	$T^{\mu 0}$, j^0
Skokov-Toneev hybrid [13]	Quark-Gluon-String-Model	ideal 3+1D, SHASTA	t_{CM} such that S/Q_B = const	not mentioned	$T^{\mu 0}$, j^0
EPOS [15]	Strings (Regge-Gribov model)	ideal 3+1D	τ	Gaussian z-contracted	Landau frame
NeXSPheRIO hybrid [16, 17]	Strings (Regge-Gribov model)	ideal 3+1D, SPH	τ = 1 fm [18]	Gaussian in x, y, $\tau\eta$	Landau frame
Gale et al [19]	IP-glasma	viscous 3+1D, MUSIC	τ = 0.2 fm/c ($\sqrt{s_{NN}}$ = 2.76 TeV)	not mentioned	Landau frame
Karpenko hybrid [20]	UrQMD cascade	viscous 3+1D	τ_{geom}	Gaussian with $\sigma_\perp$ and σ_η	$T^{\mu 0}$, j^0
Pang et al hybrid [21]	AMPT	ideal 3+1D, SHASTA	τ	Gaussian with $\sigma_\perp$ and σ_η	$T^{\mu 0}$, j^0

D. Oliinychenko and HP, *Phys.Rev.C* 93 (2016) 3, 034905

Coarse-Grained UrQMD

1. Several thousands Au+Au collisions at E_{lab} = 5-160 AGeV beam energy and different centralities
2. Calculate $T^{\mu\nu}$ on a space-time grid
3. Transform to the Landau rest frame

- Investigate locally two measures of isotropization:
  - Pressure anisotropy:

$$X \equiv \frac{|T_L^{11} - T_L^{22}| + |T_L^{22} - T_L^{33}| + |T_L^{33} - T_L^{11}|}{T_L^{11} + T_L^{22} + T_L^{33}} \ll 1$$

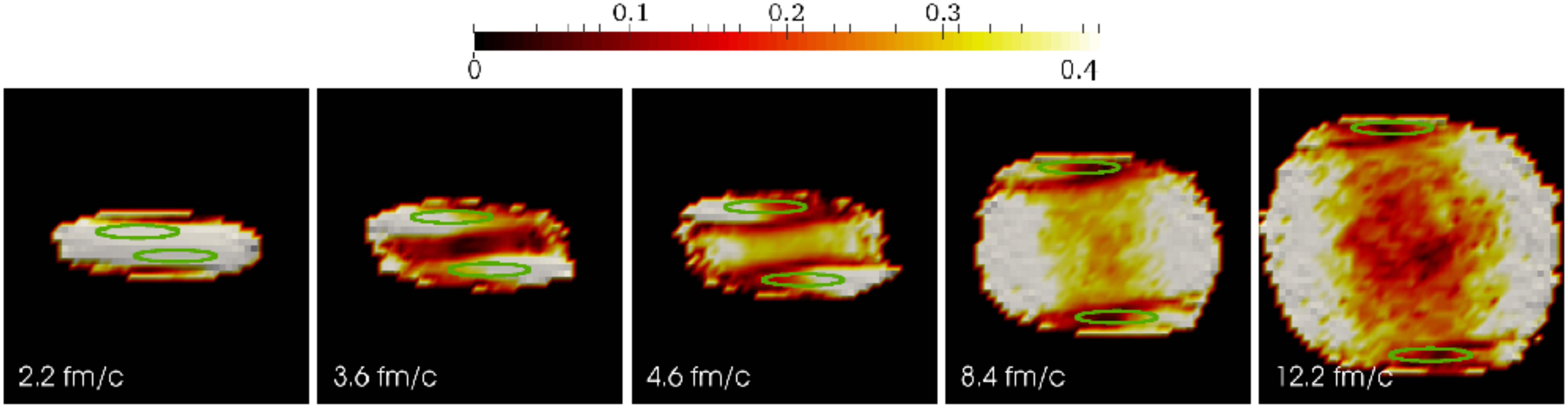
  - Off-diagonality:

$$Y \equiv \frac{3(|T_L^{12}| + |T_L^{23}| + |T_L^{13}|)}{T_L^{11} + T_L^{22} + T_L^{33}} \ll 1$$

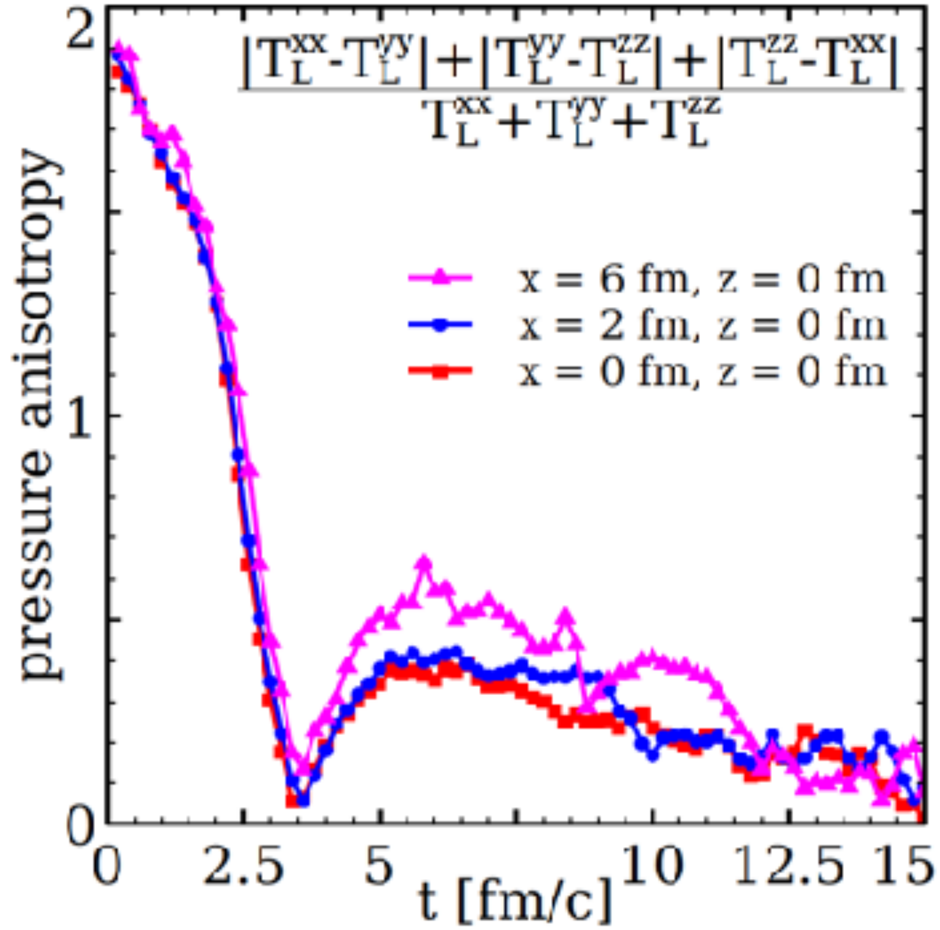
- X,Y ≤ 0.3 → viscous hydrodynamics applicable

D. Oliinychenko and HP, *Phys.Rev.C* 93 (2016) 3, 034905

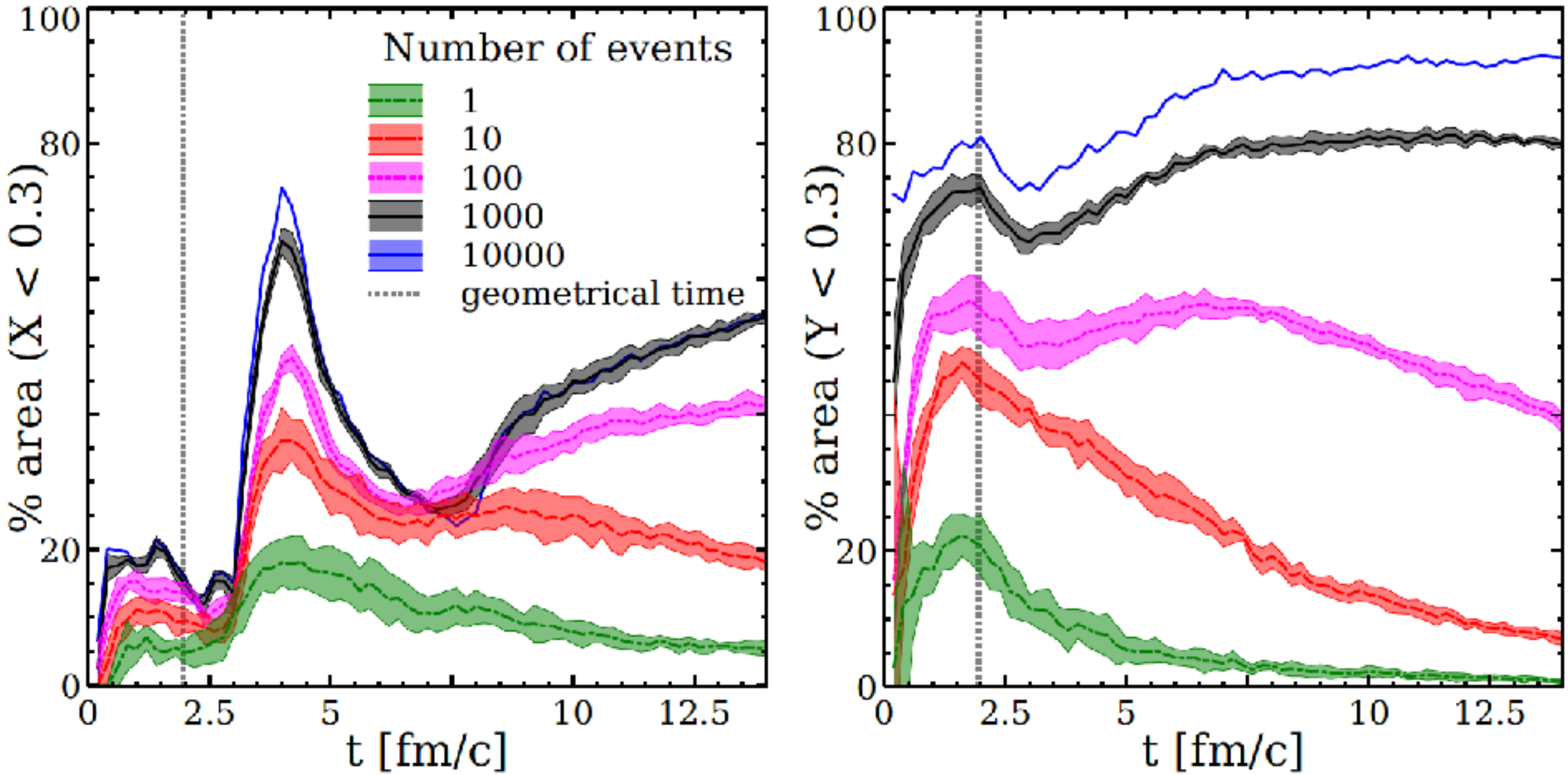
Time Evolution



- E_{lab} = 80A GeV, b=6 fm, pressure anisotropy
- After initial collisions anisotropy develops minimum over a large region in space
- Later stages: Rise due to resonance decays



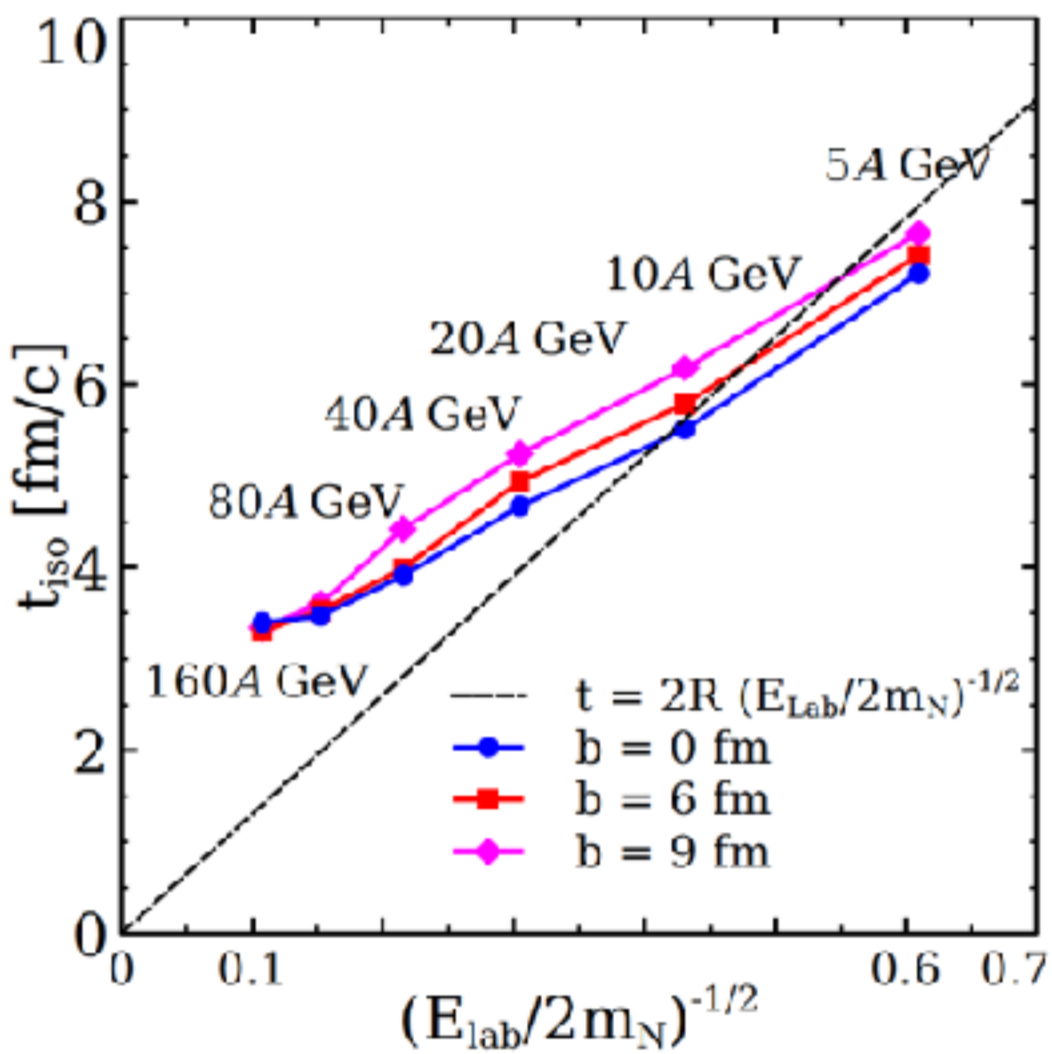
Number of Events

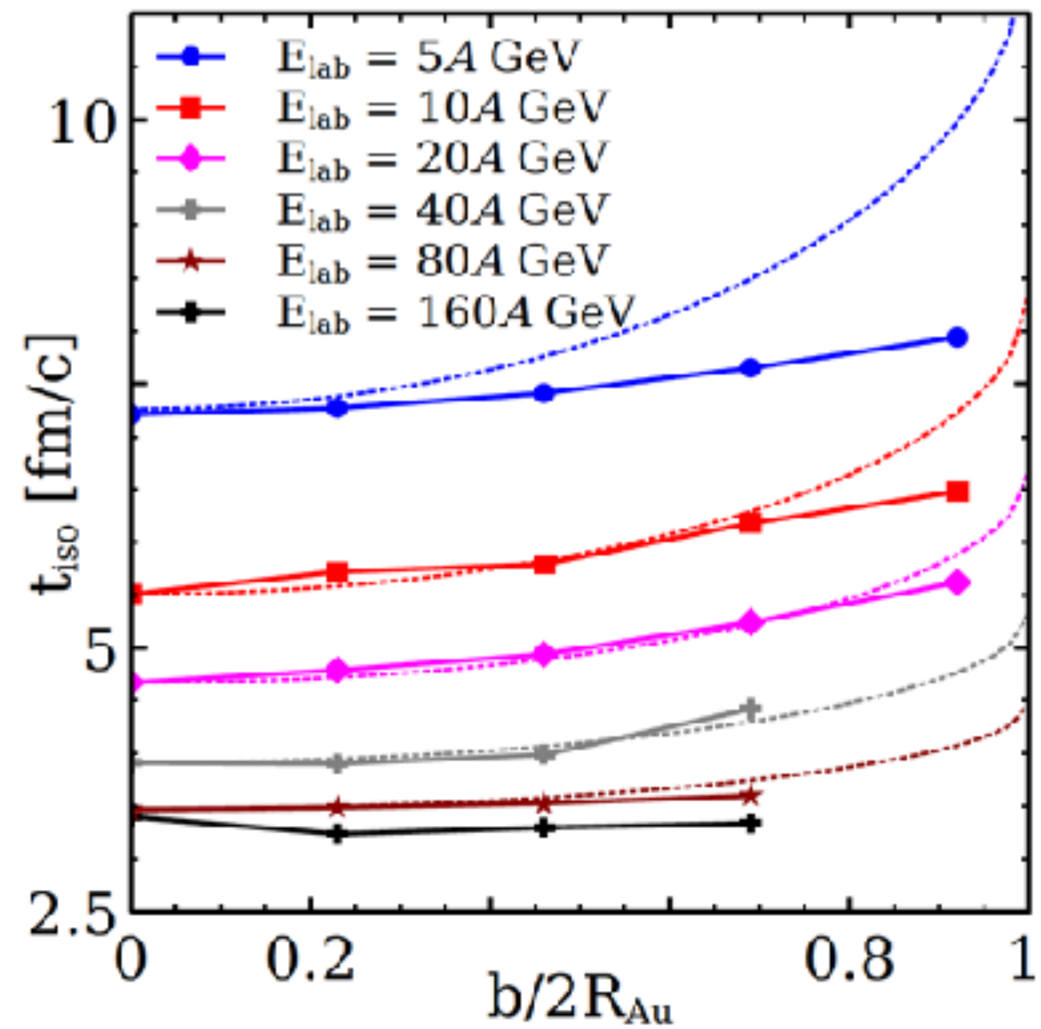


- In single events only small amount of the area is isotropic
- Off-diagonality is small in more than 80 % of area

Centrality Dependence

- Isotropization time deviates from geometrical overlap criterion for higher beam energies



