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Rossby wave instability in proto neutron stars

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Gravitational Wave Physics and Astronomy Genesis

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DIVISION OF SCIENCE
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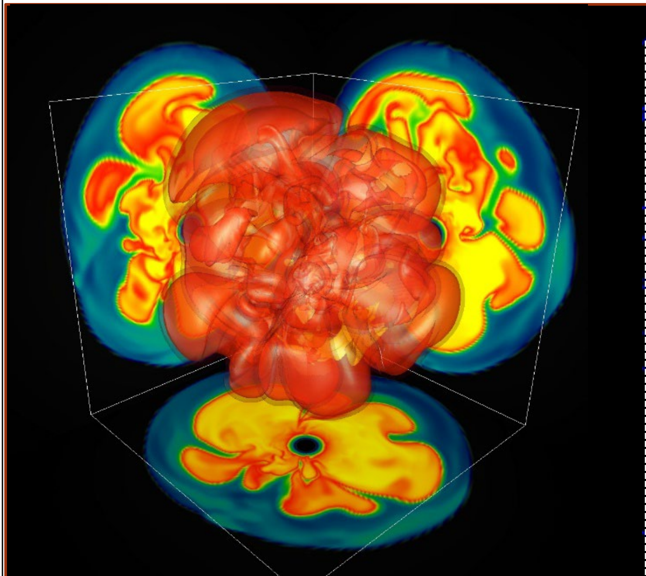
Outline

- Diversity of supernova explosions
- Introduction of low T/W instability (Rossby wave instability)
- Impact of low T/W instability, multi-messenger observation
- Mechanism of Rossby wave instability
- Unsolved issues
- Summary

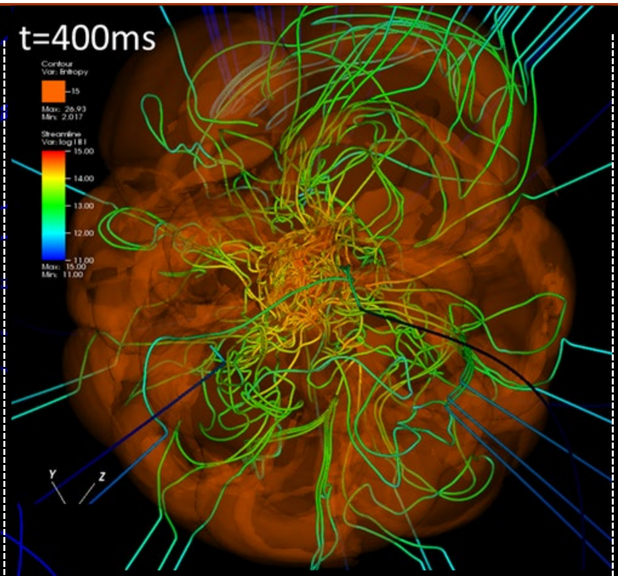
Diversity of supernovae

Main topic



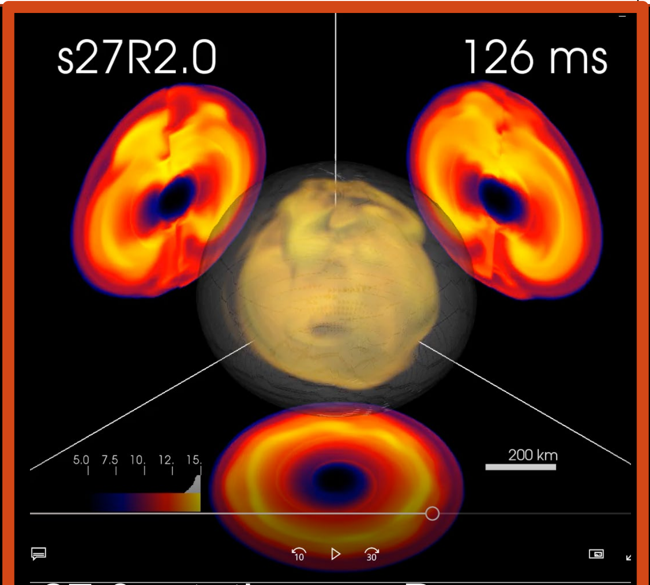
s11.2 non-rotating, no-B

Takiwaki+2016



s27.0 non-rotating, with B

Matsumoto+ sub. MNRAS

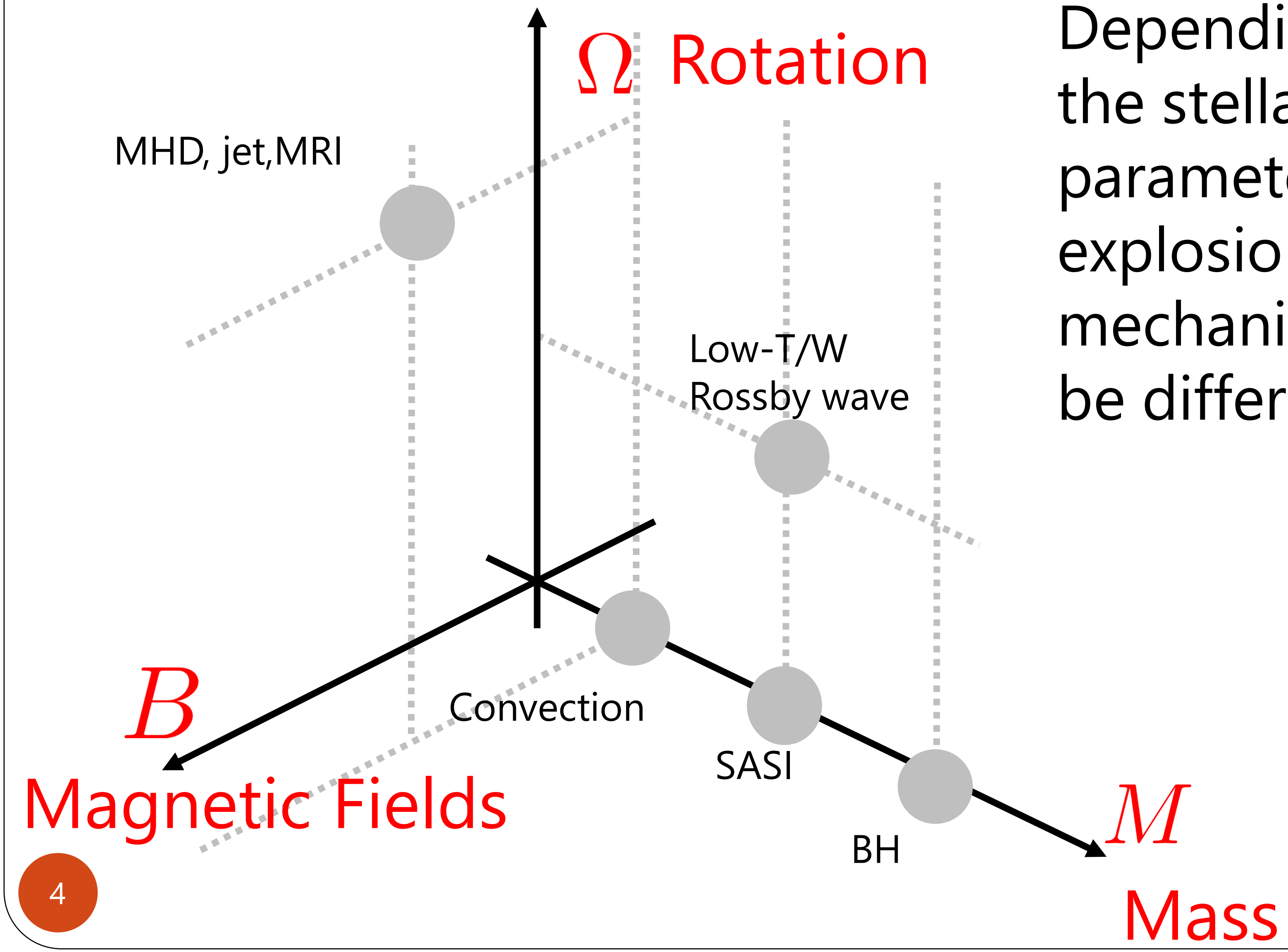


s27.0 rotating, no-B

Takiwaki+ 2021

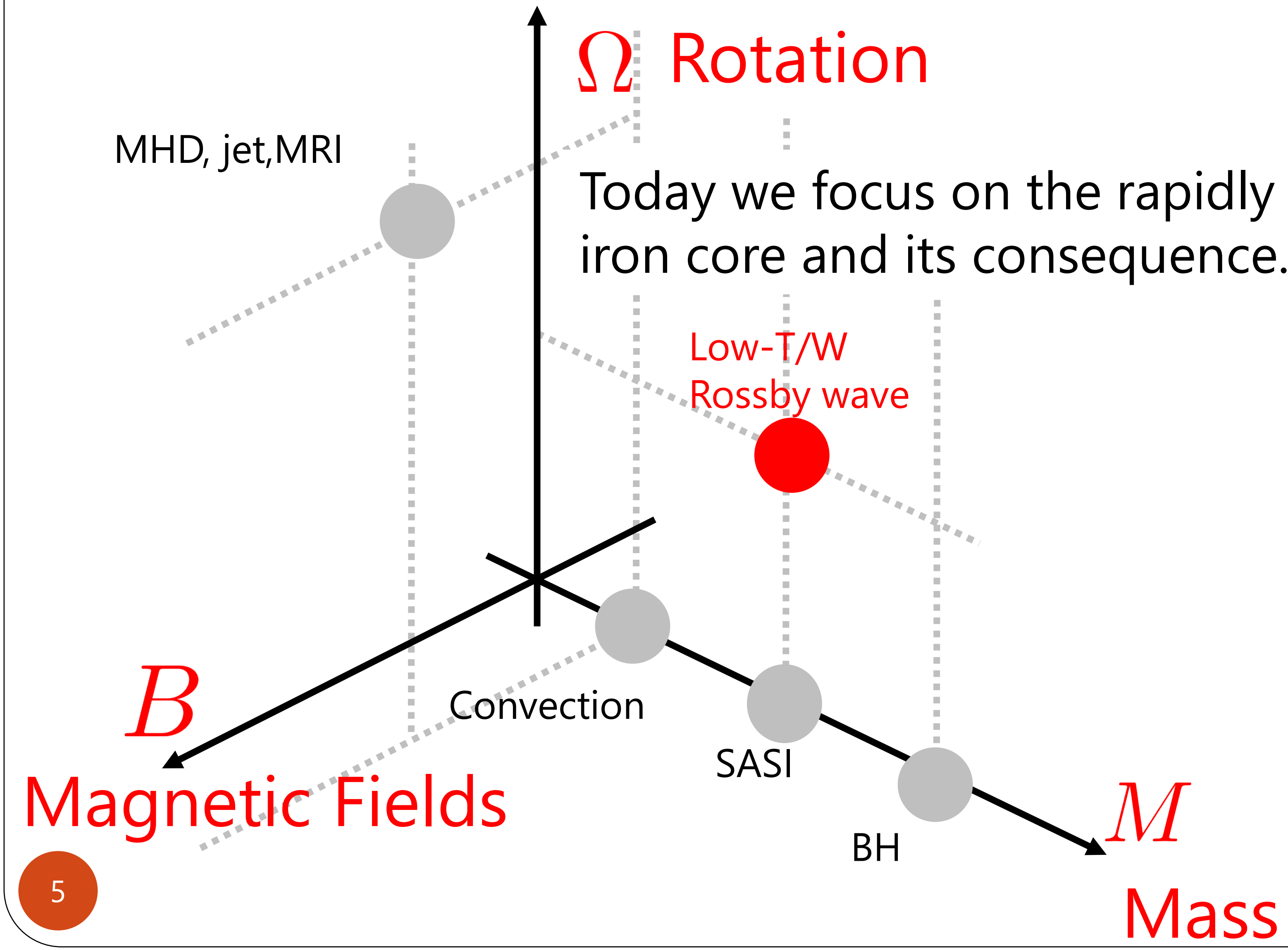
Recently various type of initial condition is employed for numerical simulations and the features of the explosions could be different.

Parameter Plane & Explosion Mechanism



Depending on the stellar parameters, explosion mechanism can be different.

Parameter Plane & Explosion Mechanism



Today we focus on the rapidly rotating iron core and its consequence.

