Delta and Omega Baryons

Old Friends and New Physics

Stefan Leupold

Uppsala University

Wrocław, April 2024

Vetenskapsrådet
Swedish Research Council

Table of Contents

Various faces of QCD

model-independent methods to explore QCD (and in general QFT):

- perturbative QCD
  - works at high energies where strong interaction is weak
- lattice QCD
  - works best around Λ_{QCD}, m_s (hadronic scale $\approx 1\,\mathrm{GeV}$)
  - light pion sees itself around the torus *if* volume is too small
  - but advantage: quark masses can be varied
- (chiral) effective field theory $\rightsquigarrow$ works at low energies
- dispersion theory $\rightsquigarrow$ works if there are only a few channels

Various faces of QCD

model-independent methods to explore QCD (and in general QFT):

- perturbative QCD
  - works at high energies where strong interaction is weak
- lattice QCD
  - works best around Λ_{QCD}, m_s (hadronic scale $\approx 1\,\mathrm{GeV}$)
  - light pion sees itself around the torus *if* volume is too small
  - but advantage: quark masses can be varied
- (chiral) effective field theory $\rightsquigarrow$ works at low energies
- dispersion theory $\rightsquigarrow$ works if there are only a few channels
- experiment!

Various faces of QCD

model-independent methods to explore QCD (and in general QFT):

- perturbative QCD
  - works at high energies where strong interaction is weak
- lattice QCD
  - works best around $\Lambda_{\rm QCD}$, m_s (hadronic scale $\approx 1\,$GeV)
  - light pion sees itself around the torus *if* volume is too small
  - but advantage: quark masses can be varied
- (chiral) effective field theory $\rightsquigarrow$ works at low energies
- dispersion theory $\rightsquigarrow$ works if there are only a few channels
- experiment! $\rightsquigarrow$ but quark masses fixed

Unitarity and analyticity

- constraints from local quantum field theory:
  partial-wave amplitudes for reactions/decays must be
  - unitary:

$$S\,S^\dagger = 1\,, \quad S = 1 + iT \quad \Rightarrow \quad 2\,\mathrm{Im}\,T = T\,T^\dagger$$

↪ note that this is a matrix equation:
$\mathrm{Im}\,T_{A\to B} = \sum_X T_{A\to X}\,T^\dagger_{X\to B}$

  - analytic (dispersion relations):

$$T(s) = \frac{1}{\pi}\int\limits_{-\infty}^{\infty} ds'\,\frac{\mathrm{Im}\,T(s')}{s'-s-i\epsilon}$$

⇝ can be used to calculate whole amplitude from imaginary part

- practical limitation: too many states X at high energies

↪ in practice dispersion theory is a low-energy method ($\lesssim 1\,\mathrm{GeV}$)

Old friends: baryon decuplet

- three lightest quark masses are small ($m_u, m_d \ll m_s < \Lambda_{\rm QCD}$)

Old friends: baryon decuplet

- three lightest quark masses are small ($m_u, m_d \ll m_s < \Lambda_{\mathrm{QCD}}$)

$\hookrightarrow$ consider "chiral" limit where they are massless

Old friends: baryon decuplet

- three lightest quark masses are small ($m_u, m_d \ll m_s < \Lambda_{\rm QCD}$)

$\hookrightarrow$ consider "chiral" limit where they are massless

$\hookrightarrow$ obtain symmetry group $SU(3)_L \times SU(3)_R$
which is spontaneously broken to $SU(3)_V$
(and explicitly broken by quark masses)

Old friends: baryon decuplet

- three lightest quark masses are small ($m_u, m_d \ll m_s < \Lambda_{\rm QCD}$)

$\hookrightarrow$ consider "chiral" limit where they are massless

$\hookrightarrow$ obtain symmetry group $\mathrm{SU}(3)_L \times \mathrm{SU}(3)_R$
which is spontaneously broken to $\mathrm{SU}(3)_V$
(and explicitly broken by quark masses)

$\hookrightarrow$ obtain hadron multiplets classified by flavor $\mathrm{SU}(3)_V$
and (very) light Goldstone bosons (pions, kaons, eta)

Old friends: baryon decuplet

- three lightest quark masses are small ($m_u, m_d \ll m_s < \Lambda_{\mathrm{QCD}}$)

$\hookrightarrow$ consider "chiral" limit where they are massless

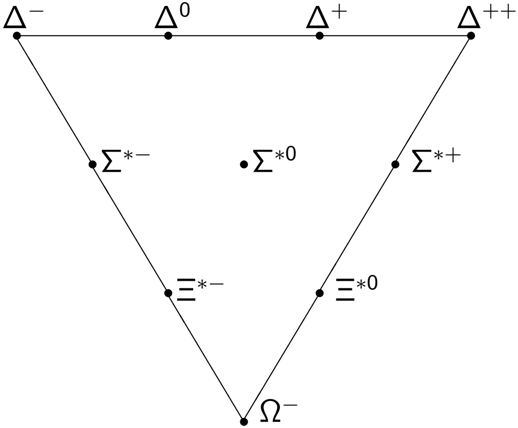
$\hookrightarrow$ obtain symmetry group $\mathrm{SU}(3)_L \times \mathrm{SU}(3)_R$
which is spontaneously broken to $\mathrm{SU}(3)_V$
(and explicitly broken by quark masses)

$\hookrightarrow$ obtain hadron multiplets classified by flavor $\mathrm{SU}(3)_V$
and (very) light Goldstone bosons (pions, kaons, eta)

$\hookrightarrow$ one multiplet is the baryon decuplet $\rightsquigarrow$ next page

$$3 \times 3 \times 3 = 10_S \oplus 8_M \oplus 8_M \oplus 1_A$$

Flavor decuplet



- Gell-Mann predicted existence and mass of Ω^- baryon

$\hookrightarrow$ Nobel Prize 1969

Old friends: baryon decuplet

some more facts and history related to the baryon decuplet

Old friends: baryon decuplet

some more facts and history related to the baryon decuplet

- in particular: Δ^{++}, Δ^- and Ω^- are states of pure flavor (note: Ω^{++}_{ccc} and Ω^-_{bbb} not found yet)

Old friends: baryon decuplet

some more facts and history related to the baryon decuplet

- in particular: Δ^{++}, Δ^{-} and Ω^{-} are states of pure flavor (note: Ω_{ccc}^{++} and Ω_{bbb}^{-} not found yet)

$\hookrightarrow$ e.g. $\Delta^{++}(J_3 = +3/2) \sim u\uparrow, u\uparrow, u\uparrow$ (flavor up, spin up)

Old friends: baryon decuplet

some more facts and history related to the baryon decuplet

- in particular: Δ^{++}, Δ^- and Ω^- are states of pure flavor (note: Ω_{ccc}^{++} and Ω_{bbb}^- not found yet)

$\hookrightarrow$ e.g. $\Delta^{++}(J_3 = +3/2) \sim u\uparrow, u\uparrow, u\uparrow$ (flavor up, spin up)

- to save Pauli principle:

Old friends: baryon decuplet

some more facts and history related to the baryon decuplet

- in particular: Δ^{++}, Δ^{-} and Ω^{-} are states of pure flavor (note: Ω^{++}_{ccc} and Ω^{-}_{bbb} not found yet)

$\hookrightarrow$ e.g. $\Delta^{++}(J_3 = +3/2) \sim u\uparrow, u\uparrow, u\uparrow$ (flavor up, spin up)

- to save Pauli principle:
  - color was introduced

Old friends: baryon decuplet

some more facts and history related to the baryon decuplet

- in particular: Δ^{++}, Δ^{-} and Ω^{-} are states of pure flavor (note: Ω_{ccc}^{++} and Ω_{bbb}^{-} not found yet)

$\hookrightarrow$ e.g. $\Delta^{++}(J_3 = +3/2) \sim u\uparrow, u\uparrow, u\uparrow$ (flavor up, spin up)

- to save Pauli principle:
  - color was introduced

  $\hookrightarrow$ finally we got QCD

Old friends: baryon decuplet

some more facts and history related to the baryon decuplet

- in particular: Δ^{++}, Δ^- and Ω^- are states of pure flavor (note: Ω^{++}_{ccc} and Ω^-_{bbb} not found yet)