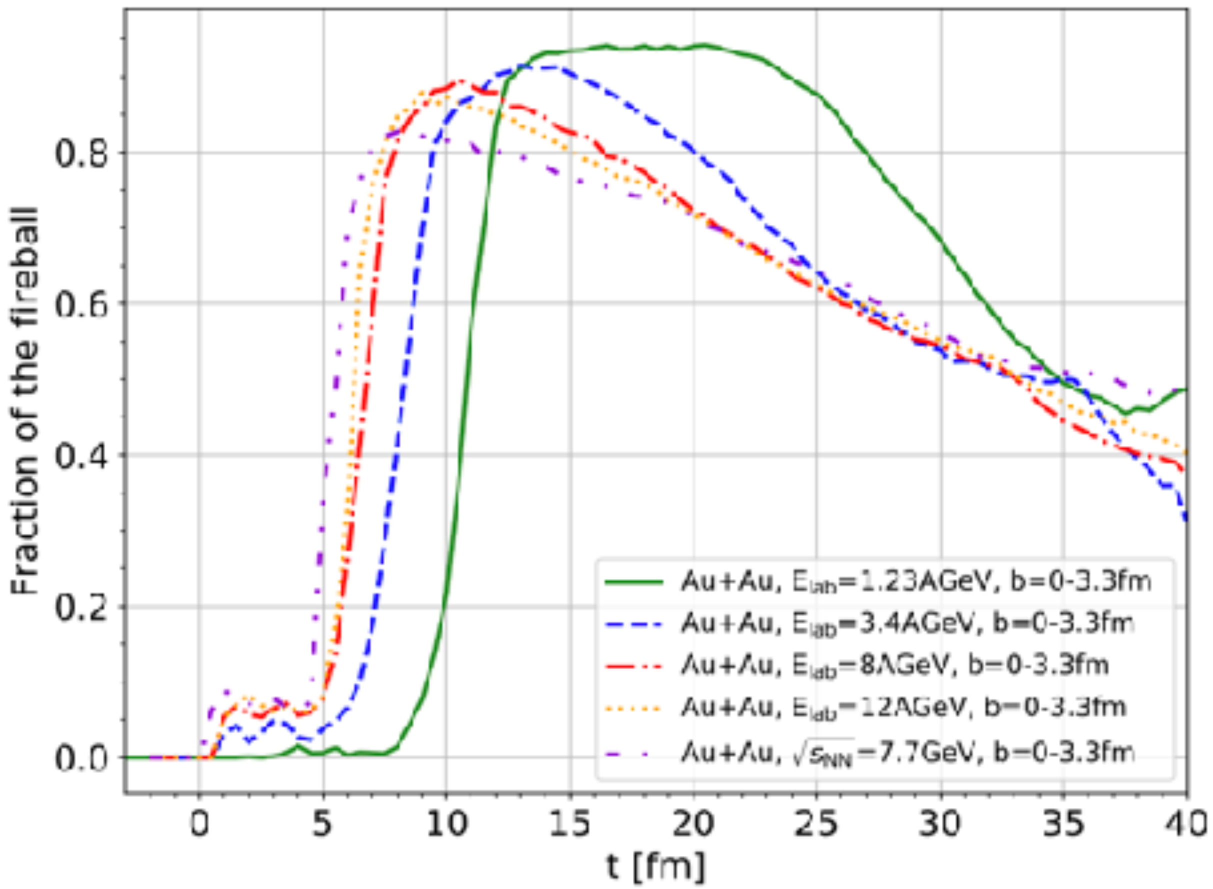


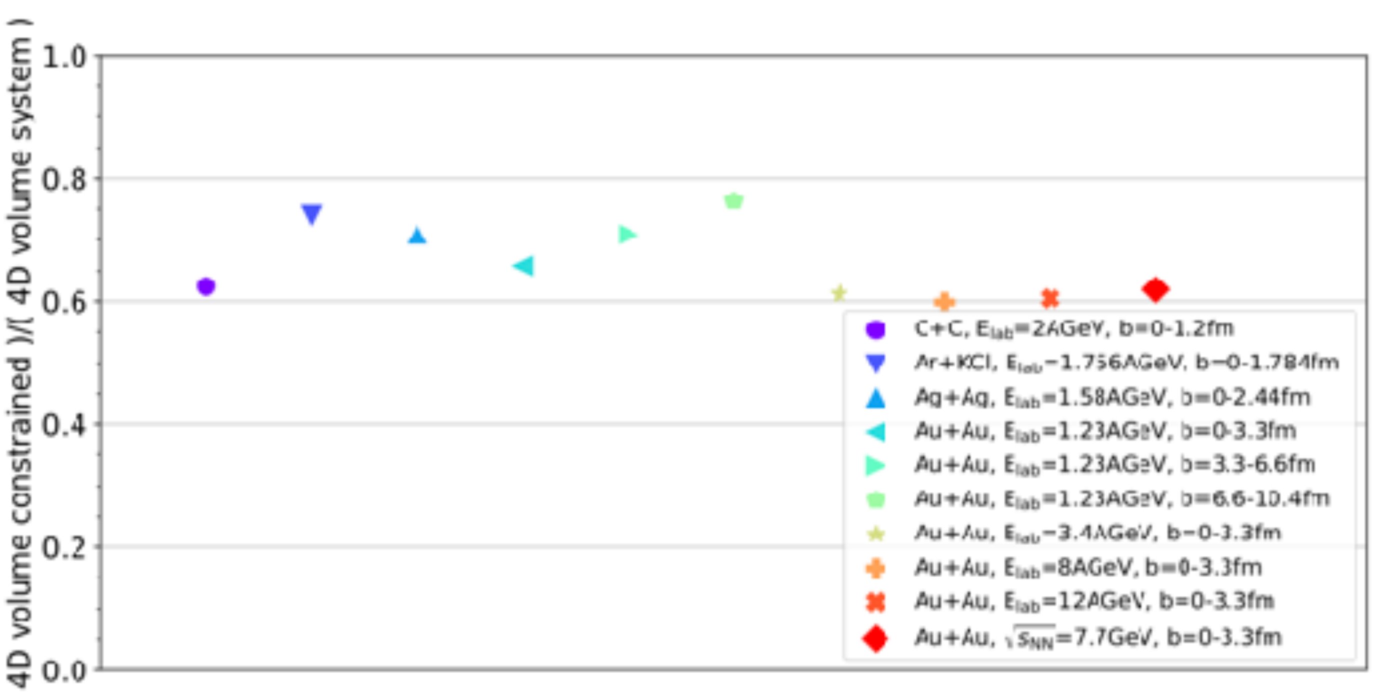
- Centrality dependence is weaker than expected from geometry

$$t_0(b) = t_0(b=0) + \frac{R}{\gamma v}\left(1 - \sqrt{1 - (b/2R)^2}\right)$$

Lower Beam Energies

G. Inghirami and HE, arXiv:2201.05934



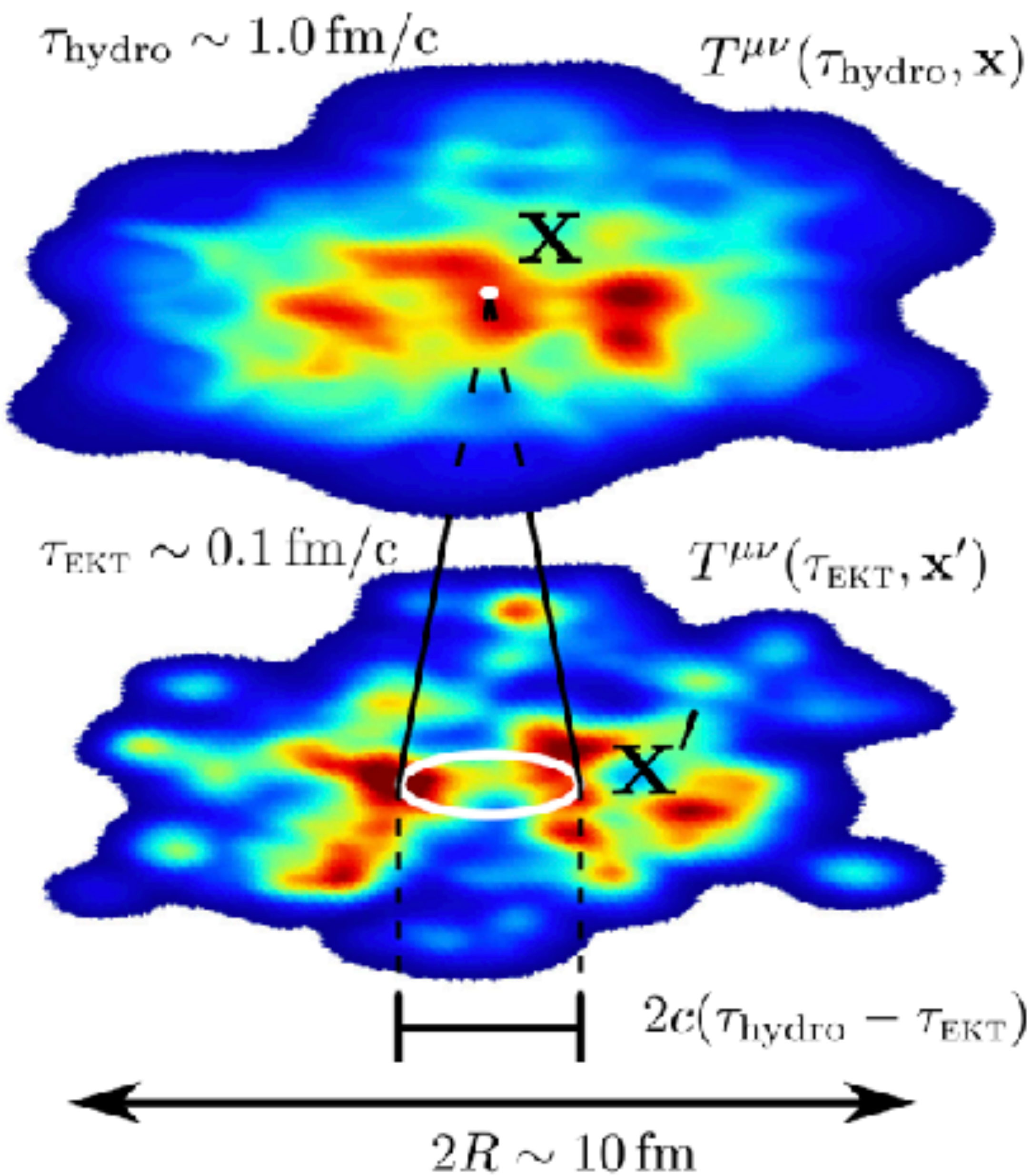


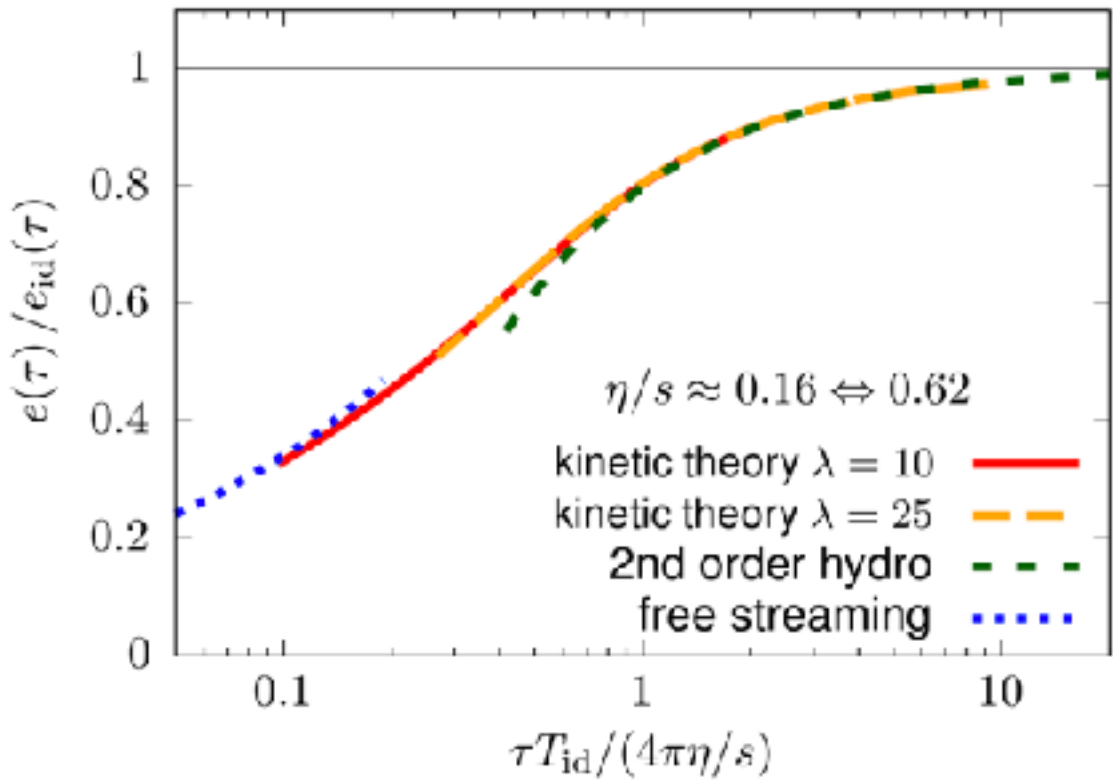
- Similar equilibration study was performed at lower beam energies within SMASH
- Significant fraction of the system is reaching X,Y<0.3
- Fraction of 4D volume is 60-80% independent of system size, energy and centrality of the collision

Kinetic theory matching..

- A lot of effort to gain a better understanding of initial non-equilibrium stages within classical Yang-Mills calculations and kinetic theory

A. Kurkela et al, Phys.Rev.Lett. 122 (2019) 12



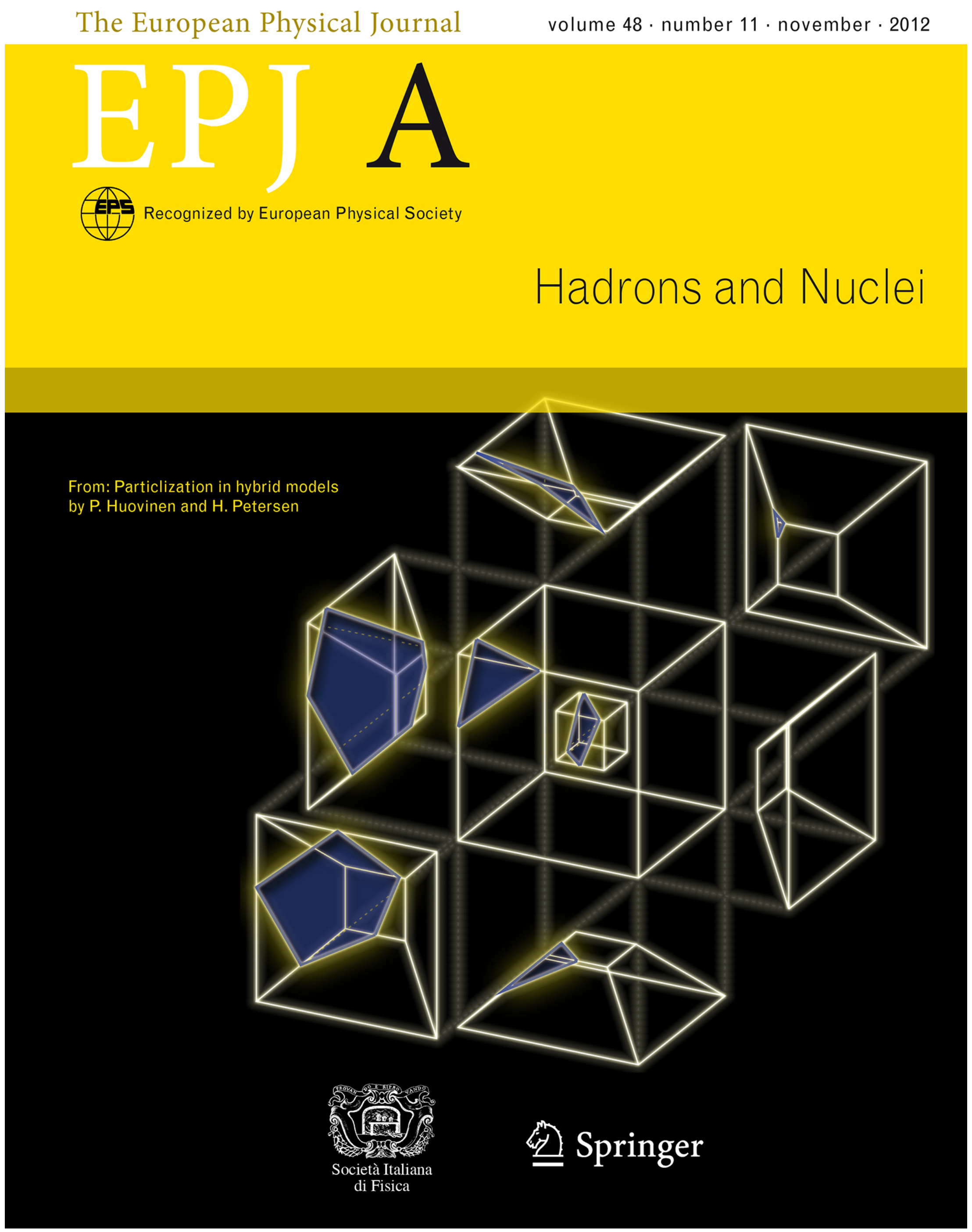


- Kompost is an example for linearised kinetic theory to allow for smoother transition to hydrodynamics

Final State Interface

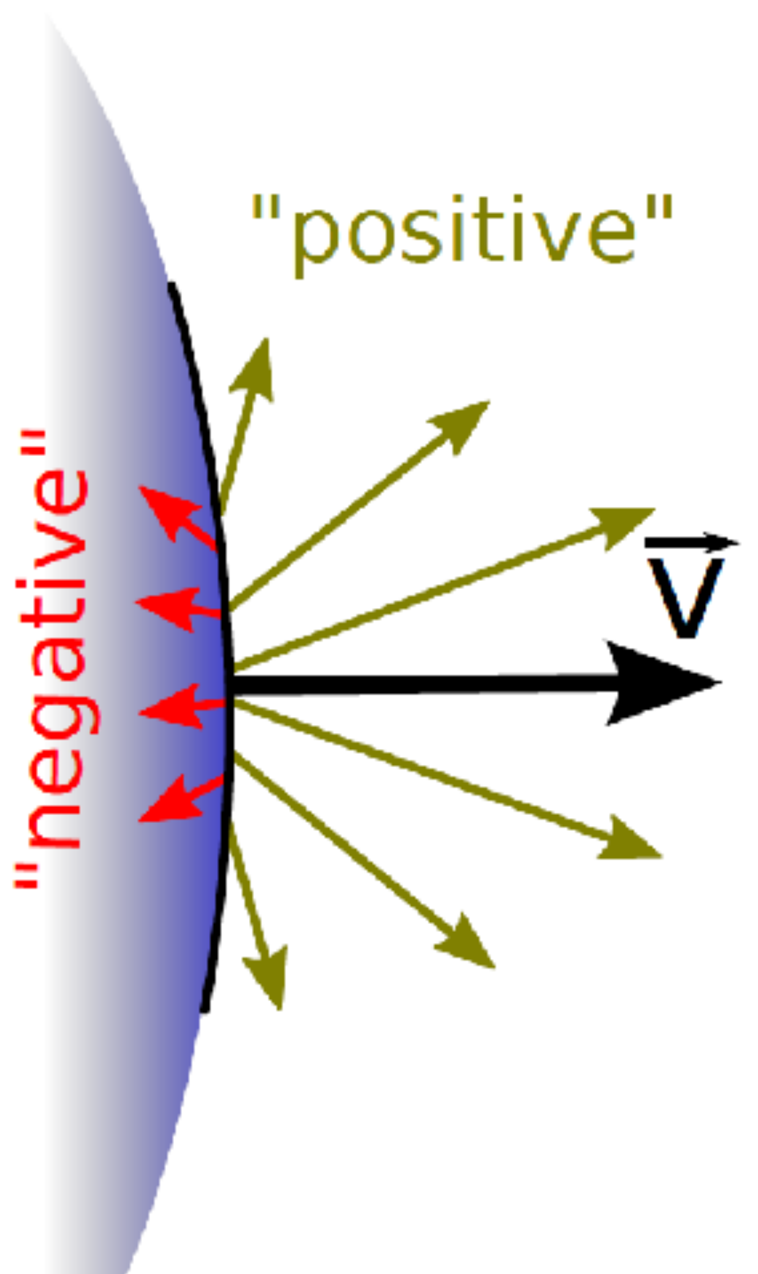
Hypersurface Finding

- Cornelius: 3D hypersurface in 4 dimensions
- Constant energy density
- Avoiding holes and double-counting
- Applicable as a subroutine during hydro run
- Input: 16-tuples of spatio-temporal information
- Output: Hypersurface vectors and interpolated thermodynamic quantities



P. Huovinen, H.P. arXiv: 1206.3371
Fortran and C++ subroutines, cornelius, implementations of this algorithm in 3D and 4D, are available at https://karman.physics.purdue.edu/OSCAR

Negative Contributions



$d\sigma_\mu$ - normal 4-vector
$u_\mu = (\gamma, \gamma \vec{v})$ - 4-velocity
T - temperature
μ - chemical potential

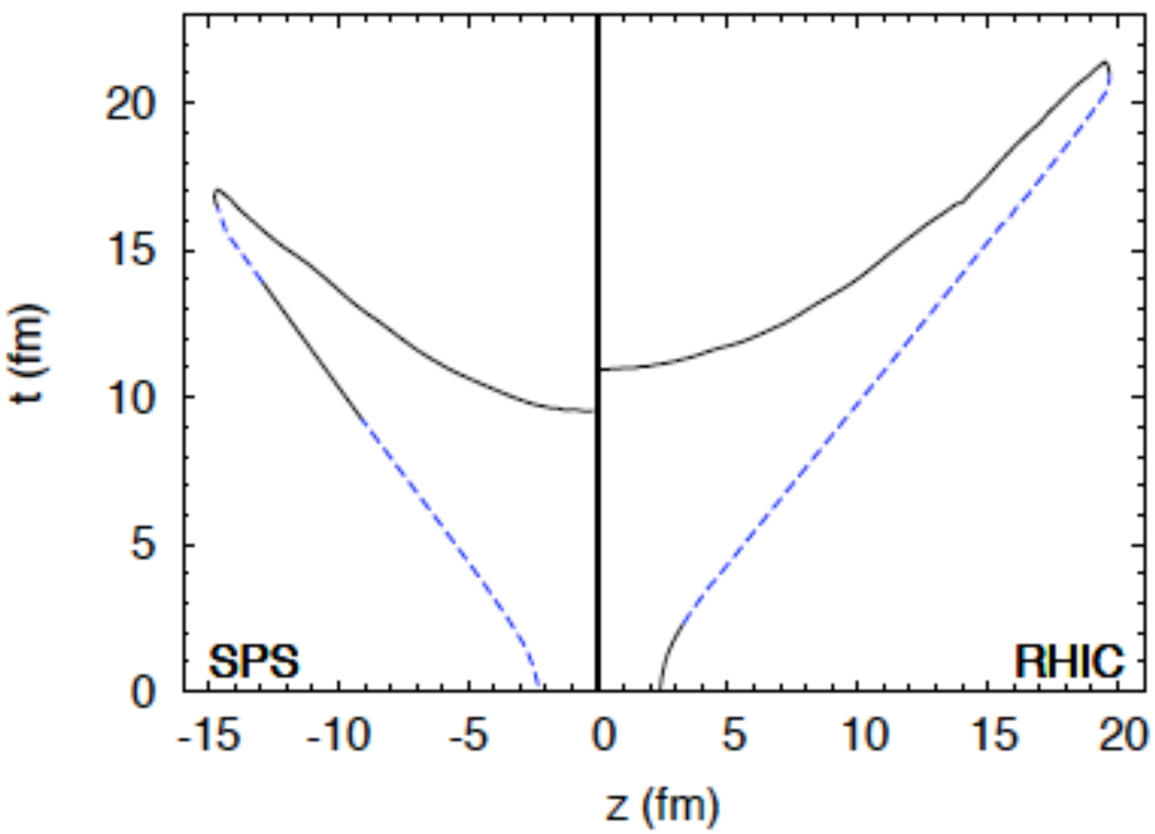
- Definition: Reference to PRC 78, 2016
  - Particles outward: $p^\mu d\sigma_\mu > 0$
  - Particles inward: $p^\mu d\sigma_\mu < 0$
- Different options:
  - Account for feedback in hydro
    K. Bugaev, Phys Rev Lett. 2003; L. Czernai, Acta Phys. Hung., 2005
  - Account effectively by weights in transport S. Pratt, Phys.Rev. C89 (2014) 2, 024910
  - Neglect them and violate conservation laws
- Systematic study of the size of negative contributions by comparison to actual transport

