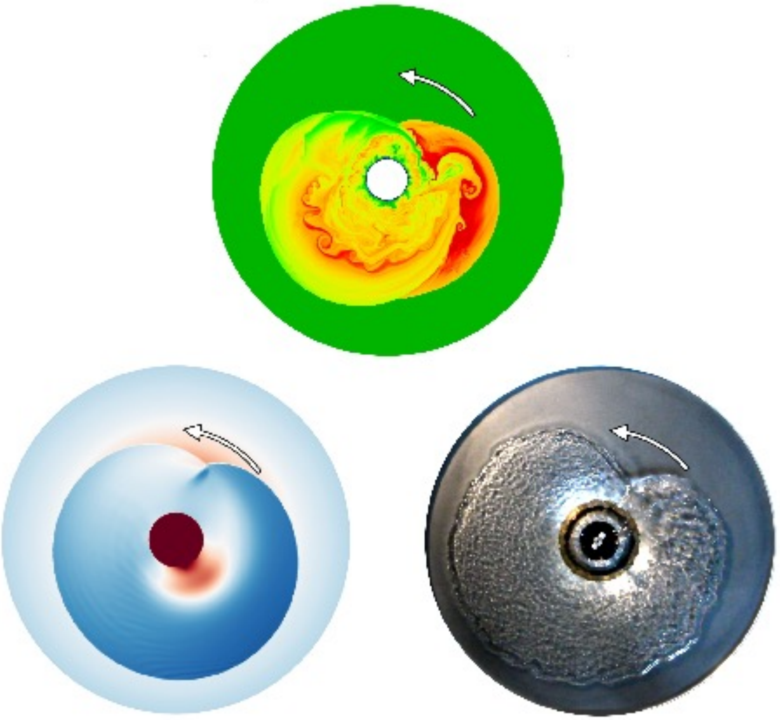

Physics of spiral instabilities during the collapse of a rotating stellar core

Thierry Foglizzo
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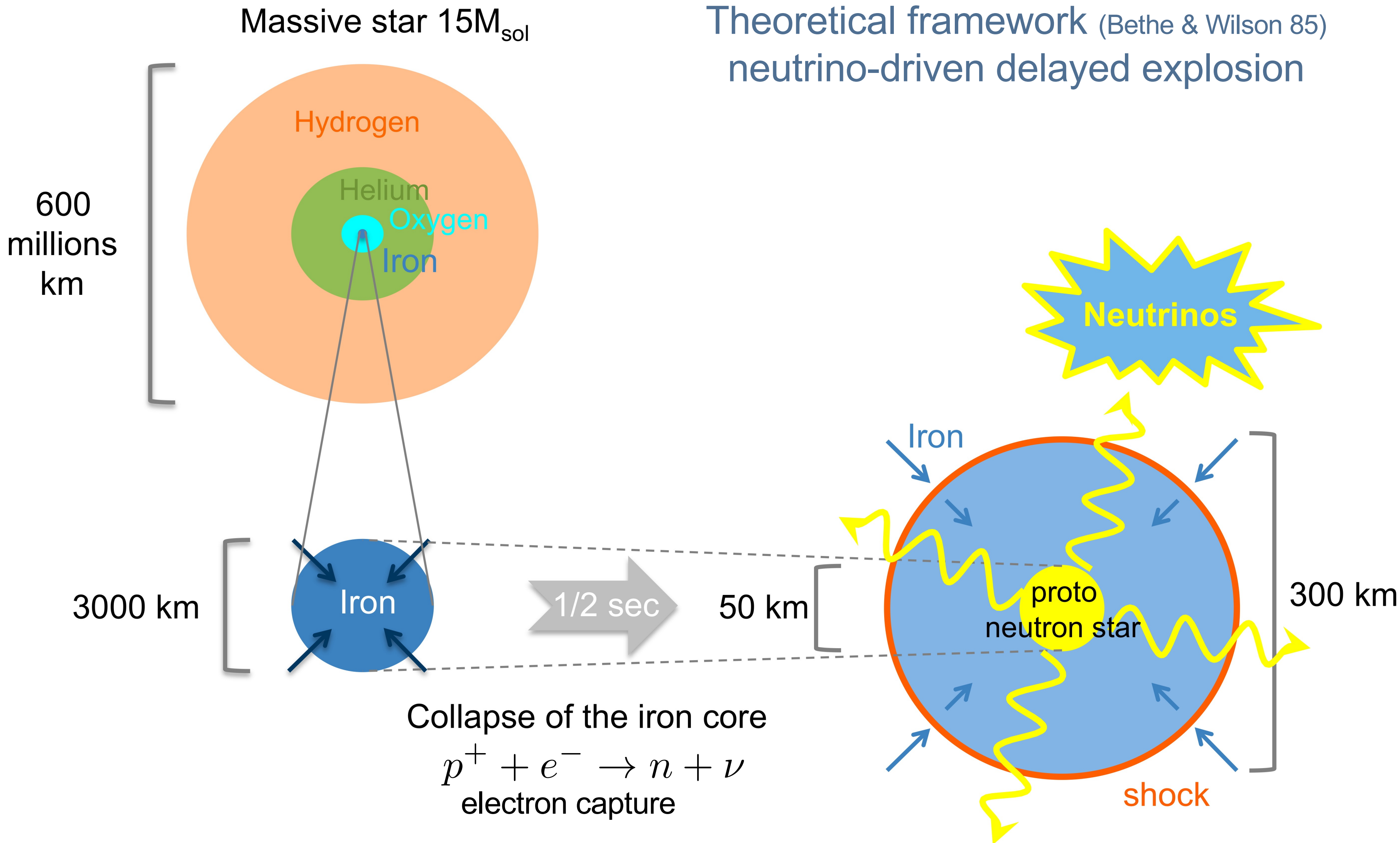
Outline

SASI vs low T/W: experimental & analytical insight
adiabatic approximation of SASI+rotation
physical model: forced oscillator
revised description of SASI with rotation

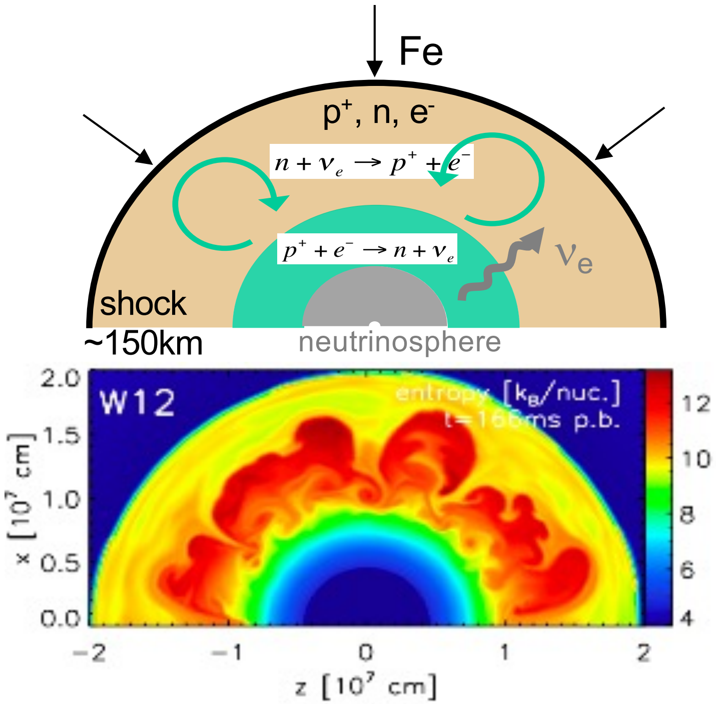
Theoretical framework (Bethe & Wilson 85)
neutrino-driven delayed explosion

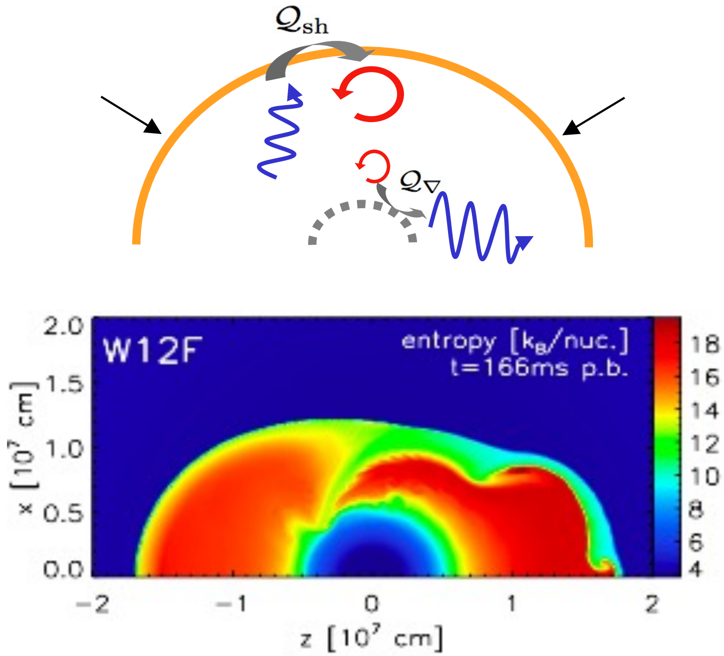


$$\frac{GM_{\rm ns}^2}{R_{\rm ns}} \sim 2\times10^{53}{\rm erg}\left(\frac{30{\rm km}}{R_{\rm ns}}\right)\left(\frac{M_{\rm ns}}{1.5M_{\rm sol}}\right)^2$$

modest energy in differential rotation: $E_{\rm diff} < E_{\rm rot} \sim 2.4\times10^{50}{\rm erg}\left(\frac{M_{\rm ns}}{1.5M_{\rm sol}}\right)\left(\frac{R_{\rm ns}}{10{\rm km}}\right)^2\left(\frac{10{\rm ms}}{P_{\rm ns}}\right)^2$

Instabilities during the phase of stalled accretion shock





Neutrino-driven convection (Herant+92, ...)