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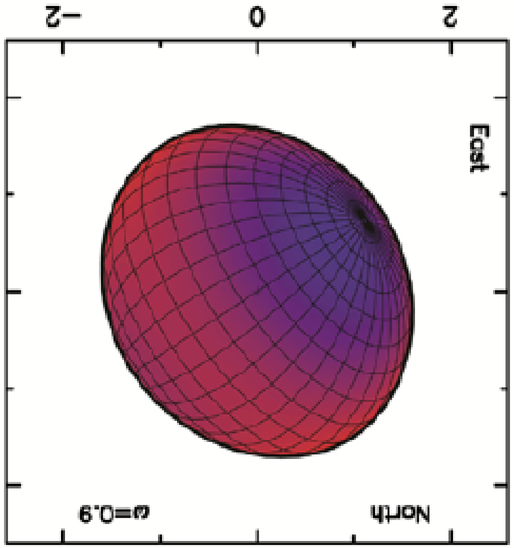
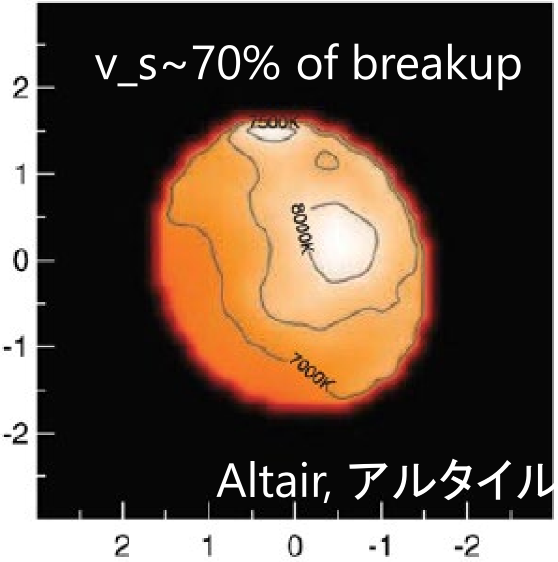
Rotational instabilities of stars

Q: How the rotation destabilize the star?



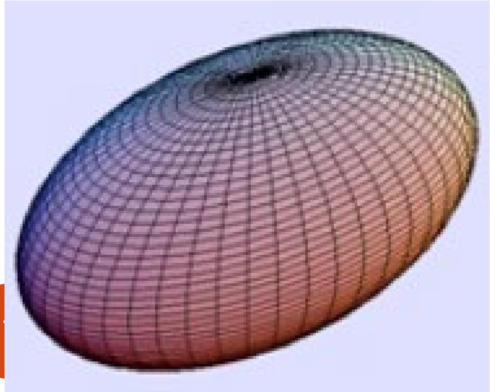


v_s~70% of breakup

Altair, アルタイル

See Belle 2012 and references therein

Rotation deforms stars.



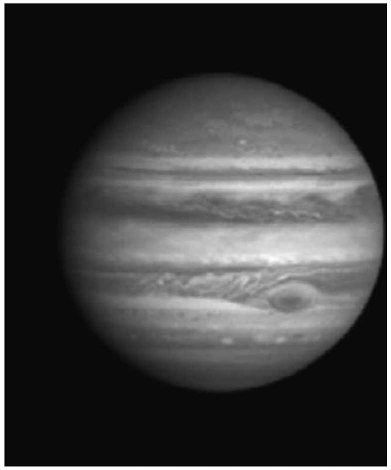
The rotation breaks spherical symmetry.

The rotation also breaks axi−symmetry.

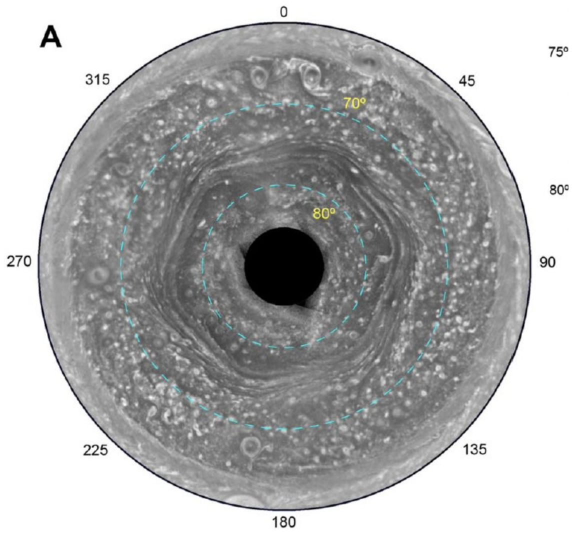
Non-axisymmetric patterns in stars and disks

Wikipedia

Earth

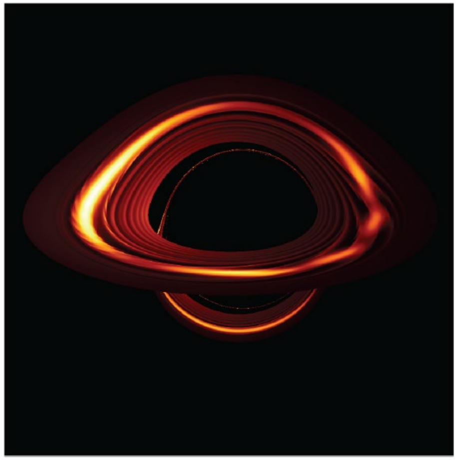


Jupitar



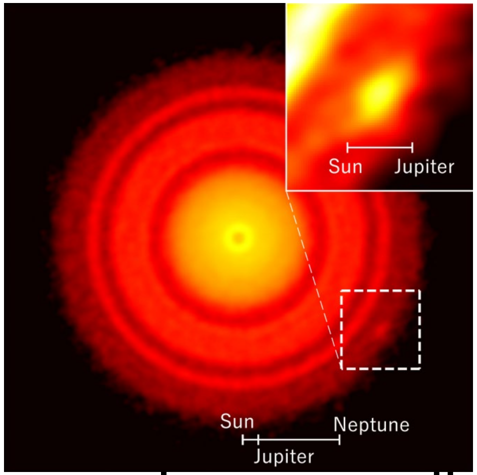
Saturn

Zaqarashvili et al. 2021



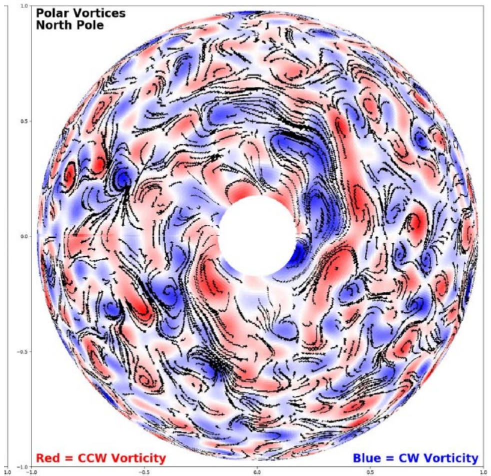
AGN

Varniere et al. 2019



protoplanetary disk

Tsukagoshi et al. 2019



Sun

Hathaway et al. 2021

History of non-axisymmetric instability

T/W is often used for the criteria.

$$\frac{T}{W} = \frac{\text{Rotational energy}}{\text{Gravitational binding energy}}$$

Classical threshold of the instability for rigidly rotating stars.

See Shapiro and Teukolsky

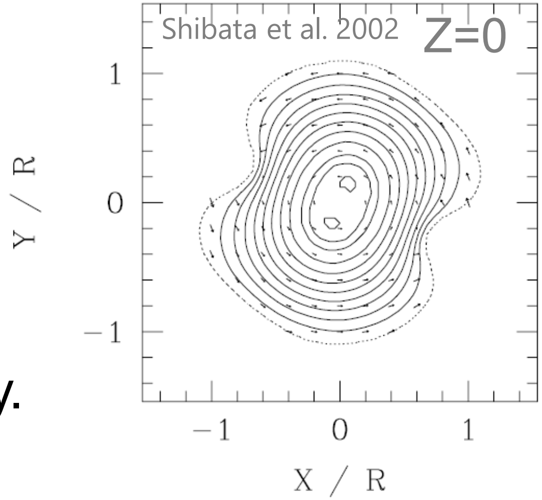
T/W ~27% (dynamical)

T/W ~ 14% (secular)

Modern calculations, for differentially rotating neutron stars

T/W ~ 1%

Differential rotation significantly relax the threshold. However, detailed mechanism was not well understood.



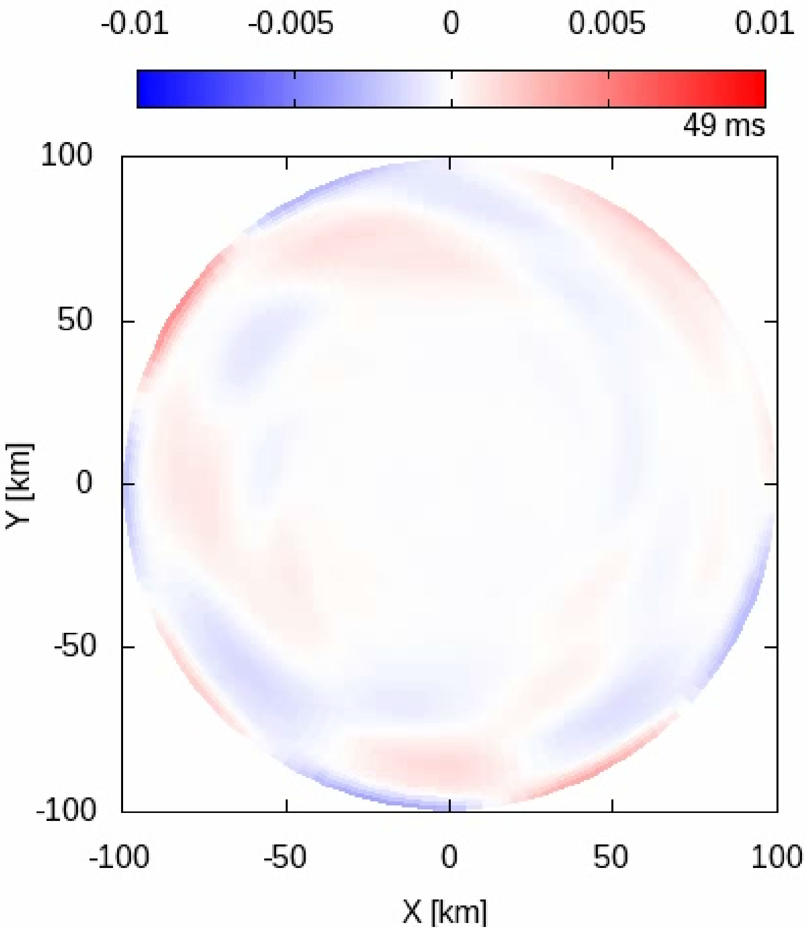
Phenomenologically, it is called low-T/W instability.
=> Later we call it Rossby wave instability.