Hypersurface Results

- Energy and net baryon number conservation on hypersurface
$$E = \int_\sigma T^{0\mu} d\sigma_\mu \quad \text{and} \quad B = \int_\sigma n_B u^\mu d\sigma_\mu,$$
- where $d\sigma_\mu T^{\mu 0} \gtrless 0$ and $d\sigma_\mu n_B u^\mu \gtrless 0$ specify the positive and negative contributions
- Results at RHIC for central and mid-central collisions

	E [GeV]			B		
	total	*pos.*	*neg.*	total	*pos.*	*neg.*
initial	5431			93.23		
final	5430	5861	-431	92.74	97.74	-5.00
initial	2327			35.84		
final	2336	2455	-119	35.80	37.10	-1.30

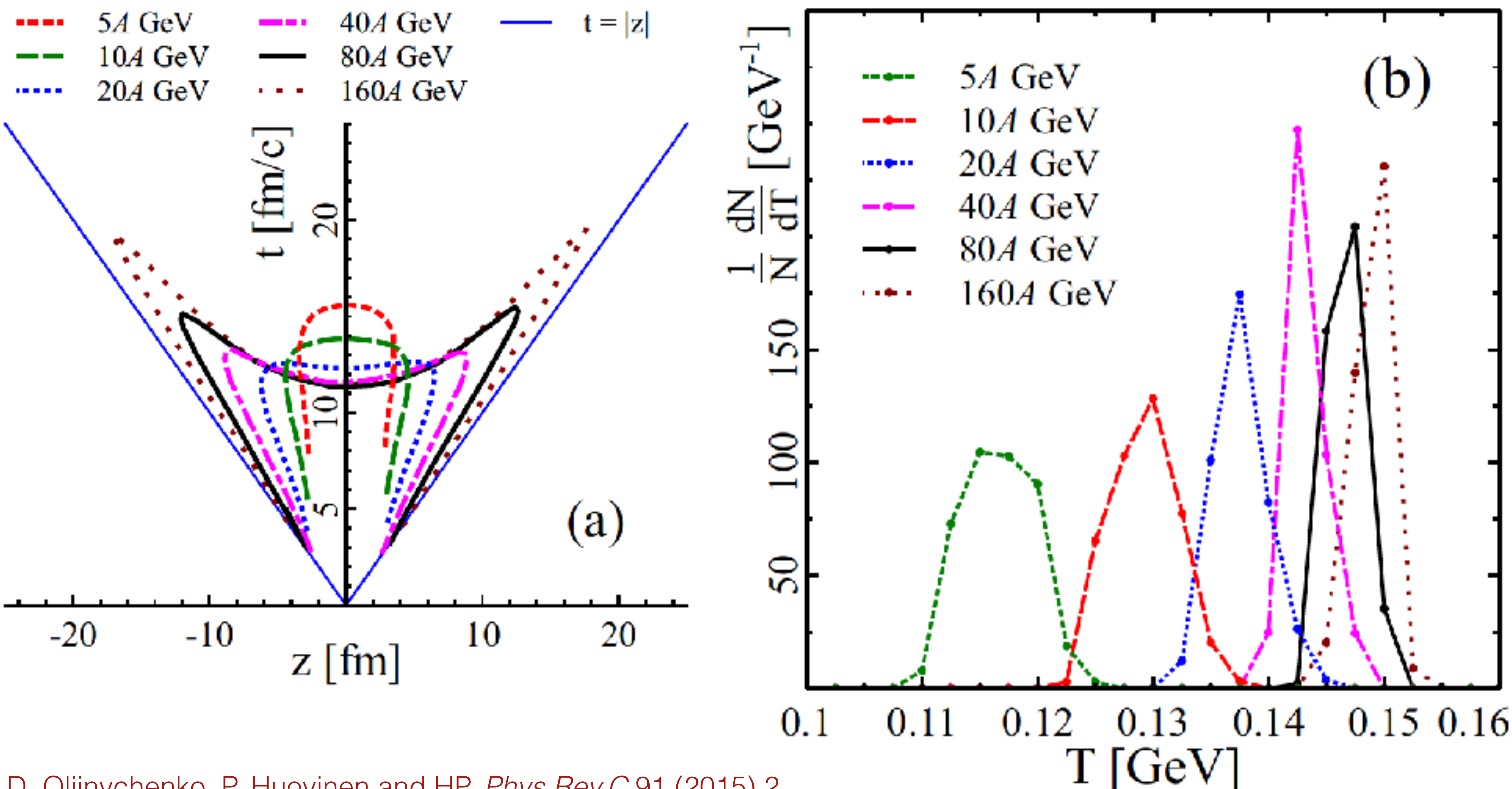
Dashed lines indicate possible positions of elements with negative contributions
$$d\sigma_\mu u^\mu < 0$$



D. Oliinychenko, P. Huovinen and HP, *Phys.Rev.C* 91 (2015) 2

Hypersurfaces

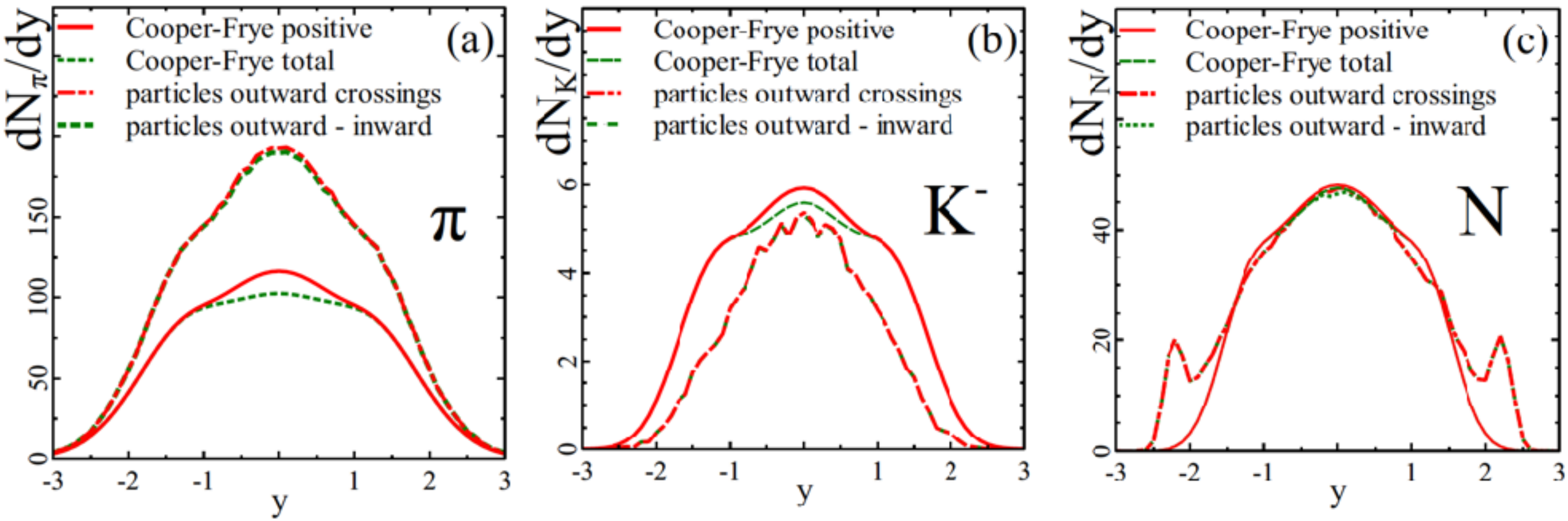
- Iso-energy density hypersurfaces (ε_C = 0.3 GeV/fm^3) represent distributions in temperature



D. Oliinychenko, P. Huovinen and HP, *Phys.Rev.C* 91 (2015) 2

Mass Dependence

- Rapidity spectra of pions, kaons and protons at E_{lab} = 40A GeV central Au+Au collisions

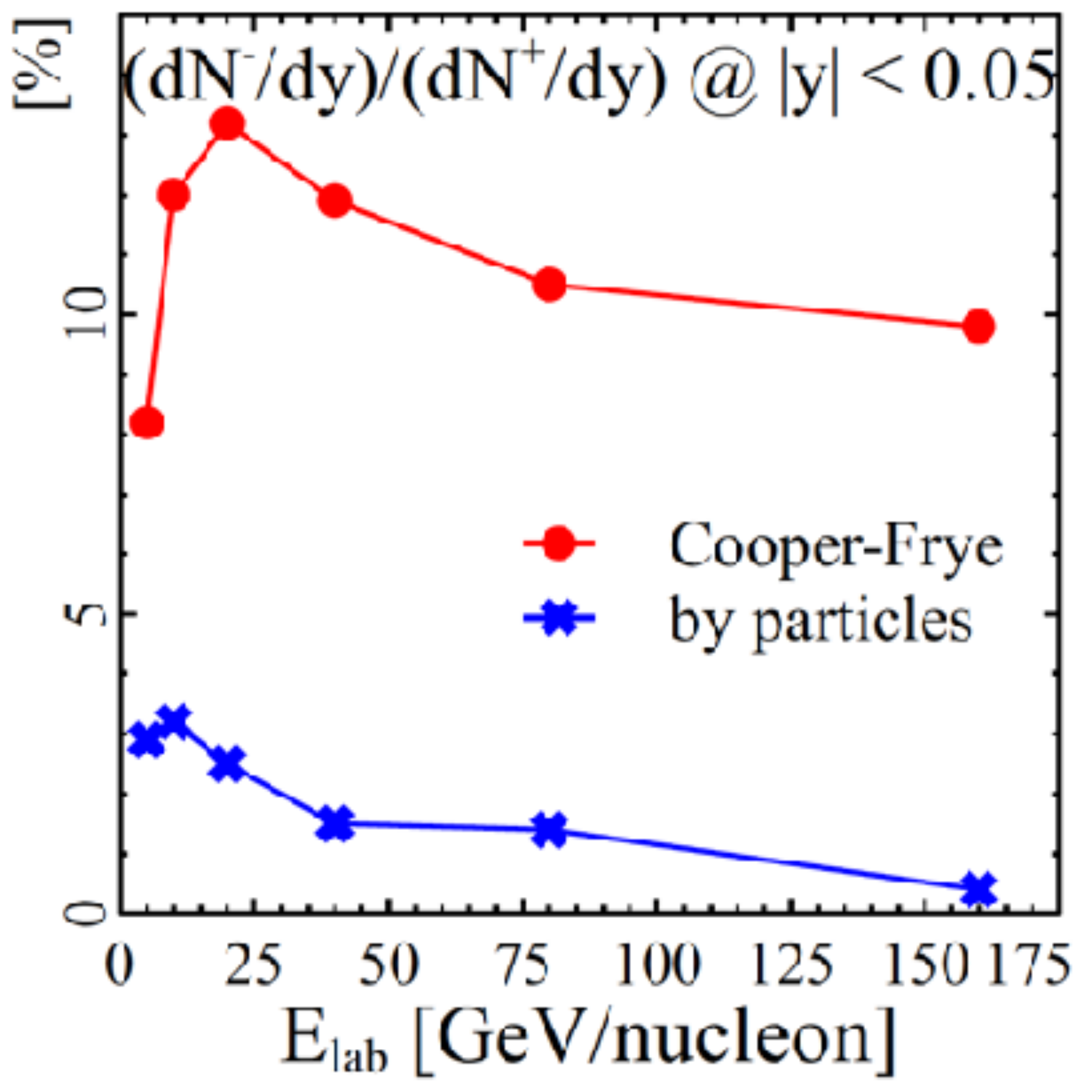


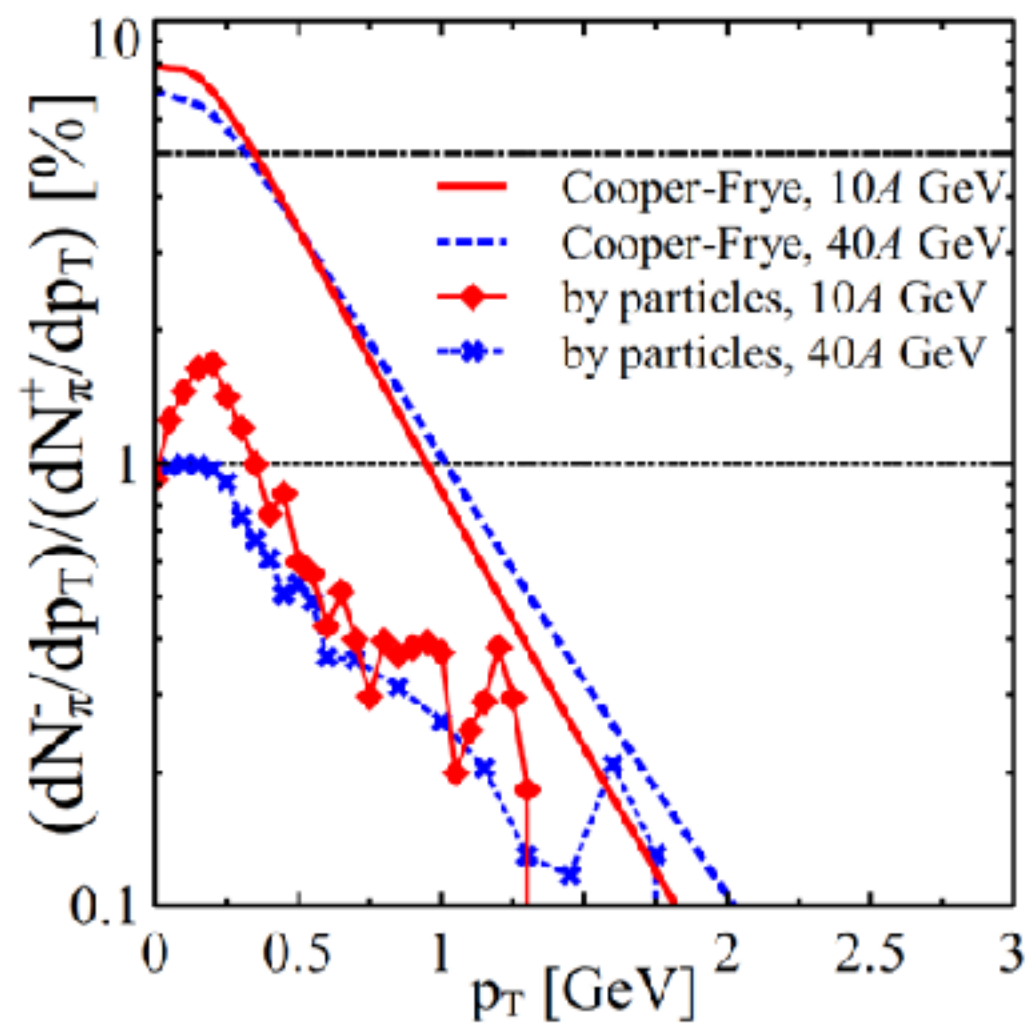
D. Oliinychenko, P. Huovinen and HP, *Phys.Rev.C* 91 (2015) 2

- Negative contributions are negligible for more massive particles
  – Concentrate on pions in the following

Energy Dependence

- Maximum at E_{lab}~25 AGeV, decreasing at higher energies
- Actual particles are always less likely to fly inward



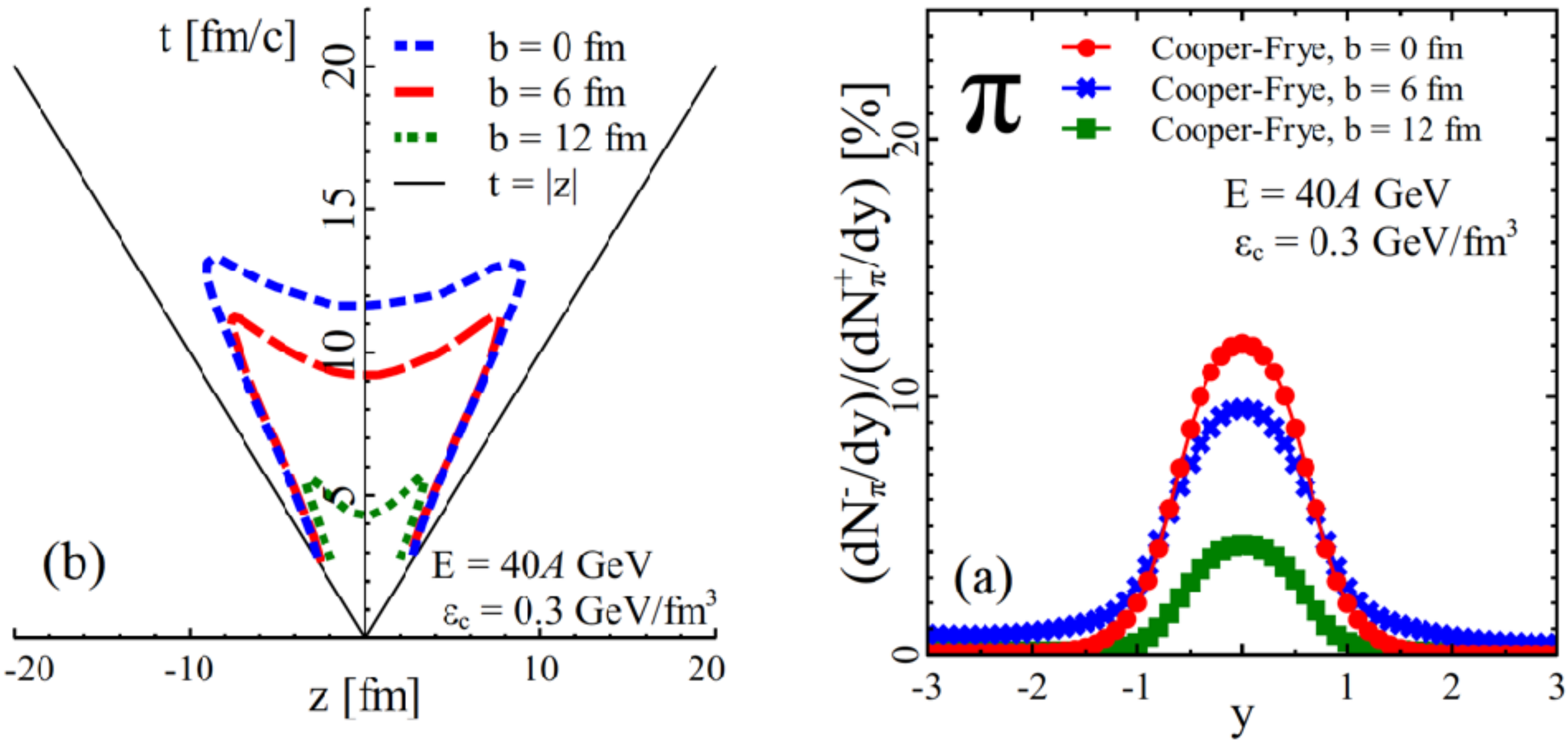


- Negative contributions are larger at small p_T

D. Oliinychenko, P. Huovinen and HP, *Phys.Rev.C* 91 (2015) 2

Centrality Dependence

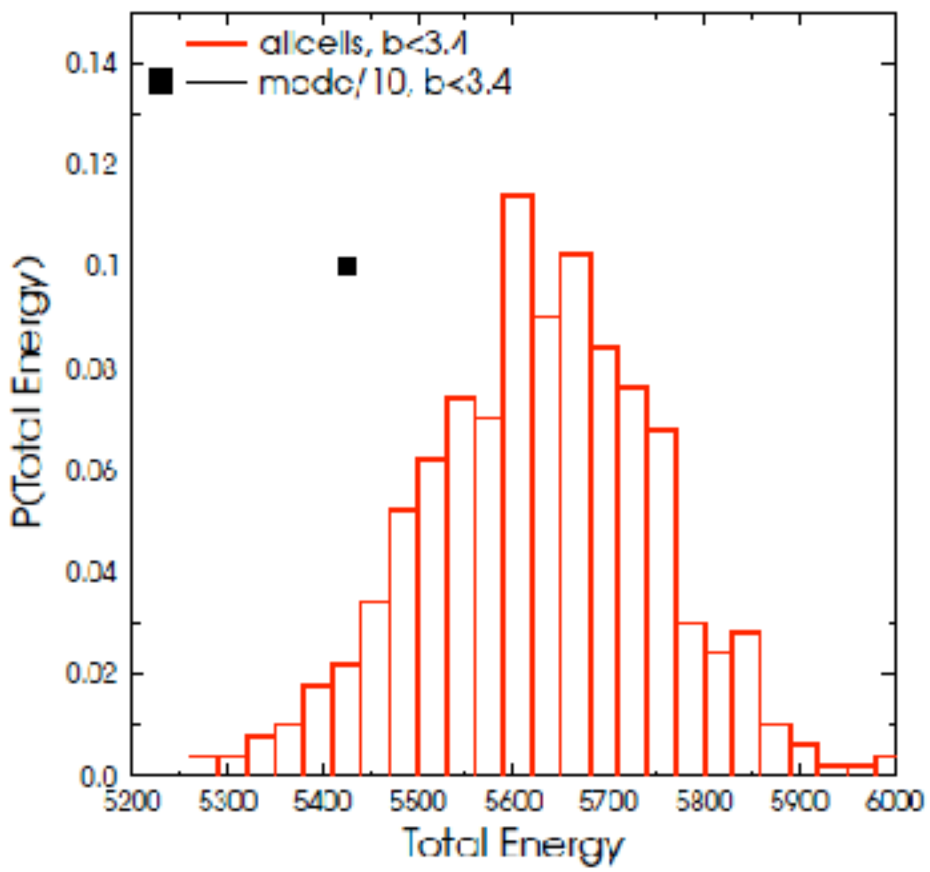
- Ratio of surface to volume emission varies with centrality
- Due to larger relative flow velocities the negative contributions are larger in more central events