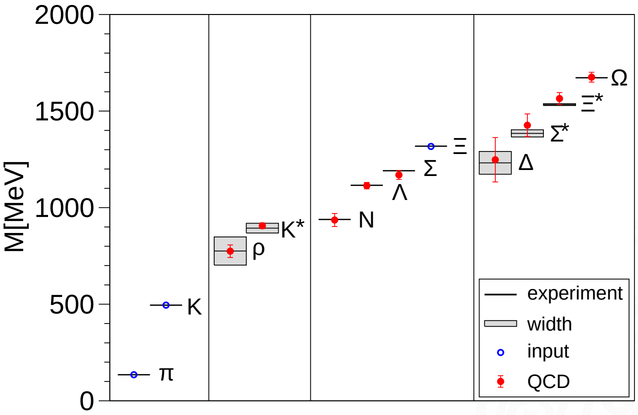
$\hookrightarrow$ e.g. $\Delta^{++}(J_3 = +3/2) \sim u\uparrow, u\uparrow, u\uparrow$ (flavor up, spin up)

- to save Pauli principle:
  - color was introduced

  $\hookrightarrow$ finally we got QCD
- Ω^- is only known long-living state with $J \geq \frac{3}{2}$
  - lowest-lying $S = -3$ state cannot decay strongly (kaon-cascade system is heavier)
  - lowest-lying $J = 3/2$ state cannot decay electromagnetically (Pauli principle forbids lower-lying $J = \frac{1}{2}$ state)

$\hookrightarrow$ all Ω decays are interesting!

Lattice QCD agrees with phenomenology



S. Dürr et al., Ab-Initio Determination of Light Hadron Masses, Science 322, 1224 (2008)

New physics: exploring neutrinos

- explore neutrino oscillations, CP violation in lepton sector, ...
- want to count how many neutrinos one has
  (of specific type and at specific distance)

New physics: exploring neutrinos

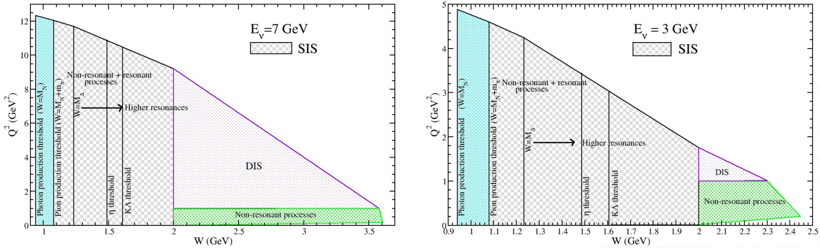
- explore neutrino oscillations, CP violation in lepton sector, ...
- want to count how many neutrinos one has
  (of specific type and at specific distance)

$\hookrightarrow$ need to know how many react with matter (nuclei!)

New physics: exploring neutrinos

- explore neutrino oscillations, CP violation in lepton sector, ...
- want to count how many neutrinos one has
  (of specific type and at specific distance)

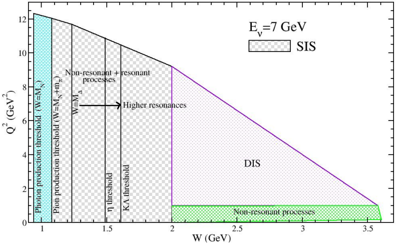
$\hookrightarrow$ need to know how many react with matter (nuclei!)

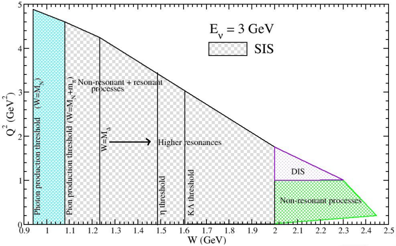


($\sqrt{Q^2}$: energy transfer to hadronic system; W: mass of produced hadronic system)

L. Alvarez Ruso et al., Contribution to Snowmass 2021, arXiv: 2203.09030 [hep-ph]

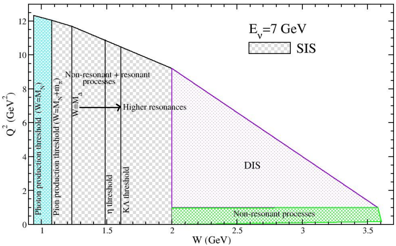
New physics: exploring neutrinos

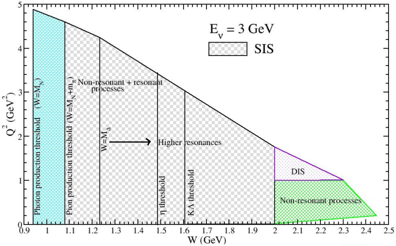




- how neutrinos interact with matter

New physics: exploring neutrinos

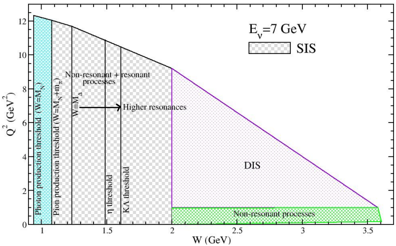


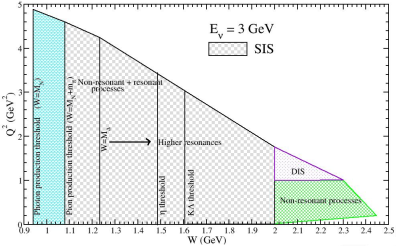


- how neutrinos interact with matter
  - at high energies: neutrinos “see” quarks

  ↪ perturbative QCD

New physics: exploring neutrinos



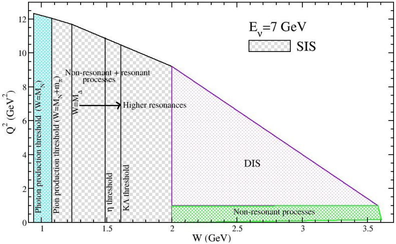


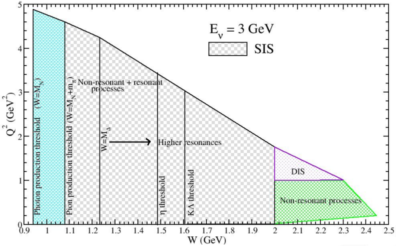
- how neutrinos interact with matter
  - at high energies: neutrinos “see” quarks

  ↪ perturbative QCD
  - at lowest energies: neutrinos “see” nuclei

  ↪ nuclear-structure physics

New physics: exploring neutrinos





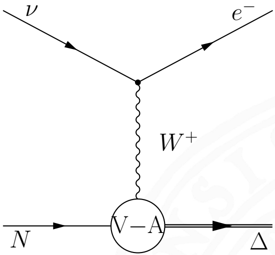
- how neutrinos interact with matter
  - at high energies: neutrinos “see” quarks
  - ↪ perturbative QCD
  - at lowest energies: neutrinos “see” nuclei
  - ↪ nuclear-structure physics
  - at low energies: neutrinos see “nucleons”
  - ↪ vector and axial-vector form factors of nucleon
    and pion production! ⇝ $\Delta(1232)$

true for all energies: need to get stuff out from the nucleus
↪ transport theory, e.g. U. Mosel, J. Phys. G 46 (2019) 11, 113001

Why is Δ interesting?

Axial-vector (and vector) transition form factors

- interesting for scattering neutrino-nucleon to electron-Δ or muon-Δ
- subsequently: $\Delta \to \pi N$
- low energies: want to know deviation from current-algebra result



NuSTEC Collaboration, L. Alvarez-Ruso et al., Prog. Part. Nucl. Phys. 100 (2018) 1;
M. Hilt, T. Bauer, S. Scherer, L. Tiator, Phys. Rev. C 97 (2018) 3, 035205;
M. Holmberg, SL, Phys. Rev. D 100 (2019) 11, 114001;
Y. Ünal, A. Küçükarslan, S. Scherer, Phys. Rev. D 104 (2021) 9, 094014;
S.K. Singh, M.J. Vicente Vacas, Phys. Rev. D 74 (2006) 053009

How about lattice QCD?

- currently under investigation using lattice QCD:
  - form factors of stable baryons
  - and their quark-mass dependence
- interpretation of results by chiral effective field theory

  e.g. M.F.M. Lutz, U. Sauerwein, R.G.E. Timmermans, Eur. Phys. J. C 80 (2020) 9, 844; Phys. Rev. D 105 (2022) 5, 054005

  F. Alvarado, L. Alvarez-Ruso, Phys. Rev. D 105 (2022) 7, 074001

How about lattice QCD?

