Phenomenological turbulent effects of core-collapse supernovae

Toward to predict progenitor dependence

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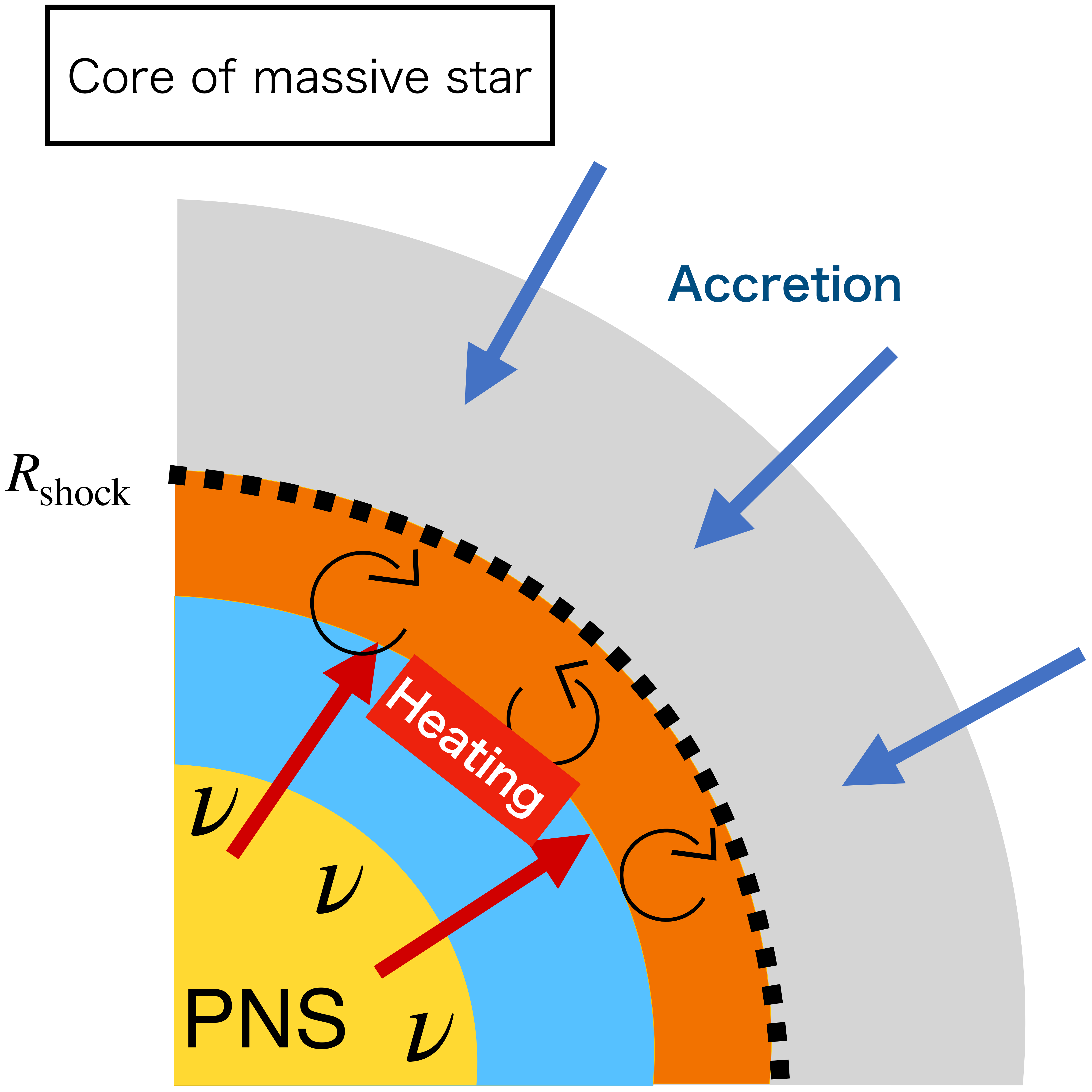
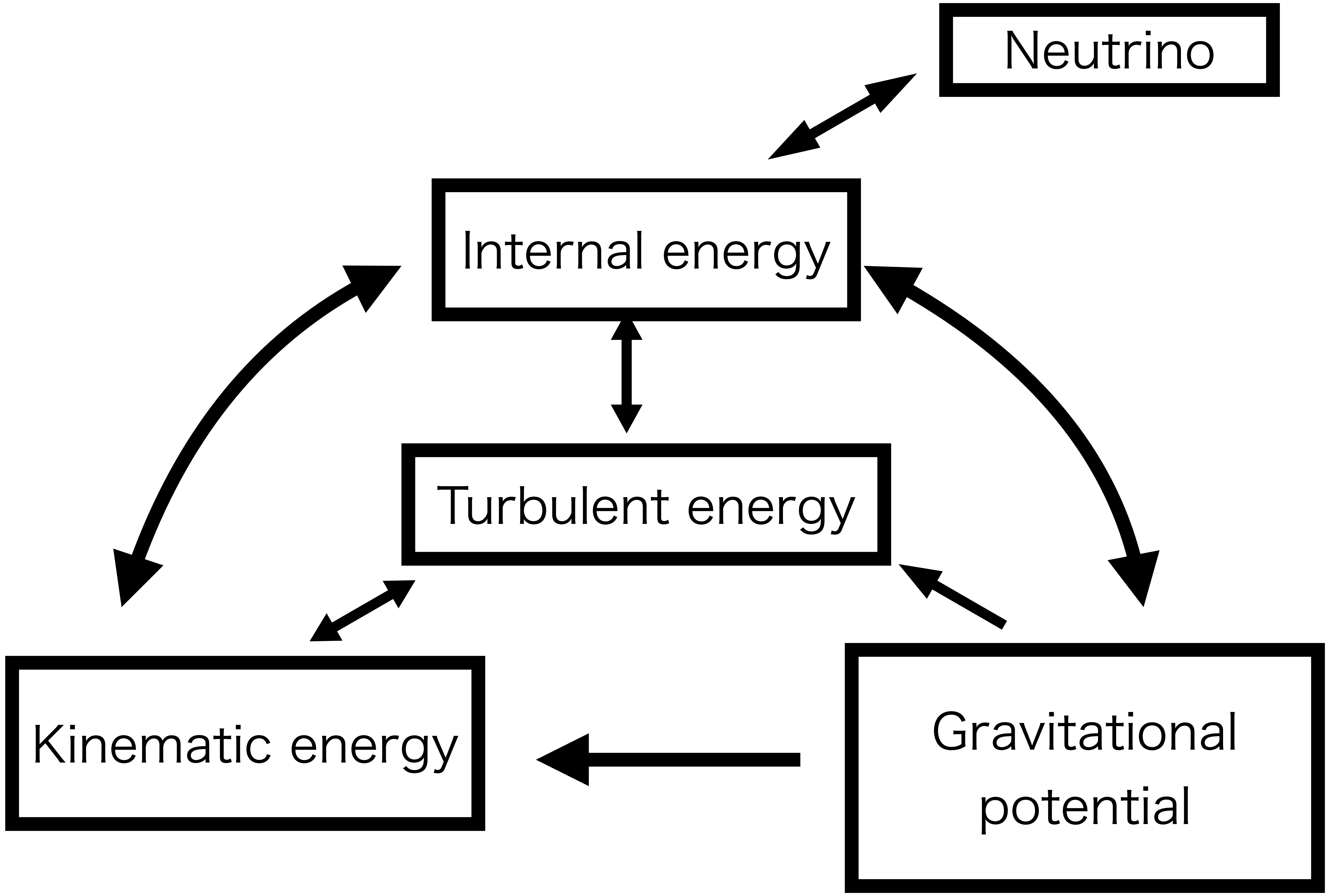
Brainstorming workshop 2022

Contents

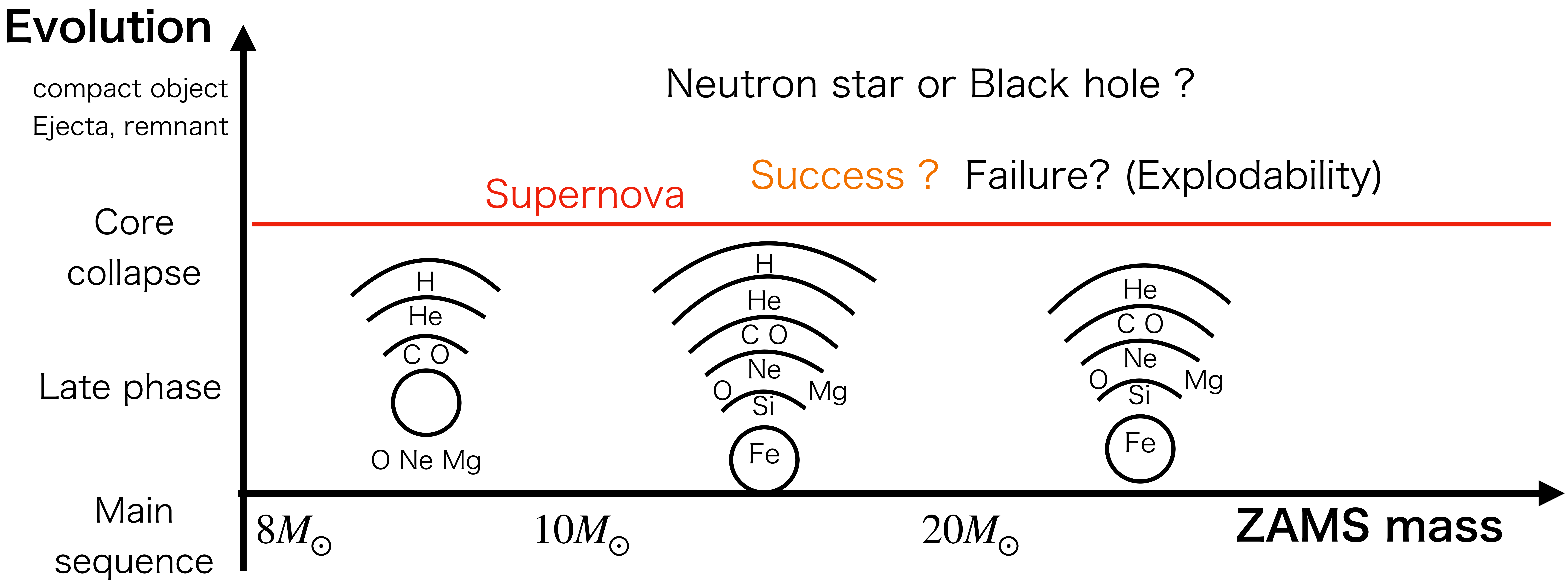
Energy exchange and mechanism





CCSN simulation calculates these energy exchanges and dynamics.

Stellar evolution and Supernovae



What is important property to predict the progenitor dependence ?

1D simulation

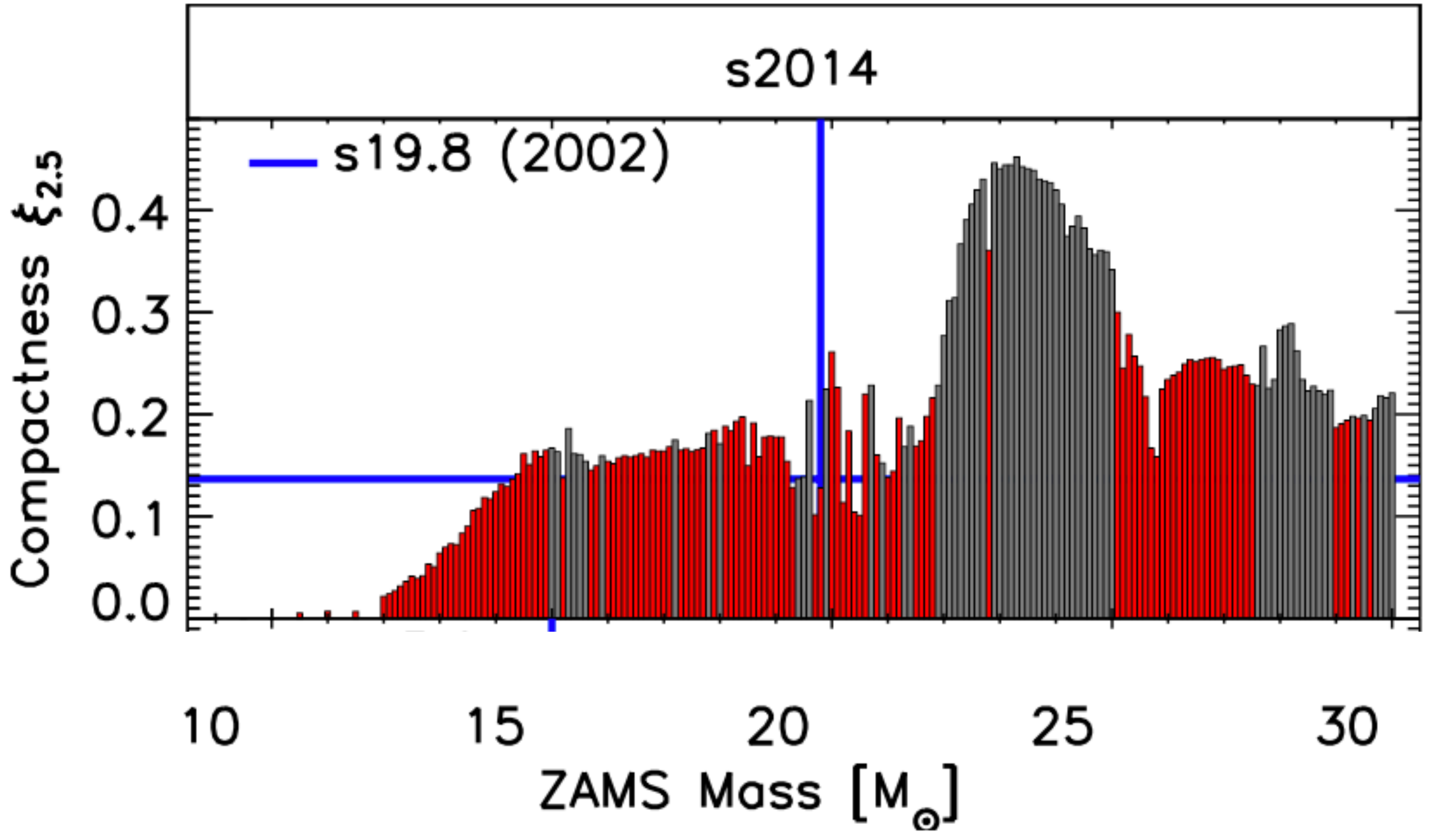
Systematic study

Compactness: $\xi_M = \dfrac{M/M_{\odot}}{R(M)/1000\mathrm{km}}$

$$M(r) = \int_0^r 4\pi\rho\, r'^2\, dr'$$

ξ_M ↑ : Black hole (black)

ξ_M ↓ : Supernova (Red)



O'Connor & Ott 2011
Ertl et al. 2016
Nakamura et al. 2015

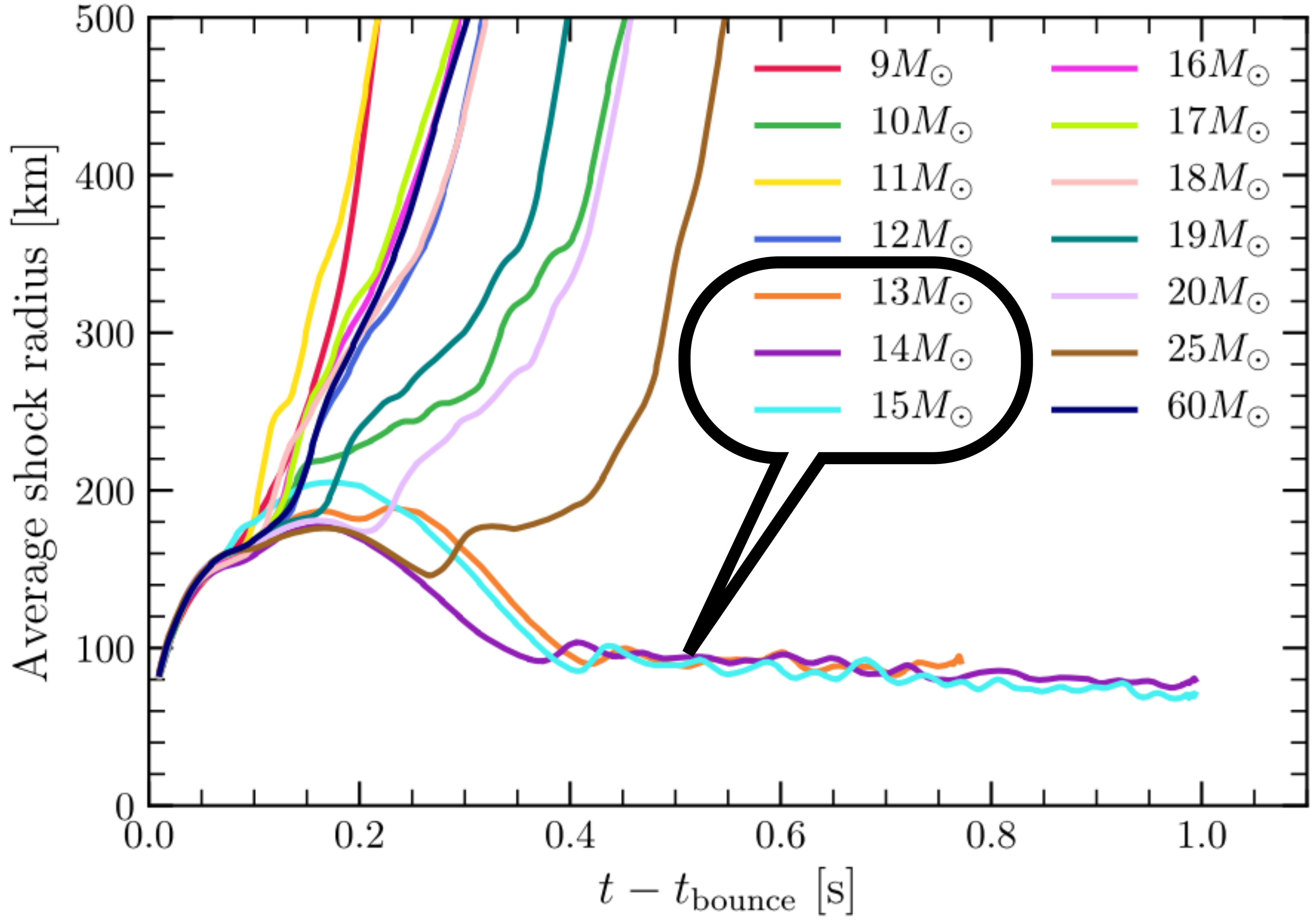
1D： phenomenological simulation

Prediction : property of progenitor like ξ_M governs supernova explosion

3D simulation

Compactness: $\xi_M = \frac{M/M_\odot}{R(M)/1000\mathrm{km}}$

Burrows et al. 2020



Progenitor ($M_\odot$)	Envelope binding energy (10^{51} erg)	Compactness (calculated at 1.75 $M_\odot$)
s9.0	0.002	3.831×10^{-5}
s10.0	0.012	2.165×10^{-4}
s11.0	0.025	7.669×10^{-3}
s12.0	0.050	2.215×10^{-2}
s13.0	0.072	5.932×10^{-2}
s14.0	0.110	0.1243
s15.0	0.144	0.1674
s16.0	0.212	0.1546
s17.0	0.251	0.1644
s18.0	0.309	0.1715
s19.0	0.341	0.1783
s20.0	0.413	0.2615
s25.0	0.865	0.3010
s60.0	0.513	0.1753

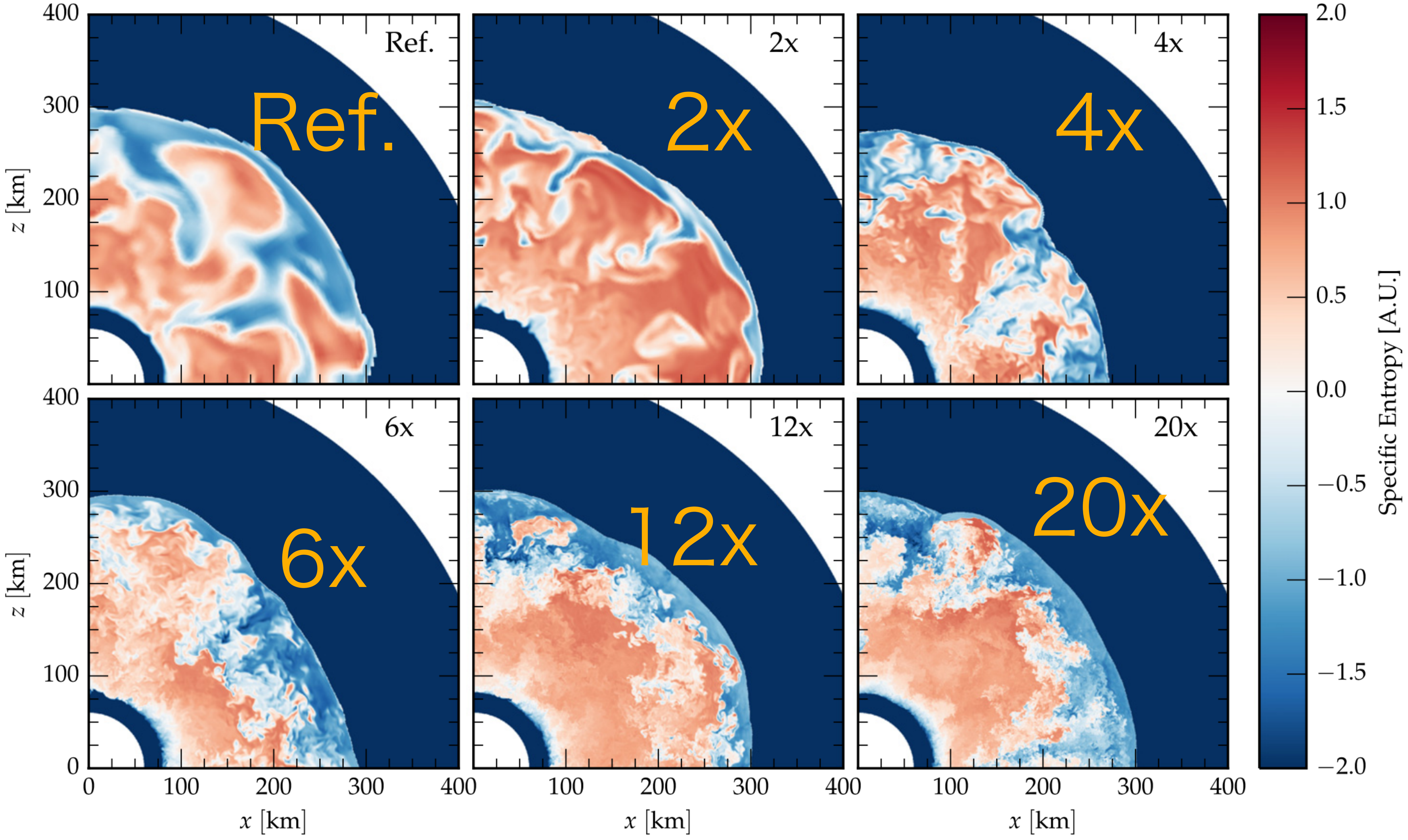
1D prediction is Not consistent with latest results of 3D