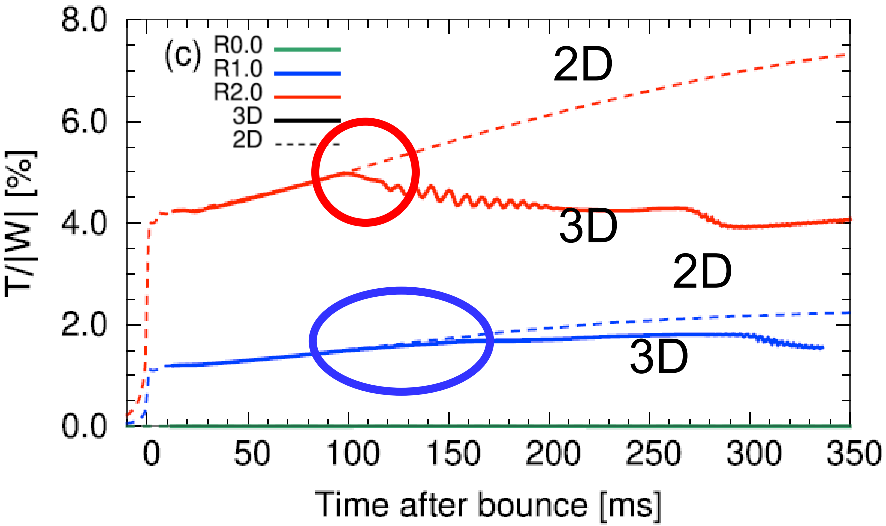
Low T/W instability in Proto-neutron stars

Takiwaki et al. 2021

$$\Delta\rho = \frac{\rho - \langle\rho\rangle}{\langle\rho\rangle}$$

z=0 plane



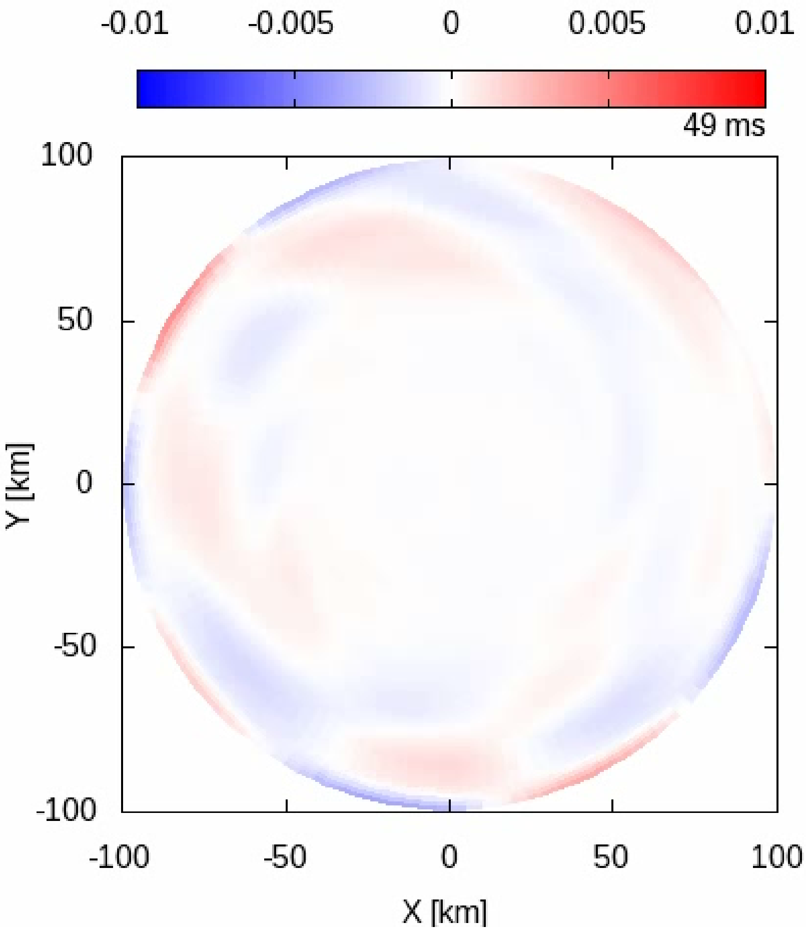


Previous slides is done
for cold star neutron star.
This is for proto-neutron star.
That is still hot and convective.

Rotation => Low T/W instability

$$\Delta\rho = \frac{\rho - \langle\rho\rangle}{\langle\rho\rangle}$$

z=0 plane



Isolated rotating star.

Rigid rotation:

T/W >27%

In cold neutrons stars, differential rotation:

T/W > 1% (Shibata+2003)

In proto-neutrons stars, convection promotes the instability (Takiwaki+2021)

I' ll show it later.

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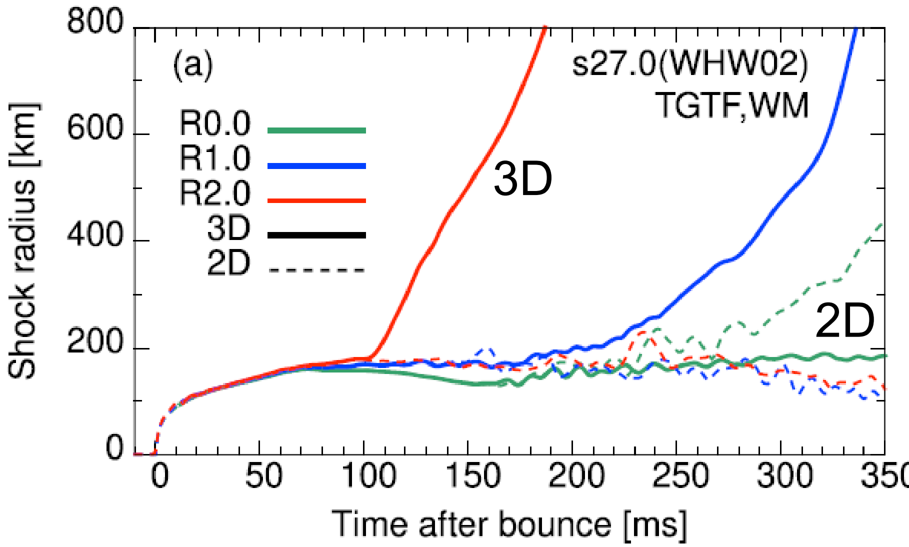
Explosion mechanism

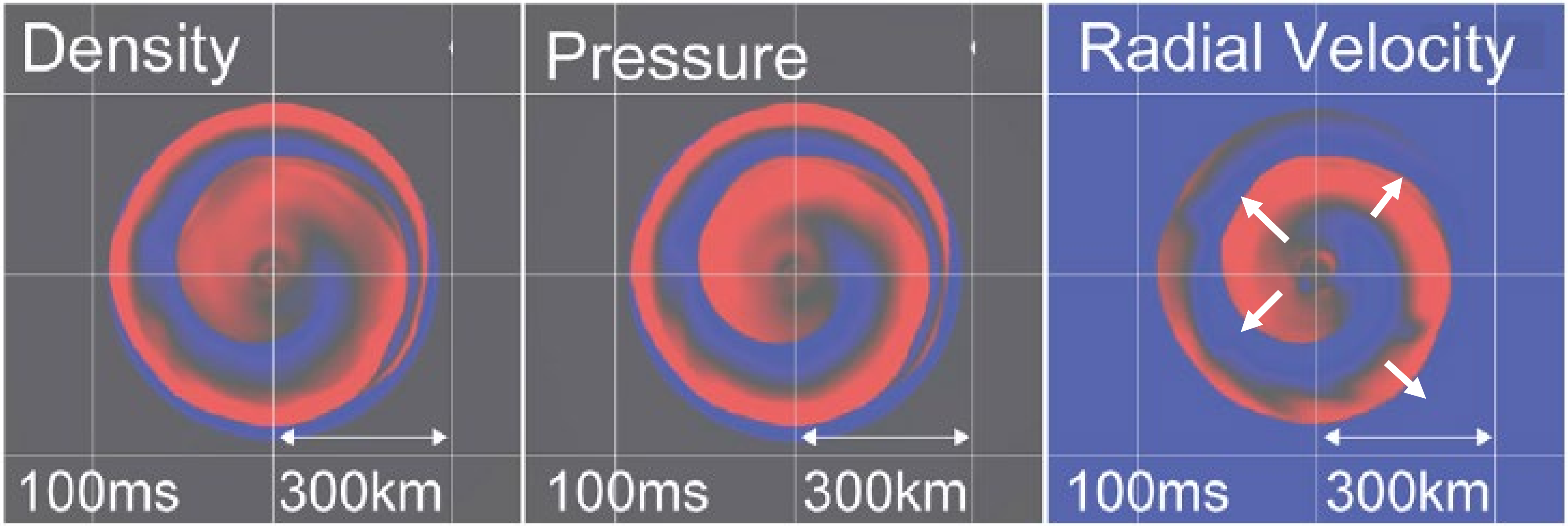
Takiwaki et al. 2016, 2021



low-T/W instability triggers a explosion.

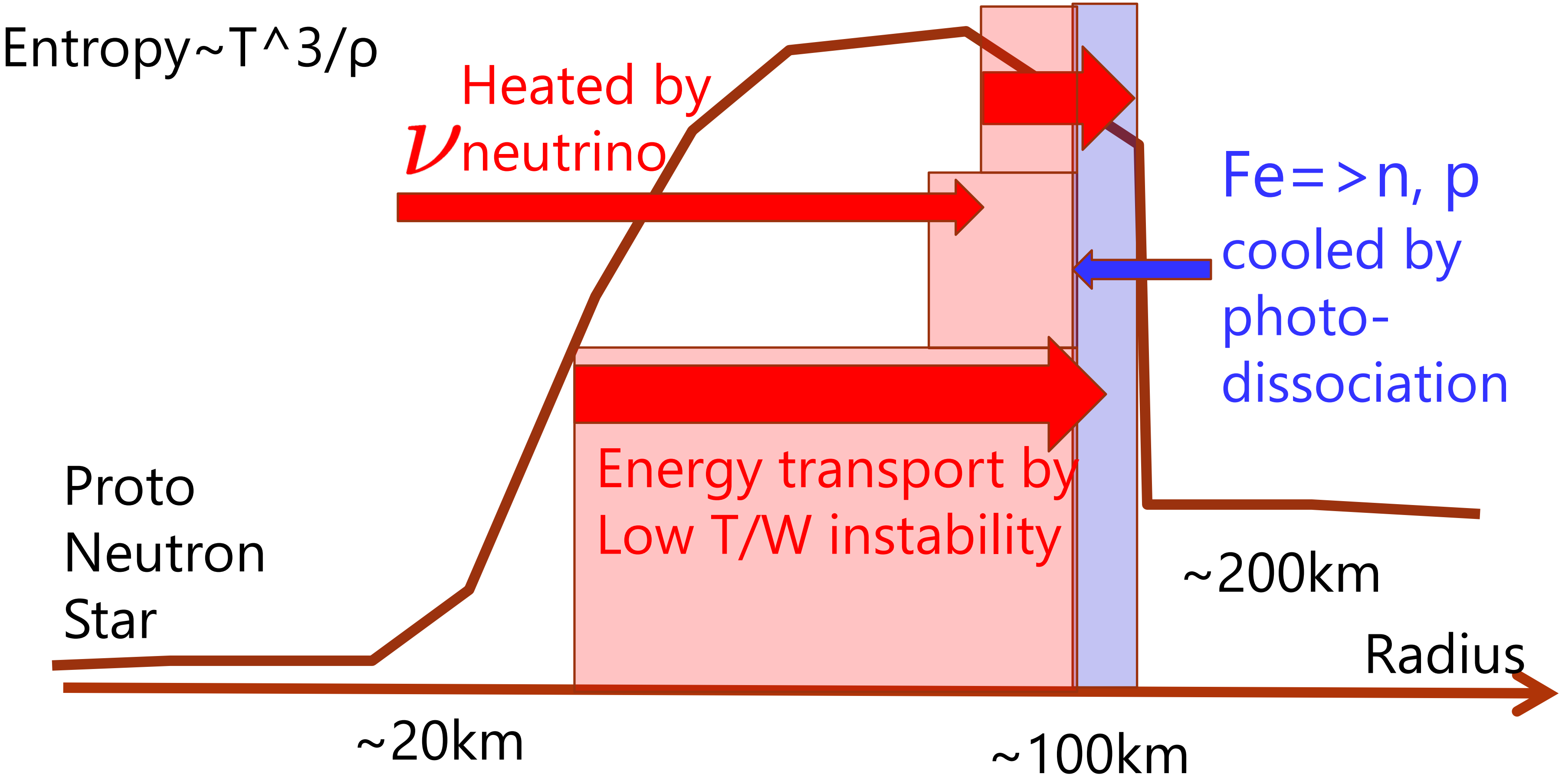
Energy transport by low T/W

Takiwaki et al. 2016



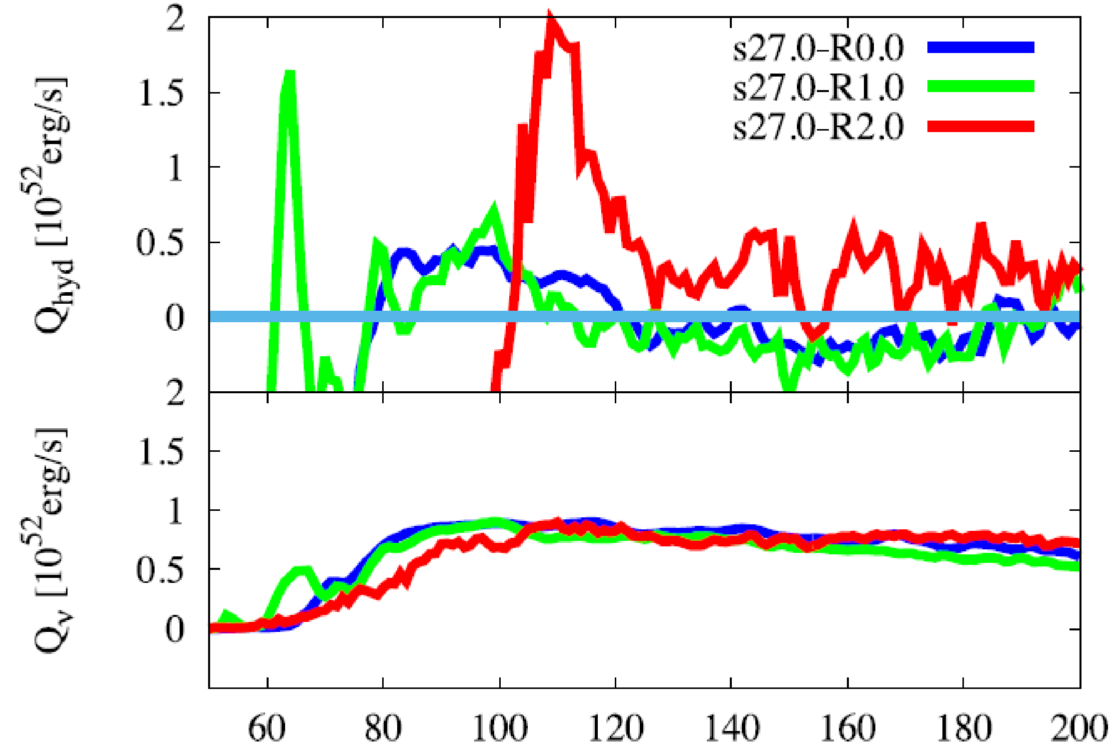
low-T/W instability launch ejecta from PNS, which has high density and pressure.

