Attractors in Quark-Gluon Plasma dynamics

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Various Faces of QCD
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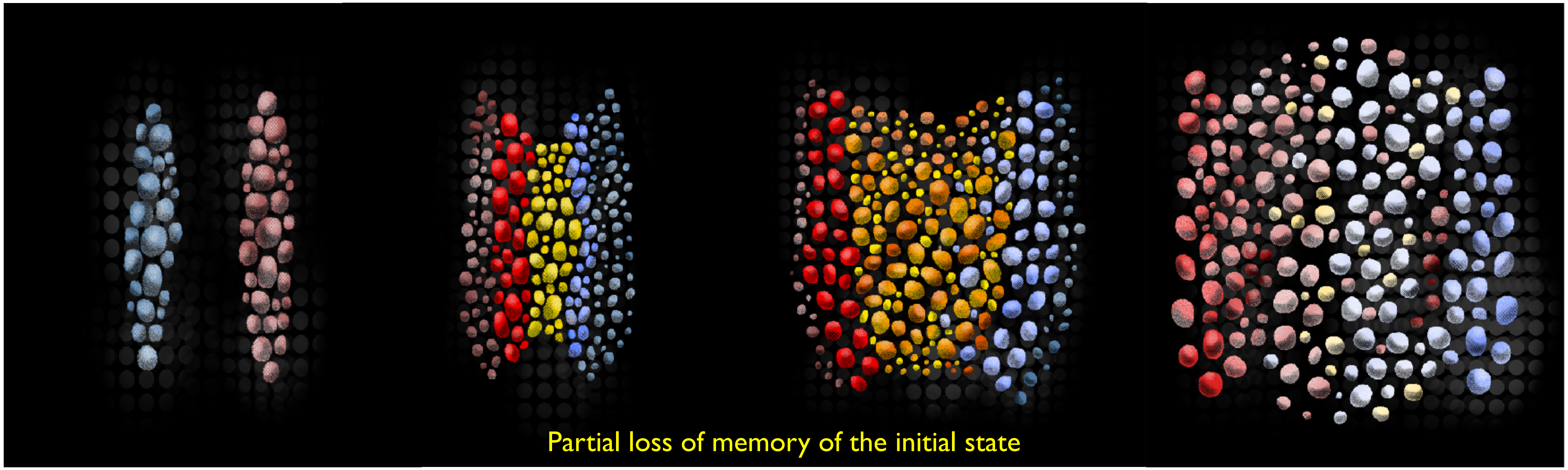
Attractors in QGP dynamics



Initial state | Hydrodynamic model | Freezeout

- Why do hydrodynamic models work far from equilibrium?
- Possible answer: pre-hydrodynamic attractors
- Attractors appear in diverse models – perhaps also in QCD
- Kinematic origin: approximate boost invariance
- Traces of the initial state survive until freeze out

This talk

- How information about the initial state survives
- A analytic approach to the attractor of RTA kinetic theory
- Extending the Bjorken model to account for transverse dynamics

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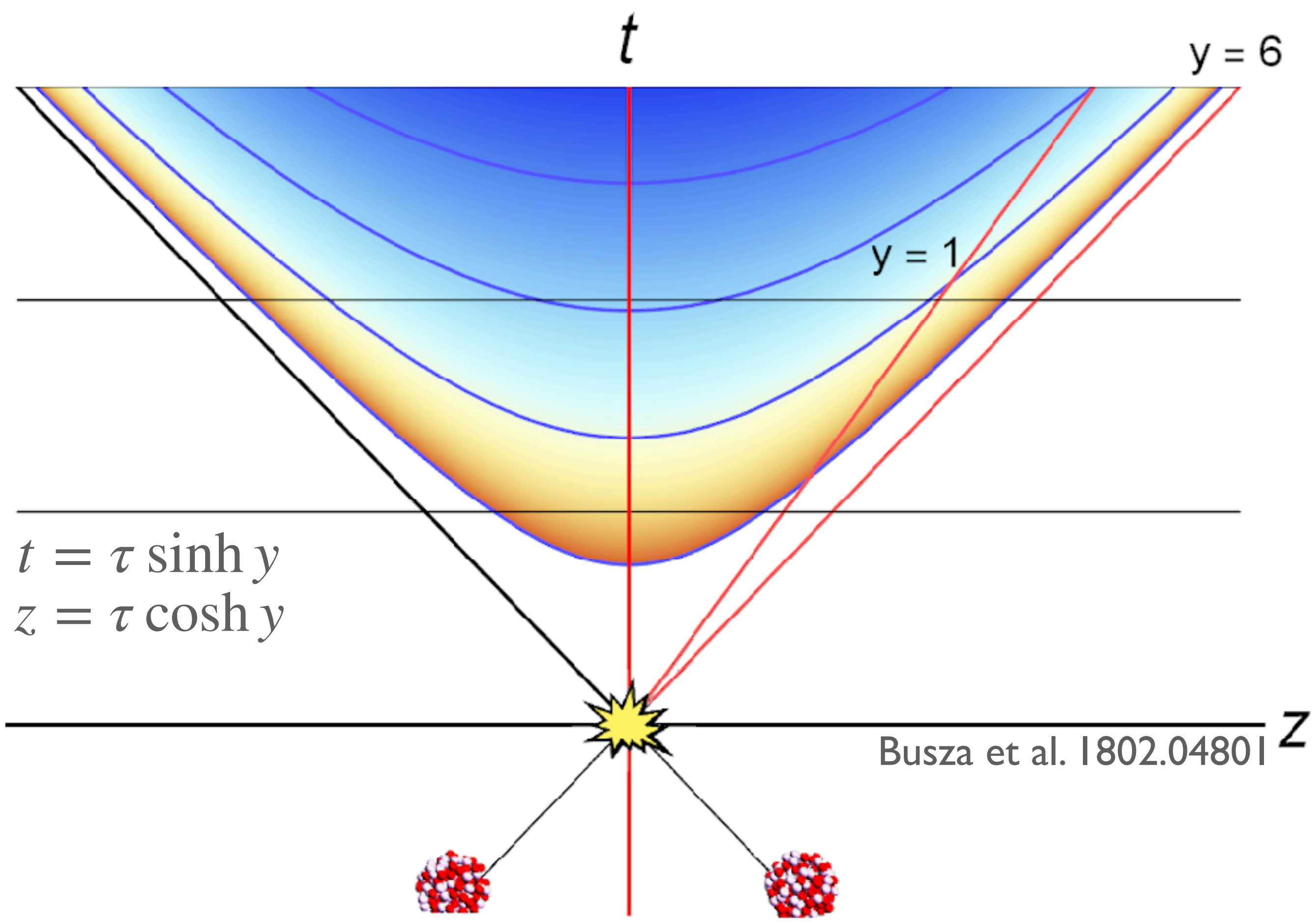
Attractors for Bjorken flow

