

Observation of nonequilibrium effects in the collective flow

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Hydrodynamic regime

- ▶ energy-momentum tensor

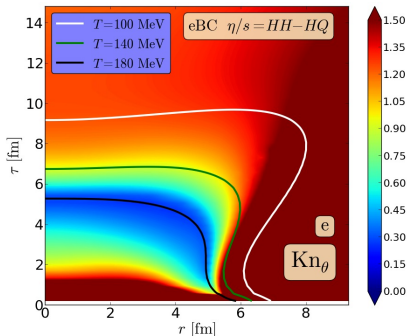
$$T_{id}^{\mu\nu} = (\epsilon + p)g^{\mu\nu} - Pu^\mu u^\nu + \pi^{\mu\nu}$$

- ▶ close to local equilibrium
- ▶ local energy density ϵ , pressure p , flow u^μ , stress (viscous) corrections $\pi^{\mu\nu}$
- ▶ local momentum distribution close to equilibrium

$$f(p) = f_{eq}(p) + \delta f(p)$$

Early times

Knudsen number $K = l_{micro}/L_{macro}$



H. Niemi, G. Denicol, , arXiv: 1404.7327

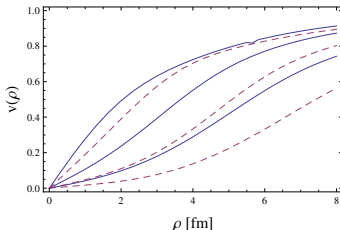
- large gradients
- viscous corrections dominant
- hydrodynamics \longrightarrow kinetic evolution

free-streaming + hydrodynamics

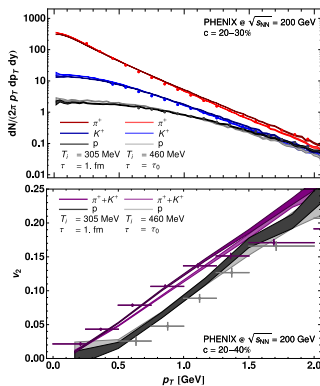
- free-streaming until T_{hydro}
- matching to fluid $T^{\mu\nu}$ at T_{hydro}
- pre-equilibrium flow

$$v_T \simeq -\frac{T_{hydro} - \tau_0}{3} \frac{\nabla_T n(x, y)}{n(x, y)}$$

universal flow J. Vredevoegt, S. Pratt, arXiv: 0810.4325



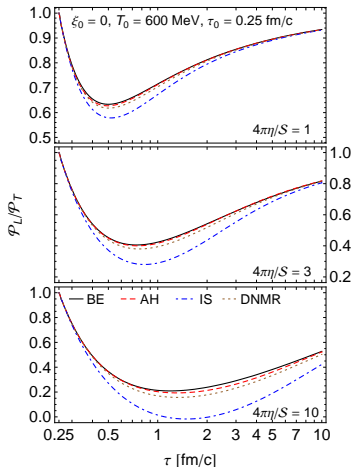
W. Broniowski, W. Florkowski, M. Chojnacki, A. Kisiel, arXiv: 0812.3393



testing hydrodynamics

- boost invariant solution of kinetic eq. in relaxation time app.

$$f(x, p) = f(t, p_{\perp}, x_{\perp}, w)$$



Nonequilibrium

$$P_L \neq P_T$$

larger viscosity η



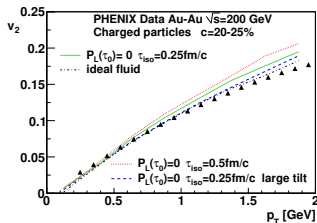
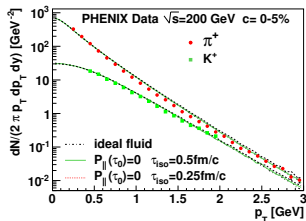
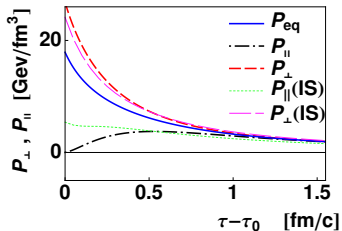
larger relaxation time τ_{relax}
(less collisions)

