Stability and Causality in Spin Hydrodynamics

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W.Florkowski, R.Ryblewski, A.Das, R.Biswas & F.Taghinavaz

PhysRevC.108.024902, PhysRevD.107.054043, PhysRevD.107.094022, PhysRevD.108.014024, arXiv: 2401.07608, 2403.04711

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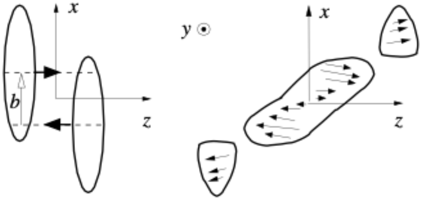
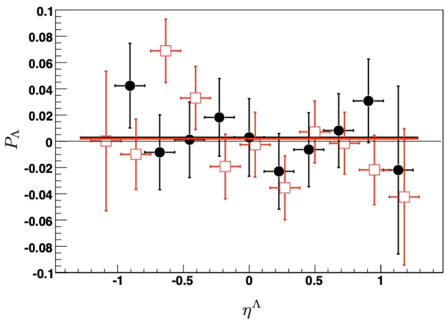
Globally Polarized Quark-Gluon Plasma in Noncentral $A + A$ Collisions

Zuo-Tang Liang and Xin-Nian Wang
Phys. Rev. Lett. **94**, 102301 – Published 14 March 2005; Erratum Phys. Rev. Lett. **96**, 039901 (2006)

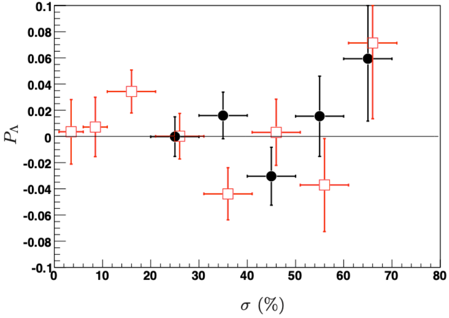


Global polarization measurement in Au+Au collisions

B. I. Abelev *et al.* (STAR Collaboration)
Phys. Rev. C **76**, 024915 – Published 29 August 2007; Erratum Phys. Rev. C **95**, 039906 (2017)



Global polarization of Λ-hyperons as a function of pseudo-rapidity η^Λ. Filled circles for $\sqrt{s_{NN}}$=200 GeV and open squares for $\sqrt{s_{NN}}$=62.4 GeV. [Phys.Rev.C76,024915].



Global polarization of Λ-hyperons as a function of centrality. Filled circles for $\sqrt{s_{NN}}$=200 GeV and open squares for $\sqrt{s_{NN}}$=62.4 Gev [Phys.Rev.C76,024915]

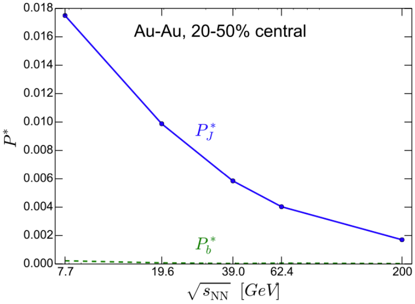
Angular momentum conservation in heavy ion collisions at very high energy

F. Becattini, F. Piccinini, and J. Rizzo
Phys. Rev. C **77**, 024906 – Published 21 February 2008

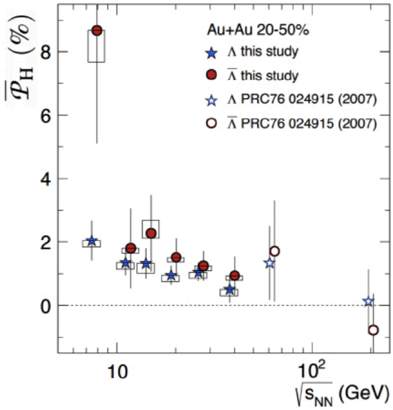
“”Rotation in hydrodynamic motion was proposed to induce polarization“”