in conformal models



$$(T^{\mu}_{\nu}) = \mathrm{diag}(-\mathcal{E}, \mathcal{P}_L, \mathcal{P}_T, \mathcal{P}_T)$$

Expressed in terms of 2 functions of proper time:

$$\mathcal{P}_L \equiv \frac{\mathcal{E}}{3}\left(1 - \frac{2}{3}\mathcal{A}\right), \quad \mathcal{P}_T \equiv \frac{\mathcal{E}}{3}\left(1 + \frac{1}{3}\mathcal{A}\right)$$

Conservation of the energy-momentum tensor:

$$\frac{d \log T}{d \log w} = \frac{\mathcal{A} - 6}{\mathcal{A} + 12}$$

$$\mathcal{E}(\tau) \sim T(\tau)^4, \quad w \equiv \tau T$$

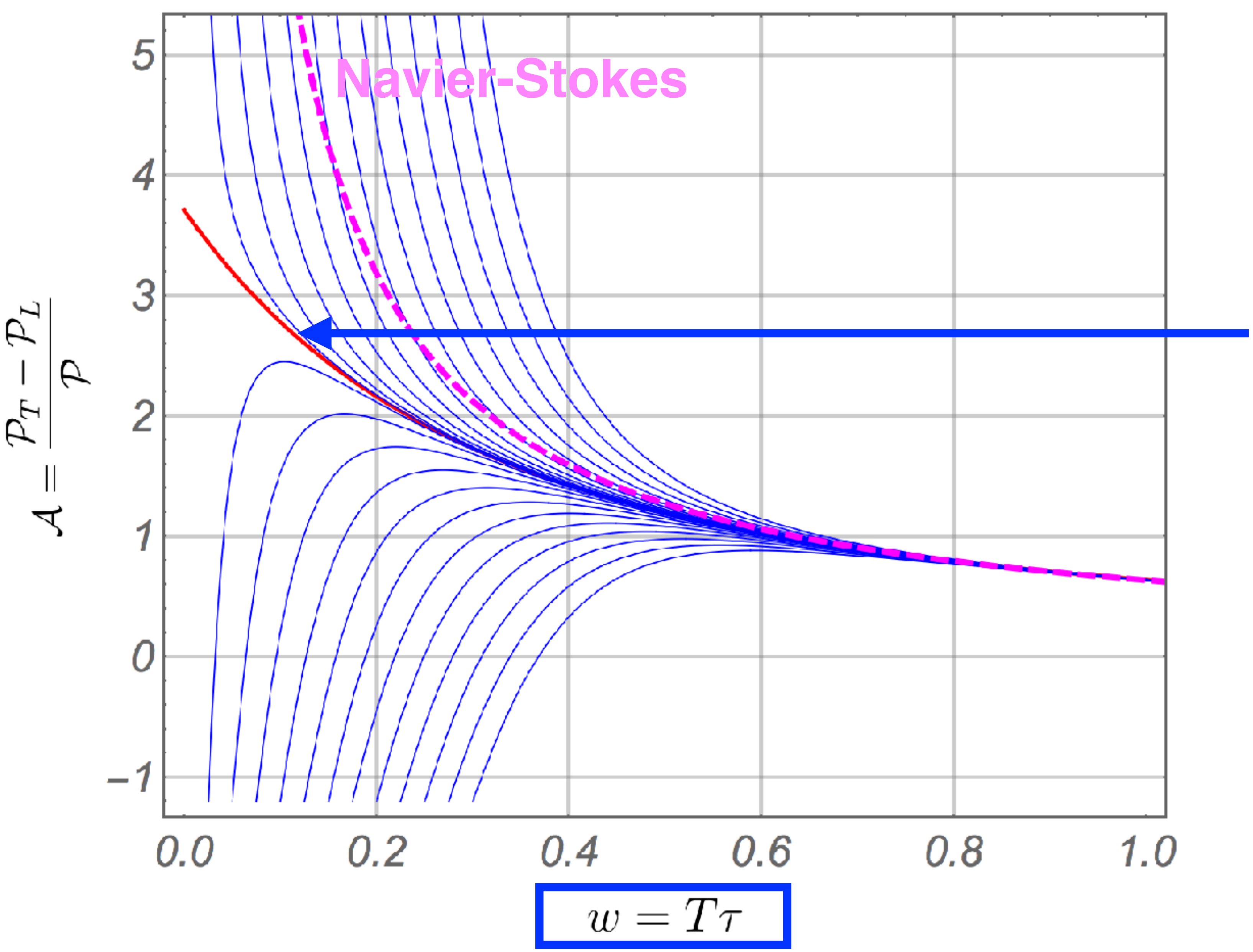
- Conservation equation introduces a single integration constant
- Remaining initial data is contained in $\mathcal{A}(w)$
- In many models $\mathcal{A}(w)$ contains an attractor