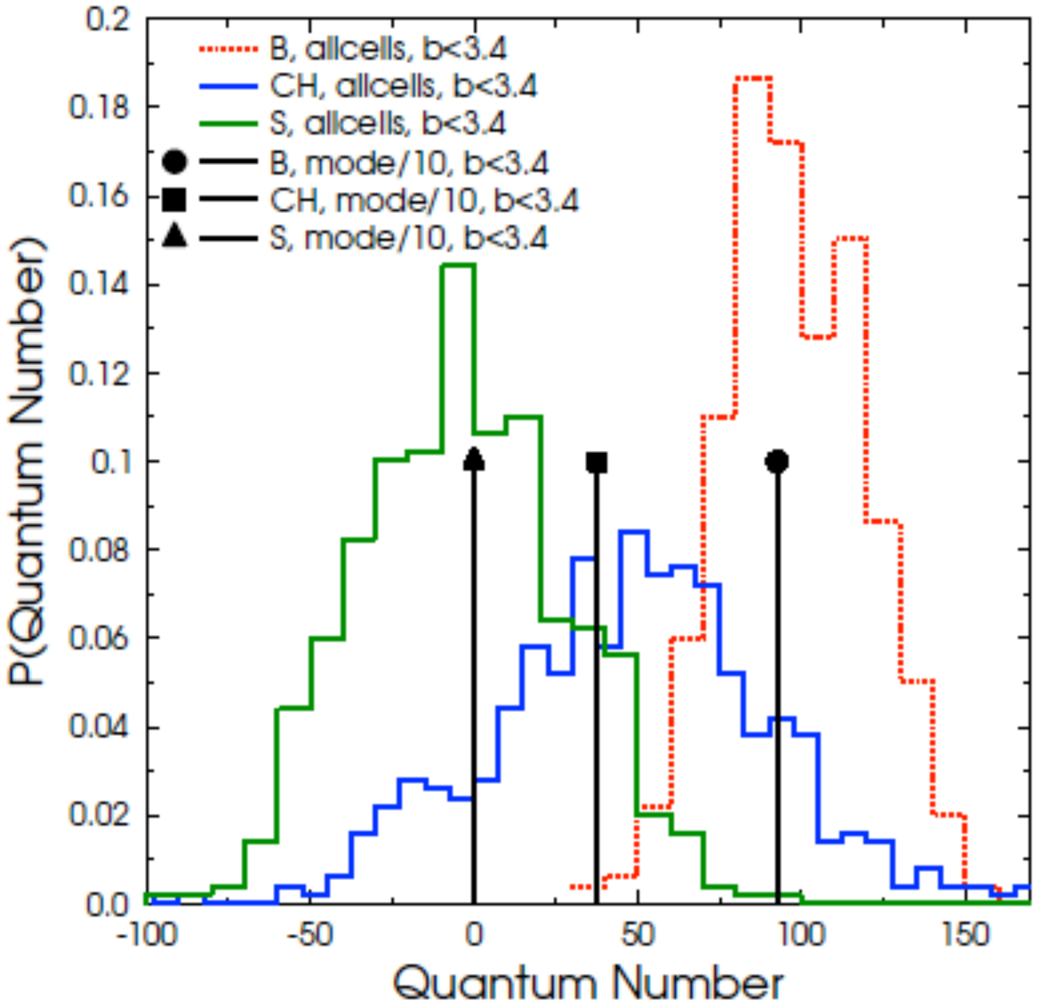


D. Oliinychenko, P. Huovinen and HP, *Phys.Rev.C* 91 (2015) 2

Sampling Procedure

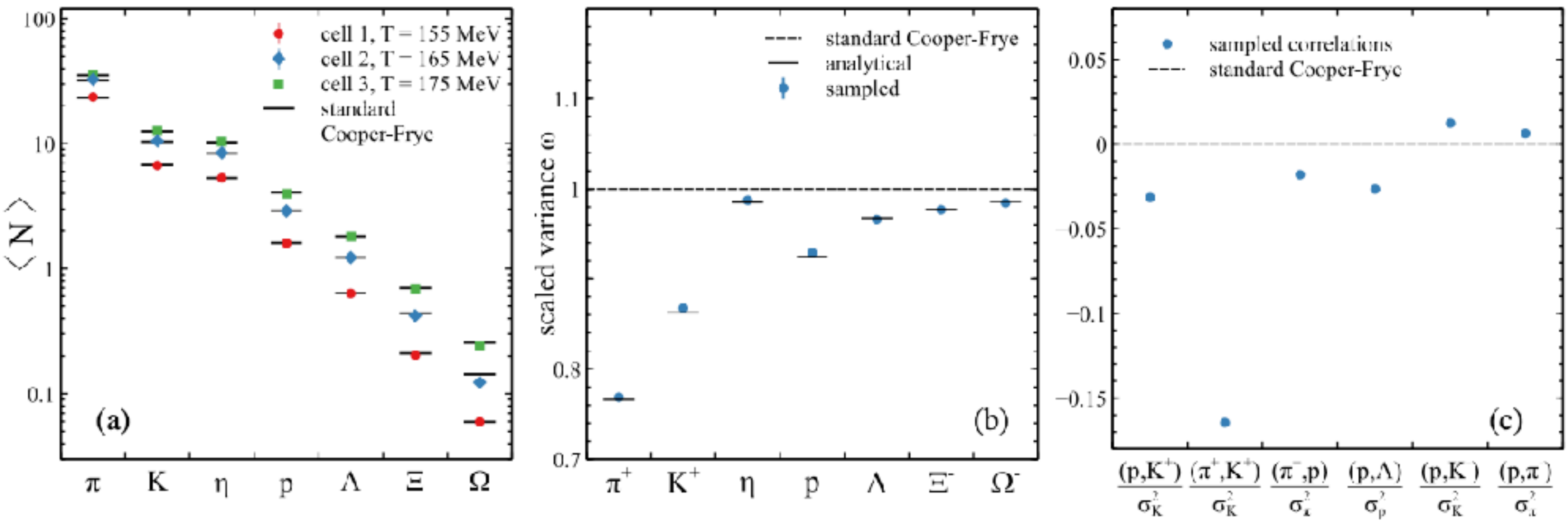
- Each event has specific quantum numbers
  - Global conservation laws are fulfilled in UrQMD hybrid
- Usually conservation laws are only fulfilled on average





Local Conservation Laws

- Micro-canonical sampler is publicly available
- Single-particle observables are not sensitive, but correlations and fluctuations are
- Sensitivity to size of region where conservation applies



D. Oliinychenko, V. Koch, Phys.Rev.Lett. 123 (2019) 18

- Extra: Include spectral functions in sampling process

SMASH-vHLLE Hybrid Approach

SMASH-vHLLE Hybrid Approach

- Modular hybrid approach for intermediate and high energy heavy-ion collisions
- Open source and public

https://github.com/smash-transport/smash-vhlle-hybrid

SMASH
- Hadronic transport approach
- Initial conditions

+

vHLLE
- 3+1 D viscous hydrodynamics (event-by-event)
- Cornelius routine for hypersurface

+

smash-hadron-sampler
- Cooper-Frye sampler
- Particlization of fluid elements

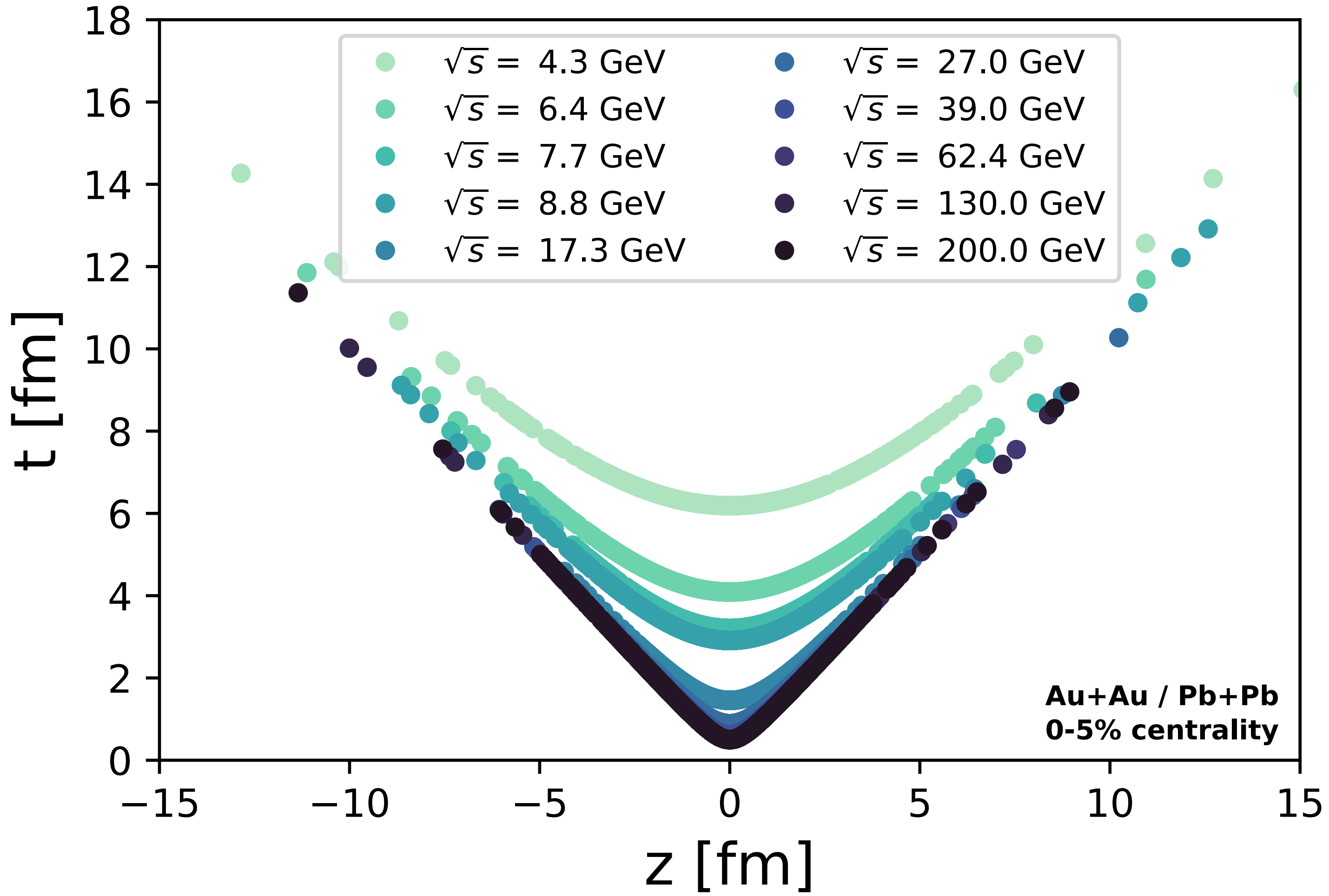
+

SMASH
- Hadronic transport approach
- Evolution of hadronic rescattering

A. Schäfer et al., arXiv: 2112.08724
Weil et al.: PRC 94 (2016)
DOI: 10.5281/zenodo.3484711
Huovinen et al.: Eur. Phys. J A 48 (2012)
Karpenko et al.: PRC 91, 064901 (2015)
Karpenko et al.: Comput. Phys. Commun. 185 (2014)

Initial Conditions from SMASH

A. Schäfer, PhD thesis



- Nuclei are initialised according to Woods-Saxon profiles
- Propagation and collisions until full overlap time

$$\tau_0 = \frac{R_p + R_t}{\sqrt{\left(\frac{\sqrt{s_{\mathrm{NN}}}}{2m_N}\right)^2 - 1}}$$

- Full energy-momentum tensor and charge distributions (B, S, Q) at constant τ hypersurface
- Fluctuations from nucleon positions and initial collisions
- Particles are smeared with Gaussian distributions

vHLLE

- 3+1 dimensional viscous hydrodynamic evolution
- Shear (and bulk) viscosity are included

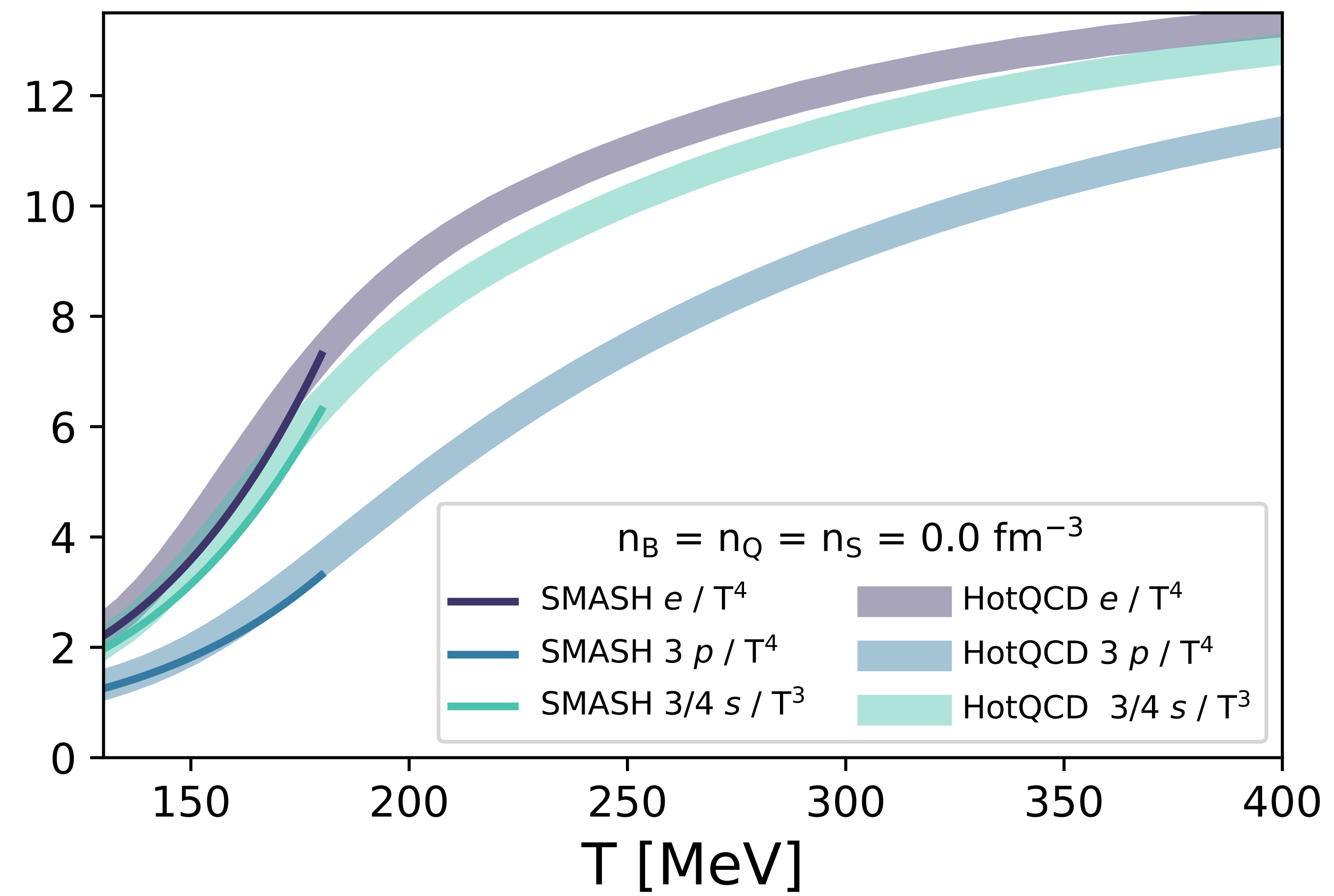
$$\partial_\mu T^{\mu\nu} = 0 \qquad \partial_\mu J_i^\mu = 0 \quad i = B, Q, S$$

- Equation of state from chiral model (update in progress)

Karpenko et al.: PRC 91, 064901 (2015)
Karpenko et al.: Comput. Phys. Commun. 185 (2014)

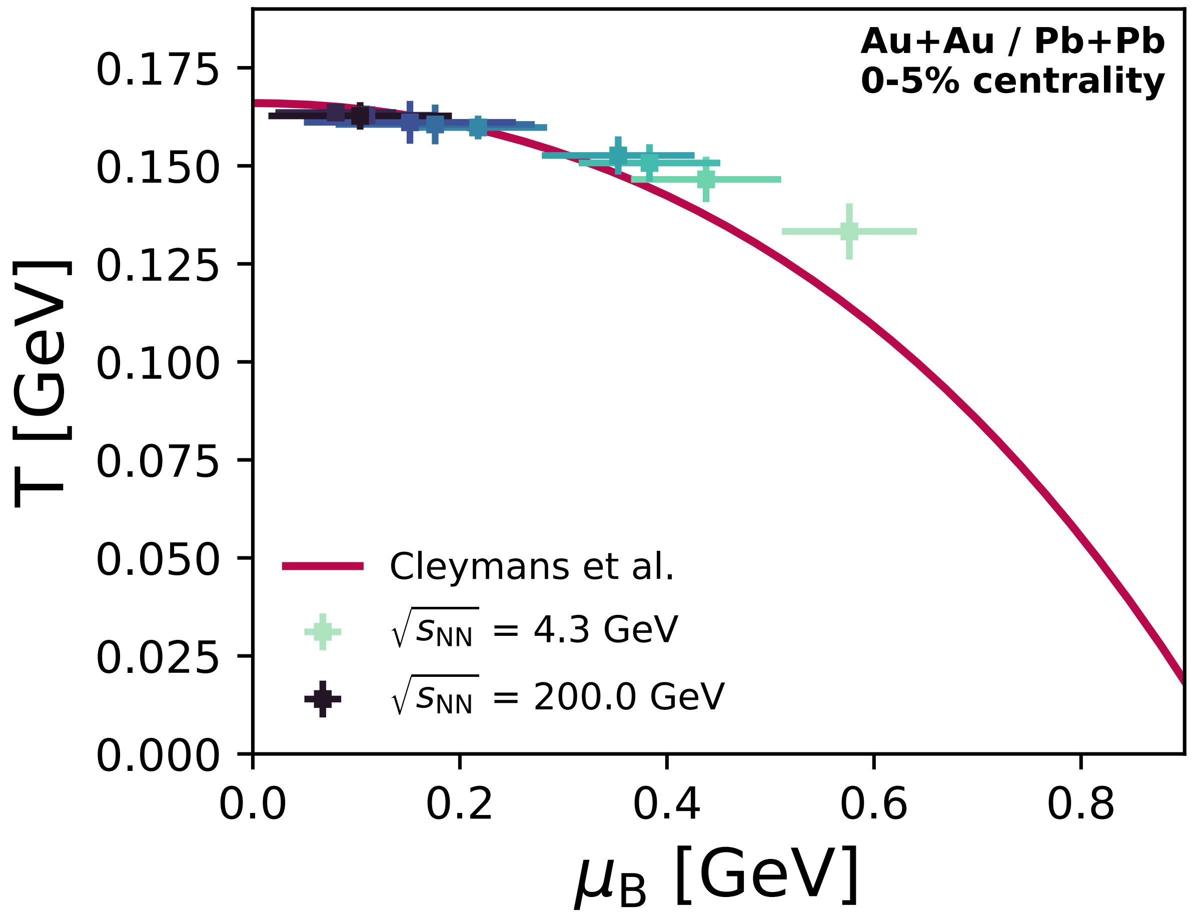
J. Steinheimer, S. Schramm and H. Stöcker, J.Phys.G 38 (2011)

- For correct mapping of degrees of freedom on hypersurface the SMASH hadron gas equation of state is used
- $(e, n_B, n_Q) \rightarrow (T, p, \mu_B, \mu_Q, \mu_S)$



Cooper-Frye Particlization

- Constant energy density hypersurface of ~2-5*ε_0 is constructed
- All SMASH hadron species are sampled according to thermal distribution functions (with δf correction for shear viscosity according to Grad 14 moment)

