Gravitational form factors of the pion: lattice QCD meets meson dominance

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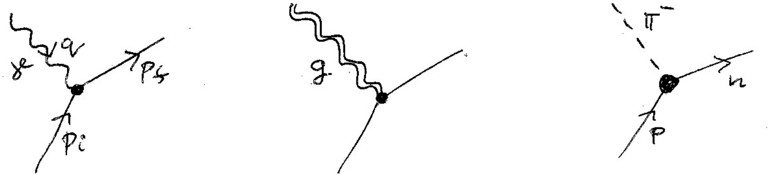
Outline

- Warm-up: EM ff of the pion
- Gravitational ff of the pion
- New: explanation of recent lattice QCD data with meson dominance
- σ meson in the scalar channel!

Different probes of the structure

electromagnetic . . . mass (gravitational) . . . composite operators . . . hadronic



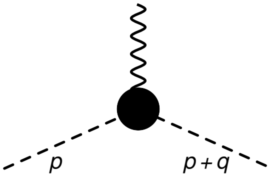
scattering amplitude = $\sum$ tensorial structure × form factor (scalar function)
Extracted from scattering data and lattice QCD

"Gravitational": no need for gravitons, e.g., $\gamma\gamma^* \to \pi^0\pi^0$ or lattice, probe with $T_{\mu\nu} = -\frac{2}{\sqrt{-g}}\frac{\delta(S_{\rm matter})}{\delta g^{\mu\nu}}$

Recent activity: gff of the electron [Berends, Gasmans, 1976, Freese, Metz, Pasquini, Rodini, 2022] ...
deuteron [He, Zahed, 2024] ... light nuclei [García Martín-Caro, Huidobro, Hatta, 2023]
... charmonium [Xu et al., 2024] ... (pion, nucleon - lots)

Field theoretic definition

On-shell matrix element of an operator at position x



Example: electromagnetic form factor of a scalar particle (like charged pion)

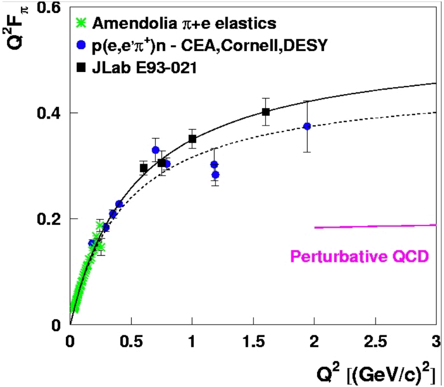
$$\langle h(p+q)|J^\mu(x)|h(p)\rangle = (2p^\mu + q^\mu)F(q^2)e^{iq\cdot x}$$

conserved, $\partial_\mu J^\mu = 0$

EM ff of the pion

$Q^2 = q^2 = -t$

- low Q^2 - χPT
- high Q^2 - pQCD: $F_\pi(Q^2)Q^2 \to 16\pi\alpha(Q^2)f_\pi^2\left[1 + 6.58\alpha(Q^2)/\pi + \ldots\right]$ (far away!)
- intermediate Q^2 - meson dominance (Sakurai ...): $F(Q^2) = \frac{1}{1+Q^2/m_\rho^2}$ (or sum over resonances)



Gravitational form factors of hadrons

... from $\langle h(p')|T^{\mu\nu}(0)|h(p)\rangle$. The stress-energy-momentum tensor is conserved, $\partial_\mu T^{\mu\nu}(x) = 0$

trace anomaly $T^\mu_\mu(x) = \frac{\beta}{2g}F^2 + \sum_f (1+\gamma_m) m_f \bar{\psi}_f \psi_f$

For the pion (spin-0) two tensor structures allowed by Lorentz covariance and conservation:

$$\langle \pi^a(p')|T^{\mu\nu}(0)|\pi^b(p)\rangle = \delta_{ab}\left[2P^\mu P^\nu A(t) + \frac{1}{2}\left(q^\mu q^\nu - g^{\mu\nu}q^2\right) D(t) + 2m_\pi^2 \left(\sum_p \bar{c}_p(t) g^{\mu\nu} = 0\right)\right]$$

a, b - isospin, $P = \frac{1}{2}(p' + p)$, $q = p' + p$, $t = q^2 = -Q^2$

Long history: Pagels 1965, K. Raman 1971, Łopuszański 1974, ..., χPT: Donoghue, Leytwyler 1991