One of the reasons : multi dimensional turbulent effects

Research of turbulence

Turbulent convection in gain region

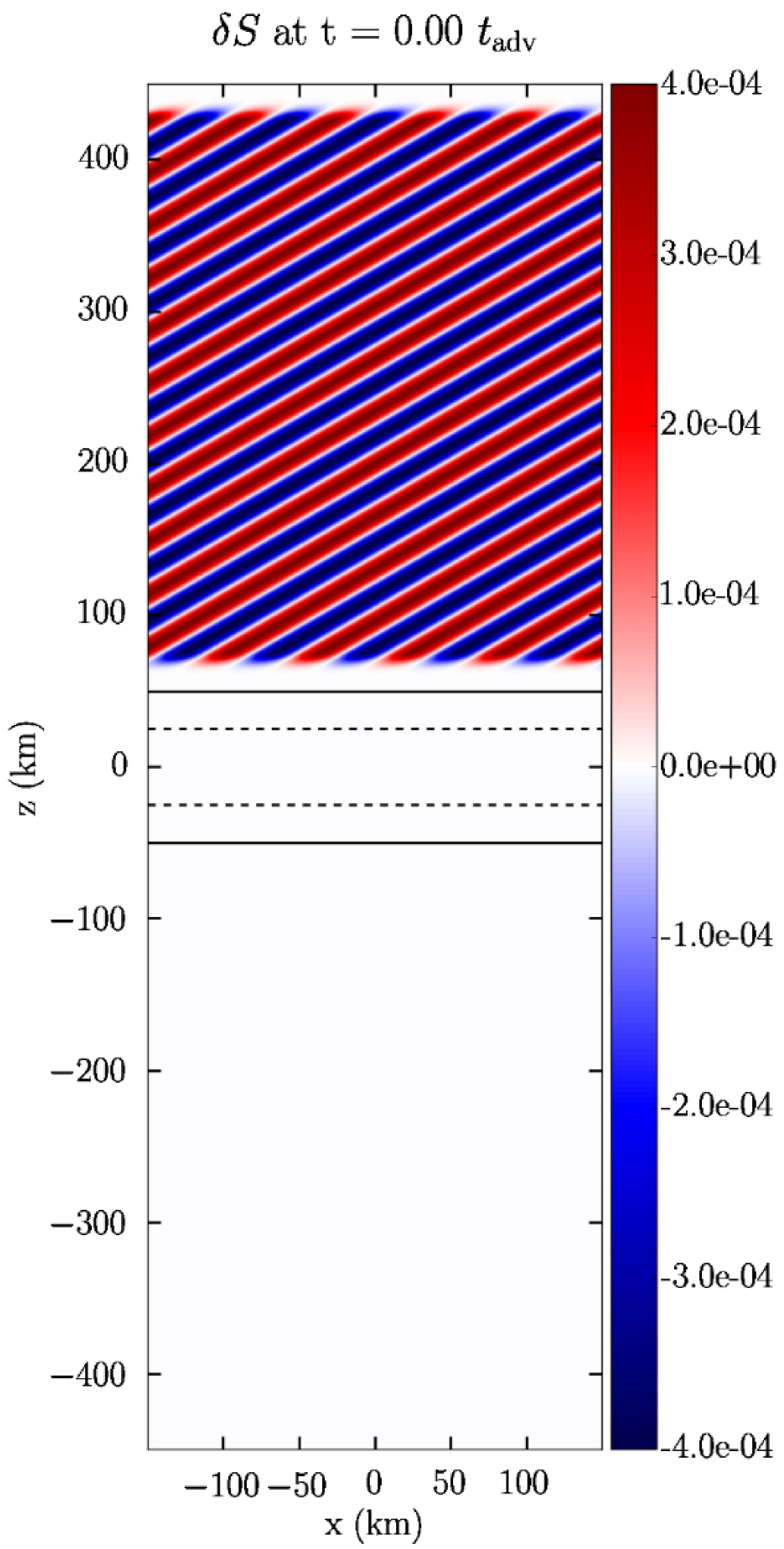
Radice et al. 2016

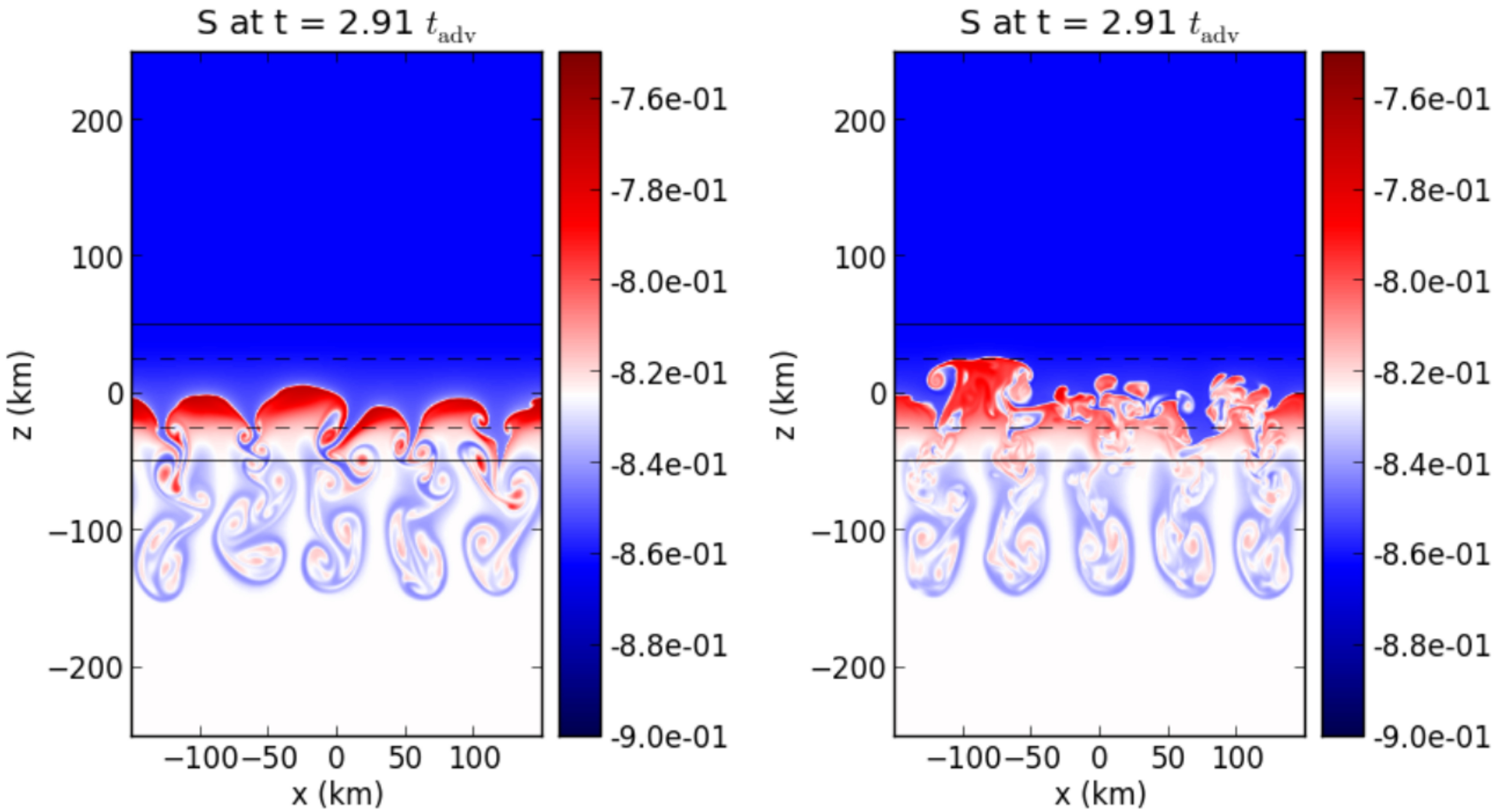


Turbulence depends on resolution, initial perturbations

Analyzing and understanding convection with multi- D simulation are difficult

Analysis of Turbulence





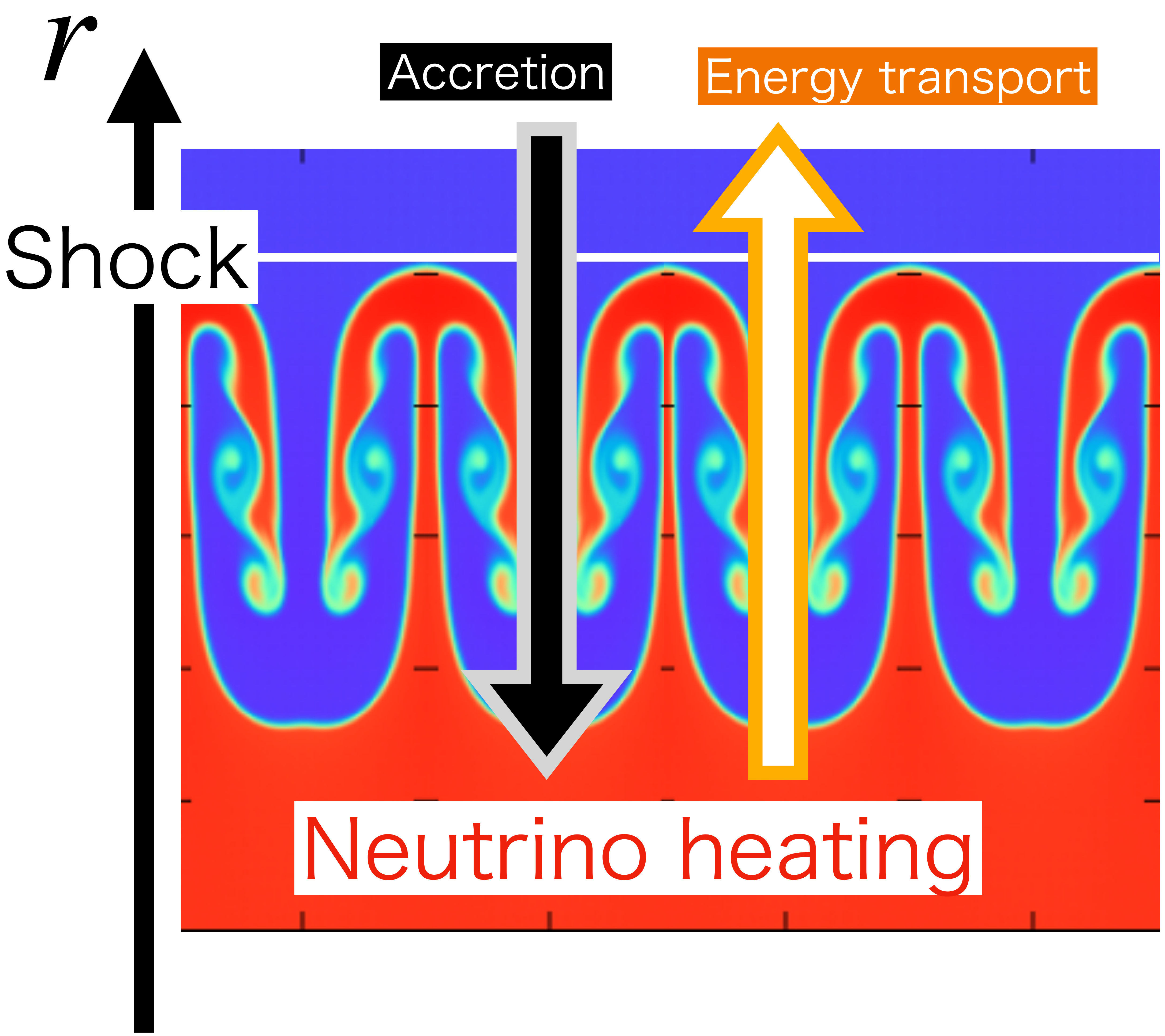
2D 3D

Kazeroni et al. 2018

non-linear analysis of turbulent effects is important to understand the mechanism.

Density perturbation triggers turbulence.

Turbulent effects



Main turbulent effects

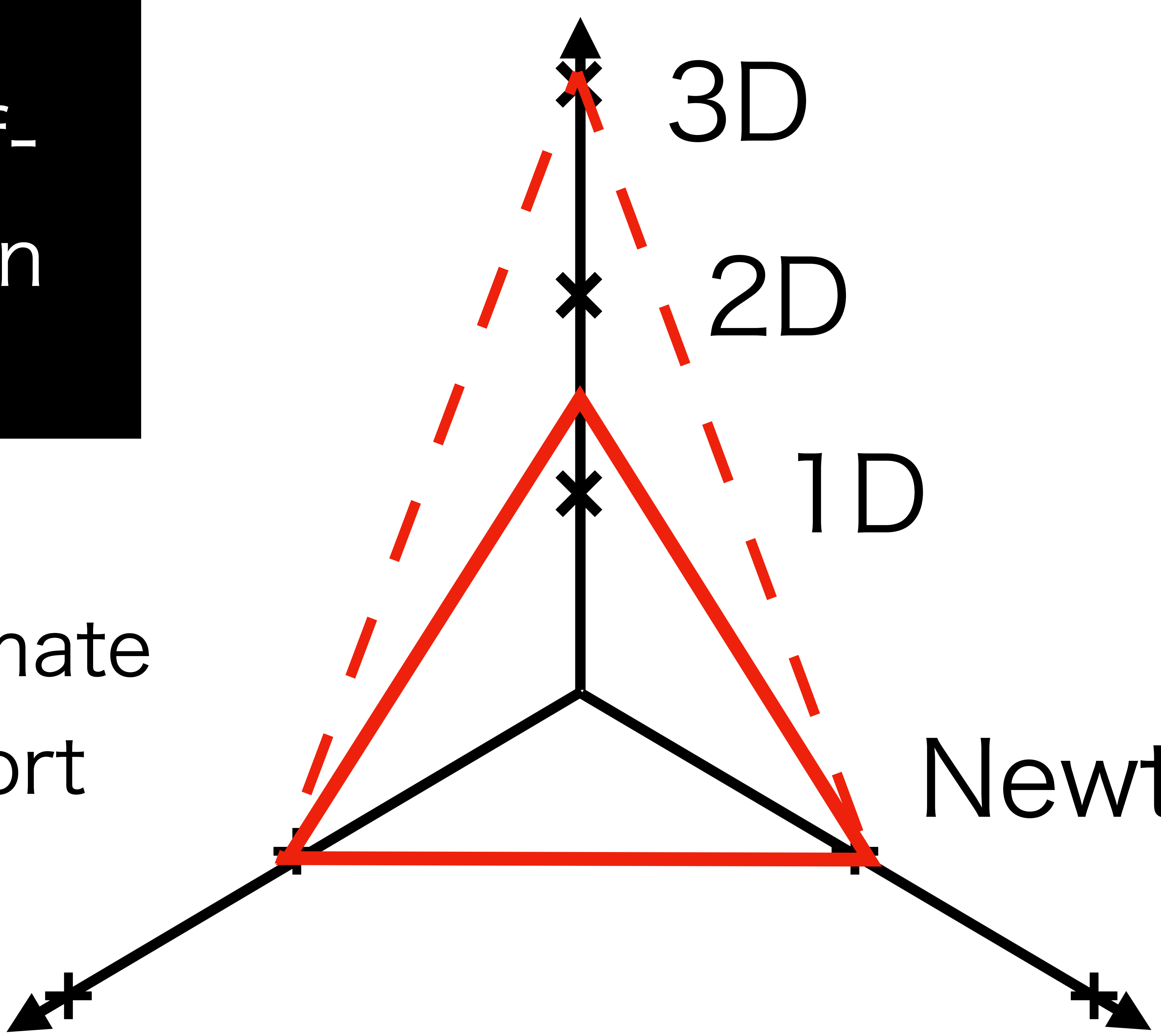
(1) Turbulent Pressure

$P_{\rm turb} = \langle \rho' v' v' \rangle$

(2) Diffusion $\langle e' v' \rangle$

(3) Dissipation $\dot{e}_{\rm dis} = \rho v'^3 / L$

Performance of self-consistent simulation

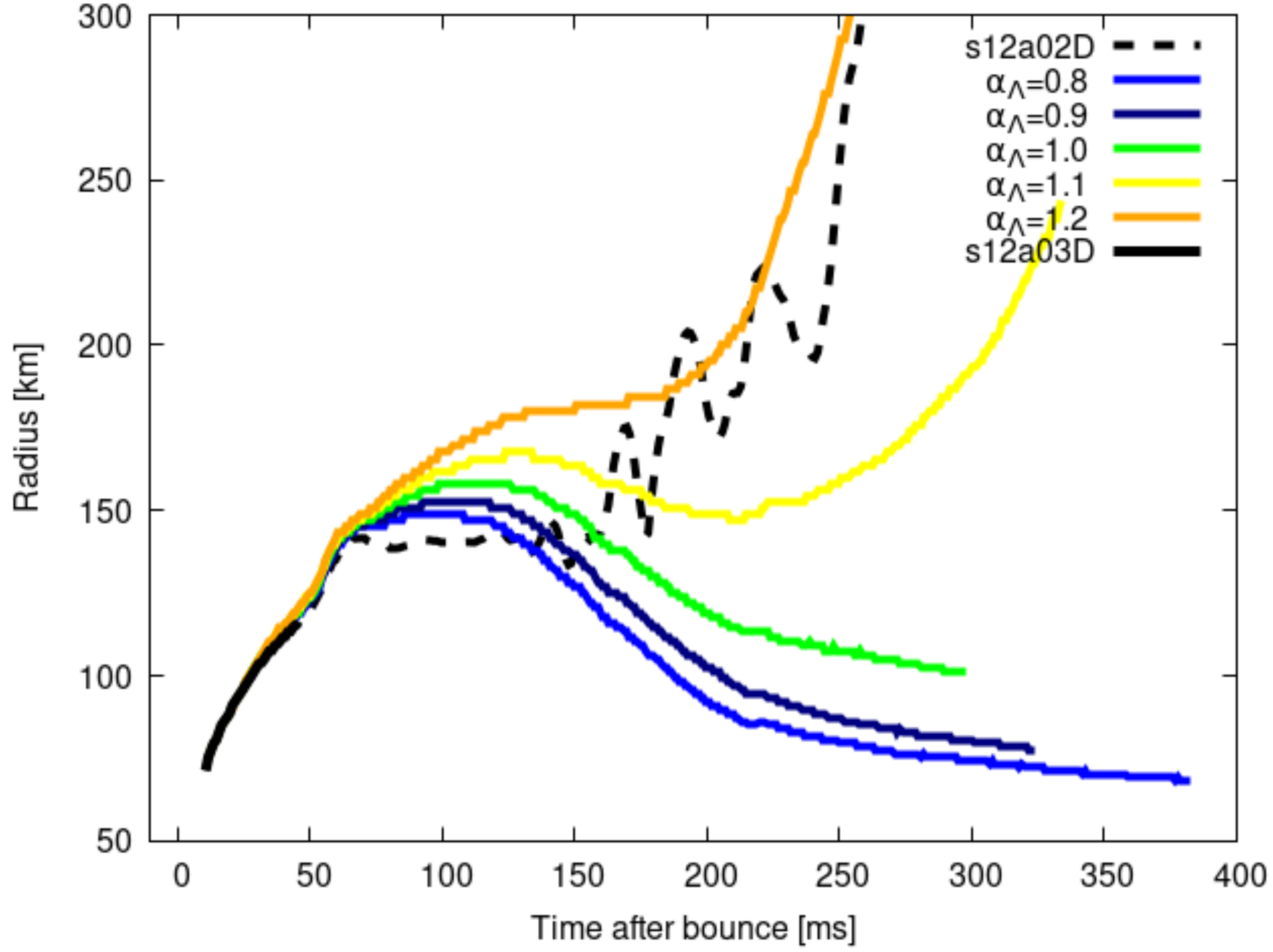


Our Goal

Accuracy : 3D = 1D+

Resource : 3D >> 1D+

1D+ simulation



Sasaki & Takiwaki

Main turbulent effects are introduced self consistent 1D simulation

(1) Turbulent Pressure

$P_{\rm turb} = \langle \rho' v' v' \rangle$

(2) Diffusion $\langle e' v' \rangle$

(3) Dissipation $\dot{e}_{\rm dis} = \rho v'^3 / L$

1D+ is low computational cost.