W. Florkowski, R. Ryblewski, M. Strickland,

arXiv 1304.0665

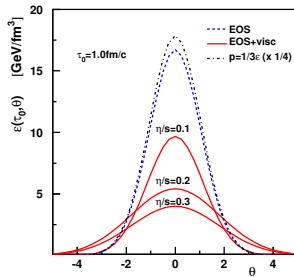
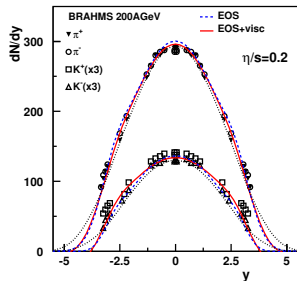
Effects of strong pressure asymmetry?

pressure asymmetry



Very small effect on transverse expansion
Early nonequilibrium cannot be observed?

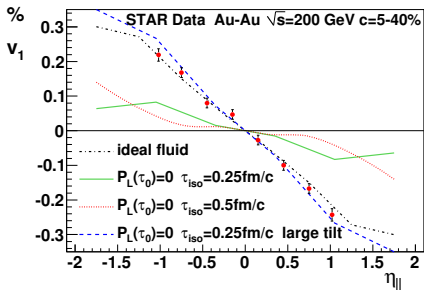
Longitudinal expansion



**Strong differences in longitudinal colling and expansion
BUT: What is the initial longitudinal profile**

Longitudinal and transverse expansion combined

Directed flow



rapidity odd directed flow is sensitive to pressure asymmetry!

Kinetic equations in RTA

- boost-invariant
- relaxation time approximation
- massless particles \rightarrow evolution of an integral (simplification)

$$f(\tau, \vec{x}_T, \vec{p}_T, p_{\parallel}, w) \rightarrow F(\tau, \vec{x}_T, \phi, v_z) = \int p^3 dp f(\tau, x_T, p_T, p_{\parallel}, w)$$

at $z = 0$

$$v = (1, \vec{v}_T, v_z) = (1, \sqrt{1 - v_z^2} \cos \phi, \sqrt{1 - v_z^2} \sin \phi, v_z)$$

$$\partial_{\tau} F + \vec{v}_T \partial_{\vec{x}_T} - \frac{v_z}{\tau} (1 - v_z^2) \partial_{v_z} F + \frac{4v_z^2}{\tau} = \frac{1}{\tau_{relax}} (F - F_{iso})$$

Energy-momentum tensor can be reconstructed from F

$$T^{\mu\nu}(\tau, \vec{x}_T) = \int_0^{2\pi} \frac{d\phi}{2\pi} \int_{-1}^1 \frac{dv_z}{2} F(\tau, \vec{x}_T, \phi, v_z)$$

A. Kurkela, U. Wiedemann, B. Wu : arXiv: 1803.02072

boost-invariant, relaxation-time approximation

- relaxation time approximation

$$p^\mu \partial_\mu f(x, p) = \frac{1}{\tau_{relax}} (f(x, p) - f_{eq}(x, p))$$

$$w = tp_{\parallel} - zE_p$$

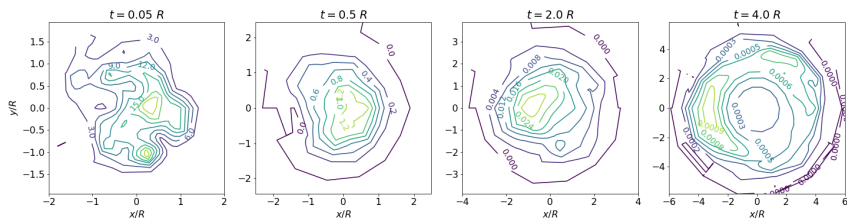
models preequilibrium transverse expansion only

Kinetic evolution in 2+1 dim

- boost-invariant
- relaxation time approximation
- massless particles \rightarrow evolution of an integral (simplification)

boost-invariant, small longitudinal pressure, $F = \delta(v_z)\tilde{F}$

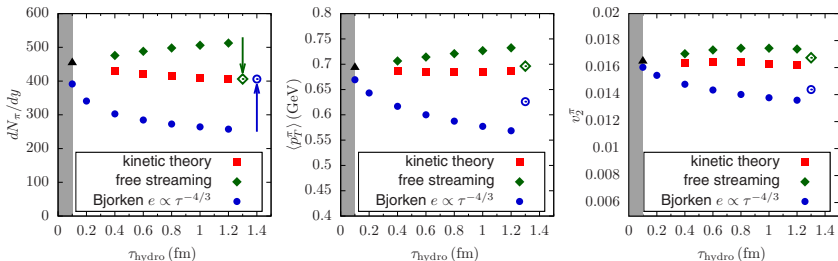
$$\partial_\tau \tilde{F} + v_T \partial_T \tilde{F} + \frac{1}{\tau} \tilde{F} = \frac{1}{\tau_{relax}} \left(\tilde{F} - \tilde{F}_{iso} \right)$$