Λ polarization in peripheral heavy ion collisions

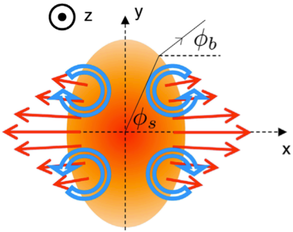
F. Becattini, L. P. Csernai, and D. J. Wang
Phys. Rev. C **88**, 034905 – Published 13 September 2013; Erratum Phys. Rev. C **93**, 069901 (2016)



Global polarization of Λ [Eur.Phys.J.C(2017)77:213]



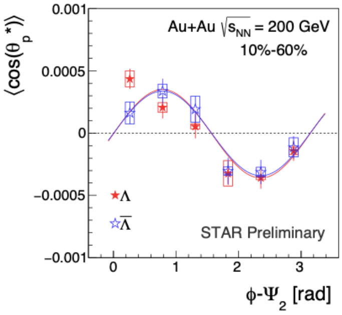
Average Λ global polarization [STAR, L. Adamczyk et al., Nature 548, 62 (2017)]



A sketch of vorticities along the beam direction (open arrows) induced by anisotropic flow (solid arrows) in the (x-y)-plane [Phys. Rev. Lett. 123, 132301]

Various complex vortical structures have been predicted to appear in heavy-ion collisions due to the collective expansion of the system [203–206] and jet-medium interaction [82, 207, 208]. Refs. [204, 205] suggest that the vorticity, consequently particle polarization, can be induced by anisotropic flow where the rotational axis is along the beam direction as shown in Fig. 4. The STAR Collaboration observed Λ ($\bar{\Lambda}$) polarization along the beam direction P_z as expected [14], and later the ALICE Collaboration confirmed it at the LHC energy [19].

[arXiv:2402.04540v1]



Pz of Λ hyperons as a function of azimuthal angle ϕ in Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV [Phys. Rev. Lett. 123, 132301]

Unlike for global polarization, local spin polarization as a function of the azimuthal angle of emission in the transverse plane turned out to be starkly different from the prediction of the combined hydrodynamic model and local equilibrium assumption [20]. This discrepancy has provided a strong motivation for theoretical studies in different

[arXiv:2402.04540v1]

Relativistic Spin hydrodynamics [Florkowski et. al]

Relativistic Quantum-Stat Spin hydrodynamics [F.Becattini, A.Daher et. al. (PhysLettB.2024.138533)]

Relativistic Kinetic theory [Risckle et. al]

Relativistic Spin hydrodynamics [Florkowski et. al]

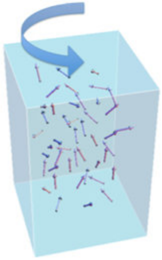
Formulation

Stability

Causality

What is a fluid with spin ?

A fluid with spin is a fluid having a macroscopic spin density and which thus needs a spin tensor $\mathcal{S}^{\lambda,\mu\nu}$ to be described, besides the stress-energy tensor $T^{\mu\nu}$



$$T^{\mu\nu}(x) = \mathrm{tr}[\widehat{\rho}\widehat{T}^{\mu\nu}(x)]$$

$$\mathcal{S}^{\lambda,\mu\nu}(x) = \mathrm{tr}[\widehat{\rho}\widehat{\mathcal{S}}^{\lambda,\mu\nu}(x)]$$

Relativistic fluids posses Minkowski spacetime cont. symmetries. By Noether's theorem:

Symmetry	Conserved Quantity
Space-time translation	Conservation of total energy and momentum
Lorentz transformation	Conservation of total angular momentum
Global U(1) symmetry	Conservation of total baryon number

$$\partial_\mu T^{\mu\nu} = 0 \;\Big|\; \partial_\lambda J^{\lambda\mu\nu} = 0 \;\Big|\; J^{\lambda\mu\nu} = L^{\lambda\mu\nu} + S^{\lambda\mu\nu} \implies \partial_\lambda S^{\lambda\mu\nu} = T^{\nu\mu} - T^{\mu\nu}$$

$$T^{\mu\nu} = (\epsilon + p) u^{\mu} u^{\nu} - p g^{\mu\nu} + T_1^{\mu\nu} \qquad \& \qquad S^{\lambda\mu\nu} = S^{\mu\nu} u^{\lambda} + S_1^{\lambda\mu\nu}$$

$$T^{\mu\nu} = (\epsilon + p) u^{\mu} u^{\nu} - p g^{\mu\nu} + T_1^{\mu\nu} \qquad \& \qquad S^{\lambda\mu\nu} = S^{\mu\nu} u^{\lambda} + S_1^{\lambda\mu\nu}$$