It is able to calculate neutrino heating and turbulent effects and to predict the progenitor dependence of CCSNe

Motivation

Motivation：to predict progenitor dependence by using 1D+

It needs to reproduce the 3D results with 1D+

⬇

1st step: researching turbulent effects in current 1D+

Contents

How to introduce turbulent effects
Fluid equations: from 3D to1D

$$\frac{\partial \rho}{\partial t} + \nabla \cdot [\rho \vec{v}] = 0$$ mass conservation

$$\frac{\partial \rho \vec{v}}{\partial t} + \nabla \cdot [\rho \vec{v}^2 + P] = -\rho g + S_\nu$$ Euler equation

$$\frac{\partial (\rho e)}{\partial t} + \nabla \cdot [\vec{v}(\rho e + P)] = -\rho \vec{v} g + Q_\nu$$ Energy conservation

Assuming spherical symmetry in the above three equations

How to introduce turbulent effects

Reynolds decomposition (Murphy & Meakin 2011)

$$\frac{\partial \rho}{\partial t} + \nabla \cdot [\rho \vec{v}] = 0$$

$$\frac{\partial (\hat{\rho} + \rho')}{\partial t} + \nabla \cdot [(\hat{\rho} + \rho')(\hat{\vec{v}} + \vec{v}')] = 0$$

$$\frac{\partial \langle \hat{\rho} + \rho' \rangle}{\partial t} + \frac{1}{r^2}\frac{\partial}{\partial r} r^2 \langle (\hat{\rho} + \rho')(\hat{v}_r + v_r') \rangle = 0$$

$$\frac{\partial \hat{\rho}}{\partial t} + \frac{1}{r^2}\frac{\partial}{\partial r} r^2 [\hat{\rho}\hat{v}_r + \langle \rho' v_r' \rangle] = 0$$

Phenomenological turbulent effect

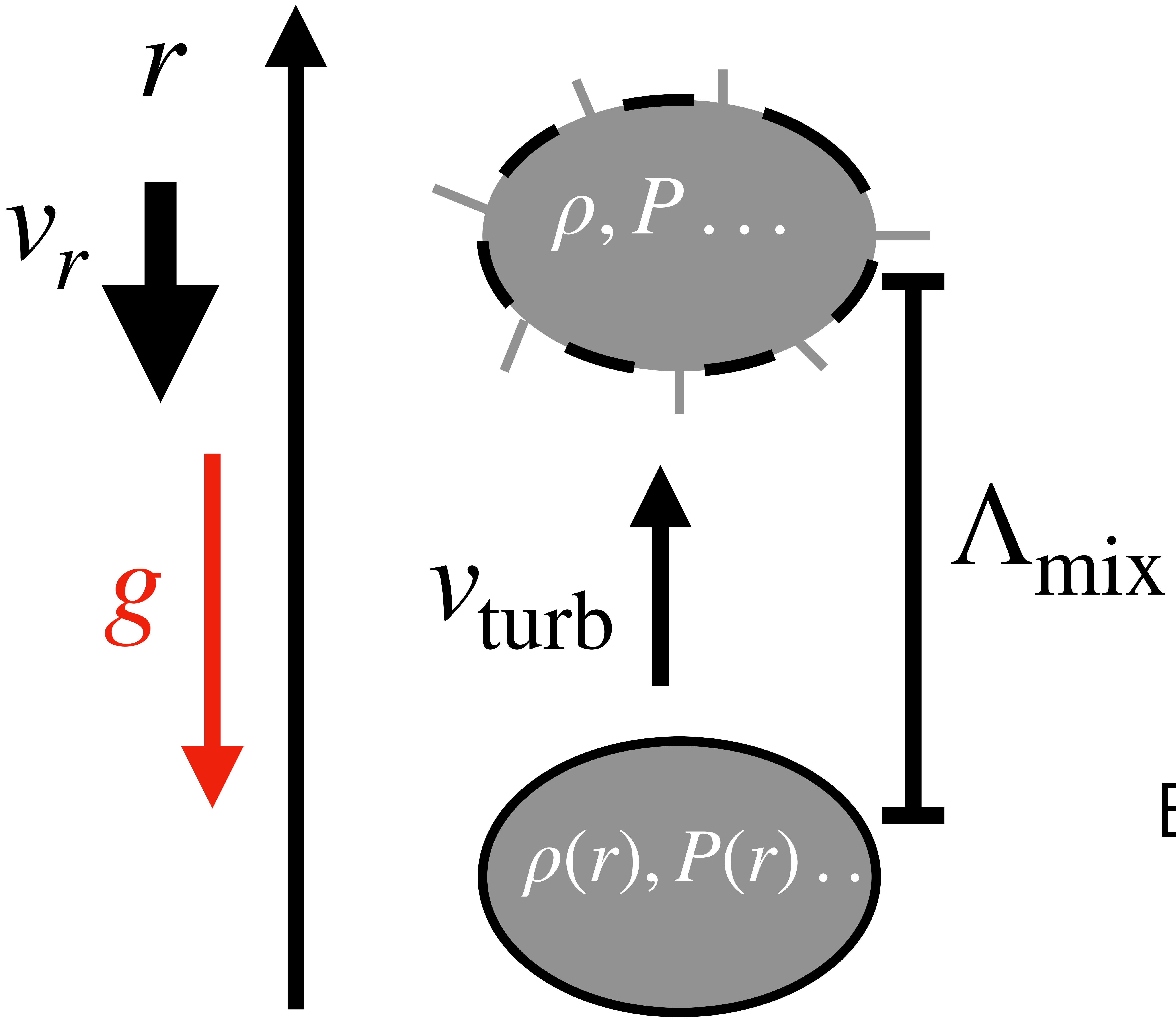
Reynolds decomposition

$$A = \hat{A} + A'$$

$$\hat{A} = \langle A \rangle = \frac{1}{4\pi}\int A(r, \theta, \varphi)\, d\Omega$$

$$\langle A' \rangle = 0$$

Model： Mixing Length Theory

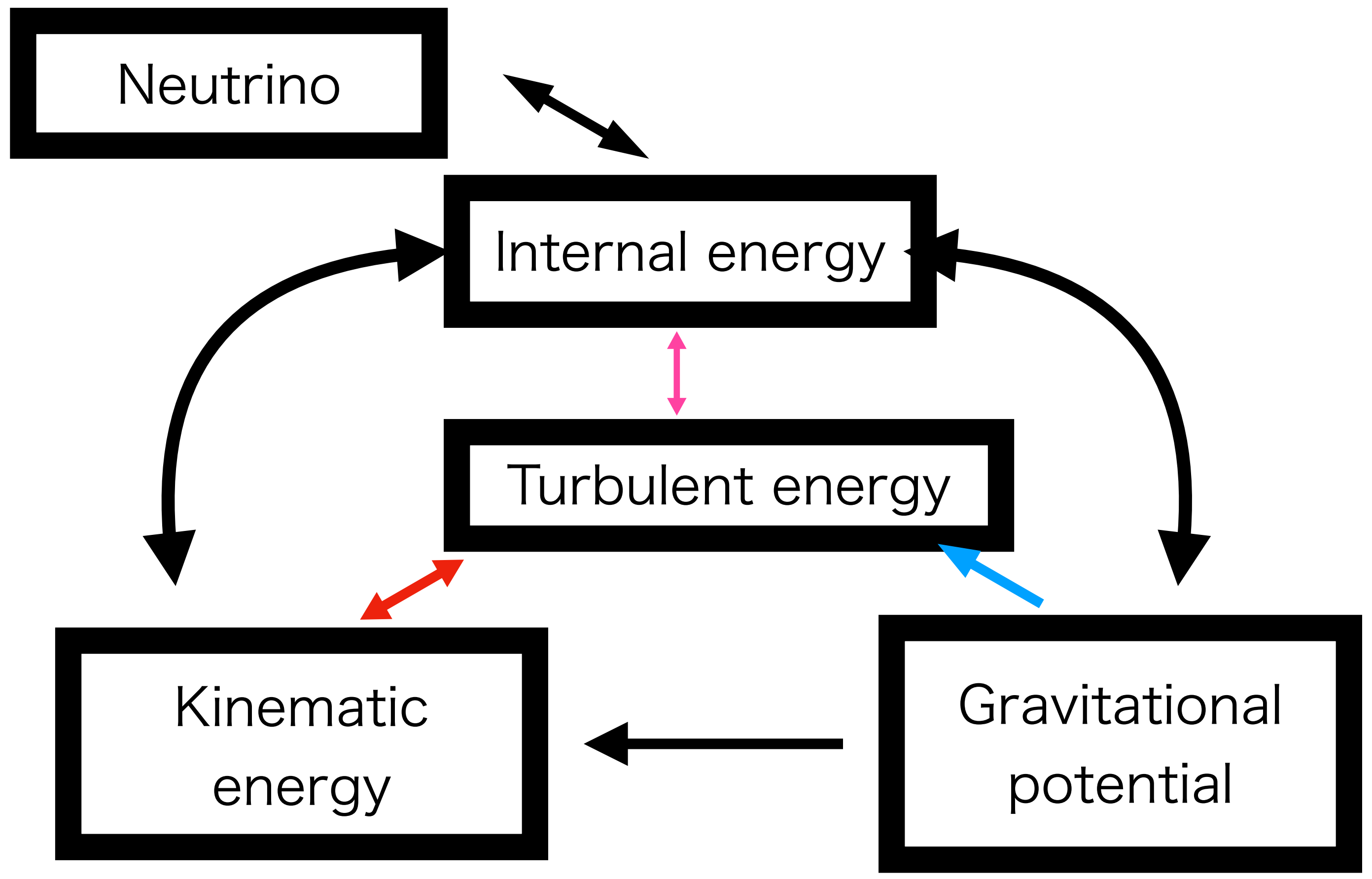


Scale of turbulence: $\Lambda_{\rm mix}$

Turbulent velocity: $v_{\rm turb}$

Element is powered by buoyancy force

Governing equation (e.g. Müller 2019, Couch et al. 2020)



Turbulent pressure

Diffusion

$$\frac{\partial \rho}{\partial t} + \frac{1}{r^2}\frac{\partial}{\partial r} r^2[\rho v_r] = 0$$ Mass conservation

$$\frac{\partial \rho v_r}{\partial t} + \frac{1}{r^2}\frac{\partial}{\partial r}[r^2(\rho v_r^2 + P + \rho v_{\mathrm{turb}}^2)] = -\rho g + S_\nu$$ Euler equation

Source of turbulence

$$\frac{\partial(\rho e)}{\partial t} + \frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2 v_r(\rho e + P + \rho v_{\mathrm{turb}}^2) - r^2\rho D_\epsilon\left(\frac{\partial \epsilon}{\partial r} + P\frac{\partial}{\partial r}\left(\frac{1}{\rho}\right)\right) - r^2\rho D_K \nabla v_{\mathrm{turb}}^2\right] = -\rho v_r g + \rho v_{\mathrm{turb}}\omega_{\mathrm{BV}}^2\Lambda_{\mathrm{mix}} + Q_\nu$$ Energy conservation

$$\frac{\partial \rho v_{turb}^2}{\partial t} + \frac{1}{r^2}\frac{\partial}{\partial r}[r^2(\rho v_{\mathrm{turb}}^2 v_r) - r^2\rho D_K \nabla v_{\mathrm{turb}}^2] = -\rho v_{\mathrm{turb}}^2 \frac{1}{r^2}\frac{\partial(r^2 v_r)}{\partial r} + \rho v_{\mathrm{turb}}\omega_{\mathrm{BV}}^2\Lambda_{\mathrm{mix}} - \rho\frac{v_{\mathrm{turb}}^3}{\Lambda_{\mathrm{mix}}}$$ Turbulent energy conservation

Dissipation

Mixing length parameters

$$\Lambda_{\mathrm{mix}} = \alpha_\Lambda H_P = \alpha_\Lambda \frac{P}{\rho g}$$ mixing length

Diffusion parameters

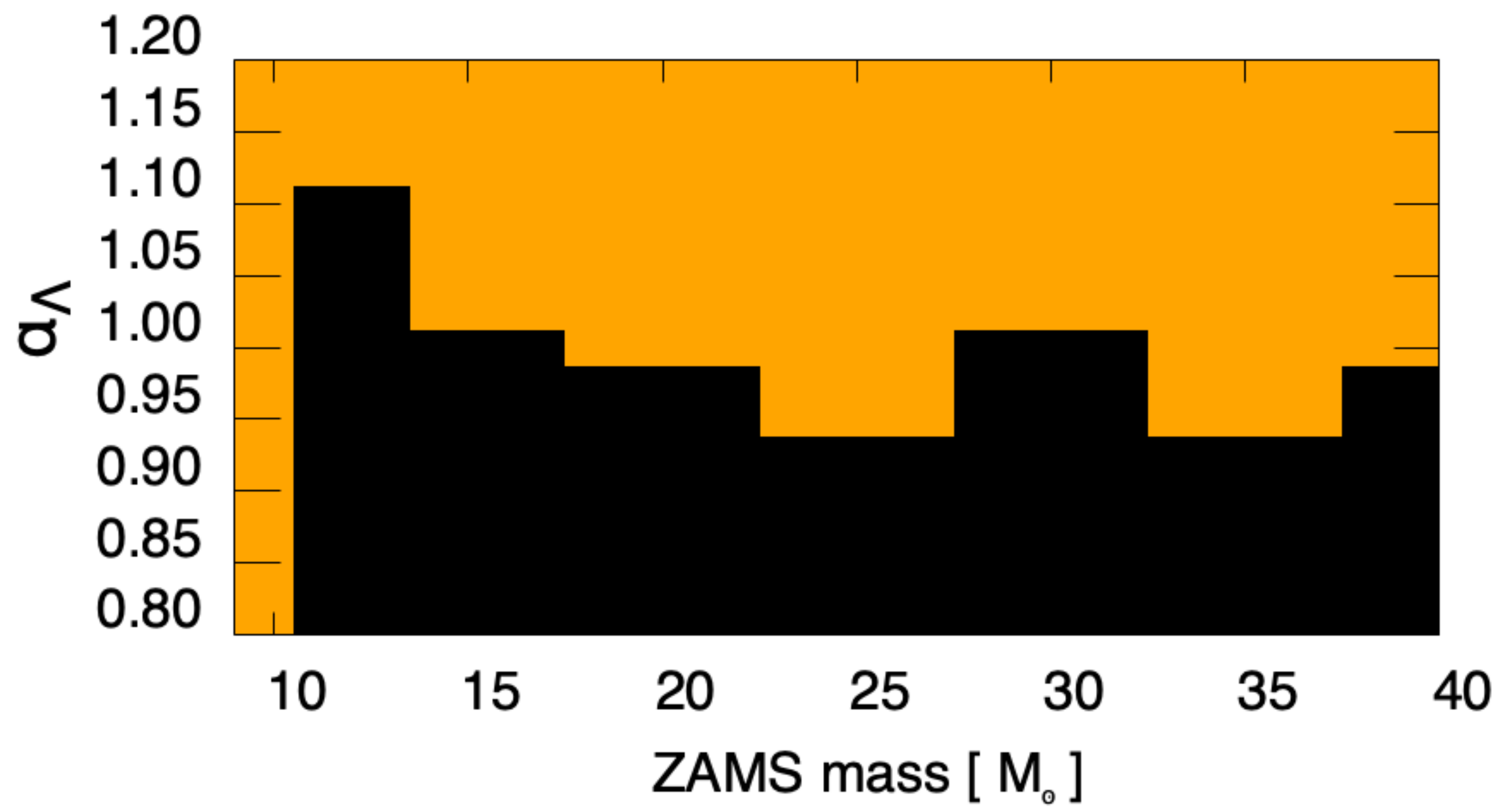
$$D_u = \alpha_u v_{\mathrm{turb}}\Lambda_{\mathrm{mix}} \quad (u = \epsilon, Y_e, K)$$

Contents

Explodability

α_Λ Mixing length parameters

$$\Lambda_{\text{mix}} = \alpha_\Lambda H_P = \alpha_\Lambda \frac{P}{\rho g}$$ mixing length



success, failure

Progenitor：$9.6, 12, 15, 20, 25, 30, 35, 40\, M_\odot$

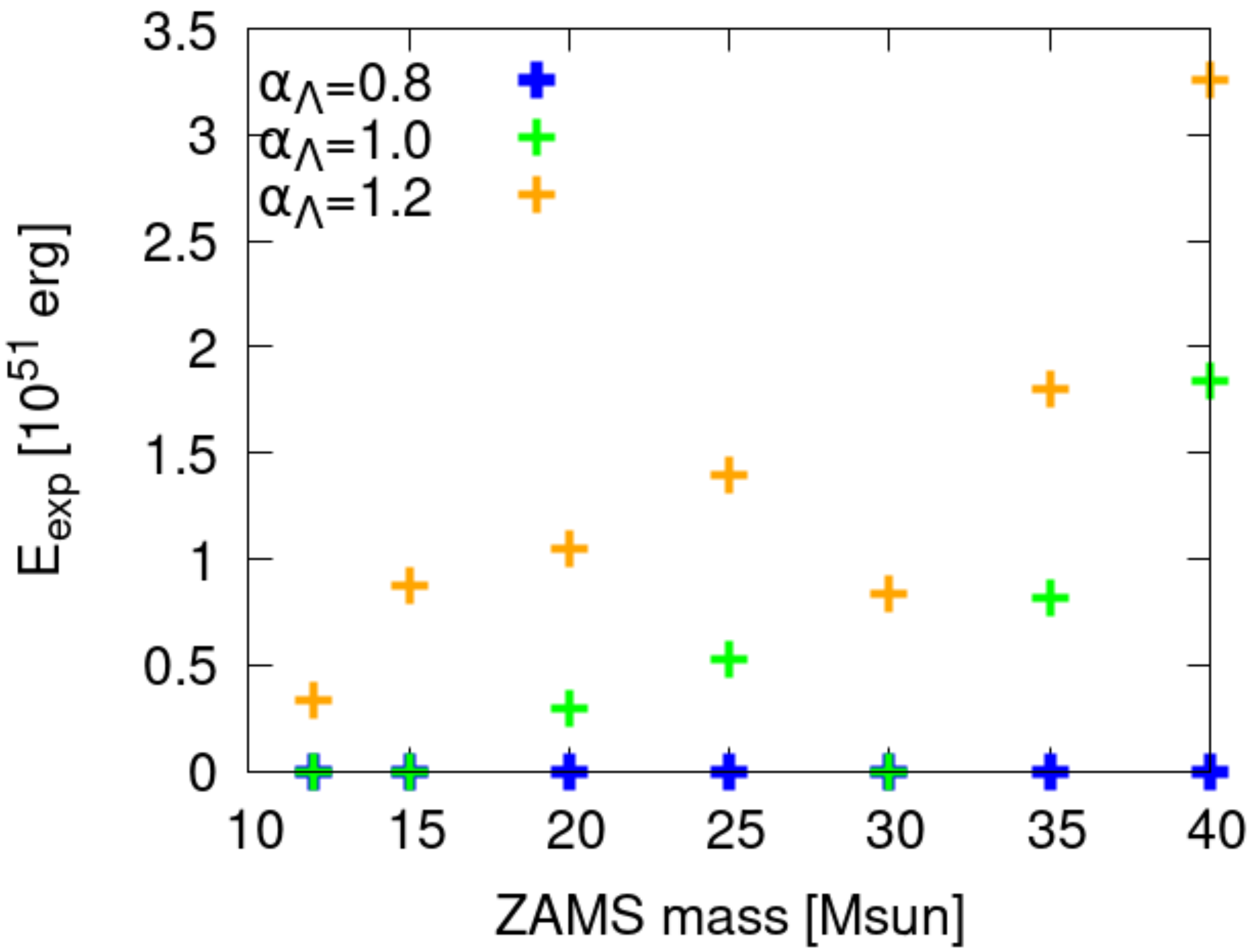
$\alpha_\Lambda = 0.8 - 1.2$ total 8x17=136 models

$12\, M_\odot$ is difficult to explode.(Vartanyan et al.2018)

$25\, M_\odot$ is easy to explode.(Couch et al.2020)

Turbulent parameter dependence

Explosion energy



Progenitor: $12, 15, 20, 25, 30, 35, 40\, M_\odot$

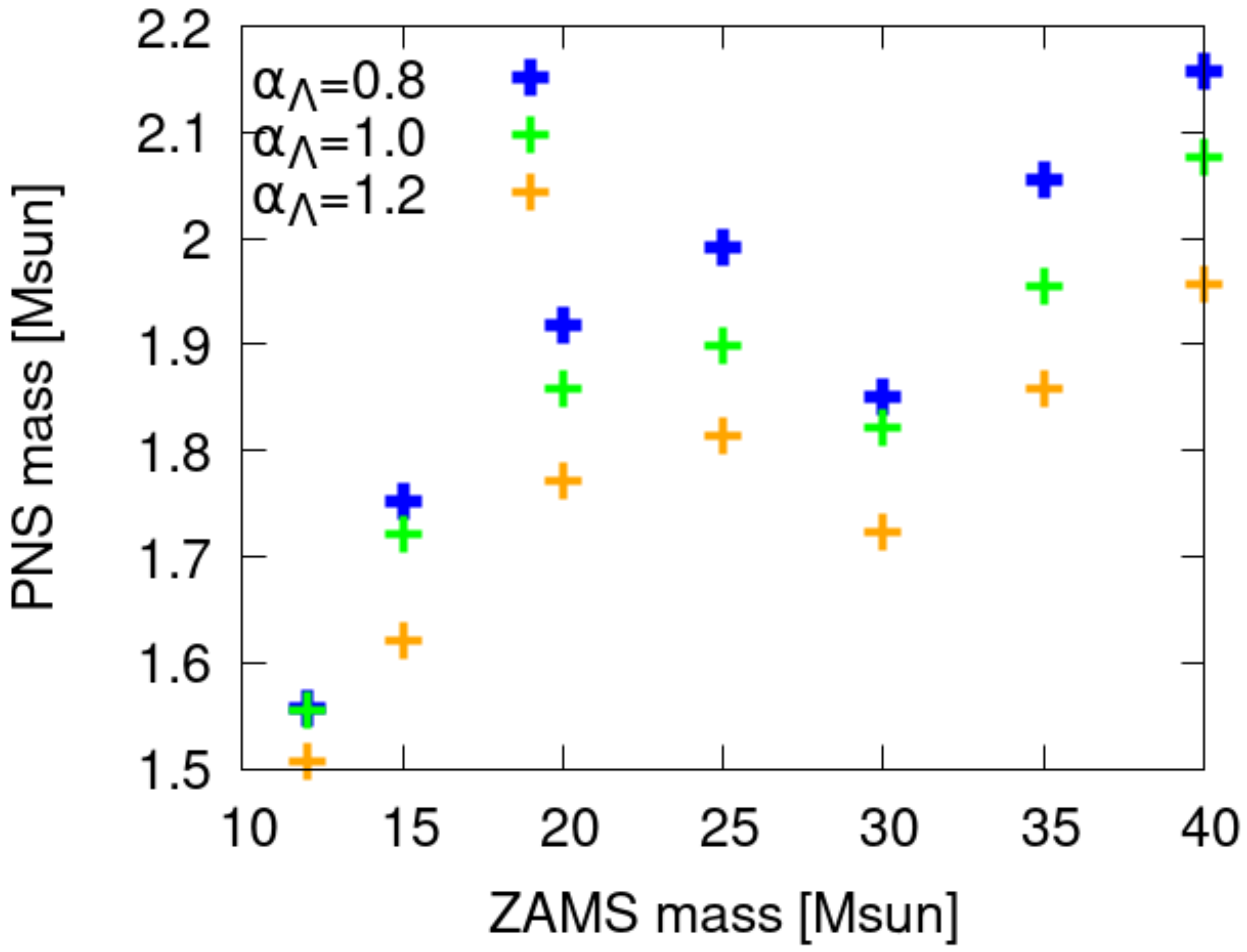
$E_{\rm exp}\left(t_{\rm b} = 0.5\right)$