Energy transport and Heating

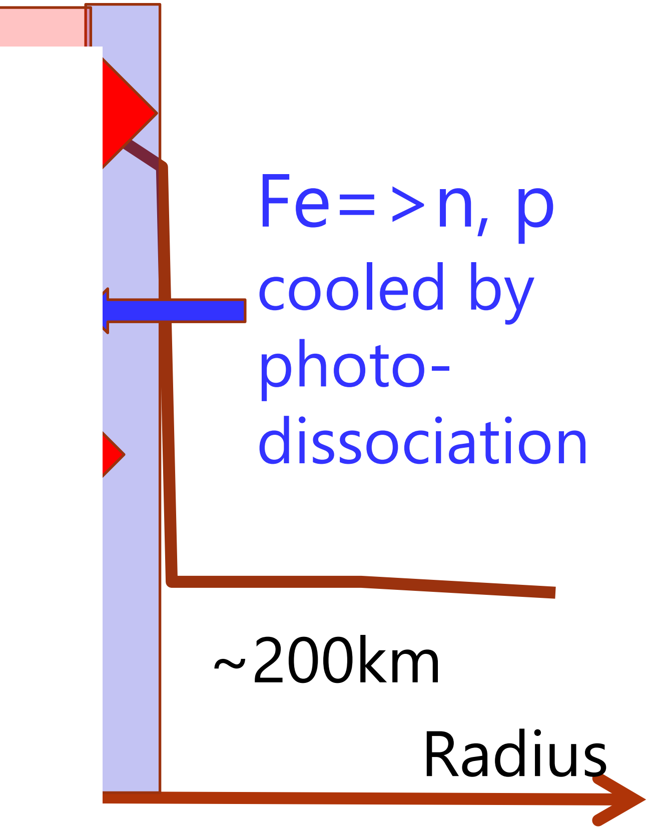


Energy transport and Heating

Energy transport by thermal convection



Takiwaki et al. 2016



Fe=>n, p cooled by photo-dissociation

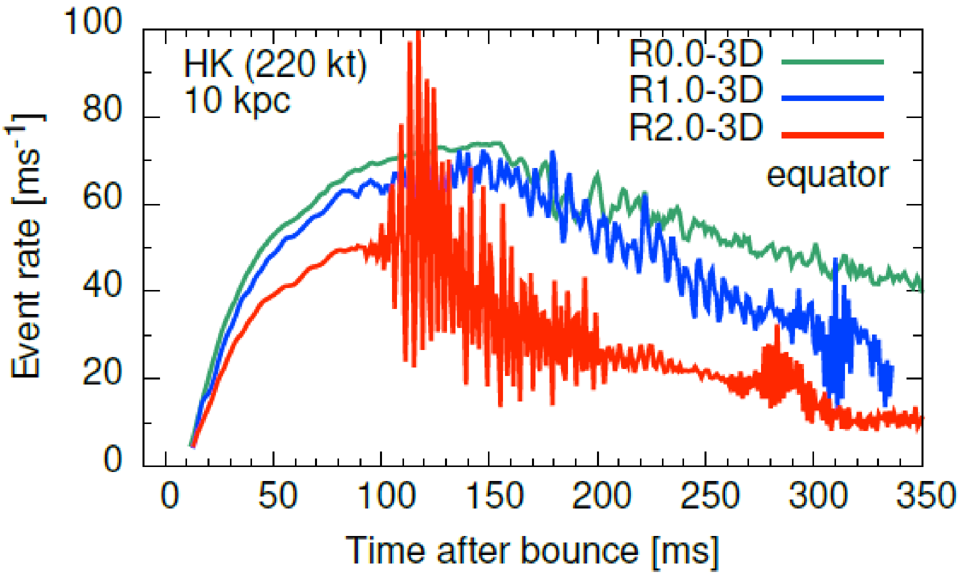
~200km

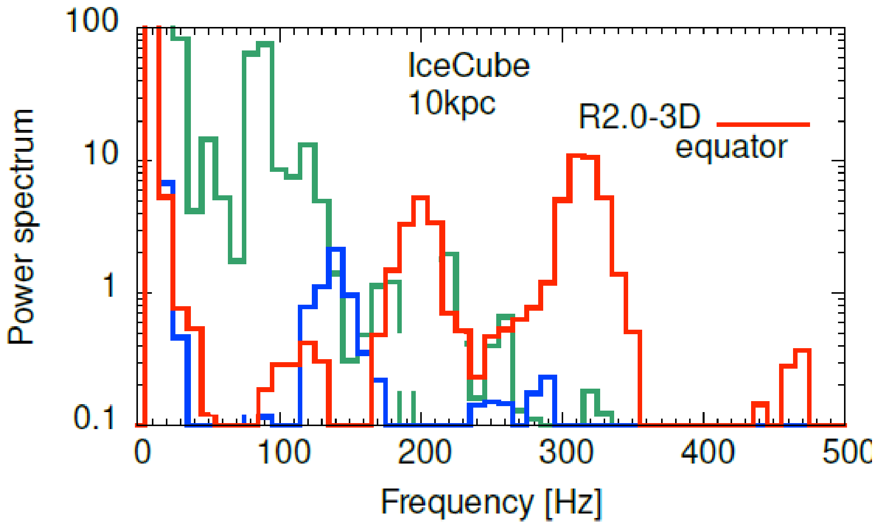
Radius

0km

Neutrino emission

Takiwaki et al. 2018, 2021





Strong time variability is found in rotating models!

Green: No rotation, SASI occurs

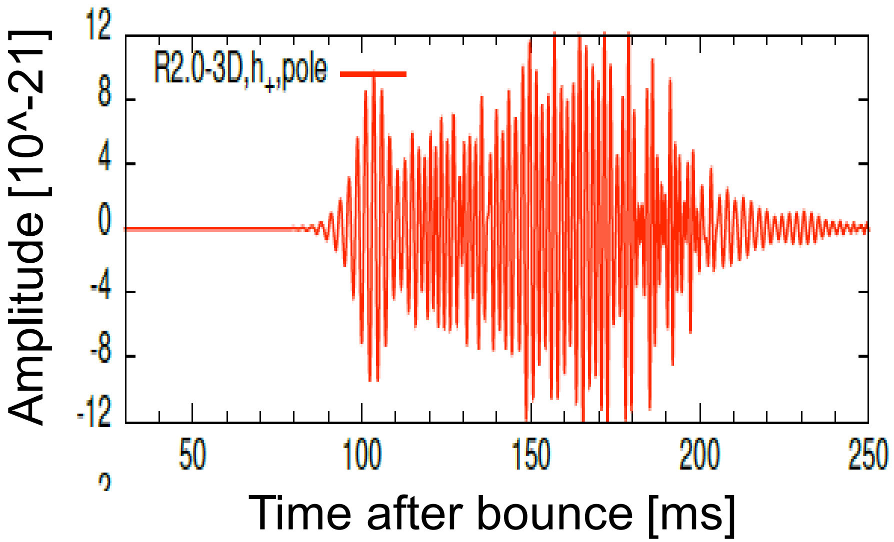
Blue: Slow rotation, has a peak at 150Hz, rotational freq.

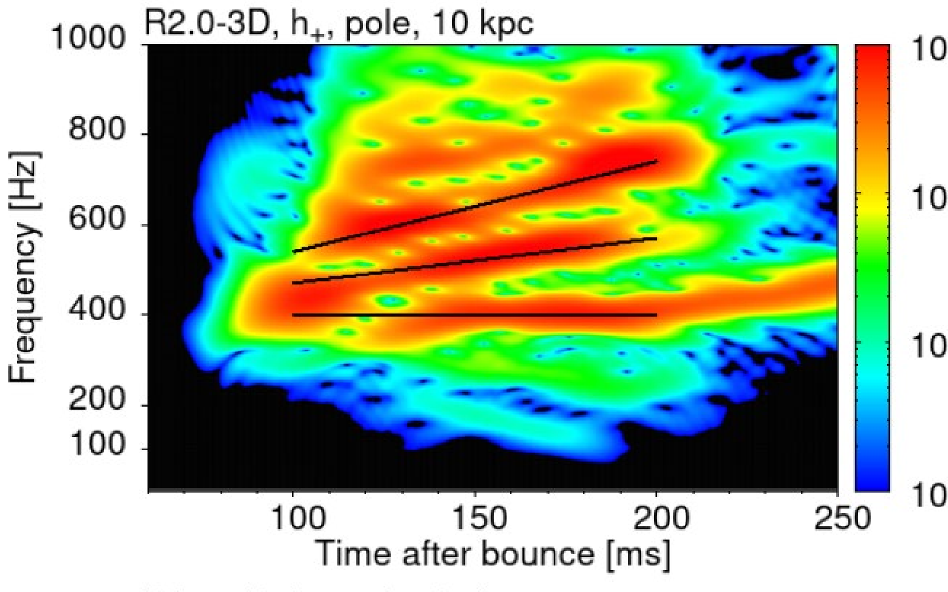
Red: Rapid rotation, has peaks at 200Hz and 300Hz , rotational freq. slightly different positions.

Gravitational wave

Takiwaki et al. 2018, 2021

Rapid rotation





Rapidly rotating model excites two hydrodynamic mode, and the three modes are excited in GWs.