- currently under investigation using lattice QCD:
  - form factors of stable baryons
  - and their quark-mass dependence
- interpretation of results by chiral effective field theory

  e.g. M.F.M. Lutz, U. Sauerwein, R.G.E. Timmermans, Eur. Phys. J. C 80 (2020) 9, 844; Phys. Rev. D 105 (2022) 5, 054005

  F. Alvarado, L. Alvarez-Ruso, Phys. Rev. D 105 (2022) 7, 074001
- but for transition form factors Δ-N:
  - much more complicated because Δ is unstable

  $\hookrightarrow$ essentially four-point function instead of three-point function

How about lattice QCD?

- currently under investigation using lattice QCD:
  - form factors of stable baryons
  - and their quark-mass dependence
- interpretation of results by chiral effective field theory

  e.g. M.F.M. Lutz, U. Sauerwein, R.G.E. Timmermans, Eur. Phys. J. C 80 (2020) 9, 844; Phys. Rev. D 105 (2022) 5, 054005

  F. Alvarado, L. Alvarez-Ruso, Phys. Rev. D 105 (2022) 7, 074001
- but for transition form factors Δ-N:
  - much more complicated because Δ is unstable

  $\hookrightarrow$ essentially four-point function instead of three-point function
- to circumvent problem: study stable flavor partners

$\hookrightarrow$ Ω-Ξ transition form factors

My suggestion for a research program

1. study Ω transition form factors in lattice QCD and experiment (and quark-mass dependence on lattice)
2. interpret results using (dispersively modified) chiral effective field theory (dimχEFT)
3. extrapolate to Δ transition form factors using (dim)χEFT and experimental data (hyperons)
4. obtain improved input for neutrino scattering (and obtain better understanding of structure of hadrons)

UU contribution: develop dimχEFT

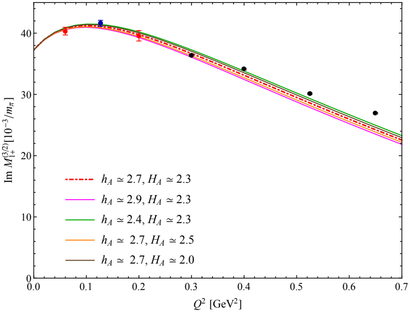
Does all this make sense?

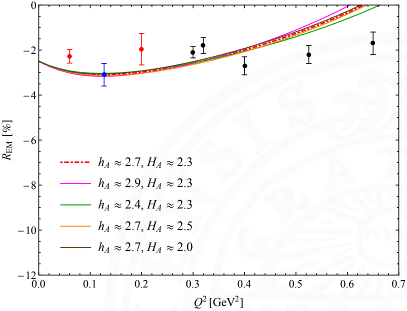
- comparison to experiment:
  - vector transition Δ-N is experimentally much better known than axial-vector

  $\hookrightarrow$ How well does dimχEFT work there?
- comparison to lattice QCD:
  - Can dimχEFT describe quark-mass dependence?

Does all this make sense?

transition form factors from nucleon to Δ
using subtracted dispersion relations
(data from JLab, Mami, . . .)

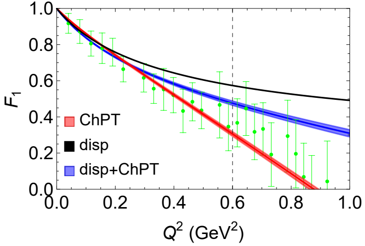




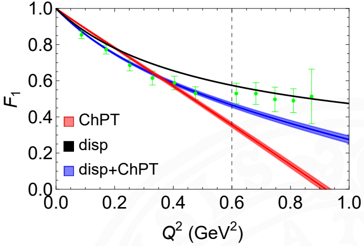
M. M. Aung, SL, E. Perotti, Y. Yan, arXiv: 2401.17756 [hep-ph]

Does all this make sense?

quark-mass and momentum dependence of nucleon Dirac form factor



$M_\pi = 0.130\,\text{GeV}$



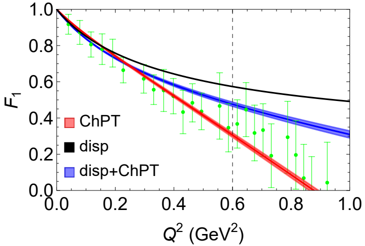
$M_\pi = 0.223\,\text{GeV}$

F. Alvarado, D. An, L. Alvarez-Ruso, SL, Phys. Rev. D 108 (2023) 11, 114021

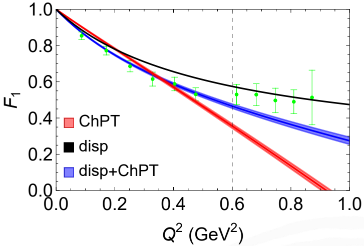
lattice data from Darmstadt-Edinburgh-Mainz group:

D. Djukanovic et al., Phys. Rev. D 103 (2021) 9, 094522

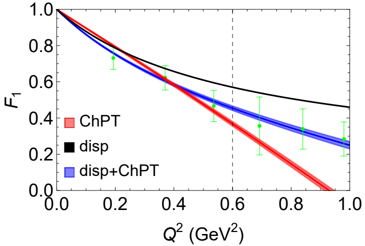
Dirac vector isovector form factor of nucleon



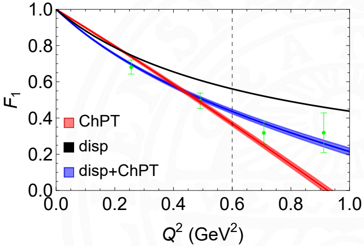
$M_\pi = 0.130\,\text{GeV}$



$M_\pi = 0.223\,\text{GeV}$



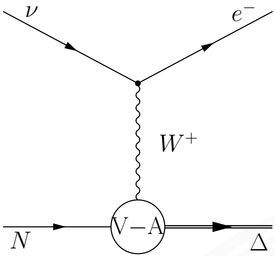
$M_\pi = 0.278\,\text{GeV}$



$M_\pi = 0.353\,\text{GeV}$

Neutrino-baryon interactions

- interesting for scattering neutrino-nucleon to electron-Delta
- low energies: want to know deviation from current-algebra result $\rightsquigarrow$ LEC c_E



- in chiral perturbation theory at next-to-leading order (NLO): only one LEC c_E for whole multiplet

$\hookrightarrow$ study $\Omega^- \to \Xi^0 e^- \bar{\nu}_e$ instead of $N\nu_e \to \Delta e^-$

lingo of low-energy effective field theory:

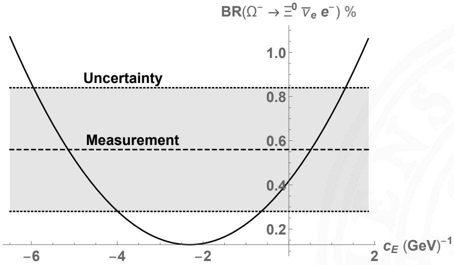
- LO: leading-order calculation
- NLO: next to leading order
- LEC: low-energy constant, i.e. coupling constant

Branching ratio $\Omega^- \to \Xi^0 e^- \bar{\nu}_e$ (measured)

- so far only next-to-leading-order (NLO) calculation finished
- contribution from LO Lagrangian ($\sim h_A$) related to $\Sigma^* \to \Sigma\pi$
- contributions from NLO Lagrangian $\sim c_M, c_E$

$\hookrightarrow$ $|c_M|$ related to $\Sigma^{*0} \to \Lambda\gamma$

$\hookrightarrow$ get constraints on c_E from measured branching ratio:

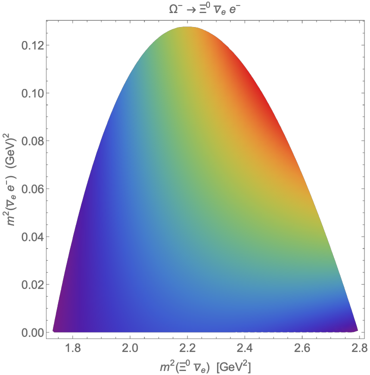


M. Holmberg, SL, Eur. Phys. J. A 54 (2018) 6, 103; Phys. Rev. D 100 (2019) 11, 114001
C.J.G. Mommers, SL, Phys. Rev. D 106 (2022) 9, 093001

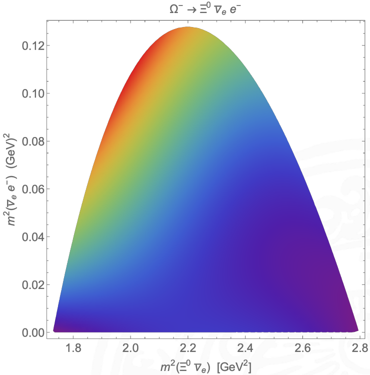
first steps beyond tree level: H. De Munck; M. Bertilsson, master theses UU 2023

Dalitz plot $\Omega^- \to \Xi^0 e^- \bar{\nu}_e$ (not measured yet)

- different values for c_E and sign of c_M influence Dalitz plot:



$$c_E = 0.52\,\text{GeV}^{-1}, \quad c_M = -1.92\,\text{GeV}^{-1}$$



$$c_E = -5.1\,\text{GeV}^{-1}, \quad c_M = -1.92\,\text{GeV}^{-1}$$

- sign change of c_M flips plots right $\leftrightarrow$ left

Back to QCD: Why are Δs interesting?