Failure to explode : $E_{\rm exp} = 0$,

$$\alpha_\Lambda \uparrow \quad \Rightarrow \quad E_{\rm exp} \uparrow$$

Turbulent parameter governs the progenitor dependence of $E_{\rm exp}$

PNS mass　(baryonic mass)
Turbulent parameter dependence

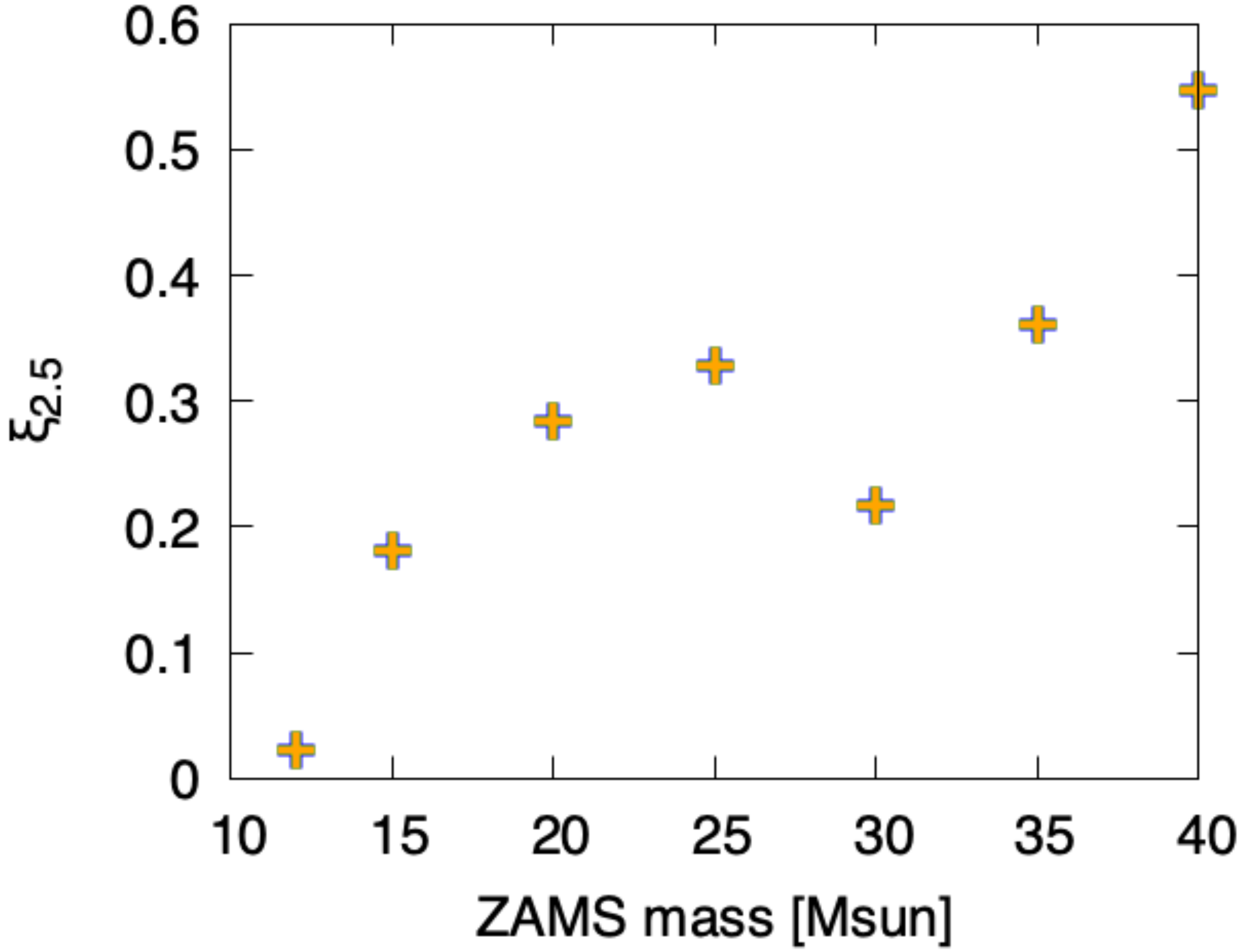


Progenitor :$12,15,20,25,30,35,40\, M_\odot$

$M_{\rm PNS}$ at $t_{\rm b} = 0.5$

$\alpha_\Lambda \uparrow \quad \Rightarrow \quad M_{\rm PNS} \downarrow$

compactness

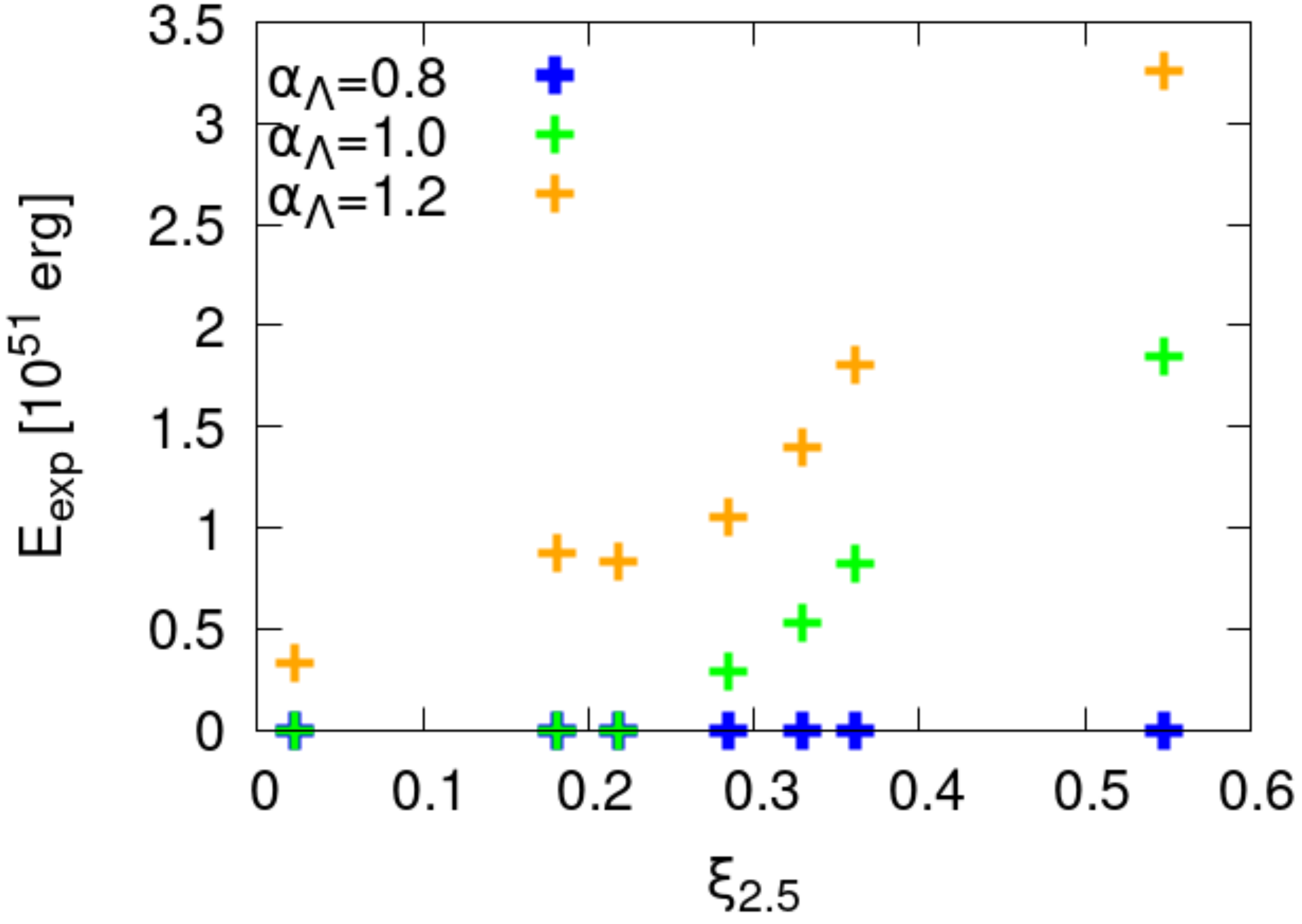


Compactness : $\xi_M = \dfrac{M/M_\odot}{R/1000\mathrm{km}}$

Progenitor : $12, 15, 20, 25, 30, 35, 40\, M_\odot$

Dependence of compactness

Explosion energy



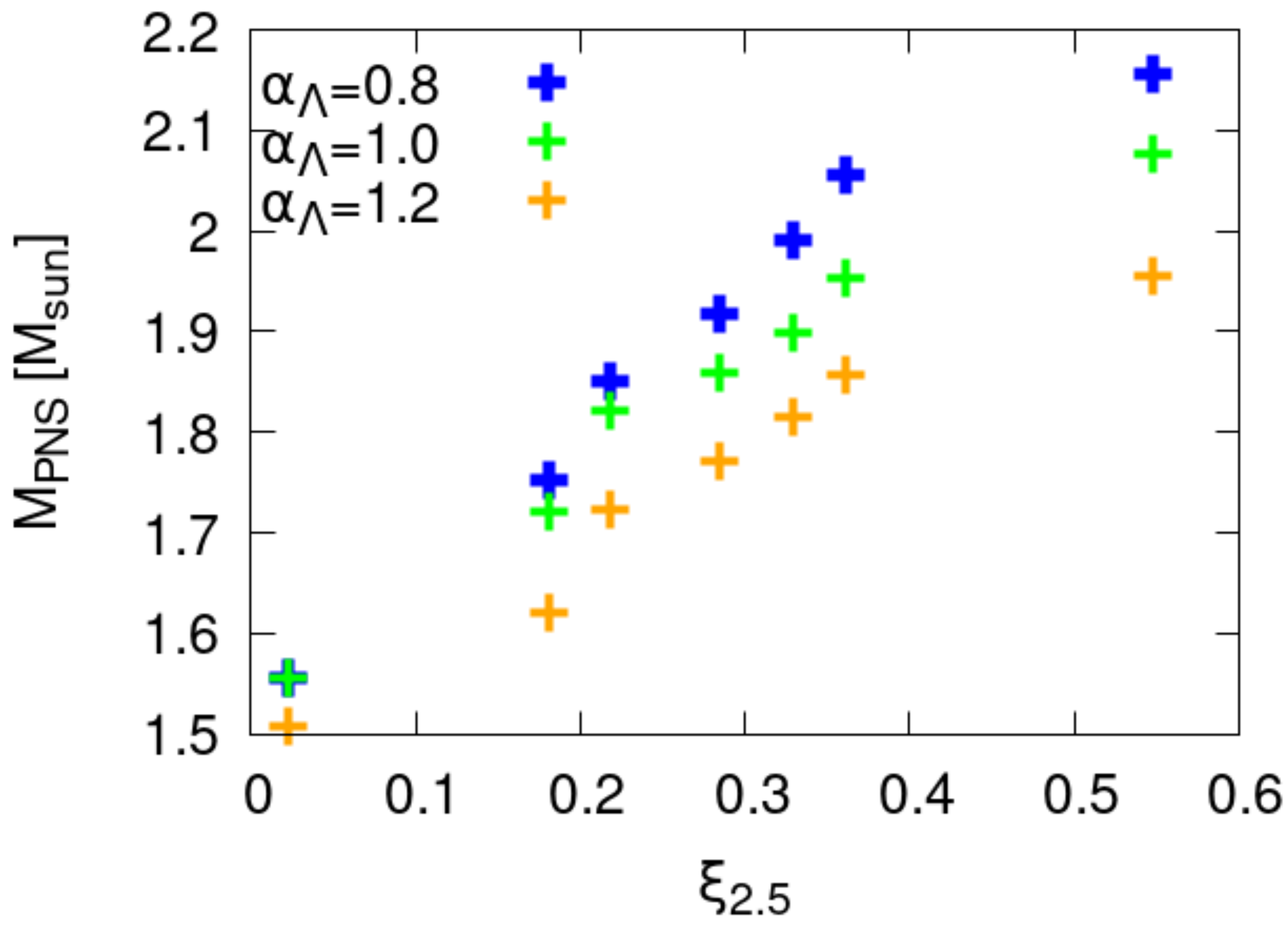
Progenitor: $12, 15, 20, 25, 30, 35, 40\, M_{\odot}$

$E_{\rm exp}\left(t_{\rm b} = 0.5\right)$

Failure to explode : $E_{\rm exp} = 0$,

Turbulent parameter governs the compactness dependence of $E_{\rm exp}$

Dependence of compactness
PNS mass　(baryonic mass)



Progenitor : $12,15,20,25,30,35,40\,M_\odot$

M_{PNS} at $t_{\mathrm{b}} = 0.5$

Summary of progenitor dependence

Explodability depends on turbulent parameter α_Λ.

Strong turbulence , which means large α_Λ , trigger strong explosion.

Focusing on ξ_M ,

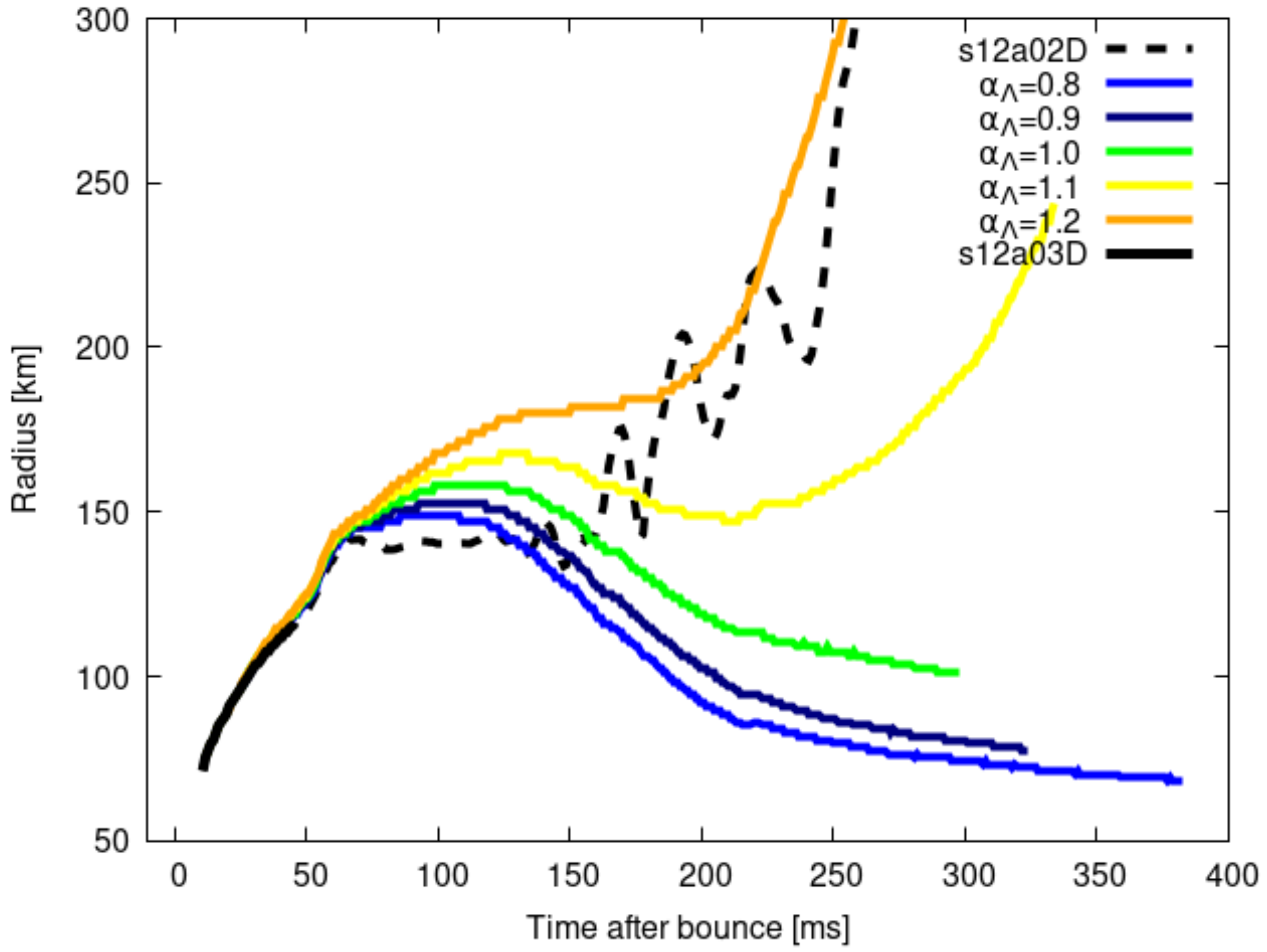
Compactness dependence of $E_{\rm exp}$ is effected by turbulence parameter, α_Λ.

Compact dependence of $M_{\rm PNS}$ is not effected by turbulence parameter, α_Λ.

Contents

Results

Evolution of Shock



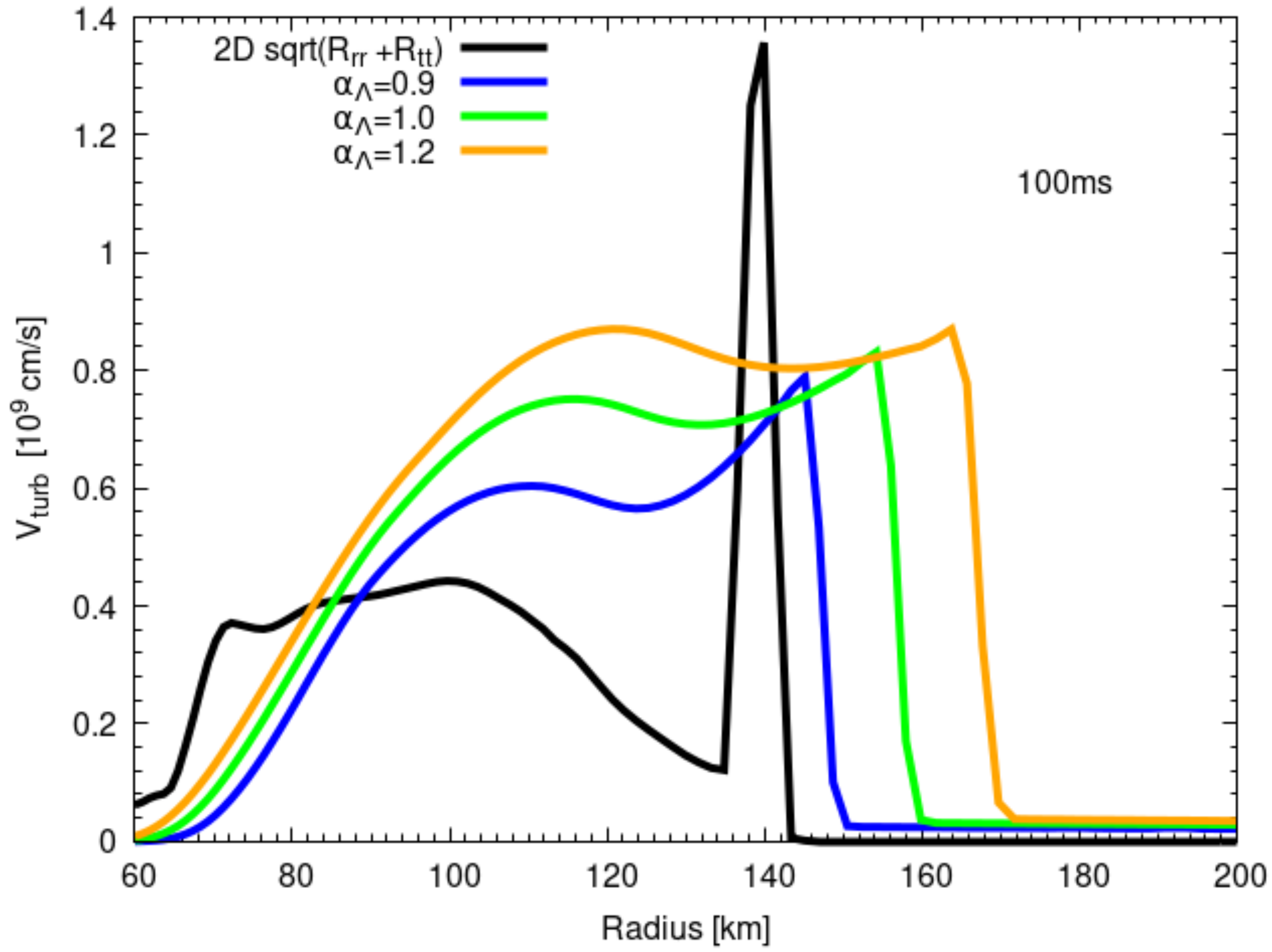
1D+ has some tasks to reproduce 3D results accurately.

One is fitting the shock propagation.

In our 1D+, turbulent parameter effects prompt convection.

Results

Turbulent velocity



Turbulent velocity of 2D;

$\sqrt{R_{rr} + R_{\theta\theta}}$

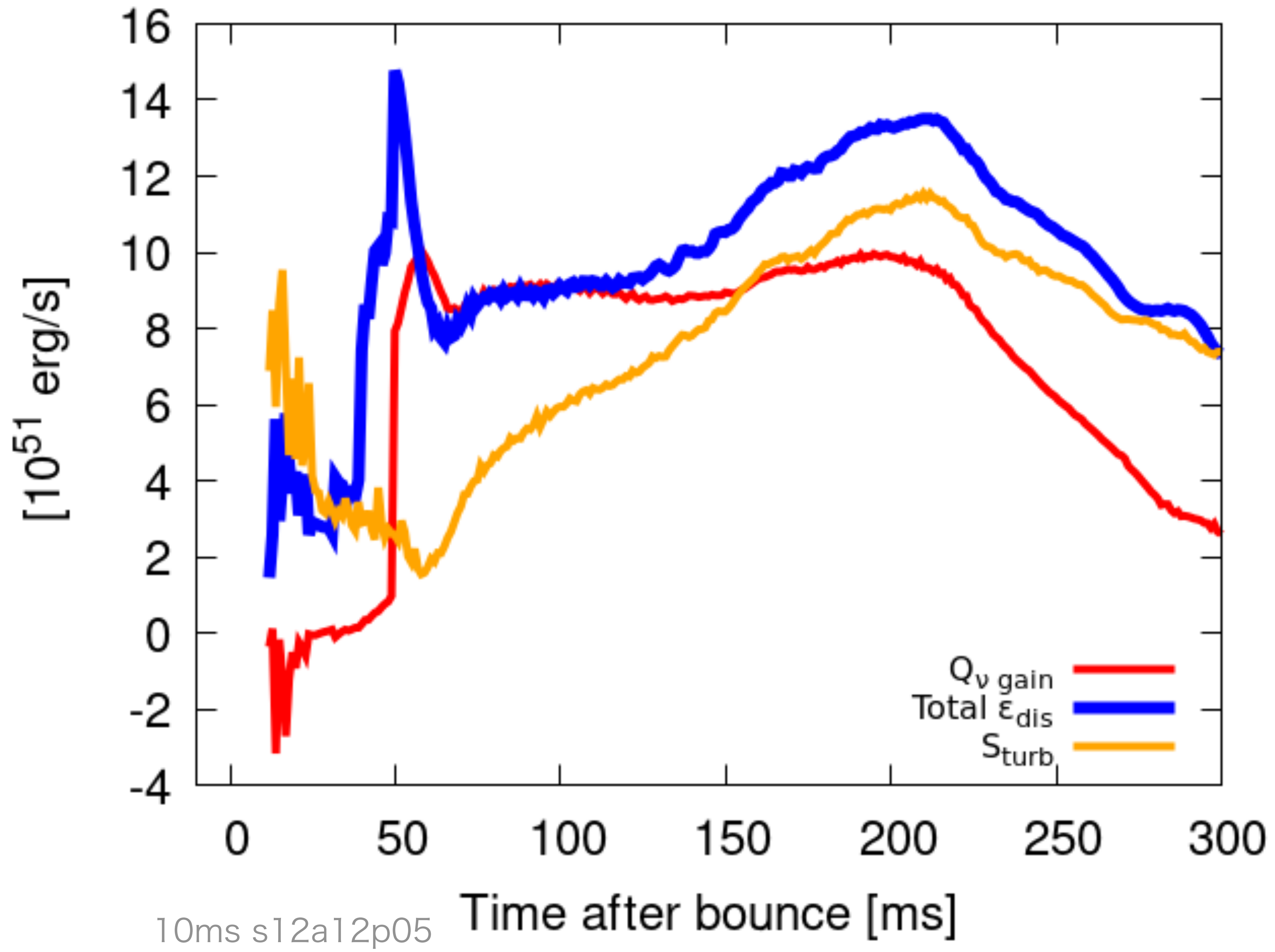
$R_{rr} = v_r' v_r' = (v_r - < v_r >)(v_r - < v_r >)$

Turbulent velocity of 1D+ is larger than that of 2D.