A. Kurkela, S.F. Taghavi, U. Wiedemann, B. Wu, arXiv: 2007.06851

also: free streaming, KOMPOST

Early dynamics gives a transverse push



A. Kurkela, A. Mazeliauskas, J. F. Paquet, S. Schlichting, D. Teaney arXiv: 1805.00961

hydrodynamics - free streaming - kinetic equations : **SMALL** differences

kinetic equations in 1+1+1 dim

- ▶ nonboost invariant dynamics
- ▶ full treatment of longitudinal and transverse momenta
- ▶ at least one nontrivial transverse dimension

set up for directed flow studies

- evolution in time t , longitudinal z , and one transverse direction x

$$\partial_t F + v_x \partial_x F + v_z F = \frac{1}{\tau_{relax}} (F - F_{iso})$$

- variables $\tau, x, \eta, \phi, v_z : F(\tau, x, \eta, \phi, v_z)$

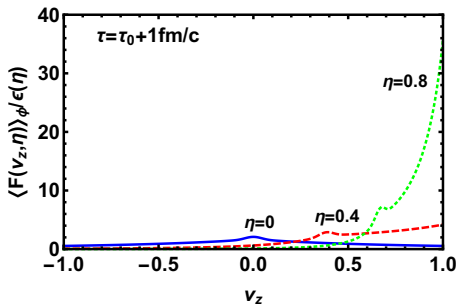
- solved in the global frame

F in boosted frame

Initial conditions $F = \epsilon(\tau_0, x, \eta) \mathbf{g}[v_z]$ (note $v_z = \cos(\theta)$)

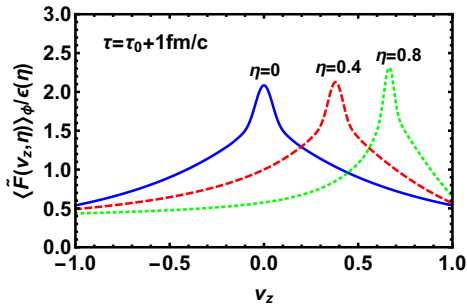
after boost by rapidity $y = \eta$ (Bjorken flow)

$$F = \epsilon(\tau_0, x, \eta) \frac{\mathbf{g}[(v_z \cosh(y) - \sinh(y)) / (\cosh(y) - v_z \sinh(y))]}{(\cosh(y) - v_z \sinh(y))^4}$$

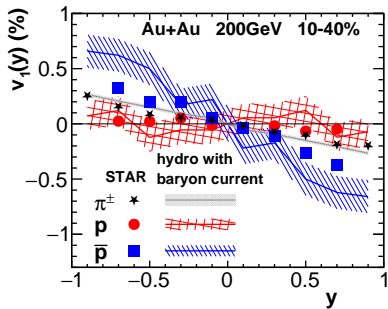
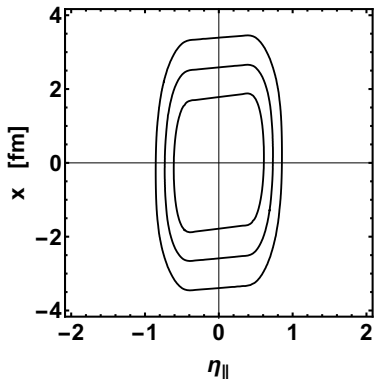