Low-energy QCD (aka chiral effective field theory)

Back to QCD: Why are Δs interesting?

Low-energy QCD (aka chiral effective field theory)

- "definition", i.e. range of applicability:
  momenta $|\vec{p}|$, pion mass $m_\pi \ll$ nucleon mass m_N

Back to QCD: Why are Δs interesting?

Low-energy QCD (aka chiral effective field theory)

- "definition", i.e. range of applicability:
  momenta $|\vec{p}|$, pion mass $m_\pi \ll$ nucleon mass m_N
- purposes:

Back to QCD: Why are Δs interesting?

Low-energy QCD (aka chiral effective field theory)

- "definition", i.e. range of applicability:
  momenta $|\vec{p}|$, pion mass $m_\pi \ll$ nucleon mass m_N
- purposes:
  - precision tests of standard model

Back to QCD: Why are Δs interesting?

Low-energy QCD (aka chiral effective field theory)

- "definition", i.e. range of applicability:
  momenta $|\vec{p}|$, pion mass $m_\pi \ll$ nucleon mass m_N
- purposes:
  - precision tests of standard model
  - ab-initio calculations of nuclei

Back to QCD: Why are Δs interesting?

Low-energy QCD (aka chiral effective field theory)

- "definition", i.e. range of applicability:
  momenta $|\vec{p}|$, pion mass $m_\pi \ll$ nucleon mass m_N
- purposes:
  - precision tests of standard model
  - ab-initio calculations of nuclei
  - description of properties of nuclear matter, neutron stars, ...
  - ...

Role of Δ in low-energy QCD?

unclear: how important are Δ baryons?

Role of Δ in low-energy QCD?

unclear: how important are Δ baryons?

- is the following the right way of ordering scales?

$$|\vec{p}|, m_\pi, m_\Delta - m_N \ll m_N$$

Role of Δ in low-energy QCD?

unclear: how important are Δ baryons?

- is the following the right way of ordering scales?

$$|\vec{p}|, m_\pi, m_\Delta - m_N \ll m_N$$

↪ must include Δ as relevant degree of freedom

Role of Δ in low-energy QCD?

unclear: how important are Δ baryons?

- is the following the right way of ordering scales?

$$|\vec{p}|, m_\pi, m_\Delta - m_N \ll m_N$$

$\hookrightarrow$ must include Δ as relevant degree of freedom

- or is rather the following sufficient?

$$|\vec{p}|, m_\pi \ll m_\Delta - m_N, m_N$$

Role of Δ in low-energy QCD?

unclear: how important are Δ baryons?

- is the following the right way of ordering scales?

$$|\vec{p}|, m_\pi, m_\Delta - m_N \ll m_N$$

$\hookrightarrow$ must include Δ as relevant degree of freedom

- or is rather the following sufficient?

$$|\vec{p}|, m_\pi \ll m_\Delta - m_N, m_N$$

$\hookrightarrow$ not enough energy to excite Δ

Role of Δ in low-energy QCD?

unclear: how important are Δ baryons?

- is the following the right way of ordering scales?

$$|\vec{p}|,\, m_\pi,\, m_\Delta - m_N \ll m_N$$

↪ must include Δ as relevant degree of freedom

- or is rather the following sufficient?

$$|\vec{p}|,\, m_\pi \ll m_\Delta - m_N,\, m_N$$

↪ not enough energy to excite Δ

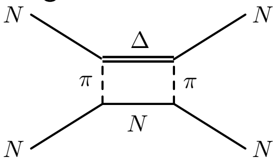
↪ need only pions and nucleons to construct low-energy QCD

Role of Δ in low-energy QCD?

consider nucleon-nucleon scattering:

Role of Δ in low-energy QCD?

consider nucleon-nucleon scattering:

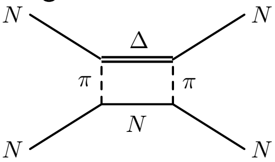


- is it necessary to consider

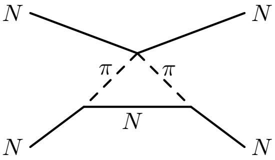
Role of Δ in low-energy QCD?

consider nucleon-nucleon scattering:

- is it necessary to consider



or is

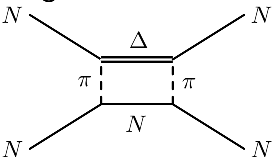


sufficient?

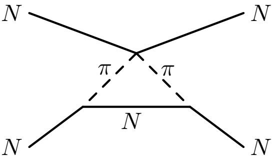
Role of Δ in low-energy QCD?

consider nucleon-nucleon scattering:

- is it necessary to consider

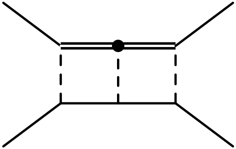


or is



sufficient?

- how important is

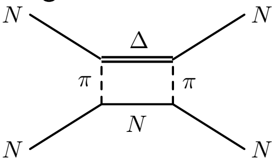


?

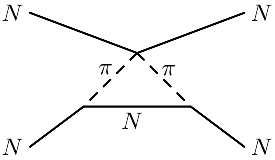
Role of Δ in low-energy QCD?

consider nucleon-nucleon scattering:

- is it necessary to consider

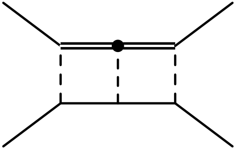


or is



sufficient?

- how important is

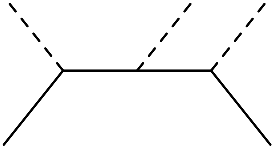


?

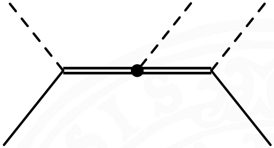
- coupling constant Δ-Δ-π • unknown!

What is unknown for Δ baryons?

- coupling constant Δ-Δ-π • completely unknown; not even sign is clear
- further example for its impact: is interference of



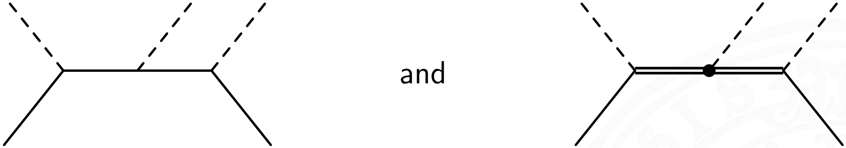
and



constructive or destructive?

What is unknown for Δ baryons?

- coupling constant Δ-Δ-π • completely unknown; not even sign is clear
- further example for its impact: is interference of

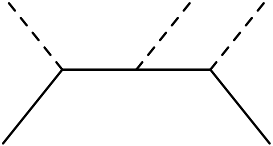


constructive or destructive?

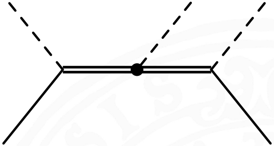
- one problem to determine Δ properties with better precision: Δs are rather broad states ($\Gamma \approx 100\,$MeV)

What is unknown for Δ baryons?

- coupling constant Δ-Δ-π • completely unknown; not even sign is clear
- further example for its impact: is interference of



and



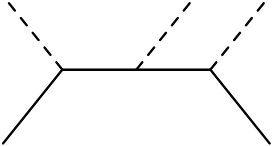
constructive or destructive?