The attractor
in conformal Mueller-Israel-Stewart theory

$$C_\tau \left(1 + \frac{\mathscr{A}}{12}\right) \mathscr{A}' + \frac{C_\tau}{3w}\mathscr{A}^2 = \frac{3}{2}\left(\frac{8C_\eta}{w} - \mathscr{A}\right)$$

The pressure anisotropy satisfies a first order ODE

$$C_\eta \equiv \eta/s, \quad C_\tau \equiv \tau_R T$$



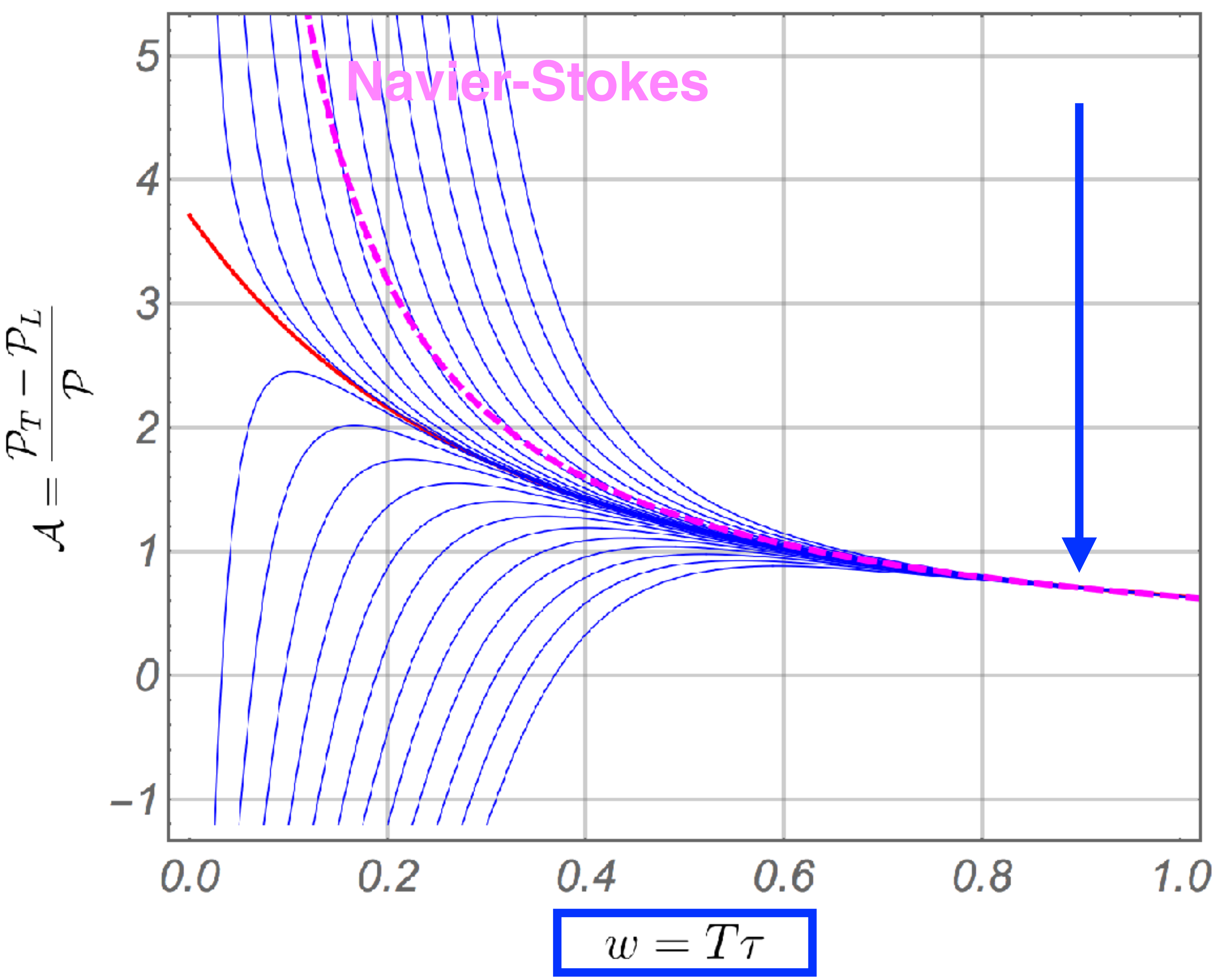
An attractor connects the early, far-from-equilibrium domain to the hydrodynamic region at late times

Solutions starting off the attractor reach its vicinity even if the pressure anisotropy is large

The attractor – the late time asymptotic view
in conformal MIS

$$\mathscr{A} = \underbrace{\frac{8C_\eta}{w}}_{\text{Navier–Stokes}} + \underbrace{\frac{16C_\eta C_\tau}{3w^2}}_{\text{2nd order}} + \ldots = \underbrace{\sum_{n>0} \frac{a_n^{(0)}}{w^n}}_{\text{gradient expansion}}$$