Results

Source terms of turbulent energy in gain region

$$\frac{\partial \rho v_{turb}^2}{\partial t} \ldots = -\rho v_{\mathrm{turb}}^2 \frac{1}{r^2}\frac{\partial (r^2 v_r)}{\partial r} + \rho v_{\mathrm{turb}} \omega_{\mathrm{BV}}^2 \Lambda_{\mathrm{mix}} - \rho \frac{v_{\mathrm{turb}}^3}{\Lambda_{\mathrm{mix}}}$$



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Source term of turbulent energy eq. :

$$S_{\mathrm{turb}} = \rho v_{\mathrm{turb}} \omega_{\mathrm{BV}}^2 \Lambda_{\mathrm{mix}}, \quad \epsilon_{\mathrm{dis}} = | -\rho \frac{v_{\mathrm{turb}}^3}{\Lambda_{\mathrm{mix}}} |$$

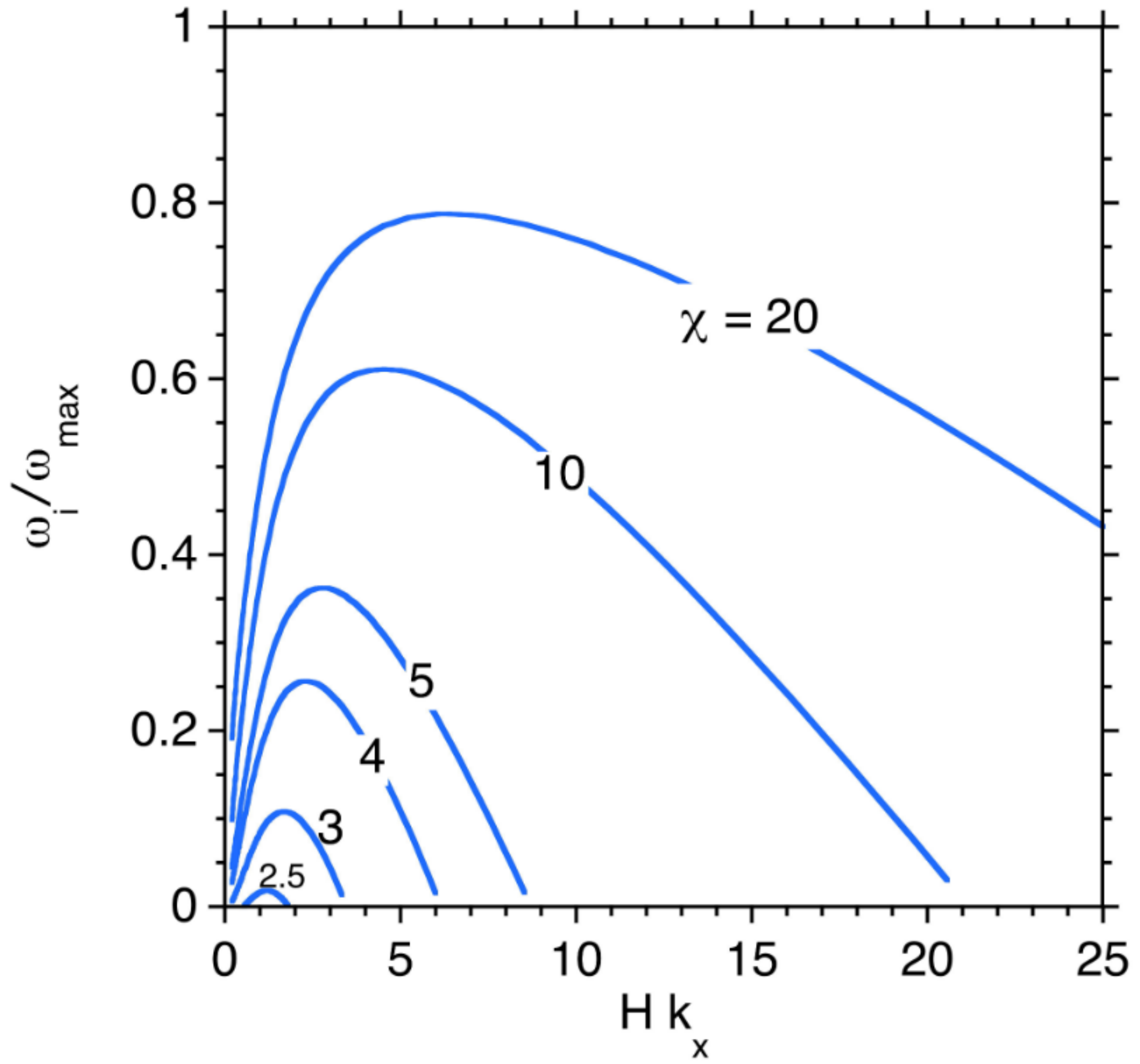
In our 1D+, dissipation term, ϵ_{dis}, is large because of prompto convection.

Discussion of turbulent effects

Phenomenological turbulent model of 1D+ is simple model. This is based on Mixing Length Theory(MLT).

However, turbulence of 3D depends on wave number. (In our model, turbulence is evaluated by only one velocity, $v_{\rm turb}$)

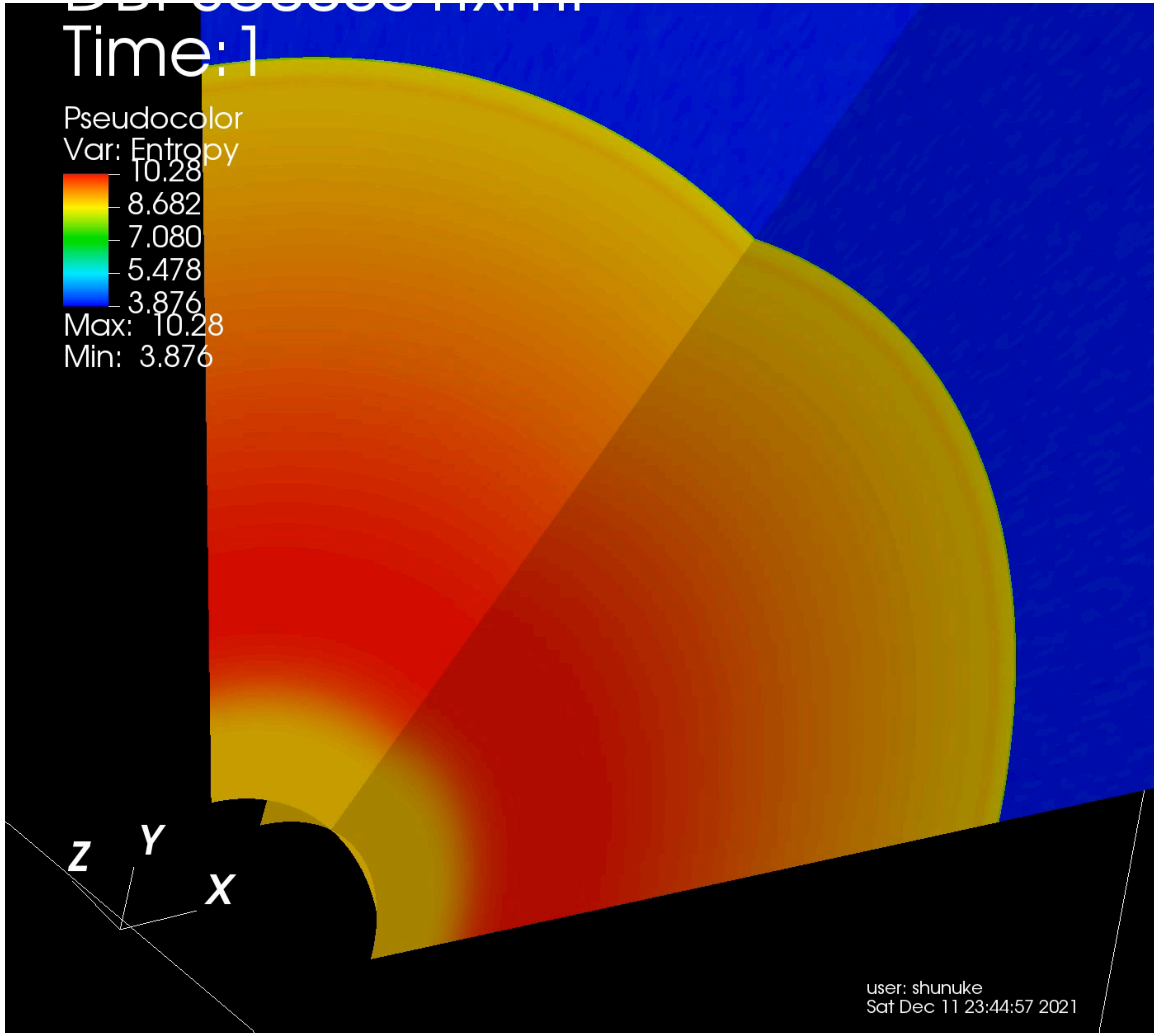
Source of turbulent energy should be also corrected.



Kazeroni et al. 2018

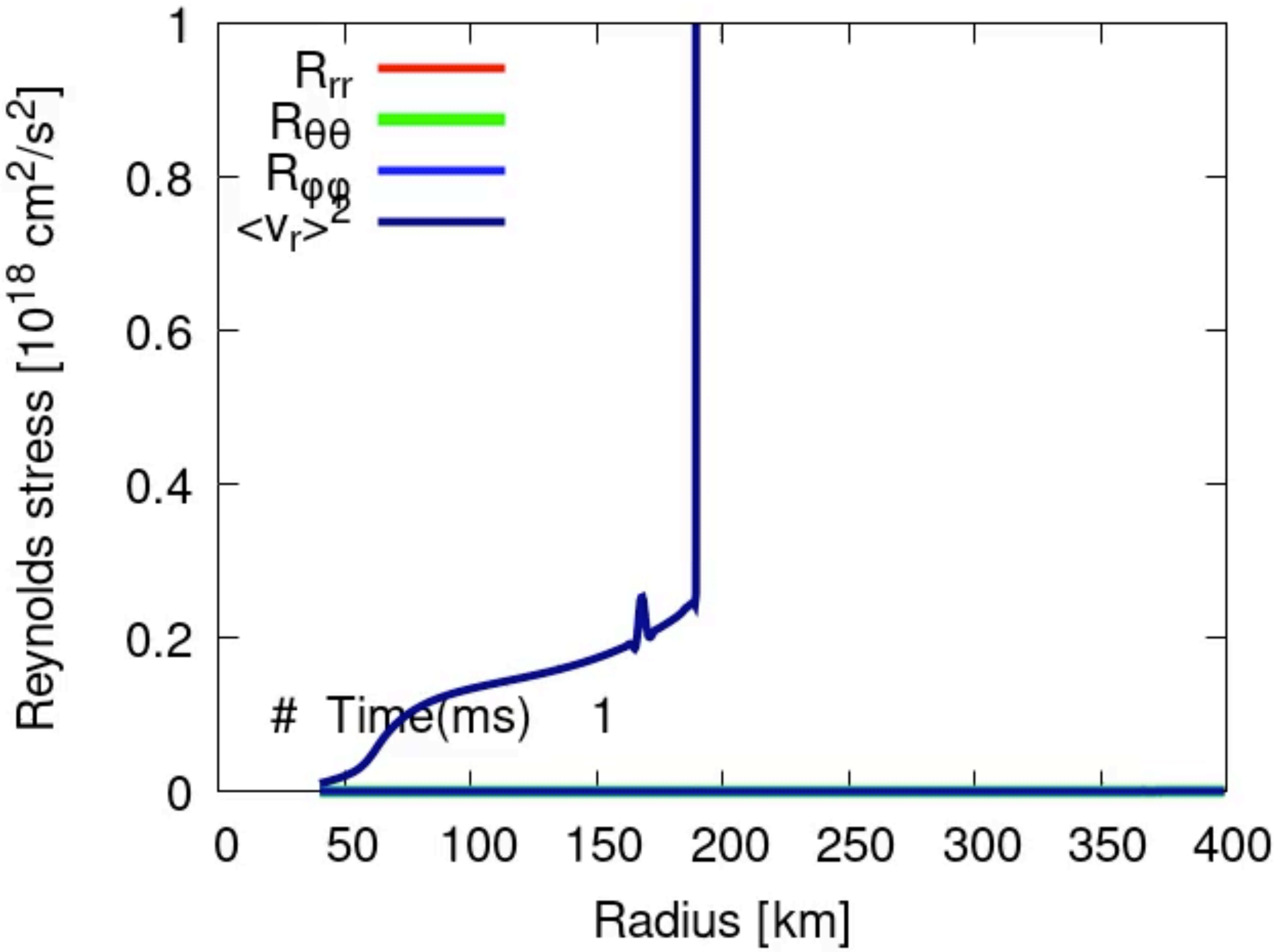
Next work

Analyzing turbulent effects with 3D



Next work

Analyzing turbulent effects with 3D



Summary

Motivation :predicting progenitor dependence

Today's talk : turbulence effects of 1D+

Results : turbulent parameter effects the progenitor dependence of ξ_M

In our 1D+ model,

Next steps are ..

Appendix

Energy source of CCSNe

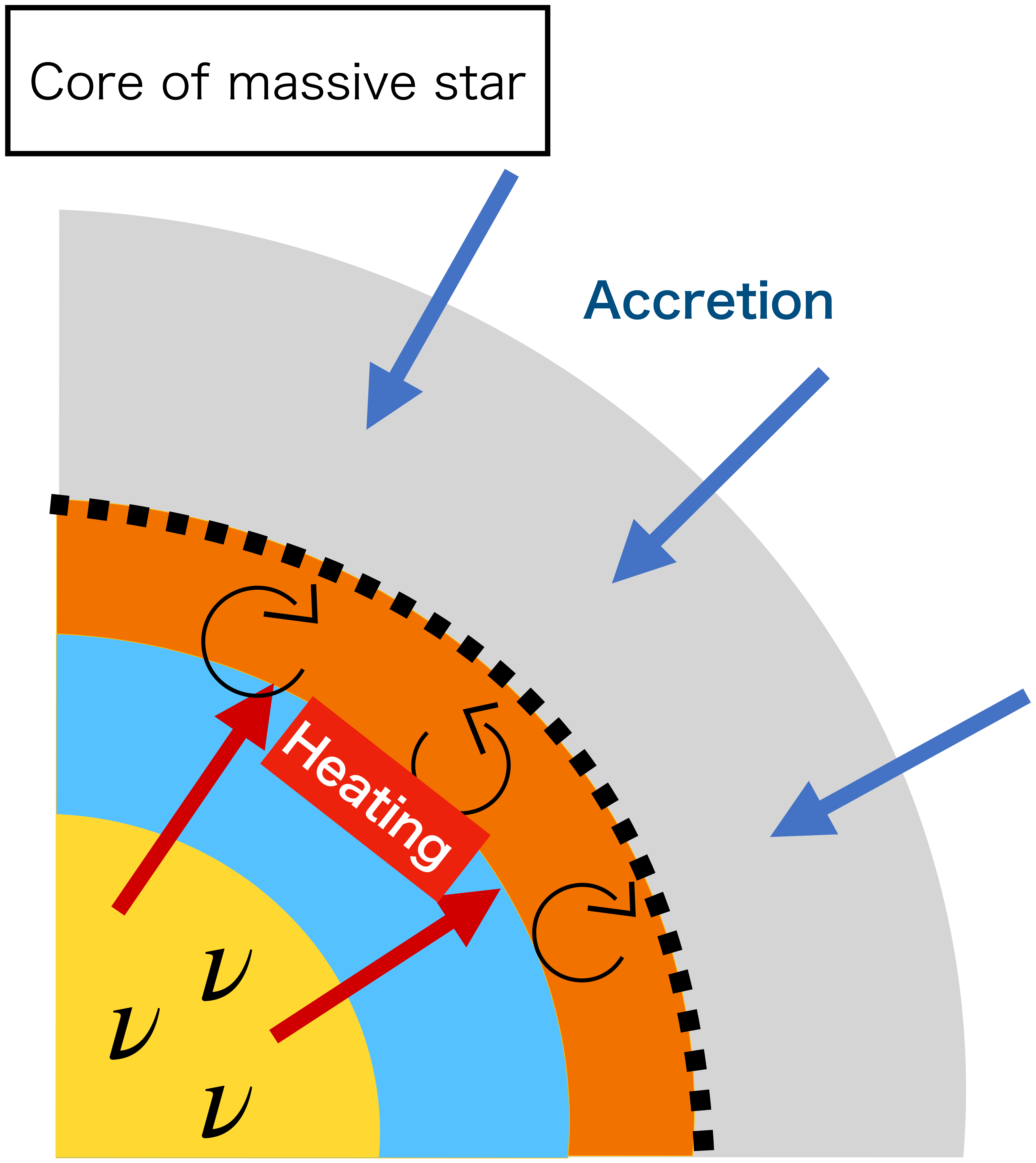
Energy source of a supernova explosion is the gravitational energy E_g released by gravitational collapse. When a core of mass M_c collapses, E_g is

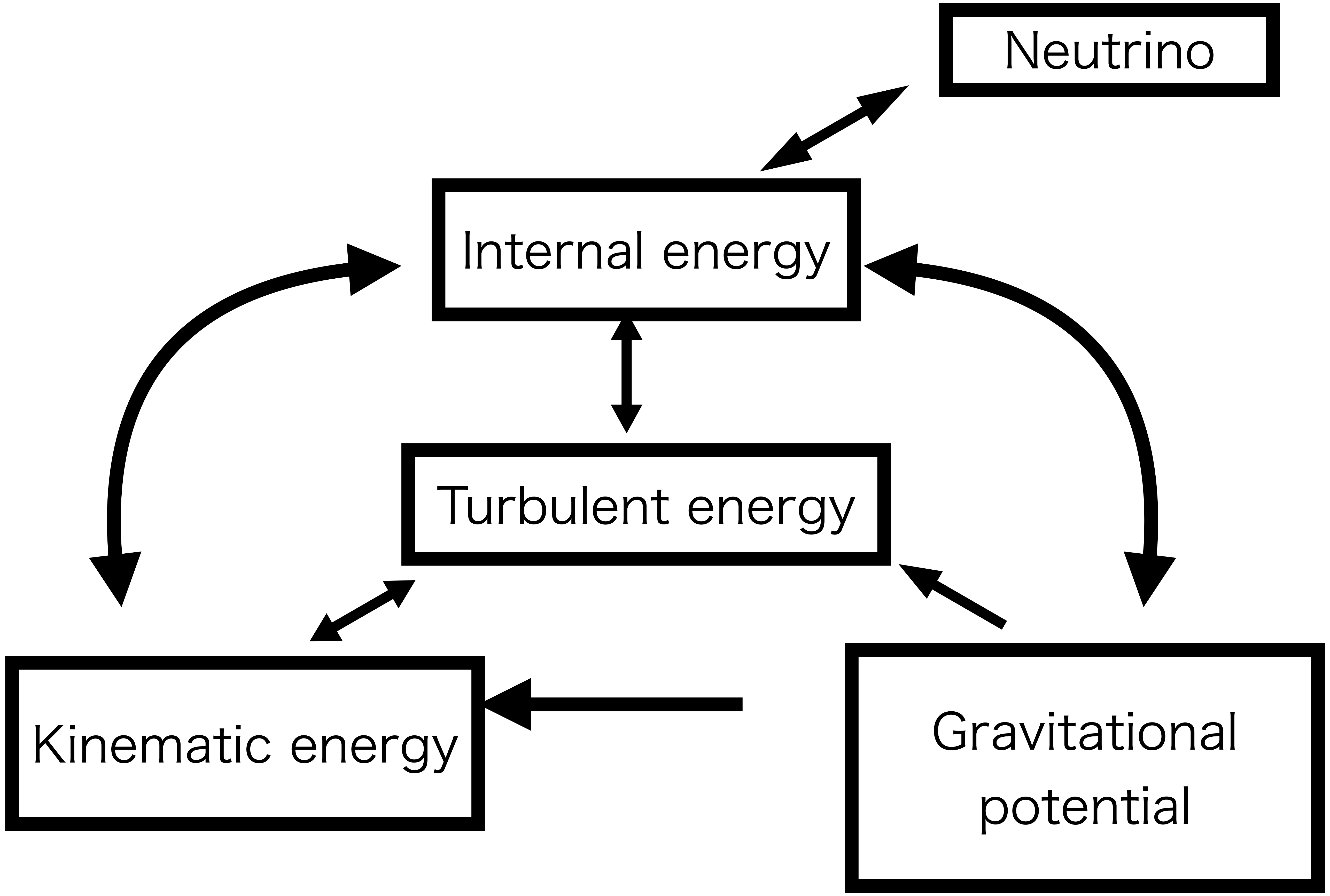
$$E_g = \left(-\frac{GM_c^2}{R_i}\right) - \left(\frac{GM_c^2}{R_f}\right) \sim \left(\frac{GM_c^2}{R_f}\right) = 3 \times 10^{53} \left(\frac{M_c}{M_\odot}\right)^2 \left(\frac{R_f}{10\mathrm{km}}\right)^{-1} \ [\mathrm{erg}]$$

$R_{i,f}$: initial (final) radius of core. ($R_i >> R_f$)

E_g is exchanged to internal energy , neutrino energy and kinematic energy. (Look the slide of mechanism)

Energy Exchange and mechanism





CCSN simulation calculates these energy exchanges.

Definition of $E_{\rm exp}, M_{\rm Ni}, < E_\nu > , M_{\rm PNS}$

$$E_{\rm exp} = \int_D \rho\epsilon + \rho v^2/2 + e_{\rm turb} + \rho\Phi \, {\rm d}V$$

$\int_D dV$: Integral of non-binding region

$$M_{\rm Ni} = \int_D \hat{\rho}\hat{X}_{\rm Ni}{\rm d}V$$

$\hat{X}_{\rm Ni}$: mass fraction of nickel

$$\langle E_\nu \rangle_{tot} = \int_{t_b=0}^{t_b=0.5} L_\nu dt$$

L_ν : neutrino luminosity

$$M_{\rm PNS} = \int_0^{R_{\rm PNS}} 4\pi r^2 \rho dr \quad , R_{PNS} = R(\rho = 10^{11}\,[{\rm g/cm^3}])$$

These properties are important to evaluate supernova mechanism.

Appendix : calculation of $f_{\rm GW}$ with 1D+

Equation 3 in Sotani et al. 2021

1D+ can evaluate PNS mass $M_{\rm PNS}$ and PNS radius R_{PNS} from density distribution.