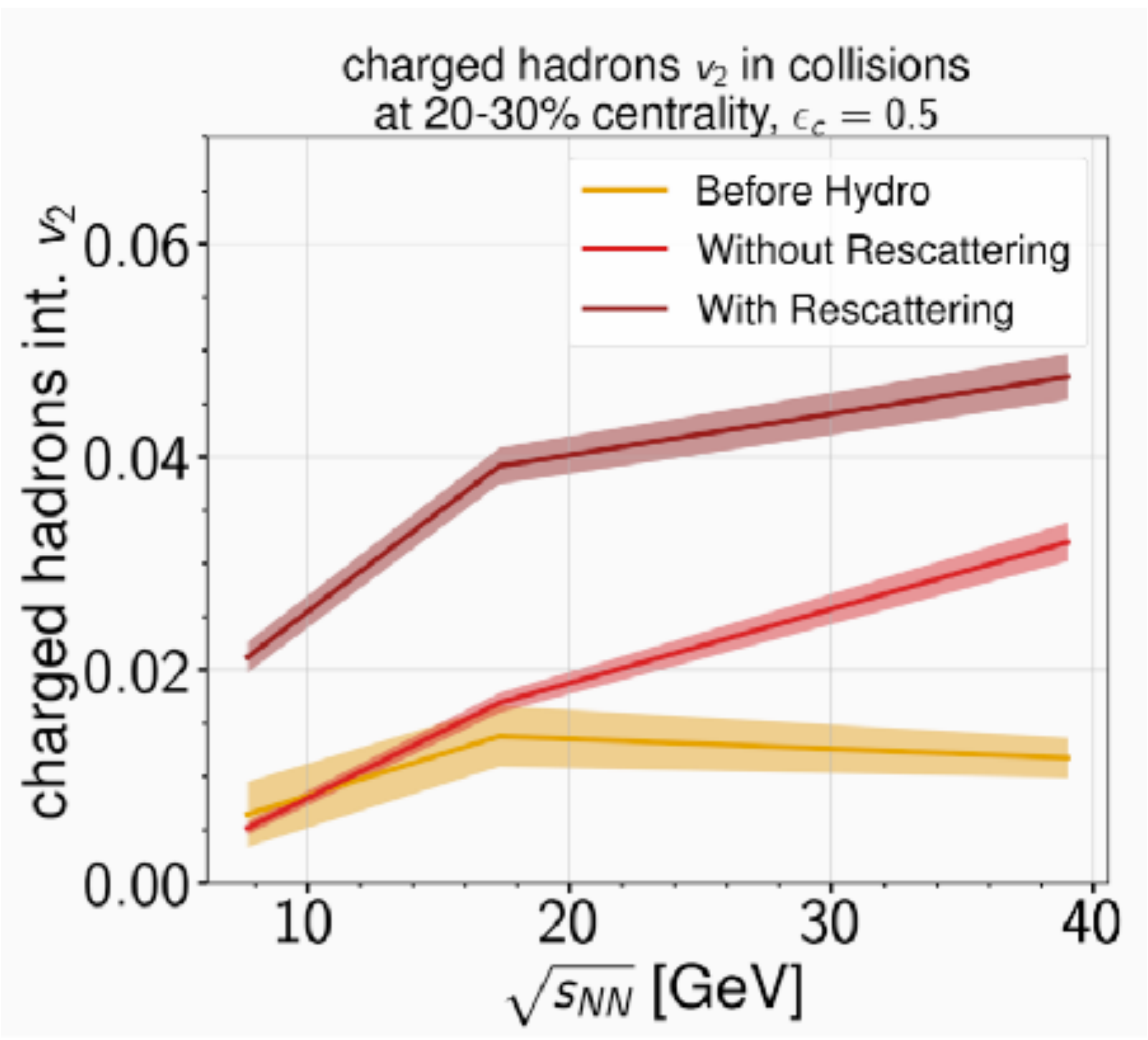


- Work in progress:
  - Sampling according to micro canonical ensemble
  - Local conservation of quantum numbers important for charge correlation observables

D. Oliinychenko, V. Koch, PRL 123 (2019)

Hadronic Rescattering

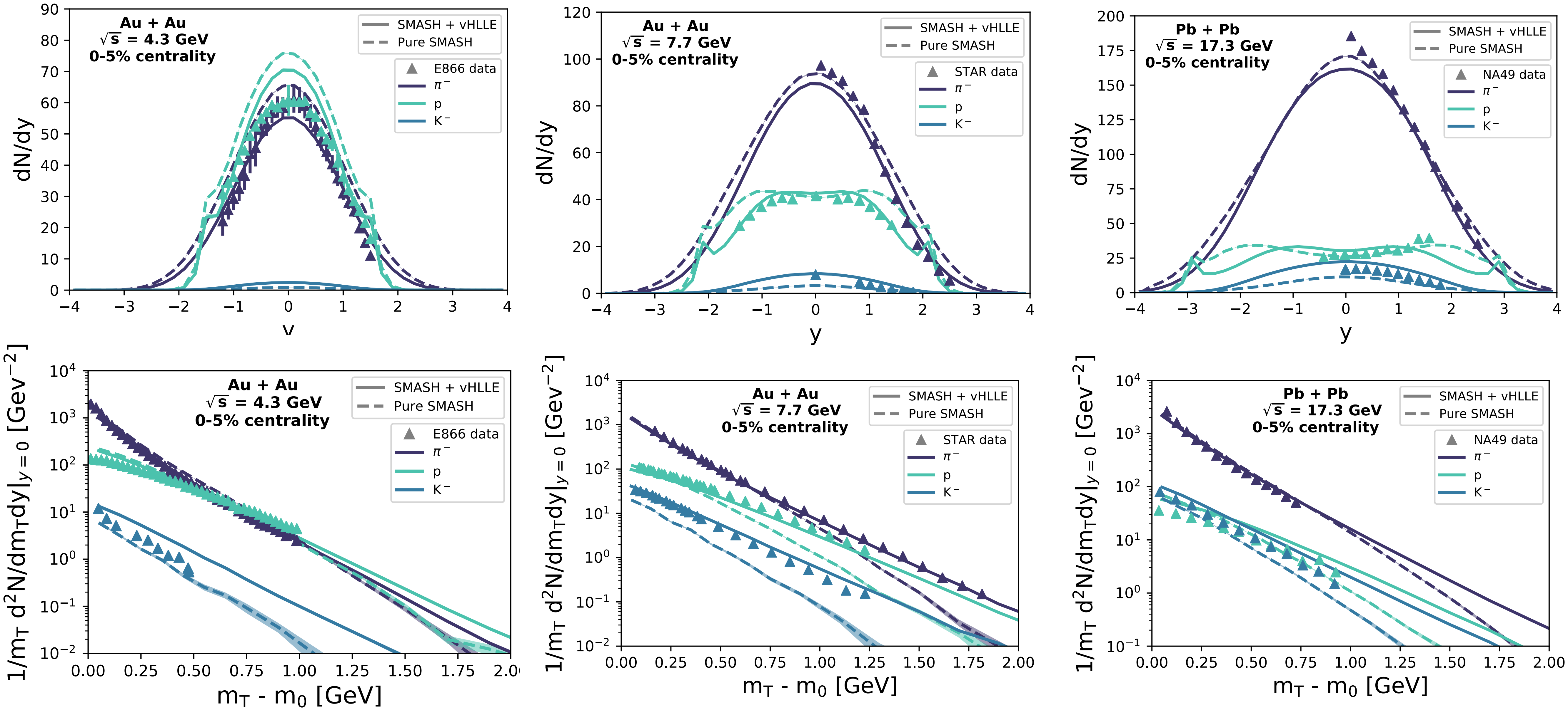
- Final state rescattering and resonance decays are handled within the hadronic transport approach SMASH
- Depending on switching transition a significant amount of elliptic flow is still generated



- Non-trivial interplay of transport and hydrodynamic evolution at low and intermediate beam energies
- Dynamical initial conditions are work in progress

N. Götz, QM 2022

Particle Spectra

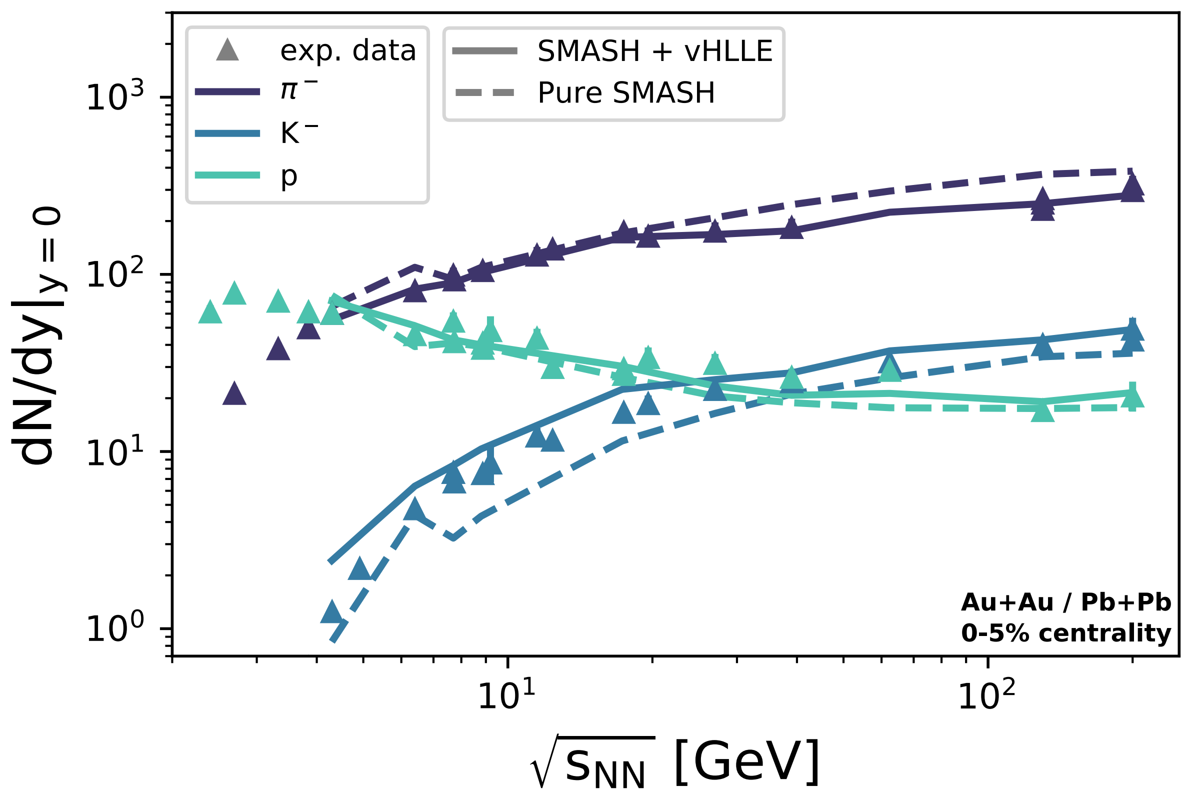


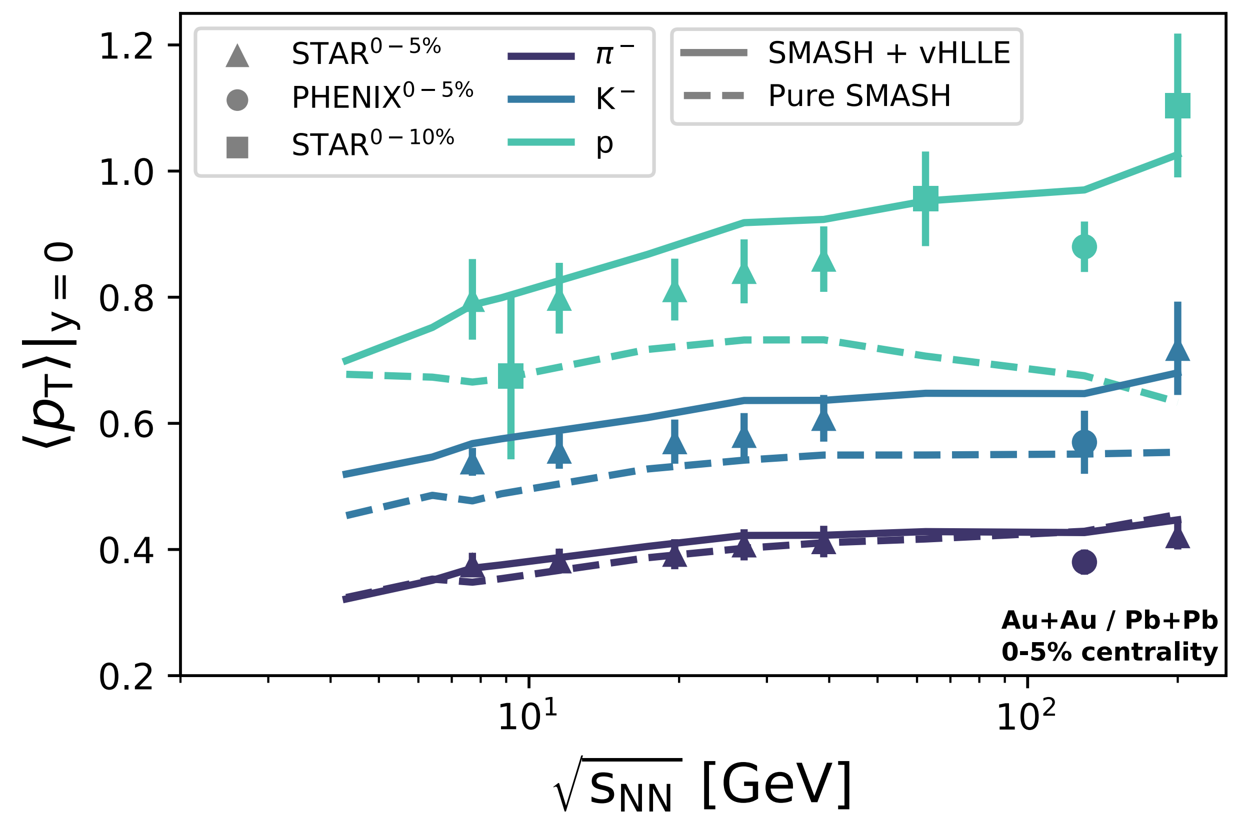
- Rapidity and transverse mass spectra of pions, kaons, protons at different energies -> Hybrid approach in decent agreement with measurements

Excitation Function

- Particle yields at midrapidity are well described over a large range of beam energies

A. Schäfer et al., arXiv: 2112.08724



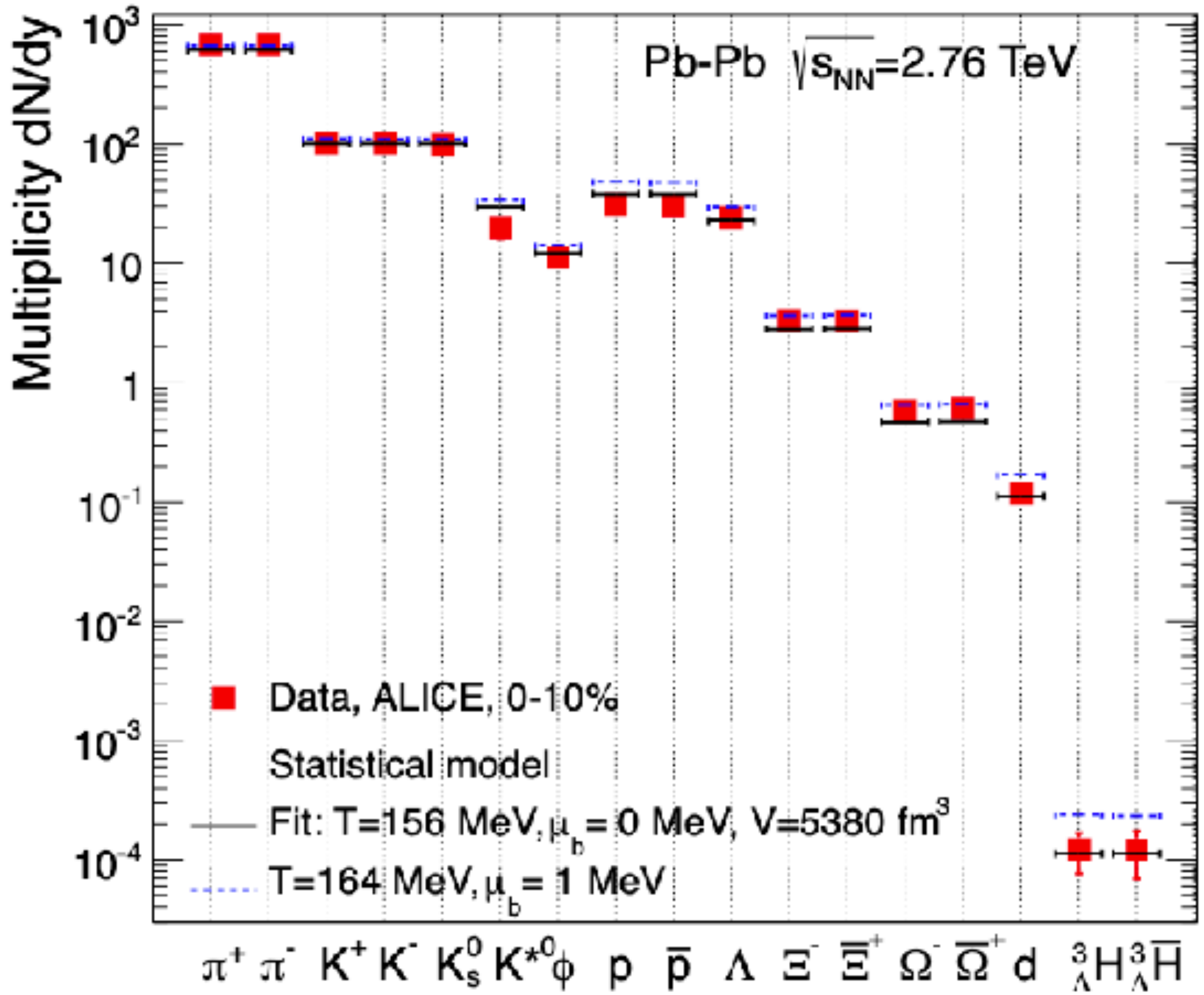


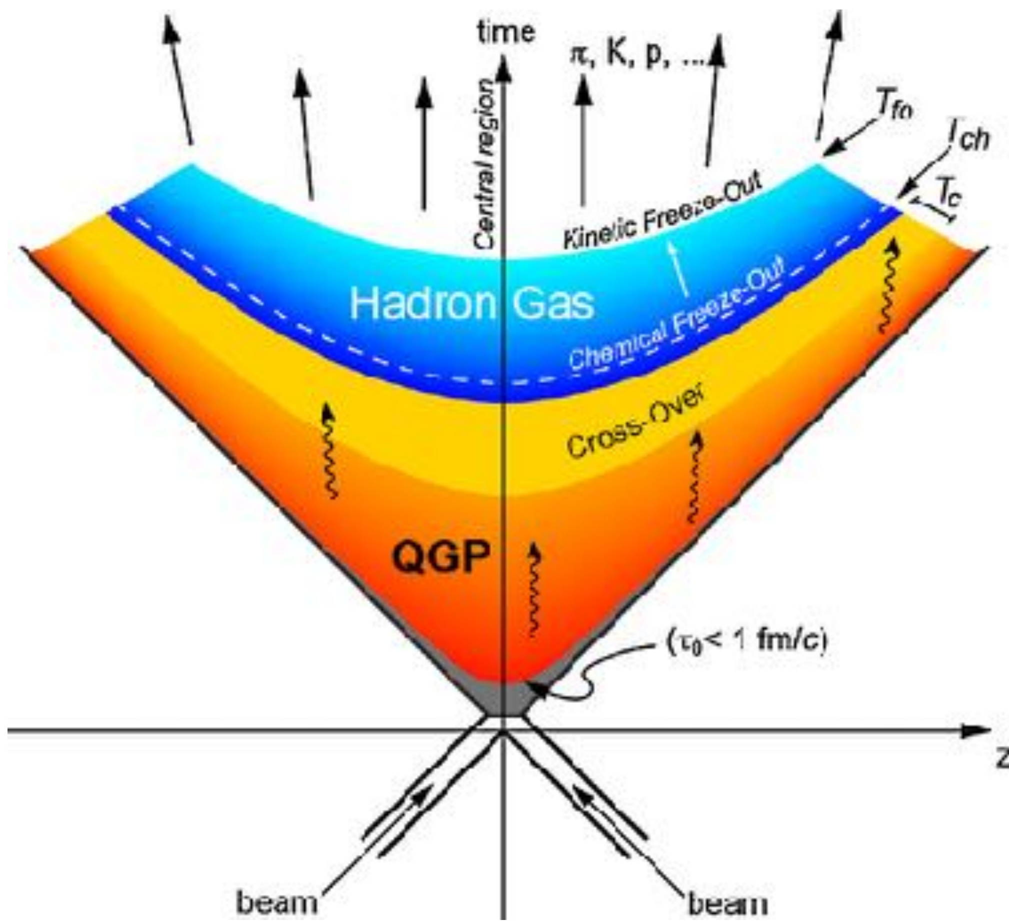
- Mean transverse momentum is also well described by the hybrid approach (too small in pure SMASH)
- More strangeness production and larger radial flow from hydrodynamics necessary from $\sqrt{s_{NN}} \sim 10$ GeV