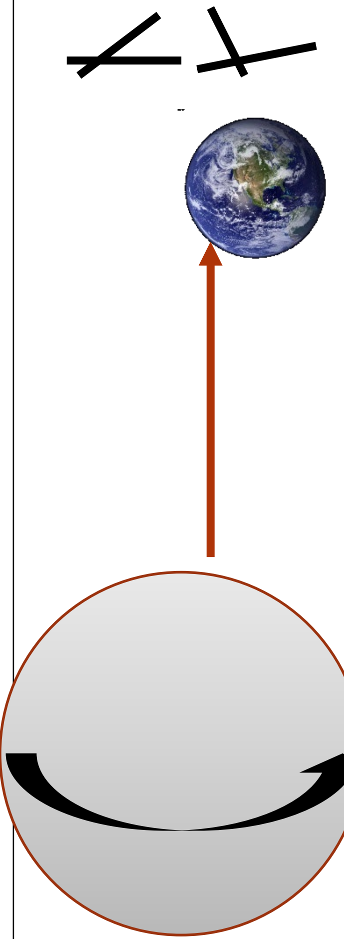
$$h \sim \frac{G}{c^4}\int \mathrm{d}V \boxed{\rho\left(v_z v_z - v_x v_x\right.} - z\partial_z\Phi + x\partial_x\Phi\Big)$$

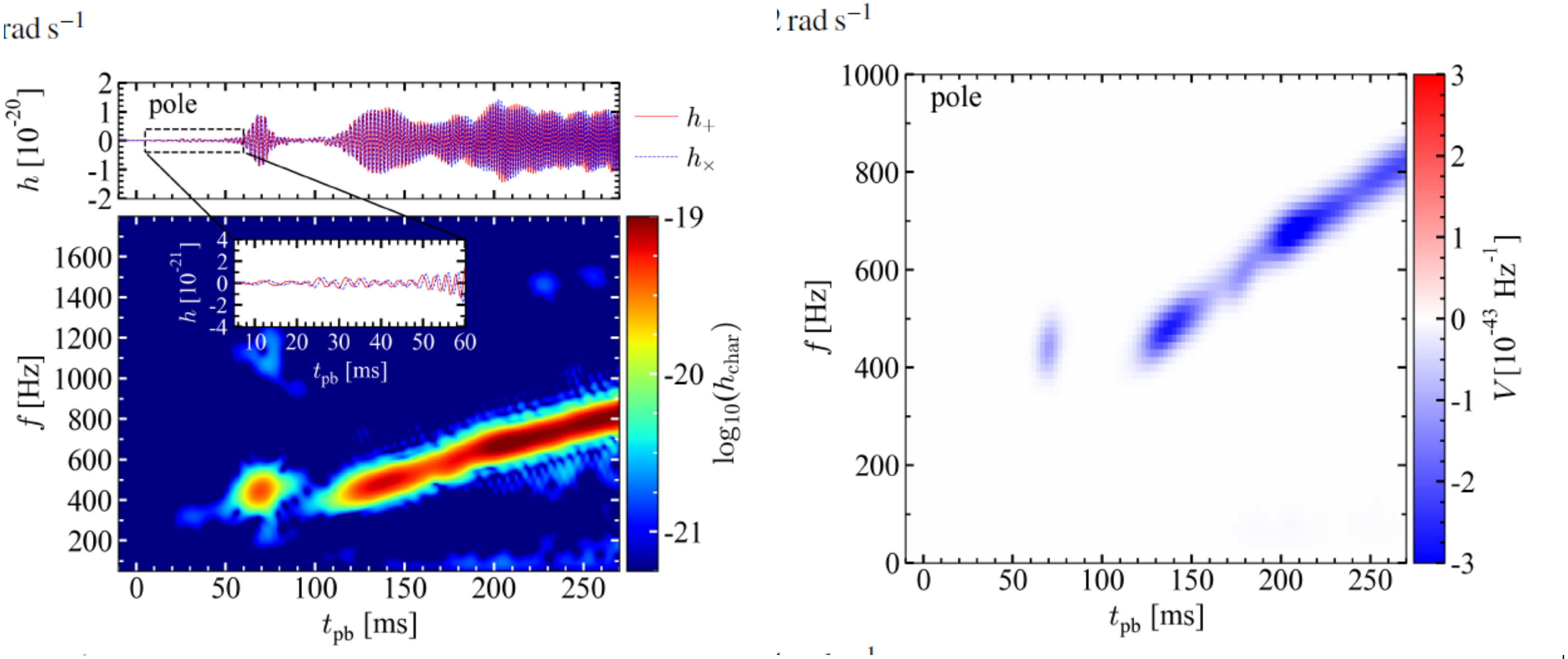
$$f_{\mathrm{gw}} = 2f_1,\ \ f_1 + f_2,\ \ 2f_2$$

200Hz & 300Hz => 400Hz, 500Hz, 600Hz

Circular Polarization

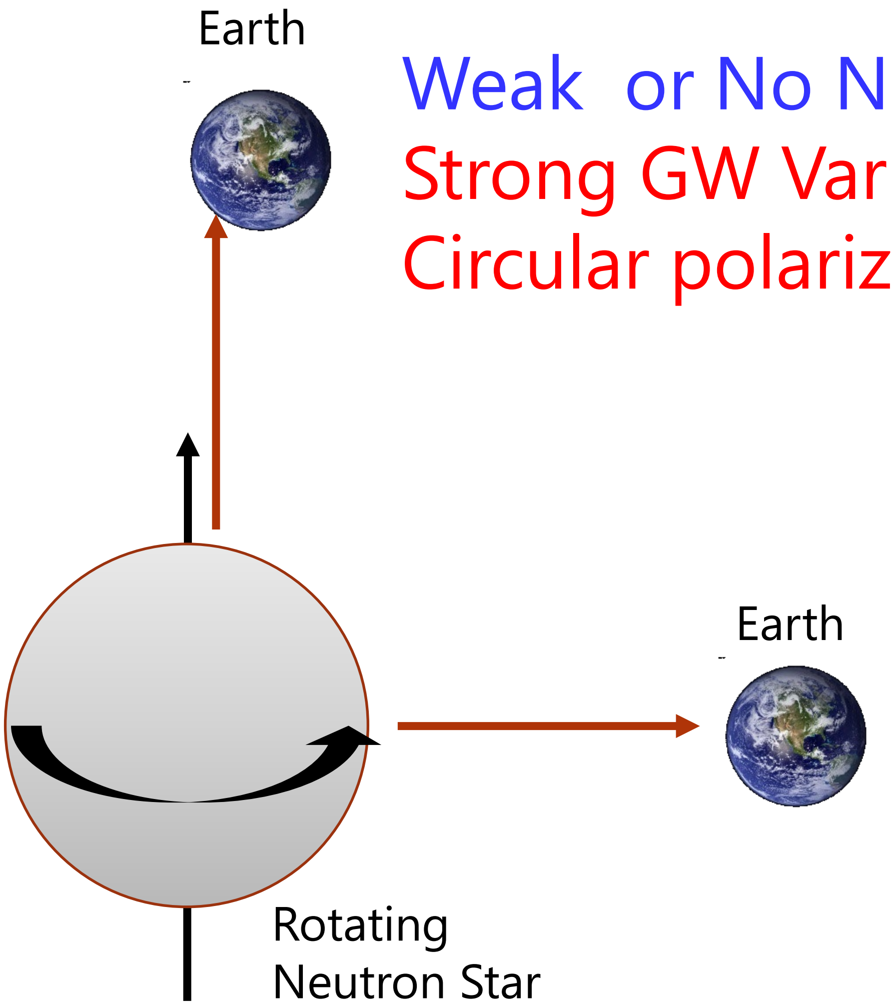
Shibagaki et al. 2021





Observing GW from the direction of north/south pole, circular polarization is found.

Direction dependence in Rotating model



Weak or No Neutrino Variability
Strong GW Variability
Circular polarization

Strong Neutrino Variability
Moderate GW Variability
Liner polarization

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Mechanism of low-T/W instability?

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T/W ~ 14% (secular)

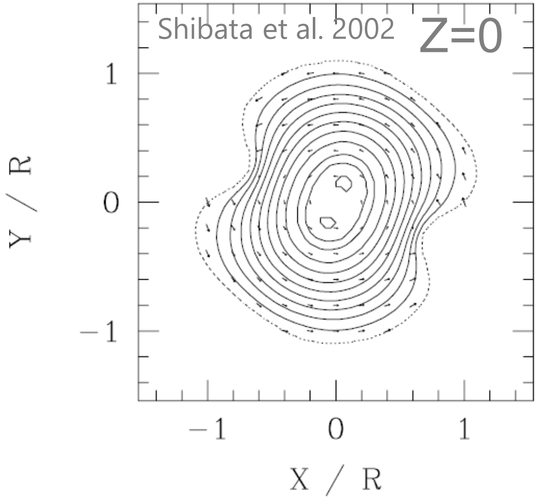
See Shapiro and Teukolsky

Modern calculations, for differentially rotating neutron stars

T/W ~ 1%

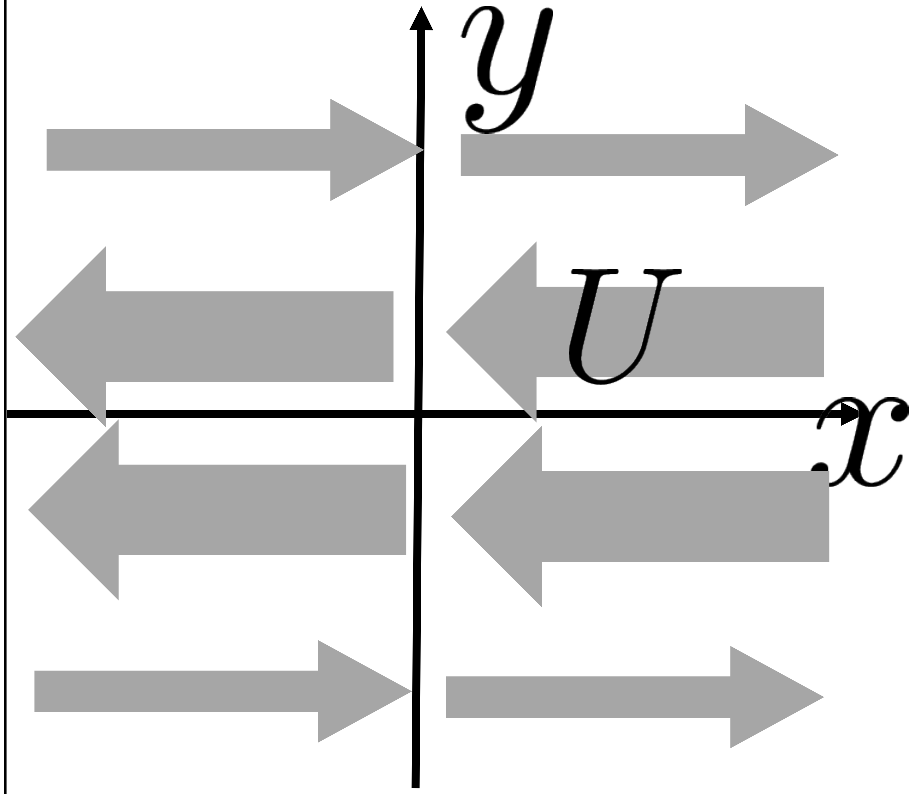
Phenomenologically, it is called low-T/W instability.

=> The mechanism would be trapping Rossby wave.



Kelvin-Helmholtz instability

Rossby wave instability is in some sense, Kelvin Helmholtz instability.



Kelvin-Helmholtz instability

Rossby wave instability is in some sense, Kelvin Helmholtz instability. In incompressible limit.

Perturbation equation.

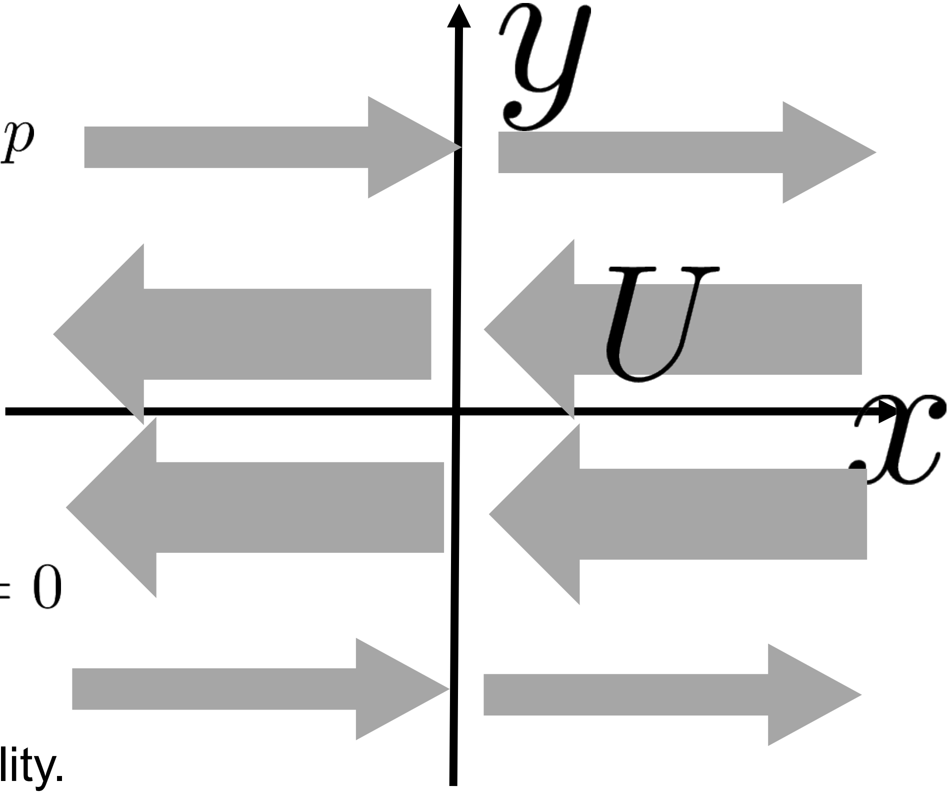
$$\begin{cases} \partial_x v_x + \partial_y v_y = 0 & \text{Continuum} \\ \partial_t v_x + U \partial_x v_x + v_y \partial_y U = -\partial_x p & \\ \partial_t v_y + U \partial_x v_y = -\partial_z p & \text{Euler eq.} \end{cases}$$

Steam function

$$(v_x, v_y) = (\partial_y \psi, -\partial_x \psi)$$

$$(\partial_t + U \partial_x) \nabla^2 \psi - \partial_y \zeta \partial_x \psi = 0$$

Advection Laplacian Source



Common structure of KH type instability.

$$\partial_y^2 U = \partial_y \zeta \quad \zeta = \nabla \times U$$

the derivative of vorticity is important.