The expansion coefficients do not depend on initial conditions

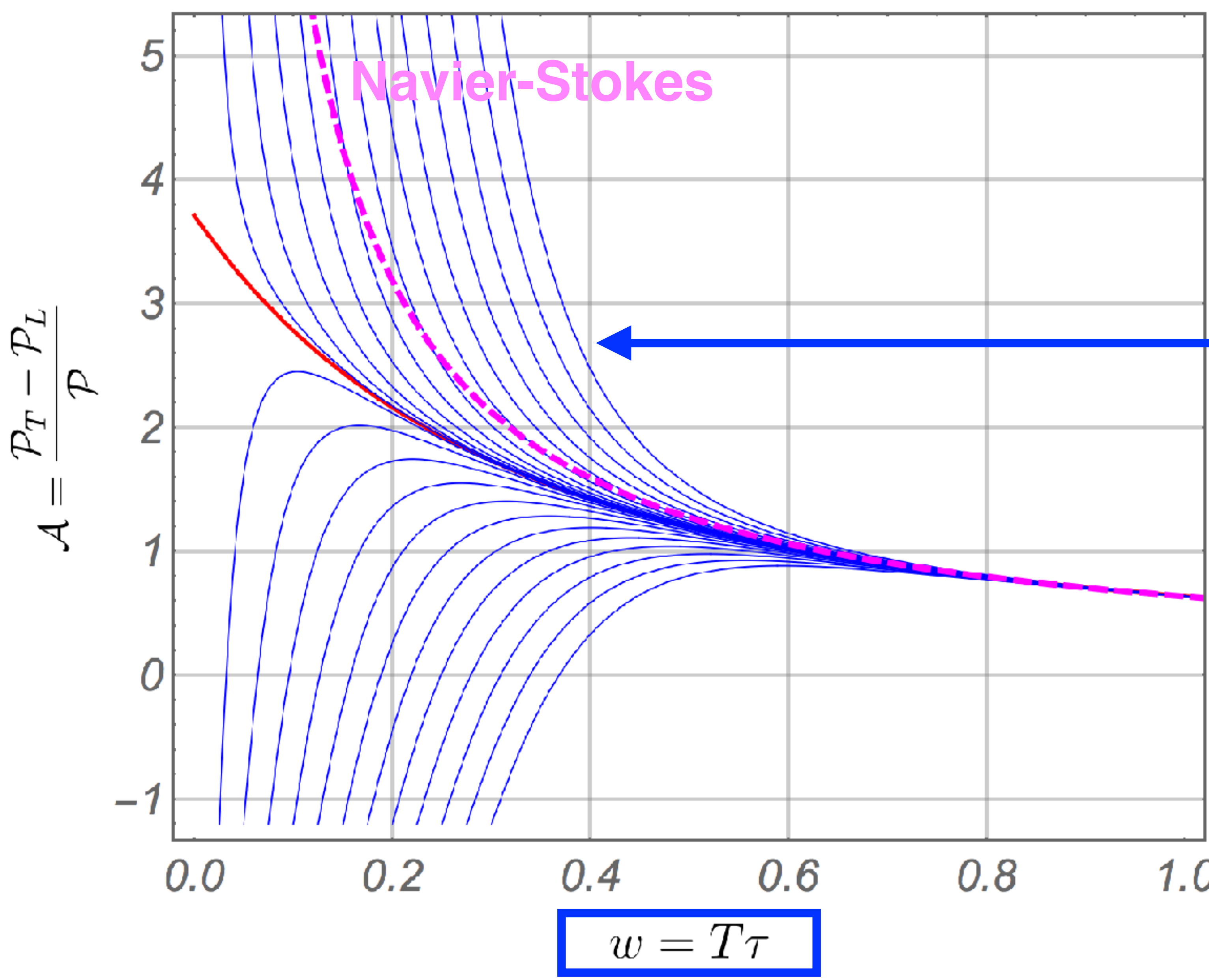


At asymptotically late times there is no memory of the initial conditions

The attractor – the transseries view

in conformal MIS

$$\mathscr{A} = \underbrace{\frac{8C_\eta}{w}}_{\text{Navier–Stokes}} + \underbrace{\frac{16C_\eta C_\tau}{3w^2}}_{\text{2nd order}} + \ldots = \underbrace{\sum_{n>0} \frac{a_n^{(0)}}{w^n}}_{\text{gradient expansion}} + \underbrace{\left(\sigma\, w^{\frac{C_\eta}{C_\tau}} e^{-\frac{3}{2C_\tau}w}\right) \sum_{n\geq 0} \frac{a_n^{(1)}}{w^n}}_{\text{1st transseries sector}} + \ldots$$