Properties:

$$A(0) = 1, \quad D(0) = -1 + \mathcal{O}(m_\pi^2)$$

Relation to GPD [see talk by K. Cichy]: $\int_{-1}^{1} dx\, x H_\pi^{I=0}(x,\xi,t)) = A(t) + \xi^2 D(t)$, $\quad \xi$ - skewness

Mechanistic interpretation

... physical meaning, "force distribution"
[M. Polyakov 2003, Polyakov, Schweitzer 2018, Ji 2021, Lorcé, Metz, Pasquini, Rodini 2021, ...]

$$\frac{\langle \pi_{\rm rest}| \int d^3r\, T^{\mu\nu}(\vec{r})|\pi_{\rm rest}\rangle}{\langle \pi_{\rm rest}|\pi_{\rm rest}\rangle} = {\rm diag}(m_\pi, 0, 0, 0)$$

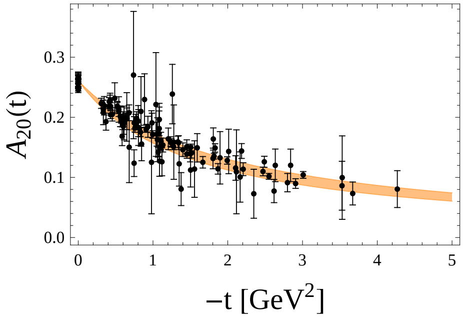
Balance of pressure, $\int d^3r\, p(r) = 0$ ($p(r)$ must change sign), $T^{00} \to$ distribution of mass

Also, $m_\pi \int d^3r\, r^2 p(r) = D(t=0)$ and for the shear forces $-\frac{4}{15} m_\pi \int d^3r\, r^2 s(r) = D(0)$

D - Druck term
Since $D(0) < 0$, $p(r)$ must change from $+$ at low r to $-$ and high r

Early estimates

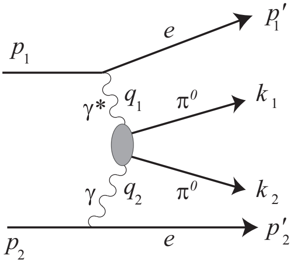
Chiral quark models: $\langle r_2 \rangle_A = \frac{1}{2} \langle r_2 \rangle_{EM}$ - mass distribution more compact than charge [WB, ERA, 2008]



Lattice [Brommel 2007] vs meson dominance [Masjuan, ERA, WB, Phys.Rev.D 87 (2013) 1, 014005]
($A_{20}(t) \equiv \frac{1}{2} A_q(t)$ - quark part)

At that time $D_q(t)$ very noisy, no gluons

Determination from the Belle data



[Belle, 2015]

[Kumano, Song, Teryaev, 2015] (GDAs, quark parts only) $\rightarrow$

$$\langle r^2 \rangle_A = (0.32 - 0.39 \text{ fm})^2$$
$$\langle r^2 \rangle_D = (0.82 - 0.88 \text{ fm})^2$$

recall $\langle r^2 \rangle_{EM} = (0.656 \pm 0.005 \text{ fm})^2$ (PDG 2021)

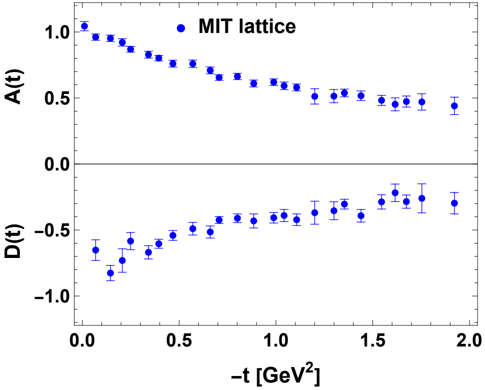
(case of A in line with our earlier quark model estimate)

MIT data

[Phys.Rev.D 108 (2023) 11, 114504 & D. Pefkou, PhD Thesis]

Unprecedented accuracy, both quarks and gluons, $m_\pi = 170$ MeV

(below the total $q+g$ used, as it corresponds to the conserved current $\to$ renorm invariant)