- one problem to determine Δ properties with better precision: Δs are rather broad states ($\Gamma \approx 100\,\text{MeV}$)

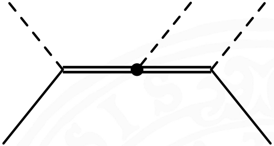
$\hookrightarrow$ worth to study strange siblings from decuplet

What is unknown for Δ baryons?

- coupling constant Δ-Δ-π • completely unknown; not even sign is clear
- further example for its impact: is interference of



and



constructive or destructive?

- one problem to determine Δ properties with better precision: Δs are rather broad states ($\Gamma \approx 100\,\mathrm{MeV}$)

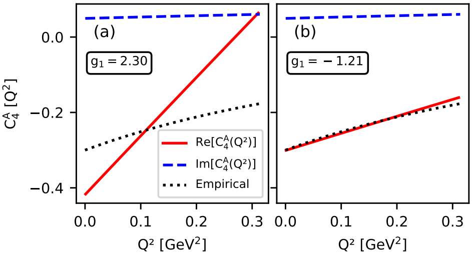
$\hookrightarrow$ worth to study strange siblings from decuplet

$\hookrightarrow$ e.g. Ω-Ξ^*-K coupling

Δ couplings and Δ-N transition form factors

- Δ loops enter calculation of Δ-N transition form factors

but coupling constant Δ-Δ-π (g_1) unknown in size and sign
(actually there are two couplings, p and f wave $\rightsquigarrow$ here: p-wave coupling)



one of the axial-vector transition form factors; g_1 enters indirectly at loop level

Y. Ünal, A. Küçükarslan, S. Scherer, Phys. Rev. D 104 (2021) 9, 094014

- quark model and large-N_c QCD suggest positive g_1 (figure: left-hand side)
- recent analysis of π-N scattering suggests negative g_1
  De-Liang Yao *et al.*, JHEP 05 (2016) 038

Decay $\Omega^- \to \Xi^{*0}\ell^-\bar{\nu}_\ell$ (not measured yet)

- provides access to sign and size of coupling constant Ω-$\Xi^*(1530)$-K via Golberger-Treiman relation
- flavor related to sign and size of coupling constant Δ-Δ-π ($H_A = g_1$)
- so far only leading-order calculation for branching ratio and forward-backward (fb) asymmetry (Wu-type experiment)
  (rest frame of dilepton, measuring angle between baryons and charged lepton)

	$\Gamma_{\Omega\to\Xi^*\ell\bar{\nu}_\ell}/\Gamma_{\Omega,\mathrm{tot}}$	$\Gamma_{\mathrm{fb}}/\Gamma_{\Omega\to\Xi^*\ell\nu}$
$\ell = e,\ H_A = +2$	$1.2\cdot10^{-4}$	$+0.011$
$\ell = e,\ H_A = 0$	$6.7\cdot10^{-5}$	-0.00043
$\ell = e,\ H_A = -2$	$1.2\cdot10^{-4}$	-0.012
$\ell = \mu,\ H_A = +2$	$4.3\cdot10^{-6}$	-0.23
$\ell = \mu,\ H_A = 0$	$2.5\cdot10^{-6}$	-0.33
$\ell = \mu,\ H_A = -2$	$4.3\cdot10^{-6}$	-0.25

- note: $\Xi^{*0}(1530)$ "easy" to reconstruct via sequence $\Xi^{*0} \to \Xi^-\pi^+$, $\Xi^- \to \Lambda\pi^-$, $\Lambda \to p\pi^-$

M. Bertilsson, SL, Phys. Rev. D 109 (2024) 3, 034028

Some fun with group theory in hot QCD

$$\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \to \mathrm{SU}(2)_V \text{ (isospin symmetry)}$$

- what happens at chiral restoration?

Some fun with group theory in hot QCD

$$\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \to \mathrm{SU}(2)_V \text{ (isospin symmetry)}$$

- what happens at chiral restoration?
- parity doublet models: assign chiral representation (L,R) to nucleon and identify its parity partner

$\hookrightarrow$ N: (2,1) $\leftrightarrow$ $N^*(1535)$: (1,2)

- note: pions and sigma live in (2,2)

Some fun with group theory in hot QCD

$$\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \to \mathrm{SU}(2)_V \text{ (isospin symmetry)}$$

- what happens at chiral restoration?
- parity doublet models: assign chiral representation (L,R) to nucleon and identify its parity partner

$\hookrightarrow$ N: (2,1) $\leftrightarrow$ $N^*(1535)$: (1,2)

- note: pions and sigma live in (2,2)
- but how about Δ?

$\hookrightarrow$ is it in (4,1) or in (3,2)? (e.g., $u_L u_L u_L$ or $u_L u_L u_R$)

Some fun with group theory in hot QCD

$$SU(2)_L \times SU(2)_R \to SU(2)_V \text{ (isospin symmetry)}$$

- what happens at chiral restoration?
- parity doublet models: assign chiral representation (L,R) to nucleon and identify its parity partner

$\hookrightarrow$ N: (2,1) $\leftrightarrow$ $N^*(1535)$: (1,2)

- note: pions and sigma live in (2,2)
- but how about Δ?

$\hookrightarrow$ is it in (4,1) or in (3,2)? (e.g., $u_L u_L u_L$ or $u_L u_L u_R$)

- pion-nucleon system:

$$(2,2) \times (2,1) = (3,2) \oplus (1,2)$$

Some fun with group theory in hot QCD

$$\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \to \mathrm{SU}(2)_V \text{ (isospin symmetry)}$$

- what happens at chiral restoration?
- parity doublet models: assign chiral representation (L,R) to nucleon and identify its parity partner

$\hookrightarrow$ N: (2,1) $\leftrightarrow$ $N^*(1535)$: (1,2)

- note: pions and sigma live in (2,2)
- but how about Δ?

$\hookrightarrow$ is it in (4,1) or in (3,2)? (e.g., $u_L u_L u_L$ or $u_L u_L u_R$)

- pion-nucleon system:

$$(2,2) \times (2,1) = (3,2) \oplus (1,2)$$

$\hookrightarrow$ no three-point coupling Δ-N-π if Δ lives in (4,1)

Some fun with group theory in hot QCD

$$\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \to \mathrm{SU}(2)_V \text{ (isospin symmetry)}$$

- what happens at chiral restoration?
- parity doublet models: assign chiral representation (L,R) to nucleon and identify its parity partner

↪ N: (2,1) ↔ $N^*(1535)$: (1,2)

- note: pions and sigma live in (2,2)
- but how about Δ?

↪ is it in (4,1) or in (3,2)? (e.g., $u_L u_L u_L$ or $u_L u_L u_R$)

- pion-nucleon system:

$$(2,2) \times (2,1) = (3,2) \oplus (1,2)$$

↪ no three-point coupling Δ-N-π if Δ lives in (4,1)

- in vacuum, coupling is effectively generated by Δ-N-π-$\langle\sigma\rangle$

$$(2,2) \times (2,2) \times (2,1) = (4,3) \oplus (4,1) \oplus 2*(2,3) \oplus 2*(2,1)$$

C. Kummer, L. von Smekal, SL, work in progress

Summary and outlook

D elta (transition) form factors are interesting for neutrino physics and low-energy QCD

Summary and outlook

- D elta (transition) form factors are interesting for neutrino physics and low-energy QCD
- O mega transition form factors are a better starting point

Summary and outlook

- D elta (transition) form factors are interesting for neutrino physics and low-energy QCD
- O mega transition form factors are a better starting point
- L attice QCD can tackle these (plus experimental guidance)

Summary and outlook

- D elta (transition) form factors are interesting for neutrino physics and low-energy QCD
- O mega transition form factors are a better starting point
- L attice QCD can tackle these (plus experimental guidance)

$$\Omega^- \to \Xi^0\, \ell^- \bar{\nu}\,, \qquad \Omega^- \to \Xi^{*0}(1530)\, \ell^- \bar{\nu}$$

- C hiral effective field theory can interpret results

Summary and outlook

- D elta (transition) form factors are interesting for neutrino physics and low-energy QCD
- O mega transition form factors are a better starting point
- L attice QCD can tackle these (plus experimental guidance)

$$\Omega^- \to \Xi^0\, \ell^- \bar{\nu}\,, \qquad \Omega^- \to \Xi^{*0}(1530)\, \ell^- \bar{\nu}$$

- C hiral effective field theory can interpret results
- E xtrapolation to Delta (EFT plus experimental guidance)

Summary and outlook

- **D** elta (transition) form factors are interesting for neutrino physics and low-energy QCD
- **O** mega transition form factors are a better starting point
- **L** attice QCD can tackle these (plus experimental guidance)

$$\Omega^- \to \Xi^0 \, \ell^- \bar{\nu} \,, \qquad \Omega^- \to \Xi^{*0}(1530) \, \ell^- \bar{\nu}$$

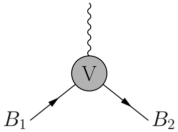
- **C** hiral effective field theory can interpret results
- **E** xtrapolation to Delta (EFT plus experimental guidance)

$\hookrightarrow$ DOLCE

Spare slides

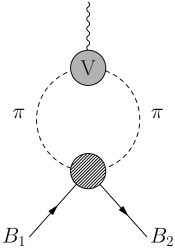
dimχEFT: electromagnetic baryon form factors

- how to obtain a form factor?