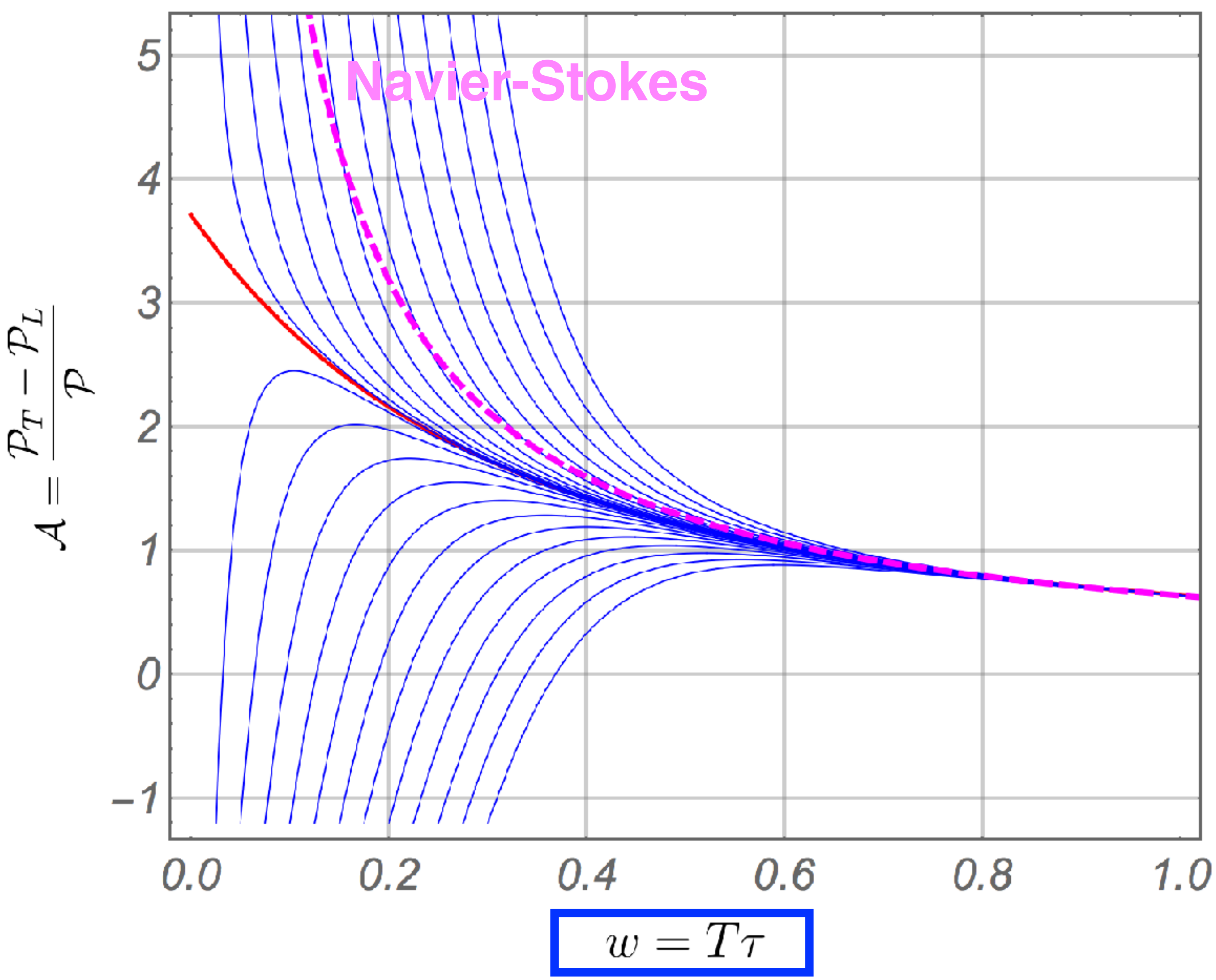
Different initial conditions are captured by exponentially-suppressed corrections to the asymptotic gradient series through the transseries parameter

The structure of the transseries reflects the large order behaviour of the gradient expansion

The attractor – three stages

in conformal MIS

- Expansion-dominated, pre-hydro, asymptotic
- Freeze-out takes place before the late-time asymptotic region



The expansion-dominated stage depends weakly on the parameters

The pre-hydro stage depends on both the parameters and the initial state

The asymptotic stage is independent of initial conditions

The attractor of RTA kinetic theory
for Bjorken flow

- Boltzmann Equation in the Relaxation Time Approximation

$$\left(\frac{\partial}{\partial\tau} - \frac{p_z}{\tau}\frac{\partial}{\partial p_z}\right) f(\tau,p) = \frac{f_{\rm eq}(\tau,p) - f(\tau,p)}{\tau_R}, \qquad \tau_R = \gamma T(\tau)^{-\Delta}$$

- Moments of the distribution function (as in Blaizot & Yan)

$$\mathscr{L}_n \equiv \int \frac{d^3p}{(2\pi)^3} p_0 P_{2n}(\cos\psi)\, f(\tau,p), \qquad \forall n \geq 0, \qquad \cos\psi = p_z/p_0$$

- Dimensionless moments

$$\mathscr{M}_n \equiv \frac{\mathscr{L}_n}{\mathscr{L}_0}, \qquad n \geq 0.$$

- The pressure anisotropy is $\mathscr{A} = -3\mathscr{M}_1$
- The moments satisfy an infinite set of coupled ODEs

The attractor of RTA kinetic theory

the generating function approach and series at early times

- We introduce a generating function for moments

$$G_{\mathscr{M}}(x,w) = \sum_{n=0}^{+\infty} x^n \mathscr{M}_n(w)$$

- The generating function satisfies a PDE which determines the moments as power series valid at early/late times

- The attractor is given by a convergent series valid for early times, which can be analytically continued to late times

$$\mathscr{A} = \sum_{n=0}^{\infty} c_n w^n \qquad \Longrightarrow \qquad \text{Pade}[\mathscr{A}]_{(70,71)} \sim \frac{1.60004}{w} + \frac{0.27348}{w^2} + \ldots$$