- difficult numerics at large rapidities

Eqs. solved for $\tilde{F} = F (\cosh(y) - v_z \sinh(y))^4$



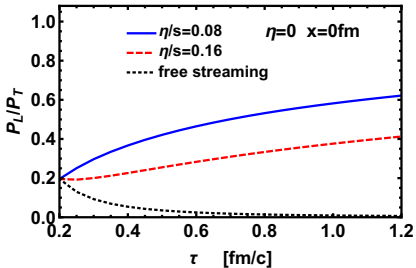
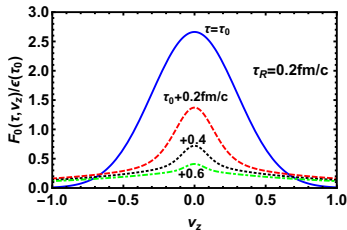
tilted fireball initial conditions



can describe the directed flow with early start of hydrodynamics

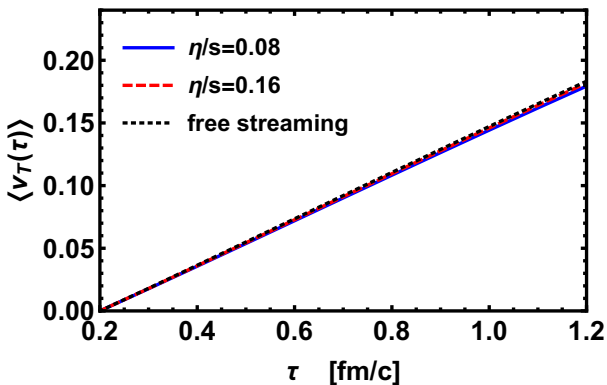
$$\tau_{hydro} = 0.2 \text{ fm}/c$$

evolution of momentum distribution



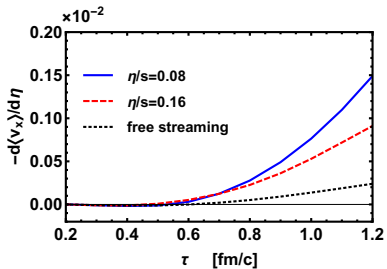
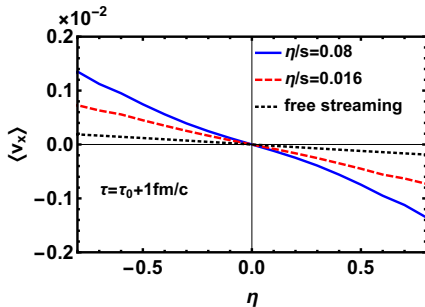
- redshift \rightarrow reduction of longitudinal pressure
- collisions \rightarrow isotropization

transverse flow



universal transverse flow J. Vredevoogt, S. Pratt, arXiv: 0810.4325

directed flow



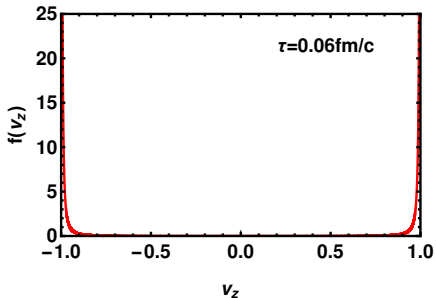
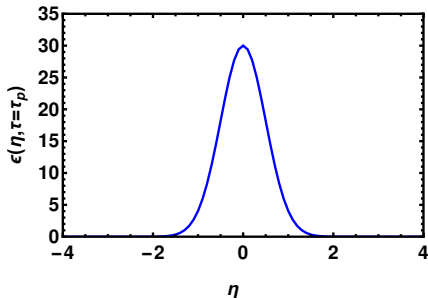
directed flow probes equilibration rate !

- ▶ Kinetic equation for early dynamics
- ▶ Non-boost invariant solutions
- ▶ Early formation of directed flow
- ▶ Directed flow **sensitive to pressure anisotropy**
- ▶ Preequilibrium dynamics in 3+1D ?

pre-Bjorken flow in 1+1 dim

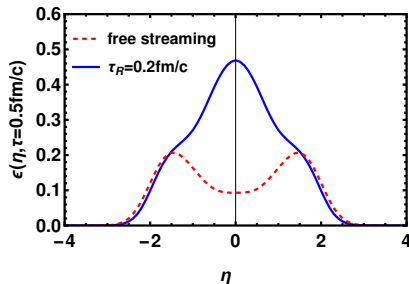
two streams of partons in the initial state