Particle Yields LHC

- Multiplicities can be fitted with thermal model
- At LHC the proton yield is rather low

J. Stachel et al, JPCS 509 (2014)





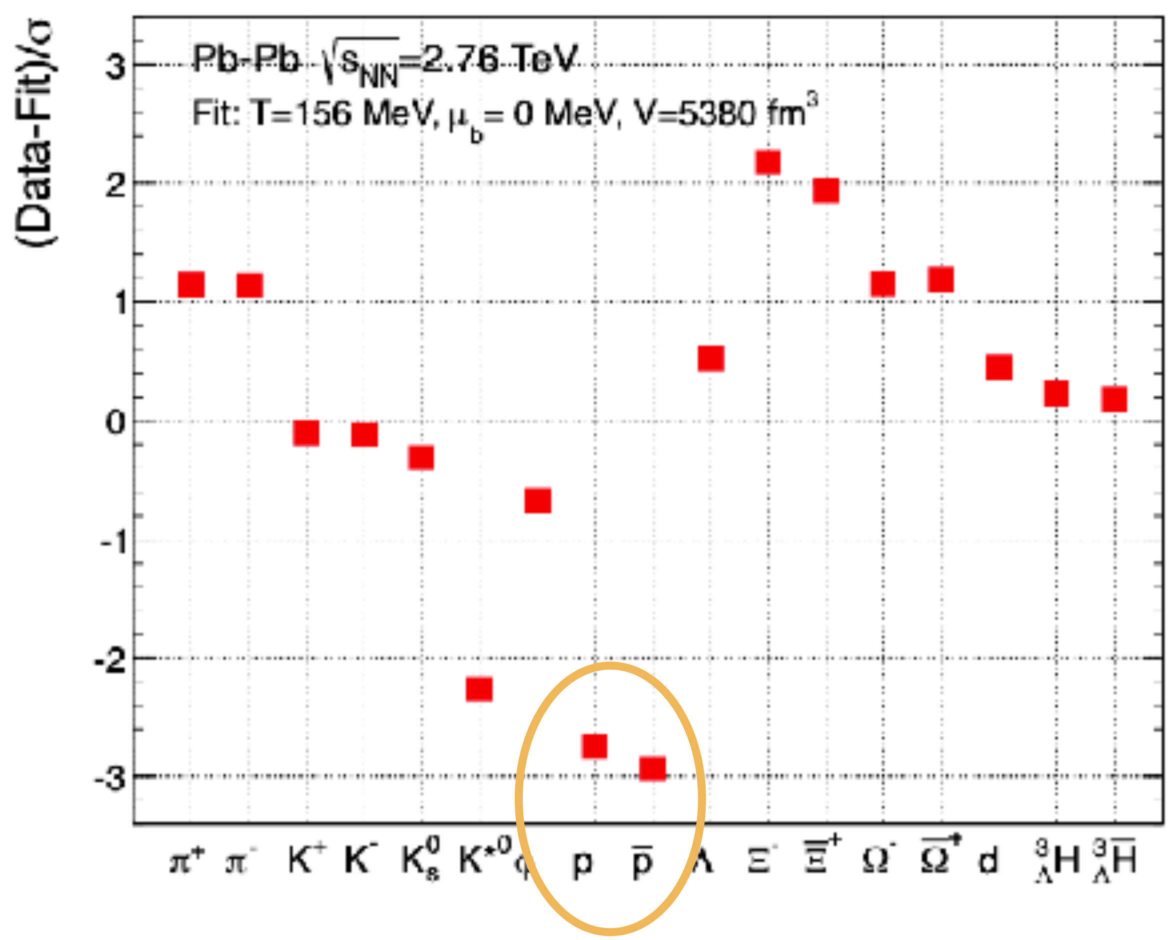
Sketch of time evolution of heavy-ion collisions

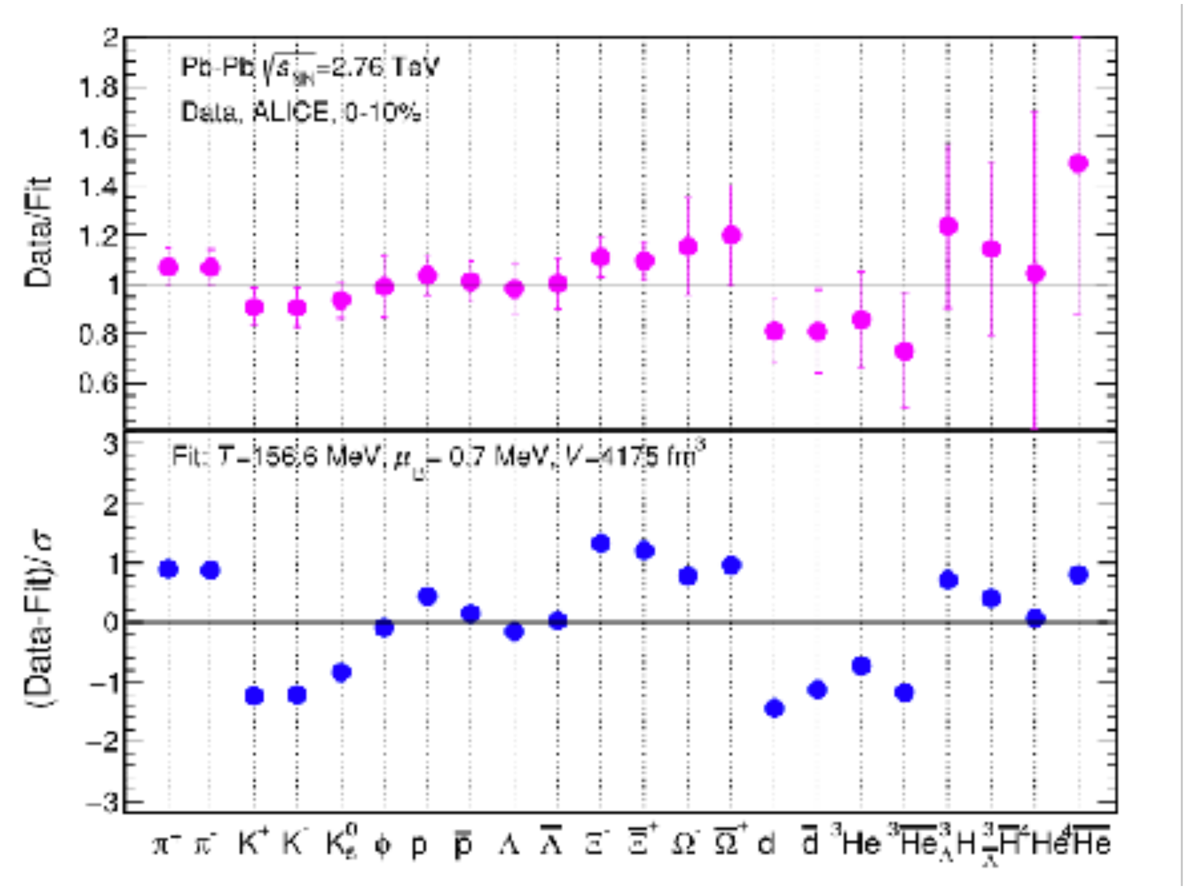
Particle Yields LHC

- Multiplicities can be fitted with thermal model
- At LHC the proton yield is rather low

A. Andronic et al, PLB 792 (2019)

J. Stachel et al, JPCS 509 (2014)



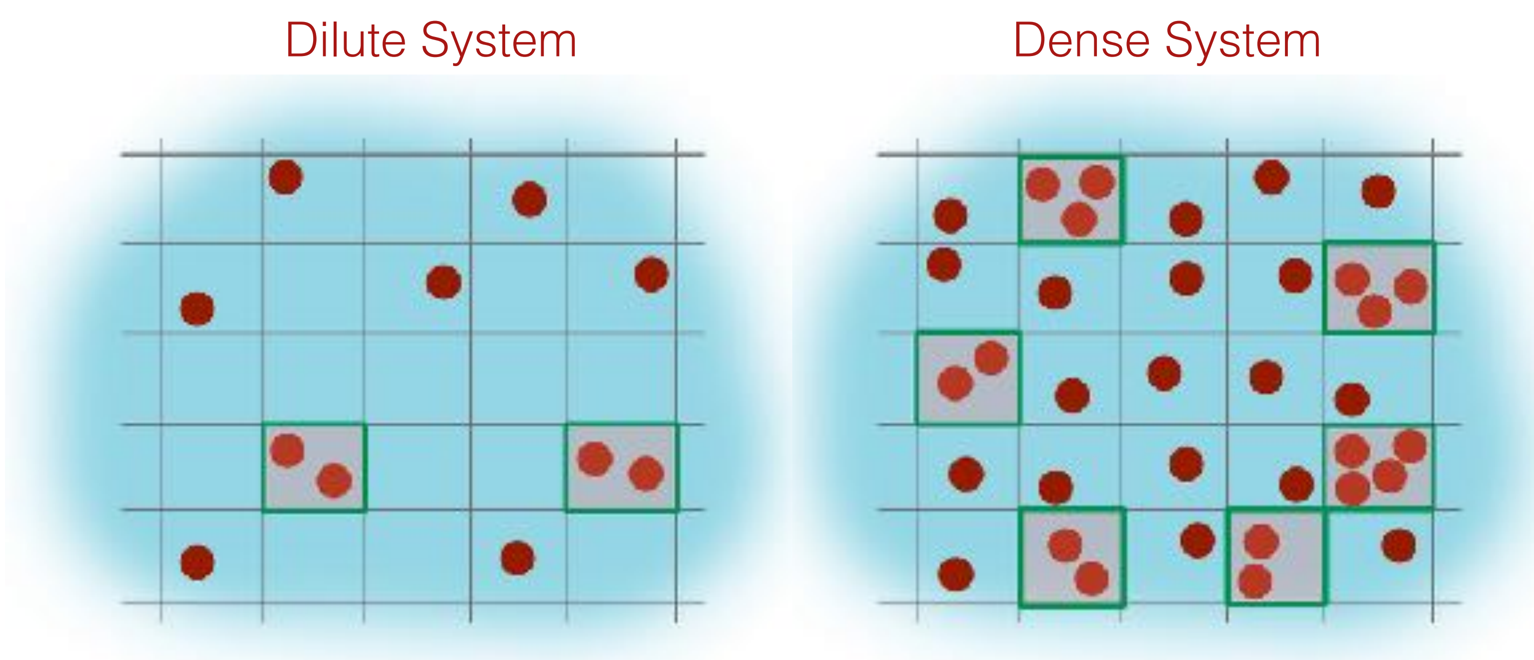


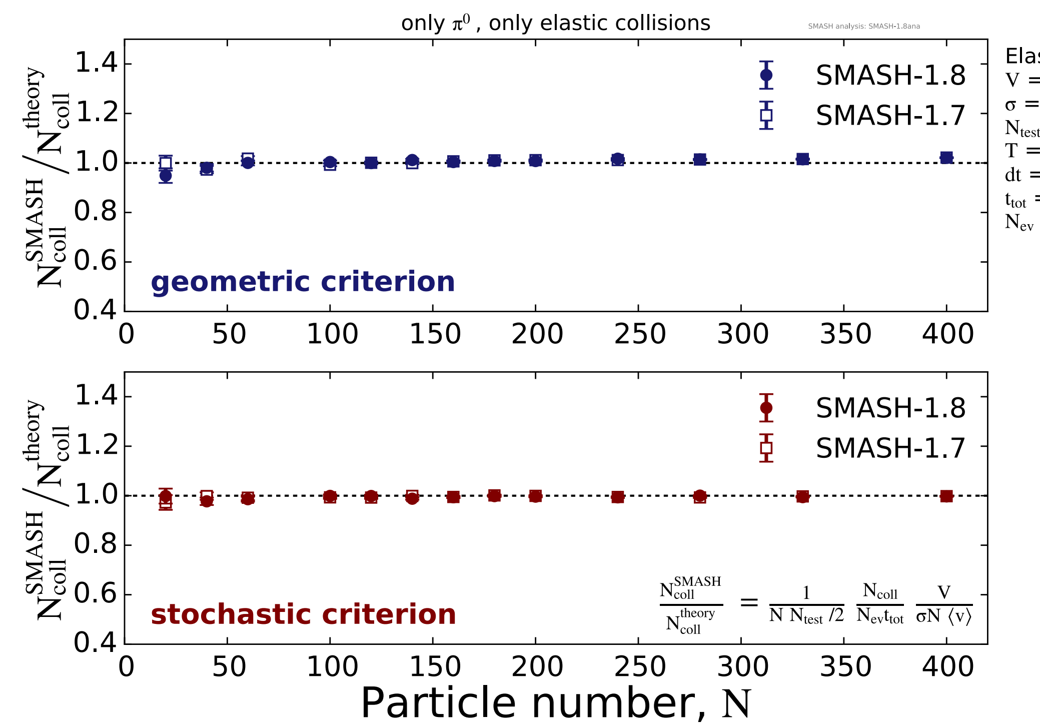
- Remedied by S-matrix approach
- Alternative explanation: Baryon annihilations
- Light nuclei are reproduced despite small binding energies

Multi-Particle Interactions

- At high densities multiparticle interactions will become relevant

PhD thesis, J. Staudenmaier 2021





  - $\omega \leftrightarrow 3\pi,\ B\bar{B} \leftrightarrow 5\pi,\ M \leftrightarrow N$

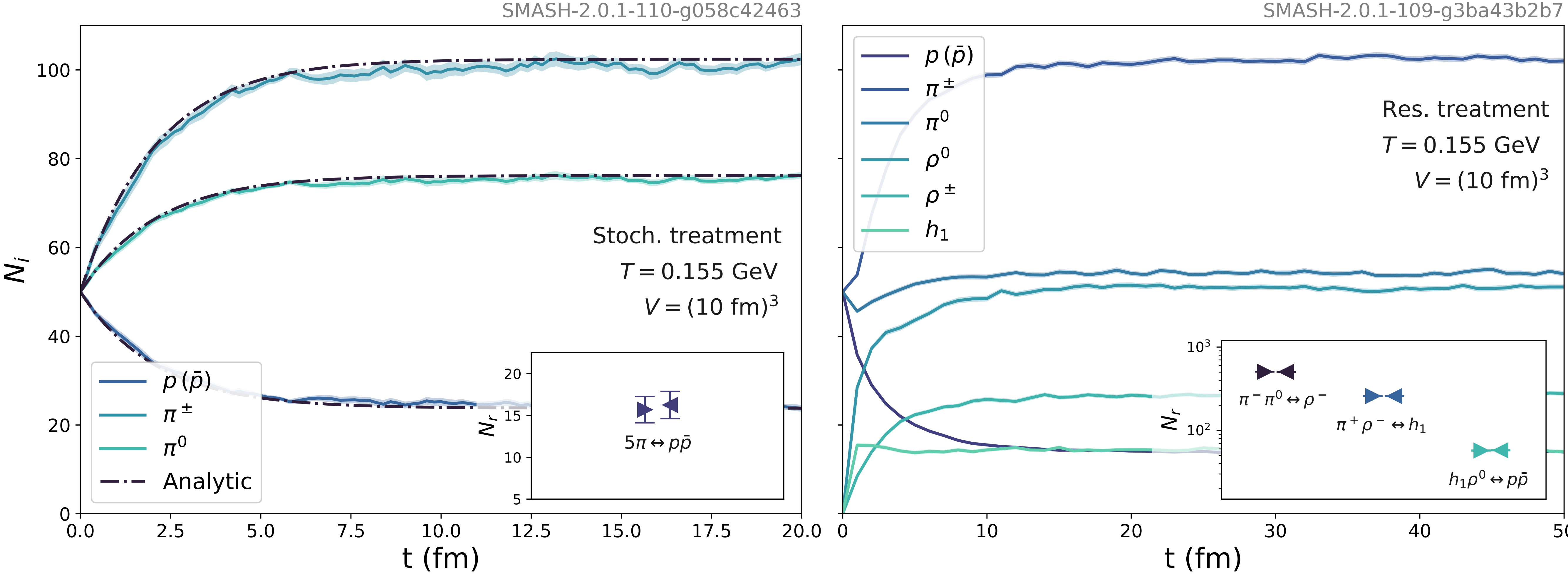
- 2<->2, 2<->1, 3<->1, 2<->3 and 5<->2 are implemented
- Application to deuteron production and $p\bar{p}$ annihilation in the following

Proton Annihilation at LHC

- First implementation of direct back reaction of 5 pions to proton-antiproton

O. Garcia-Montero et al., arXiv:2107.08812

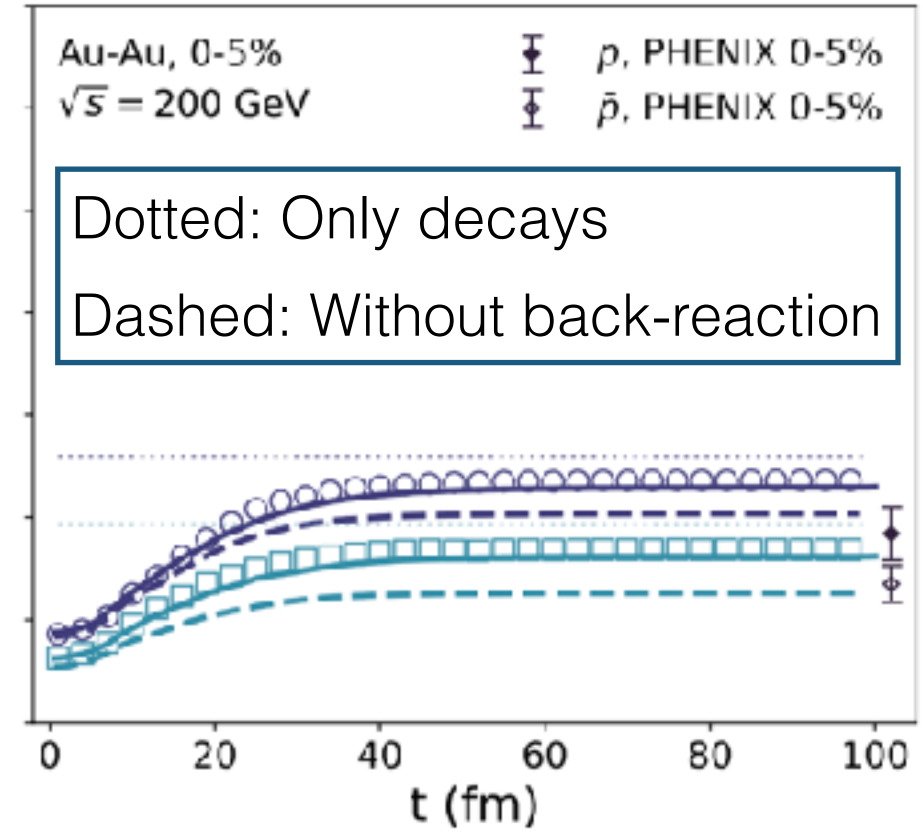
- Alternative treatment via intermediate resonances

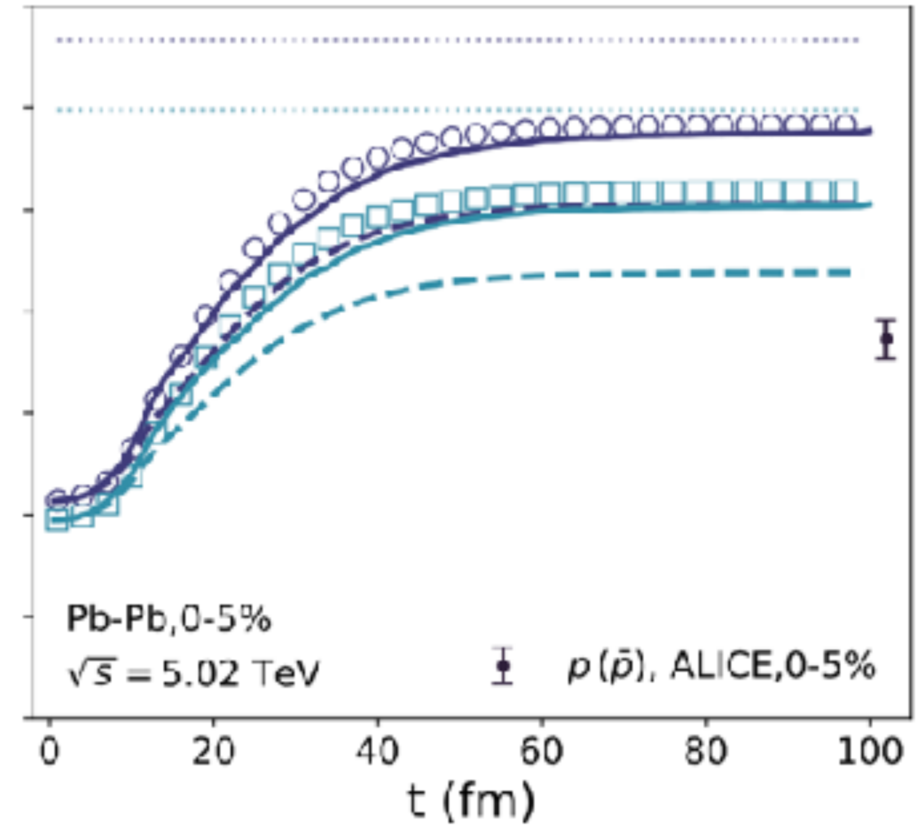


- Stochastic rates agree with the rate equation results
- Detailed balance is fulfilled in infinite hadronic matter