- The analytic continuation matches hydrodynamics very well

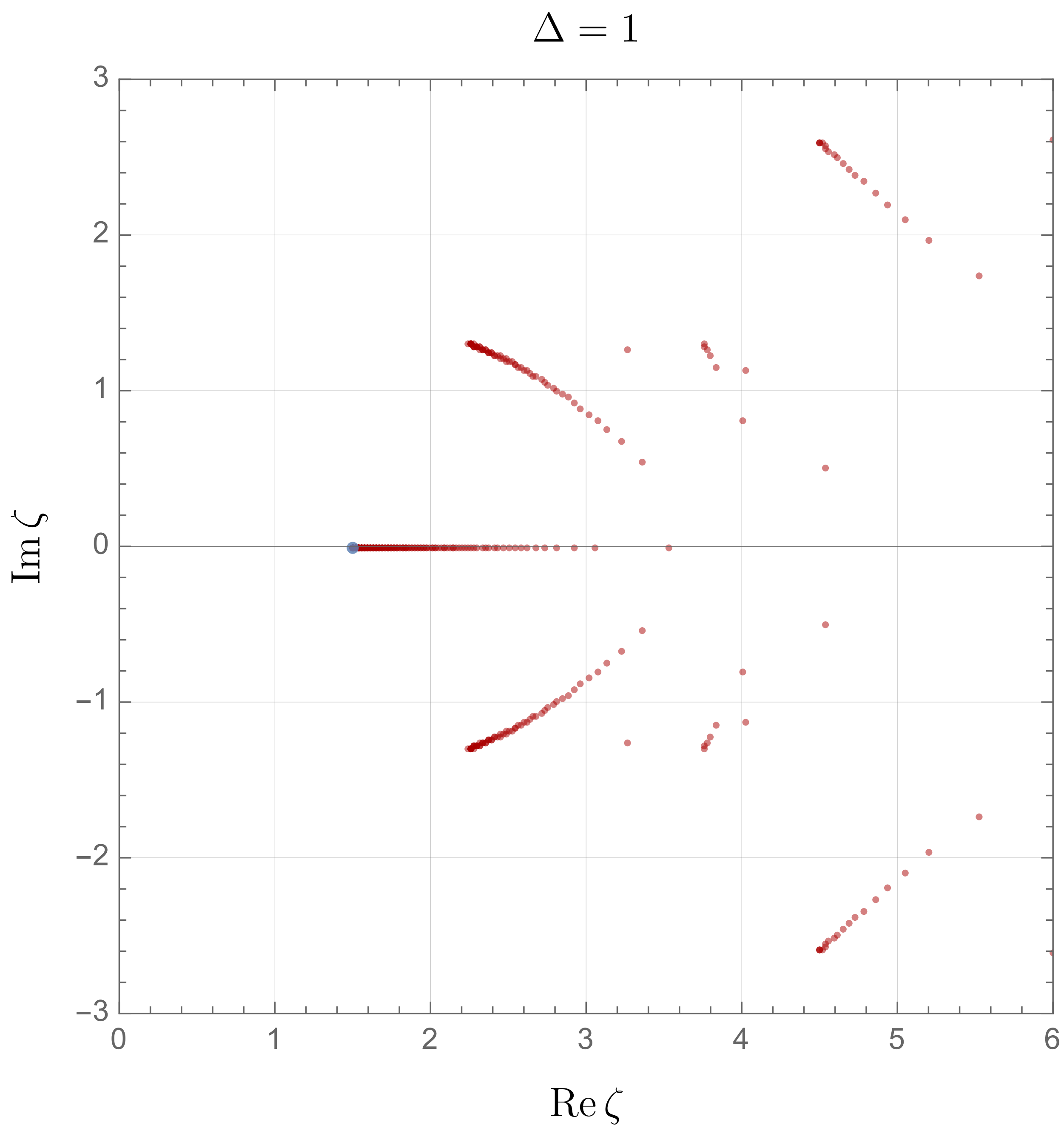
$$\mathscr{A} \sim \frac{8/5}{w} + \frac{32/105}{w^2} + \ldots$$

- All moments follow the attractor

The attractor of RTA kinetic theory

the series at late times

- The structure of transseries corrections which encode initial state data is still not completely clear



Branch points of the analytic continuation of the Borel transform of the gradient expansion suggest a non-hydrodynamic sector very different from what is seen in MIS theory

Off-axis branch points do not appear in hydrodynamic truncations of the moment equations

Transverse dynamics as perturbations

a semi-analytic extension of the Bjorken model

- Expansion around the homogenous attractor background

$$T(\tau, \mathbf{x}) = T(\tau) + \delta T(\tau, \mathbf{x})$$

- Normalised fields

$$\delta \hat{T}(\tau, \mathbf{x}) = \frac{\delta T(\tau, \mathbf{x})}{T(\tau)}$$