- entropy gradient, fed by neutrino absorption
- inhibited if the advection time is too short (Foglizzo+06)

$$\chi \equiv \int_{\rm sh}^{\rm gain} \omega_{\rm BV} \frac{{\rm d}r}{v_r} < 3$$

SASI: Standing Accretion Shock Instability

(Blondin+03 ...)

- advective-acoustic cycle
- oscillatory, large angular scale l=1,2:
  pulsar kick, nucleosynthesis,
  gravitational waves & neutrino signatures

Uncertain instabilities induced by moderate rotation

-low T/|W| instability?

(Shibata+02, Watts+05, Passamonti & Andersson 15, Takiwaki+21)

- corotation radius
- vorticity gradient? mid lattitude Rossby waves?

-Papaloizou-Pringle instability?

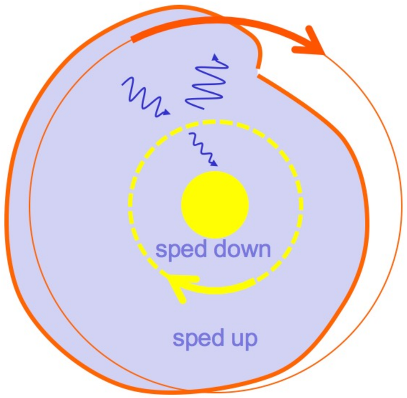
(Papaloizou & Pringle 84, Goldreich & Narayan 85)

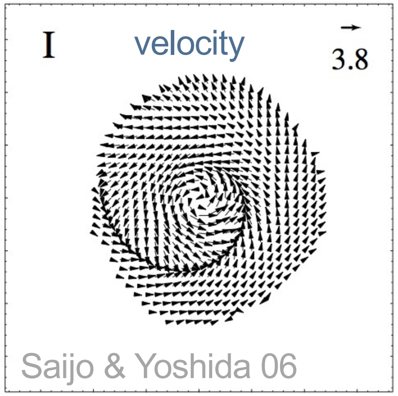
- corotation radius
- reflecting boundaries

-spiral mode of SASI?

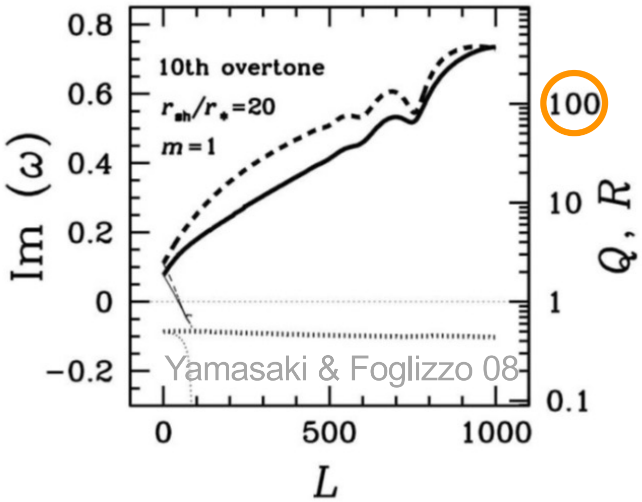
(Blondin & Mezzacappa 07, Yamasaki & Foglizzo 08)

- stationary shock
- rotation-enhanced advective-acoustic cycle?





?



stellar parameters: progenitor mass, compactness, angular momentum, inhomogeneities

\+

puzzling dynamics: SASI, ν-driven convection, low T/|W|, PNS dynamo

\+

uncertain physics: reaction rates, EOS, neutrino interactions, magnetic fields

\+

numerical approximations: neutrino transport, 2D vs 3D, turbulence

Can gravitational waves and neutrino signatures disentangle so many processes ?

Additional instabilities induced by moderate rotation

-low T/|W| instability?

(Shibata+02, Watts+05, Passamonti & Andersson 15, Takiwaki+21)

- corotation radius
- vorticity gradient?

-Papaloizou-Pringle instability?

(Papaloizou & Pringle 84, Goldreich & Narayan 85)

- corotation radius
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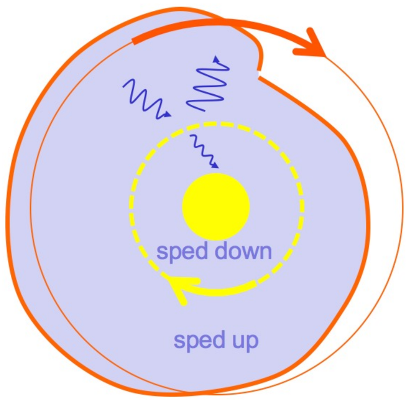
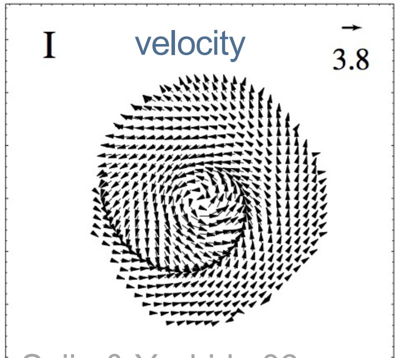
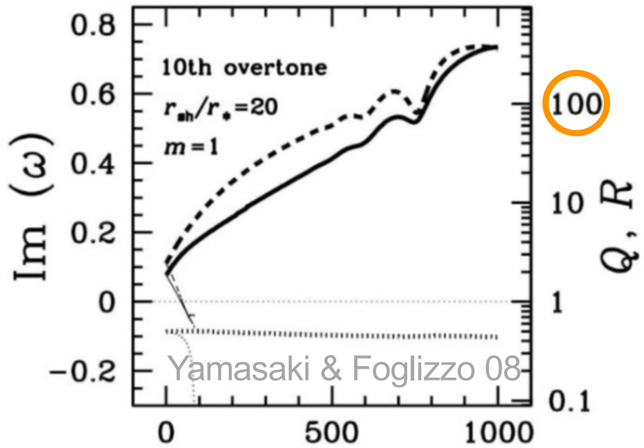




Saijo & Yoshida 06

?



Yamasaki & Foglizzo 08

L = r Ω

stellar parameters: progenitor mass, compactness, angular momentum, inhomogeneities

\+ **puzzling dynamics:** SASI, ν-driven convection, low T/|W|, PNS dynamo

\+ **uncertain physics:** reaction rates, EOS, neutrino interactions, magnetic fields

\+ **numerical approximations:** neutrino transport, 2D vs 3D, turbulence

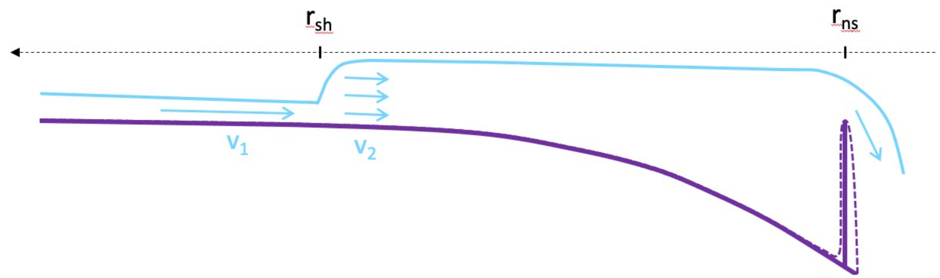
Can gravitational waves and neutrino signatures disentangle these processes ?

Work in progress at Saclay

-analytical insight on SASI with rotation

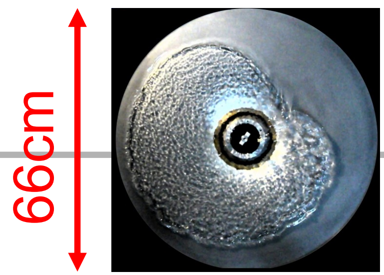
-why is rotation so efficient at destabilizing the prograde mode of SASI ?
-what is the mechanism of spiral SASI with a corotation radius?