Dissipative Variable	Description
Shear Stress (πμν)	Force per unit area due to internal friction between fluid layers with different velocities.
Bulk Pressure (Π)	Pressure resisting the change in volume of the fluid.
Rotational Viscosity (Φμν)	

$$T^{\mu\nu} = (\epsilon + p) u^{\mu} u^{\nu} - p g^{\mu\nu} + T_1^{\mu\nu} \qquad \& \qquad S^{\lambda\mu\nu} = S^{\mu\nu} u^{\lambda} + S_1^{\lambda\mu\nu}$$

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Rotational Viscosity (Φμν)	

$\epsilon + p = Ts + \omega_{\mu\nu} S^{\mu\nu}$ | Spin Equation of State: $S^{\mu\nu} = S(T)\, \omega^{\mu\nu}$

What do we expect?

- $T^{\mu\nu} \longrightarrow$ 16 D.O.F (T, u^μ, $\pi^{\mu\nu}$, Π, q^μ, $\phi^{\mu\nu}$), with 16 evolution equations.
- $S^{\lambda\mu\nu} \longrightarrow$ 24 D.O.F ($\omega^{\mu\nu}$, Φ, $\tau^{\mu\nu}_{(s)}$, $\tau^{\mu\nu}_{(a)}$, $\Theta^{\lambda\mu\nu}$) with 24 evolution equations.

$$D\varepsilon + (\varepsilon + p)\theta = \pi^{\mu\nu}\partial_\mu u_\nu + \Pi\theta - \nabla \cdot q + \phi^{\mu\nu}\partial_\mu u_\nu \,,$$

$$\begin{aligned}(\varepsilon + p)Du^\alpha - \nabla^\alpha p = & - \Delta^\alpha_\nu \partial_\mu \pi^{\mu\nu} - \Delta^{\mu\alpha}\partial_\mu \pi + \pi D u^\alpha - q^\mu \partial_\mu u^\alpha \\ & + \Delta^\alpha_\nu D q^\nu + q^\alpha \theta - \Delta^\alpha_\nu \partial_\mu \phi^{\mu\nu} \,,\end{aligned}$$

$$\tau_\Pi D\Pi + \Pi = \zeta \left[\theta + T a_1 \Pi\theta + T\Pi D a_1\right] ,$$

$$\tau_\pi \Delta^{\mu\nu}_{\alpha\beta} D\pi^{\alpha\beta} + \pi^{\mu\nu} = 2\eta \left[(\nabla^{(\mu}u^{\nu)} - \frac{1}{3}\theta\Delta^{\mu\nu}) + T a_2 \theta \pi^{\mu\nu} + T\pi^{\mu\nu} D a_2\right] ,$$

$$\tau_q \Delta^\mu_\nu D q^\nu + q^\mu = \lambda \left[(\beta\nabla^\mu T + Du^\mu - 4\omega^{\mu\nu}u_\nu) - T a_4 q^\mu \theta - T q^\mu D a_4\right] ,$$

$$\tau_\phi \Delta^{[\mu\nu]}_{[\alpha\beta]} D\phi^{\alpha\beta} + \phi^{\mu\nu} = \gamma \left[(\beta\nabla^{[\mu}u^{\nu]} + 2\beta\Delta^{\mu\alpha}\Delta^{\nu\beta}\omega_{\alpha\beta}) + a_5 \theta\phi^{\mu\nu} + \phi^{\mu\nu} D a_5\right] ,$$