- Fourier modes

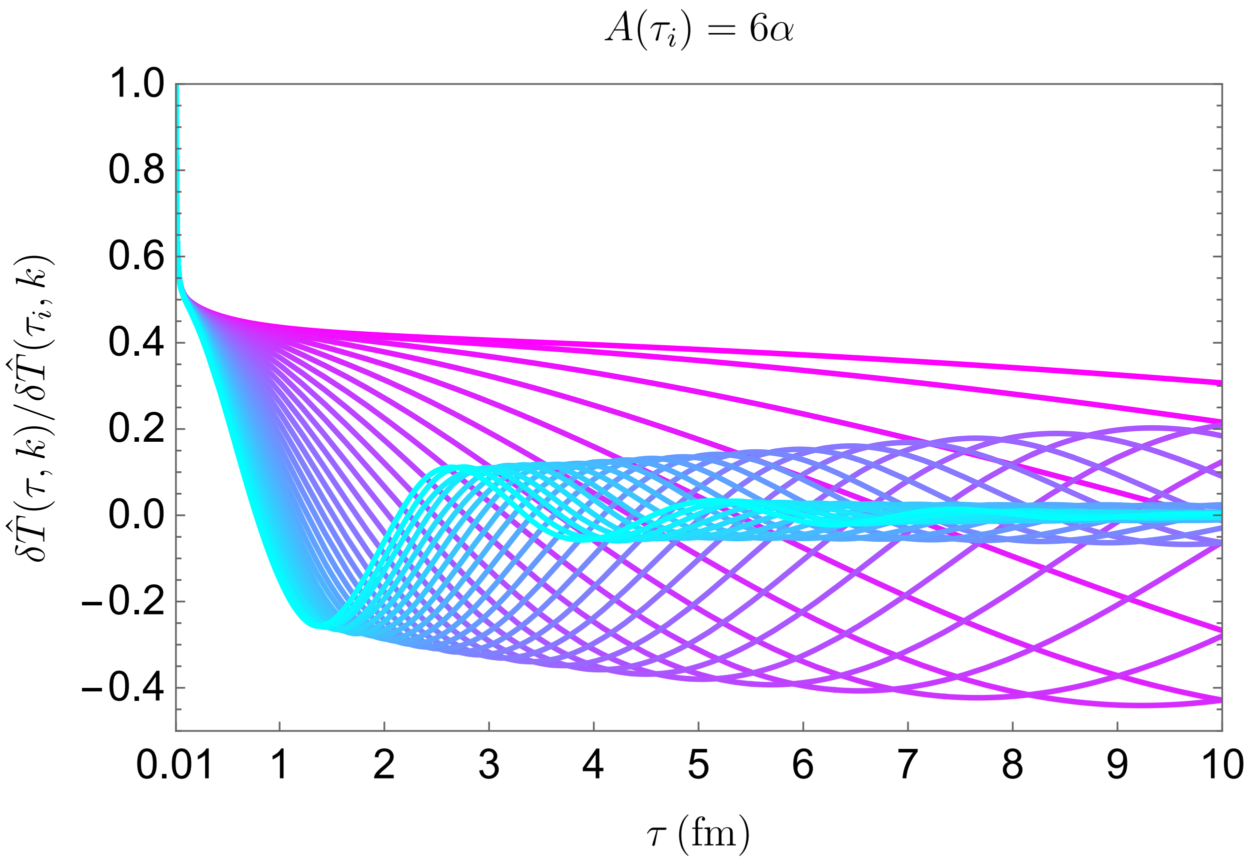
$$\hat{\phi}(\tau, \mathbf{x}) = \int \frac{d^2k}{(2\pi)^2}\; e^{i\mathbf{k}\cdot\mathbf{x}} \hat{\phi}(\tau, \mathbf{k})$$

- Linearised MIS equations: a set of 6 coupled ODEs for each k

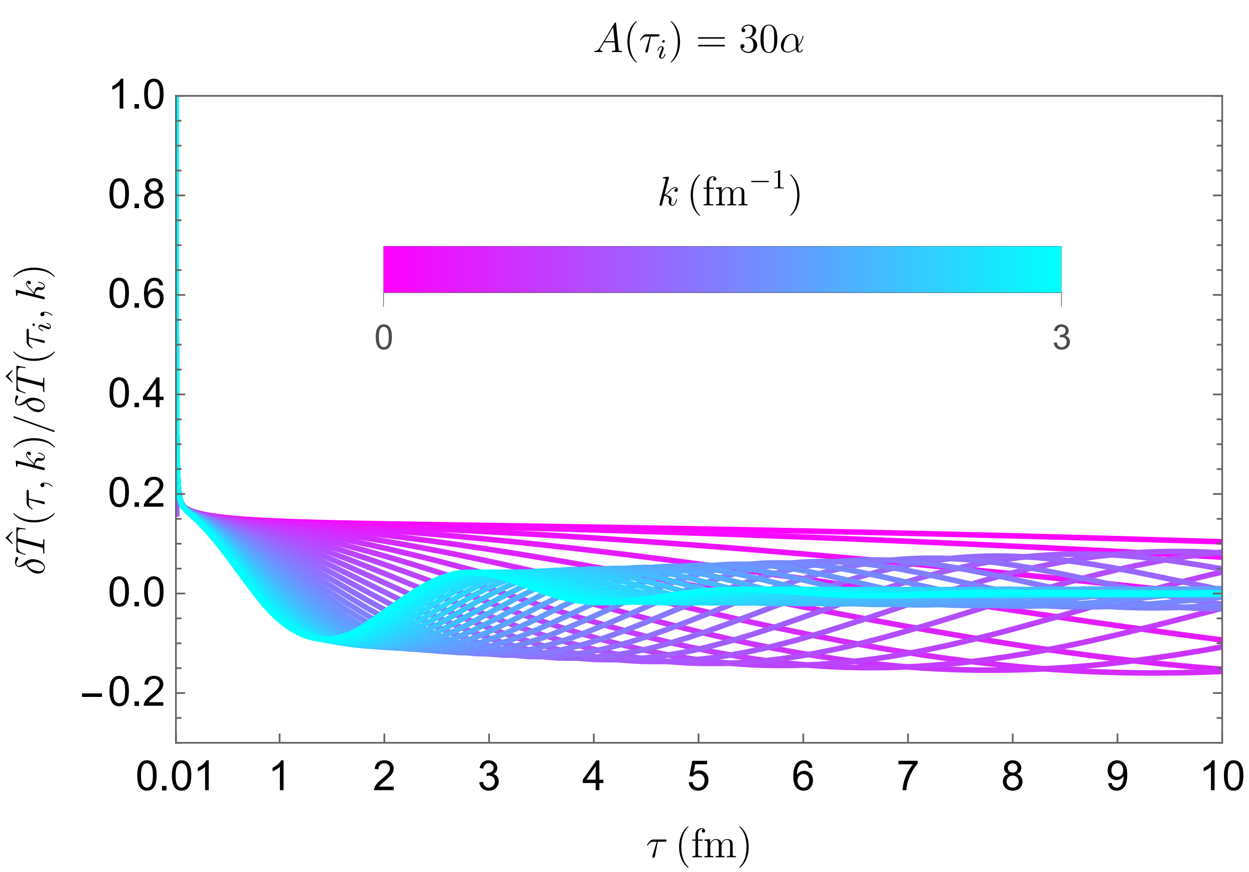
- Initial states provide initial conditions for the modes