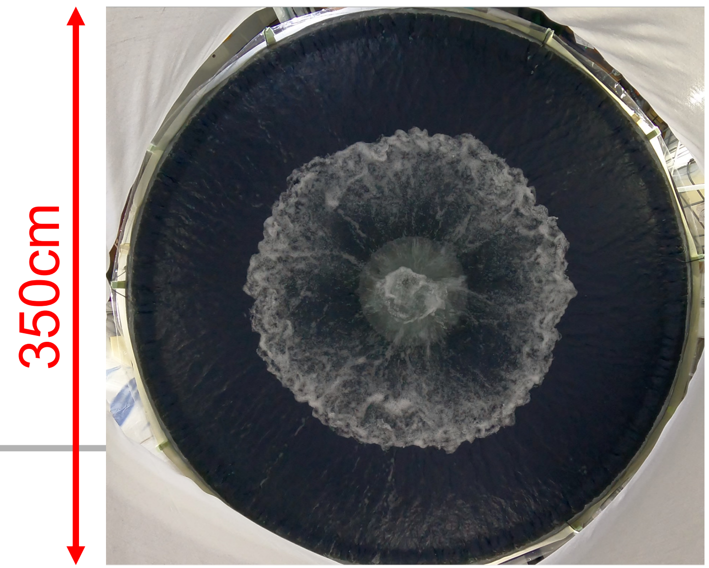
METHOD
spherical/cylindrical comparison
+
adiabatic model of gas accretion
inspired by experimental and
perturbative results in shallow water



-impact of turbulence on SASI dynamics

- without rotation, why turbulent SASI @ 100L/s
seems less unstable than viscous SASI @1L/s?



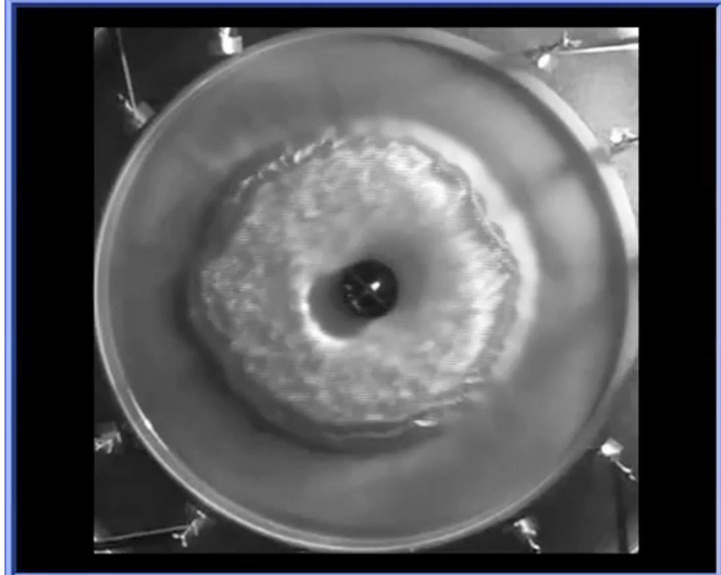


Dynamics of water in the fountain

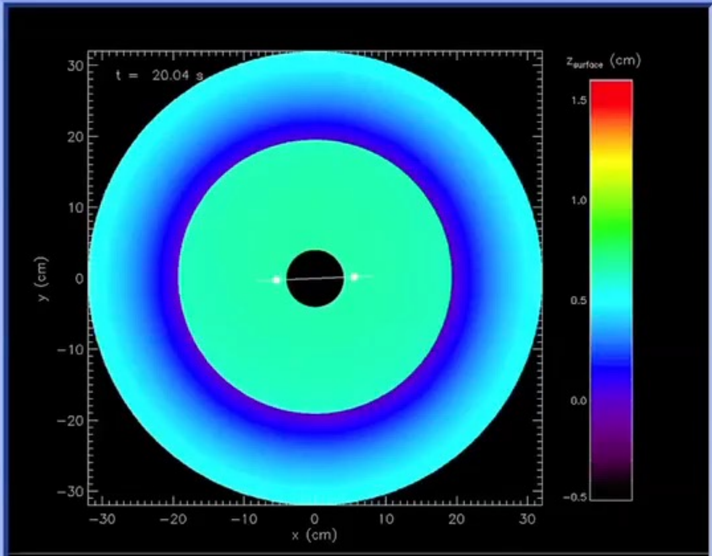
Dynamics of the gas in the supernova core

diameter 40cm ← 1 000 000 x bigger → diameter 400km

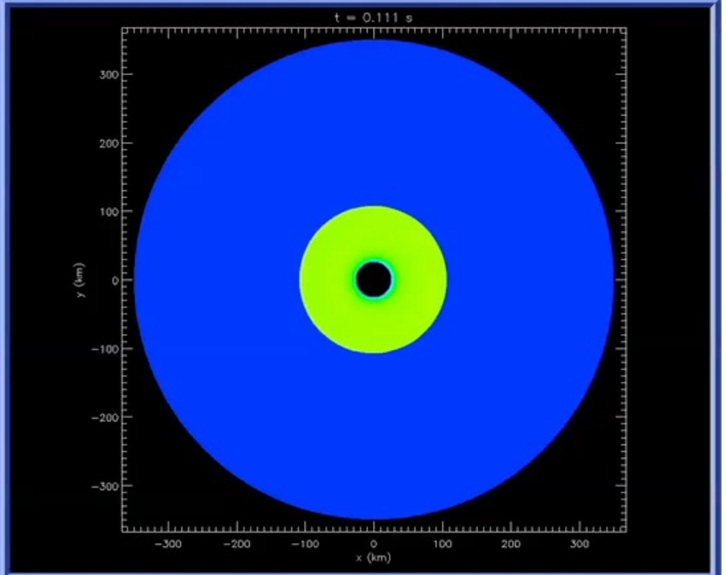
3s/oscillation ← 100 x faster → 0.03s/oscillation



Expérience hydraulique



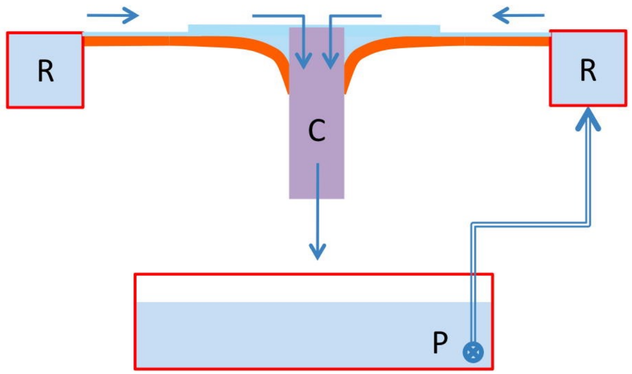
Simulation numérique de l'expérience hydraulique

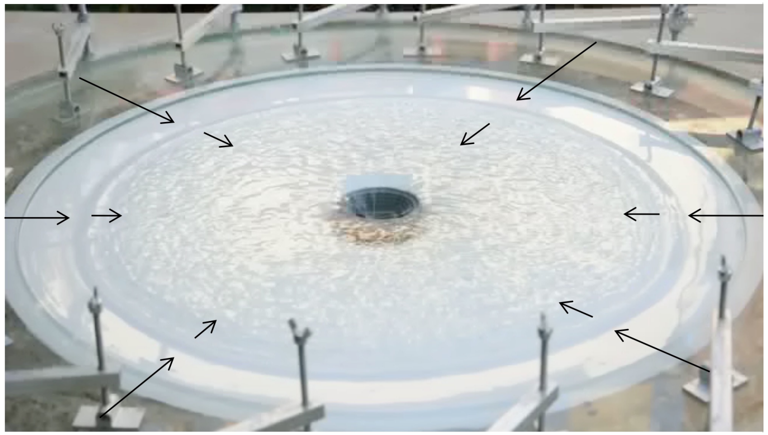


Simulation numérique de l'onde de choc dans le coeur de la supernova

SASI dynamics seems to be adiabatic

Shallow water analogy





adiabatic gas

$$c_s^2 \equiv \frac{\gamma P}{\rho}$$

$$\Phi \equiv -\frac{GM_{ns}}{r}$$

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho v) = 0$$

$$\frac{\partial v}{\partial t} + (\nabla \times v) \times v + \nabla \left(\frac{v^2}{2} + \frac{c_s^2}{\gamma - 1} + \Phi \right) = \frac{c_s^2}{\gamma} \nabla S$$

Inviscid shallow water is analogue to an isentropic gas γ=2

St Venant

$$c_{sw}^2 \equiv gH$$

$$\Phi \equiv gH_\Phi$$

$$\frac{\partial H}{\partial t} + \nabla \cdot (Hv) = 0$$

$$\frac{\partial v}{\partial t} + (\nabla \times v) \times v + \nabla \left(\frac{v^2}{2} + c_{sw}^2 + \Phi \right) = 0$$