Accuracy allows for more stringent tests and general understanding

Spin decomposition

Recall $\langle\pi(p')|T^{\mu\nu}(0)|\pi(p)\rangle = 2P^\mu P^\nu A(t) + \frac{1}{2}\left(q^\mu q^\nu - g^{\mu\nu}q^2\right) D(t)$

contains spin-0 (traceful) and spin-2 (traceless) parts, which must be properly identified

Spin-0: $\langle\pi(p')|T^\mu_\mu(0)|\pi(p)\rangle \equiv \Theta(t) = 2\left(m_\pi^2 - \frac{t}{4}\right)A(t) - \frac{3t}{2}D(t)$
Spin-2: $A(t)$

from where

$$D(t) = \frac{(4m_\pi^2 - t)A(t) - 2\Theta(t)}{3t} \quad \text{(mixes quantum numbers!)}$$

The low-energy constraints $A(0) = 1$, $D(0) = -1 + \mathcal{O}(m_\pi^2) \rightarrow$

$$\Theta(0) = 2m_\pi^2, \quad d\Theta(t)/dt|_{t=0} = 1 + \mathcal{O}(m_\pi^2)$$

Dispersion relations

gffs satisfy dispersion relations. Once-subtracted form:

$$A(s) = 1 + \frac{1}{\pi}\int_{4m_\pi^2}^{\infty} ds' \frac{s}{s'}\frac{\mathrm{Im}A(s')}{s'-s-i\epsilon} \qquad D(s) = D(0) + \frac{1}{\pi}\int_{4m_\pi^2}^{\infty} ds' \frac{s}{s'}\frac{\mathrm{Im}D(s')}{s'-s-i\epsilon}$$

pQCD: $\lim_{Q^2\to\infty} A(-Q^2) = \lim_{Q^2\to\infty} D(-Q^2) = 0$ (vanish as $1/Q^2$ mod logs) $\rightarrow$

$$0 = 1 - \frac{1}{\pi}\int_{4m_\pi^2}^{\infty} ds' \frac{\mathrm{Im}A(s')}{s'} \qquad 0 = D(0) - \frac{1}{\pi}\int_{4m_\pi^2}^{\infty} ds' \frac{\mathrm{Im}D(s')}{s'}$$

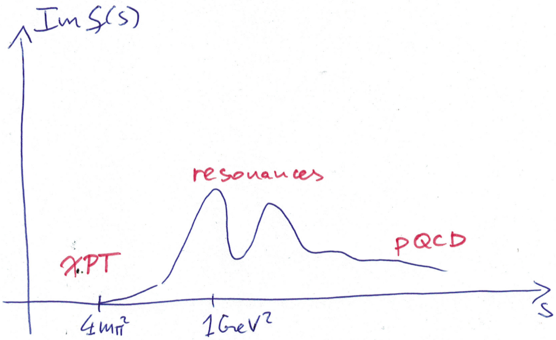
Correspondingly,

$$\lim_{Q^2\to\infty} \Theta(-Q^2) = 2m_\pi^2 - \frac{1}{\pi}\int_{4m_\pi^2}^{\infty} ds' \frac{\mathrm{Im}\Theta(s')}{s'}$$

Asymptotically, it goes to a constant (!) mod $\log(Q^2)$ [X.-B. Tong, J.-P. Ma, F. Yuan 2021, 2022]
(but sign change in $D(-Q^2)$)

Absorptive parts

Genericallly



Question of modeling/using the spectral density

Large N_c and meson dominance

t Hooft, Witten: At large-N_c, amplitudes are saturated by tree-level diagrams with towers of mesons in intermediate states $\rightarrow$

$$\mathrm{Im}A(s) = \sum_T c_T M_T^2 \pi \delta(M_T^2 - s), \quad \mathrm{Im}\Theta(s) = \sum_S c_S M_S^4 \pi \delta(M_S^2 - s)$$

$+$ dispersion relations $\rightarrow$

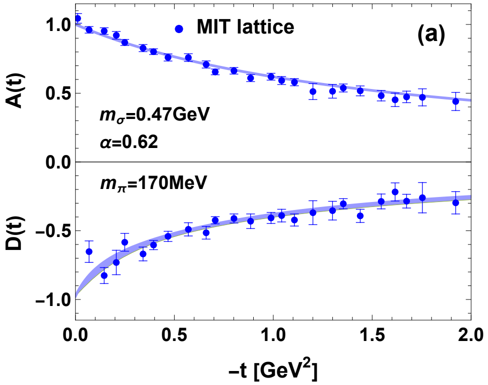
$$A(-Q^2) = 1 - \sum_T \frac{c_T Q^2}{M_T^2 + Q^2}, \quad \Theta(-Q^2) = 2m_\pi^2 - \sum_S \frac{c_S Q^2 M_S^2}{M_S^2 + Q^2}$$