Time Evolution

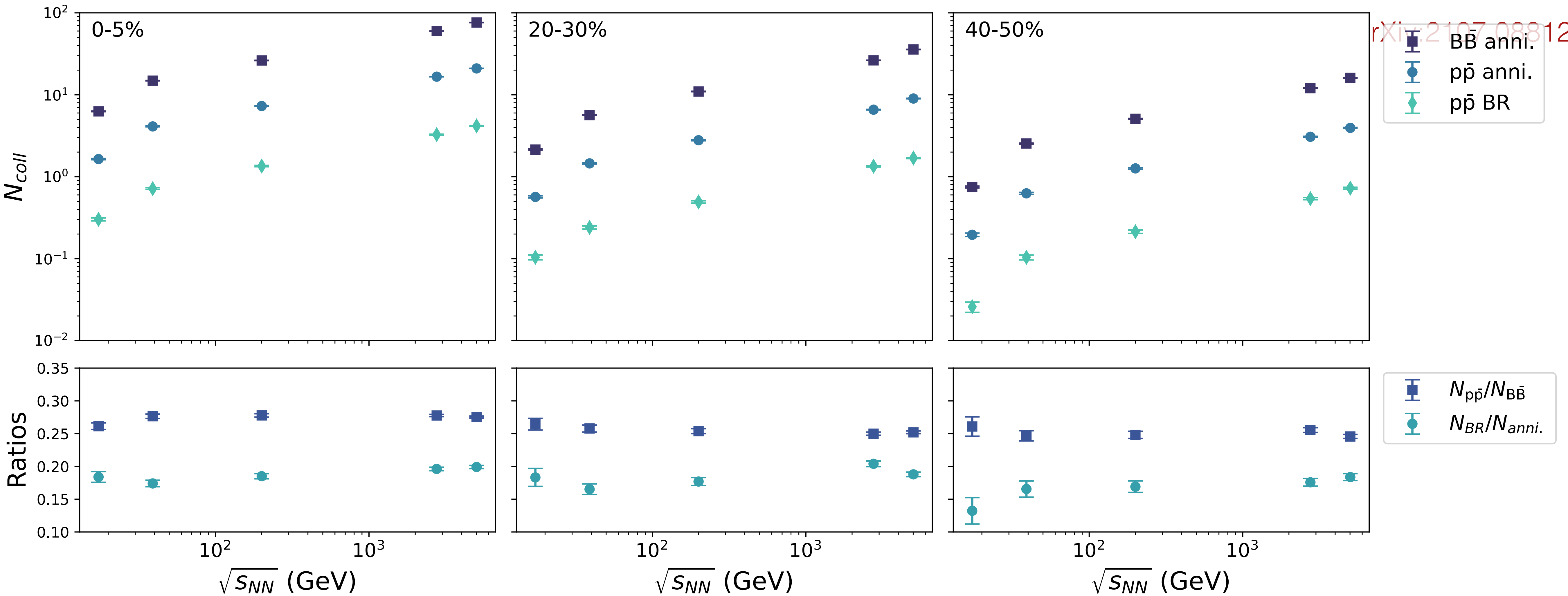
- Results at lower beam energies are not affected, since there are only few anti-baryons in the system
- Results from resonance approach (open symbols) and stochastic rates (lines) agree surprisingly well
- Backreaction restores about 50% of the annihilated proton yield
- Hadronic rescattering affects proton yields





O. Garcia-Montero et al., arXiv:2107.08812

Fraction of Backreaction

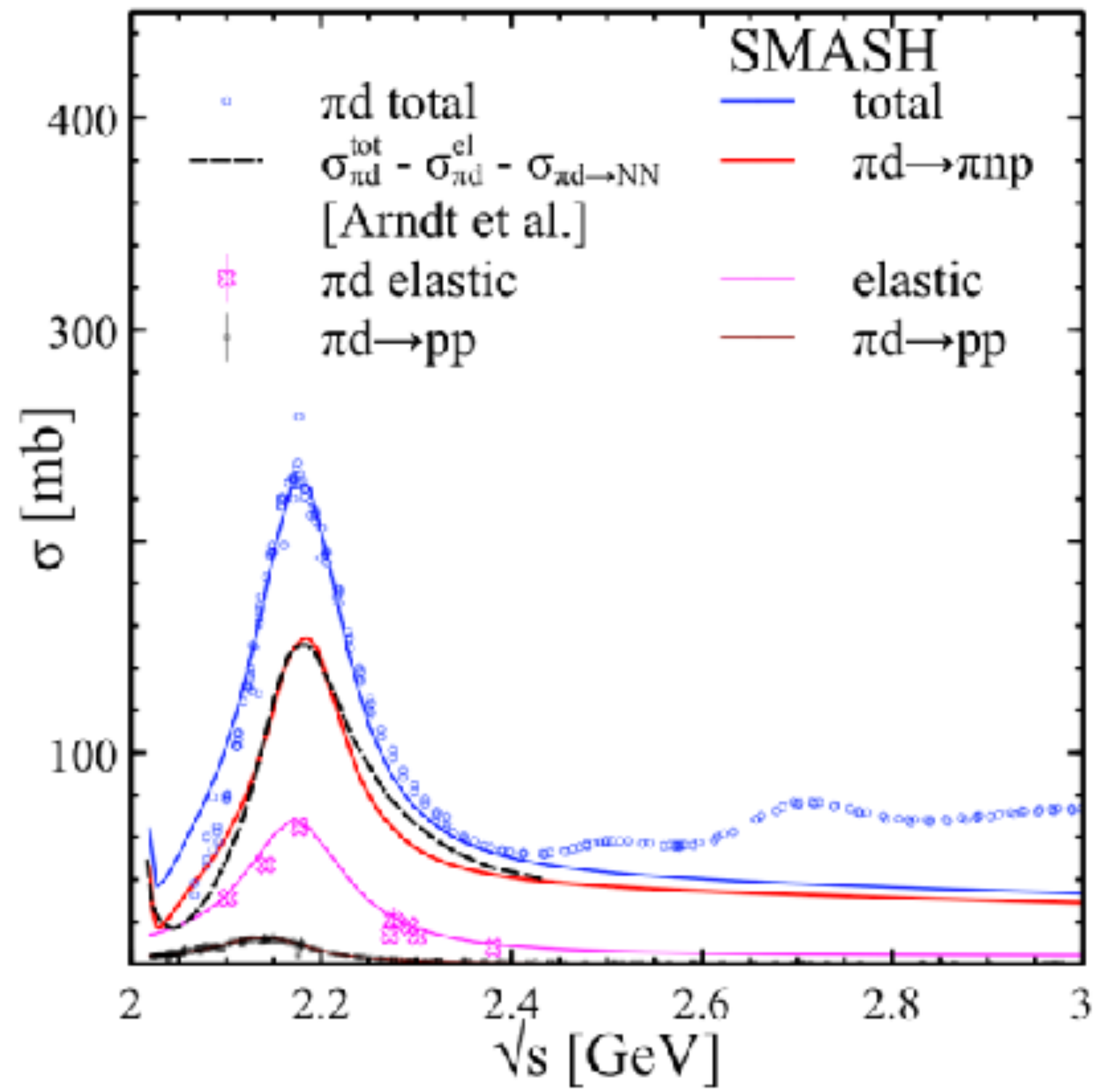


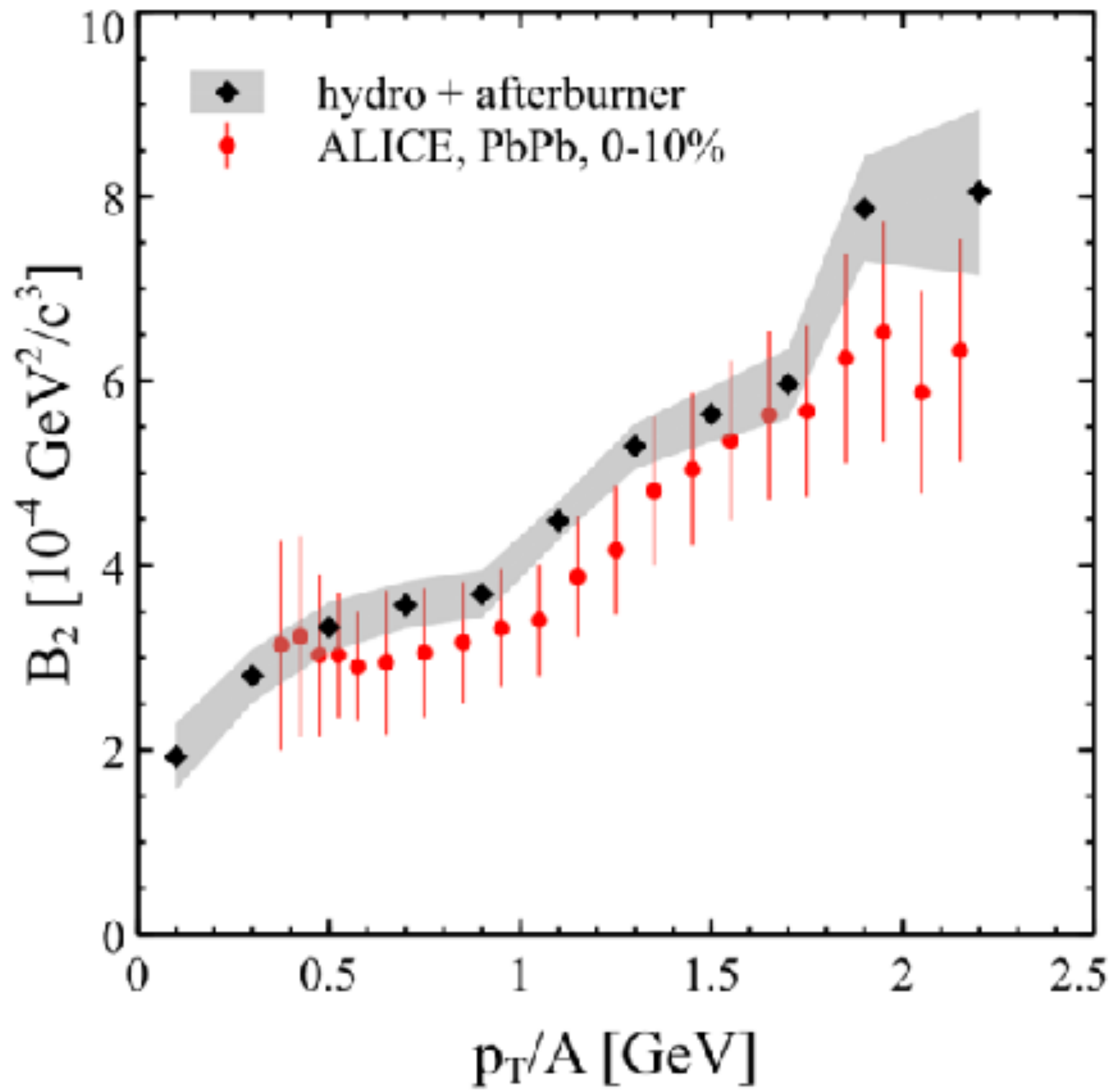
- Independent of beam energy and centrality the backreaction happens in ~20% of the cases 5π-> $p\bar{p}$
- $p\bar{p}$ annihilations can serve as a proxy for $B\bar{B}$ annihilations in general

Deuteron Production

- Deuterons and their cross-sections are implemented in SMASH
- During rescattering at LHC interactions with pions dominate

D. Oliinychenko, LongGang Pang, HE and V. Koch, PRC99 (2019)



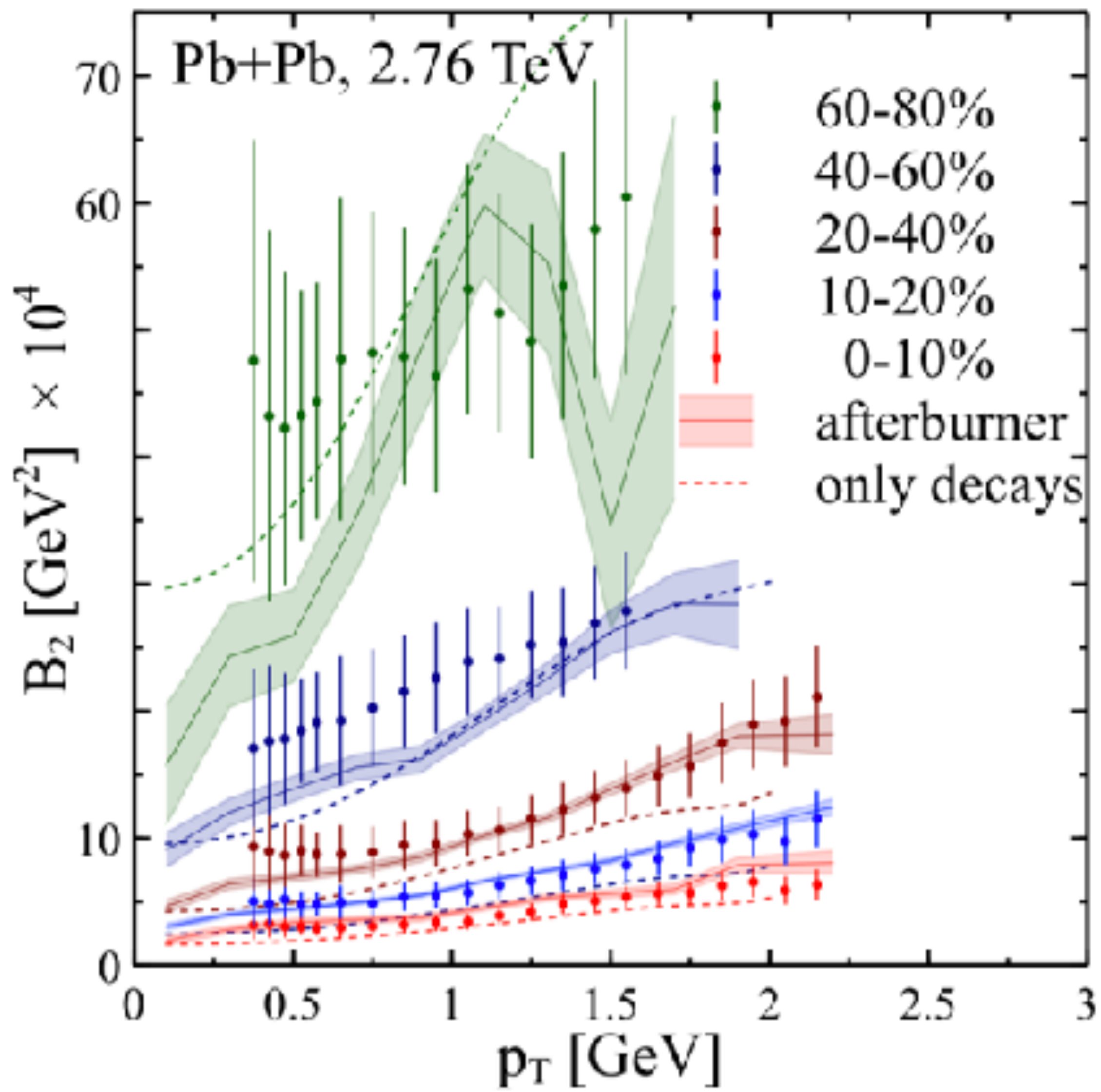


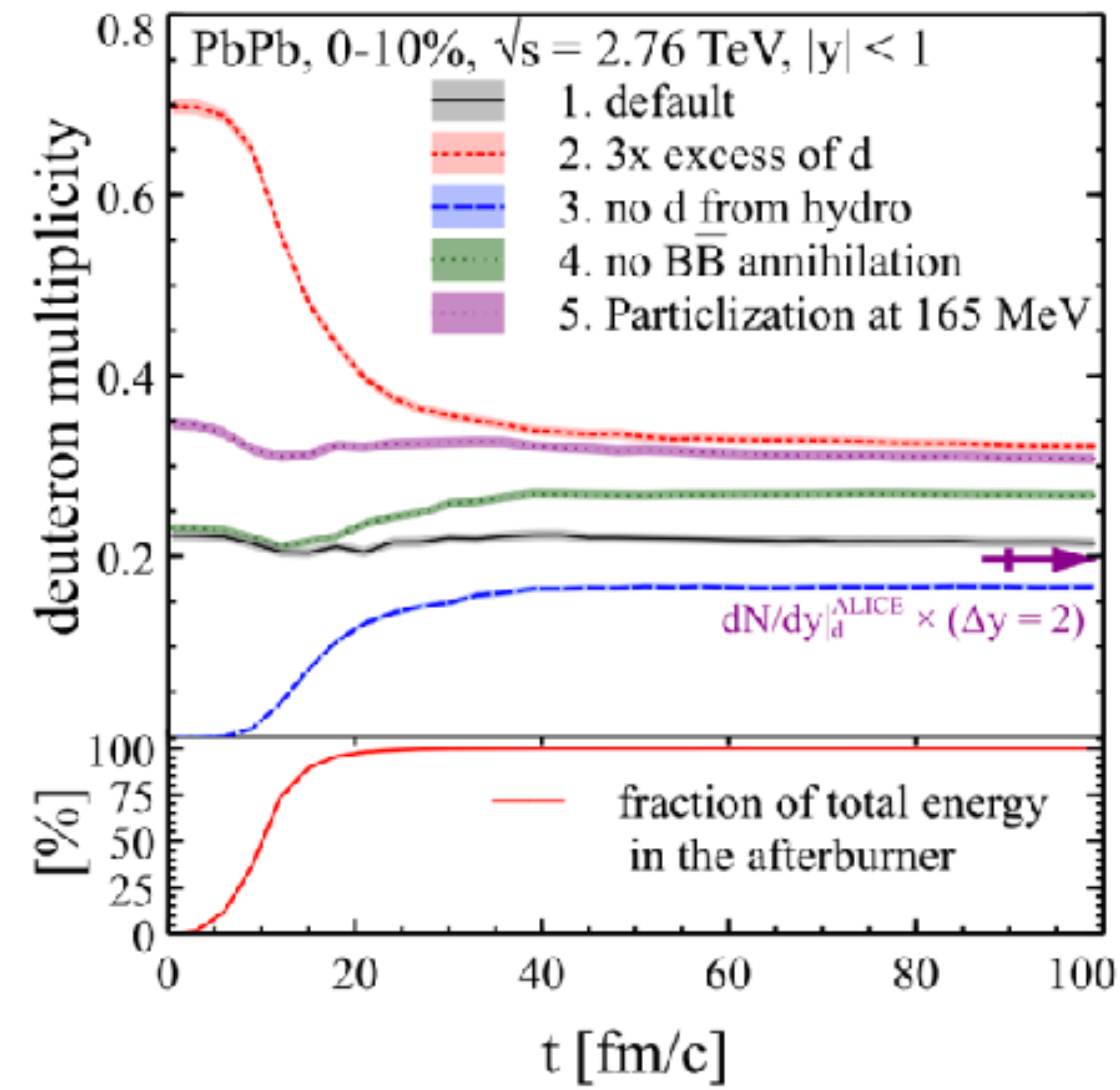
- Full microscopic non-equilibrium evolution in contrast to statistical production or coalescence approaches