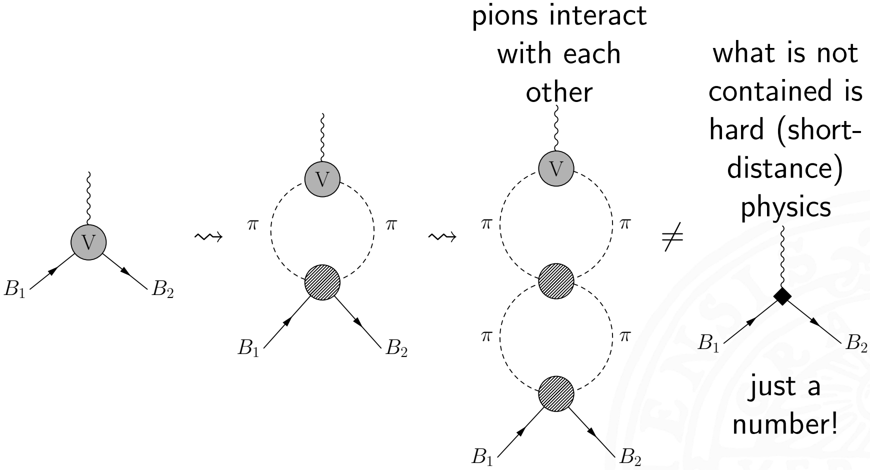
- need to resolve at least the finite size $\lesssim 1\,\mathrm{fm}$
- but inverse size of a hadron is larger than pion mass
- first one probes something universal (independent of $B_{1,2}$):

the "pion cloud":

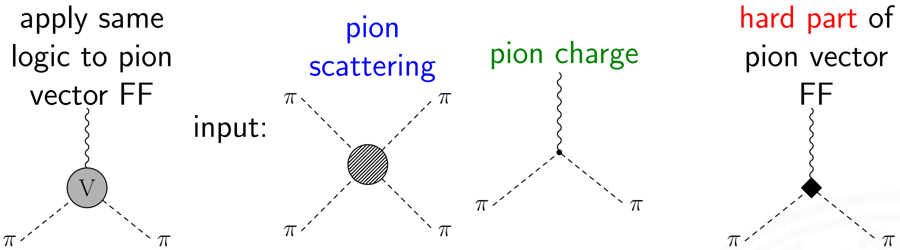


- now we are in the game with dispersion theory

Deconstruct a form factor



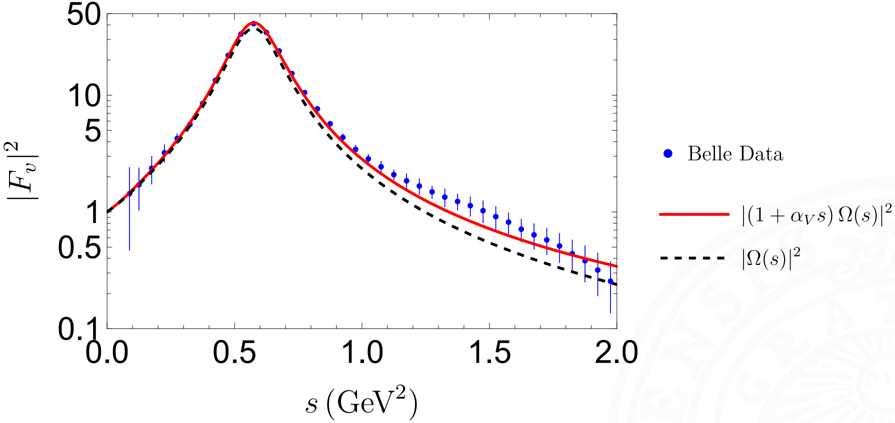
How to get the pion vector form factor?



$$F_V(s) = (1 + \alpha_V\, s)\, \exp\left\{ s \int\limits_{4m_\pi^2}^{\infty} \frac{ds'}{\pi} \frac{\delta(s')}{s'\,(s'-s-i\epsilon)} \right\}$$

with pion phase shift δ
and $\alpha_V \approx 0.12\,\mathrm{GeV}^{-2}$ (from fit to FF data)

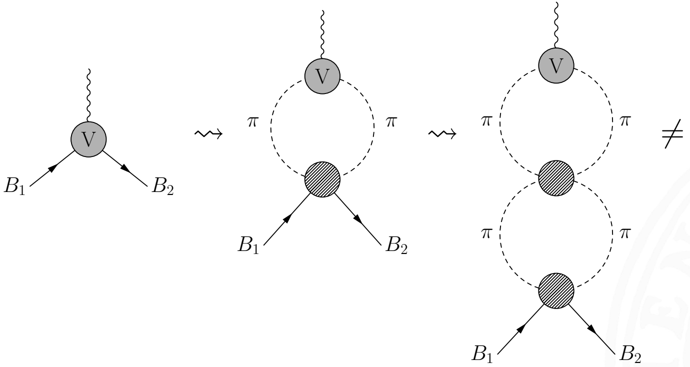
Pion vector form factor and data



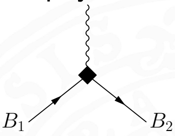
Alvarado/An/Alvarez-Ruso/SL, Phys. Rev. D 108 (2023) 11, 114021

Deconstruct a form factor

pions interact with each other



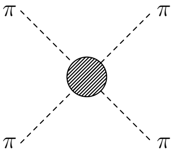
what is not contained is hard (short-distance) physics



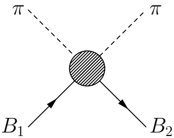
just a number!

Scattering processes

from data plus dispersion theory:

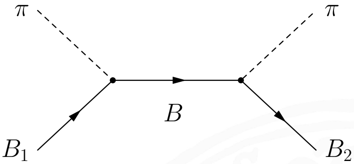


part that is not pion rescattering:

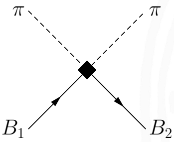


⇝

low-energy approximation:

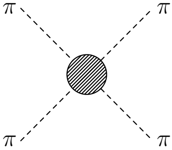


what is not covered is hard physics (contact terms)

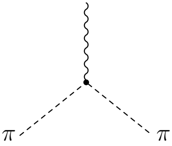


Known input

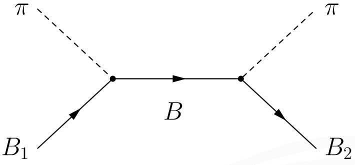
pion scattering



pion charge



low-energy approximation

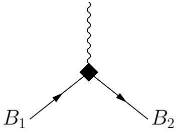


- baryon-pion coupling constants from decay widths

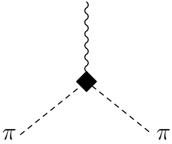
$\hookrightarrow$ sometimes only moduli known

Unknown: some numbers

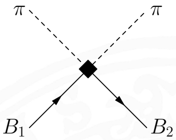
part without two intermediate pions



intermediate state with more than two pions



intermediate state that is not one baryon



$\hookrightarrow$ fit to data (now) or calculate with quark-gluon based methods (future)