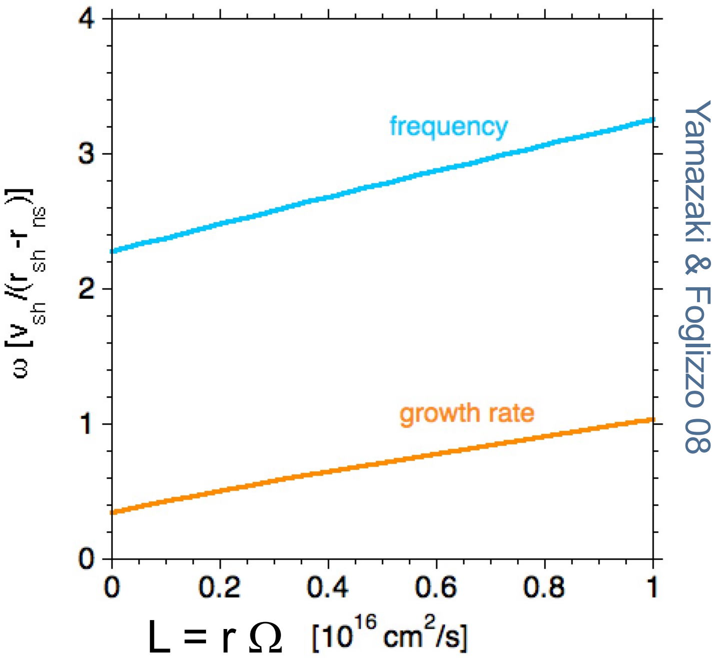
acoustic waves shock wave density ρ	⟷	surface waves hydraulic jump depth H

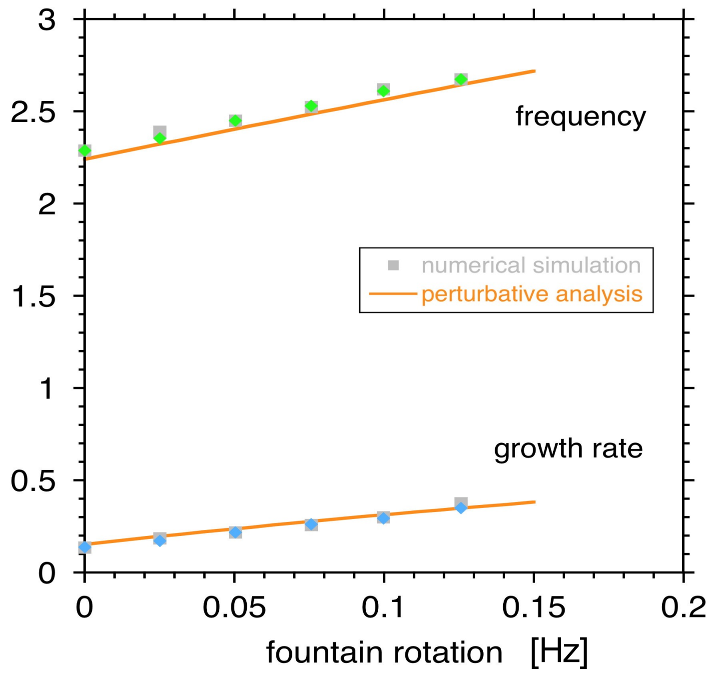
expected scaling $\quad \dfrac{t_{ff}^{sh}}{t_{ff}^{jp}} \equiv \left(\dfrac{r_{sh}}{r_{jp}} \right) \left(\dfrac{r_{sh} g H_\Phi^{jp}}{GM_{ns}} \right)^{\frac{1}{2}} \sim 10^{-2}$

shock radius $\times 10^{-6}$ 200 km $\to$ 20 cm

oscillation period $\times 10^2$ 30 ms $\to$ 3 s

Perturbative analysis: same destabilizing effect of rotation in adiabatic shallow water and in non-adiabatic gas dynamics





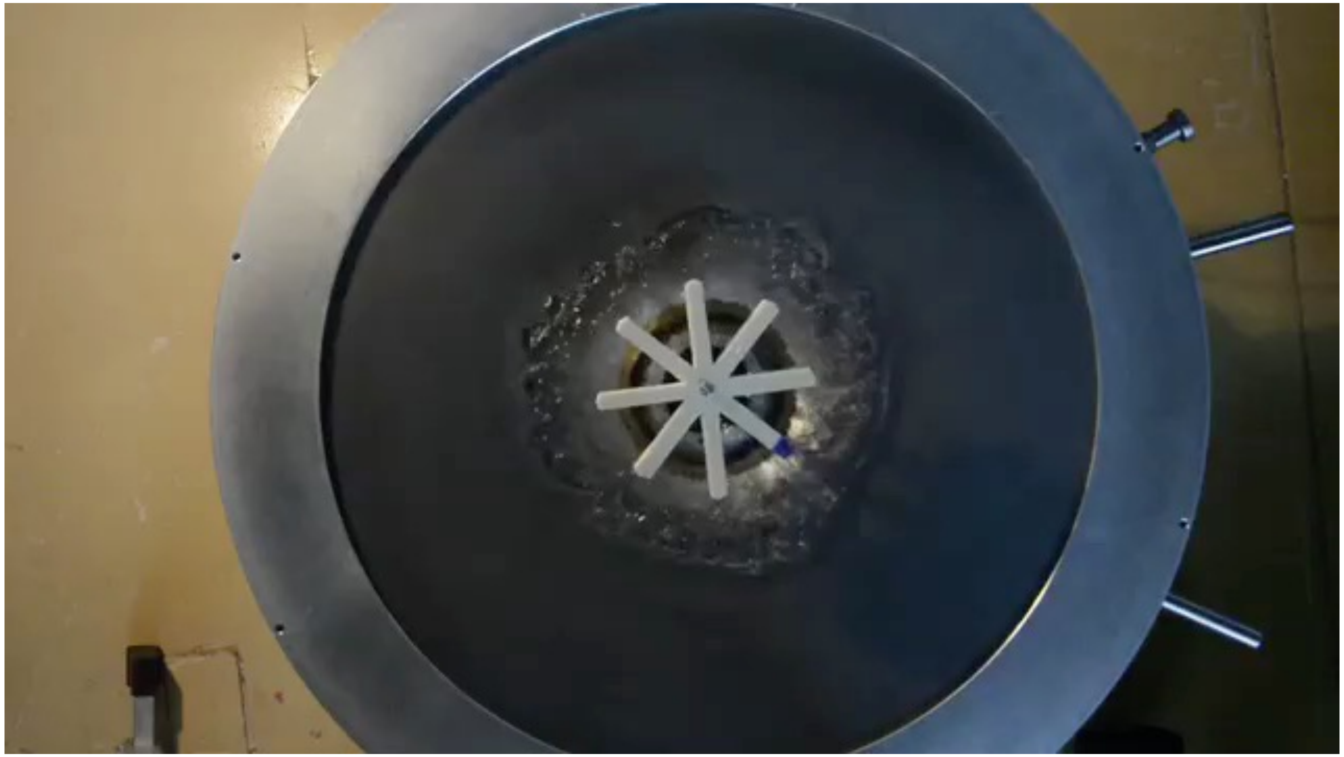
shocked gas dynamics

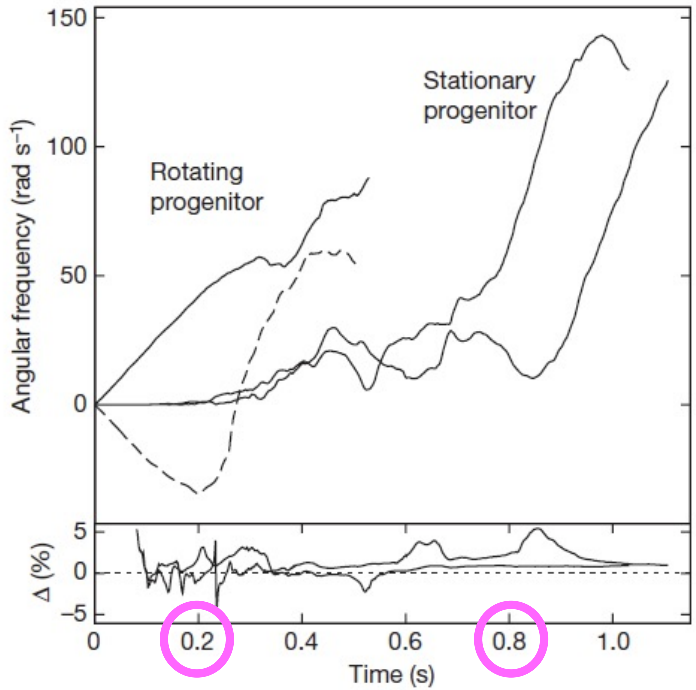
shallow water dynamics

$$\omega' \equiv \omega - \frac{mL}{r^2}$$

Why is the prograde mode of SASI linearly destabilized by rotation?