Finding the radius that is $\rho = 10^{11}$ [g/cm^3] → R_{PNS} $M_{\rm PNS} = \int_0^{R_{\rm PNS}} 4\pi r^2 \rho dr$

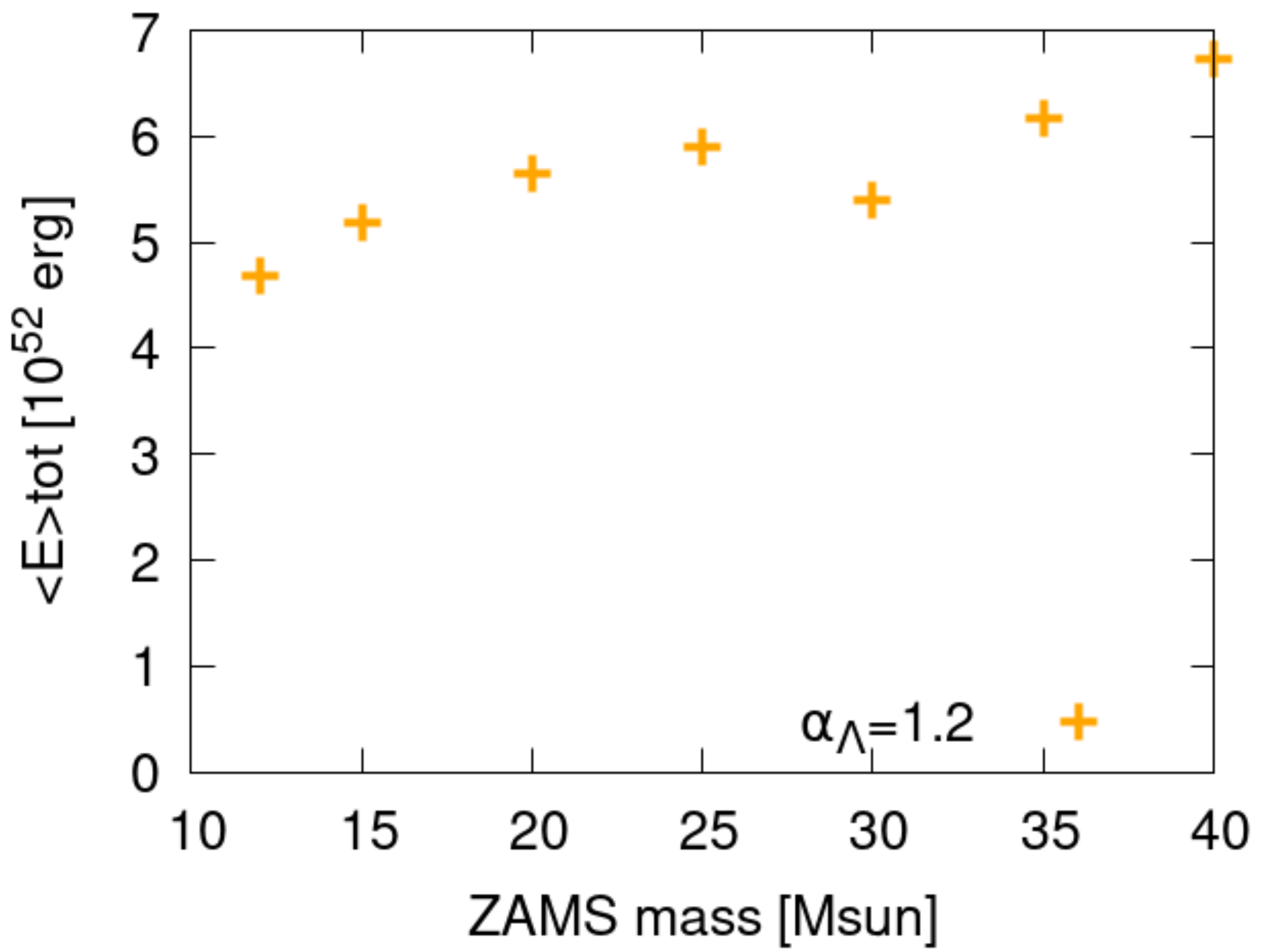
Derive the GW frequency $f_{\rm GW}$ from the following equation (Sotani et al. 2021)

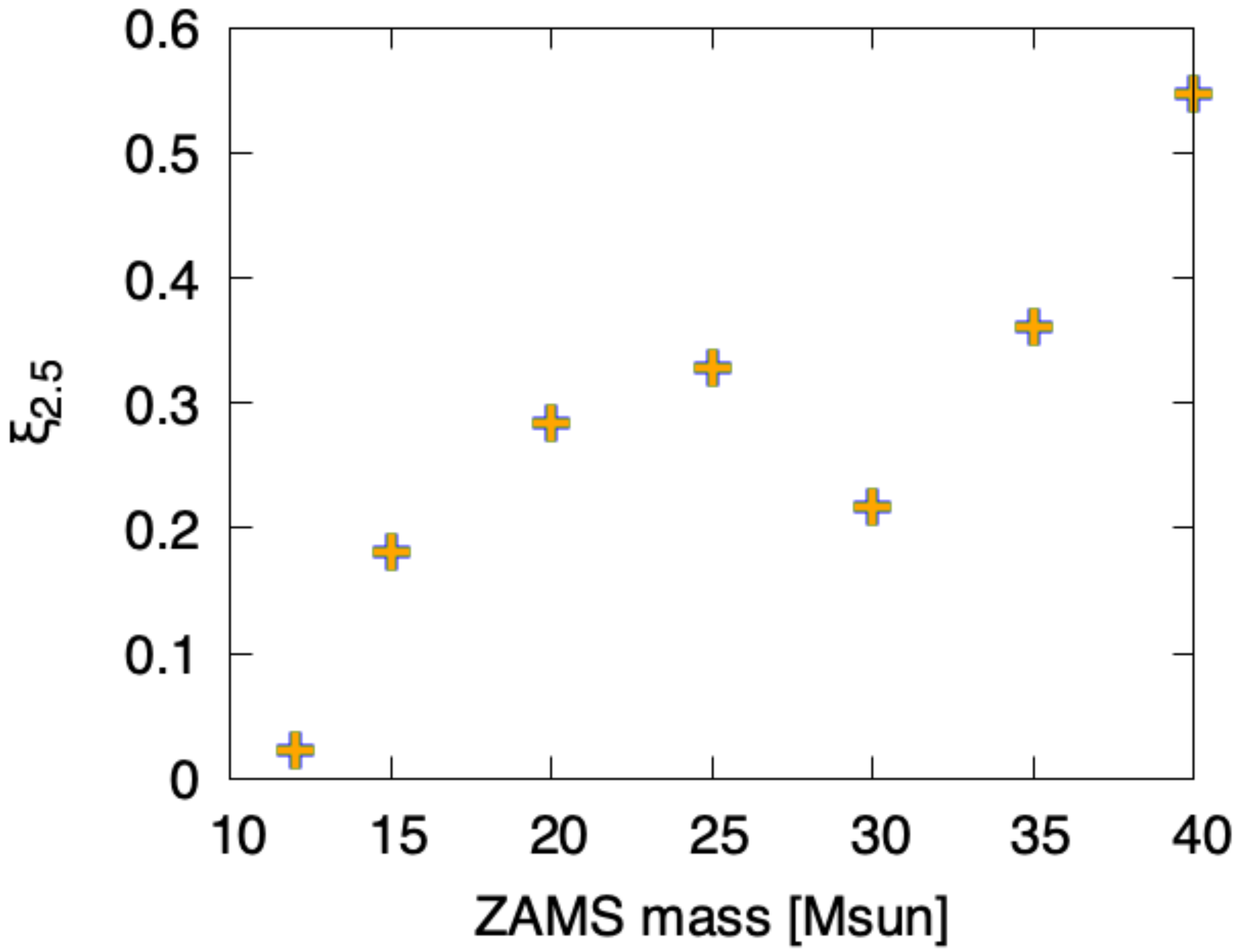
$$f[\mathrm{kHz}] = -1.410 - 0.443\ln(x) + 9.337x - 6.714x^2\ ,\ x \equiv \left(\frac{M_{\rm PNS}}{1.4M_\odot}\right)^{1/2}\left(\frac{R_{\rm PNS}}{10\ \mathrm{km}}\right)^{-3/2}$$

Total emitted neutrino energy

$$\langle E_\nu \rangle_{tot} = \int_{t_b=0}^{t_b=0.5} L_\nu dt$$

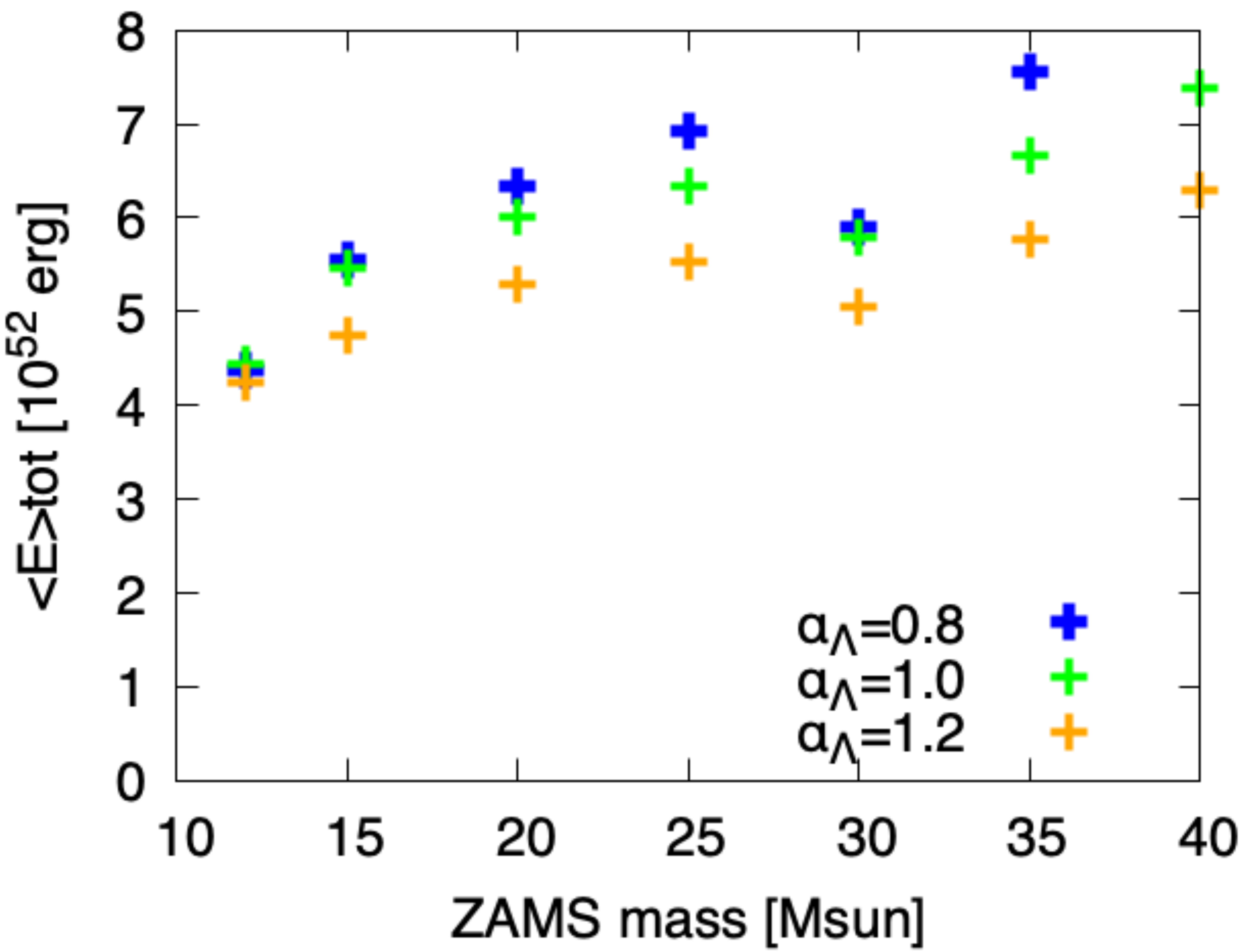
Comparison with Compactness





The progenitor dependence of $< E_\nu >_{\rm tot}$ tends to be similar to compactness.

Total emitted neutrino energy



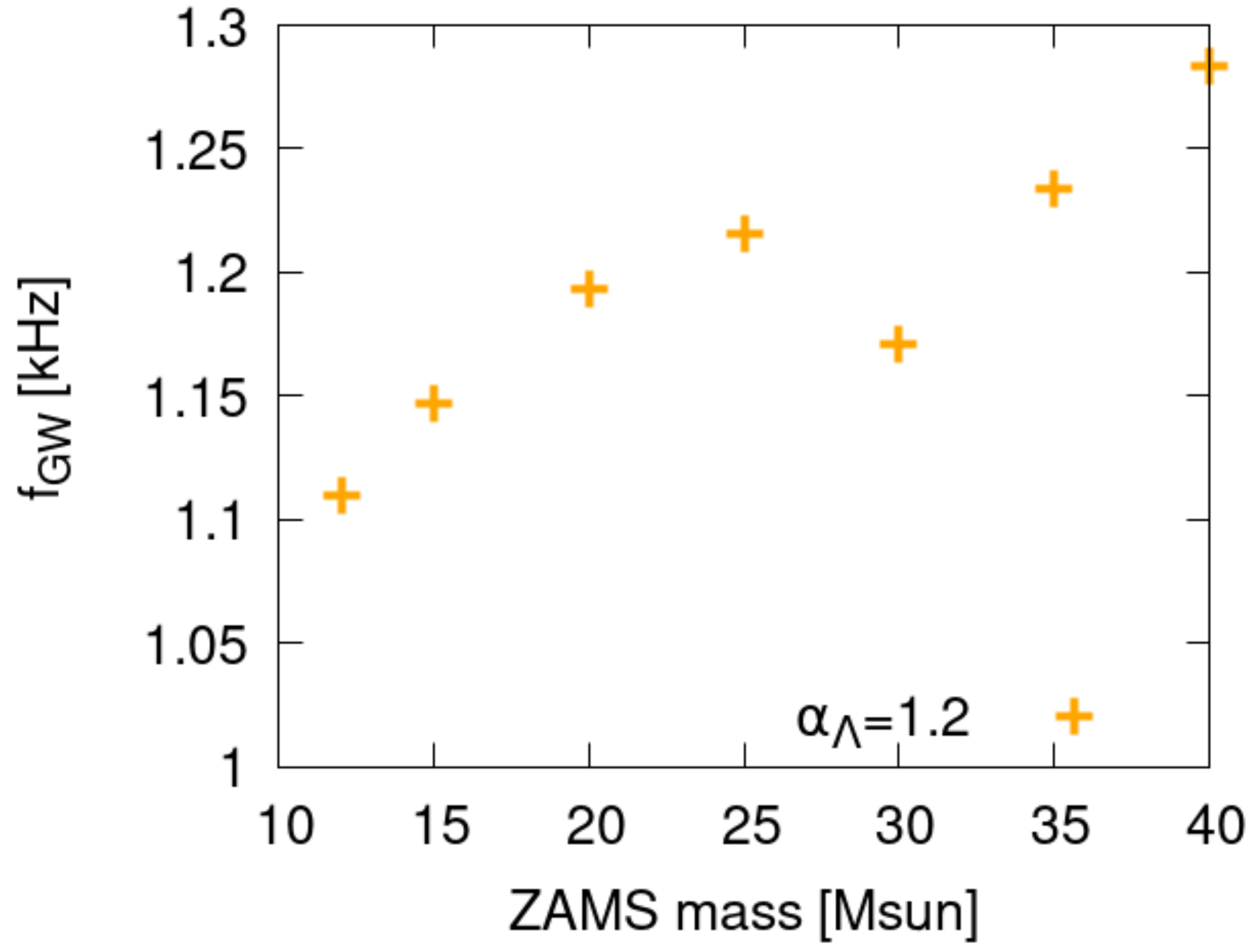
Progenitor :$12,15,20,25,30,35,40\,M_{\odot}$

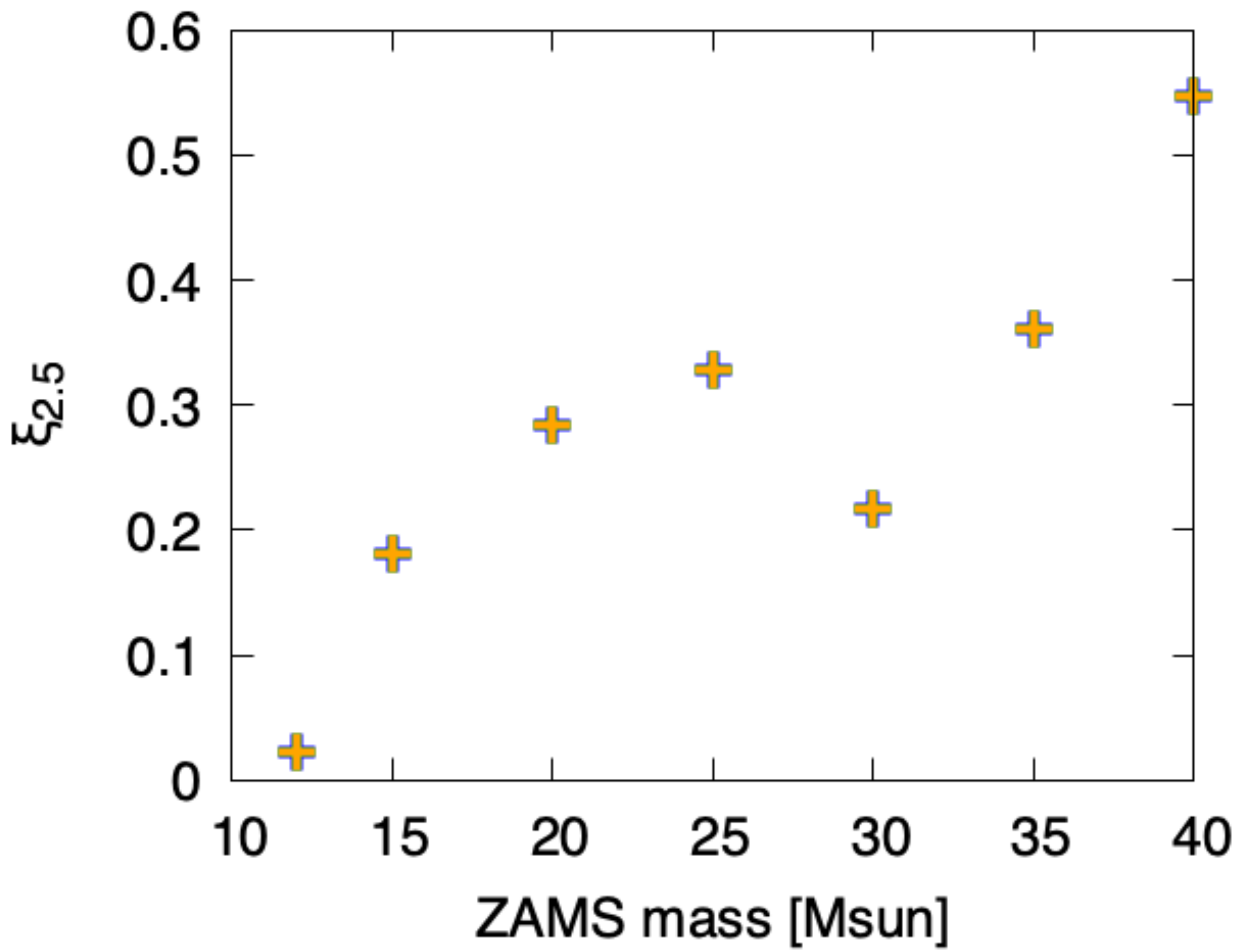
$$\langle E_\nu \rangle_{tot} = \int_{t_b=0}^{t_b=0.5} L_\nu dt$$

$$\alpha_\Lambda \uparrow \quad \Rightarrow \quad \langle E_\nu \rangle_{tot} \downarrow$$

Frequency of GW

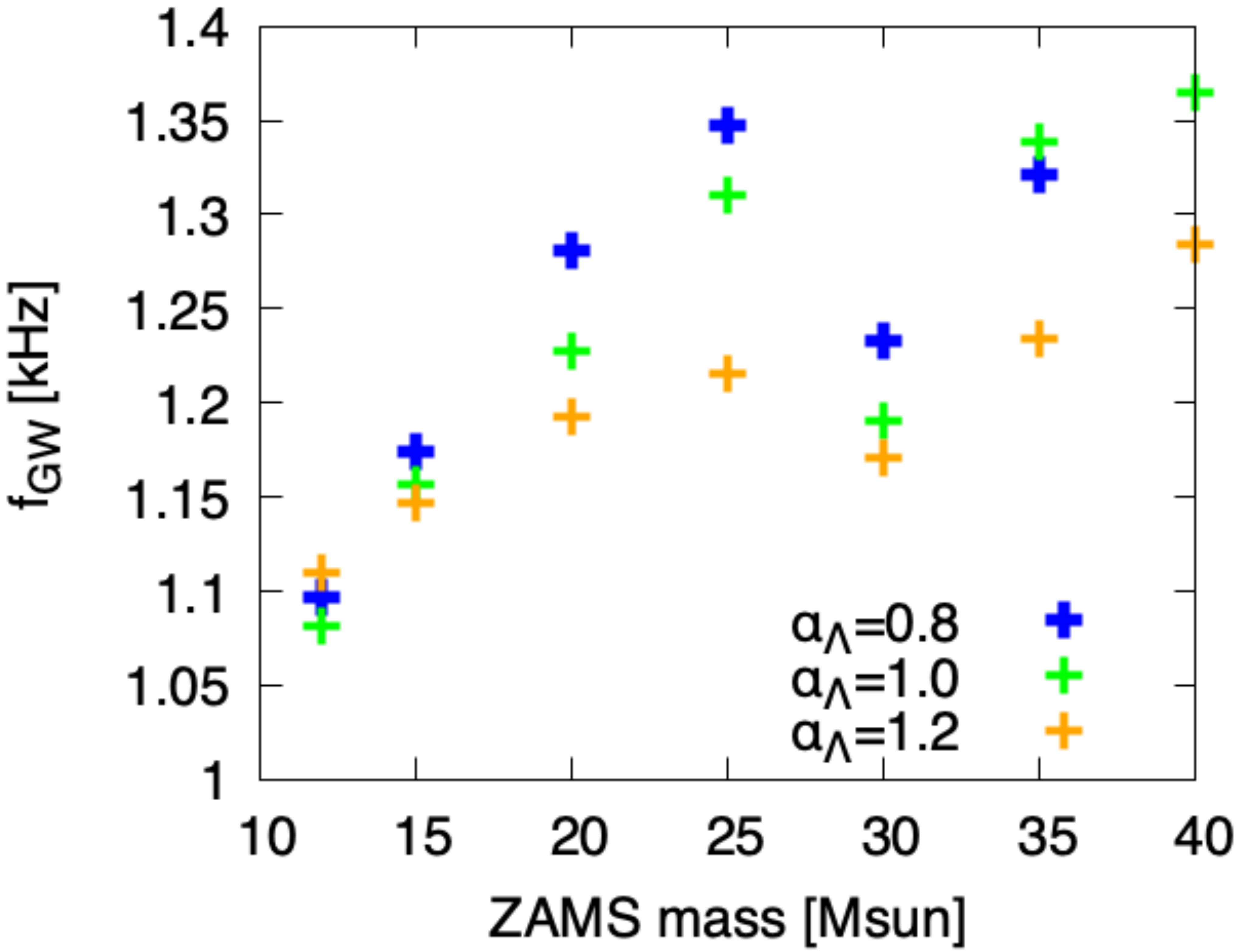
Comparison with Compactness





The progenitor dependence of f_{GW} tends to be similar to compactness.