$\tau = 0.06\text{fm}/c$, passage time for the colliding nuclei

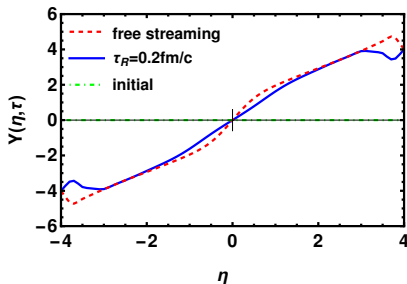


$\tau=0.5\text{fm}/c$, after kinetic evolution

stopping due to relaxation



close to Bjorken flow

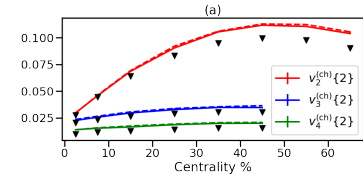


pre-equilibrium longitudinal flow

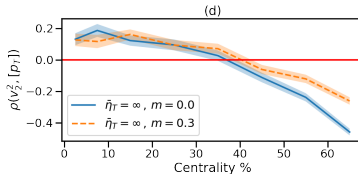
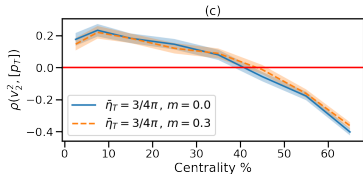
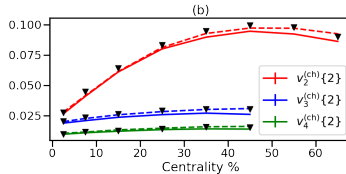
non-Bjorken initial flow for hydrodynamics

RTA pre-equilibrium + hydrodynamics

relaxation time isotropization

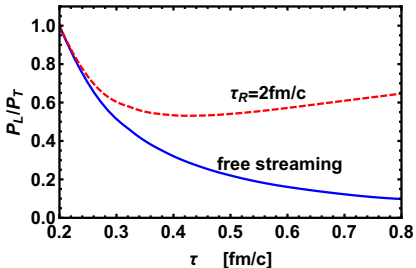
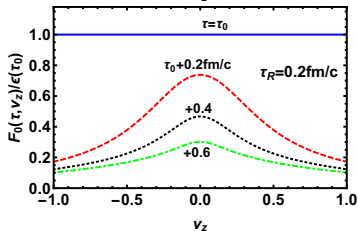
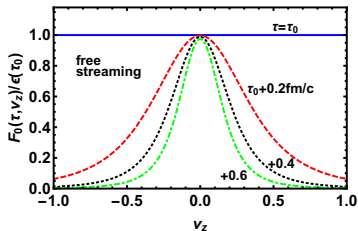


free-streaming



D. Liyanage, D. Everett, C. Chattopadhyay, U. Heinz arXiv: 2205.00964

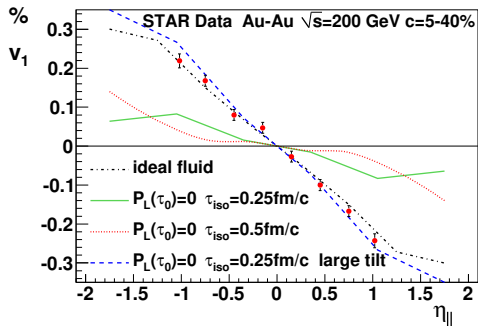
longitudinal evolution



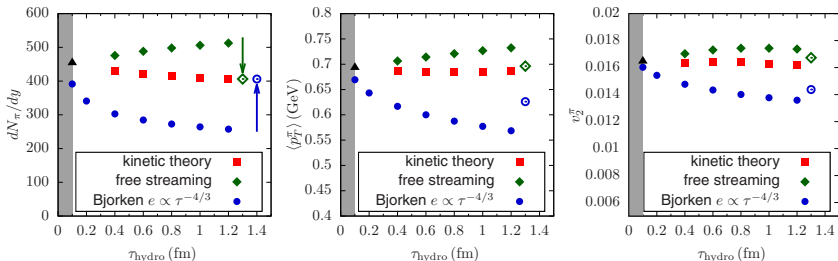
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Kinetic equation

equation for the phase-space distribution $f(x, p)$

$$p^\mu \partial_\mu f(x, p) = C[f(x, p)]$$

$C[f(x, p)]$ collisions integral ($2 \leftrightarrow 2$ and $1 \leftrightarrow 2$ processes)

- for on-shell particles $f(t, \mathbf{x}, \mathbf{p})$ 7-dimensional function
- more general than viscous hydrodynamics
- most solutions for boost-invariant geometry