Kelvin-Helmholtz instability

As usual, let us take Fourier component.

$$\begin{cases} \psi = \hat{\psi}(y)\exp(i(kx-\omega t)) \\ (\partial_t + U\partial_x)\nabla^2\psi - \partial_y\zeta\partial_x\psi = 0 \end{cases}$$

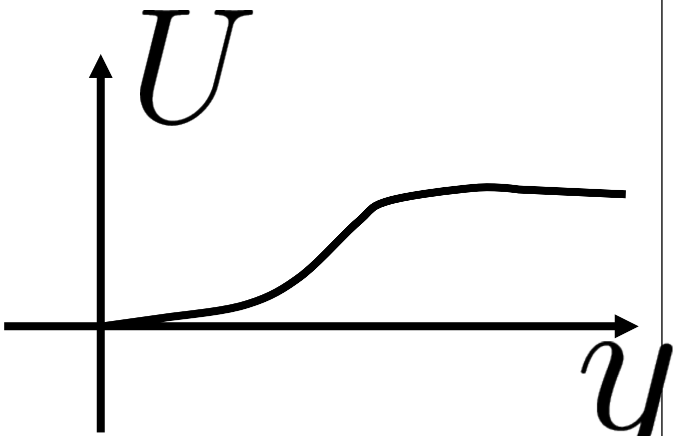
⇒ $$\imath\left(Uk-\omega\right)\hat{\nabla}^2\hat{\psi} - \imath k\partial_y\zeta\hat{\psi} = 0$$

⇒ $$F = \hat{\nabla}^2\hat{\psi} - \frac{\partial_y\zeta}{(U-\omega/k)}\hat{\psi} = 0$$

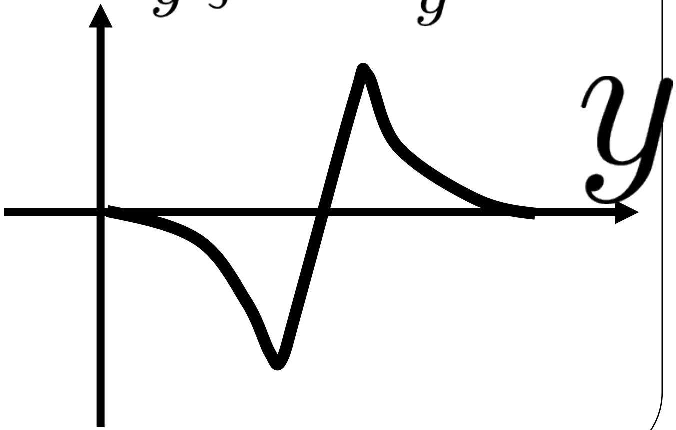
$\int \mathrm{d}y\,[\psi^* F] = 0$ And some mathematics

⇒ $\mathrm{Im}[\omega] \neq 0$ system is unstable

If $\partial_y\zeta$ change the sign somewhere in y.



$$\partial_y\zeta = \partial_y^2 U$$



Kelvin-Helmholtz instability

Intuitive analogy is as follows.

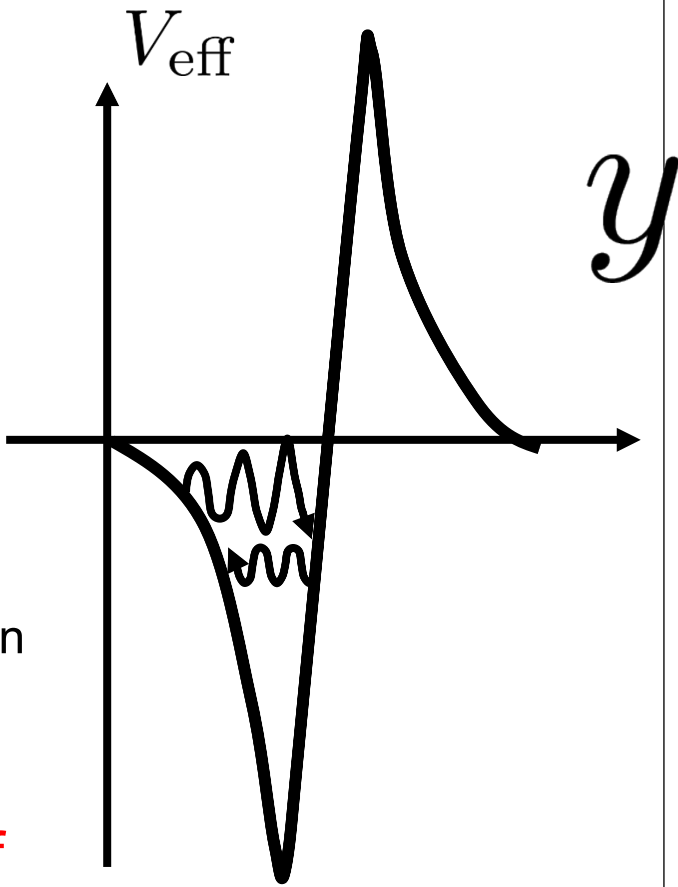
$$\begin{cases} \hat{\psi}(y)\exp(i(kx-\omega t)) \\ \hat{\nabla}^2\hat{\psi} - \frac{\partial_y \zeta}{(U-\omega/k)}\hat{\psi} = 0 \end{cases}$$

The equation is similar to Schrodinger equation.

$$\left[\frac{-\hbar^2}{2m}\nabla^2 + V_{\mathrm{eff}}\right]\Psi = 0$$

$\partial_y \zeta$ change the sign, potential energy change the sign

Then the wave is trapped and amplified.



KH instability is initiated when the sign of derivative of vorticity changes.

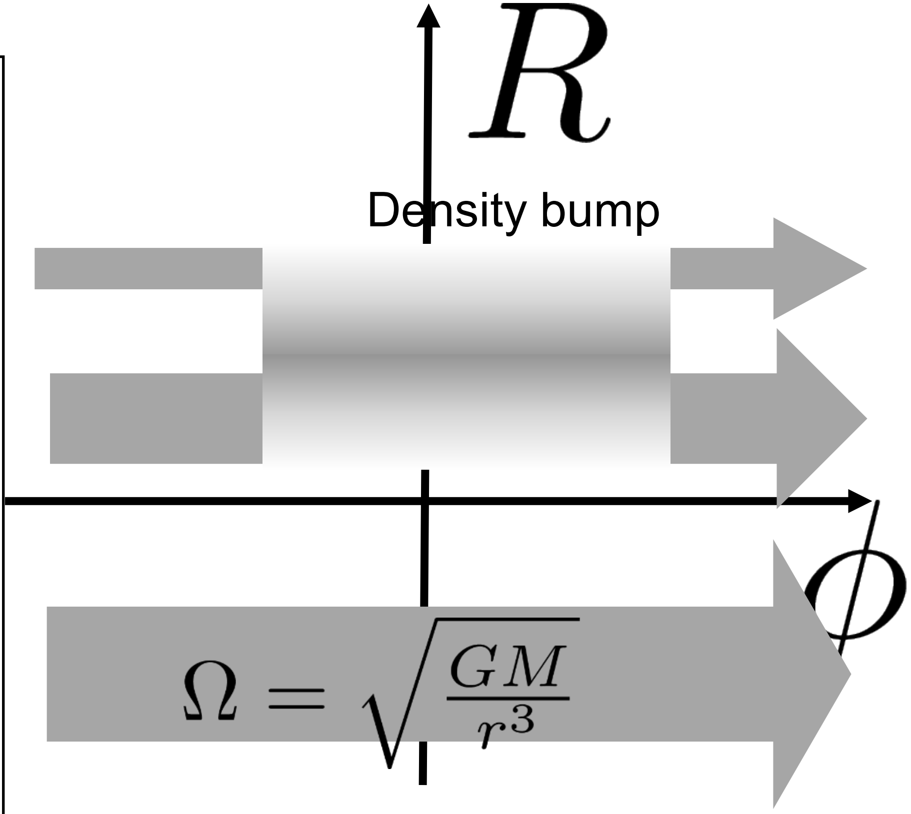
V_{eff} becomes huge when $\omega = Uk$ => idea of corotation.

Rossby wave instability in disk

Lovelace et al. 1999 Ono et al. 2018

In the disk, density bump triggers KH-like instability.
=> Rossby wave instability.

Density



$$\Omega = \sqrt{\frac{GM}{r^3}}$$

Rossby wave instability in disk

Ono et al. 2018

We obtain similar perturbation equation.

$$(\partial_t + \Omega\partial_\phi)\nabla^2\psi - \partial_y \Pi \partial_\phi \psi = 0$$

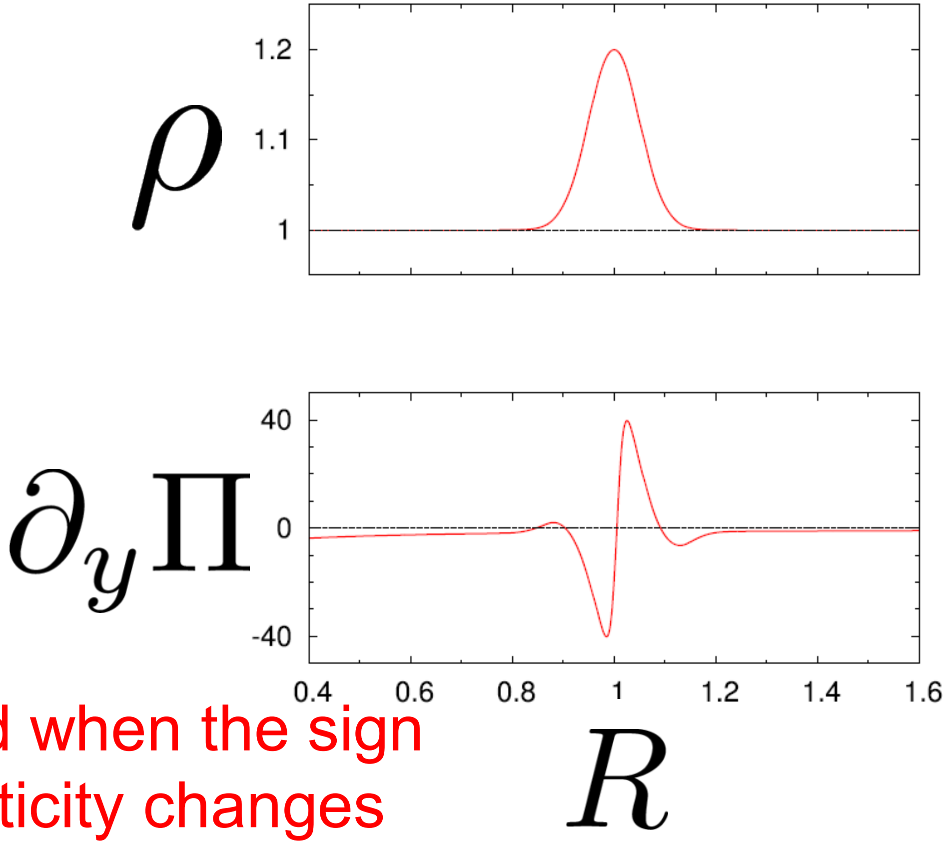
Advection Laplacian Source

$\Pi \sim \zeta/\rho$ Potential vorticity

In the case of isothermal EOS

$$\psi = \hat{\psi}(R)\exp(i(m\phi - \omega t))$$

$$\hat{\nabla}^2\hat{\psi} - \frac{m\partial_y\Pi}{(m\Omega - \omega)}\hat{\psi} = 0$$

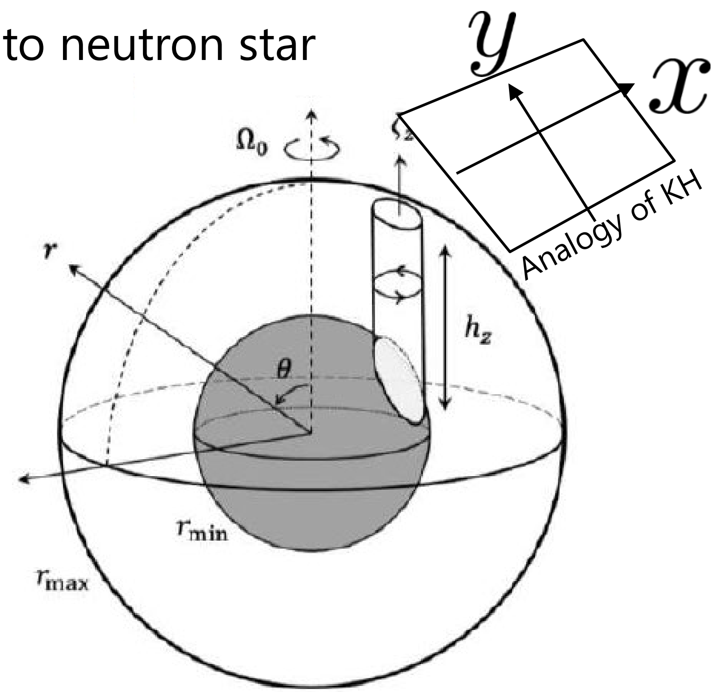


Rossby instability is initiated when the sign of derivative of potential vorticity changes and pattern frequency matches angular frequency.