Frequency of GW
Turbulent parameter dependence



Progenitor:$12,15,20,25,30,35,40\, M_\odot$

$$f_{\rm GW}\left(t_{\rm b}=0.5\right)$$

$$\alpha_\Lambda \uparrow \quad \Rightarrow \quad f_{\rm GW} \downarrow$$

Correlation study

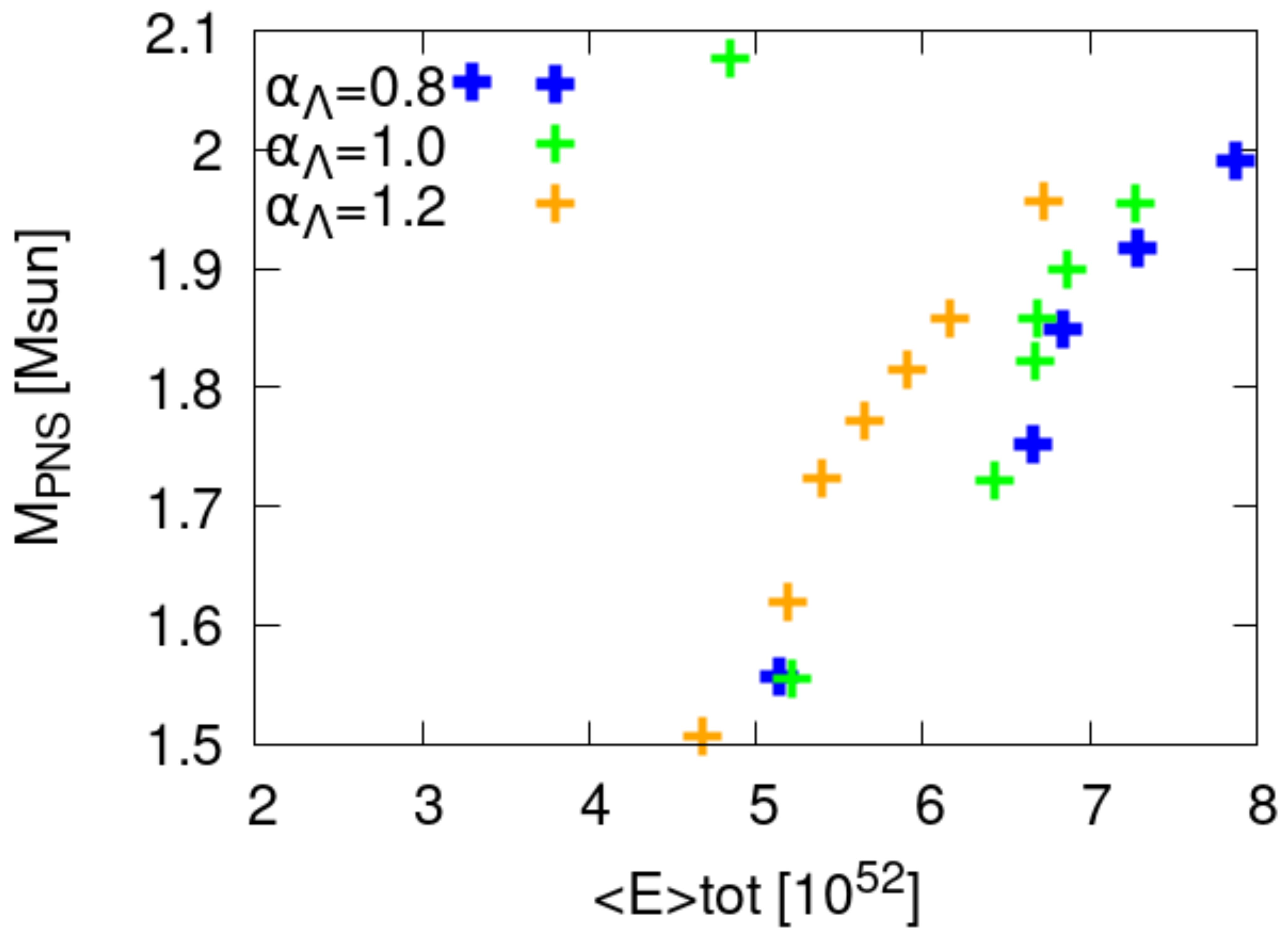
Table of correlation coefficient

The following is a test calculation of correlation coefficients

	ZAMSmass	compactness	PNSmass	<E>	Eexp	t(r_s=400)	f_GW
ZAMSmass	1.000	0.869	0.901	0.881	0.847	0.259	0.876
ompactness	0.869	1.000	0.985	0.987	0.950	0.397	0.972
PNSmass	0.901	0.985	1.000	0.968	0.903	0.398	0.994
<E>	0.881	0.987	0.968	1.000	0.974	0.329	0.946
Eexp	0.847	0.950	0.903	0.974	1.000	0.186	0.861
t(r_s=400)	0.259	0.397	0.398	0.329	0.186	1.000	0.485
f_GW	0.876	0.972	0.994	0.946	0.861	0.485	1.000

Correlation study

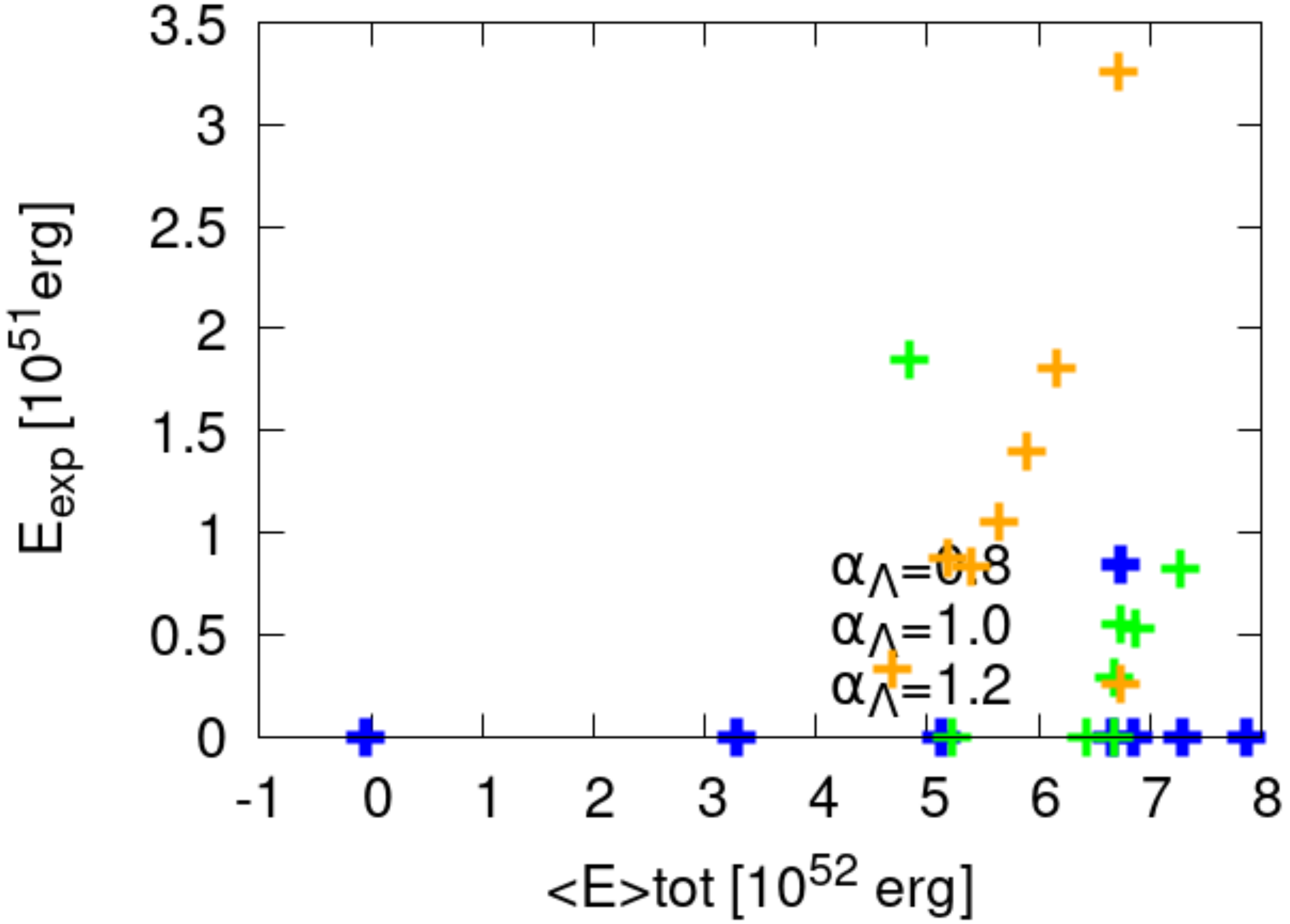
Correlation with Total neutrino energy

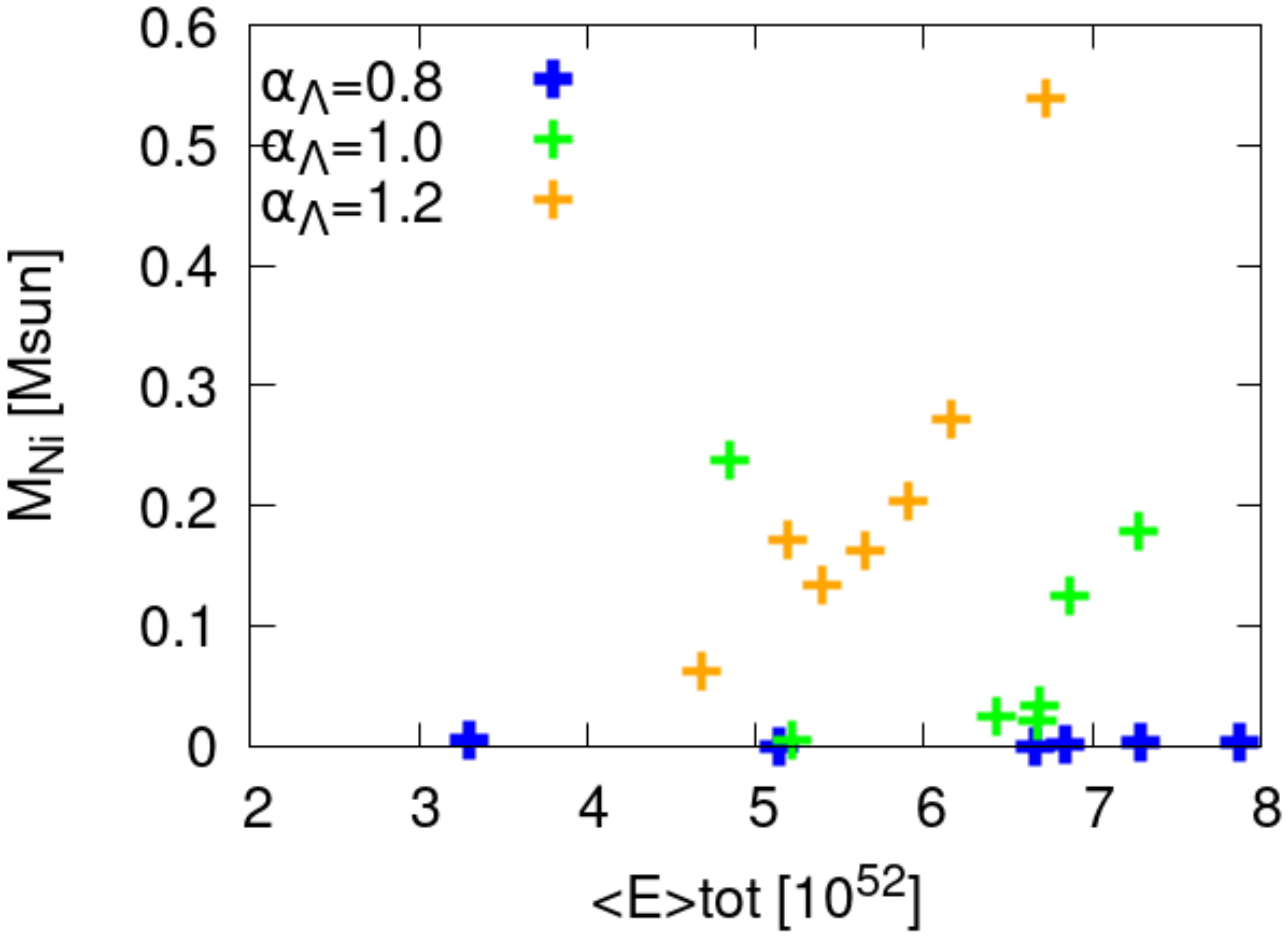


$M_{\rm PMS}$ **has strongly positive correlation with** $<E_\nu>_{\rm tot}$ **.**

Correlation study

Correlation with Total neutrino energy



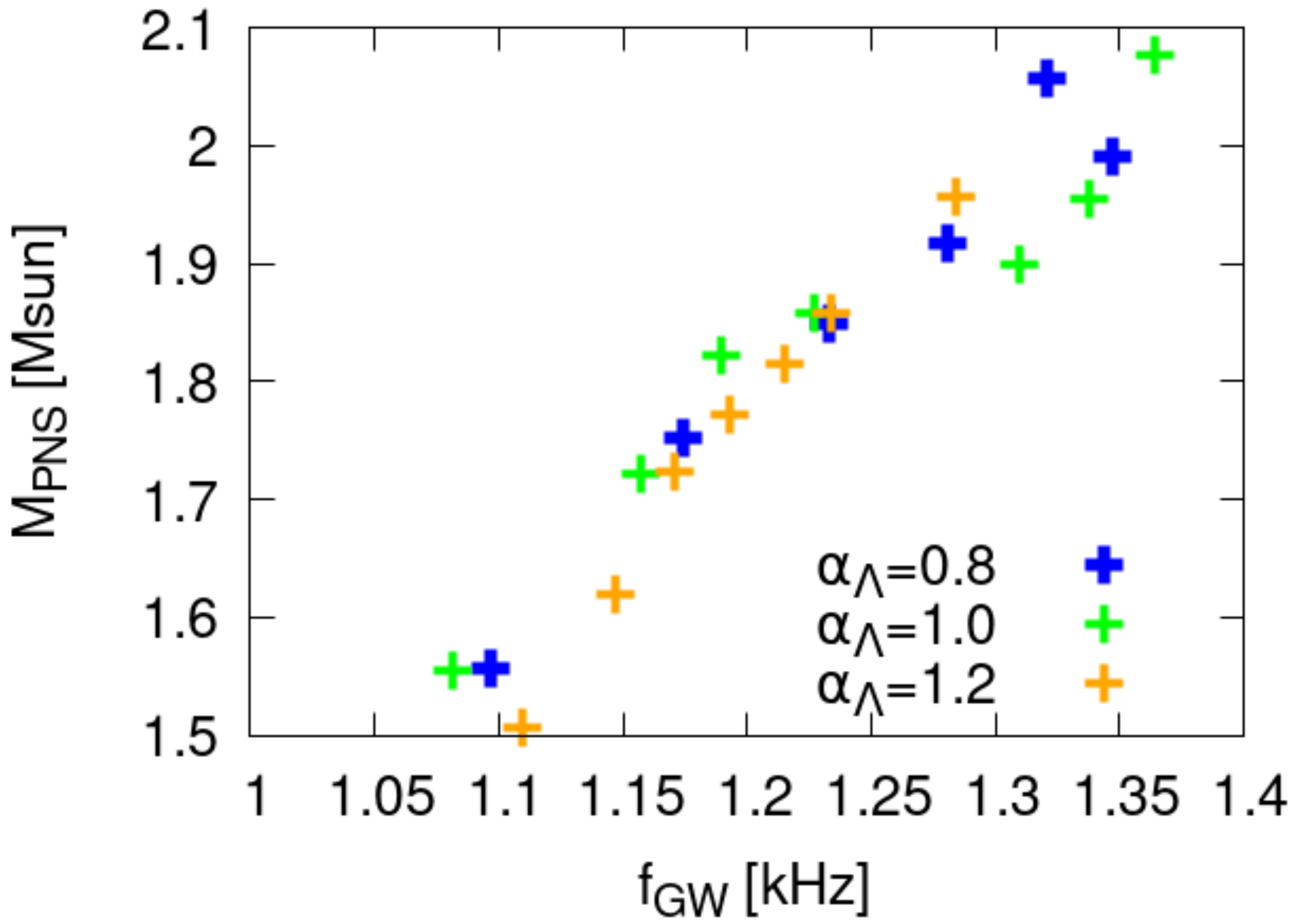


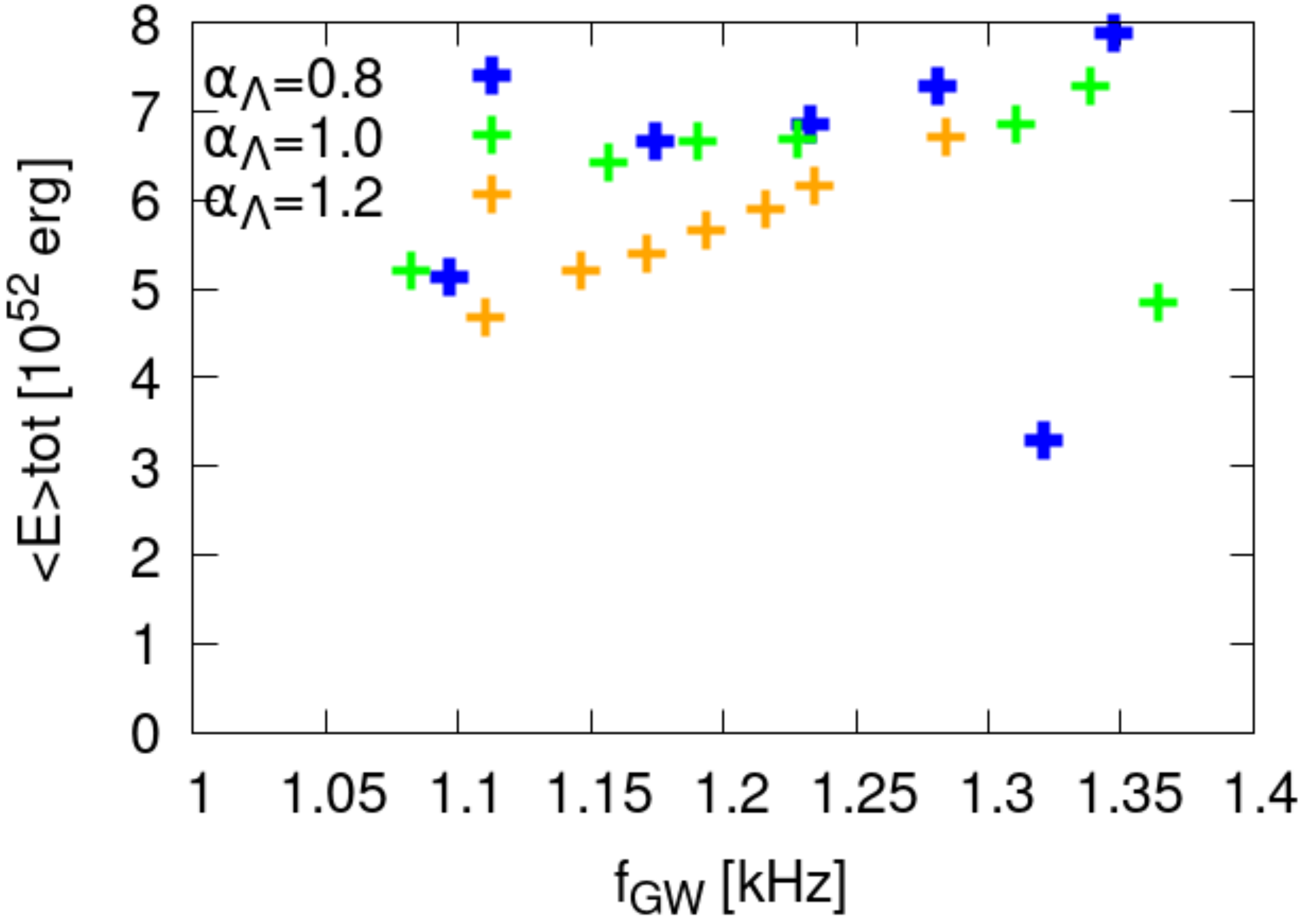
E_{exp} and M_{Ni} are positively correlated with $< E_\nu >_{\mathrm{tot}}$.

But, these correlations depend turbulent parameter.