$\sum_T c_T = 1$ (since $A(-\infty) = 0$ and (in the chiral limit) $\sum_S c_S = 1$ ($\Theta'(0) = 1$)
We take one tensor meson, $f_2(1275)$, and two scalar mesons, $f_0(975)$ and σ [see talk by R. Kamiński]

$$A(-Q^2) = \frac{m_{f_2}}{m_{f_2}^2 + Q^2}, \quad \Theta(-Q^2) = 2m_\pi^2 - \alpha \frac{Q^2 m_\sigma^2}{m_\sigma^2 + Q^2} - (1-\alpha) \frac{Q^2 m_{f_0}^2}{m_{f_0}^2 + Q^2}$$

Formula for $D(-Q^2)$ follows. Only m_σ and α are fitted.

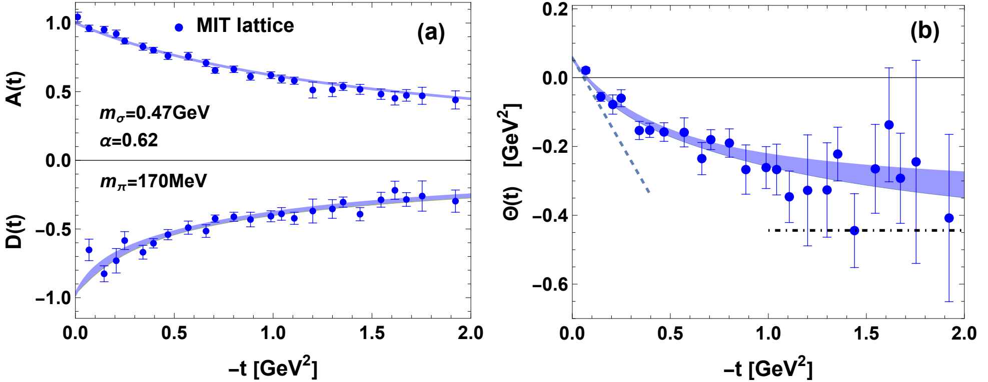
Result of the fit



band width in D - 68% CL

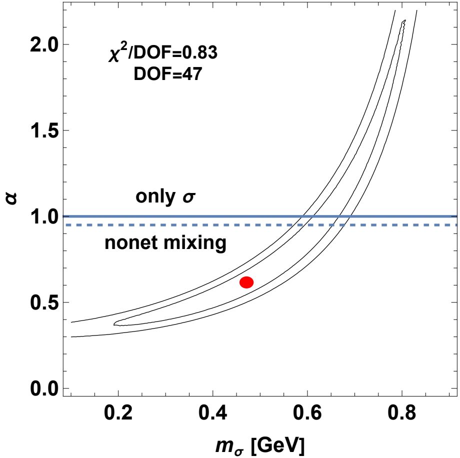
Result of the fit

(our way of taking advantage of the MIT data)



for Θ, errors added in quadrature, lines: $2m_\pi^2 + t$ and asymptotics (constant!)

Fit parameters



Strong correlation between m_σ and $\alpha \to$ ambiguity, more information needed

Properties of the model

$$D(0) = -1 + \frac{4m_\pi^2}{2m_{f_2}^2} + \mathcal{O}(m_\pi^2), \quad \langle r^2 \rangle_A = \frac{6}{m_{f_2}^2}, \quad \langle r^2 \rangle_D = \frac{4}{m_{f_2}^2} + \frac{2\alpha}{m_\sigma^2} + \frac{2(1-\alpha)}{m_{f_0}^2} + \mathcal{O}(m_\pi^2)$$

Numerically (at the lattice value $m_\pi = 170$ MeV)

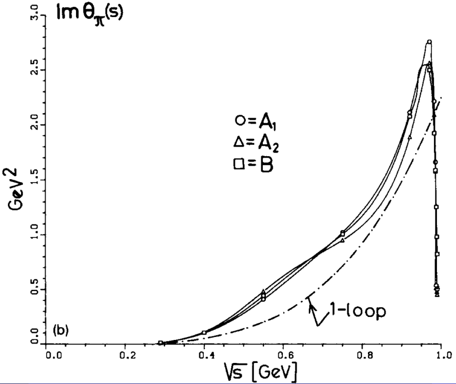
$$D(0) = -0.976, \quad \langle r^2 \rangle_A = (0.38 \text{ fm})^2, \quad \langle r^2 \rangle_D = (0.74 \text{ fm})^2$$

(radii in line with Kumano et al.)