Transverse dynamics as perturbations

stability of perturbations around the attractor



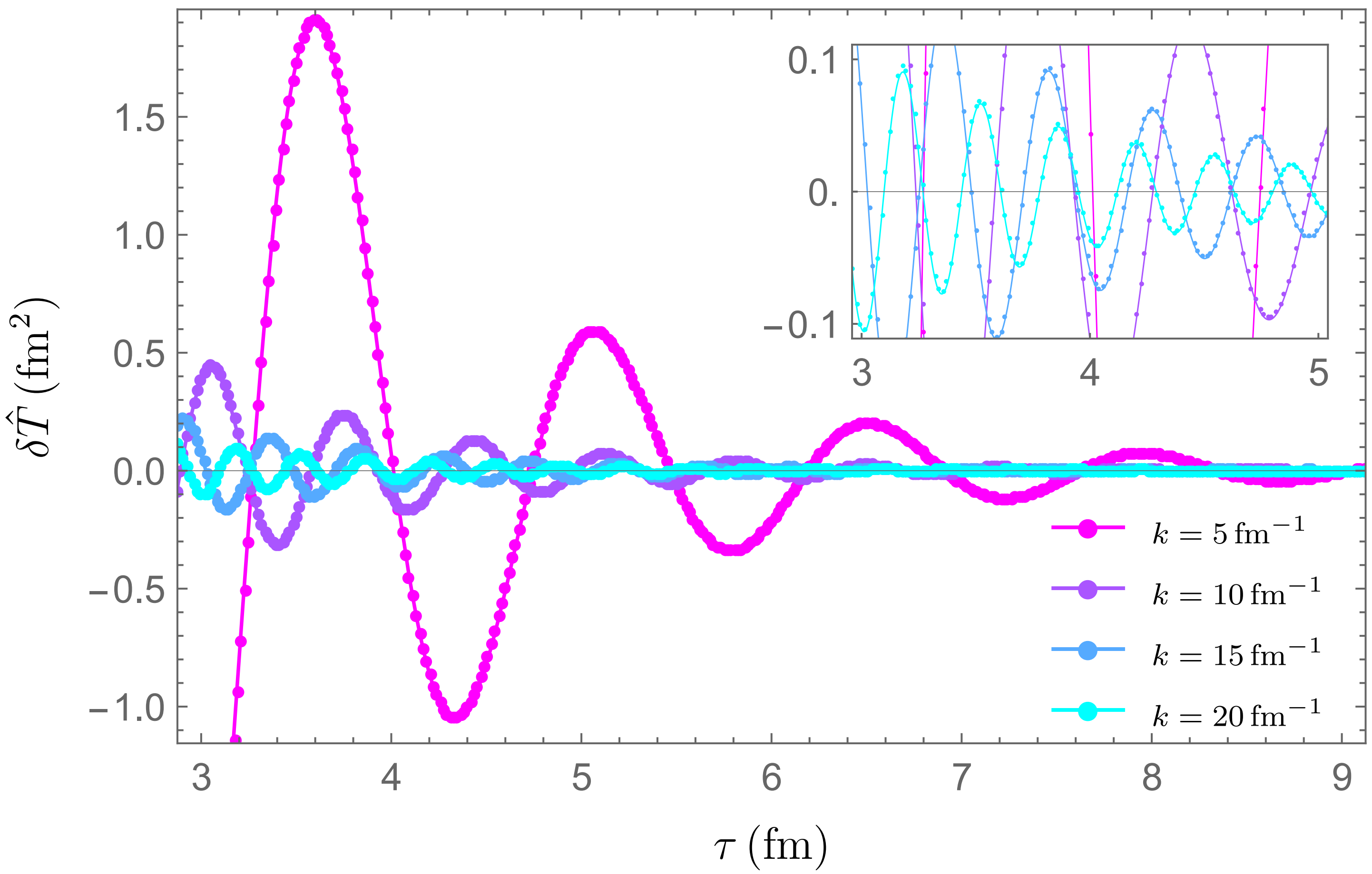
Perturbations initialised on the attractor



Perturbations initialised off the attractor

Transverse dynamics as perturbations

late time asymptotics



$$\delta\hat{T} = \sum_{i=1}^{4} \sigma_i (\Lambda\tau)^{\beta_i} e^{-i\omega_i\tau - A_i(\Lambda\tau)^{2/3}}$$

Different initial conditions are reflected by the amplitudes which determine the physics at freeze-out time

$$A_1 = A_2 = \frac{\alpha^2}{C_\tau c_\infty^2}, \quad A_3 = \frac{3}{2C_\tau}, \quad A_4 = \frac{1}{2C_\tau c_\infty^2}, \quad A_5 = A_6 = \frac{3}{4C_\tau},$$

$$\omega_1 = -\,\omega_2 = c_\infty k \left[1 + \frac{2\alpha^2}{3c_\infty^2}\left(2C_\tau(1-\alpha^2) - \frac{(1+\alpha^2)\Lambda^2}{C_\tau^2 c_\infty^4 k^2}\right)(\Lambda\tau)^{-2/3}\right], \quad \omega_3 = \omega_4 = 0$$

$$c_\infty = \sqrt{\frac{1}{3}\left(1 + 4\frac{C_\eta}{C_\tau}\right)}, \qquad \alpha \equiv \sqrt{\frac{C_\eta}{C_\tau}}$$

Summary and Outlook

- Details of the initial state are encoded in transseries corrections to boost-invariant, transversely-homogeneous attractors
- Generating function methods may lead to a better picture of how initial data survives until freeze-out
- Incorporation of transverse dynamics at the linearised level provides a semi-analytic extension of the Bjorken model