Correlation study

Correlation with GW frequency

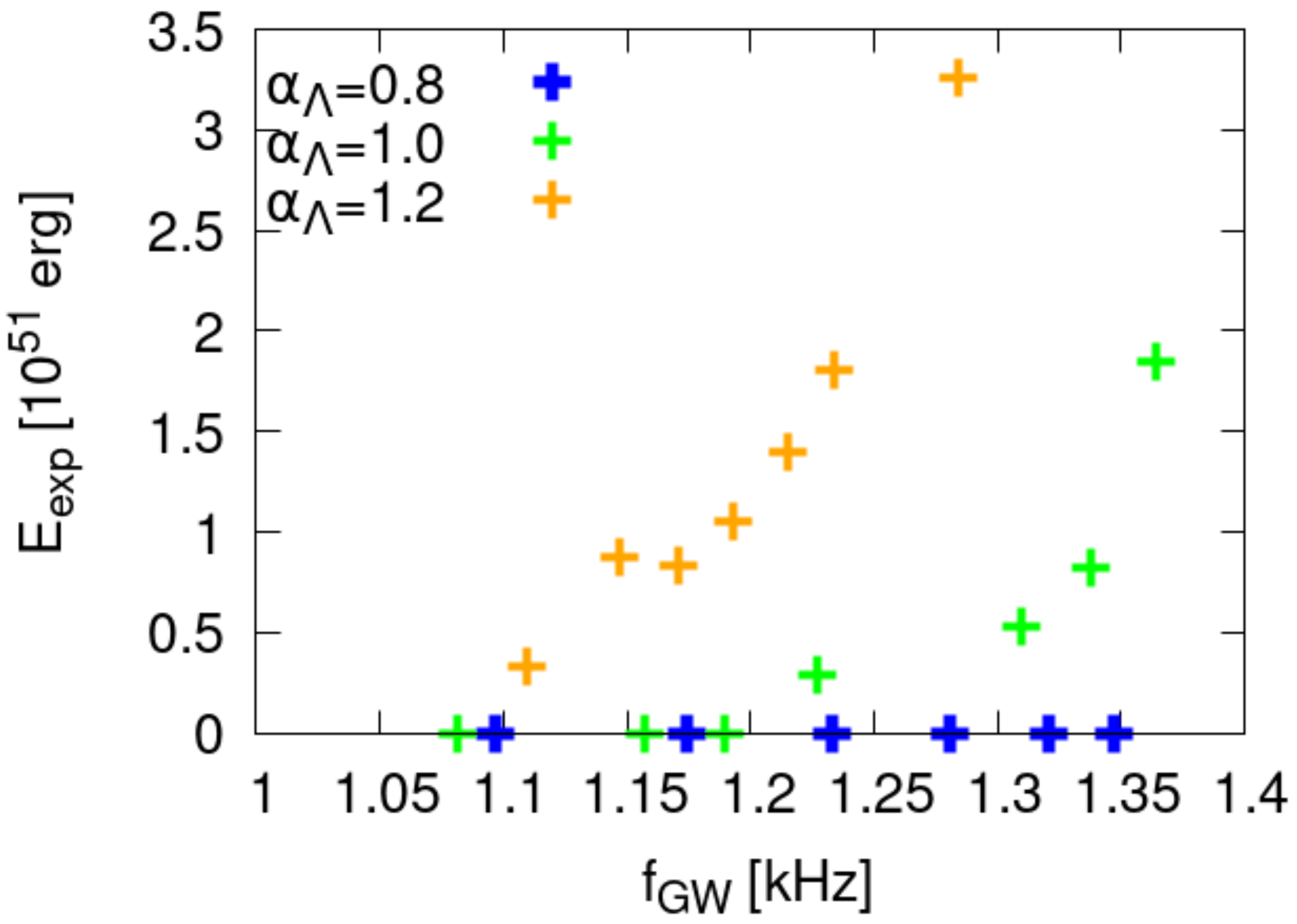


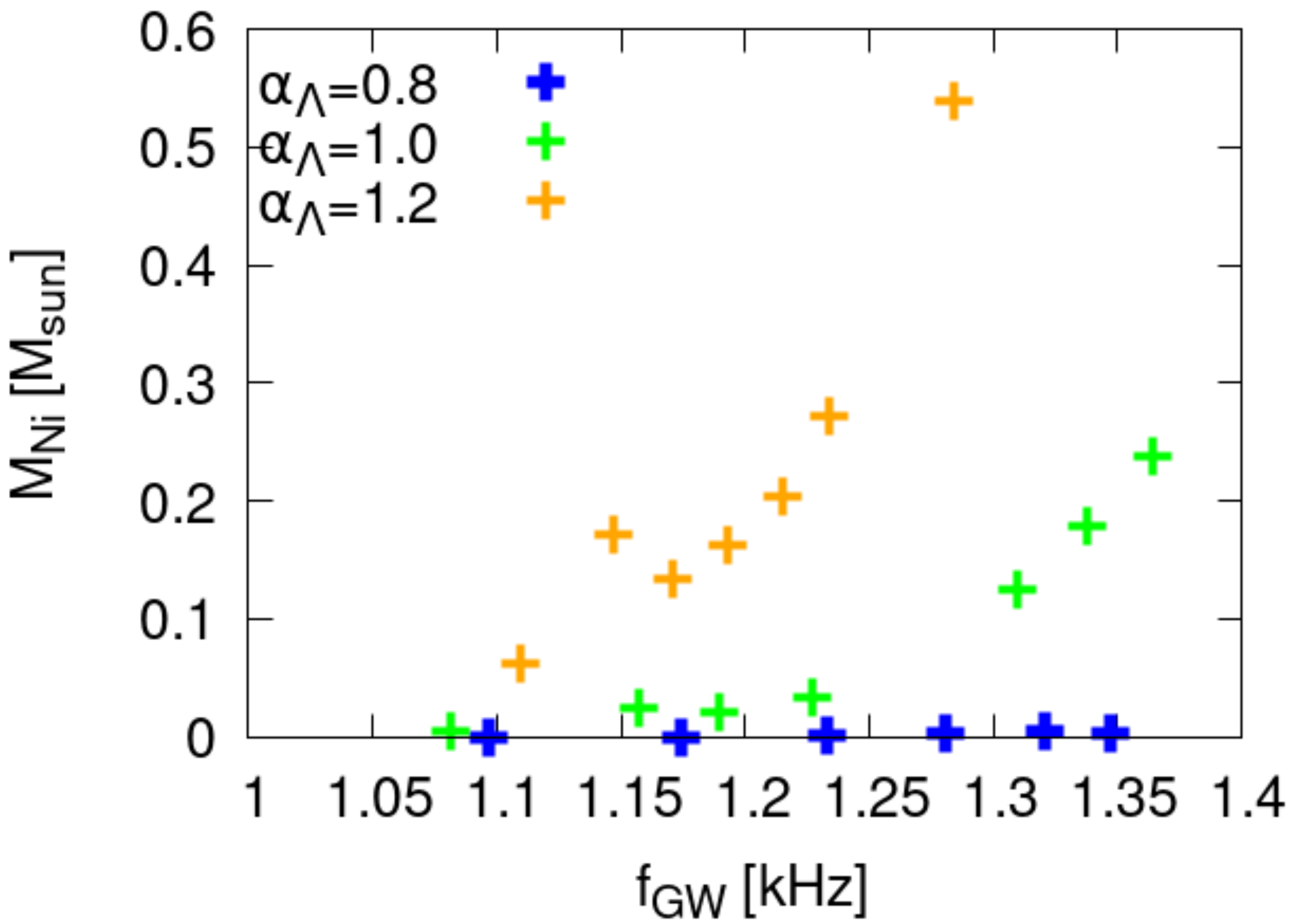


M_{PMS} **has strongly positive correlation with** f_{GW}**.**

Correlation study

Correlation with GW frequency





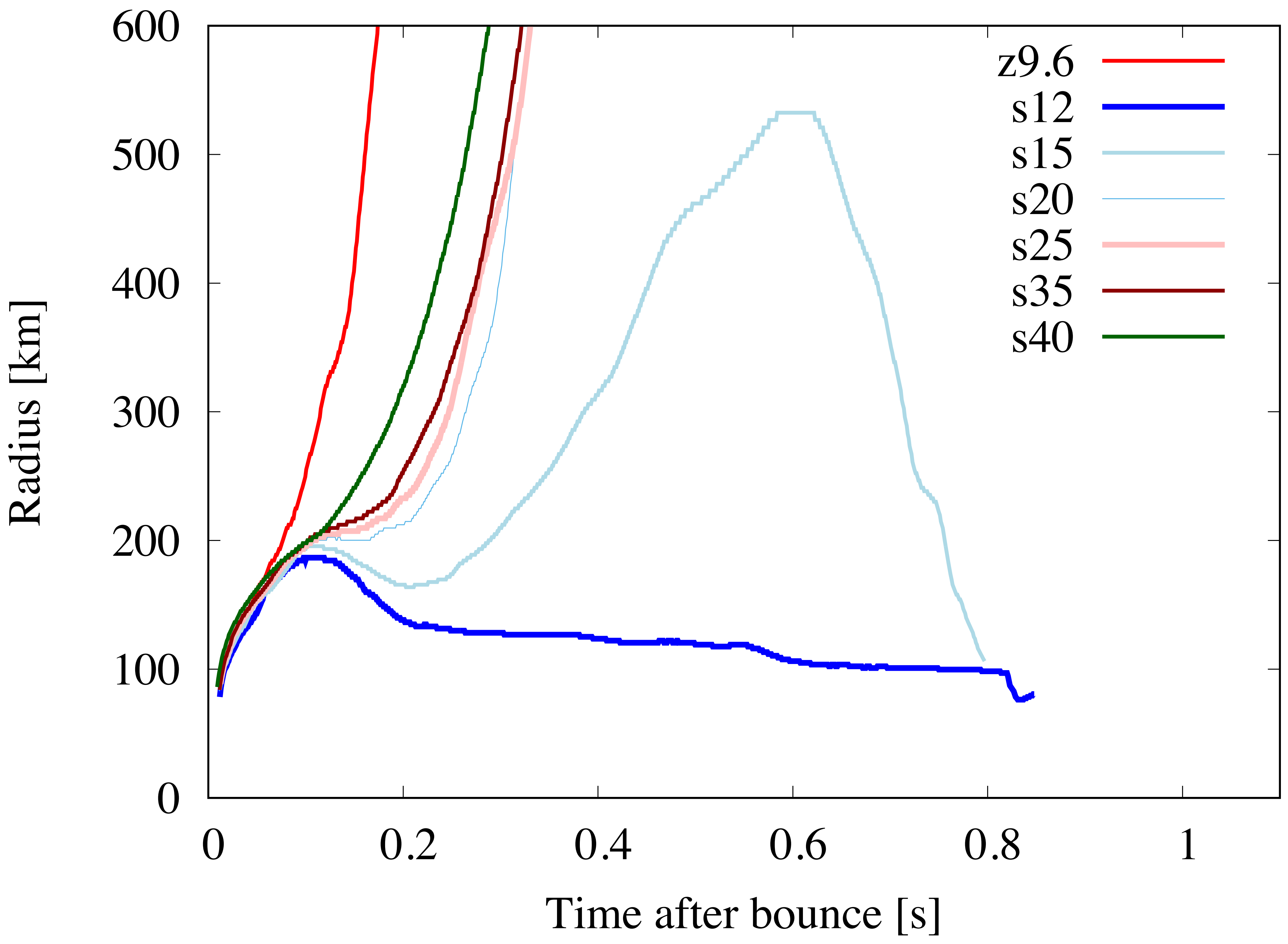
$E_{\rm exp}$ and $M_{\rm Ni}$ are positively correlated with $f_{\rm GW}$.

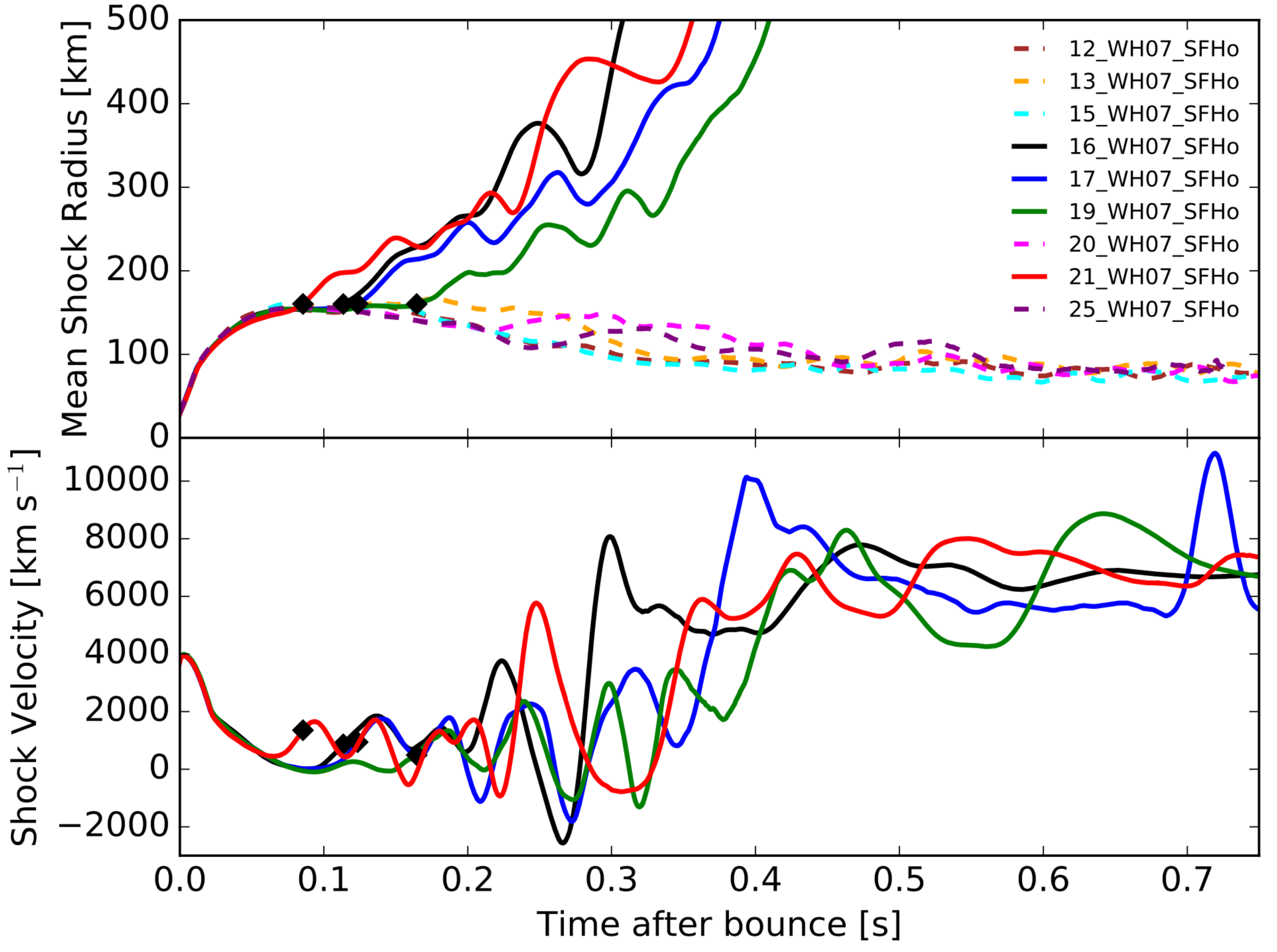
But, these correlations depend turbulent parameter.

Discussion

Compared to 3D

Results of 2D (Vartanyan et al. 2018)

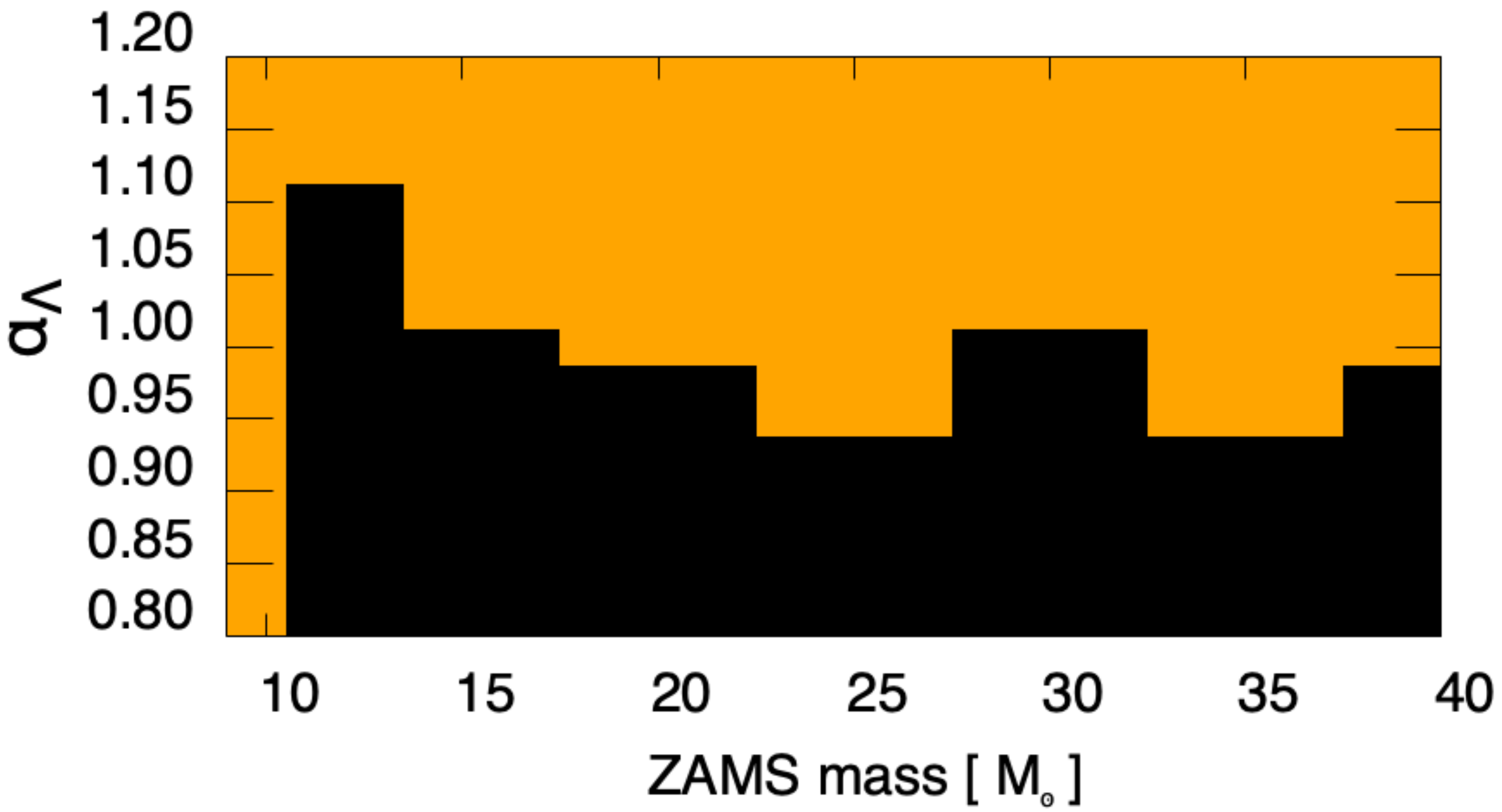




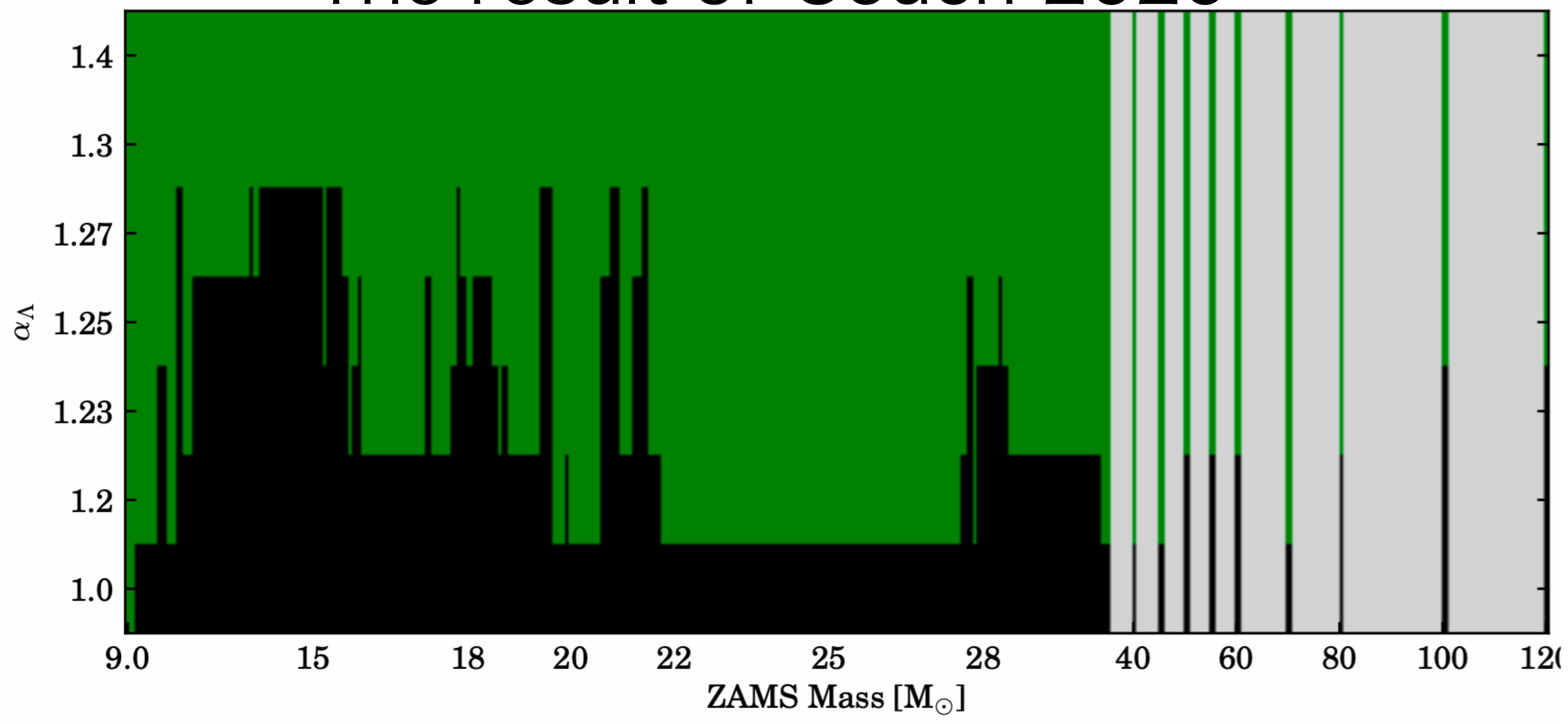
Our results of $\alpha_\Lambda = 1.0$ is consistent to the result of 3D simulation

Discussion

Compared explodability to Couch et al. 2020



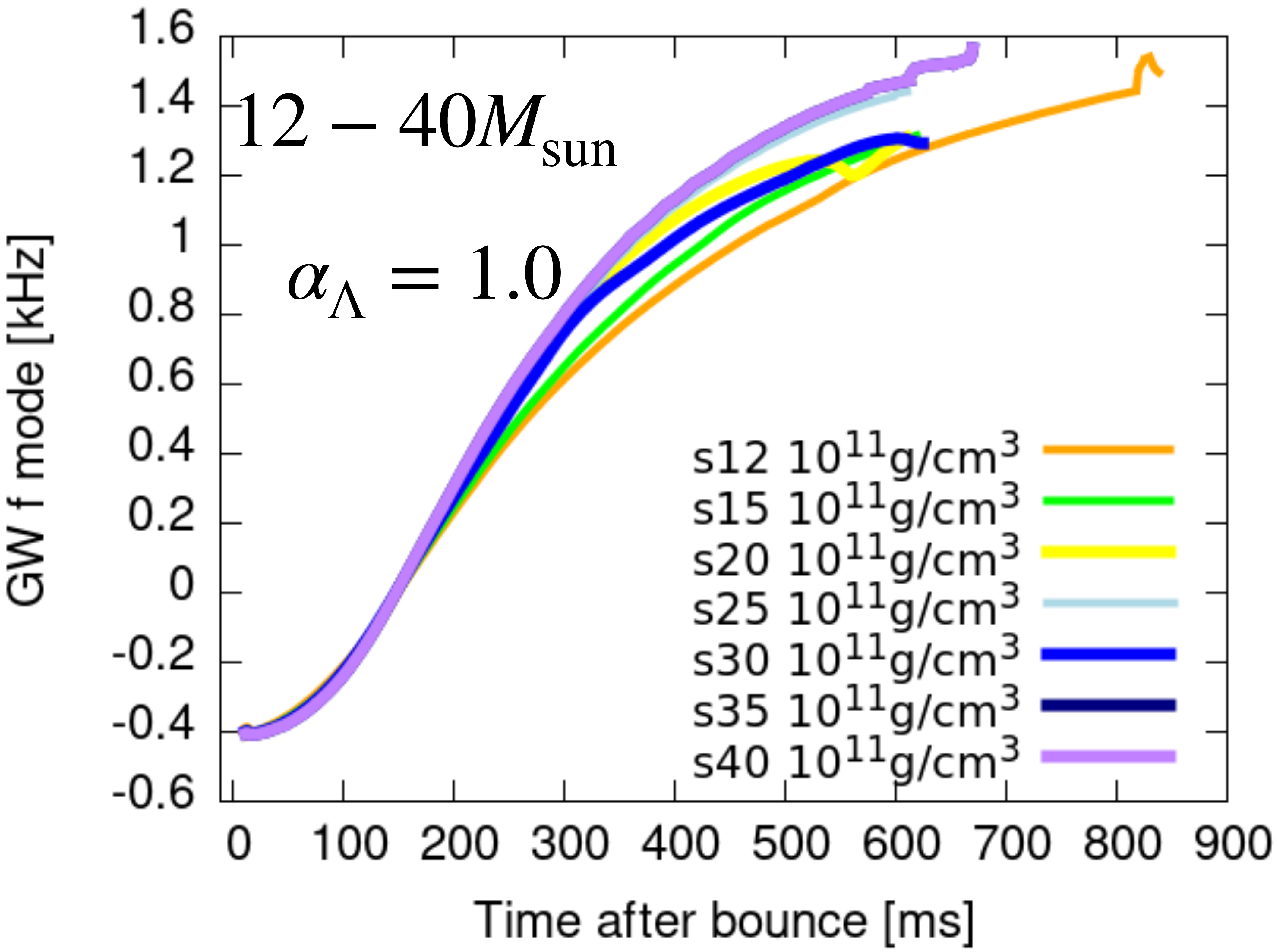
The result of Couch 2020



Our feature is slightly different from previous work

However, the landscape of explodability is similar.

Time dependence



the fitting formula of Gravitational wave (GW) by Sotani et al. 2021,

$$f(\mathrm{kHz}) = -1.410 - 0.443\ln(x) + 9.337x - 6.714x^2 ,$$

$$x \equiv \left(\frac{M_{\mathrm{PNS}}}{1.4M_\odot}\right)^{1/2}\left(\frac{R_{\mathrm{PNS}}}{10\ \mathrm{km}}\right)^{-3/2}$$

All properties of CCSNe depends on time.

Time dependence is also important factor for correlation study.

-> time-domain astronomy