Rossby wave instability in PNS

Complex version of Kelvin Helmholtz instability.

Proto neutron star



Consider stellar rotation and density, entropy and Ye, structures. We obtain similar perturbation equation.

$$(\partial_t + \Omega\partial_\phi)\left[\Delta\Psi + \cdots\right] + \partial_r \Pi \frac{\partial_\phi \Psi}{r} = 0$$

$$\Pi = \zeta \cdot \nabla\lambda/\rho$$

Potential vorticity.

In the case of adiabatic EOS

$$\zeta = \nabla \times \vec{v}$$

vorticity

$$\lambda = s, Y_e$$

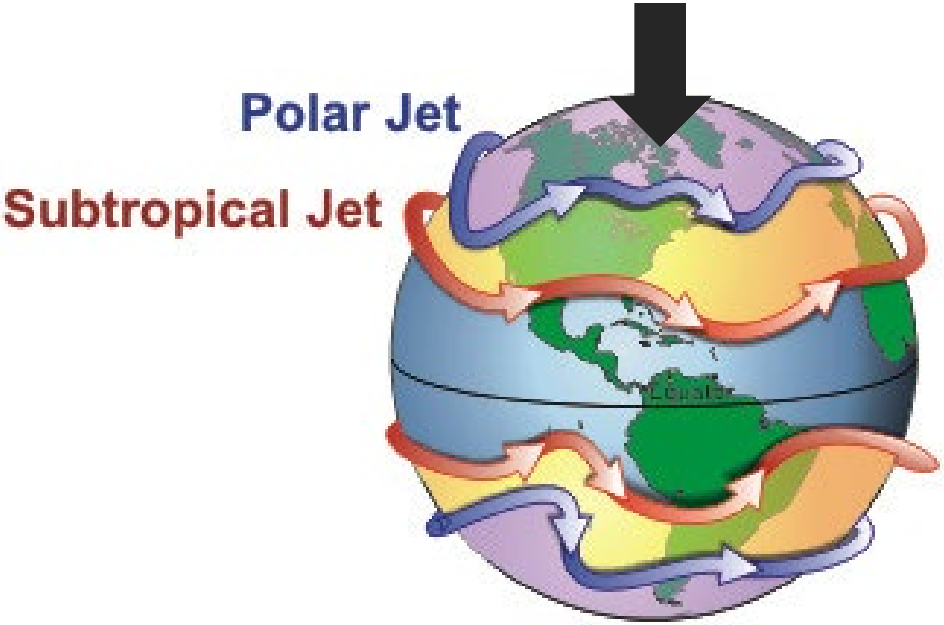
$$\Psi = \psi \exp\left(\imath m\phi - \imath\omega t\right)$$

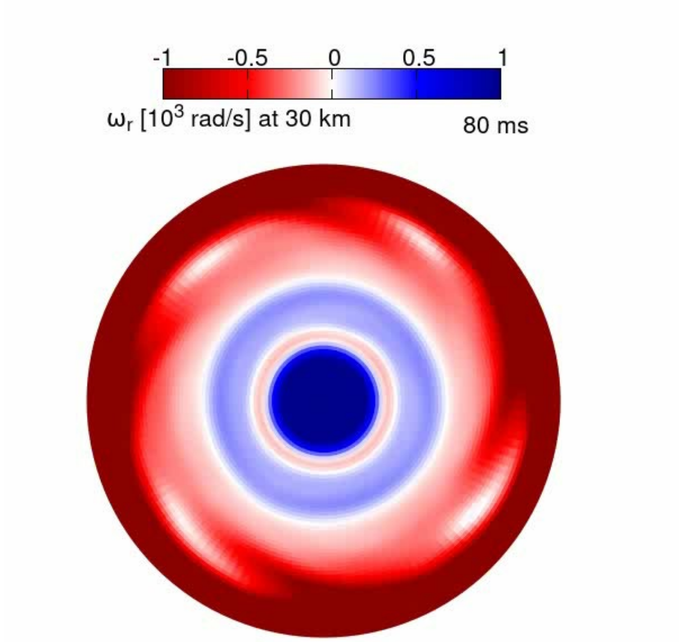
$$\Delta\psi + \cdots = \frac{m\partial_r \Pi}{r(\omega - m\Omega)}\psi = V_{\rm eff}\psi$$

Important point here : the effective potential is function of rotation velocity and density and entropy and Ye.

Typical evolution

Takiwaki et al. 2021



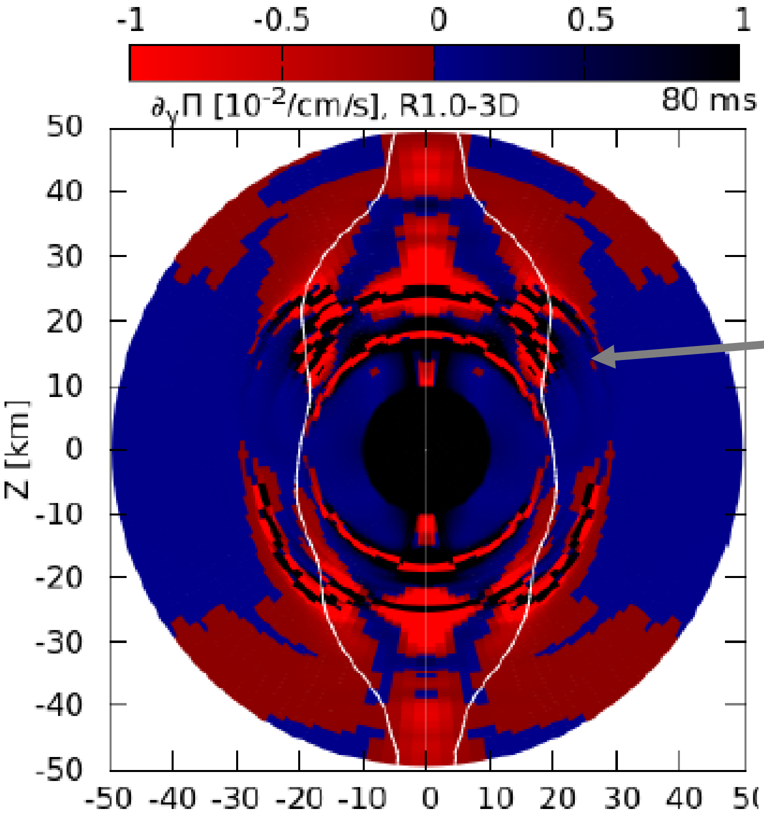


In our case, m=4 modes appears and immediately it becomes m=1 mode.

azimuthal equidistant projection from north pole

New stability analysis

Takiwaki et al. 2021



derivative of potential vorticity is important

White line is corotation point. There, color change from blue to red. This satisfy the condition of Rossby wave trapping.

unique in PNS compared to cold NS.

$$\partial_y \Pi = \frac{2\Omega}{r}\sin\theta\left[1+2\cos^2\theta\frac{r}{\rho}\partial_r\left(\rho\frac{\Omega^2}{\Omega_{\mathrm{BV}}^2}\alpha\right)\right]+\Delta_{\theta\theta}v_\phi,$$

Small BV makes this term large!,
This instability is expected near convection zone in general.

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Unsolved issues

Theory, concept and name of Rossby wave instability are not well organized.

Potential vorticity = Vortensity

The name of the trapped wave is not identified.

r–mode: restoring force is stellar rotation, Rossby wave

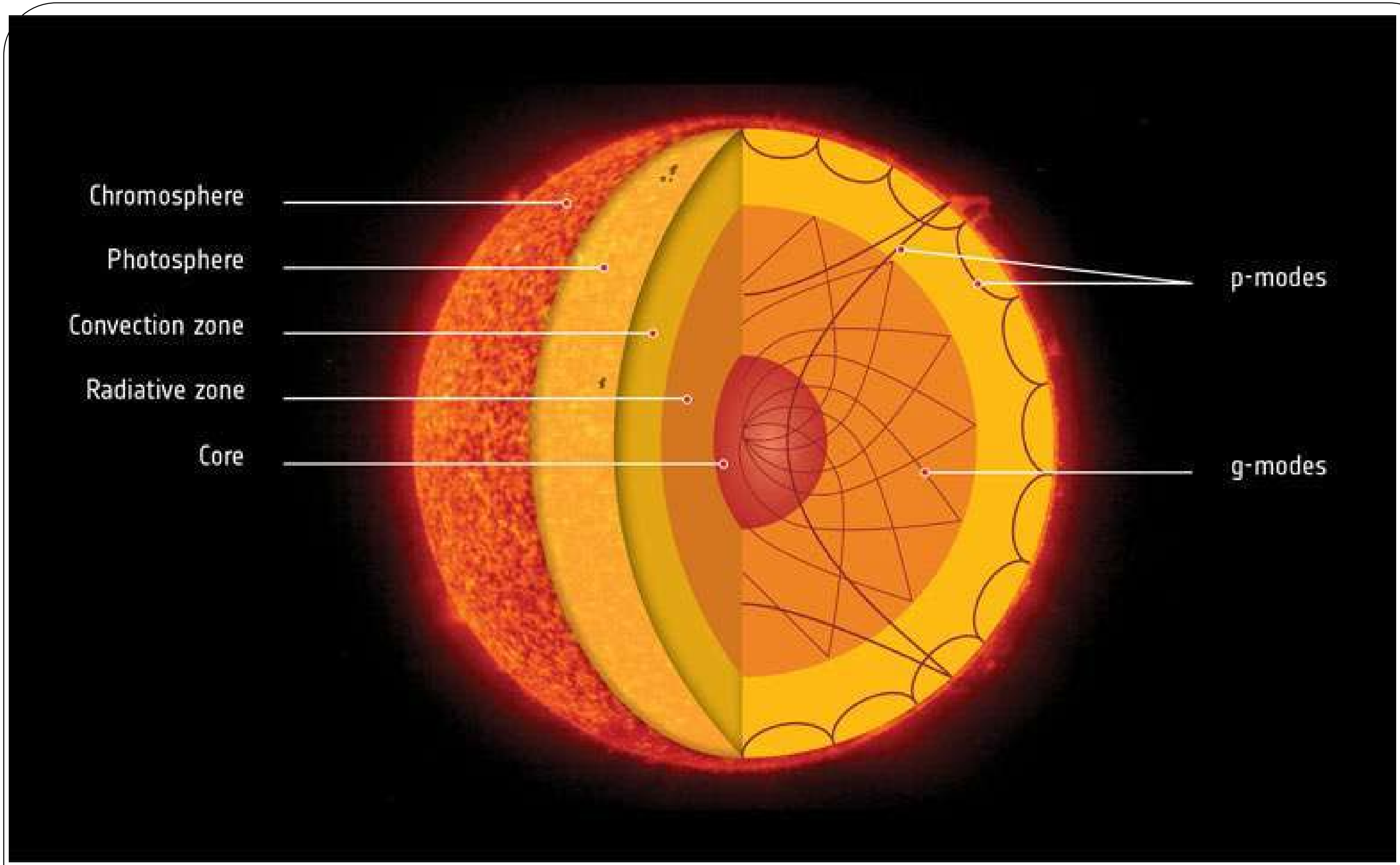
p–mode: restoring force is pressure, sound wave, acoustic wave

g–mode: restoring force is pressure and gravity, gravity wave

f–mode: p–mode or g–mode or mixture of it. It depends context.