Centrality Dependence

- Centrality dependence agrees well with ALICE data

D. Oliinychenko, L.-G. Pang, HE, V. Koch, MDPI Proc. 10, 2019 and PRC 99, 2019

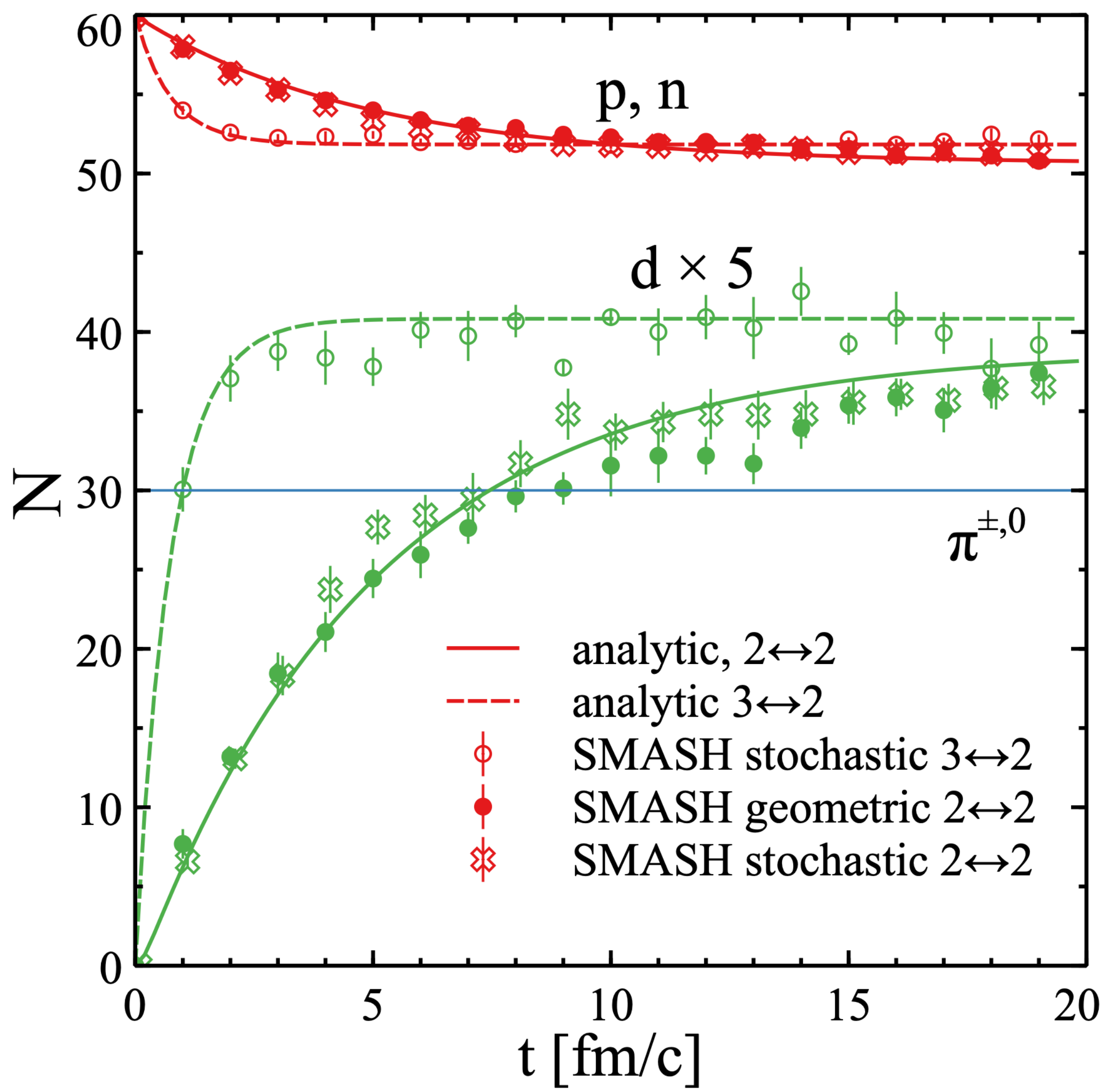




- The amount of deuterons on the hyper surface does not matter for the final yields

Deuterons at Low Beam Energies

- Direct 3<->2 reaction lead to even faster equilibration



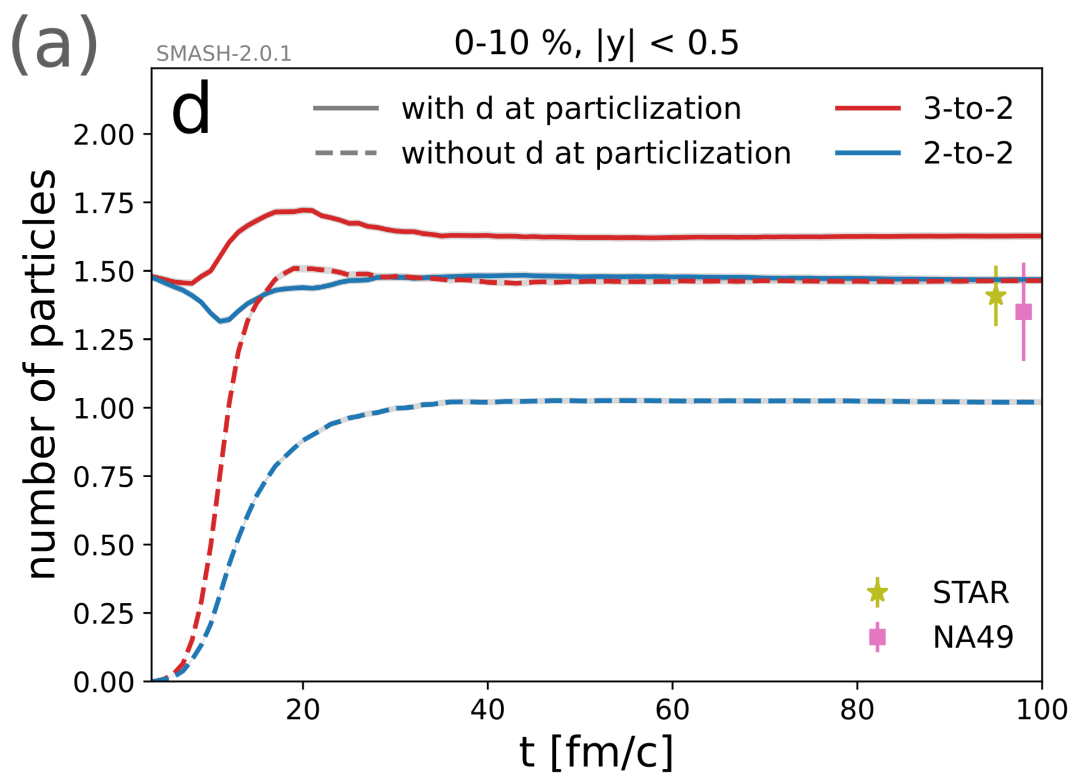
J. Staudenmaier et al, PRC104, 2021

Implemented reactions

$\pi d \leftrightarrow \pi np$ $\quad Nd \leftrightarrow Nnp$

$\pi d \leftrightarrow NN$ $\quad \bar{N}d \leftrightarrow \bar{N}np$

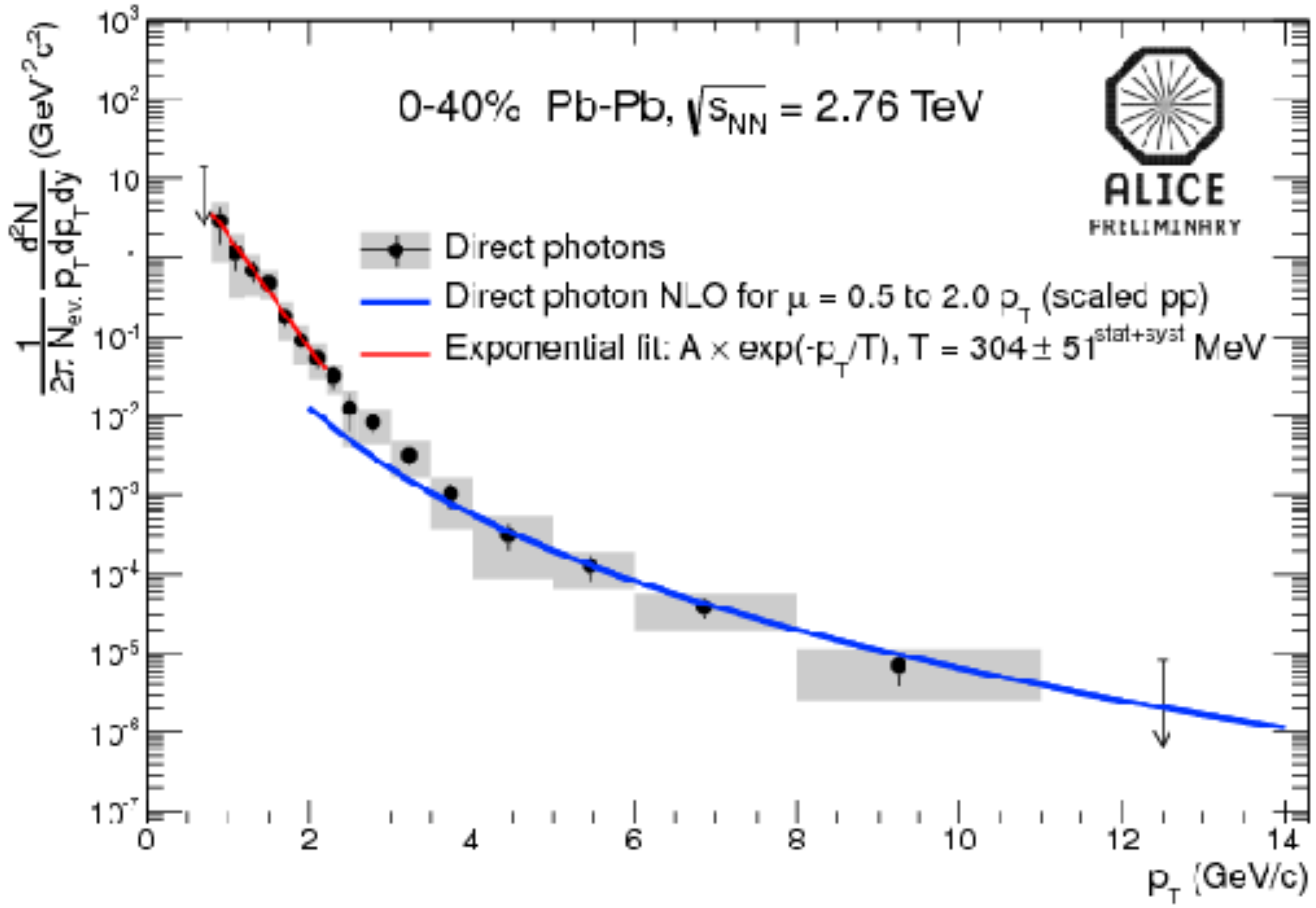
+elastic channels



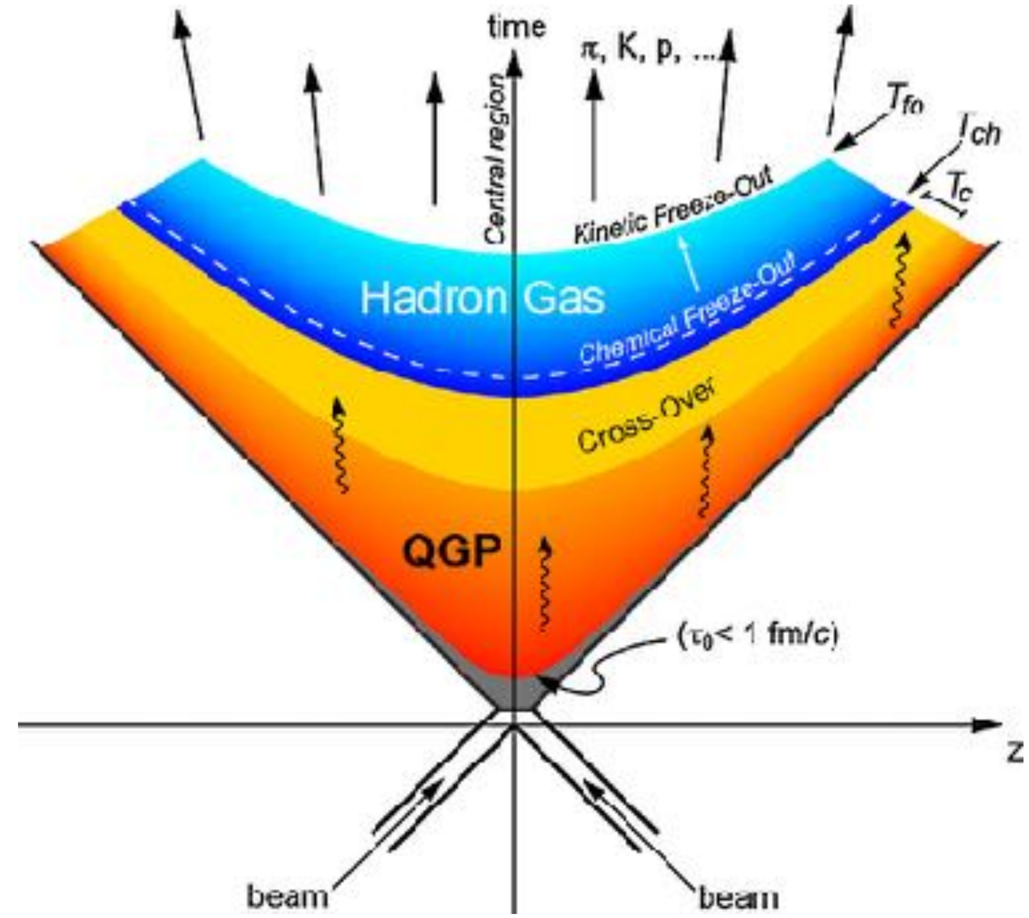
- Perfect match to rate equation results in a box

Hottest Matter in World

- Temperature from photon spectra



arXiv:1401.2481



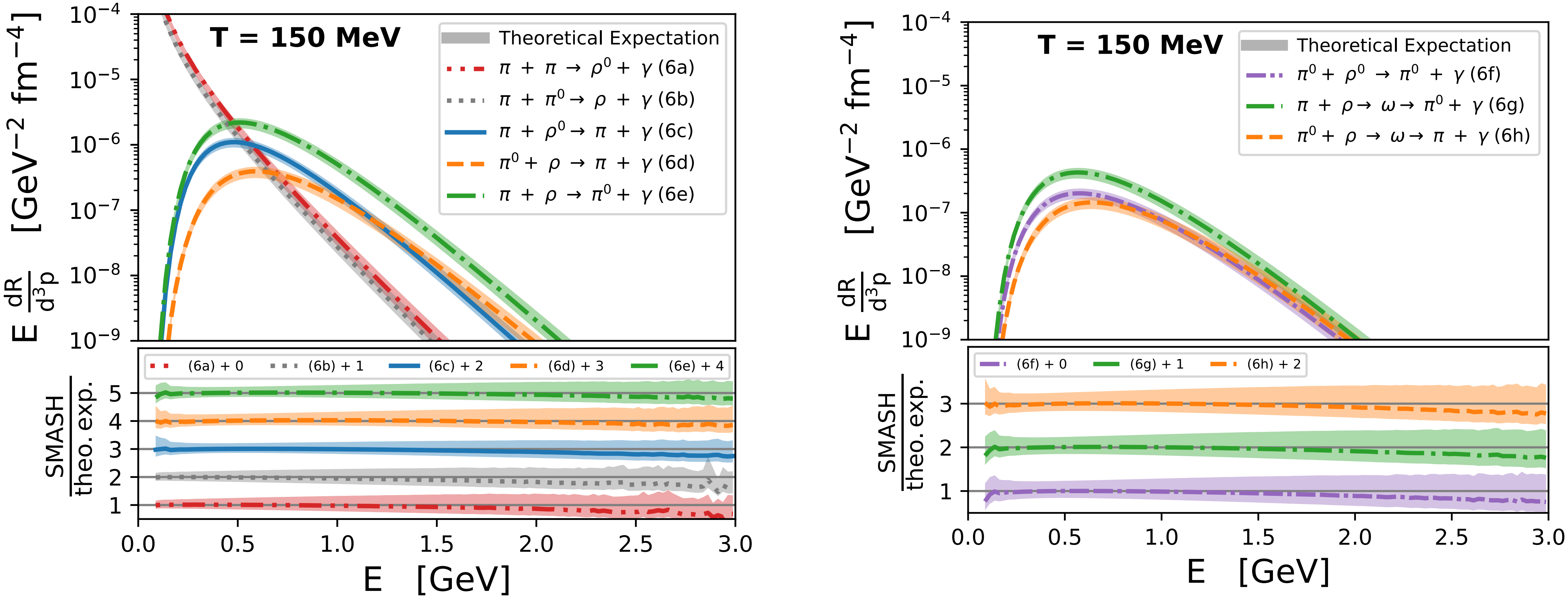
- This is worth an entry in the Guinness book of records
- ~3000 times as hot as the inner core of the sun
- Photons are emitted throughout all stages and do not interact strongly

Photons

- Perturbative photon production in hadronic scatterings of pions and ρ mesons

Turbide et al.: Int.J.Mod.Phys. A19 (2004)

- Cross-sections calculated within effective field theory

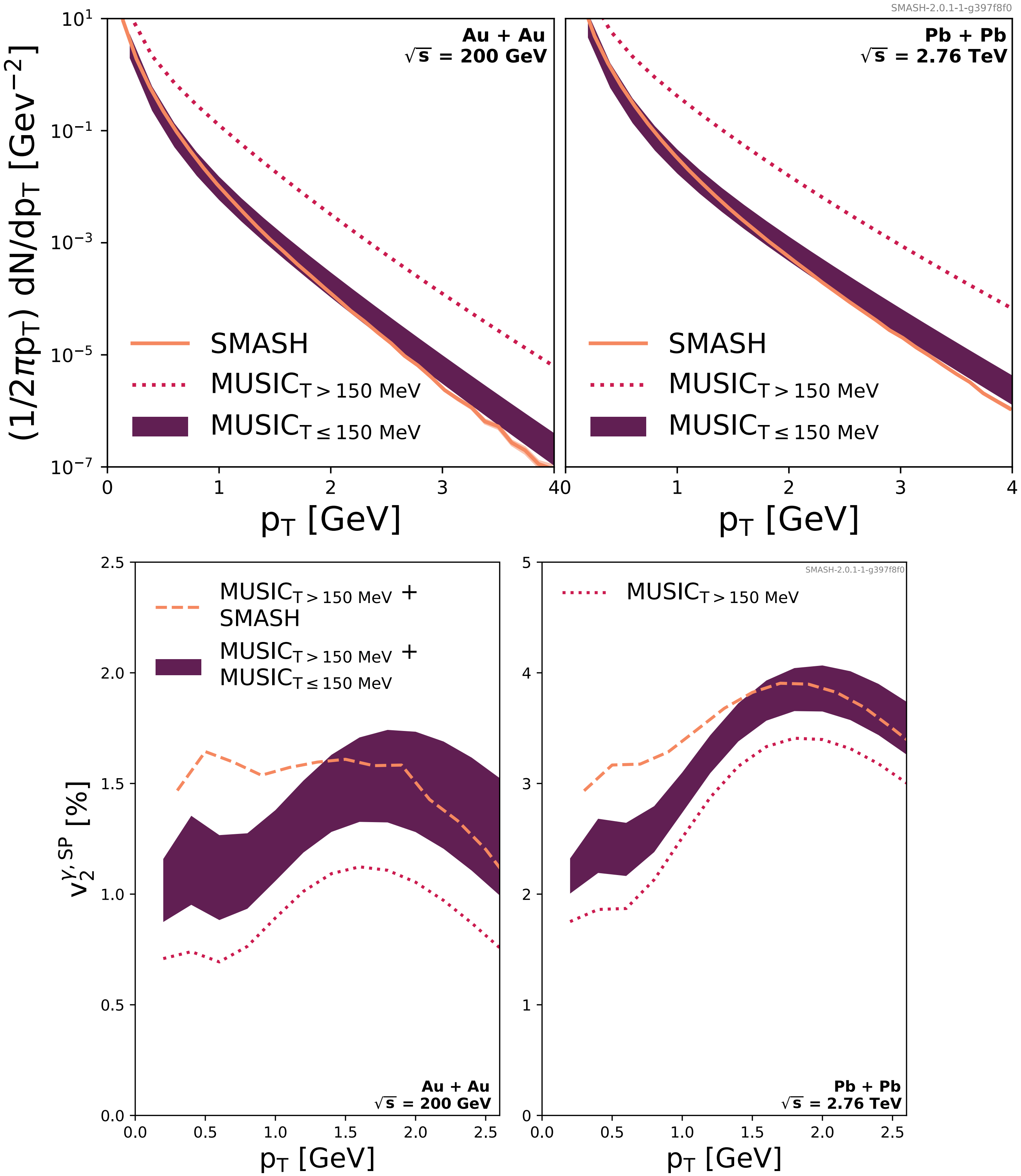


- Rates in thermal box nicely reproduced

A. Schäfer et al, PRD 99 (2019)

- Bremsstrahlung from mesonic processes has been added

Photon Emission Hadronic Stage



- Photon production from MUSIC+SMASH at RHIC and LHC energies
- Direct photons from hadronic scatterings and bremsstrahlung are consistently implemented
- Total elliptic flow is affected by photons emitted from the hadronic rescattering

A. Schäfer et al., PRC 105 (2022)

Summary

- Hybrid approaches based on relativistic hydrodynamics and hadron transport provide realistic dynamical description
- Over the last ~20 years hybrid approaches have become more and more sophisticated
- State-of-the-art is 3+1 D viscous hydrodynamics with hadronic transport for final rescattering and varying initial conditions
- Interfaces are crucial to understand sensitivities
  - Initial non-equilibrium evolution needs further investigations
  - Final state matching on Cooper-Frye hypersurface
- SMASH-vHLLE hybrid approach is publicly available
- Hadronic rescattering matters at RHIC and LHC for particle spectra, mass ordering of elliptic flow, proton yields, light nuclei production and direct photon flow
- Many Bayesian analysis are applied based on hybrid approaches

Questions

- What are the boundary conditions for hydrodynamics?
- What are the advantages and disadvantages of hydrodynamics and transport?
- What are negative contributions on the Cooper-Frye hypersurface and how can one deal with them?
- How is hydrodynamization achieved so fast?
- Which conservation laws should be fulfilled in the dynamical time evolution? Are they?
- What are the main effects of hadronic rescattering in high energy heavy-ion collisions?
- What are the sources of fluctuations in the initial state?
- What are the challenges for hybrid approaches at lower beam energies?