Impact of spiral SASI on the NS spin:
accreted angular momentum changes its sign as SASI grows
same as in shocked gas dynamics





Blondin & Mezzacappa 07
Kazeroni +16,17

fountain rotation period: 246s
injection slit: 0.55mm
flow rate: 1.17L/s

increased angular momentum in the post shock flow
+
decreased angular momentum in the neutron star
= 0

Comparing stationary shocked accretion in spherical and cylindrical geometries

Postshock subsonic accretion flow ~ quasi-hydrostatic

Same density, pressure, acoustic structures

Different radial velocity = different advection times

$$\left(\frac{r}{r_{\rm sh}}\right)^2 \frac{\rho}{\rho_{\rm sh}}\frac{v}{v_{\rm sh}} = 1 \text{ spherical,} \qquad \frac{v}{v_{\rm sh}} = \left(\frac{r}{r_{\rm sh}}\right) \text{ spherical,}$$

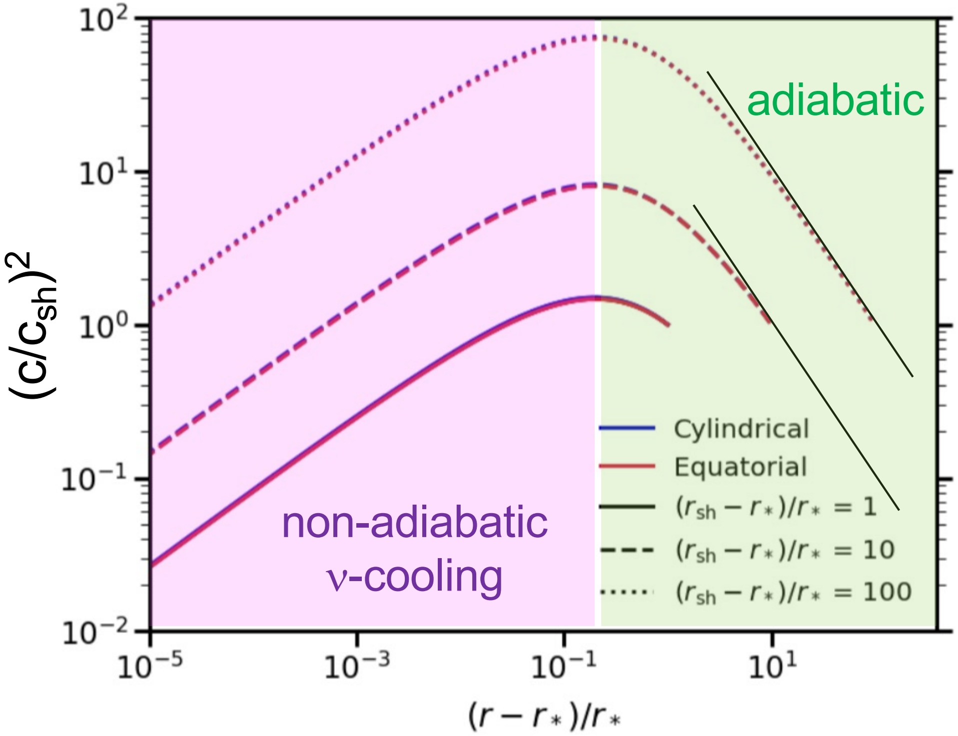
$$\left(\frac{r}{r_{\rm sh}}\right) \frac{\rho}{\rho_{\rm sh}}\frac{v}{v_{\rm sh}} = 1 \text{ cylindrical.} \qquad \frac{v}{v_{\rm sh}} = \left(\frac{r}{r_{\rm sh}}\right)^2 \text{ cylindrical.}$$

γ=4/3

$$\frac{\rho}{\rho_{\rm sh}} = \left(\frac{c}{c_{\rm sh}}\right)^{\frac{2}{\gamma-1}} \sim \left(\frac{r_{\rm sh}}{r}\right)^{\frac{1}{\gamma-1}}$$

adiabatic Bernoulli $c^2 \sim (\gamma-1)GM/r$

$$\tau_{\rm adv} \equiv \int_{\rm sh}^{r} \frac{{\rm d}r}{v},$$
$$\sim \frac{r_{\rm sh}}{|v_{\rm sh}|}\log\frac{r_{\rm sh}}{r} \text{ spherical,}$$
$$\sim \frac{r_{\rm sh}}{|v_{\rm sh}|}\left(\frac{r_{\rm sh}}{r}-1\right) \text{ cylindrical.}$$

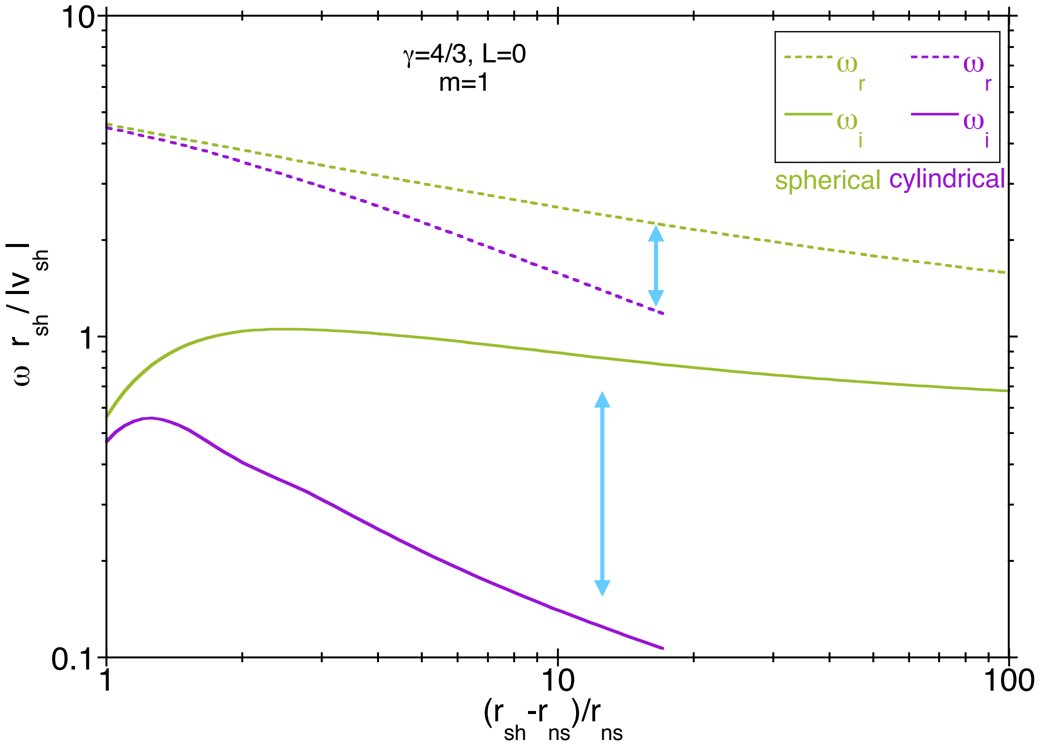


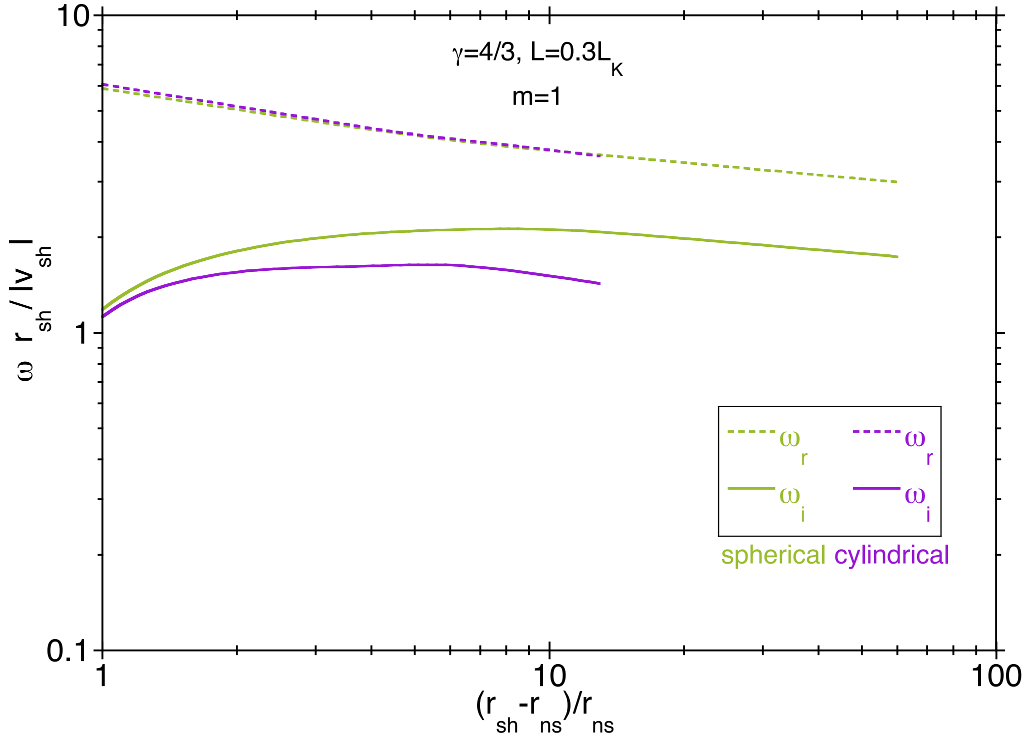
The layer of non-adiabatic cooling dominated by ν-emission is localized near the NS surface