Inertial wave: mixture or Rossby wave and g–mode

Yoshida+ (2017) consider f–mode and p–mode trapping in cold neutron stars.


Chromosphere
Photosphere
Convection zone
Radiative zone
Core
p-modes
g-modes

Rossby wave

Governing equation in rotating frame

$$\frac{\partial \vec{v}}{\partial t} + \vec{v} \cdot \nabla \vec{v} = -\nabla p/\rho + \nabla \Phi \underbrace{- 2\vec{\Omega} \times \vec{v}}_{\text{Coriolis force}}$$

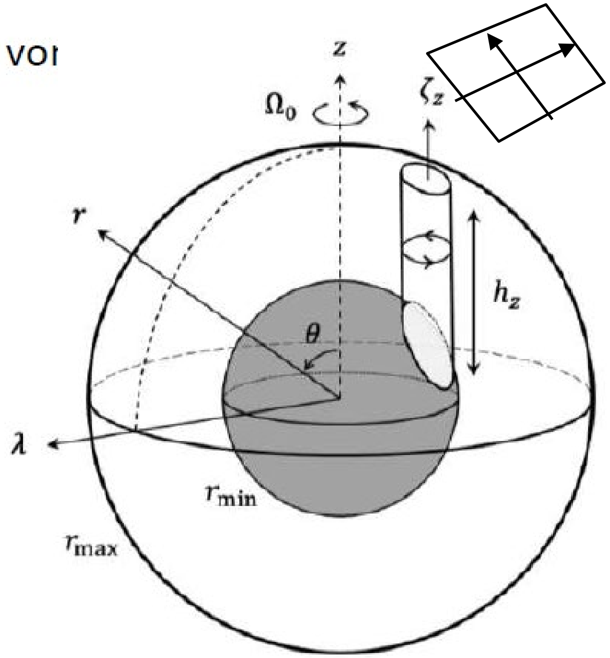
Some algebra with several approximations

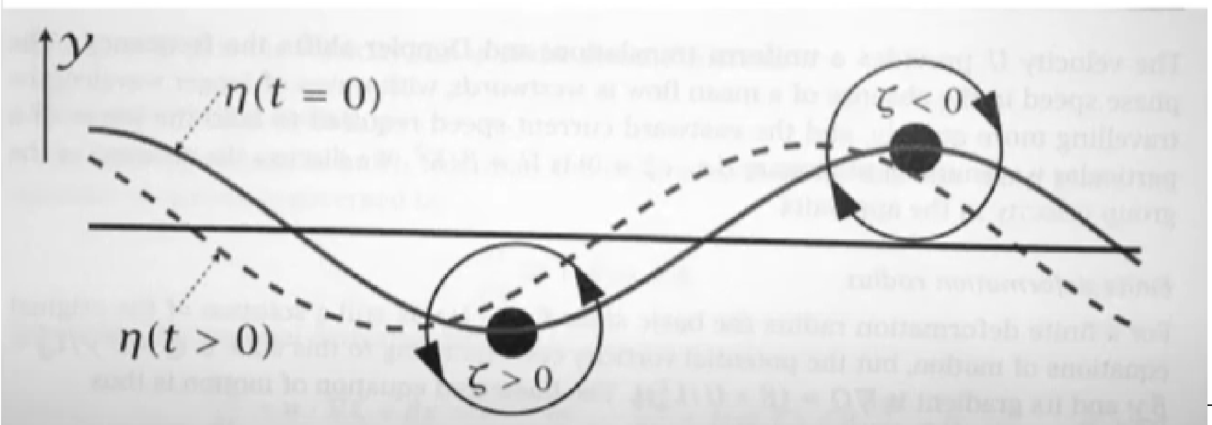
=> conservation of absolute vorticity

$$\zeta = \nabla \times \vec{v}$$

$$\frac{\partial \zeta_\mathrm{a}}{\partial t} + \vec{v} \cdot \nabla \zeta_\mathrm{a} = 0$$

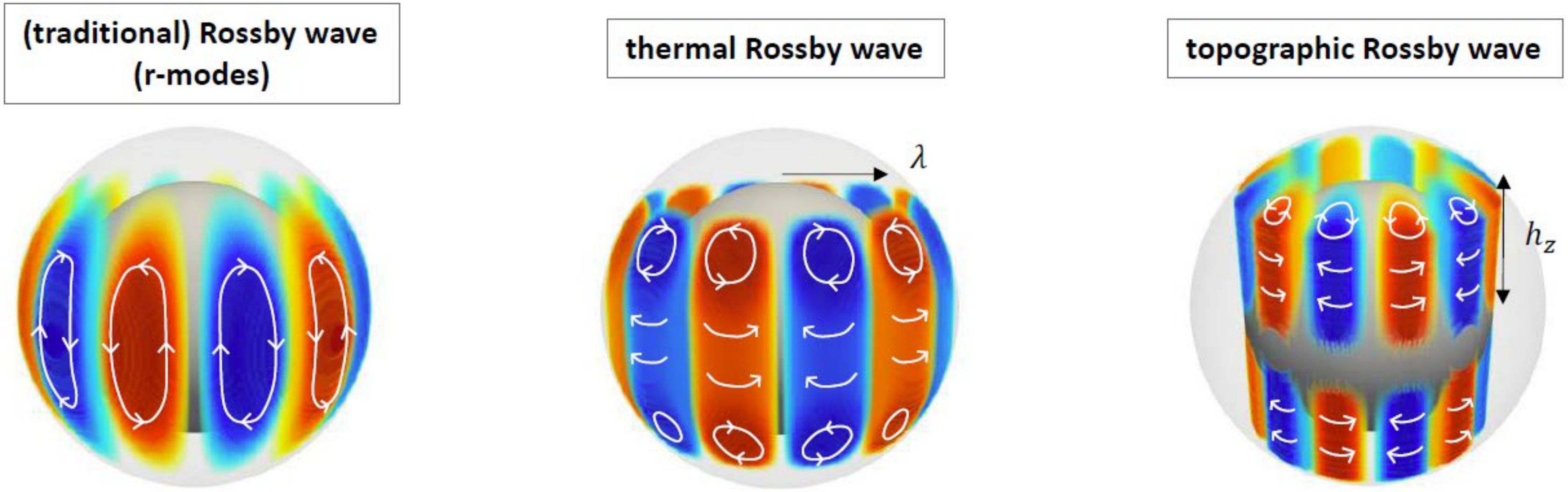
$$\zeta_\mathrm{a} = 2\Omega \cos\theta + \zeta$$





A few types of Rossby waves

Bekki et al. 2022



We do not know the detail of Rossby wave.

Our morphology looks like topographic Rossby wave.

Unsolved issues

Can Rossby wave and Rossby wave instability survive in convective zone?

Classical argument: No.

Recently: Yes. (e.g., Callies and Ferrari 2018)

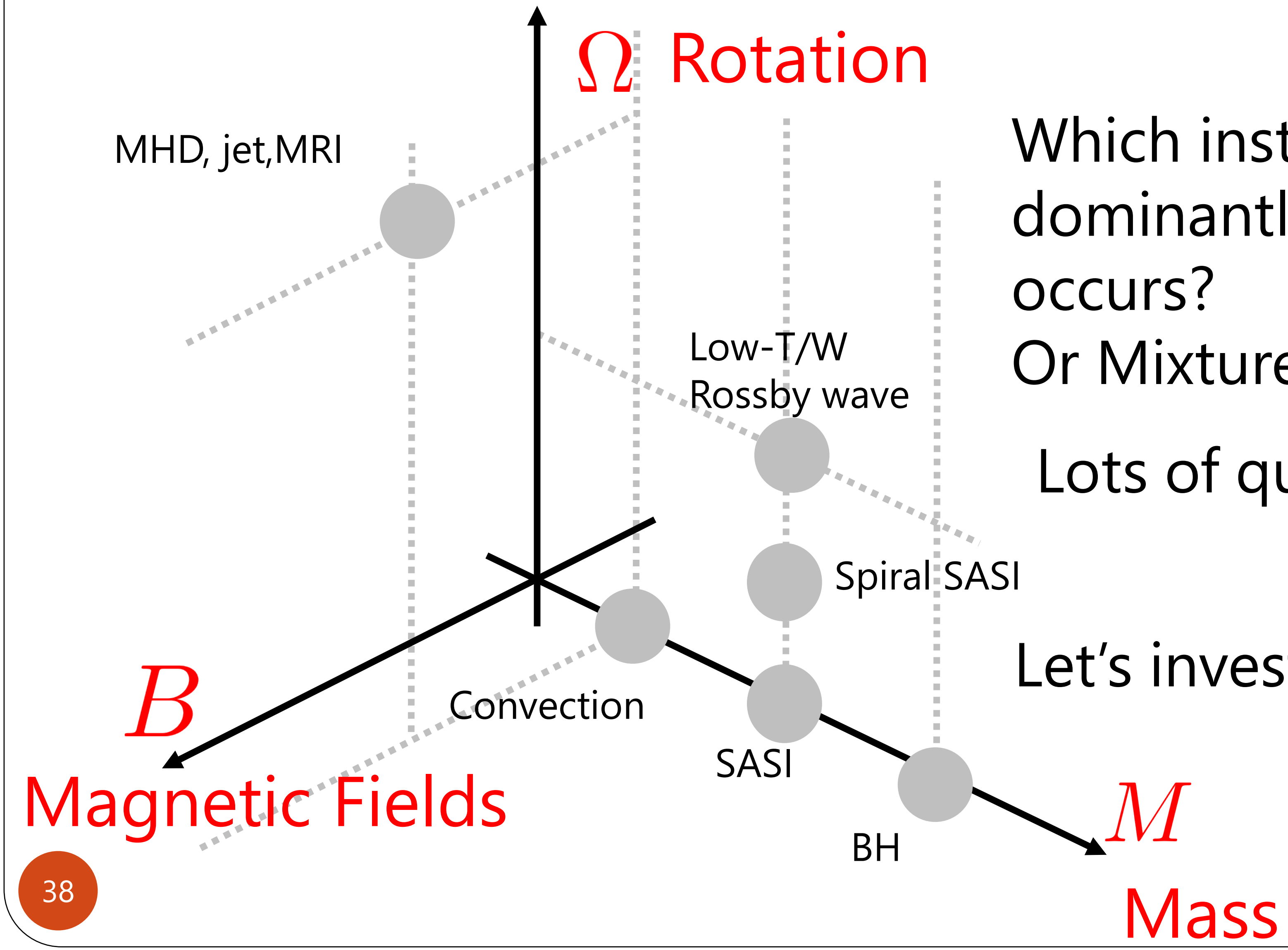
$$\partial_y \Pi = \frac{2\Omega}{r}\sin\theta\left[1 + 2\cos^2\theta\frac{r}{\rho}\partial_r\left(\rho\frac{\Omega^2}{\Omega_{\mathrm{BV}}^2}\alpha\right)\right] + \Delta_{\theta\theta}v_{\phi},$$

$\Omega^2/\Omega_{\mathrm{BV}}^2$ Is important. But not sure this formula is applicable if $\Omega_{\mathrm{BV}}^2 < 0$

And how much angular frequency is necessary to trigger the instability?

Does the B-fields suppress the Rossby wave instability?

Parameter Plane & Explosion Mechanism



Which instability dominantly occurs?
Or Mixture?

Lots of question.

Let's investigate it.

Summary

- The low–T/W instability or Rossby wave instability is found in proto neutron stars.
- The instability triggers explosion! Imposes large time–variability in neutrino observation and emits strong gravitational waves.
- The mechanism is identified as Rossby wave instability, which is governed by potential vorticity . It is similar to Kelvin–Helmholtz instability, which is governed by vorticity.

  $$\Pi = \zeta_a \cdot \nabla \lambda / \rho \quad \lambda = s, Y$$

  Differential rotation, density, entropy or induce it.
- Unsolved issues is competition to buoyancy. The angular velocity should be comparable to BV frequency but no quantitative answer is obtained.