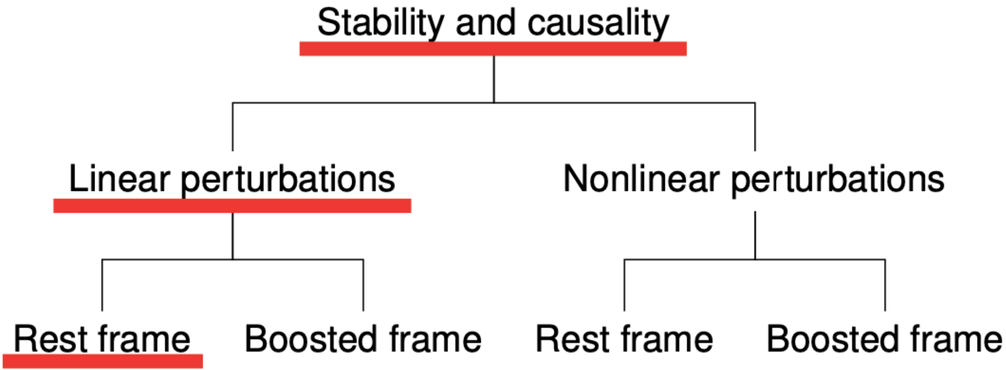
$$DS^{\alpha\beta} + S^{\alpha\beta}\theta + \partial_\mu S_1^{\mu\alpha\beta} = -2(q^\alpha u^\beta - q^\beta u^\alpha + \phi^{\alpha\beta}),$$

$$\tau_\Phi D\Phi + \Phi = \chi_1 \left[-2u^\alpha \nabla^\beta (\beta\omega_{\alpha\beta}) + \tilde{a}_1 \theta\Phi + \Phi D\tilde{a}_1\right] ,$$

$$\tau_{\tau_s} \Delta^{\mu\nu}_{\alpha\beta} D\tau_s^{\alpha\beta} + \tau_s^{\mu\nu} = \chi_2 \left[- u^\alpha (\Delta^{\gamma\mu}\Delta^{\rho\nu} + \Delta^{\gamma\nu}\Delta^{\rho\mu} - \frac{2}{3}\Delta^{\gamma\rho}\Delta^{\mu\nu})\nabla_\gamma(\beta\omega_{\alpha\rho}) + \tilde{a}_2 \theta \tau_s^{\mu\nu} + \tau_s^{\mu\nu} D\tilde{a}_2\right],$$

$$\tau_{\tau_a} \Delta^{[\mu\nu]}_{[\alpha\beta]} D\tau_a^{\alpha\beta} + \tau_a^{\mu\nu} = \chi_3 \left[-u^\alpha (\Delta^{\gamma\mu}\Delta^{\rho\nu} - \Delta^{\gamma\nu}\Delta^{\rho\mu})\nabla_\gamma(\beta\omega_{\alpha\rho}) + \tilde{a}_3 \theta\tau_a^{\mu\nu} + \tau_a^{\mu\nu} D\tilde{a}_3\right] ,$$

$$\tau_\Theta \Delta^\alpha_\lambda \Delta^\mu_\sigma \Delta^\nu_\beta D\Theta^{\lambda\sigma\beta} + \Theta^{\alpha\mu\nu} = -\chi_4 \left[-\Delta^{\delta\mu}\Delta^{\rho\nu}\Delta^{\gamma\alpha}\nabla_\gamma(\beta\omega_{\delta\rho}) + \tilde{a}_4 \theta\Theta^{\alpha\mu\nu} + \Theta^{\alpha\mu\nu} D\tilde{a}_4\right] ,$$


Stability and causality
Linear perturbations
Nonlinear perturbations
Rest frame
Boosted frame
Rest frame
Boosted frame

Concept	Description
Stability	Mathematical tool that investigates how small perturbations evolve in time (decay or diverge).
Causality	Fundamental principle that ensures no propagation travels faster than the speed of light within the fluid.

Linear Perturbation

$$\varepsilon(x) \to \varepsilon_0 + \delta\varepsilon(x), \qquad u^\mu \to u_0^\mu + \delta u^\mu(x) = (1,\vec{0}) + (0,\delta\vec{v}),$$
$$\omega^{\mu\nu}(x) \to 0 + \delta\omega^{\mu\nu}(x), \qquad S^{\mu\nu}(x) = 0 + \delta S^{\mu\nu}(x),$$
$$X(x) \to 0 + \delta X(x).$$

Fourier Transformation: (P).Differntial Eqs. $\rightarrow$ Algebraic Eqs.