Most of the flow is adiabatic if $r_{sh} >> r_{ns}$

Walk, Foglizzo, Tamborra, in prep

Growth rate and oscillation frequency of SASI for a large shock radius: cylindrical vs spherical geometry





without rotation,
the spherical and cylindrical eigenfrequencies
diverge as the shock distance increases

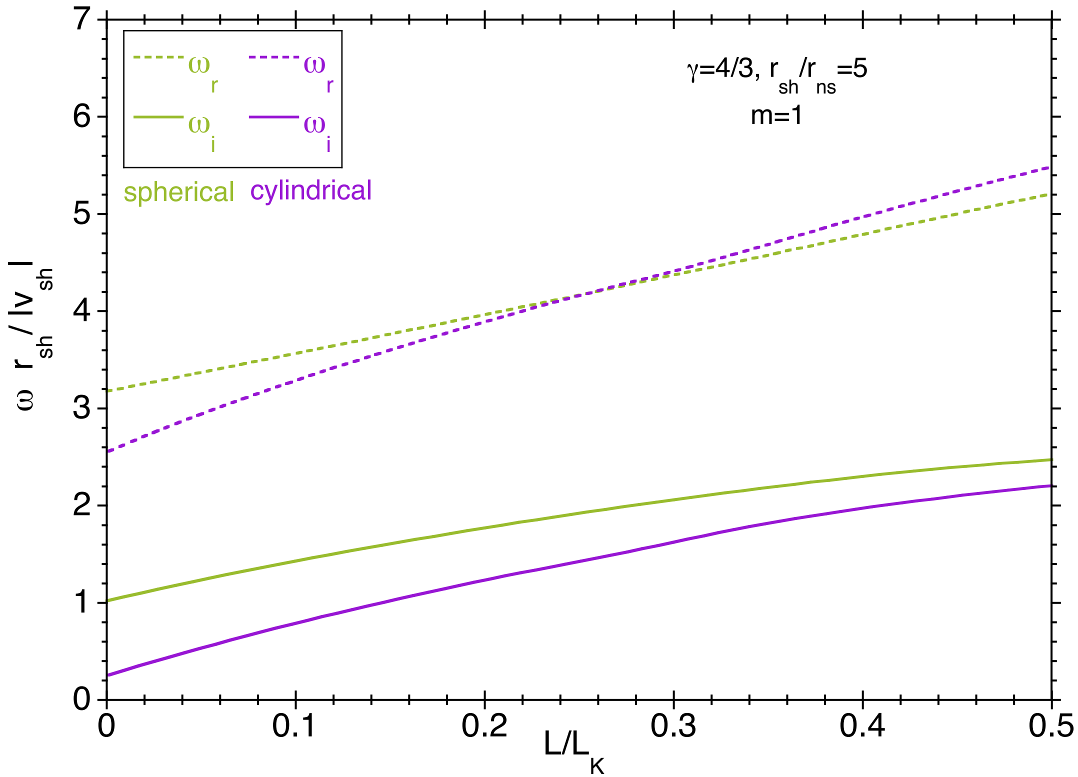
with 30% Keplerian rotation,
the spherical and cylindrical eigenfrequencies
remain similar

as expected from an advective-acoustic mechanism

unexpected for an advective-acoustic mechanism??

Walk, Foglizzo, Tamborra, in prep

Growth rate and oscillation frequency of SASI for a large shock radius: cylindrical vs spherical geometry



the spherical and cylindrical eigenfrequencies are surprisingly similar despite very different advection times (x2.5)

unexpected for an advective-acoustic mechanism??

$$\tau_{\rm adv} \equiv \int_{\rm sh}^{r} \frac{{\rm d}r}{v},$$
$$\sim \frac{r_{\rm sh}}{|v_{\rm sh}|} \log \frac{r_{\rm sh}}{r} \quad \text{spherical},$$
$$\sim \frac{r_{\rm sh}}{|v_{\rm sh}|} \left(\frac{r_{\rm sh}}{r} - 1\right) \quad \text{cylindrical}.$$

Walk, Foglizzo, Tamborra, in prep

adiabatic inner boundary condition
inspired by the shallow water experiment

Stellar SASI:

- spherical geometry
- γ=4/3
- buoyancy effects
- neutronization at the NS surface

non adiabatic ν-processes

➢ 4th order differential system

$$\omega' \equiv \omega - \frac{mL}{r^2}$$

Yamasaki & Foglizzo 08

$$\begin{cases} \frac{\partial}{\partial r}(r\delta v_\phi) = \frac{im}{v_r}\left(v_r\delta v_r - \frac{\delta K}{m^2} + \frac{c^2}{\gamma}\delta S\right) \\ \frac{\partial \delta h}{\partial r} = \frac{i\omega'}{v_r}\frac{\delta\rho}{\rho} - \frac{im}{rv_r}\delta v_\phi \,, \\ \left(\frac{\partial}{\partial r} - \frac{i\omega'}{v_r}\right)\delta S = \delta\left(\frac{\mathcal{L}}{p v_r}\right) \,, \\ \left(\frac{\partial}{\partial r} - \frac{i\omega'}{v_r}\right)\frac{\delta K}{m^2} = \delta\left(\frac{\mathcal{L}}{\rho v_r}\right) \,. \end{cases}$$

Shallow water analogue:

- cylindrical geometry
- γ=2
- isentropic fluid
- adiabatic inner boundary

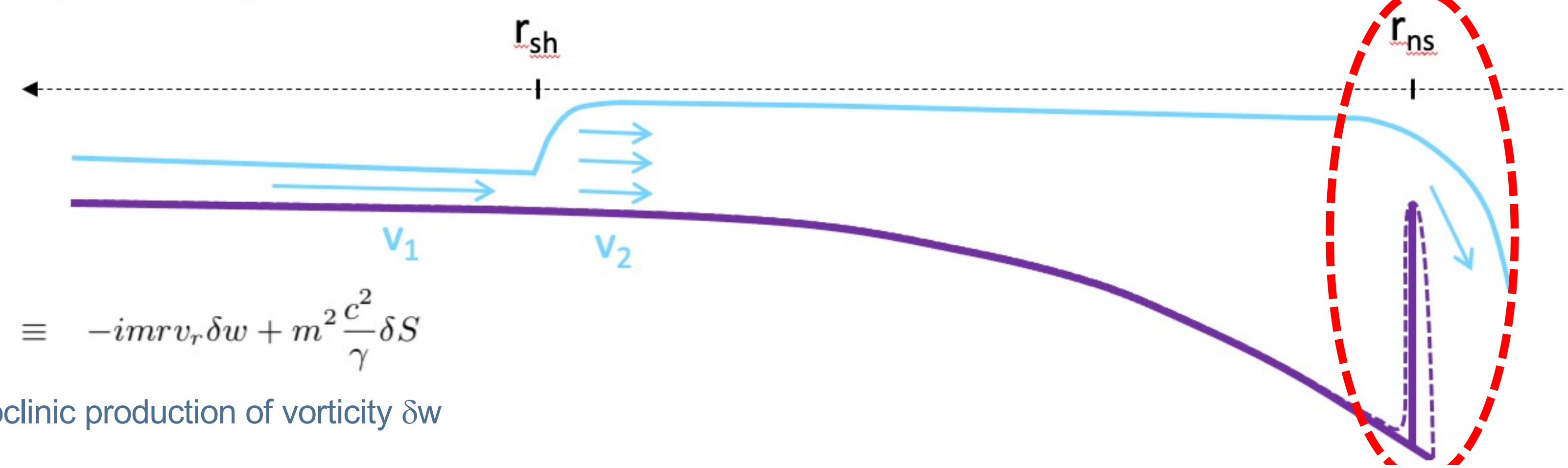
adiabatic evolution

➢ conservation of "vorticity" δK + entropy δS

➢ 2nd order differential system

➢ analytic approximation

$$\begin{cases} \frac{\partial^2 Y}{\partial X^2} + \left[\frac{\omega'^2}{c^2} - \frac{m^2}{r^2}(1-\mathcal{M}^2)\right]\frac{Y}{v_r^2} = \mathcal{S}, \\ \mathcal{S} \equiv -\frac{r_{\rm sh}}{v_{\rm sh}}\delta w_{\rm sh} e^{\int_{\rm sh}\frac{i\omega'}{c^2}{\rm d}X}\frac{\partial}{\partial X}\left(\frac{\mathcal{M}_{\rm sh}^2}{\mathcal{M}^2}e^{\int_{\rm sh}\frac{i\omega'}{v_r}{\rm d}r}\right) \end{cases}$$