Why all decays of Ω^- are interesting

- $\Omega^- \to \Xi^0\, e^- \bar{\nu}_e$ (Br $= 6 \cdot 10^{-3}$; experiment)
  flavor related to neutrino-nucleon scattering

Why all decays of Ω^- are interesting

- $\Omega^- \to \Xi^0 \, e^- \bar{\nu}_e$ (Br $= 6 \cdot 10^{-3}$; experiment)
  flavor related to neutrino-nucleon scattering
- $\Omega^- \to \Xi^{*0} \, e^- \bar{\nu}_e$ (Br $\approx 10^{-4}$; our estimate)
  flavor related to Δ-Δ-π via Goldberger-Treiman relation

Why all decays of Ω^- are interesting

- $\Omega^- \to \Xi^0 \, e^- \bar{\nu}_e$ (Br $= 6 \cdot 10^{-3}$; experiment)
  flavor related to neutrino-nucleon scattering
- $\Omega^- \to \Xi^{*0} \, e^- \bar{\nu}_e$ (Br $\approx 10^{-4}$; our estimate)
  flavor related to Δ-Δ-π via Goldberger-Treiman relation
- $\Omega^- \to \Lambda K^-$ (Br $= 68\%$)
  has decay asymmetry $\alpha = 0.0157 \pm 0.0021 \neq 0$

Why all decays of Ω^- are interesting

- $\Omega^- \to \Xi^0 \, e^- \bar{\nu}_e$ (Br $= 6 \cdot 10^{-3}$; experiment)
  flavor related to neutrino-nucleon scattering
- $\Omega^- \to \Xi^{*0} \, e^- \bar{\nu}_e$ (Br $\approx 10^{-4}$; our estimate)
  flavor related to Δ-Δ-π via Goldberger-Treiman relation
- $\Omega^- \to \Lambda K^-$ (Br $= 68\%$)
  has decay asymmetry $\alpha = 0.0157 \pm 0.0021 \neq 0$

$\hookrightarrow$ probably accuracy can be improved; search for CP violation ...

Why all decays of Ω^- are interesting

- $\Omega^- \to \Xi^0\, e^- \bar{\nu}_e$ (Br $= 6 \cdot 10^{-3}$; experiment)
  flavor related to neutrino-nucleon scattering
- $\Omega^- \to \Xi^{*0}\, e^- \bar{\nu}_e$ (Br $\approx 10^{-4}$; our estimate)
  flavor related to Δ-Δ-π via Goldberger-Treiman relation
- $\Omega^- \to \Lambda K^-$ (Br $= 68\%$)
  has decay asymmetry $\alpha = 0.0157 \pm 0.0021 \neq 0$

$\hookrightarrow$ probably accuracy can be improved; search for CP violation ...

- $\Omega^- \to \Xi^- \pi^0, \Xi^0 \pi^-$ has suspicious ratio of branching fractions

Why all decays of Ω^- are interesting

- $\Omega^- \to \Xi^0\, e^-\bar{\nu}_e$ (Br $= 6 \cdot 10^{-3}$; experiment)
  flavor related to neutrino-nucleon scattering
- $\Omega^- \to \Xi^{*0}\, e^-\bar{\nu}_e$ (Br $\approx 10^{-4}$; our estimate)
  flavor related to Δ-Δ-π via Goldberger-Treiman relation
- $\Omega^- \to \Lambda K^-$ (Br $= 68\%$)
  has decay asymmetry $\alpha = 0.0157 \pm 0.0021 \neq 0$

$\hookrightarrow$ probably accuracy can be improved; search for CP violation . . .

- $\Omega^- \to \Xi^-\pi^0, \Xi^0\pi^-$ has suspicious ratio of branching fractions
- $\Omega^- \to \Xi^-\pi^+\pi^-$ (Br $= 4 \cdot 10^{-4}$)
  - exp: seems not to show $\Xi^*(1530)$ as intermediate state

  $\hookrightarrow$ flaw in resonance saturation?

  $\hookrightarrow$ EFT construction:
  What are the relevant degrees of freedom in Ω decays?

C.J.G. Mommers, SL, Phys. Rev. D 106 (2022) 9, 093001

Non-leptonic decays

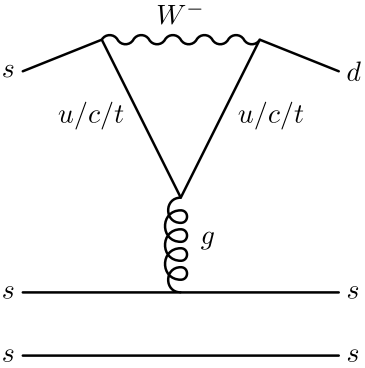
- standard picture for non-leptonic decays $\Omega \to$ meson + baryon

$\hookrightarrow$ dominated by transition $s \to d$, e.g. penguin diagram

Non-leptonic decays

- standard picture for non-leptonic decays $\Omega \to$ meson $+$ baryon

$\hookrightarrow$ dominated by transition $s \to d$, e.g. penguin diagram

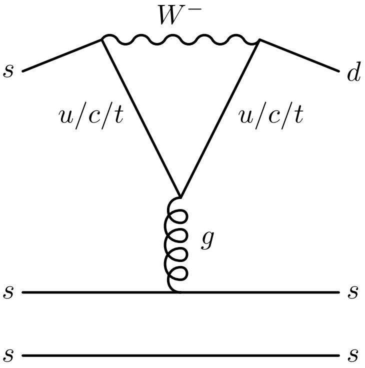


$\rightsquigarrow$

Non-leptonic decays

- standard picture for non-leptonic decays $\Omega \to$ meson $+$ baryon

$\hookrightarrow$ dominated by transition $s \to d$, e.g. penguin diagram



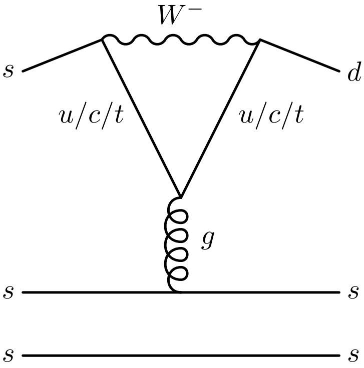
$\rightsquigarrow$

$\hookrightarrow$ thus intermediate state is an offshell Ξ^{*-} with $J = \frac{3}{2}$, $I = \frac{1}{2}$

Non-leptonic decays

- standard picture for non-leptonic decays $\Omega \to$ meson + baryon

$\hookrightarrow$ dominated by transition $s \to d$, e.g. penguin diagram



$\rightsquigarrow$

$\hookrightarrow$ thus intermediate state is an offshell Ξ^{*-} with $J = \frac{3}{2}$, $I = \frac{1}{2}$

$\hookrightarrow$ $\Delta I = \frac{1}{2}$ rule

Non-leptonic decays

- consider decays:
  - $\Omega^- \to \Xi^- \pi^0$ (Br = 9%)
  - and $\Omega^- \to \Xi^0 \pi^-$ (Br = 24%)

Non-leptonic decays

- consider decays:
  - $\Omega^- \to \Xi^- \pi^0$ (Br = 9%)
  - and $\Omega^- \to \Xi^0 \pi^-$ (Br = 24%)

$\hookrightarrow$ determine Clebsch-Gordans for $\Xi^{*-} \to \Xi^- \pi^0, \Xi^0 \pi^-$:

- $\Omega^- \to (\Xi^{*-} \to)\Xi^- \pi^0$:

$$\left\langle I = \frac{1}{2}, I_3 = -\frac{1}{2} \,\middle|\, 1, 0; \frac{1}{2}, -\frac{1}{2} \right\rangle = \frac{1}{\sqrt{3}}$$

- $\Omega^- \to (\Xi^{*-} \to)\Xi^0 \pi^-$:

$$\left\langle I = \frac{1}{2}, I_3 = -\frac{1}{2} \,\middle|\, 1, -1; \frac{1}{2}, +\frac{1}{2} \right\rangle = -\sqrt{\frac{2}{3}}$$

Non-leptonic decays

- consider decays:
  - $\Omega^- \to \Xi^- \pi^0$ (Br = 9%)
  - and $\Omega^- \to \Xi^0 \pi^-$ (Br = 24%)

$\hookrightarrow$ determine Clebsch-Gordans for $\Xi^{*-} \to \Xi^- \pi^0, \Xi^0 \pi^-$:

- $\Omega^- \to (\Xi^{*-} \to) \Xi^- \pi^0$:

$$\left\langle I = \frac{1}{2}, I_3 = -\frac{1}{2} \middle| 1, 0; \frac{1}{2}, -\frac{1}{2} \right\rangle = \frac{1}{\sqrt{3}}$$

- $\Omega^- \to (\Xi^{*-} \to) \Xi^0 \pi^-$:

$$\left\langle I = \frac{1}{2}, I_3 = -\frac{1}{2} \middle| 1, -1; \frac{1}{2}, +\frac{1}{2} \right\rangle = -\sqrt{\frac{2}{3}}$$

$\hookrightarrow$ prediction

$$\frac{\Gamma(\Omega^- \to \Xi^0 \pi^-)}{\Gamma(\Omega^- \to \Xi^- \pi^0)} \approx 2$$

Non-leptonic decays

- consider decays:
  - $\Omega^- \to \Xi^- \pi^0$ (Br = 9%)
  - and $\Omega^- \to \Xi^0 \pi^-$ (Br = 24%)

$\hookrightarrow$ determine Clebsch-Gordans for $\Xi^{*-} \to \Xi^- \pi^0, \Xi^0 \pi^-$:

- $\Omega^- \to (\Xi^{*-} \to) \Xi^- \pi^0$:

$$\left\langle I = \frac{1}{2}, I_3 = -\frac{1}{2} \,\middle|\, 1, 0; \frac{1}{2}, -\frac{1}{2} \right\rangle = \frac{1}{\sqrt{3}}$$

- $\Omega^- \to (\Xi^{*-} \to) \Xi^0 \pi^-$:

$$\left\langle I = \frac{1}{2}, I_3 = -\frac{1}{2} \,\middle|\, 1, -1; \frac{1}{2}, +\frac{1}{2} \right\rangle = -\sqrt{\frac{2}{3}}$$

$\hookrightarrow$ prediction

$$\frac{\Gamma(\Omega^- \to \Xi^0 \pi^-)}{\Gamma(\Omega^- \to \Xi^- \pi^0)} \approx 2 \neq \frac{24.}{9.}$$

Some details about main decays of Ω^-

- $\mathrm{Br}(\Omega^- \to \Lambda K^-) = (67.8 \pm 0.7)\%$
- $\mathrm{Br}(\Omega^- \to \Xi^0 \pi^-) = (23.6 \pm 0.7)\%$
- $\mathrm{Br}(\Omega^- \to \Xi^- \pi^0) = (8.6 \pm 0.4)\%$
- measured at SPS ($\approx$ 1984), recently confirmed by BES III

  Phys. Rev. D 108 (2023) 9, L091101

- if $\mathrm{Br}(\Omega^- \to \Lambda K^-) \approx 68\%$ and if $\Delta I = 1/2$ rule holds:

$$\mathrm{Br}(\Omega^- \to \Xi^0 \pi^-) \approx 21.3\% \,, \qquad \mathrm{Br}(\Omega^- \to \Xi^- \pi^0) \approx 10.7\%$$

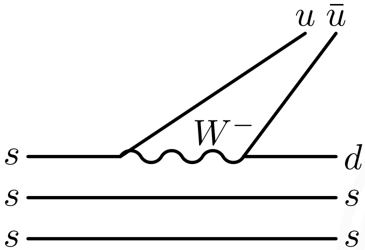
$\hookrightarrow$ deviation by about 3σ and 5σ, respectively

$\hookrightarrow$ worth to check $\Delta I = 1/2$ rule, e.g. in $\Omega \to \Xi\pi\pi$

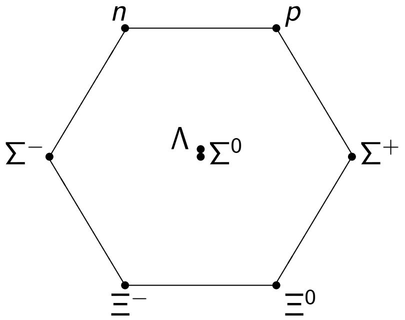
C.J.G. Mommers, SL, Phys. Rev. D 106 (2022) 9, 093001

Not everything is a penguin

example for a non-penguin diagram (contribution to $\Omega^- \to \Xi^- \pi^0$)

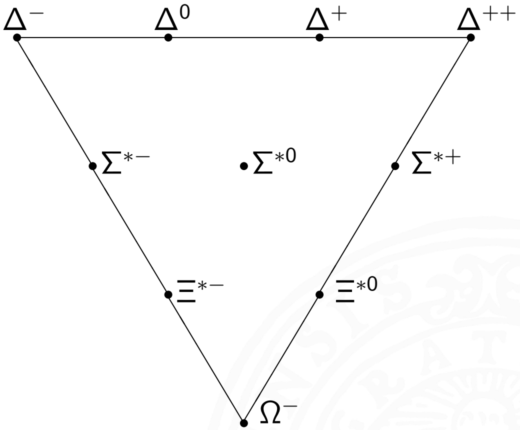


Reminder about multiplets



baryon octet

- antisym. in color
- antisym. in flavor, spin



baryon decuplet

- antisym. in color
- sym. in flavor, spin

Large-N_c QCD

for a large number of colors, $N_c \to \infty$,

Large-N_c QCD

for a large number of colors, $N_c \to \infty$,

- baryons consist of N_c quarks

Large-N_c QCD

for a large number of colors, $N_c \to \infty$,

- baryons consist of N_c quarks
- states with $J = I = \frac{1}{2}, \frac{3}{2}, \ldots, \frac{N_c}{2}$ are mass degenerate

Large-N_c QCD

for a large number of colors, $N_c \to \infty$,

- baryons consist of N_c quarks
- states with $J = I = \frac{1}{2}, \frac{3}{2}, \ldots, \frac{N_c}{2}$ are mass degenerate
- couplings to pions are related to each other (Dashen, Jenkins, Manohar)

Large-N_c QCD

for a large number of colors, $N_c \to \infty$,

- baryons consist of N_c quarks
- states with $J = I = \frac{1}{2}, \frac{3}{2}, \ldots, \frac{N_c}{2}$ are mass degenerate
- couplings to pions are related to each other (Dashen, Jenkins, Manohar)

$\hookrightarrow$ nucleon ($I = J = \frac{1}{2}$) and Δ ($I = J = \frac{3}{2}$) are (approximately) equal in mass

Large-N_c QCD

for a large number of colors, $N_c \to \infty$,

- baryons consist of N_c quarks
- states with $J = I = \frac{1}{2}, \frac{3}{2}, \ldots, \frac{N_c}{2}$ are mass degenerate
- couplings to pions are related to each other (Dashen, Jenkins, Manohar)

$\hookrightarrow$ nucleon ($I = J = \frac{1}{2}$) and Δ ($I = J = \frac{3}{2}$) are (approximately) equal in mass

$\hookrightarrow$ can be useful to treat them on equal footing for low-energy QCD

Large-N_c QCD

for a large number of colors, $N_c \to \infty$,

- baryons consist of N_c quarks
- states with $J = I = \frac{1}{2}, \frac{3}{2}, \ldots, \frac{N_c}{2}$ are mass degenerate
- couplings to pions are related to each other (Dashen, Jenkins, Manohar)

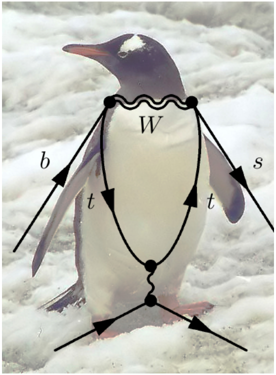
$\hookrightarrow$ nucleon ($I = J = \frac{1}{2}$) and Δ ($I = J = \frac{3}{2}$) are (approximately) equal in mass

$\hookrightarrow$ can be useful to treat them on equal footing for low-energy QCD

$\hookrightarrow$ extension to three flavors (baryon octet and decuplet) also useful

(expansion in $1/N_c$ could be as meaningful/meaningless as expansion in electric charge $e \approx 0.3$)

A penguin and its diagram



by Quilbert - own work derived from a LaTeX source code given in

http://cnlart.web.cern.ch/cnlart/221/node63.html (archived) (slightly modified) and

Image:Pygoscelis papua.jpg by User:Stan Shebs, CC BY-SA 2.5,

https://commons.wikimedia.org/w/index.php?curid=2795824