$$\delta\varepsilon = \widetilde{\delta\varepsilon}\; e^{-i\omega t + i\vec{k}\cdot\vec{x}}, \quad \delta\Pi = \widetilde{\delta\Pi}\; e^{-i\omega t + i\vec{k}\cdot\vec{x}},$$

$$\delta S^{ij} = \widetilde{\delta S}^{ij}\; e^{-i\omega t + i\vec{k}\cdot\vec{x}}, \quad \delta\Theta^{ijk} = \widetilde{\delta\Theta}^{ijk}\; e^{-i\omega t + i\vec{k}\cdot\vec{x}}$$

Goal: Solve, i.e., find the 40 ω(s)

$$
\begin{pmatrix}
-i\,\omega & i\,kz & 0 & i\,E0\,kz & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
i\,c^2\,kz & i\,\omega & 0 & -i\,E0\,\omega & -i\,kz & -i\,kz & -i\,kz & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
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\end{pmatrix}
$$

Stability Analysis

To guarantee that perturbations of the equilibrium state do not grow exponentially with time, we demand that they satisfy the condition

$$\mathrm{Im}[\omega(k_z)] < 0.$$

Low- and large-momentum limit

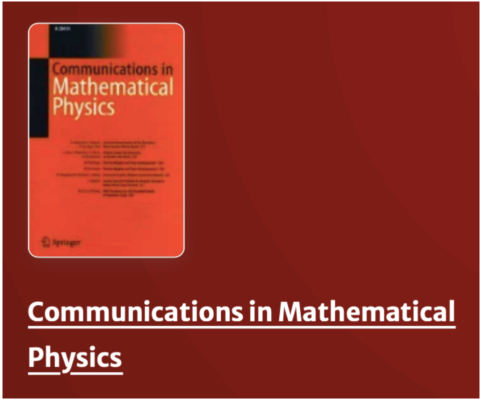
$$\omega_{1,2} = \pm c_s k_z - i\frac{(\frac{4}{3}\eta + \zeta)}{2(\varepsilon_0 + p_0)} k_z^2,$$

$$\omega_1 = -\frac{i(\frac{4}{3}\eta + \xi)}{\frac{4}{3}\eta\tau_\Pi + \xi\tau_\pi} + \mathcal{O}(\frac{1}{k_z^2}) + \mathcal{O}(\frac{1}{k_z^4}),$$

Observation

$$S^{\mu\nu} = S(T)\,\omega^{\mu\nu} \;\longrightarrow\; S^{\gamma\delta} = S_1\left(k^{\gamma}u^{\delta} - k^{\delta}u^{\gamma}\right) + S_2\epsilon^{\gamma\delta\rho\sigma}u_{\rho}\omega_{\sigma}\;;\quad \omega.u = 0 \;\&\; k.u = 0$$

$$S_1 = -C\,\frac{T^3}{\pi^2}\left[4K_2(x) + xK_1(x)\right] \quad \& \quad S_2 = C\,\frac{T^3}{2\pi^2}\left[(8 + x^2)K_2(x) + 2xK_1(x)\right]\;;\quad x = m/T$$



Eckhard Krotscheck and Wolfgang Kundt (1978)

$$v_f := sup\left[\lim_{k_z\to\infty}\left(\frac{Re\,\omega(k_z)}{k_z}\right)\right].$$

$$sup\left[\lim_{k_z\to\infty}\left(\frac{Re\,\omega(k_z)}{k_z}\right)\right] \leq 1,$$

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$$sup\left[\lim_{k_z\to\infty}\left(\frac{Re\,\omega(k_z)}{k_z}\right)\right] \leq 1,$$

$$\omega_{8,9} = -\frac{i}{2\tau_\Theta} \pm \sqrt{\frac{\tilde{\chi}_4}{\tau_\Theta}}\,k_z + \mathcal{O}(\frac{1}{k_z}).$$

$$0 \leq \frac{\tilde{\chi}_4}{\tau_\Theta} \leq 1.$$

Conclusion and Outlook

Our theory is stable in both the low- and high-momentum limits and is also causal.

The very next step is to extend these studies into a boosted frame

Use kinetic theory to study the physics of all the new variables (transport coefficients)

Translate all of the above to physical obervables

Thank You!

NARODOWE CENTRUM NAUKI

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