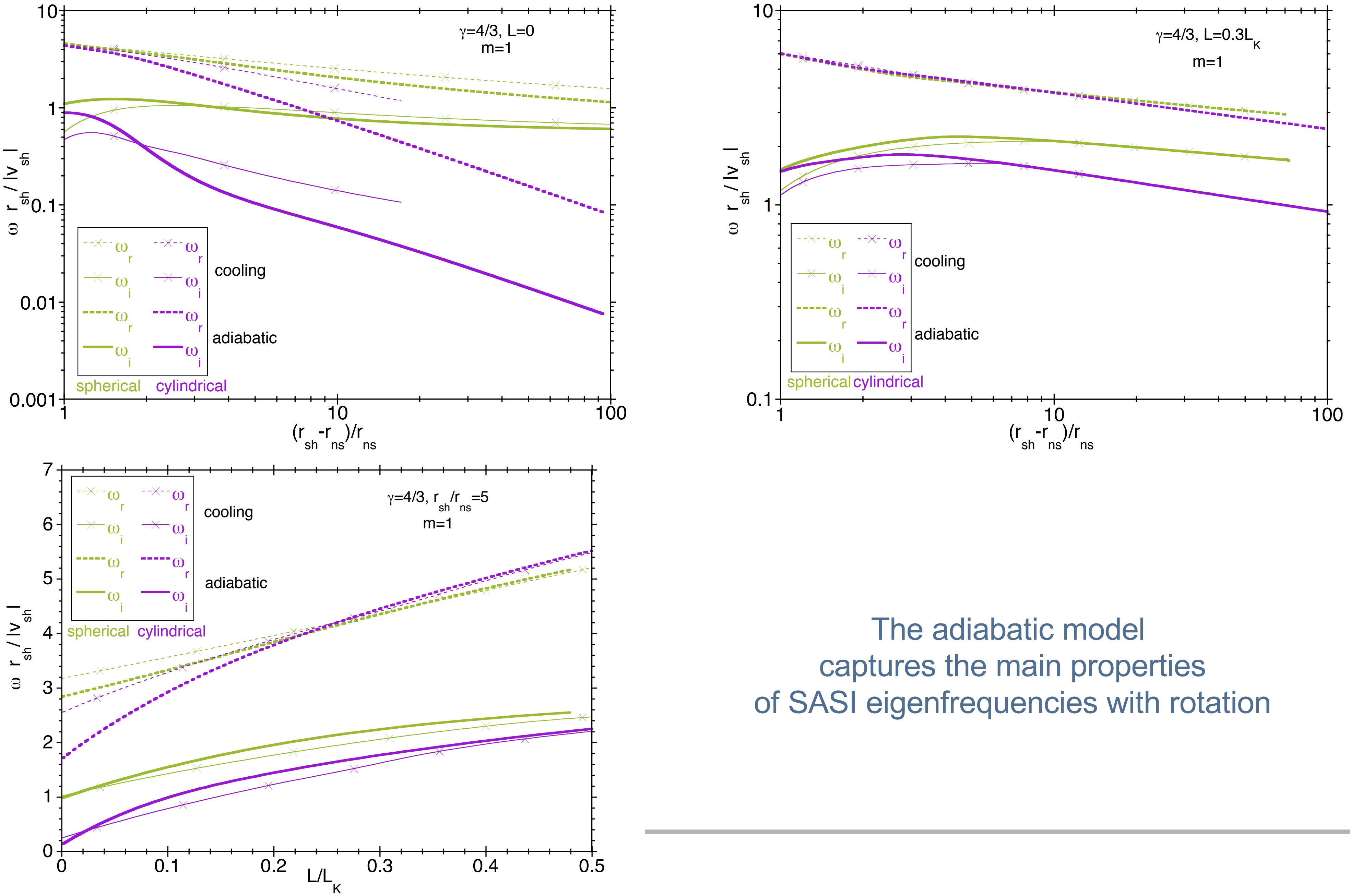
$$\delta K \equiv -imrv_r\delta w + m^2\frac{c^2}{\gamma}\delta S$$

baroclinic production of vorticity δw

$${\rm d}X \equiv \frac{v_r}{1-\mathcal{M}^2}{\rm d}r,$$

$$Y \equiv r\delta v_\phi e^{\int_{\rm sh}\frac{i\omega'\mathcal{M}^2}{1-\mathcal{M}^2}\frac{{\rm d}r}{v_r}},$$

Comparison of the non-adiabatic and adiabatic models



The adiabatic model
captures the main properties
of SASI eigenfrequencies with rotation

The adiabatic model can be interpreted physically

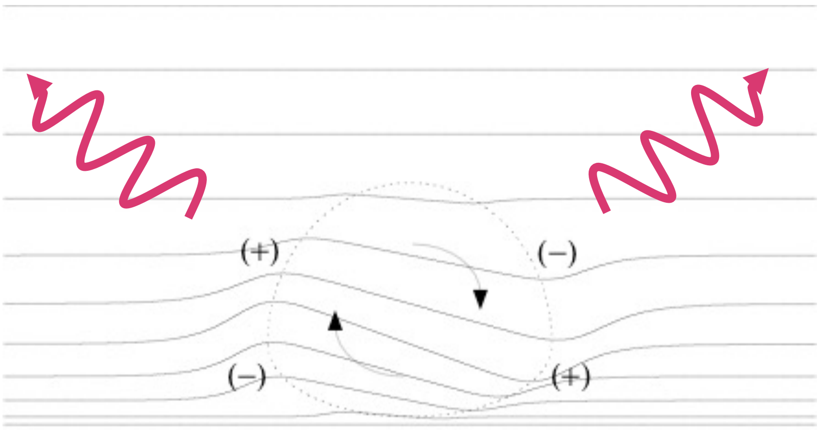
$$\mathrm{dX} \equiv \frac{v_r}{1-\mathcal{M}^2}\mathrm{dr},$$

$$\omega' \equiv \omega - \frac{mL}{r^2} \qquad\qquad Y \equiv r\delta v_\phi \mathrm{e}^{\int_{\mathrm{sh}} \frac{i\omega' \mathcal{M}^2}{1-\mathcal{M}^2}\frac{\mathrm{dr}}{v_r}},$$

Forced oscillator 🙂

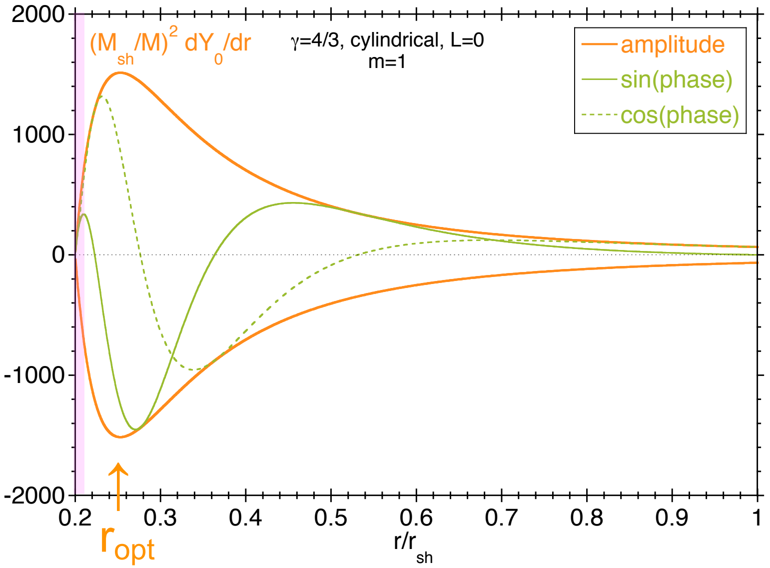
$$\left\{\begin{array}{l} \dfrac{\partial^2 Y}{\partial X^2} + \left[\dfrac{\omega'^2}{c^2} - \dfrac{m^2}{r^2}(1-\mathcal{M}^2)\right]\dfrac{Y}{v_r^2} = \mathcal{S}, \\ \mathcal{S} \quad \equiv \quad -\dfrac{r_{\mathrm{sh}}}{v_{\mathrm{sh}}}\delta w_{\mathrm{sh}} \mathrm{e}^{\int_{\mathrm{sh}} \frac{i\omega'}{c^2}\mathrm{dX}} \dfrac{\partial}{\partial X}\left(\dfrac{\mathcal{M}_{\mathrm{sh}}^2}{\mathcal{M}^2}\mathrm{e}^{\int_{\mathrm{sh}} \frac{i\omega'}{v_r}\mathrm{d}r}\right) \end{array}\right.$$