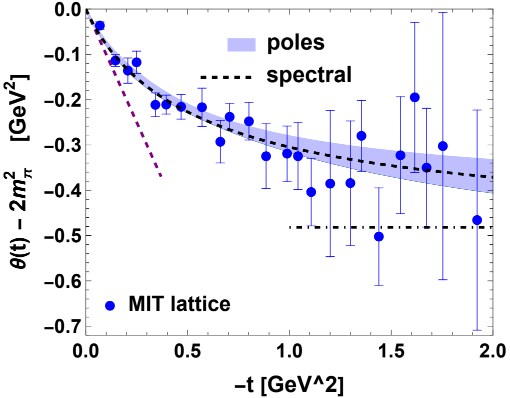
Scalar spectral function

$$\Theta(-Q^2) = 2m_\pi^2 - \frac{Q^2}{\pi}\int_{4m_\pi^2}^{\infty} ds \frac{\mathrm{Im}\Theta(s)}{s(s+Q^2)}$$

Use, e.g., the spectral density from physical (CERN-Munich) phase shifts [Donoghue, Gasser, Leutwyler, Nucl.Phys.B 343(1990)341, Fig. 3] (Watson's theorem, Omnès-Muskhelishvili coupled equations)

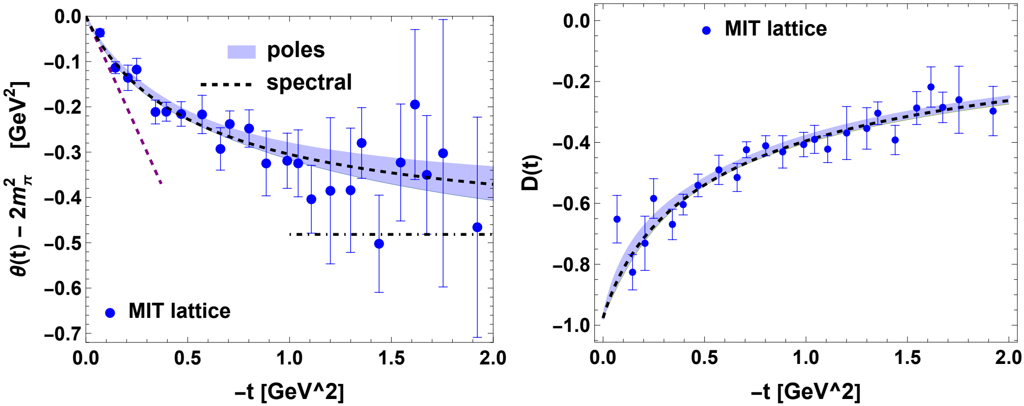


Spectral modeling vs lattice



(here we subtract $2m_\pi^2$, as $m_\pi = 170$ MeV for the data and 140 MeV for the spectral modeling)

Spectral modeling vs lattice



Lattice consistent with "meson physics", also and in particular in the scalar channel

Conclusions

1. Lattice results for gravitational ff of the pion fully compatible with meson dominance at "intermediate" values of Q^2
2. Important to look at the data in good spin channels - all expected features satisfied: $\Theta(0) = 2m_\pi^2$, $\Theta'(0) = 1 + \mathcal{O}(m_\pi^2)$, or $D(0) = -1 + \mathcal{O}(m_\pi^2)$
3. $D(t)$ (the Druck term) is a combination of good spin form factors
4. Tensor channel: just $f_2(1275)$
5. Scalar channel: $\Theta(-Q^2)$ goes to a negative constant at accessible values of Q^2. Higher Q^2 desired approach pQCD ... Modeling involves the broad σ meson!
6. For the nucleon similar story expected, cf. [Masjuan, ERA, WB, 2013]

One sees mesons all over the lattice!

Happy Birthday, Ludwik !!!