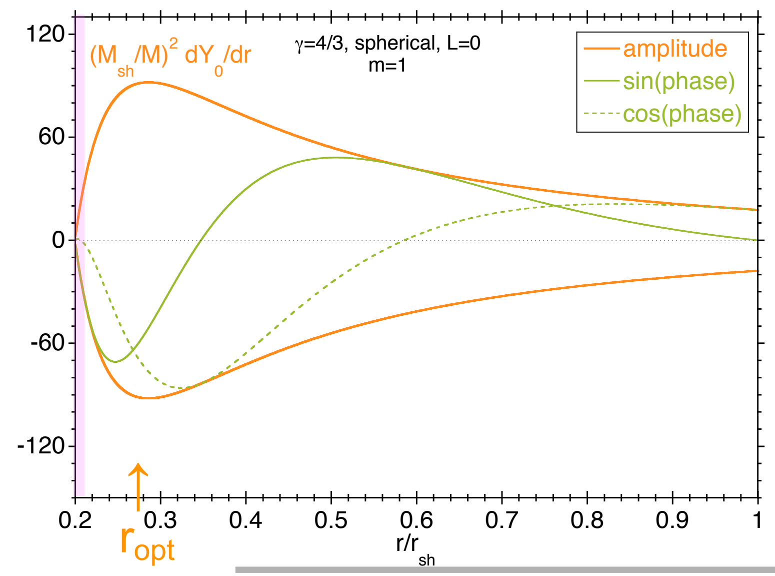
The acoustic cavity is forced by the radial advection of vorticity perturbations δw



vortical-acoustic coupling

The effect of moderate rotation is clarified





$r_{opt}>r_{ns}$ → insensitive to the cooling layer

The forcing efficiency depends on

- the forcing amplitude $\frac{\partial Y_0}{\partial r}\frac{\mathcal{M}_{sh}^2}{\mathcal{M}^2}$ increases inward maximal at r_{opt}
- the phase match $e^{\int_{sh}\frac{i\omega'}{v_r}dr}$ phase mixing as $v_r \to 0$

🙂
$$\left\{\begin{aligned}
&\frac{\partial^2 Y}{\partial X^2}+\left[\frac{\omega'^2}{c^2}-\frac{m^2}{r^2}(1-\mathcal{M}^2)\right]\frac{Y}{v_r^2}=\mathcal{S},\\
&\mathcal{S}\equiv -\frac{r_{sh}}{v_{sh}}\delta w_{sh}e^{\int_{sh}\frac{i\omega'}{c^2}dX}\frac{\partial}{\partial X}\left(\frac{\mathcal{M}_{sh}^2}{\mathcal{M}^2}e^{\int_{sh}\frac{i\omega'}{v_r}dr}\right)
\end{aligned}\right.$$

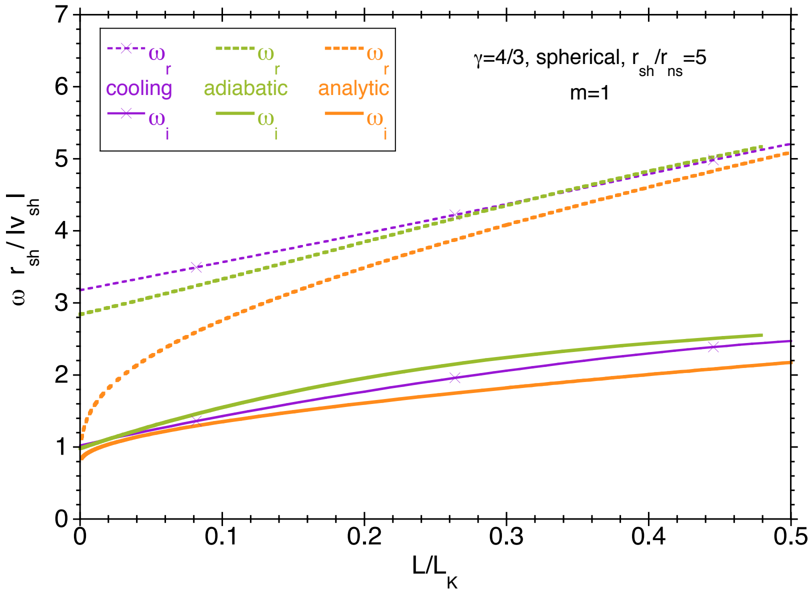
$Y_0(r)$ = (acoustic) solution to the homogeneous equation

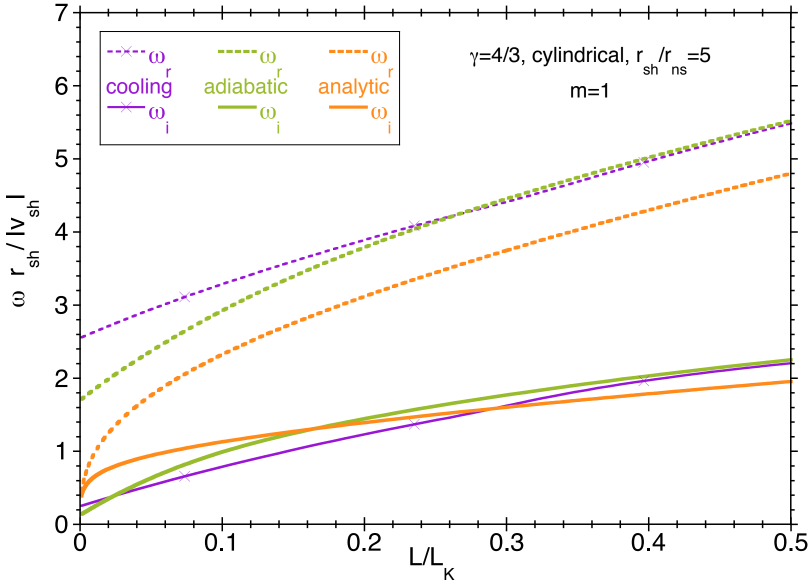
$$a_1' Y_0^{sh}+a_2'\left(\frac{\partial Y_0}{\partial X}\right)_{sh}+a_3' Y_0^{ns}=$$
$$-\int_{ns}^{sh}\frac{\partial}{\partial r}\left(Y_0 e^{\int_{sh}\frac{i\omega'\mathcal{M}^2}{1-\mathcal{M}^2}\frac{dr}{v_r}}\right)\frac{\mathcal{M}_{sh}^2}{\mathcal{M}^2}e^{\int_{sh}\frac{i\omega'}{v_r}dr}dr,$$

$$\boxed{\omega'\equiv\omega-\frac{mL}{r^2}}$$

Reduced phase mixing near the PNS explains the efficient destabilizing effect of moderate rotation on the prograde mode of SASI

The mechanism of spiral SASI is clarified





$$a_1' Y_0^{\rm sh} + a_2' \left(\frac{\partial Y_0}{\partial X}\right)_{\rm sh} + a_3' Y_0^{\rm ns} = -\int_{\rm ns}^{\rm sh} \frac{\partial}{\partial r}\left(Y_0 {\rm e}^{\int_{\rm sh} \frac{i\omega' \mathcal{M}^2}{1-\mathcal{M}^2}\frac{{\rm d}r}{v_r}}\right) \frac{\mathcal{M}_{\rm sh}^2}{\mathcal{M}^2} {\rm e}^{\int_{\rm sh}\frac{i\omega'}{v_r}{\rm d}r}\, {\rm d}r,$$

$$\boxed{\omega' \equiv \omega - \frac{mL}{r^2}} \qquad \boxed{\omega_r \equiv \frac{mL}{r_{\rm co}^2},}$$

$$\Psi \equiv \int_{\rm sh}^{r} \omega' \frac{{\rm d}r}{v}$$

Taylor expansion

$$\Psi \sim \Psi_{\rm co} - \left(\frac{r-r_{\rm co}}{\Delta r}\right)^2$$

The stationary phase approximation captures the dominant coupling at the corotation radius $\omega'=0$

$$\boxed{a_1' Y_0^{\rm sh} + a_2' \left(\frac{\partial Y_0}{\partial X}\right)_{\rm sh} + a_3' Y_0^{\rm ns} = -{\rm e}^{i\Psi_{\rm co}} \pi^{\frac{1}{2}} {\rm e}^{-i\frac{\pi}{4}} \frac{\mathcal{M}_{\rm sh}^2}{\mathcal{M}_{\rm co}^2} \left(\frac{\partial Y_0}{\partial r}\right)_{\rm co} \Delta r.}$$

$r_{co} >> r_{ns}$ explains:

- cylindrical/spherical similarity
- adiabatic/non-adiabatic similarity

Conclusions

Most massive stars explode in a non-spherical manner

A diversity of dynamical evolutions is expected based on the interplay of SASI, ν-driven convection and rotation.

Neutrinos and GW carry direct information on the explosion engine

Improved understanding of spiral SASI inspired by the shallow water analogy

- SASI mechanism is mostly adiabatic
- SASI is best understood as a forced oscillator

- prograde SASI mode destabilized by rotation = reduced phase mixing near the PNS
- corotating SASI spiral = stationary phase approximation at the corotation radius
- warning on using the cylindrical approximation

In progress:
Interpreting turbulent experimental results using this new framework
Effective extraction of the stellar parameters, including rotation, from ν and GW frequencies?